DYNAMIC MODELLING OF A FOUR-DEGREE-OF-FREEDOM ROBOTIC MANIPULATOR

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ABSTRACT

The paper is concerned with deriving a dynamic model of a four-degree- of freedom robotic manipulator. The equations of motion of the arm with respect to a non-stationary coordinate system are derived initially. This analysis is then extended to a robot arm, and consideration of inertial effects included. The dynamic model developed is useful without modification for arm speed control purposes. However, further work is required to include the effect of bending movements in the arm links before the analysis is generally applicable in accurate position control applications.

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Notation

OXYZ	Fixed co-ordinate system
O'xyz	Moving co-ordinate system
$\bar{i}', \bar{j}', \bar{k}'$	Unit vectors in X,Y, and Z directions
$\bar{i}, \bar{j}, \bar{k}$	Unit vectors in x,y, and z directions
$\bar{r}, \dot{\bar{r}}, \ddot{\bar{r}}$	Absolute displacement, velocity, and acceleration of the moving point
$\bar{R}, \dot{\bar{R}}, \ddot{\bar{R}}$	Absolute displacement, velocity, and acceleration of the moving origin
$\bar{\rho}, \dot{\bar{\rho}}, \ddot{\bar{\rho}}$	Displacement, velocity, and acceleration of the moving point relative to O'xyz
$\Psi, \dot{\Psi}, \ddot{\Psi}$	Rotation, angular velocity, and acceleration of the kinematic chain about the Y(or y) - axis, (rad, rad/S, rad/S^2).
$\beta_n, \dot{\beta}_n, \ddot{\beta}_n$	Total rotation, angular velocity, and acceleration of the link n about the z-axis, taken from xz- plane.
$\theta_n, \dot{\theta}_n, \ddot{\theta}_n$	Rotation, angular velocity, and acceleration of the link n about the axis through the centre of the joint n.
$\gamma, \dot{\gamma}, \ddot{\gamma}$	Rotation, angular velocity, and acceleration of the absolute displacement vector, $\bar{r}$, about z-axis.
$\bar{\omega}, \dot{\bar{\omega}}$	Angular velocity and acceleration of the moving co-ordinate system relative to OXYZ, (rad/S,rad/S^2).
$r_x, [r_y]$	Component of absolute displacement, $\bar{r}$, in x $\lvert$Y(or y)$\rvert$ - direction.
q_n	Mass density of link n, (kg/m)
m_n	Mass of motor n, (kg)
ℓ_n	Length of link n, (m)
d_n	Distance between the centre of the gravity of the link n and joint n, (m).

g	Acceleration due to gravity, (m/S^2)
$a_{x_{\ell n}}$ $(a_{x_{mn}})$	Acceleration of the centre of the gravity of link (motor) n in x-direction.
$a_{y_{\ell n}}$ $(a_{y_{mn}})$	Acceleration of the centre of the gravity of link (motor) n in y-direction.
$a_{z_{\ell n}}$ $(a_{z_{mn}})$	Acceleration of the centre of the gravity of link (motor) n in z-direction.
c.G.n.(C.G.m.n.)	Centre of gravity of link (motor)n.
$I_{z_{\ell n}}$ $(I_{z_{mn}})$	Moment of inertia of the link (motor)n about the z-axis.
$I_{y_{\ell n}}$ $(I_{y_{mn}})$	Moment of inertia of the link (motor) n, about the y-axis.
M_{z_n}	Moment about z-axis of joint n.
M_{y_n}	Moment about y-axis at joint n.

1. INTRODUCTION

In the past few years, many scientific, technological, economic, and humanitarian considerations have brought forth the need to augment or replace human manipulative capabilities by 'intelligent' Computer-Controlled Manipulators (CCM). The demand for such general purpose manipulators has originated primarily from the need to automate industrial processes, such as in the automotive industries. Explorations and operations in space or deep sea and sophisticated handling requirements in nuclear reactor or other hot laboratory environments are two of many challenging application domains for more automonous control of mechanical arms. Other application areas are comprehensive industrial automation for increased productivity and more dextorous prosthetic aids for the handicapped [1],[2].

At present, the use of robots for accurate assembly of mechanical parts is still at the beginning of its development. Current industrial robots, even the most accurate, are unable to perform most of the desired assembly tasks in an open loop manner due to the rigid structure of the part-bearing mechanisms [3].

The need for the quantitative modelling of a robot and the tasks it is to perform, in measurable, calculable and controllable terms has been suggested before [4]. This type of modelling relies on physical laws, empirical rules, and mathematical techniques, and this paper is an attempt to formulate one such model for a four link - four joint mechanical arm (Fig. 1.1).

In section (2) the general formula for the absolute acceleration of a moving point in free space is derived [5]. This is extended in Section (3), where general formula for the torque applied to a joint is derived in terms of inertia and masses.

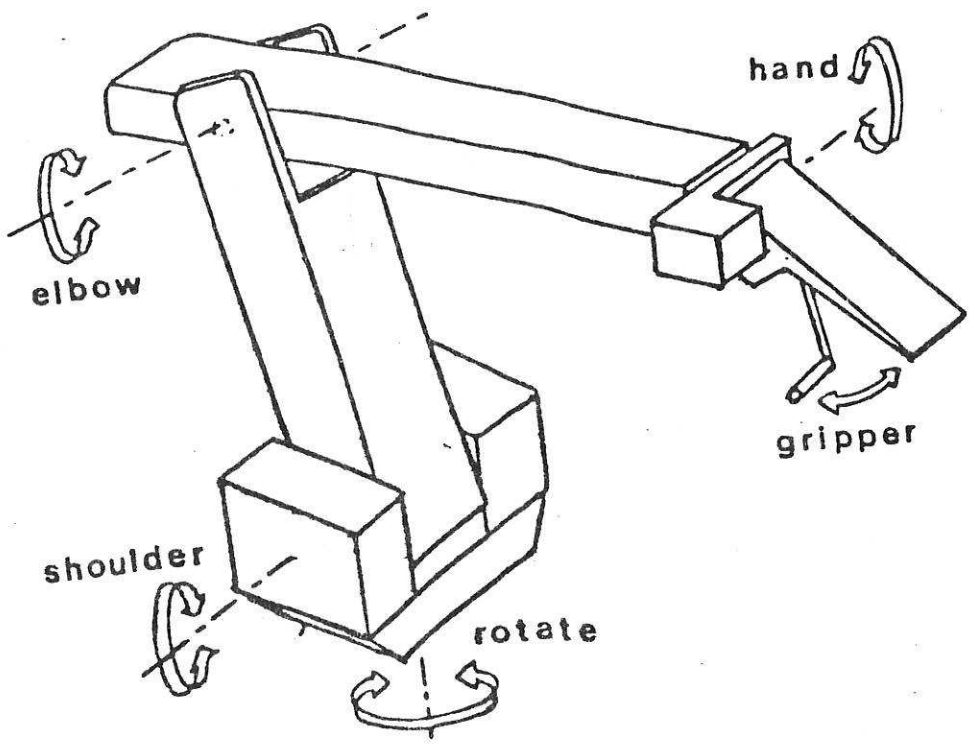


FIG 1.1

2. MOTION REFERRED TO A MOVING CO-ORDINATE SYSTEM

Suppose that the position of a point P(Fig. 2.1) is determined with respect to an xyz co-ordinate system, while at the same time the origin of this co-ordinate system moves with a translational velocity $\dot{\bar{R}}$ and an angular velocity $\bar{\omega}$ with respect to a 'fixed' XYZ co-ordinate system.

