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THE STABILITY OF ACCELERATING REPETITIVE SYSTEMS

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SUMMARY

The spatial stability of a sequence of accelerating repetitive operations is investigated using the output spectral-density in a frequency-band enclosing the first resonance-peak as a stability indicator. The operations of the sequence are identical in dynamic structure but subject to a constant rate of acceleration between operations thus resembling the rolling of metal strip and other repetitive manufacturing processes such as machining. The technique readily yields a value for the critical number of operations within which stability can be expected to be achieved, within the chosen frequency band. Simulation of a variety of systems confirms the physical usefulness of this number which correlates well with the observed number of operations found to be necessary to achieve an output profile that is adequately stable from a practical viewpoint.

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1. List of Symbols and Abbreviations

$G_n(s)$ = transfer-function of nth operation in a repetitive sequence $G_o(s)$, $G_1(s) \ldots G_{N-1}(s)$

k = static gain of second order process

k_1, k_2 = gain parameters in metal rolling process model

L_n = length of metal strip after nth operation

ℓ_n = distance from leading end of strip to slice of interest after nth operation

L = normalised strip length = L_o

ℓ = normalised value of $\ell_n = \ell_n L_o/L_n$

n = integer representing number of repetitive operations undergone by workpiece

N = total number of operations in repetitive sequence

n_c = critical value of n above which spectral-density of output of $G_n(s)$ {in response to impulse applied to $G_o(s)$} will not increase at any frequency in band $\omega_a \leq \omega \leq \omega_b$

r = constant = 1/acceleration rate or process dynamics between operations

s = Laplace variable

$S_n(\omega)$ = spectral-density of output of $G_n(s)$

X = fixed transport delay distance in metal rolling (between rolls and gauge sensor)

X_n = n-dependent delay distance in hypothetical rolling model

$y_n(\ell)$ = output from nth operation at normalised distance ℓ

$\tilde{y}_n(s)$ = Laplace transform of $y_n(\ell)$ in s w.r.t. ℓ

ω	=	angular natural frequency in radians p.u. normalised distance ℓ.
ω'	=	lowest value of ω making $\lvert G_n(j\omega)\rvert = 1.0$
ω_a	=	lowest value of ω at which $\lvert G_o(j\omega)\rvert = 1.0$
ω_b	=	next higher value of ω at which $\lvert G_o(j\omega)\rvert = 1.0$
ω_n	=	undamped natural frequency of n^{th} second-order operation
ζ	=	fixed damping ratio of second-order operation.

2. Introduction

In the course of manufacture it is common for each individual workpiece flowing along a section of production line to be subjected to a sequence of operations $G_o(s)$, $G_1(s)$ .... $G_n(s)$ .... $G_{N-1}(s)$, in being converted from its initial (rough) state to its final (finished) state. Examples of such sequential processes include the machining of precision components, the production of metal strip by rolling, etc. In the interests of standardisation, the individual operations $G_o(s)$,... $G_{N-1}(s)$ may be extremely similar to one another.

Because each operation may require a significant floor-space, the cost of which is ill-afforded in a depressed economic climate, there is considerable incentive to explore the possibility of minimising the total number N of such operations. Consequent time-saving is clearly an important additional consideration.

One important characteristic of the operation sequence is that the dynamics of each successive unit operation, whilst retaining the same basic form of frequency response (e.g. analogous modes having identical damping ratios), tend to become progressively faster as n increases from 0 to N-1. This is fairly readily appreciated in the case of metal rolling[(1),(2)] where the dynamics of each rolling operation are dominated by the transport-delay resulting from a fixed spacing, X, between the rolls and the output-thickness (gauge) sensor used for automatic gauge-control. Although X is fixed, due to the progressive lengthening of the metal strip with each rolling operation, the relative magnitude of the delay, viewed from any given vertical slice of material, appears to progressively shorten with each operation, i.e. as n increases. Furthermore, if rolling takes place repetitively at constant speed through the same (or identical) rolling stands, the fixed dynamics of unit rolling system (i.e. the complex spring/ mass network representing the roll-structure and the gauge-setting servo) will also appear to vibrate faster and faster when viewed from any chosen vertical slice of the metal strip: a phenomenon again resulting from the progressive lengthening of the strip.

In a sequence of similar machining operations[3], this progressive step-wise 'acceleration' of the dynamics of each successive operation may result from the use of progressively lighter-cutting-tools and their associated support-structures as the average thickness of metal removed is reduced as the final finishing cut is approached.

