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LINEAR DISCRETE SMOOTHING OF MULTIPASS PROCESSES

by

J.B. Edwards and A.M.S.R. Yazdi

Department of Control Engineering,
University of Sheffield,
Mappin Street, Sheffield S13JD.

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1. Introduction

All previous analyses of interpass smoothing in multipass systems (1) have been confined to smoothing phenomena described by spatial differential equations (i.e. continuous interpass processes). In practice, however, the interpass process is frequently of a discrete nature: examples including rigid bricks layed on a continuous bed of mortar in brick-laying and rigid steel conveyor-trays resting on the continuous coal-floor profile produced by a longwall coal-cutter. In the latter example, considerable discrepancy has been revealed (1)(2) in the multipass stability predictions based on (a) simulation of a piecewise rigid conveyor and (b) analysis and simulation of a continuous model in which the piecewise rigid structure is approximated by a continuous elastic beam along the full pass length. Indeed, simulations of the rigid conveyor have failed to produce multipass stability whereas sufficient elastic beam stiffness will stabilise an otherwise unstable multipass system.

Being based on general dynamic programming, rigid conveyor simulations are expensive and there therefore exists a strong motivation to attempt a determination of piecewise rigid smoothing effects by purely analytical means. This report examines analytically the effect of such an interpass process on the multipass behaviour of an idealised coal-cutter steering process described by a simple delay equation that has proved to be illuminating in previous investigations (1). The analytical predictions made are confirmed by simulation and used to guide the choice of parameters in the full (true) process simulation. By this method, the possibilities of stabilising the true process, by enlarging conveyor tray lengths, is revealed.

2. Stability criterion for a discrete multipass process

A discrete process H(z) that is the z-transform of a sampled linear single-pass process G(s) cascaded with an interpass smoothing process may be examined for multipass stability by solution of the characteristic equation

$$z^{-m}H(z) = 1.0 \tag{1}$$

where

$$m = L/X_p \tag{2}$$

Fig. 1 illustrates the entire process. L is the pass length, X_p the sampling distance interval and m is taken to be a very large integer. The usual long-pass, stable-G assumptions are, of course, implied.

Provided H(z) contains no unstable zero then, for multipass stability, it follows that the locus of $z^{-m}H(z)$ described in its own plane as z describes the contour shown in Fig. 2 -, viz:

(a) $z = 1 \angle \theta \quad , 0 \leq \underset{\rightarrow}{\theta} \leq 2\pi$

and (b) $z = R_{\infty} \angle \theta \quad , 2\pi \geq \underset{\rightarrow}{\theta} \geq 0$ (3)

around the entire unstable region of the z-plane, should make zero net encirclements of the point -1 + j0. Since z^{-m} collapses to zero during portion (b) of the z contour, it follows that the locus of $z^{-m}H(z)$ during portion (a) should not encircle the point -1 + j0. The vector $z^{-m}H(z)$ will rotate very rapidly if m is large (as assumed) being slowly modulated by H(z) so that, since $|z| = 1.0$, to avoid the critical point, $|H(z)| < 1.0$ for all θ in the range 0-2π.

The multipass stability criterion may therefore be expressed thus:

$$|H(z)| < 1.0 \text{ for } z = 1 \angle \theta,\ 0 \leq \theta \leq 2\pi \tag{4}$$

3. The linear discrete smoothing model

Fig. 3 illustrates the behaviour assumed for the piecewise rigid conveyor. The conveyor profile $d(n+1,\ell)$ is assumed to be supported on cut-floor profile $y(n,\ell)$ only at the ends of the trays i.e.

$$d(n+1,\ i\ X_p) = y(n,\ i\ X_p)\ ,\ i = 0,1,2,\ldots m \quad (5)$$

and

$$d(n+1,\ell) = \left[y\{n,\ i\ X_p\}\{(i+1)X_p - \ell\} + y\{n,(i+1)X_p\}\{\ell - i\ X_p\}\right]/X_p$$

$$i\ X_p \leq \ell \leq (i+1)X_p \quad (6)$$

Some floor penetration is therefore allowed, unlike the dynamic programming (dp) model and like the elastic beam, whilst tray rigidity is retained (like the d.p. model and unlike the elastic beam model). This investigation should therefore allow some isolation of the causes of discrepancy between previous model predictions.

4. The single-pass process model, G(s)

The automatic vertical steering process G(s) is here represented by the simple delayed feedback configuration illustrated in Fig. 4. The only dynamics here included are coal-sensor delay distance X. The controller height gain is k. This model is well justified and has been frequently used in earlier studies. The process equation is

$$y(n,\ell) = d(n,\ell) - k\ y(n,\ell - X) \quad (7)$$

and in this study we assume that

$$X_p = r\ X \quad (8)$$

where r is some integer, 1,2,3,... to be selected.

5. Calculation of the composite discrete process H(z)

The open-loop z-transfer function of the system of Fig. 1 is $H(z)\ z^{-m}$ and, for stability studies, the order of the elements may be re-arranged to the form shown in Fig. 5, the siting of long delay term z^{-m} being immaterial. To find H(z) we must first determine the response of $y^*(n,\ell)$ (i.e. at point Q in Fig. 5) to a unit impulse applied at point P. The shape of the impulse response of the smoother above is depicted in Fig. 5 and we must now calculate its effect on G(s) at delay intervals X

i.e. at $\ell = i\ X \quad ,\quad i = -r,\ -(r-1)\ \ldots\ 0,1,2,$ etc. $\quad (9)$

For this purpose we use the following simplified notation

$$d(i) = d(n,iX) \tag{10}$$

$$y(i) = y(n,iX) \tag{11}$$

argument n being dropped since it is identical for both variables of interest. The calculations may be carried out analytically in the step-wise manner of Table 1 from which values of the sampled function $y(n,\ell)$ are selected at intervals X_p (= rX) i.e. at i = -r,o,r,2r,3r etc. to produce the output of process H(z). From the table we deduce that $y^*(n,\ell)$ (the output of H(z)) may be expressed thus

$$y^*(n,\ell) = Ar^{-1}\delta(\ell) + \sum_{q=1}^{\infty} (-k)^{(q-1)r}\{B+A(-k)^r\}r^{-1}\delta(\ell - q\ rX) \tag{12}$$

where, again from Table 1, the parameters A and B are given by

$$A = \sum_{i=1}^{r-1} (r-i)(-k)^i \tag{13}$$

and

$$B = \sum_{i=o}^{r-1} i(-k)^i \tag{14}$$

Taking Laplace transforms in s w.r.t. ℓ gives

$$\tilde{y}^*(s) = A\ r^{-1} + \frac{\{B+A(-k)^r\}}{(-k)^r r} \sum_{q=1}^{\infty} \{(-k)^r e^{-rXs}\}^q \tag{15}$$

Now the sampling distance is X_p (= rX), not X so that for the z-transform $\tilde{y}(z)$ of the output of H(z) we must set

$$e^{rXs} = z \tag{16}$$

in (15) to give

$$\tilde{y}(z) = Ar^{-1} + \frac{\{B+A(-k)^r\}}{(-k)^r r} \sum_{q=1}^{\infty} \{(-k)^r z^{-1}\}^q$$

$$= -B(-k)^{-r}r^{-1} + \frac{\{B+A(-k)^r\}}{(-k)^r r} \sum_{q=o}^{\infty} \{(-k)^r z^{-1}\}^q$$

$$= -B(-k)^{-r}r^{-1} + \frac{\{B+A(-k)^r\}z}{(-k)^r r\ \{z-(-k)^r\}} \tag{17}$$

Now since the input to H(z) is a unit impulse at ℓ = o in the above analysis, z-transfer function $H(z) = \tilde{y}(z)$ as given by equation (17). Simplifying