We shall now derive a general expression for the acceleration of a point referred to a co-ordinate system which itself is moving.

In the analysis to follow, we shall always measure the vectors $\bar{R}$ and $\bar{r}$ in the fixed XYZ system. The unit vectors $\bar{i}$, $\bar{j}$, and $\bar{k}$ always have the direction of the moving co-ordinate axis, while the unit vectors $\bar{i}'$, $\bar{j}'$, and $\bar{k}'$, always have the direction of the fixed co-ordinate axes.

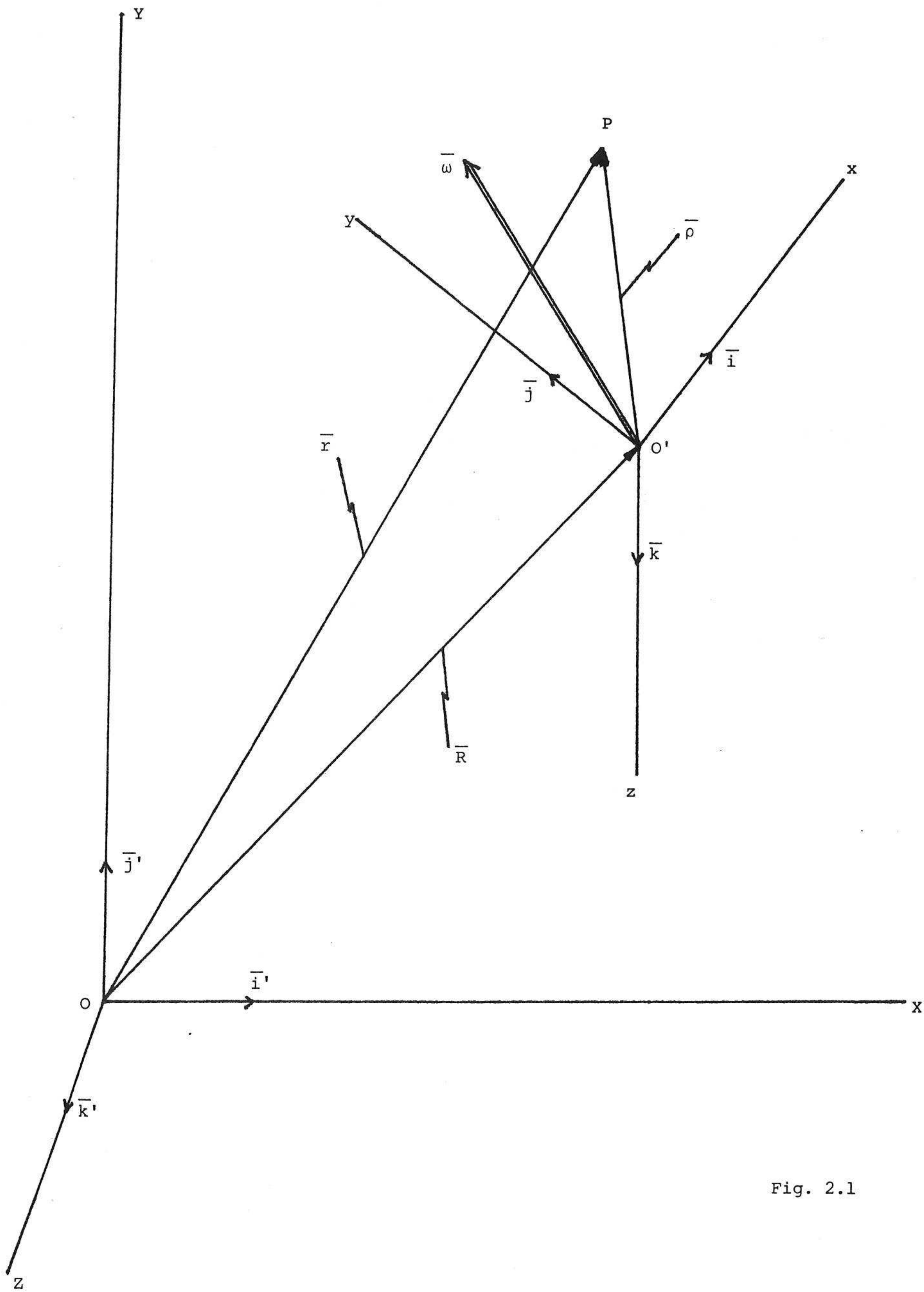
We also note that

$$\bar{i}x\bar{i} = \bar{i}'x\bar{i}' = \bar{j}x\bar{j} = \bar{j}'x\bar{j}' = \bar{k}x\bar{k} = \bar{k}'x\bar{k}' = \bar{O}$$

$$\begin{aligned} &\bar{i}x\bar{j} = \bar{k} \quad \text{and} \quad \bar{j}x\bar{i} = -\bar{k} \\ &\bar{i}'x\bar{j}' = \bar{k}' \text{and} \quad \bar{j}'x\bar{i}' = -\bar{k}' \\ &\bar{j}x\bar{k} = \bar{i} \quad \text{and} \quad \bar{k}x\bar{j} = -\bar{i} \\ &\bar{j}'x\bar{k}' = \bar{i}' \text{and} \quad \bar{k}'x\bar{j}' = -\bar{i}' \\ &\bar{k}x\bar{i} = \bar{j} \quad \text{and} \quad \bar{i}x\bar{k} = -\bar{j} \\ &\bar{k}'x\bar{i}' = \bar{j}' \text{and} \quad \bar{i}'x\bar{k}' = -\bar{j}' \end{aligned} \qquad (2.1)$$

By the absolute displacement $\bar{r}$ of the point P is meant the displacement measured with respect to the fixed XYZ system. By differentiating this absolute displacement we obtain the absolute velocity $\dot{\bar{r}}$ and the absolute acceleration $\ddot{\bar{r}}$ of point P.

$$\begin{aligned} &\bar{r} = X\bar{i}' + Y\bar{j}' + Z\bar{k}' \\ &\dot{\bar{r}} = \dot{X}\bar{i}' + \dot{Y}\bar{j}' + \dot{Z}\bar{k}' \\ &\ddot{\bar{r}} = \ddot{X}\,\bar{i}' + \ddot{Y}\,\bar{j}' + \ddot{Z}\,\bar{k}' \end{aligned} \qquad (2.2)$$



Y
P
$\overline{\omega}$
x
y
$\overline{\rho}$
$\overline{i}$
$\overline{j}$
$\overline{r}$
O'
$\overline{k}$
$\overline{R}$
z
$\overline{j}'$
$\overline{i}'$
O
X
$\overline{k}'$
Z

Fig. 2.1

During these differentiations, the unit vectors $\bar{i}', \bar{j}'$, and $\bar{k}'$ are treated as constants, since neither their magnitudes nor their directions change with time.