Unless machines of enormous mass and rigidity are employed, there will always exist some dynamic interaction between spatial profile $y_n(\ell)$ produced during operation n-1 and that, i.e. $y_{n+1}(\ell)$ produced on operation n, and it is this interaction which is here represented by transfer-function $G_n(s)$. The length coordinate, ℓ, will clearly lie within a finite range, i.e.

$$0 \leq \ell \leq L \tag{1}$$

in practical processes where L is the normalised* length of the workpiece but it is usual for L to greatly exceed the wave-length of the slowest mode of $G_n(s)$ so that, for practical purposes, $G_n(s)$, if stable may be regarded as a continuous process over all time. Thus, by taking Laplace transforms in s w.r.t. ℓ we get:

$$\tilde{y}_{n+1}(s) = G_n(s)\ \tilde{y}_n(s) \quad , \qquad o \leq n \leq N-1 \tag{2}$$

where superscript ~ denotes the Laplace transform of the associated spatial variable. The overall system may therefore be represented by the block-diagram of Fig. 1 which implies a single continuous process of transfer-function $G_o(s)\ G_1(s)\ G_2(s) \ldots\ldots G_n(s) \ldots . G_{N-1}(s)$. Such a representation is appropriate provided the boundary conditions can be engineered to induce

* In metal rolling, L would be the length of the strip prior to its first rolling operation and ℓ would be a normalised quantity related to real distance ℓ_n (of the slice in question after pass n) by the relation $\ell = \ell_n L/L_n$ where L_n is the strip-length after pass n.

no transients at the start of each subprocess or if attention is confined to process variables at distances far from either end of the workpiece.

3. Stability

Now although $G_n(s)$ may be stable for all values of n, the sequence of processes $G_o(s)$, $G_1(s)$.... $G_n(s)$.... $G_{N-1}(s)$ may, in a practical sense, constitute an unstable process overall. In particular, any impulse in $y_o(\ell)$ may generate oscillations in $y_1(\ell)$, and hence in $y_2(\ell)$,...$y_n(\ell)$.... $y_N(\ell)$, that grow in number and/or amplitude with increasing n: clearly an undesirable state of affairs.

3.1 Repetitive Systems of Constant (n-independent) Dynamics

In the special case $G_n(s) = G_{n+1}(s) = G(s)$, (n=0,1...,N-1) then, to avoid instability in the sense described above, G(s) must satisfy the condition that:

$$|G(j\omega)| < 1.0, \text{ for all real } \omega \tag{3}$$

This is because the spectral density of $y_n(\ell)$ is $|G^n(j\omega)|^2$ {in response to a unit impulse in $y_o(\ell)$} and for this to reduce, over the entire range of ω, with increasing n, criterion (3) must be satisfied. Fig. 2 illustrates a frequency response $|G(j\omega)|$ that would produce instability in the repetitive system $G^N(s)$ since within the frequency band

$$\omega_a \leq \omega \leq \omega_b \tag{4}$$

criterion 3 is clearly contravened.

In a special-case, delay-dominated metal-rolling example:

$$G(s) = k_2/\{1 + k_1\exp(-Xs)\} \tag{5}$$

where k_1, k_2 are constant gain parameters and X the constant normalised* delay distance, it is readily shown[1] that criterion (3) reduces to simply

$$k_1 < 1 - k_2 \tag{6}$$

* This example involves the rather hypothetical case of an increasing transport delay-distance, X_n, with each rolling operation, proportional to strip length L_n such that normalised delay $X = X_n L/L_n$, where $L = L_o$, the initial strip length.

The result is confirmed by the computed transient responses of Fig. 3 for $k_1 = 0.4$, $k_2 = 0.5$, i.e. a stable case and Fig. 4 for $k_1 = 0.75$, $k_2 = 0.5$, i.e. an unstable case.

Criterion (3) has been developed more fully elsewhere[(1),(2)] and interpreted in a variety of different ways. The spectral-density concept used above however is the most important in the present context of systematically varying processes $G_n(s)$, $n = 0,1,2....N-1$.

3.2 Systems of Accelerating Dynamics $G_n(s) = G_o(sr^n)$

If the unit process transfer-functions are interlated thus

$$G_n(s) = G_o(sr^n) \tag{7}$$

where r is a fixed positive parameter less than unity, then it is clear that all subprocesses of the repetitive system have modes that are identical in form (i.e. in damping ratio) but subject to a uniform increase in speed (i.e. in natural-frequency) with increase in n. The form of model (7) is particularly appropriate tometal rolling if r is the nominal ratio of output- to input-strip-thickness on each pass(= ratio imput- to output-strip-length if the strip-width remains unchanged). The reasons for this have been outlined in Section 2 and are more fully treated in reference (2). The form does have more general application in manufacturing systems, however, as has already been mentioned.

