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CONTROL SYSTEMS DESIGN FOR UNCERTAIN DYNAMICAL SYSTEMS

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APPLICATION OF MULTIVARIABLE SYSTEMS THEORY, OCTOBER 1982

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The use of approximate plant models for computer-aided controller design for multivariable systems with uncertain or unknown dynamic model is considered. The techniques use simple graphical analysis of plant and model step response data as the basis for a pseudo-classical inverse Nyquist stability criteria. Two illustrative examples are described.

INTRODUCTION

It would appear to be self-evident (see, for example, Rosenbrock (1), Owens (2), (6), MacFarlane (3),(4) and Harris and Owens (5)) that multivariable systems design theory has reached some degree of maturity in that, given an (assumed exact) system model in state-space or transfer function matrix form, a number of highly successful computer-aided-design methodologies exist for the design of stable, high-performance feedback systems. It is equally clear that it is commonly the case in practice that the plant model is subject to severe uncertainty or may even be unknown. This is not too much of a problem if the errors are small enough to be within the tolerances implicit in the standard model-based techniques of ensuring adequate gain and phase margins during the design. However, if the observed errors are large, it is not possible to make confident predictions about the stability and performance of the real implemented control scheme in terms of the dynamic characteristics of the plant model observed off-line. There is clearly therefore an incentive to obtain a fairly accurate plant model to reduce this problem to negligible proportions. Unfortunately, accurate plant models tend to be of fairly high dynamic order leading to an increase in the complexity of the design process (particularly in the multivariable case) that may be regarded as undesirable or unnecessary. In contrast, it is frequently possible to construct a low-order (but noticably inaccurate) model of plant dynamics that avoids these complications but leads to the vitally important problem of confidently assessing the stability and performance of the real implemented scheme in terms of the dynamics of the inaccurate model. It is of course possible to adopt the philosophical viewpoint that the inadequacies of even a crude model of plant dynamics can always be accommodated by final on-line tuning of the control scheme. There are however problems associated with this pragmatic approach as it tends, in general, to mean a reduction in control gains (and hence decreased performance) and could lead to difficulties in multivariable systems where on-line tuning is not the straightforward business that it is for single-input/single-output systems. Based on these (and other) considerations the authors believe that the use of approximate and possibly rough-and-ready models is a valuable possibility in control design but that it requires the development of a modified design methodology that explicitly includes observed dynamic differences in plant and model characteristics in the design process.

This paper is concerned with controller design when a plant model is not available in the sense that

(a) the plant model is not known but open-loop plant step responses are available from plant tests, or

(b) the plant model is known but is so complex that design calculations other than simulations are not regarded as feasible (or necessary) with available computing facilities.

In both cases the plant model is unknown from the designers viewpoint and the design must proceed using some other basis. The procedure followed here is described in Owens and Chotai (7) and is based on preliminary work reported by, for example, Edwards and Owens (8), Owens (9) and Owens and Chotai (10). In essence the following procedure is followed:

(i) construct an approximate model G_A of the real plant G by whatever means is available. It is not assumed that the modelling error $G-G_A$ is small, hence allowing the designer to reach his own compromise between accuracy and simplicity,

(ii) design a controller K for the model G_A to ensure that the approximate feedback scheme illustrated in Fig.1(a) is stable with the desired stability and performance characteristics, and finally

(iii) provide easily checked conditions that ensure that the resultant controller stabilizes the real plant G in the real, implemented configuration of Fig.1(b).

(Note: unity feedback is assumed but the incorporation of measurement dynamics is easily accomplished as described in ref.(7)).

Steps (i) and (ii) are standard parts of design off-line whereas step (iii) is a new addition introduced to guarantee the successful implementation of a stable control system despite the known plant/model differences. The details are described in the following sections together with illustrative examples of applications to multivariable process plant.

Finally, we observe that this field of 'non-adaptive unknown systems control' has only recently been identified as a theoretically feasible proposition with contributions from Davison (11), Pentinnen and Koivo (12), Porter (13), Astrom (4) and Owens and Chotai (7), (10), (15), (16) and is continuing to develop at a rapid rate.