If we wish to express the absolute motion in terms of motion measured in the moving xyz system, we have

$$\bar{r} = \bar{R} + \bar{\rho} = \bar{R} + x\bar{i} + y\bar{j} + z\bar{k} \qquad (2.3)$$

where the directions of the $\bar{i}, \bar{j}$, and $\bar{k}$ unit vectors are known with respect to the fixed system. However, the unit vectors are changing direction with time, since they rotate with the xyz system. In taking derivatives $\dot{\bar{r}}$ and $\ddot{\bar{r}}$, therefore, the time derivatives of these unit vectors must be included.

Differentiating (2.3) with respect to time gives:

$$\dot{\bar{r}} = \dot{\bar{R}} + \dot{x}\bar{i} + x\dot{\bar{i}} + \dot{y}\bar{j} + y\dot{\bar{j}} + \dot{z}\bar{k} + z\dot{\bar{k}} \qquad (2.4)$$

The derivatives of the unit vectors are given by

$\dot{\bar{i}} = \bar{\omega}\, x\bar{i}$

$\dot{\bar{j}} = \bar{\omega} x\bar{j}$

$\dot{\bar{k}} = \bar{\omega} x\bar{k}$

So that

$\dot{\bar{r}} = \dot{\bar{R}} + \dot{x}\bar{i} + \dot{y}\bar{j} + \dot{z}\bar{k} + \bar{\omega} x(x\bar{i} + y\bar{j} + z\bar{k})$

The quanitity $(\dot{x}\bar{i} + \dot{y}\bar{j} + \dot{z}\bar{k})$ represents the translational velocity of the point P, measured relative to the moving co-ordinate system, which we shall call the relative velocity $\dot{\bar{\rho}}$. Using this notation, the expression for $\dot{\bar{r}}$ becomes

$$\dot{\bar{r}} = \dot{\bar{R}} + \dot{\bar{\rho}} + \bar{\omega} \; x \; \bar{\rho} \qquad (2.5)$$

The acceleration of P may be found by a second differentiation of equation (2.4):

$$\ddot{\bar{r}} = \ddot{\bar{R}} + (\ddot{x}\bar{i} + \ddot{y}\bar{j} + \ddot{z}\bar{k}) + (\dot{x}\dot{\bar{i}} + \dot{y}\dot{\bar{j}} + \dot{z}\dot{\bar{k}}) + \dot{\bar{\omega}} x(x\bar{i} + y\bar{j} + z\bar{k})$$

$$+ \bar{\omega} x(\dot{x}\bar{i} + \dot{y}\bar{j} + \dot{z}\bar{k}) + \bar{\omega} x(x\dot{\bar{i}} + y\dot{\bar{j}} + z\dot{\bar{k}}) \qquad (2.6)$$

Writing $(\ddot{x}\bar{i} + \ddot{y}\bar{j} + \ddot{z}\bar{k}) = \ddot{\bar{\rho}}$, which we call the relative acceleration of the point P, the expression for $\ddot{\bar{r}}$ can be written as

$$\ddot{\bar{r}} = \ddot{\bar{R}} + \bar{\omega} \times (\bar{\omega} \times \bar{\rho}) + \dot{\bar{\omega}} \times \bar{\rho} + \ddot{\bar{\rho}} + 2\bar{\omega} \times \dot{\bar{\rho}} \qquad (2.7)$$

The first three terms in this expression for $\ddot{\bar{r}}$ represent the absolute accelerations of a point attached to the moving co-ordinate system, coincident with the point P at any given time. This may be seen by noting that for a point fixed in the moving system $\dot{\bar{\rho}} = \ddot{\bar{\rho}} = \bar{O}$. The fourth term $\ddot{\bar{\rho}}$ represents the acceleration of P relative to the moving system. The last term $2\bar{\omega} \times \dot{\bar{\rho}}$ is sometimes called the acceleration of Coriolis, after G. Coriolis.

The equation of motion for a mass m in terms of the moving co-ordinate system may thus be written as

$$\bar{F} = m\ddot{\bar{R}} + m\bar{\omega} \times (\bar{\omega} \times \bar{\rho}) + m\dot{\bar{\omega}} \times \bar{\rho} + m\ddot{\bar{\rho}} + 2m\bar{\omega} \times \dot{\bar{\rho}} \qquad (2.8)$$

3. APPLICATION TO A FOUR-LINK ROBOT ARM

In this section we consider the movement of a kinematic chain, that of a robot arm with four links and four joints. Joints one (that between links one and two), two (between links two and three), and three (between links three and four) rotate about z-axis and joint four(between link four and the base) rotates about Y(or y)-axis, as shown in Figure 3.1.

With angle Ψ(rotation of joint four) fixed at Ψ_1, the movement of the arm is limited to the rotations of joints 1,2, and 3 only, and the end effector of the robot arm moves in a plane perpendicular to the XZ-plane, namely the xY-plane. x-axis is the projection of the arm on XZ-plane.

Figure (3.2) shows the general situation in which Ψ can take any value. In this case we have a co-ordinate system oxyz (or oxYz) which rotates about axis Y(or y) as angle Ψ changes. The changes in angles of joints 1,2, and 3 will not affect the state of joint 4 (angle Ψ). Therefore we have a fixed co-ordinate system OXYZ and a moving one oxyz, and the analysis of section (1) can be applied as follows.