3.2.1 The Stability-Band Concept

Fig. 5 illustrates the frequency responses of the n individual sub-processes contributing the sequence $G_o(s), G_1(s)$ .... $G_{n-1}(s)$ when these are subject to condition (7). The spectra clearly spread and shift towards the higher frequency domain as n increases. Had r been unity then all the spectra would have been identical to $|G_o(j\omega)|$ and the repetitive sequence would have been unstable for the case illustrated since

$$|G_o(j\omega)| \geq 1.0 \quad , \omega_a \leq \omega \leq \omega_b \tag{8}$$

Because of the shifting operation resulting from $r < 1.0$, however, it is clear from Fig. 5 that, in response to a unit impulse in $y_o(\ell)$, the output signal's spectral-density, $S_n(\omega)$, from $G_n(s)$, given by:

$$S_n(\omega) = \prod_{i=o}^{n-1} |G_i(j\omega)|^2 \tag{9}$$

will increase with increasing n, within the narrowing frequency-band:

$$\omega' \leq \omega \leq \omega_b \quad , \quad \omega' > \omega_a \tag{10}$$

3.2.2 The Critical Number of Repetitions, n_c

The progressive increase of spectral-density at any frequency within the band

$$\omega_a \leq \omega \leq \omega_b \tag{11}$$

will cease, however, when n reaches a value such that

$$|G_n(j\omega_b)| \leq 1.0 \triangleq |G_o(j\omega_a)| \tag{12}$$

By combining this condition with equation (7) we can therefore obtain the critical value (n_c) for n beyond which no spectral-density, in the band $\omega_a \leq \omega \leq \omega_b$, will continue to increase. This value is thus obtained by setting

$$|G_o(j\omega_b r^n)| = |G_o(j\omega_a)| \tag{13}$$

$$\text{i.e., } \omega_b r^n = \omega_a \tag{14}$$

so that we get

$$\boxed{n_c = \frac{\log(\omega_b/\omega_a)}{\log(r^{-1})}} \tag{15}$$

So far as signals within frequency band (11) are concerned therefore, n_c, as calculated from equation 15, and rounded up to the nearest integer, provides a useful upper-bound on the number of repetitive subprocesses which will produce instability. Before this number of operations is complete, natural oscillations within band (11) may be expected to begin attenuation. Before contemplating higher frequency effects it is profitable, at this stage, to examine specimen simulation results.

4. Results and Discussion

The foregoing analysis has presupposed a process $G_o(s)$ having a single resonance peak. Such processes may well have high order but it is sensible to investigate first the simplest of such systems, namely a second-order-lag process:

$$G_o(s) = k\,\omega_o^2/(s^2 + 2\,\zeta\omega_o s + \omega_o^2) \tag{15}$$

so that

$$G_n(s) = k\,\omega_n^2/(s^2 + 2\,\zeta\omega_n s + \omega_n^2) \tag{16}$$

where $\omega_n r^n = \omega_o$ (17)

Fig. 6 shows the spatial response for $k = 0.65$, $\zeta = 0.3$, and $r = 0.9$, (for which $\omega_a = 0.52\,\omega_o$ and $\omega_b = 1.12\,\omega_o$) giving a calculated value for $n_c = 6.6$. Clearly the process stability deteriorates* only for the first 6-7 operations and n_c, as calculated, provides a good estimate of the number of operations for a stable output to be achieved. Experiments conducted on a wide variety of systems seem to indicate the general validity of this conclusion so that n_c has an important practical significance as well as being an easily calculated mathematical parameter. Fig. 8 confirms this finding for the metal-rolling example examined in Section 2 but here with r set to 0.95 rather than unity. We therefore have

$$G_n(s) = k_2/\{1 + k_1 \exp(-X\,s\,r^n)\} \tag{18}$$

from which we deduce that

$$\omega_a, \omega_b = X^{-1} \cos^{-1}\left[-\left\{\frac{(k_1^2 + 1 - k_2^2}{2k_1}\right\}\right] \tag{19}$$

and with $k_1 = 0.75$ and $k_2 = 0.5$ (as for Figs. 4 and 8) the values for these 'unit-gain frequencies' work out to be $\omega_a = 2.62X^{-1}$ and $\omega_b = 3.67X^{-1}$ so yielding a value for n_c (via equation 15) of 6.8. From observation of Fig. 8, stability clearly begins to improve after n = 3 or 4 and is again achieved for practical purposes by $n = n_c$.

* Deterioration here means the spreading of the distance overwhich oscillation of significant amplitude appears. Comparing Figs. 6 (r=0.9) and 7(r=1.0), clearly demonstrates the stabilising effect of accelerating dynamics: The instability in Fig.7 is manifested as much by the spread as by the amplitude of $y_n(\ell)$.

This example again demonstrates the practical usefulness of parameter n_c but now on a system that has a multiplicity of resonances : the numerical values of ω_a and ω_b calculated above applying strictly to only the first resonance peak. The results given in Fig. 8 clearly shows the shift of the system's spectra towards higher and higher frequencies but the relative amplitude of the high-frequency ripple that develops with increasing n is obviously small and of minor practical importance. Concentrating analysis on the first resonance peak would therefore seem to be no less valid than using describing-functions in general nonlinear systems analysis[(4)]. This conclusion is reinforced by the realisation that, in practice, low-pass filtering processes, (justifiably excluded in the mathematical-modelling of $G_o(s)$ because of their high bandwidths) are likely to occur in positions interposed in the process sequence $G_o(s)$, $G_1(s) \ldots. G_n(s)$ of Fig. 1. The effect of these filters would be to prevent the shift of system energy to ever higher frequency domains as n increases.