STABILITY THEORY FOR UNCERTAIN SYSTEMS

It is the purpose of this section to outline and illustrate by applications the main results obtained in ref.(7) in the form of a computer-aided-design technique. The details of the theory can be found in ref.(7) and its generalization in references (15) and (16).

Stability Theory for Uncertain Continuous Plant

Suppose that the plant has ℓ-inputs, m-outputs and is linear, time-invariant with $m\times\ell$ strictly proper transfer function matrix G(s). The plant approximate model to be used for controller design is supposed to have $m\times\ell$ strictly proper transfer function matrix $G_A(s)$. The forward path controller has $\ell\times m$ proper transfer function matrix K(s) which is assumed to be designed by any means available to ensure the stability and acceptable performance characteristics of the approximating feedback system of Fig.1(a). The basic theoretical problem outlined in the introduction is the derivation of conditions on G_A, K and G which ensure that the stability of Fig.1(a) implies the stability of Fig.1(b). The answer to the problem clearly depends upon the modelling error $G(s)-G_A(s)$ but this is by assumption unknown due to the uncertainty in our knowledge of the plant G(s)! However, even though G(s) may not be known in detail it is frequently the case that the plant step response matrix

$$Y(t) = \begin{pmatrix} Y_{11}(t) & \cdots & Y_{1\ell}(t) \\ \vdots & & \vdots \\ Y_{m1}(t) & \cdots & Y_{m\ell}(t) \end{pmatrix} \qquad .....(1)$$

is known from plant trials or complex model simulations. Here, $Y_{ij}(t)$ is the system response from zero initial conditions of the i^{th} output $y_i(t)$ to a unit step in the j^{th} input $u_j(t)$ with all other inputs held to zero. Clearly Y(t) could be used (by curve fitting or other identification methods) as the basic data to fit a 'best' approximate model G_A to the real plant G but we will take the broader view that observations of Y(t) lead to a 'convenient' model G_A but the errors are not necessarily small. Given a choice of G_A, however, simulation methods will then yield its step response matrix $Y_A(t)$ and the 'modelling error matrix'

$$E(t) = Y(t) - Y_A(t) \qquad .....(2)$$

is easily computed by computing elements $E_{ij}(t) = Y_{ij}(t) - (Y_A(t))_{ij}$.

The error E contains all possible information on $G-G_A$ but it is important to extract only sufficient information to solve our problem and to extract it in a convenient (and preferably graphical form). The general case (including the effect of measurement nonlinearities) is described in references (15) and (16). For simplicity, however, we restrict attention to the special case when the modelling error $G-G_A$ is monotone (see ref (7)).

Definition: An m-output/ℓ-input, strictly proper, linear system is monotonic if, for all (i,j), the response of the i^{th} output from zero initial conditions to a unit step in the j^{th} input is either monotonically increasing or monotonically decreasing.

(Note: the definition used in ref (7) extends to sign-definite systems but this generalization is not needed here).

An important practical point is that the monotonicity of $G-G_A$ can be observed by visual inspection of the time-variation of the elements $E_{ij}(t)$ of E(t).

The important consequence of monotonicity is that frequency domain characteristics of the modelling error can be bounded in terms of its steady-state characteristics observed in the time domain ie.

Proposition (see ref (7)): If $E_{ij}(t)$ is monotonically increasing or decreasing and bounded, then

$$|G_{ij}(i\omega) - (G_A(i\omega))_{ij}| \leq E_{ij}(\infty) \qquad .....(3)$$

An important consequence of this result is the following theorem proved in ref (7).

Theorem 1: If the unknown plant G is approximated by a model G_A with the properties that $G-G_A$ is stable and monotonic and the controller K is designed to stabilize the model G_A in the configuration shown in Fig.1(a), then the same controller will also stabilize the real unknown plant G in the implemented scheme of Fig.1(b) if

(i) KG is controllable and observable, and

(ii) the inequality

$$\lambda \triangleq \sup_{s \in D} \max_{1 \leq i \leq \ell} \sum_{j=1}^{\ell} \sum_{k=1}^{m} |((I_m + K(s)G_A(s))^{-1}K(s))_{ik}| \cdot |E_{kj}(\infty)| < 1 \qquad .....(4)$$

is satisfied where D is the usual Nyquist contour in the complex plane.