Let us consider figures (2.1) and (3.1) to (3.4). Because the origins of the fixed and moving co-ordinate systems have been shown above to be coincident

$$\bar{R} = \dot{\bar{R}} = \ddot{\bar{R}} = \bar{O}$$

$\bar{\omega}$, the rotation (angular velocity) vector, is due to rotation of joint 4 (angle Ψ) about the Y (or y) - axis

$$\therefore \; \bar{\omega} = \dot{\Psi} \, \bar{j}$$

and

$$\dot{\bar{\omega}} = \ddot{\Psi} \, \bar{j}$$

also

$$\bar{\rho} = \bar{r} = r.\cos\gamma.\bar{i} + r.\sin\gamma.\ \bar{j}$$

$$\dot{\bar{\rho}} = (\dot{r}.\cos\gamma + r\dot{\gamma}.\sin\gamma)\bar{i} + (\dot{r}.\sin\gamma + r\dot{\gamma}.\ \cos\gamma)\bar{j}$$

$$\ddot{\bar{\rho}} = (\ddot{r}.\cos\gamma - \dot{r}\dot{\gamma}.\sin\gamma - \dot{r}\dot{\gamma}.\sin\gamma - r\ddot{\gamma}.\sin\gamma - r\dot{\gamma}^2.\cos\gamma)\bar{i} +$$

$$(\ddot{r}.\sin\gamma + \dot{r}\dot{\gamma}.\cos\gamma + \dot{r}\dot{\gamma}.\cos\gamma + r\ddot{\gamma}.\cos\gamma - r\dot{\gamma}^2.\sin\gamma)\bar{j}$$

Using equations (2.1)

$$\bar{\omega}\ x\ \bar{\rho} = -r\dot{\Psi}.\cos\gamma.\bar{k}$$

$$\bar{\omega}\ x\ (\bar{\omega}x\bar{\rho}) = -\ r\dot{\Psi}^2.\ \cos\psi.\ \bar{i}$$

$$\dot{\bar{\omega}}\ x\ \bar{\rho} = -\ r\ddot{\Psi}.\ \cos\gamma.\ \bar{k}$$

$$\bar{\omega}\ x\ \dot{\bar{\rho}} = -\ \dot{\Psi}(r.\ \cos\gamma - r\dot{\gamma}.\ \sin\gamma)\bar{k}$$

$$2\bar{\omega}\ x\ \dot{\bar{\rho}} = 2\dot{\Psi}(r\dot{\gamma}.\ \sin\gamma - \dot{r}.\cos\gamma)\bar{k}$$

Hence from equation (2.7) we obtain

$$\begin{aligned}\ddot{\bar{r}} = &\left[(\ddot{r}-r\dot{\gamma}^2 - r\dot{\Psi}^2)\cos\gamma - (r\ddot{\gamma} + 2\dot{r}\dot{\gamma})\sin\gamma\right]\ \bar{i} + \\ &\left[(\ddot{r}-r\dot{\gamma}^2)\sin\gamma + (r\gamma + 2\dot{r}\gamma)\cos\gamma\right]\ \bar{j} + \\ &\left[2r\dot{\gamma}\dot{\Psi}.\sin\gamma - (2\dot{\Psi}\dot{r} + r\ddot{\Psi})\cos\ \gamma\right]\bar{k}\end{aligned} \qquad (3.1)$$

From figures (3.1) and (3.3) we get

$$x = \ell_1.\cos\ \beta_1 + \ell_2.\cos\ \beta_2 + \ell_3\ \cos\beta_3$$

$$y = \ell_1.\sin\beta_1 + \ell_2\ \sin\beta_2 + \ell_3\ \sin\beta_3 + \ell_4 \qquad (3.2)$$

where

$$\beta_1 = \theta_1 + \theta_2 + \theta_3$$

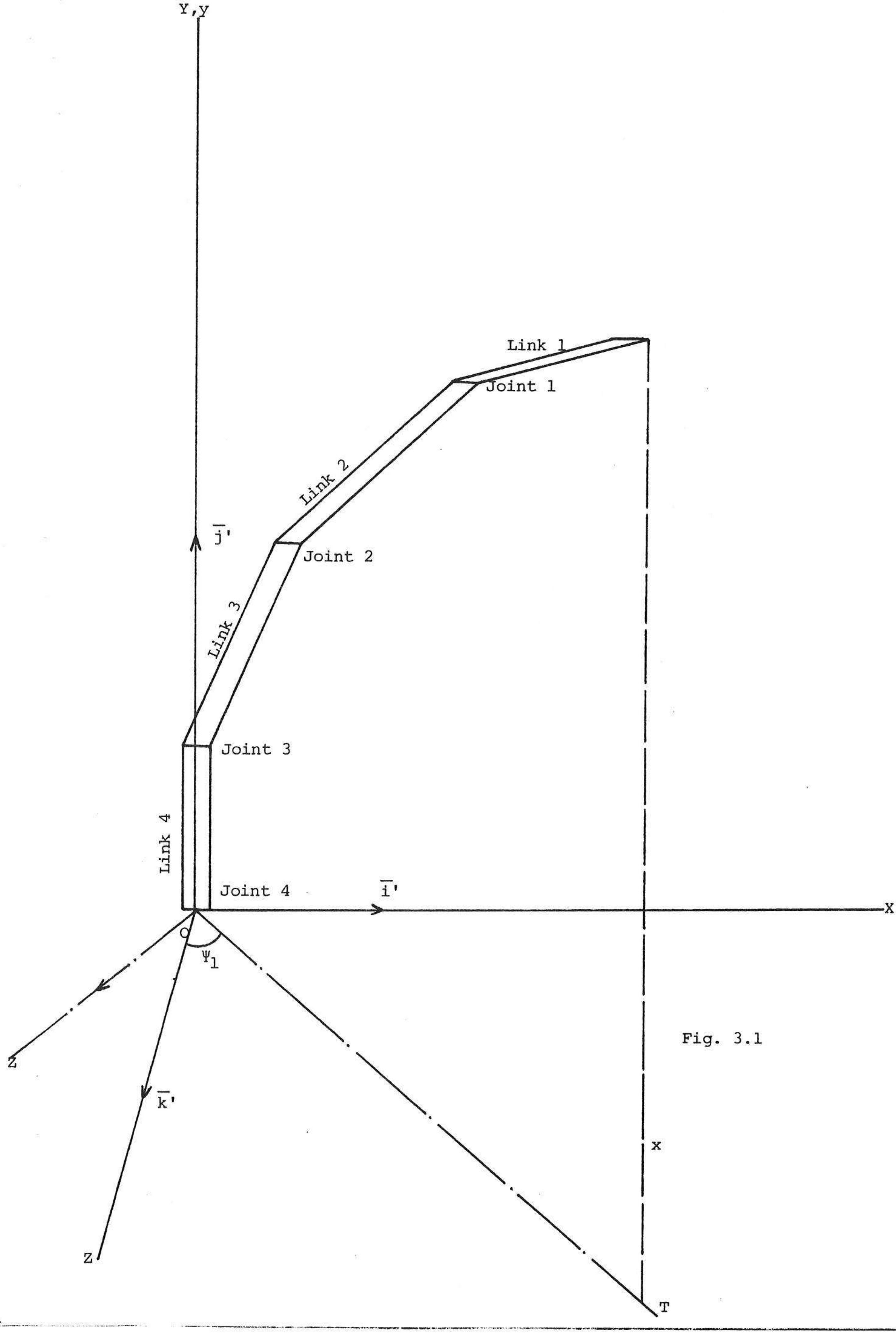
$$\beta_2 = \theta_2 + \theta_3 \qquad (3.3)$$

$$\beta_3 = \theta_3$$

Let

$$\theta_4 = \Psi \qquad (3.4)$$

By differentiating equations (3.2) to (3.4) with respect to time



Y,y
Link 1
Joint 1
Link 2
j'
Joint 2
Link 3
Joint 3
Link 4
Joint 4
i'
X
O
Ψ_1
Z
k'
x
Z
T