5. Conclusions

The stability of an important calss of repetitive processes encountered in metal rolling and manufacturing generally and described by a sequence of transfer-functions $G_o(s)$, $G_1(s), \ldots G_n(s), \ldots. G_{N-1}(s)$ has been investigated. The individual subprocesses are inter-related thus: $G_n(s) = G_o(sr^n)$, $0 < r < 1$, and the sequence may therefore be described as one of accelerating dynamics. It has been shown that, if the first resonance peak of $G_o(s)$ exceeds unity, within the frequency band $\omega_a \leq \omega \leq \omega_b$, the spectral-density of the output of $G_n(s)$ {in response to a unit-impulse applied to $G_o(s)$} will continue to increase at some frequencies within this band, as n increases, until n exceeds a critical value $n_c = \{\log(\omega_b/\omega_a)\}/\log(r^{-1})$. This value is therefore an upper bound on the number of operations for which instability persists in the sequence output, within the band $\omega_a \leq \omega \leq \omega_b$.

Simulation examples of single-and multiple-resonance systems have demonstrated good correlation between the easily-caculated parameter n_c and the number of operations actually needed for a stable output to be produced. The parameter would therefore seem to be of considerable practical importance, particularly in view of the economic need to keep the total number N of operations to minimum.

The theory's disregard for oscillation in higher-frequency bands has been shown by the simulation to be justifiable and in practice may be further excused by the existence of high-bandwidth low-pass filtering processes interposed between the subsystems. The approximation differs little from that underlying describing-function methods for nonlinear systems analysis.

6. References

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(4) Atherton, D.P. 'Stability of nonlinear systems', J. Wiley, (Research Studies Press), Letchworth, 1981, 231 pp.

7. Acknowledgements

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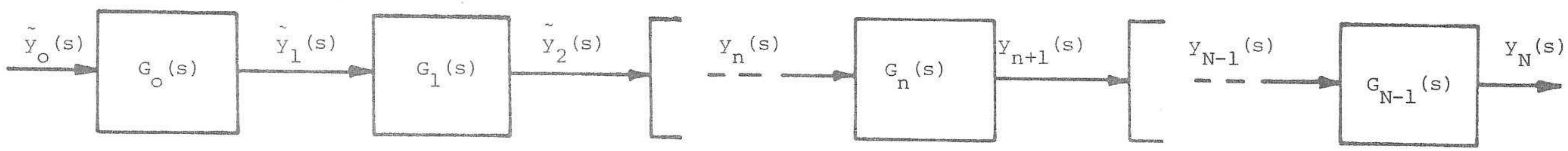


Fig. 1 Block-diagram representation of N-stage Repetitive Process

Fig. 2 Form of Frequency Response $|G(j\omega)|$, that would Produce Instability in N-Stage Sequence $G^N(s)$.

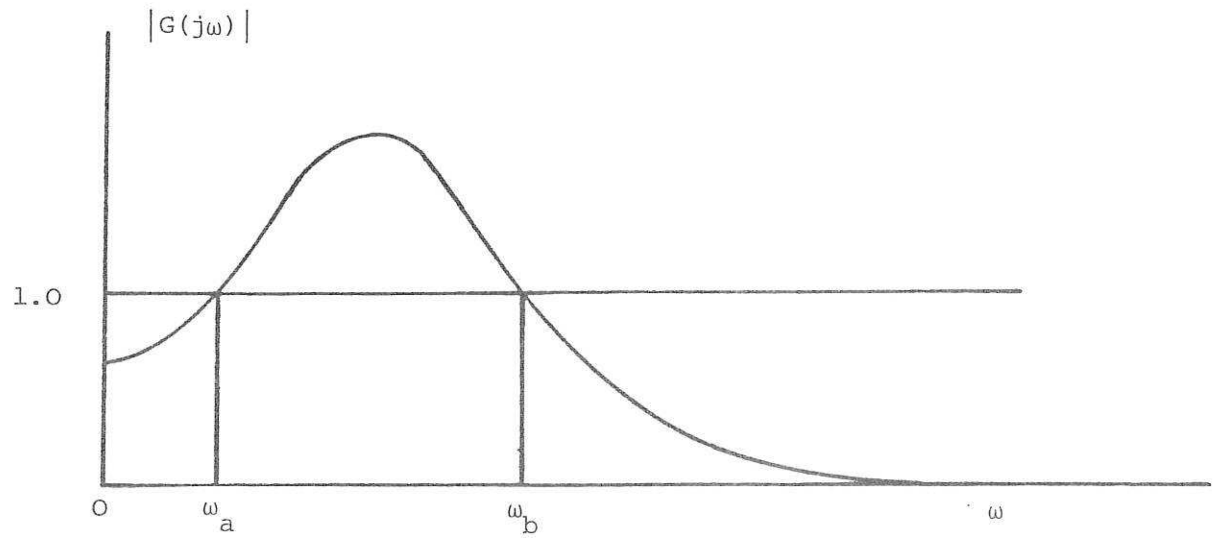


Fig. 3 Time-Response of Stable Metal-Rolling System with Fixed Normalised Delay. ($k_1 = 0.4$, $k_2 = 0.5$)