TABLE 1 Calculation of unit impulse response of H(z)

i	d(i)	y(i) = d(i) - k y(i-1) (eq^{n}. 7)
$-r$	0	0
$-(r-1)$	r^{-1}	r^{-1}
$-(r-2)$	$2r^{-1}$	$(2-k)r^{-1}$
$-(r-3)$	$3r^{-1}$	$(3-2k + k^{2})r^{-1}$
.		
.		
.		
0	$rr^{-1} = 1$	$\{r+(r-1)(-k) + (r-2)(-k)^{2} \ldots\ldots (-k)^{r-1}\} = Ar^{-1}$
1	$(r-1)r^{-1}$	$\{(r-1) - k\ A\}r^{-1}$
2	$(r-2)r^{-1}$	$\{(r-2) + (r-1(-k) + A(-k)^{2}\}r^{-1}$
3	$(r-3)r^{-1}$	$\{(r-3) + (r-2)(-k) + (r-1)(-k)^{2} + A(-k)^{3}\}r^{-1}$
.		
.		
.		
r	0	$\{(-k) + 2(-k)^{2} \ldots (r-1)(-k)^{r-1} + A(-k)^{r}\}r^{-1}$
		$= \{B + A(-k)^{r}\}r^{-1}$
$r+1$	0	$-\ k\{B + A(-k)^{r}\}r^{-1}$
$r+2$	0	$(-k)^{2}\{B + A(-k)^{r}\}r^{-1}$
.		
.		
.		
$2r$	0	$(-k)^{r}\{B+A(-k)^{r}\}r^{-1}$
.		
.		
.		
$3r$	0	$(-k)^{2r}\{B + A(-k)^{r}\}r^{-1}$
qr	0	$(-k)^{(q-1)r}\{B+A(-k)^{r}\}r^{-1}$

the R.H.S. of the equation therefore we obtain finally:

$$\boxed{H(z) = \frac{zA + B}{r\{z - (-k)^r\}}} \qquad (18)$$

6. The effect of tray/delay ratio r on multipass stability

Criterion (4) may now be used to examine the multipass stability of H(z) and hence the effect of varying parameter r. Applying this criterion we deduce that, for multipass stability,

$$|zA + B| < r|z - (-k)^r|$$

$$\text{for } z = 1 \angle \theta \quad , \quad 0 \le \theta \le 2\pi \qquad (19)$$

Now as shown in Appendix 1, A + B and A - B are both positive for a single-pass stable system (i.e. for $0 < k < 1.0$) for r = 1,2,3, etc..... so that vectors $zA + B$ and $r\{z - (-k)^r\}$ both describe circles counterclockwise enclosing the origin as illustrated in Fig. 6 for multipass stability therefore it is necessary and sufficient that

$$\left.\begin{array}{l} A + B < r(1-k^r) \\ A - B < r(1+k^r) \end{array}\right\} \quad r = 2,4,6 \text{ etc.} \qquad (20)$$

(and)

whilst

$$\left.\begin{array}{l} A + B < r(1+k^r) \\ A - B < r(1-k^r) \end{array}\right\} \quad r = 1,3,5 \text{ etc.} \qquad (21)$$

(and)

As shown in Appendix 1, the series for A and B may be summed to give

$$A + B = r\{1 - (-k)^r\}/(1+k) \qquad (22)$$

and

$$A - B = \{r + k(2+r) + r(-k)^r + (2-r)(-k)^{r+1}\}/(1+k)^2 \qquad (23)$$

We now consider the cases of r-even and r-odd separately as they yield somewhat different results.

6.1 r-even Combining results (20), (22) and (23) we deduce that for multipass stability

$$\frac{r(1-k^r)}{(1+k)} < r(1 - k^r) \tag{24}$$

which is clearly satisfied in the region of interest, $o < k < 1$, and

$$(r-2) + kr + (r+2)k^r + r\,k^{r+1} > 0 \tag{25}$$

which is also satisfied within this range of gain provided

$$r \geq 2 \tag{26}$$

6.2 r-odd In this case, combining results (21), (22) and (23) we deduce that for multipass stability

$$\frac{r(1+k^r)}{1+k} < r(1+k^r) \tag{27}$$

which is again satisfied for $o < k < 1$ (i.e. for all single-pass stable systems) and

$$r-2 + r\,k > (2+r)k^r + r\,k^{r+1} \tag{28}$$

6.2.1 Special case r = 1

Setting $r = 1$ in (28) the stability criterion becomes

$$k^2 + 2k + 1 < 0 \tag{29}$$

which is clearly impossible for any $k > o$ so that we may conclude that making the conveyor tray length, X_p = delay distance, X will not stabilise the multipass system.

6.2.2 Special case r = 3

Setting $r = 3$ in (28) yields the stability criterion

$$1 + 3k > 5k^3 + 3k^4 \tag{30}$$

which is satisfied if

$$0 < k < (1 + \sqrt{13})/6 = 0.768 \tag{31}$$

The setting of r = an odd integer has thus produced a somewhat tighter restriction on controller gain k than pertains for r-even. The reason for this is probably that, when r is odd, the discrete linear smoothing process picks up some of the peaks of the oscillatory response of G(s) yielding a somewhat oscillatory process overall whereas, when r is even, the smoothing process picks up only the troughs, yielding a non-oscillatory process overall. Fig. 7 illustrates the two situations for $r = 2$ and 3. When r is odd therefore, some restricted proportion of the oscillatory modes of G(s) are transmitted from pass to pass so demanding a reduction in k for multipass stability. Criterion (28) is however, not expressed in the most convenient form to calculate the critical gain. In the case of $r = 3$, for instance, a quartic equation must be solved (see 30) to give result (31).

6.2.3 A more convenient expression of the stability criterion for r-odd

Rather than using the analytically-derived solution (23) for A-B in criterion (21), a criterion of lower order is produced if the original series-summation definitions (13) and (14) for A and B are used, giving instead:

$$\sum_{i=o}^{r-1} (r-2i)(-k)^{i} < r(1-k)^{r} \qquad (32)$$

which, for r=3, becomes

$$3\,k^{2} < k+1 \qquad (33)$$

now necessitating only the solution of a quadratic equation to produce result (31).

7. Confirmatory simulation results

Fig. 8 illustrates the stable recovery of the multipass system analysed in the report from a disturbed initial profile (specified by N.C.B) to the desired zero-height profile. The parameters used are $k = 0.8$ and $r = 2$. The result is clearly in accord with theoretical prediction and indeed other simulations confirm that, for r-even, stability is achieved for all k in the range $o < k < 1$.

Fig. 9 shows the expected instability of the system when r is set at unity. The value of k used here is 0.5, but as expected, no value of k will yield multipass stability in this case.

Figs. 10 and 11 show the system behaviour for r=3 with k set at 0.95 and 0.65 respectively (i.e. values either side the predicted critical value of 0.768). As expected, stability is obtained only for the lower gain situation.