Fig. 3.1

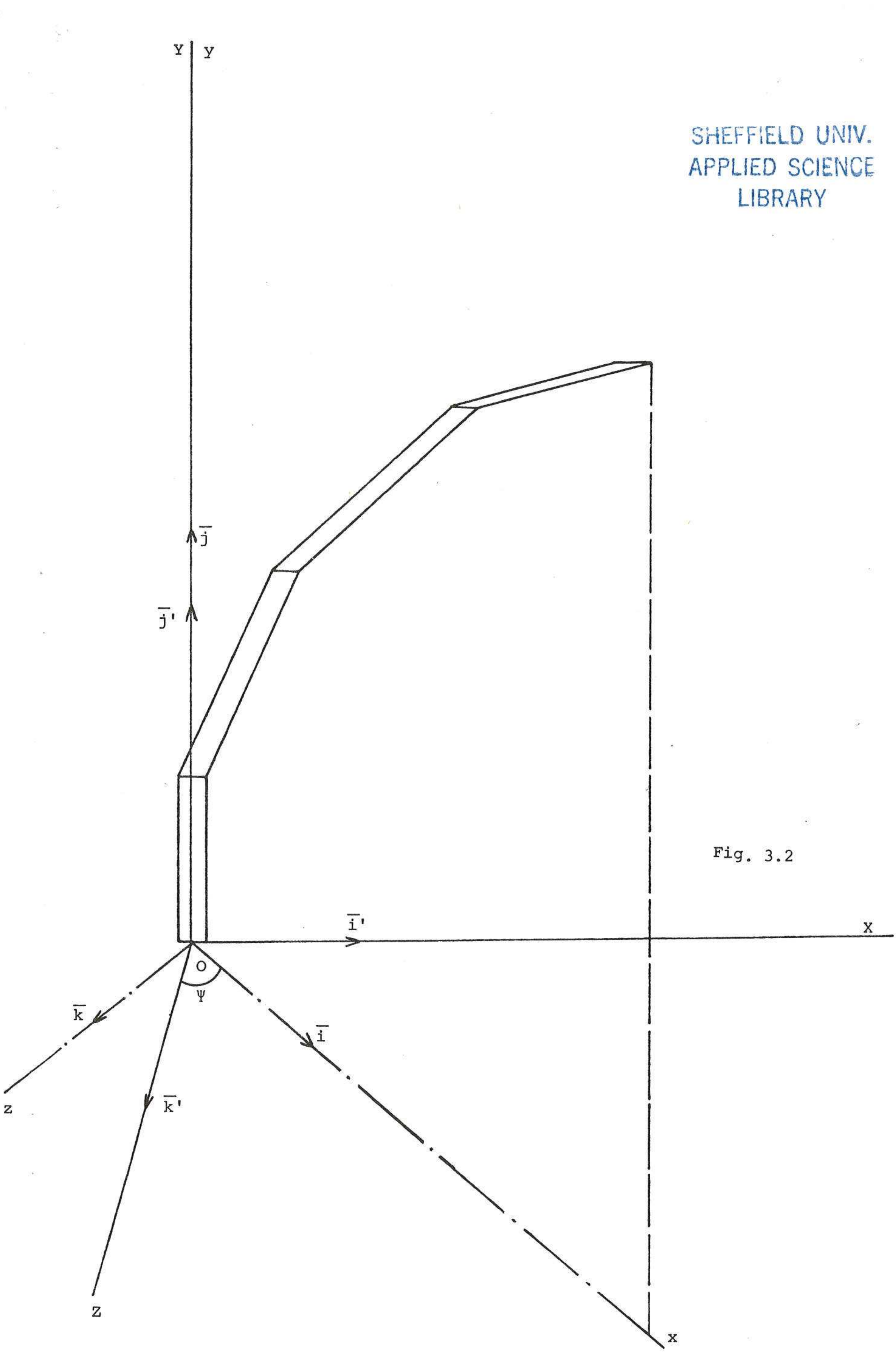


Fig. 3.2

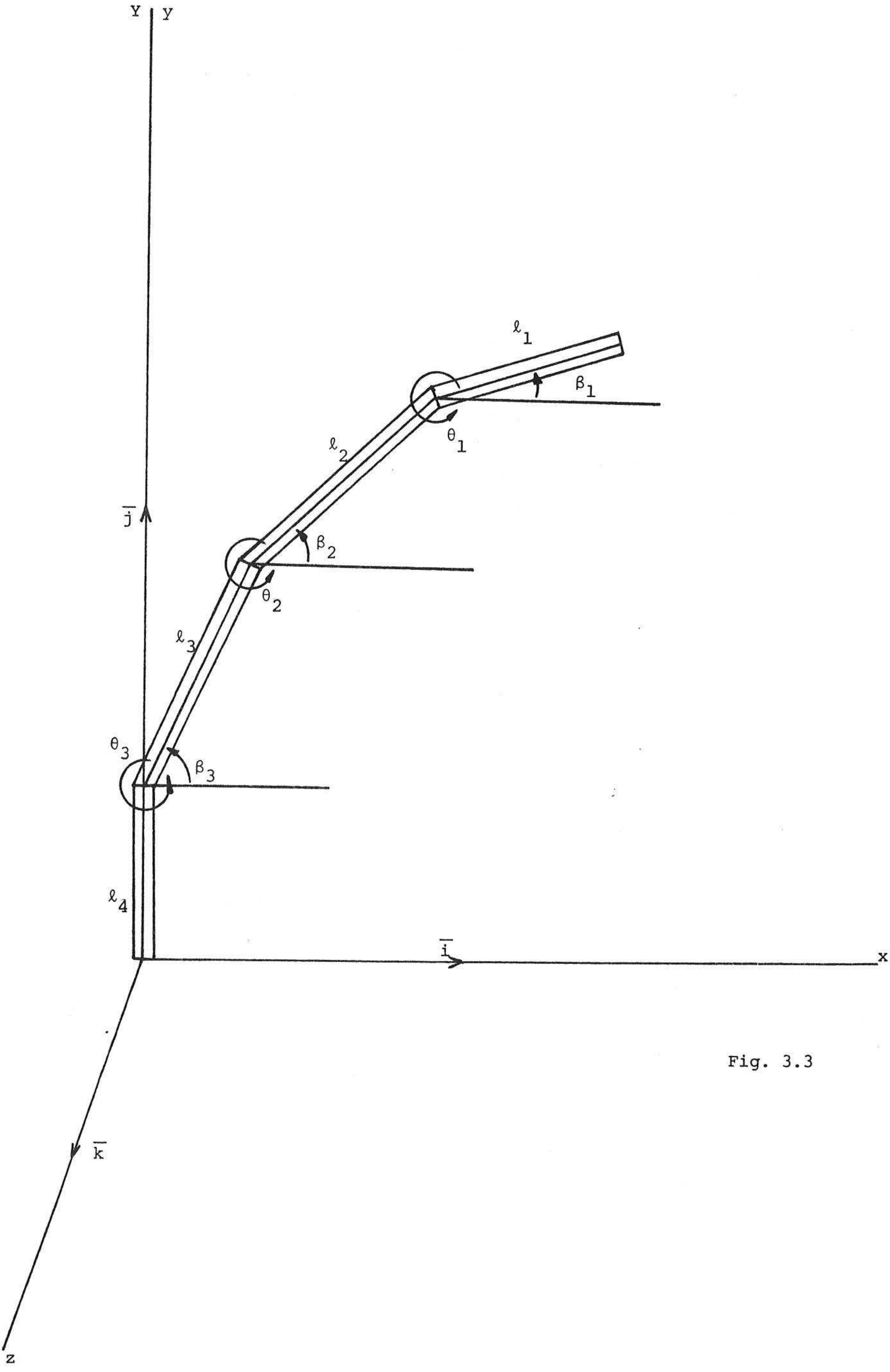
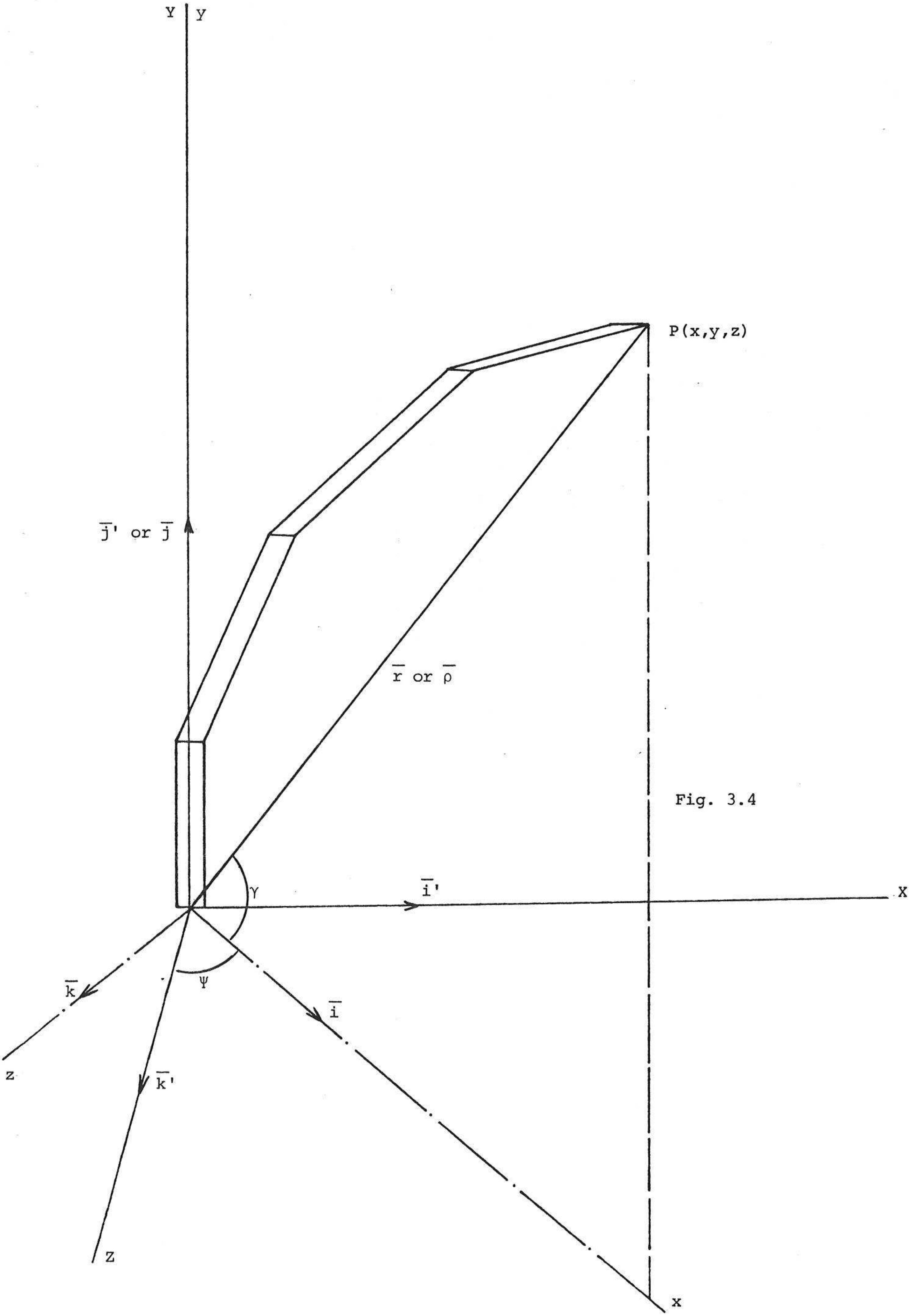


Fig. 3.3



Y
y
P(x,y,z)
$\bar{j}'$ or $\bar{j}$
$\bar{r}$ or $\bar{\rho}$
γ
$\bar{i}'$
X
$\bar{k}$
ψ
$\bar{i}$
z
$\bar{k}'$
Z
x

Fig. 3.4

$$\left.\begin{aligned}
\dot{x} &= -\ell_1\dot{\beta}_1 \sin\beta_1 - \ell_2\dot{\beta}_2 \sin\beta_2 - \ell_3\dot{\beta}_3 \sin\beta_3 \\
\dot{y} &= \ell_1\dot{\beta}_1 \cos\beta_1 + \ell_2\dot{\beta}_2 \cos\beta_2 + \ell_3\dot{\beta}_3 \cos\beta_3 \\
\dot{\beta}_1 &= \dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3 \\
\dot{\beta}_2 &= \dot{\theta}_2 + \dot{\theta}_3 \\
\dot{\beta}_3 &= \dot{\theta}_3 \\
\dot{\theta}_4 &= \dot{\Psi}
\end{aligned}\right\} \qquad (3.5)$$