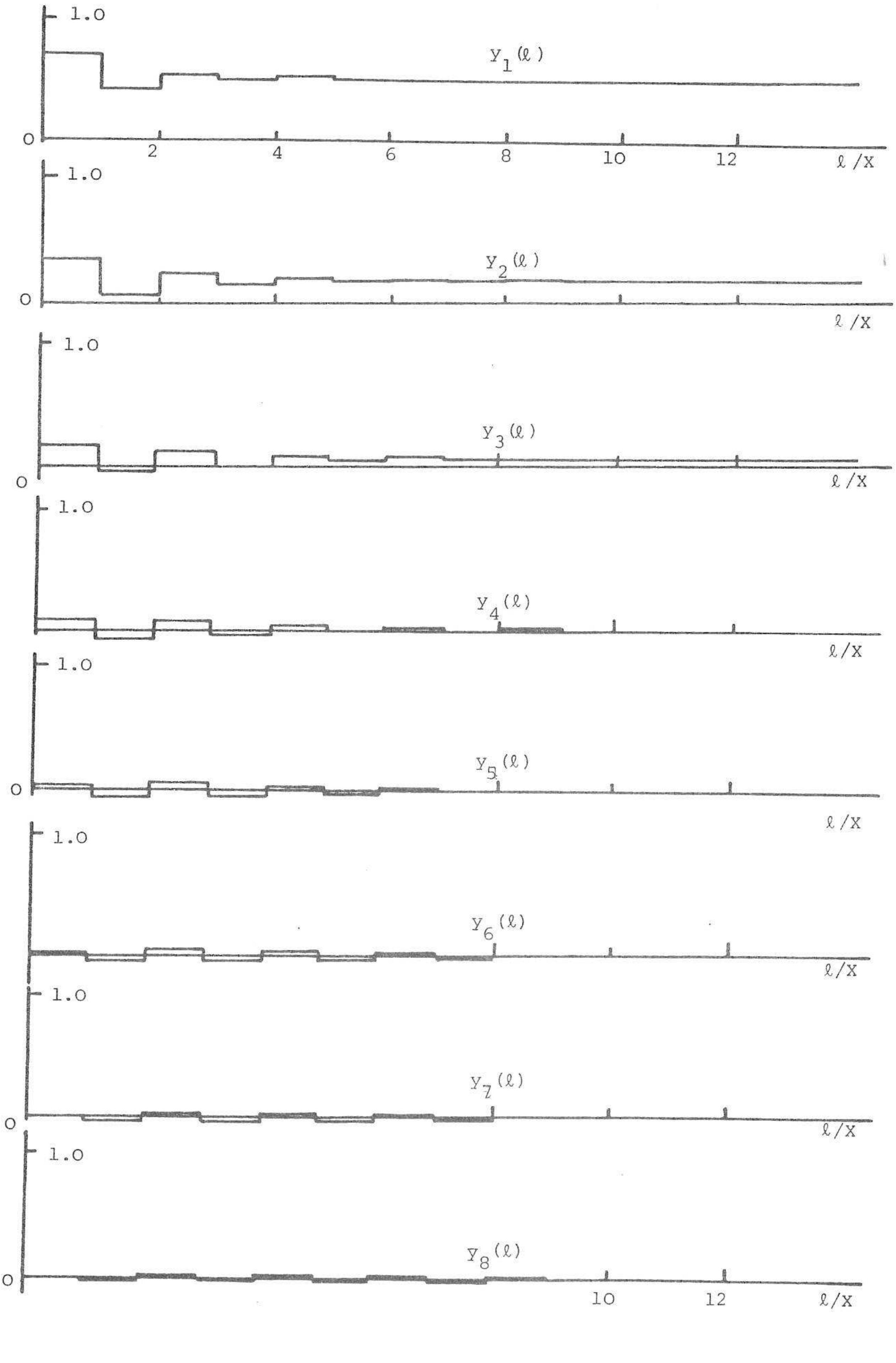


Fig. 4 Time-Response of Unstable Metal Rolling System with Fixed Normalised Delay (k_1=0.75,k_2=0.5)