The result has the following natural design interpretation:

Step 1: Obtain the plant step response $Y_{ij}(t)$, Step 2: Choose an approximate model G_A such that the errors $E_{ij}(t)$ are all monotonic (see ref (7) for some general techniques of achieving this objective).

Step 3: Design the controller K for the model G_A to obtain the required stability and performance characteristics for the approximating feedback scheme of Fig.1(a),

Step 4: Check the inequality in equation (4) by numerical or graphical means,

Step 5: Check that KG is controllable and observable.

Steps 1-3 are easily undertaken as they are simply a special case of the classical design process. It is steps 4 and 5 that enable the designer to check stability of the implemented scheme despite the uncertainty in the plant G! Step 4 does not, in the general form given, have a classical frequency domain structure but presents no real problem as it only requires knowledge of G_A, K and the steady state modelling error $E(\infty)$. Step 5 could present a problem in principle as it requires structural knowledge of G but, as observed in ref (7), its validity can very often be deduced from physical considerations or, as controllability and observability is generic, its validity could be assumed until the unlikely event of it being proved otherwise.

Rather than dwell on the general case above, we will focus our attention on an important special case that covers single-input/single-output systems and has the flavour of the inverse Nyquist array (refs (1)-(3)) by choosing models and controllers of diagonal (ie. non-interacting) structure. In such a case, theorem 1 has the following form (see ref (7)):

Theorem 2: The conclusions of theorem 1 remain valid with the extra constraints that $m = \ell$ and G_A and K are diagonal when (4) is replaced by the equivalent conditions

$$\lim_{|s|\to\infty} |K_{kk}(s)| < \frac{1}{\sum_{j=1}^{m} |E_{kj}(\infty)|} , \quad 1 \le k \le m \qquad \ldots\ldots(5)$$

and

$$|1 + ((G_A(s))_{kk} K_{kk}(s))^{-1}| > d_k(s)$$

$$1 \le k \le m , \quad \forall \; s \in D_I \qquad \ldots\ldots(6)$$

where D_I is the positive imaginary axis component of D and the 'confidence radius'

$$d_k(s) \triangleq |(G_A(s))_{kk}^{-1}| \sum_{j=1}^{m} |E_{kj}(\infty)|, \; 1 \le k \le m \qquad \ldots\ldots(7)$$

Conditions (5) and (6) have a much nicer graphical interpretation than (4). Condition (5) provides a preliminary bound on the proportional component of the loop gains whilst (6) can be checked by plotting the standard inverse Nyquist locus Γ_k of $(G_A)_{kk}K_{kk}$ and superimposing 'confidence circles' (expressing the confidence we can have in the predictions of the model G_A) at each frequency point of radius $d_k(s)$. This procedure is illustrated in Fig.2. Note that (6) is satisfied if, and only if, the (-1,0) point does not lie in the confidence band generated by the confidence circles.

Noting that the stability of Fig.1(a) can be undertaken by classical inverse Nyquist analysis of Γ_k, it is clear that the result has an identical structure to the multivariable inverse Nyquist array technique with the Gershgorin circles being replaced by confidence circles reflecting our uncertainty in G. In this sense, the result is a generalization of the INA to systems with uncertain dynamics. Note however in the form stated, the result requires that $G-G_A$ must be monotonic which implies, in particular, that $E_{ij}(t) = Y_{ij}(t)$ must be monotonic for $i \neq j$ ie. the interaction effects in G must be monotonic. This constraint is removed in references (15) and (16). An example of the application of these ideas is described below.