By differentiating equation (3.5) with respect to time we get

$$\ddot{x} = -\ell_1\ddot{\beta}_1 \sin\beta_1 - \ell_1\dot{\beta}_1^2\cos\beta_1 - \ell_2\ddot{\beta}_2\sin\beta_2 - \ell_2\dot{\beta}_2^2\cos\beta_2 - \ell_3\ddot{\beta}_3\sin\beta_3 - \ell_3\dot{\beta}_3^2\cos\beta_3$$

$$\ddot{y} = \ell_1\ddot{\beta}_1\cos\beta_1 - \ell_1\dot{\beta}_1^2\sin\beta_1 + \ell_2\ddot{\beta}_2\cos\beta_2 - \ell_2\dot{\beta}_2^2\sin\beta_2 + \ell_3\ddot{\beta}_3\cos\beta_3 - \ell_3\dot{\beta}_3^2 \sin\beta_3$$

$$\ddot{\beta}_1 = \ddot{\theta}_1 + \ddot{\theta}_2 + \ddot{\theta}_3$$

$$\ddot{\beta}_2 = \ddot{\theta}_2 + \ddot{\theta}_3$$

$$\ddot{\beta}_3 = \ddot{\theta}_3$$

$$\ddot{\theta}_4 = \ddot{\Psi} \qquad (3.6)$$

We also have

$$r^2 = x^2 + y^2 \qquad (3.7)$$

$$\tan\gamma = \frac{y}{x} \qquad (3.8)$$

Differentiating (3.7) and (3.8) with respect to time

$$2r\dot{r} = 2x\dot{x} + 2y\dot{y}$$

$$\dot{r} = \frac{\dot{x}x + \dot{y}y}{r} \qquad (3.9)$$

$$\dot{\gamma}\sec^2\gamma = \frac{\dot{y}x - y\dot{x}}{x^2}$$

$$\sec^2\gamma = 1+\tan^2\gamma = 1 + \frac{y^2}{x^2} = \frac{x^2+y^2}{x^2} = \frac{r^2}{x^2}$$

$$\dot{\gamma} = \frac{\dot{y}x - y\dot{x}}{r^2} \qquad (3.10)$$

Differentiating (3.9) and (3.10) with respect to time

$$\ddot{r} = \frac{1}{r^2}\left[(\ddot{x}x + \dot{x}^2 + \ddot{y}y + \dot{y}^2)r - (\dot{x}x + \dot{y}y)\dot{r}\right]$$

$$\ddot{r} = \frac{1}{r^3}\left[(\ddot{x}x + \dot{x}^2 + \ddot{y}y + \dot{y}^2)r^2 - (\dot{x}x + \dot{y}y)^2\right] \qquad (3.11)$$

$$\ddot{\gamma} = \frac{1}{r^4}\left[(\ddot{y}x + \dot{y}\dot{x} - \dot{y}\dot{x} - y\ddot{x})r^2 - (\dot{y}x - y\dot{x})(2r\dot{r})\right]$$

$$\ddot{\gamma} = \frac{1}{r^4}\left[(\ddot{y}x - y\ddot{x})r^2 - 2(\dot{y}x - y\dot{x})(\dot{x}x + \dot{y}y)\right] \qquad (3.12)$$

Let

$$\left.\begin{aligned} A &= r^3\ddot{r} = (\ddot{x}x + \dot{x}^2 + \ddot{y}y + \dot{y}^2)r^2 - (\dot{x}x + \dot{y}y)^2 \\ B &= (\ddot{y}x - y\ddot{x})r^2 \\ C &= (\dot{x}x + \dot{y}y) \\ D &= (x\dot{y} - y\dot{x}) \end{aligned}\right\} \qquad (3.13)$$

Substituting (3.13) in equation (3.1) and noting that

$\cos\gamma = \frac{x}{r}$

$\sin\gamma = \frac{y}{r}$

$$\therefore \quad \ddot{\bar{r}} = \left\{\left[\frac{A}{r^3} - \frac{D^2}{r^3} - r\dot{\psi}^2\right]\frac{x}{r} - \left[\frac{B}{r^3} - \frac{2}{r^3}(CD) + 2\left(\frac{C}{r}\right)\left(\frac{D}{r^2}\right)\right]\frac{y}{r}\right\}\bar{i} +$$

$$\left\{\left[\frac{A}{r^3} - \frac{D^2}{r^3}\right]\frac{y}{r} + \left[\frac{B}{r^3} - \frac{2}{r^3}(CD) + 2\left(\frac{C}{r}\right)\left(\frac{D}{r^2}\right)\right]\frac{x}{r}\right\}\bar{j} +$$

$$\left\{\frac{2}{r}\left[Dx\frac{y}{r} - C\,x\,\frac{x}{r}\right]\dot{\psi} - r\ddot{\psi}x\frac{x}{r}\right\}\bar{k}$$

Let

$$E = A - D^2$$

and

$$F = 2(Dy - Cx) \qquad (3.14)$$

Then

$$\ddot{\bar{r}} = \left(\frac{1}{r^4}(Ex - By) - \dot{\Psi}^2 x\right) \bar{i} + \frac{1}{r^4} \left(Ey + Bx\right) \bar{j} +$$

$$- \left(\frac{F}{r^2} \dot{\Psi} - x\ddot{\Psi}\right)\bar{k} \qquad (3.15)$$

Equation (3.15) is the general formula for the acceleration of any point on the arm. We can now formulate the forces and therefore moments acting on joints and centre of gravity of links. Consider figure (3.5)

$$M_{z1} = (I_{z_{\ell 1}} + I_{z_{m1}})\ddot{\beta}_1 + q_1 \ell_1 d_1 \left[(a_{y_{\ell 1}} + g)\cos\beta_1 - a_{x_{\ell 1}} \sin\beta_1\right] \qquad (3.16)$$

$$M_{z2} = (I_{z_{\ell 2}} + I_{z_{m2}})\ddot{\beta}_2 + M_{z1} + \ell_2\{\left(q_2 d_2 (a_{y_{\ell 2}} + g) + q_1 \ell_1 (a_{y_{\ell 1}} + g)\right.$$

$$\left. + m_1 (a_{y_{m1}} + g)\right)\cos\beta_2 - \left(q_2 d_2 a_{x_{\ell 2}} + q_1 \ell_1 a_{x_{\ell 1}} + m_1 a_{x_{m1}}\right)\sin\beta_2\} \qquad (3.17)$$