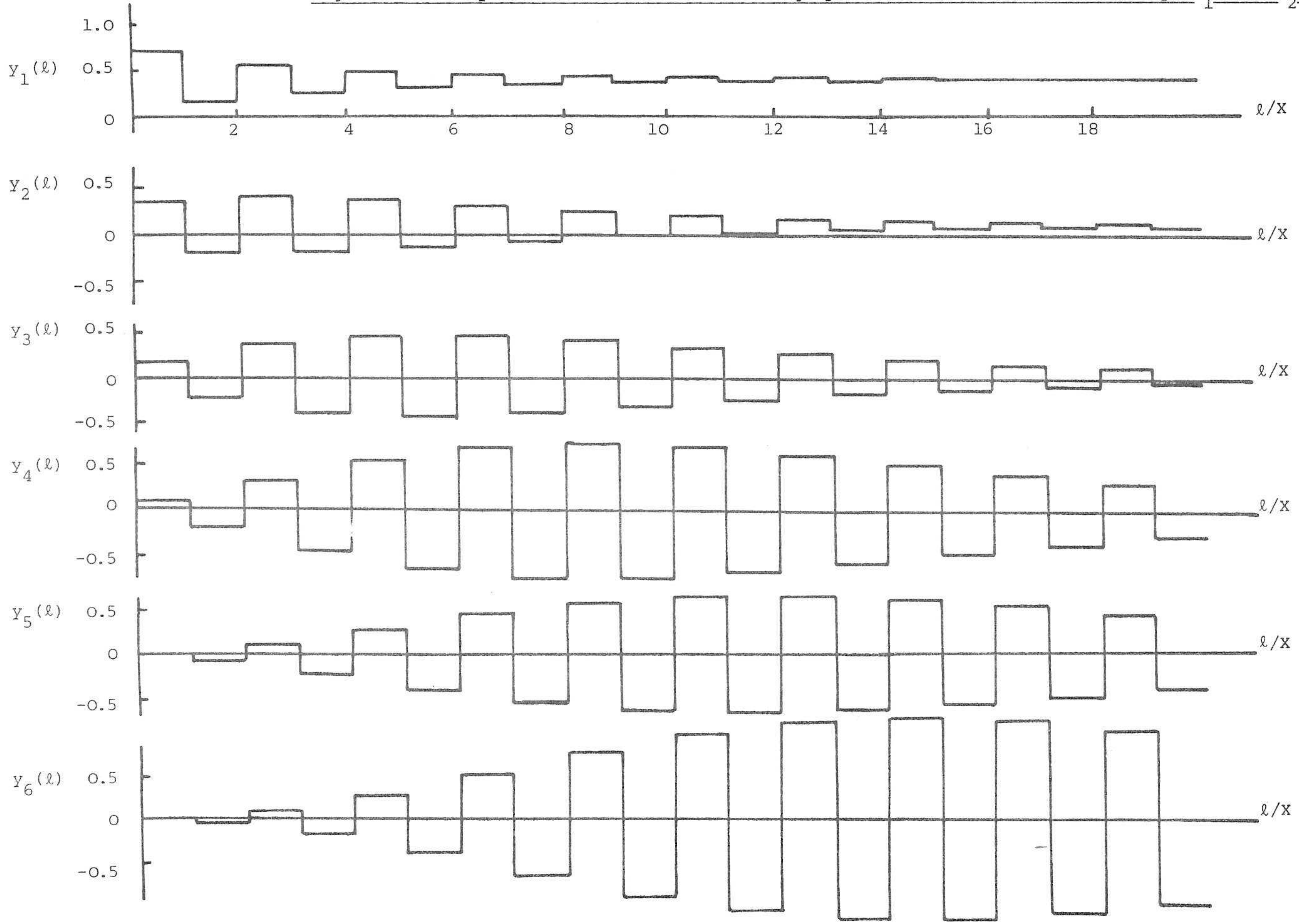


Fig. 5 Showing the Shifting And Spreading of the Spectrum of $G_{n-1}(s)$ as n increases