Example: To illustrate the ideas outlined above consider the simple liquid storage system illustrated in Fig.3 with input flows u_1 and u_2, states x_i equal to the deviation of the liquid level in vessel i from a known equilibrium position and outputs $y_1 = x_1, y_2 = x_3$. Each vessel is assumed to be cylindrical with cross-section areas a_i and the pipework is assumed to have linear pressure/flow characteristics in the area of interest characterized by a resistance parameter. The state-variable model of the process has the structure

$$\dot{x}(t) = \begin{pmatrix} -0.5 & 0.17 & 0.0 \\ 0.25 & -1.75 & 1.0 \\ 0.0 & 2.0 & -3.0 \end{pmatrix} x(t) + \begin{pmatrix} 0.33 & 0.0 \\ 0.0 & 0.5 \\ 0.0 & 1.0 \end{pmatrix} u(t)$$

$$y(t) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} x(t) \qquad .....(8)$$

and, with the data shown, the open-loop step responses shown in Fig.4. Note that we will only need the step responses during the design and hence a knowledge of the model structure and data is not required for completion of a successful design. We will however use the known model to check closed-loop characteristics.

Examination of Fig.4 indicates that all responses $Y_{ij}(t)$ are monotonic and hence that we can obtain a monotonic modelling error by ignoring interaction terms and modelling diagonal terms accurately. More precisely, we will use an approximate model of the form

$$G_A(s) = \begin{pmatrix} g_1(s) & 0 \\ 0 & g_2(s) \end{pmatrix} \qquad .....(9)$$

where $g_i(s)$ is an accurate model of the i^{th} diagonal term of G(s). The g_i can be computed from G(s) if it is known or by model fitting to satisfy the equation

$$Y_{ii}(t) = \mathcal{L}^{-1}\{g_i(s)\frac{1}{s}\} \quad , \quad i = 1,2 \qquad .....(10)$$

These considerations lead to the choice of

$$g_1(s) = \frac{0.33s^2 + 1.568s + 1.07}{s^3 + 5.25s^2 + 5.58s + 1.50}$$

$$g_2(s) = \frac{s^2 + 3.25s + 1.33}{s^3 + 5.25s^2 + 5.58s + 1.50} \qquad .....(11)$$

and the steady state modelling error matrix

$$E(\infty) = \begin{pmatrix} 0 & 0.284 \\ 0.11 & 0 \end{pmatrix} \qquad .....(12)$$

The magnitude of the interaction terms Y_{12} and Y_{21} in Fig.4 are less than 30% of the diagonal terms in transient magnitude. We can therefore expect, on intuitive grounds, that a diagonal model will be successful and, of course, the benefits in the design of K will be enormous.

The diagonal structure of G_A suggests that the controller K can be chosen to be diagonal,

$$K(s) = \begin{pmatrix} k_1(s) & 0 \\ 0 & k_2(s) \end{pmatrix} \qquad .....(13)$$

where, assuming PI control, the loop transfer functions have the structure

$$k_i(s) = k_i\left(1 + \frac{1}{T_i s}\right) \quad , \quad i = 1,2 \qquad .....(14)$$

The stabilization of Fig.1(a) boils down to ensuring that the scalar feedback systems $g_i k_i/(1+g_i k_i)$, $i = 1,2$, are stable. Choosing the data $k_1 = 2.5$, $k_2 = 2.5$, $T_1 = 1.67$, $T_2 = 1.67$, we can now check the stability of the designed controller when implemented on the real plant described by equation (8) by checking conditions (5) and (6) of theorem 2. Condition 5 boils down to

$$|k_1| < 3.52 \quad , \quad |k_2| < 9.09 \qquad .....(15)$$

which are clearly satisfied whilst condition (6) is checked by plotting the confidence bands as shown in Fig.5. As the (-1,0) point does not lie in either of the confidence bands we conclude that all the conditions of theorem 2 are satisfied if KG is both controllable and observable. This last condition could be checked if the plant model is available or assumed if unavailable. In either case we conclude that the real plant will be stable with the designed controller. This is verified by examination of the closed-loop step responses shown in Fig.6.