$$M_{z3} = (I_{z_{\ell 3}} + I_{z_{m3}})\ddot{\beta}_3 + \ell_3\{\left(q_3 d_3 (a_{y_{\ell 3}} + g) + q_2 \ell_2 (a_{y_{\ell 2}} + g) + \right.$$

$$\left. m_2 (a_{y_{m2}} + g) + q_1 \ell_1 (a_{y_{\ell 1}} + g) + m_1 (a_{y_{m1}} + g)\right)\cos\beta_3 - \left(q_3 d_3 a_{x_{\ell 3}}\right.$$

$$\left. + q_2 \ell_2 a_{x_{\ell 2}} + m_2 a_{x_{m2}} + q_1 \ell_1 a_{x_{\ell 1}} + m_1 a_{x_{m1}}\right)\sin\beta_3\} + M_{z2} \qquad (3.18)$$

$$M_{y_4} = \left(I_{y_{\ell 4}} + I_{y_{\ell 3}} + I_{y_{\ell 2}} + I_{y_{\ell 1}} + I_{y_{m4}} + I_{y_{m3}} + I_{y_{m2}} + I_{y_{m1}}\right)\ddot{\theta}_4 -$$

$$\{q_1 \ell_1 d_1 a_{z_{\ell 1}} \cos\beta_1 + \ell_2 \left(q_1 \ell_1 a_{z_{\ell 1}} + m_1 a_{z_{m1}} + q_2 d_2 a_{z_{\ell 2}}\right)\cos\beta_2 + \ell_3$$

$$\left(q_1 \ell_1 a_{z_{\ell 1}} + m_1 a_{z_{m1}} + q_2 \ell_2 a_{z_{\ell 2}} + m_2 a_{z_{m2}} + q_3 d_3 a_{z_{\ell 3}}\right) \cos \beta_3\} \qquad (3.19)$$

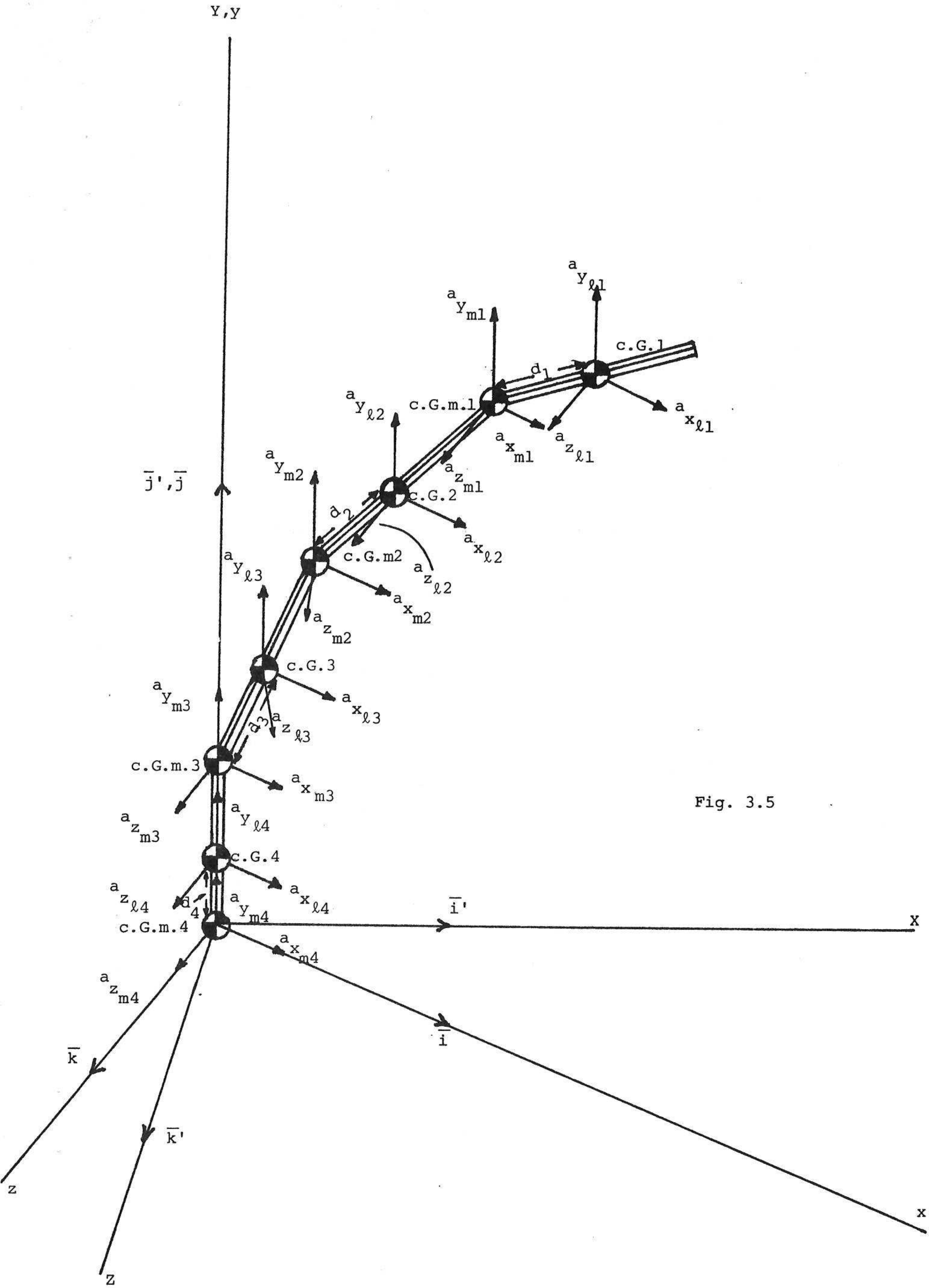


Fig. 3.5

Where (applied to each case)

$$a_x = \frac{1}{r^4}\left(E_x - B_y\right) - \dot{\Psi}^2 x$$

$$a_y = \frac{1}{r^4}\left(E_y + B_x\right) \qquad (3.20)$$

$$a_z = \frac{F}{r^2}\dot{\Psi} - \ddot{\Psi}x$$

CONCLUSION

The stated aim of deriving a quantitative model for a mechanical arm has been fulfilled. This was developed by using moving co-ordinate system analysis of the mechanical arm. The torque/inertia model has been shown to be highly complex and non-linear. It is obvious that an analytical solution to the equation of the torque is not possible at this time and therefore a numerical solution must be used [6].

The derived manipulator model is useful for speed control purposes without modification. However, where accurate position control is required, the omission of bending moment calculations in the analysis cannot be disregarded. Further work to include consideration of the effect of bending moments is planned.

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