8. Conclusions for more-elaborate models

The foregoing analysis and simulation has been carried out for a much simplified process model: The interpass (conveyor) model has been assumed to generate piecewise-straight profiles, as in real-life, but has been allowed to penetrate the output-(cut-floor) profile. Conventional conveyor structures should produce little floor penetration, however, if the cut material is reasonably hard, and a simulation-model[(2),(3)] rigorously based on potential-energy minimisation by general dynamic programming (d.p.) has been developed in earlier research. In view of the prediction (based on the simplified model) of Section 6 that $r \geq 2$ should produce stability, it is therefore tempting to investigate whether or not the criterion should carry over as a 'rule-of-thumb' for the elaborated (d.p.) model. Subsequent simulation has in fact revealed that stability is not achieved with r=2 but can be obtained for r=4 as demonstrated by Fig. 12, for which the controller gain, k, was set to 0.5.

Elaborating the single-pass (steering system) model to include, say, transducer and actuator lags X_1 and X_2, reveals that their inclusion increases the critical value of X_p/X and again, as a 'rule-of-thumb', simulation suggests that real-life stability is achievable with the $r > 4$ criterion provided r is now defined as $X_p/(X + X_1 + X_2)$, $(X_1, X_2 < X)$. Thus, for real-system stability, the criterion should be

$$X_p > 4(X + X_1 + X_2)$$

and the controller gain k should be set somewhat < 1.0. We therefore deduce that multipass stability can be generated by intercoupled, piecewise-straight interpass smoothing sections provided their individual length exceeds some four times the total distance-lag of the single-pass system. The result had not been discovered by cut-and-try simulation prior to the present analysis and the value of simplified analysis to guide the course of simulation experiments is therefore clearly demonstrated here.

That multipass stability can indeed be produced by spatially discrete smoothing is an extremely important deduction that accords with the predictions of earlier spatially-continuous interpass models based on elastic beams etc. The conclusion generates incentive to search for a continuous equivalent of the discrete process thereby simplifying analytical work in this area in the future.

9. References

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(3) Edwards, J.B., 'Modelling of coal and mineral extraction processes', Chap. 7, Vol. 2 of Modelling of Dynamical Systems, (Ed. H. Nicholson), Peter Peregrinus, I.E.E., Control Engineering Series 13, 1981, pp. 178-233.

APPENDIX 1

Calculation of A + B and A - B

$$\left.\begin{aligned} \text{Given } A &= \sum_{i=o}^{r-1} (r-i)\ a^i \\ \text{and } \quad B &= \sum_{i=o}^{r-1} i\ a^i \end{aligned}\right\} \qquad \text{(A1)}$$

From equations (13) and (14) with -k set = a for convenience we deduce that

$$A + B = \sum_{i=o}^{r-1} r\ a^i = r\ G_{r-1}$$

where G_{r-1} represents the series $1, a, a^2 \ldots$ summed to include a^{r-1}. Now it is well known that

$$G_{r-1} = (1 - a^r)/(1 - a) \qquad \text{(A2)}$$

since $(1 - a)\ G_{r-1} = (1 + a + a^2 \ldots . a^{r-1}) - (a + a^2 \ldots\ a^r) = 1 - a^r$

Hence $$\boxed{(A + B) = \frac{r(1-(-k)^r)}{(1 + k)}} \qquad \text{(A3)}$$

and it is clear that $A + B > O$ for $O < k < 1.O$ (A4)

Result (A3) is stated as equation (22) in the main text).

Now consider A-B = A+B - 2B

$$= r\ G_{r-1} - 2\ \{a + 2a^2 + 3a^3 \ \ldots\ (r-1)a^{r-1}\} \qquad \text{(A5)}$$

Now let $$S_{r-1} = a + 2a^2 + 3a^3 \ \ldots\ldots\ (r-1)a^{r-1} \qquad \text{(A6)}$$

$$= a\{1 + 2a + 3a^3 \ \ldots\ (r-1)a^{r-2}\}$$

$$= a\left[1 + a + a^2 \ldots a^{r-2} + \{a + 2a^2 + 3a^3 \ldots (r-2)a^{r-2}\}\right]$$

$$\therefore \quad S_{r-1} = a(G_{r-2} + S_{r-2}) \qquad \text{(A7)}$$

Now $$S_{r-2} = S_{r-1} - (r-1)a^{r-1} \qquad \text{(A8)}$$

so that from (A7) and (A8) we get

$$S_{r-1} = \{aG_{r-2} - (r-1)a^r\}/(1-a) \qquad \text{A9}$$

Thus eliminating S_{r-1} from (A5) we get

$$A-B = r\,G_{r-1} - 2\{aG_{r-2} - (r-1)a^r\}/(1-a) \qquad \text{(A10)}$$

and since $G_{r-2} = G_{r-1} - a^{r-1}$ (A11)

we deduce from (A10) (A11) and (A2) that

$$A-B = \{r - a(2+r) + r\,a^r + (2-r)(a)^{r+1}\}/(1 - a)^2$$

i.e.

$$\boxed{A-B = \{r + k(2+r) + r(-k)^r + (2-r)(-k)^{r+1}\}/(1+k)^2} \qquad \text{(A12)}$$

Equation (23) in the main text is thus derived.

If $o < k < 1$ it is clear that A-B will be positive, (a) for r even and ≥ 2 and also, (b) for r odd and ≥ 1 since $r > r\,k^r$ and $r\,k > r\,k^{r+1}$.