Stability Theory for Uncertain Discrete Plant

All of the ideas outlined above carry over with trivial modifications to the case of ℓ-input/m-output, linear, time-invariant discrete systems with synchronous input actuation and output sampling (see refs (7), (15) and (16) for details). The only real changes are that continuous time data such as $Y(t)$ is replaced by its sampled-data counter-part $Y(k)$, transfer function matrices are replaced by z-transfer function matrices and the Nyquist contour D is replaced by the two circles $|s| = 1$ and $|s| = R$ with R 'very large'. In fact the basic theorem 1 for the continuous case carries over to the discrete case with these simple modifications and theorem 2 also carries over with D_I replaced by the unit semi-circle $\{s : |s| = 1, \text{Im } s \leq 0\}$. The details can be found in the references.

Example: To illustrate the application of the theory to the discrete case, we consider the four-input/four-output boiler furnace introduced in ref (1) described by the transfer function matrix

$$G(s) = \begin{pmatrix} \frac{1.0}{1+4s} & \frac{0.7}{1+5s} & \frac{0.3}{1+5s} & \frac{0.2}{1+5s} \\ \frac{0.6}{1+5s} & \frac{1.0}{1+4s} & \frac{0.4}{1+5s} & \frac{0.35}{1+5s} \\ \frac{0.35}{1+5s} & \frac{0.4}{1+5s} & \frac{1.0}{1+4s} & \frac{0.6}{1+5s} \\ \frac{0.2}{1+5s} & \frac{0.3}{1+5s} & \frac{0.7}{1+5s} & \frac{1.0}{1+4s} \end{pmatrix} \qquad .....(16)$$

The interaction effects are clearly monotonic in continuous (and hence discrete) time. Suppose that a sampling interval of $h = 0.4$ is used and that, despite the large interaction effects, the approximate model is chosen to be diagonal and to model the (sampled) diagonal terms exactly. The relevant model clearly has z-transfer function matrix

$$G_A(z) = \frac{0.1}{z-0.9} I_4 \qquad .....(17)$$

(where I_4 is the 4x4 unit matrix) and hence

$$E(\infty) = \begin{pmatrix} 0 & 0.7 & 0.3 & 0.2 \\ 0.6 & 0 & 0.4 & 0.35 \\ 0.35 & 0.4 & 0 & 0.6 \\ 0.2 & 0.3 & 0.7 & 0 \end{pmatrix} \qquad .....(18)$$

Considering initially proportional control only with diagonal z-transfer function matrix $K(z) = k\, I_4$ $(k>0)$ reflecting the identical loop structure of G_A, it is clear that K stabilizes G_A over the fairly wide gain range of $0<k<19$. This may be misleading as a prediction of the behaviour of the real plant in the presence of such a controller however as, applying the discrete version of theorem 2, we can only guarantee stability of this configuration if (equation (5))

$$k < 1/\max_i \sum_{j=1}^{4} |E_{ij}| = 0.74 \qquad .....(18)$$

and (equation (6))

$$|1 + (10z-9)k^{-1}| > |10z-9| \max_i \sum_{j=1}^{4} |E_{ij}|$$
$$= |10z-9|\ 1.35 \qquad .....(19)$$

whenever $|z| = 1$ ie (after a little graphical analysis)

$$k < 0.71 \qquad .....(20)$$

This last constraint is clearly the dominant one so we conclude that the controller will stabilize the real plant in the configuration of Fig.1(b) if the loop gain $k < 0.71$ and KG (and hence G) is controllable and observable.

Although the approximate model has provided a range of loop gains that guarantee stability, simulations indicate that the closed-loop system is rather sluggish with large steady state errors and interaction effects. This situation is due in the main to the rather crude model G_A employed and can be improved, if required, by using a more accurate representation. Suppose however that it is decided that the disappointing performance is to be improved by using the same model but introducing integral action into the controller ie $K(z) = (k_1+k_2z/(z-1))I_4$ with $k_2 \neq 0$. Equation (5) again yields $|k_1| < 0.74$ but equation (6) requires, in particular, that $\max_i d_i(1) < 1$. This second inequality cannot be satisfied as $(G_A(1))_{ii} = 1$ for all i and $\max_i \sum_{j=1}^{4} |E_{ij}| = 1.35$. We conclude from this analysis that the diagonal approximate model of (17) is also not accurate enough to provide a basis for the design of integral controllers using the discrete version of theorem 2. Clearly, a more accurate model G_A is required for this system. Although this will most probably increase the complexity of the design of K for G_A the principles of the design technique remain unchanged except that the discrete form of theorem 1 rather than theorem 2 must be invoked. The details are omitted for brevity.