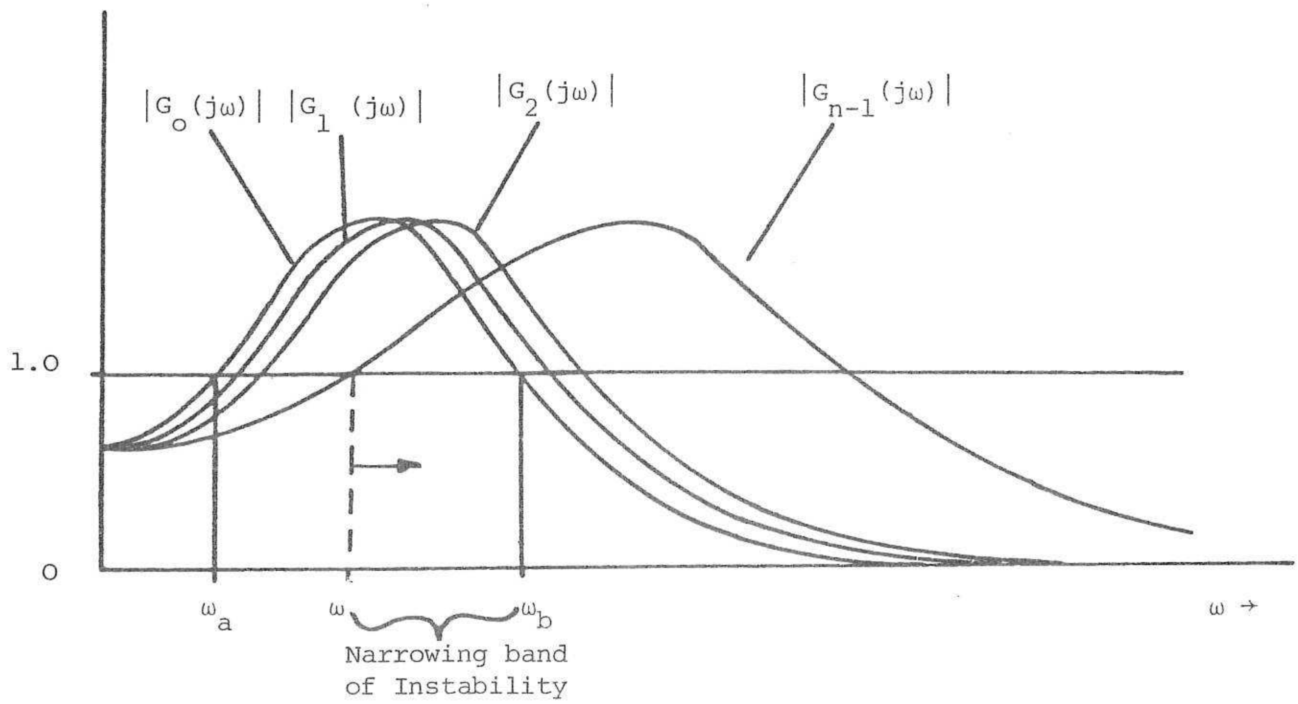
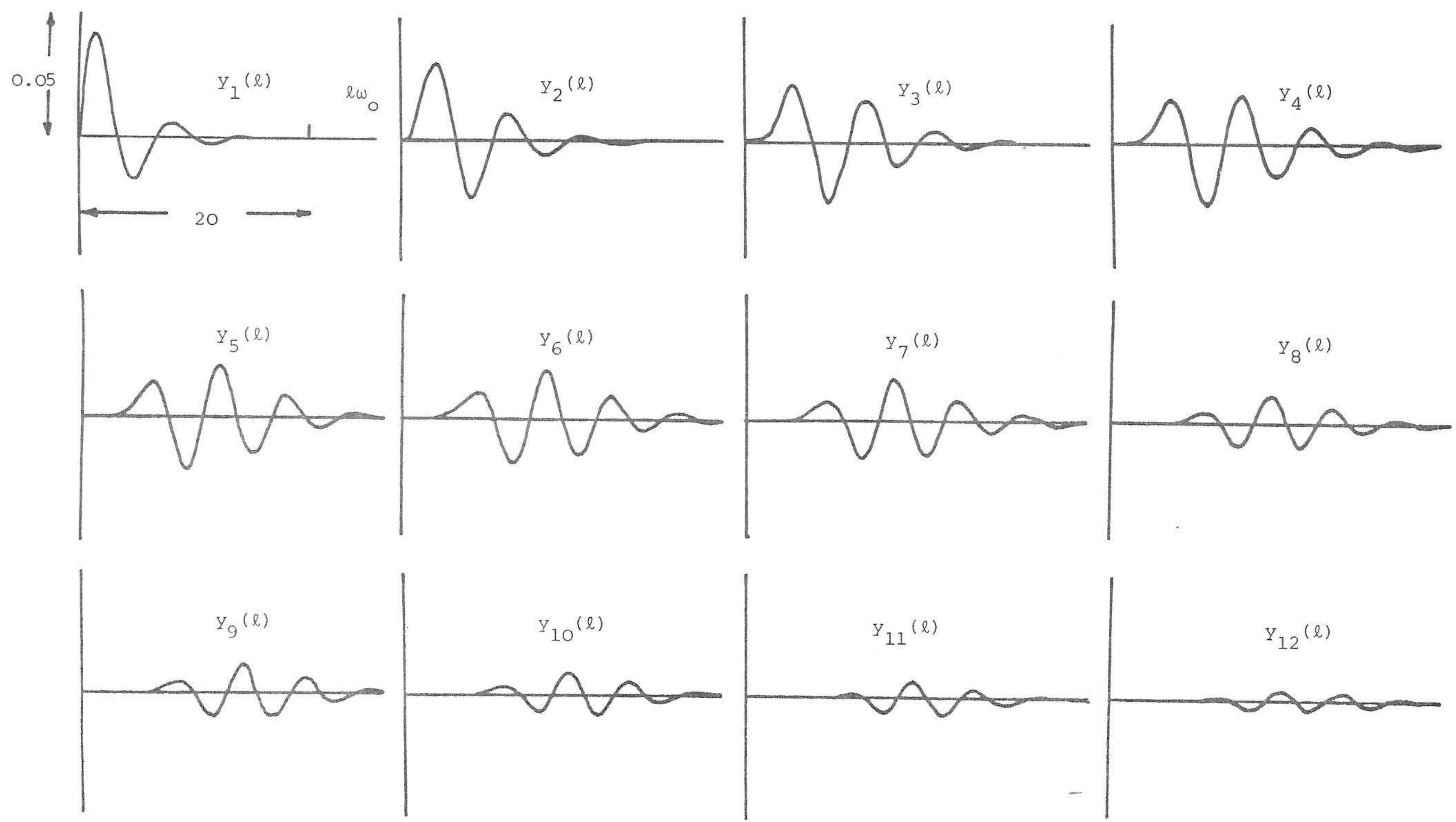