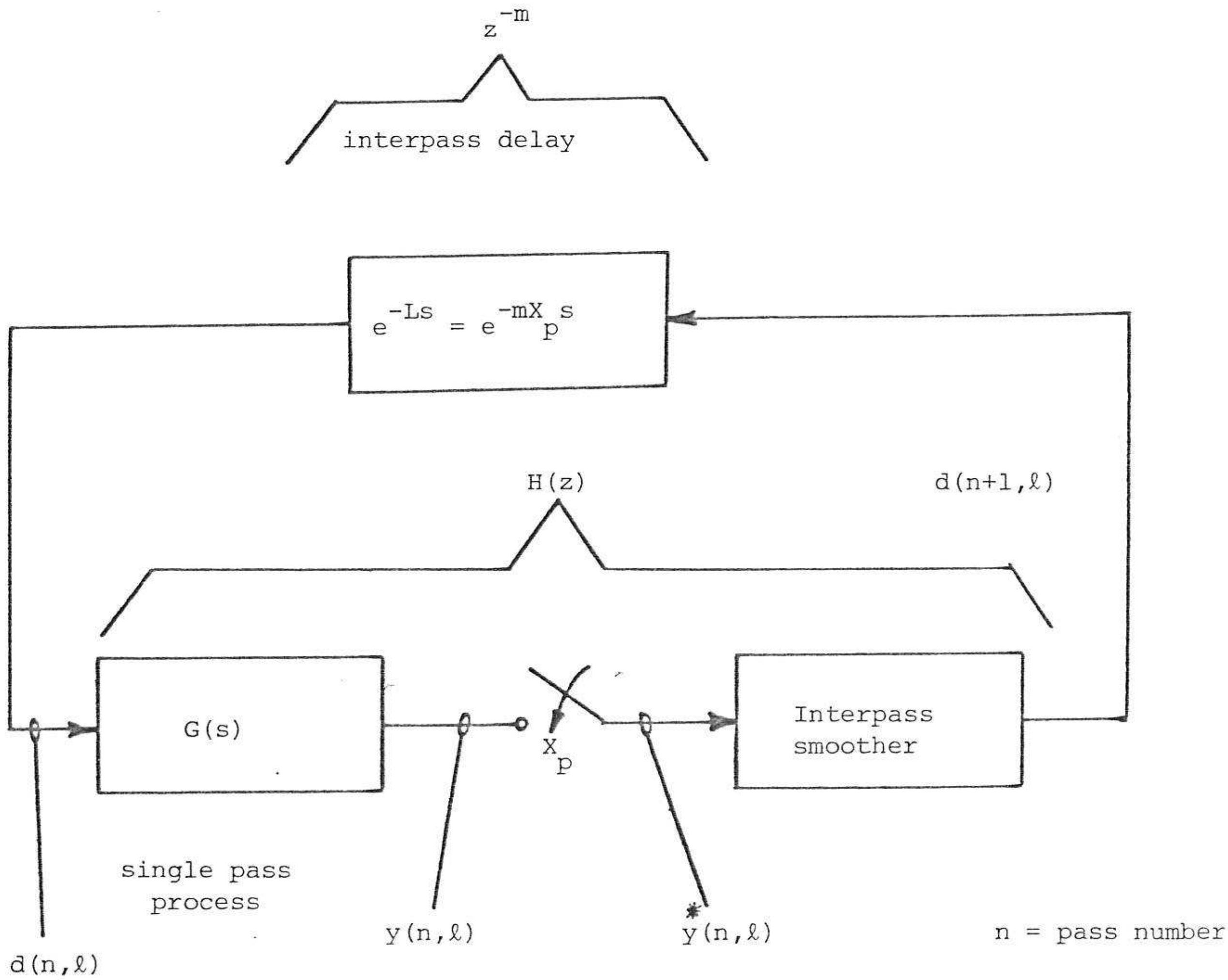


FIG. 1 Multipass process with discrete interpass smoothing

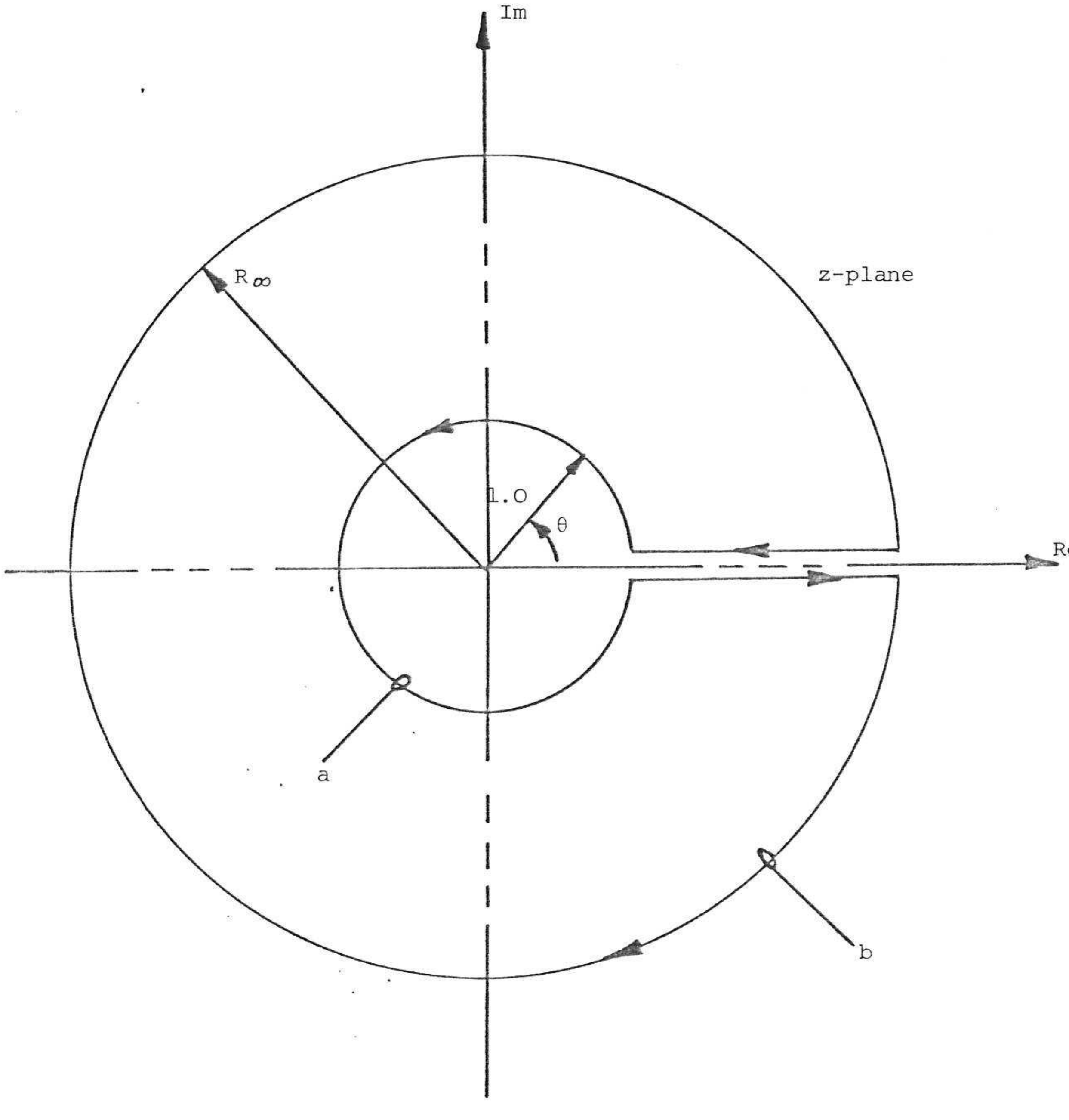


FIG. 2 Contour encircling unstable part of z-plane

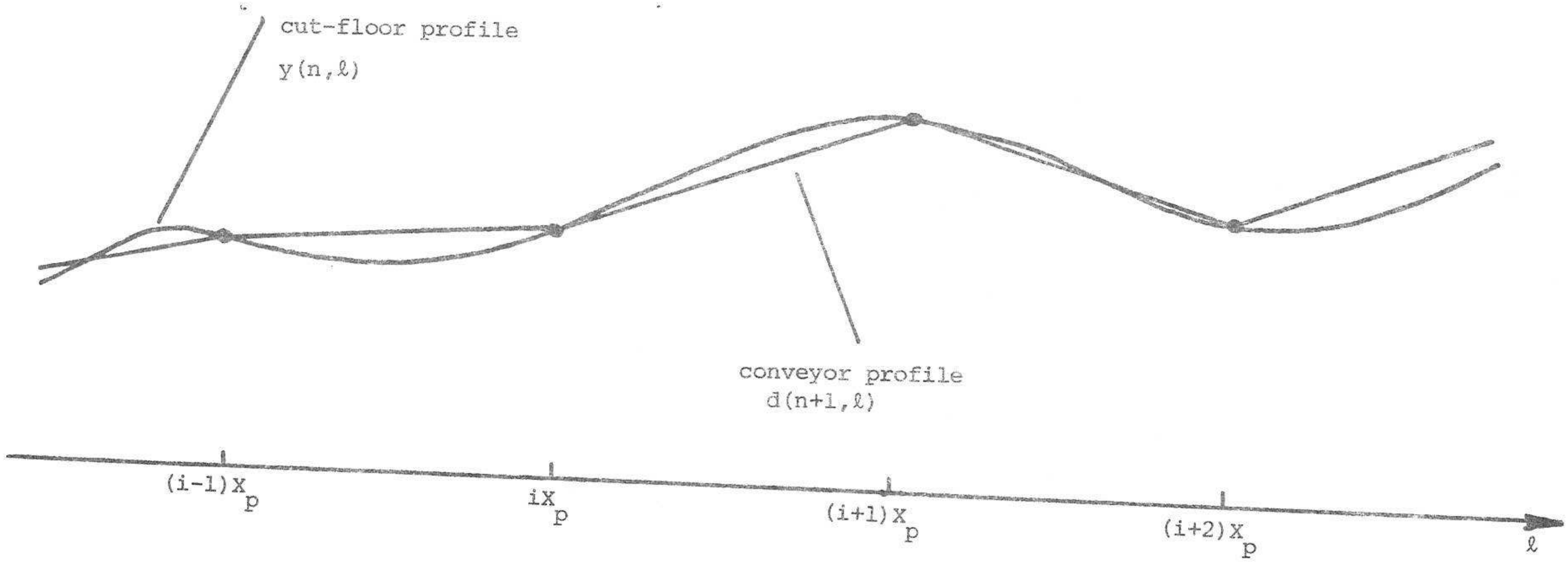