CONCLUSIONS

The paper has outlined the principles underlying a new approach to controller design in the presence of uncertainty by deliberate use of an approximate plant model for the purposes of controller design and the incorporation of observed differences in plant/model time-response characteristics into the design process using graphical techniques similar to the inverse Nyquist array. The details of the theory are given in refs (7), (15) and (16) and, at no stage in the design process, is an accurate plant model required. The techniques have been illustrated with two examples of simple-process plant by attempting design based on a diagonal approximate model. The advantages of a diagonal model are obvious and its use enables, in one case, the systematic design of loop controllers for a two-input/two-output system. It cannot be expected however that a diagonal model will always be suitable. This was demonstrated by a 4-input/4-output boiler furnace system where a diagonal model enabled the design of a rather conservative 'low-gain' proportional controller but was not suitable for high-performance design or the design of PI controls. A more accurate (non-diagonal) model is needed in this case. The theory can however easily cope with such cases.

ACKNOWLEDGEMENT

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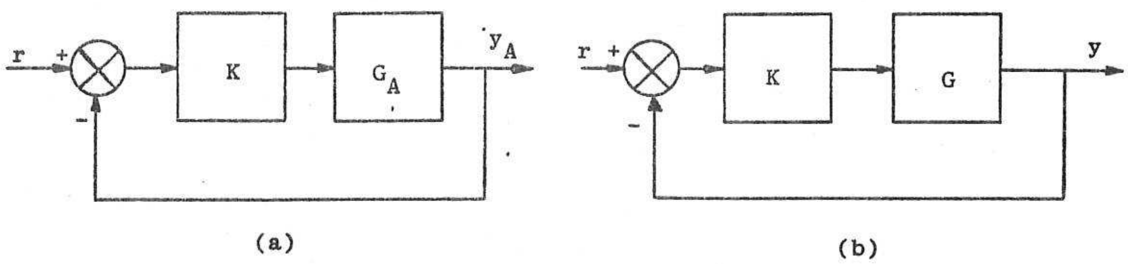