Fig. 6 Unit-Impulse Response of Repetitive Second-Order System with Accelerating Dynamics ($k=0.65, \zeta=0.3, r=0.9$)



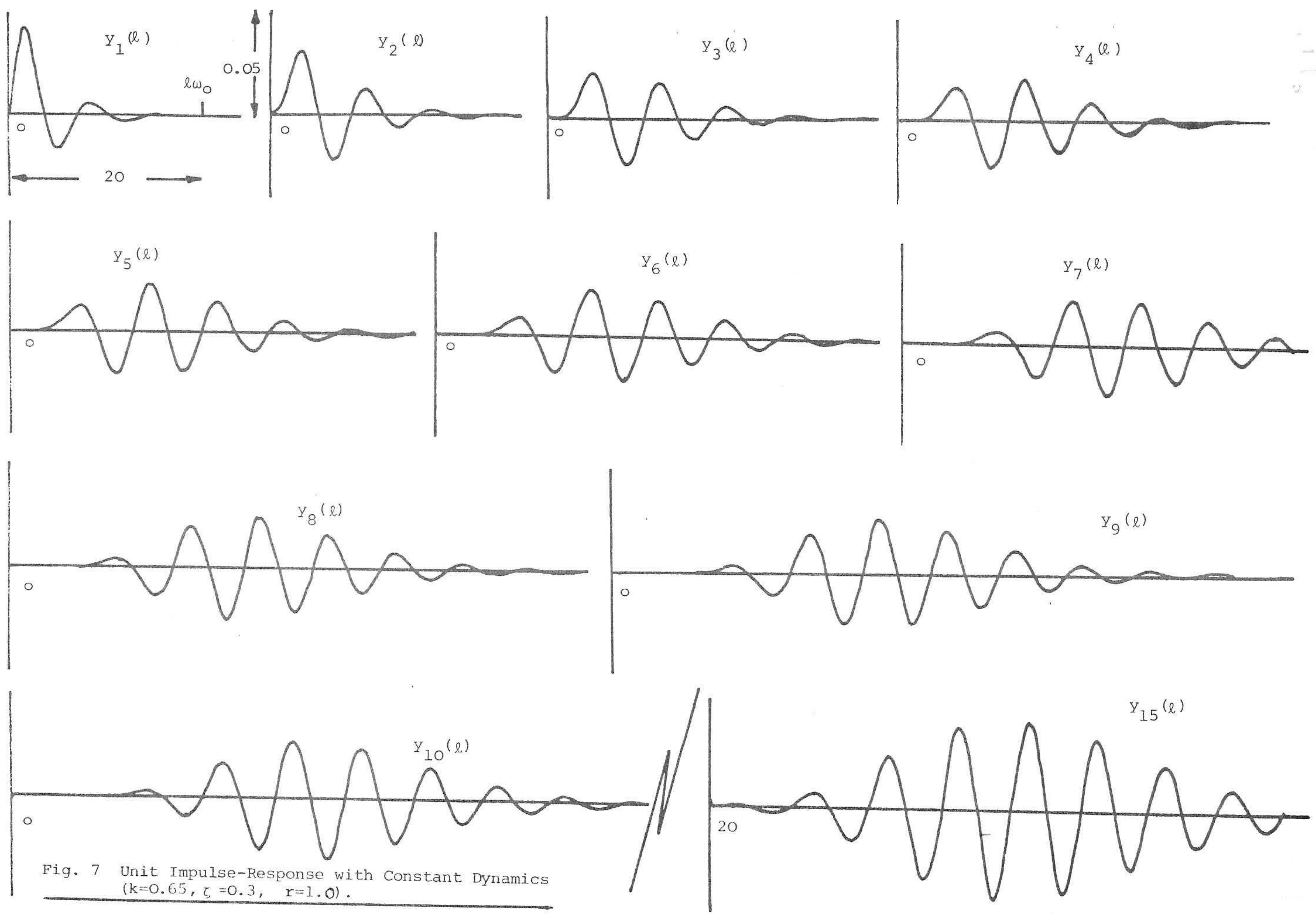


Fig. 7 Unit Impulse-Response with Constant Dynamics
($k=0.65$, $\zeta=0.3$, $r=1.0$).

Fig. 8 Response of Metal-Rolling System with Variable Normalised Delay ($k_1 = 0.75$, $k_2 = 0.5$)