Fig. 3 Showing assumed behaviour of piecewise rigid conveyor

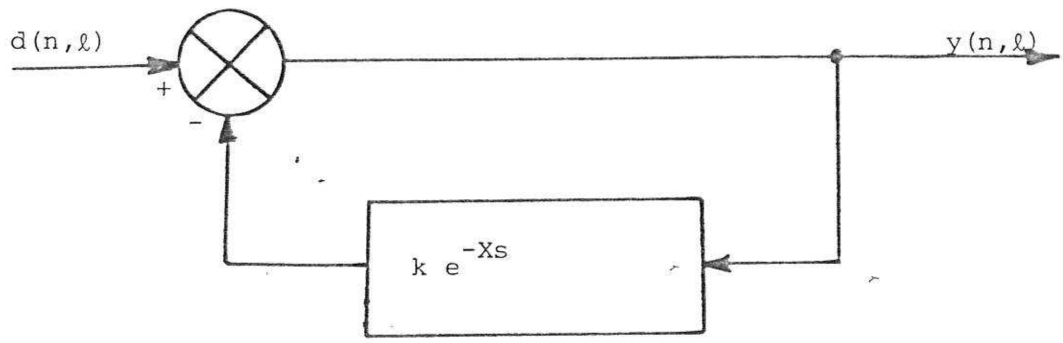


Fig. 4 Automatic steering process block-diagram

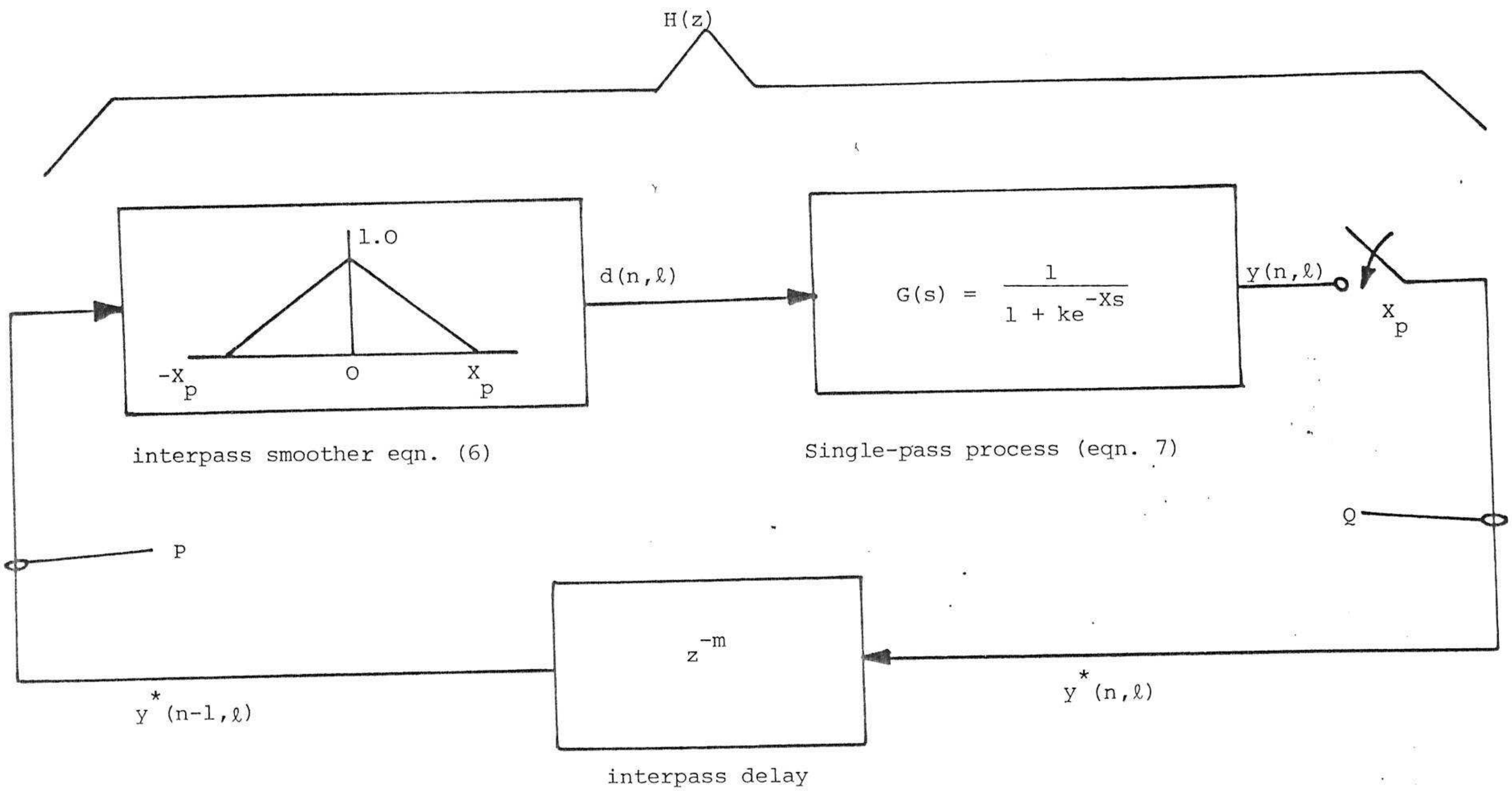
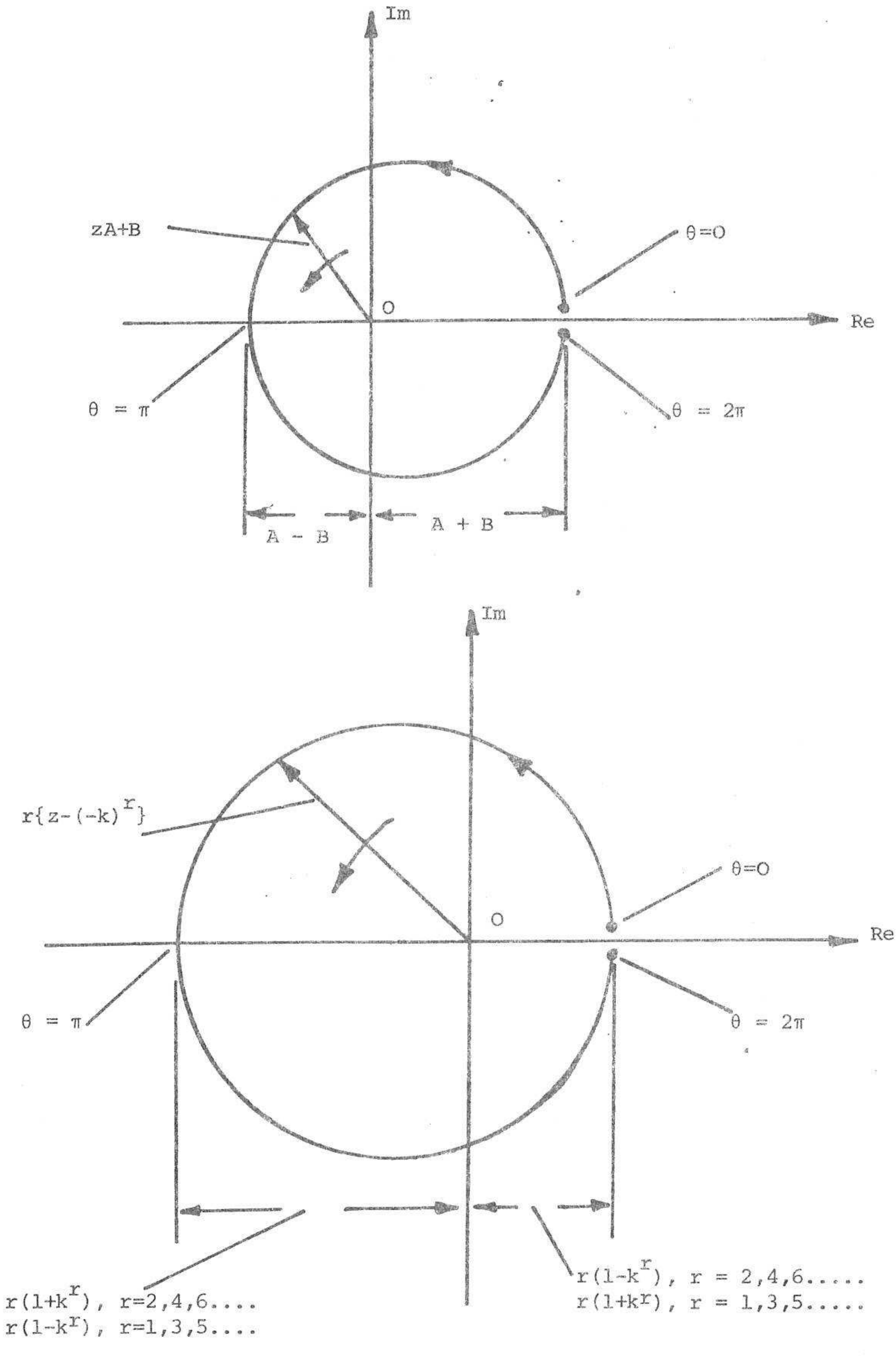