Fig. 1 (a) Approximate and (b) real feedback schemes

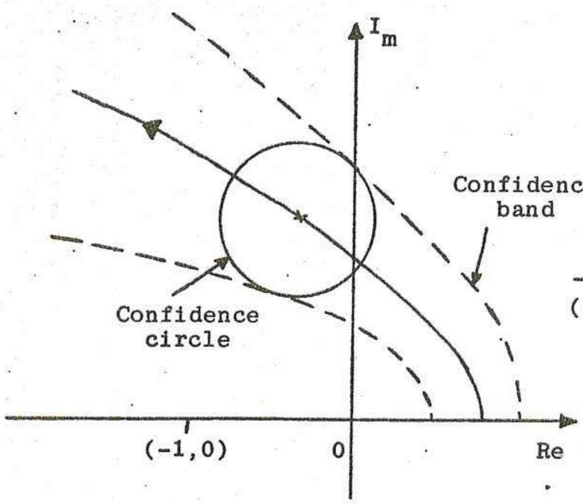

Fig. 2 Stability and the confidence band

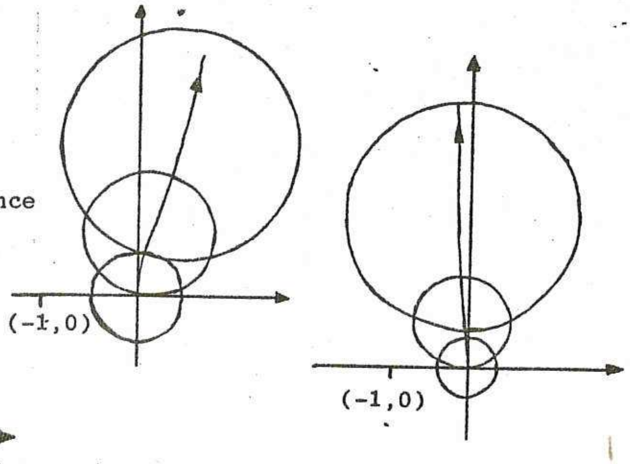

Fig. 5 Confidence bands for liquid level system

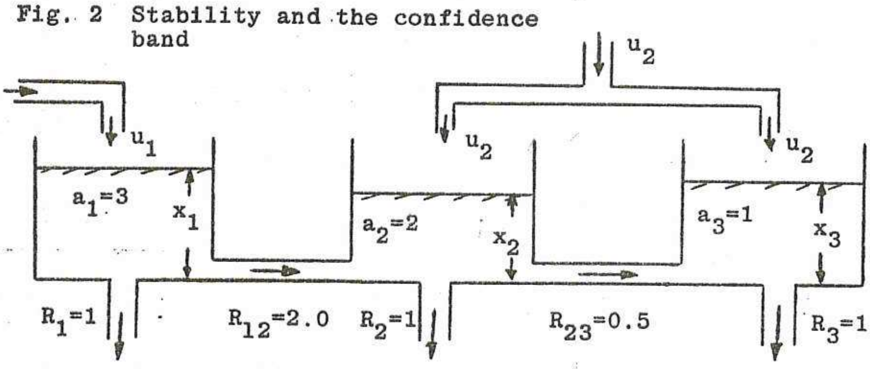

Fig. 3 Liquid level system

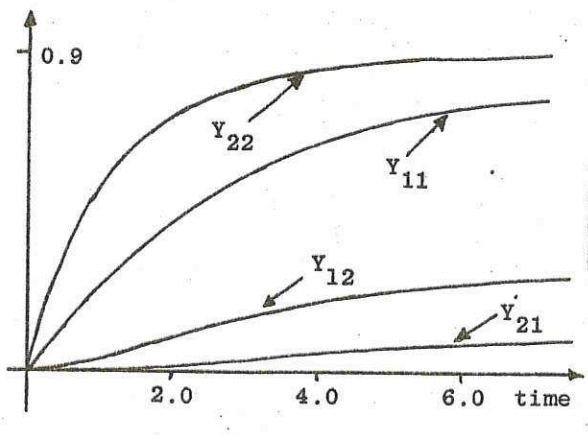

Fig. 4 Open-loop step responses for the liquid level system

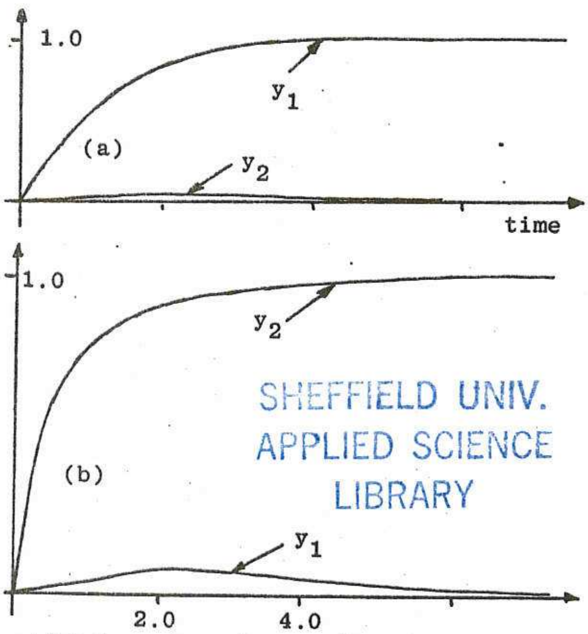

Fig. 6 Closed-loop responses to a unit step demand in output (a) one and (b) two