Fig. 5 System of Fig. 1 re-configured for calculation of H(z)

Fig. 6 Showing circular loci of zA+B and $r\{z - (-k)^r\}$ for $z = 1\angle\theta$



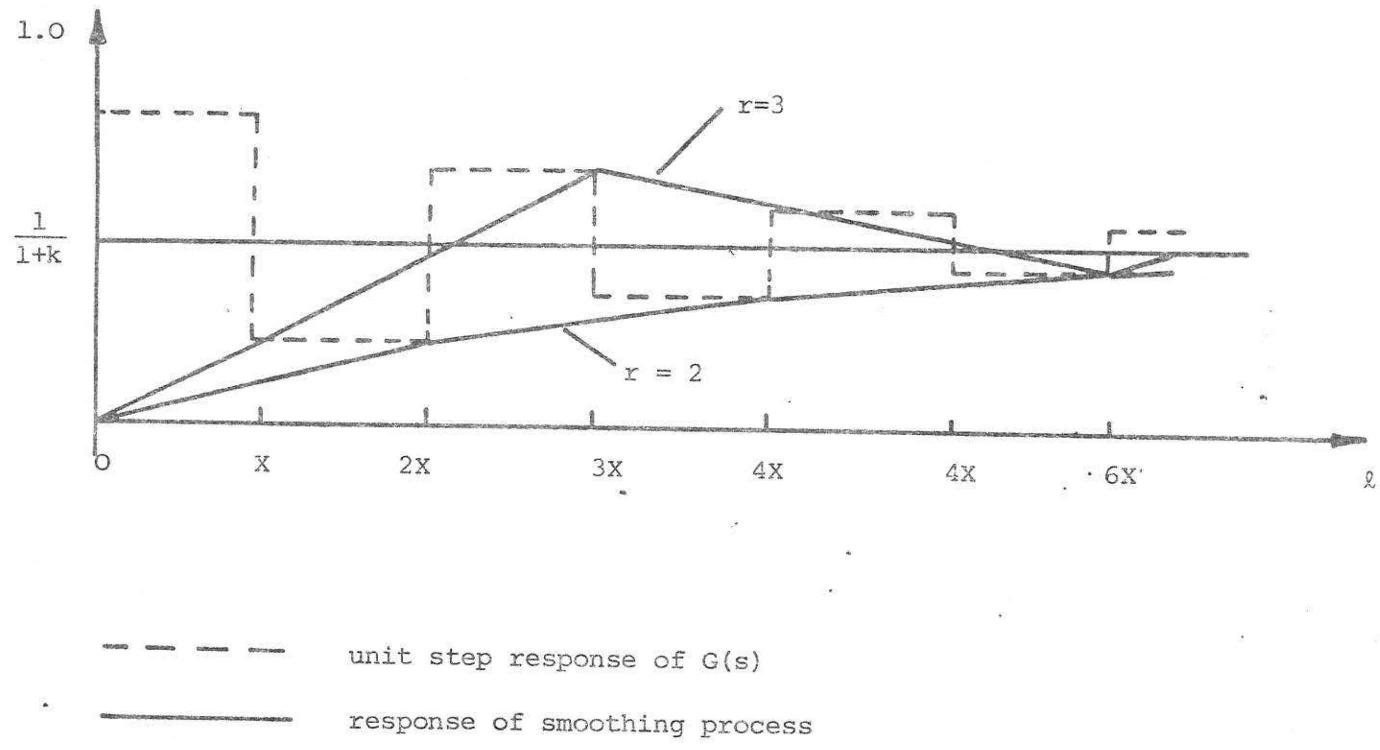
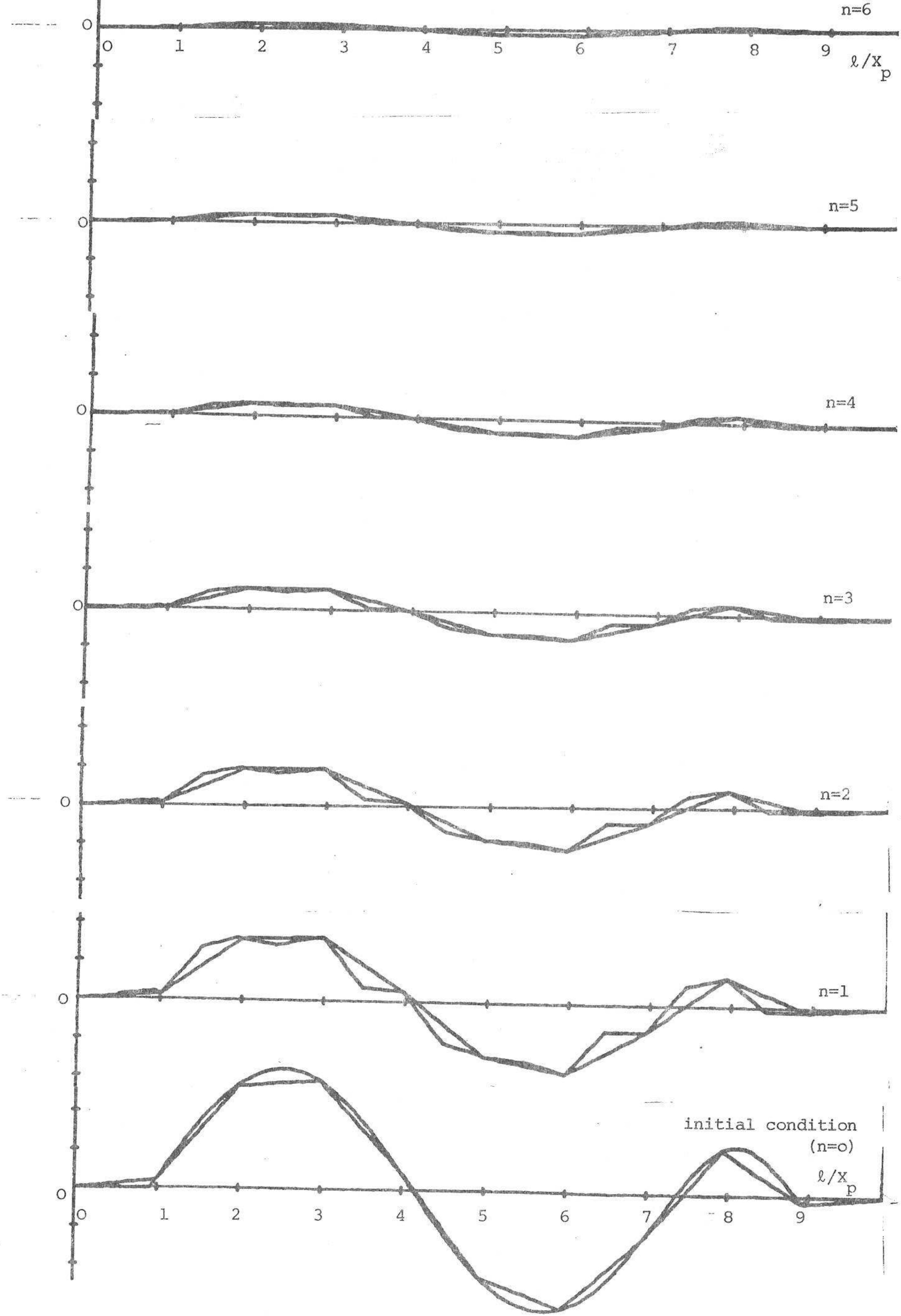


Fig. 7 Comparing oscillatory and non-oscillatory step responses for r-odd and r-even

Fig. 8 Traces of $y(n,\ell)$ and $d(n+1,\ell)$ showing multipass stability (r=2, k=0.8)



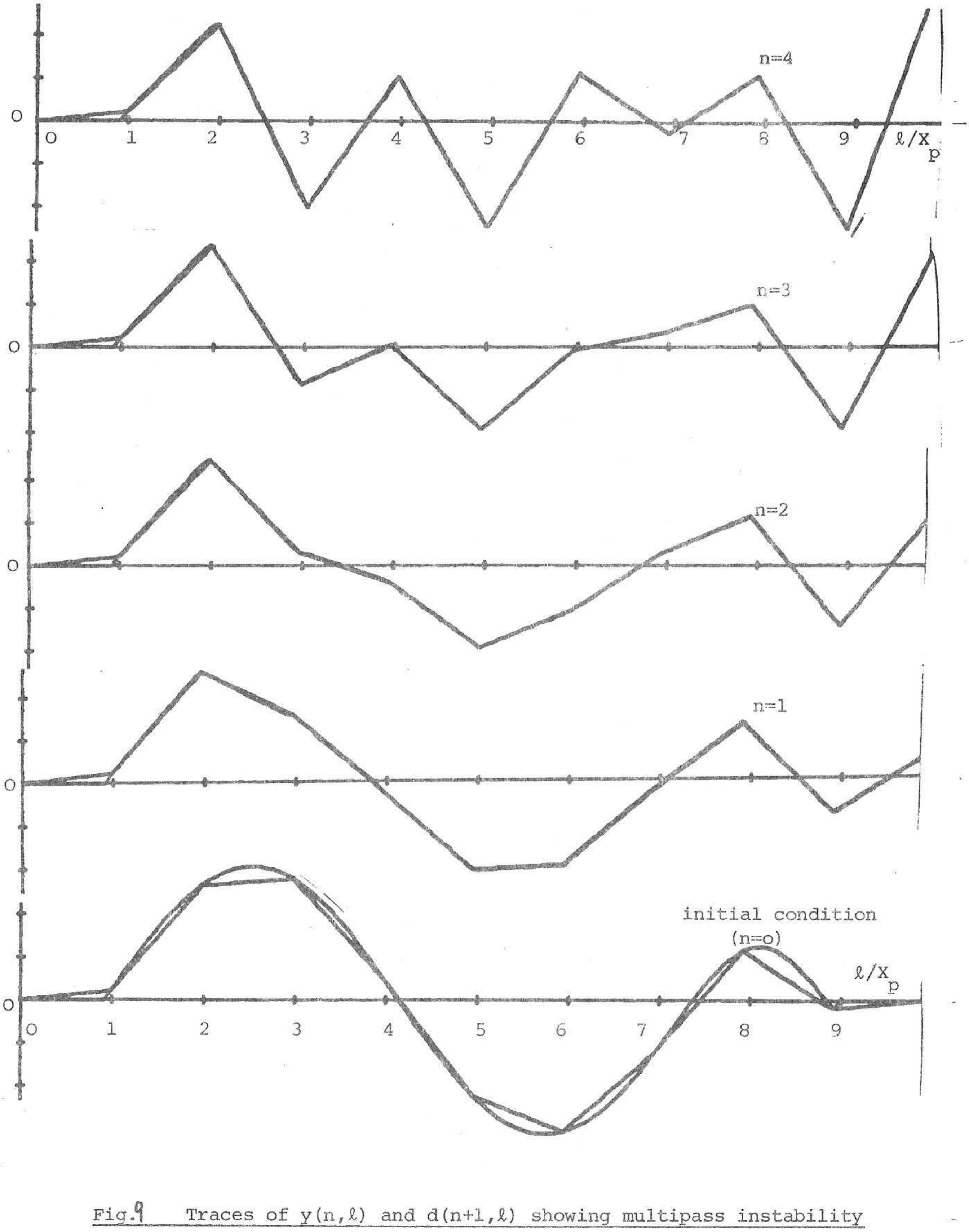


Fig.9 Traces of $y(n,\ell)$ and $d(n+1,\ell)$ showing multipass instability

($r=1$, $k= 0.5$)

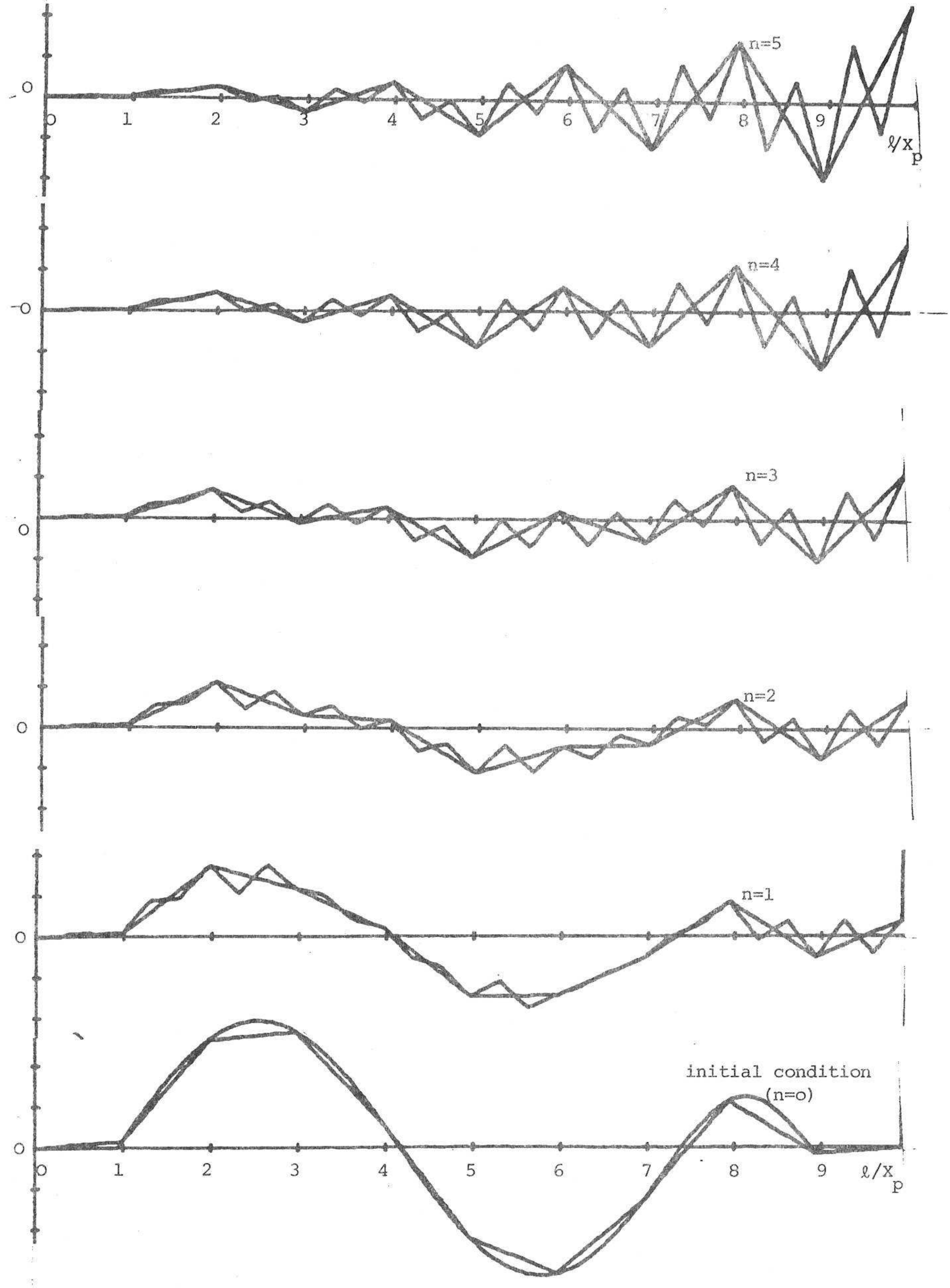


Fig. 10 Traces of y(n,ℓ) and d(n+1,ℓ) showing multipass instability (r=3, k = 0.95)

Fig. 11 Traces of y(n,λ) and d(n+1,λ) showing multipass instability (r=3, k=0.65)

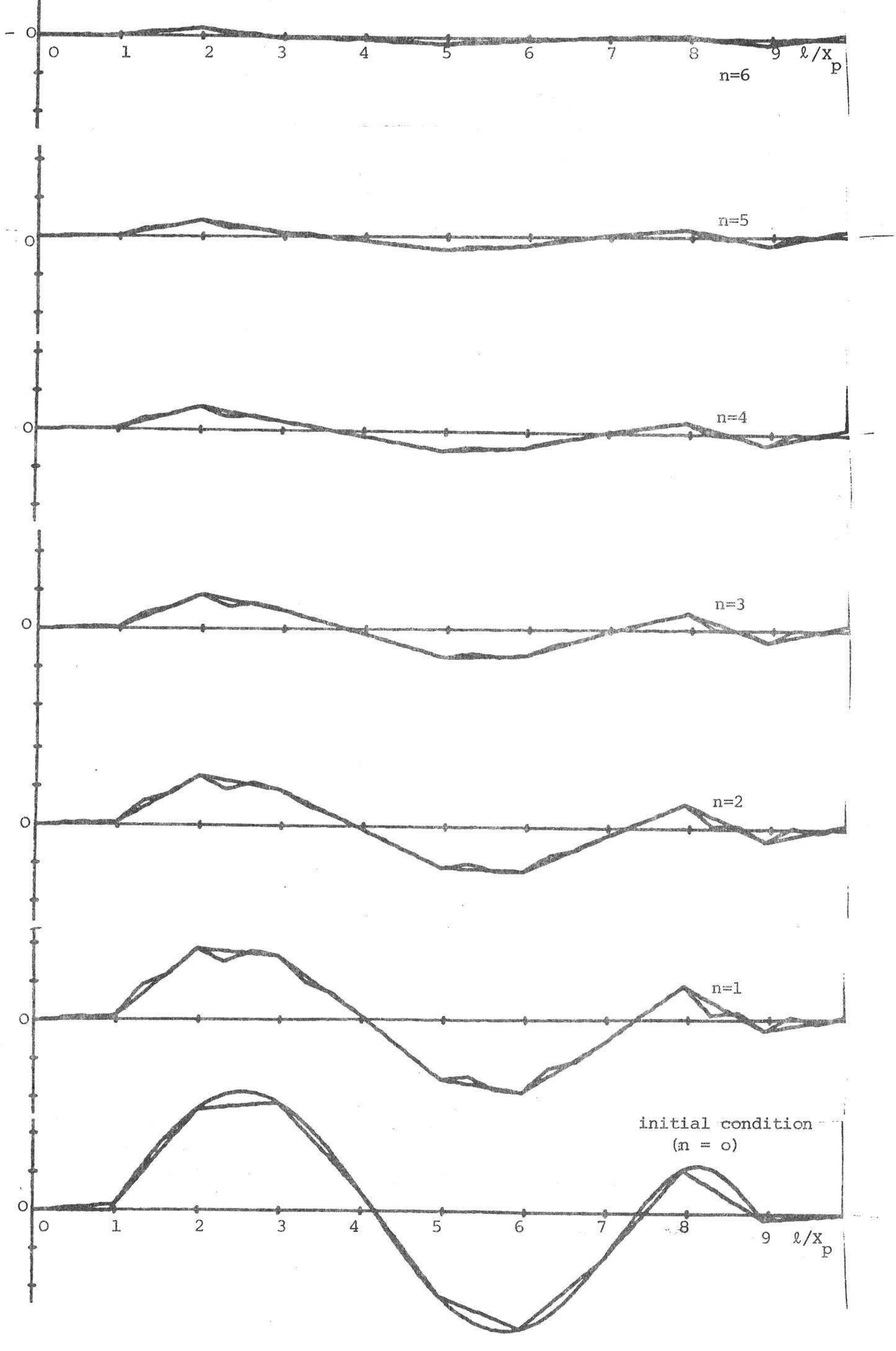


Fig. 12 Traces of $y(n,\ell)$ and $d(n+1,\ell)$ showing stability of rigorous dynamic programming model (r=4, k=0.5)