Monograph:
Edwards, J.B. and Guilandoust, M (1981) The Influence of Vapour Capacitance on the Composition Dynamics of Packed Distillation Columns. Research Report. ACSE Report 166 . Department of Control Engineering, University of Sheffield, Mappin Street, Sheffield

THE INFLUENCE OF VAPOUR CAPACITANCE ON THE COMPOSITION DYNAMICS OF PACKED DISTILLATION COLUMNS

by

Edwards, J.B. and Guilandoust, M.

Department of Control Engineering,
University of Sheffield,
Mappin Street, Sheffield S1 3JD.

Research Report No. 166

November 1981

Summary

A general transfer-function matrix (T.F.M.) expression is derived for the composition dynamics of ideal packed columns that are statically symmetrical but are dynamically asymmetric: i.e. having a nonunity vapour/liquid capacitance ratio, c. For the special case of c = o, the system T.F.M. is derived completely analytically to produce a parametric T.F.M. resembling that for c = 1 previously derived[1,2,3] by the authors. The result is successfully tested at high-and zero-frequencies against the initial and final values of the system step-responses determined (a) analytically and (b) by time-domain simulation. Complete inverse Nyquist diagrams are presented and compared with the c = 1 case.

It is noted that (whereas for c = 1, the system response, for long columns, is nonminimum phase), for c = o, the system behaves in a non-strictly-proper fashion with an abrupt initial negative response of magnitude similar to the nonminimum phase dip produced when c = 1.

We conclude that the variation of c does not greatly influence the general response characteristics of packed columns.

THE INFLUENCE OF VAPOUR CAPACITANCE ON THE COMPOSITION DYNAMICS OF PACKED DISTILLATION COLUMNS

Edwards, J.B. and Guilandoust, M.

1. Introduction

Previous analyses[1,2,3] have produced precise, parametric transfer-function matrices for ideal packed columns that are symmetrical statically and dynamically. Static symmetry demands equality of the geometry and packing characteristics of the rectifier and stripper and nominal vapour and reflux flow rates chosen to produce even loading throughout the column. An equilibrium curve, symmetrical about the -45° line and mixed liquid/vapour feed compositions located at the intersection of these two lines are also prerequisites for static symmetry. None of these assumptions are grossly unreal and indeed represent ideal operating conditions. They have frequently been used explicitly or implicitly in analytical studies by previous workers[4,5,6,7]

Dynamic symmetry implies equality of terminal vessel capacitances, which is not too serious a restriction, and, more importantly, equality of vapour- and liquid-capacitances in rectifier and stripper (and vice versa). Whereas these conditions can be approached[8] (or even attained) in real columns, the assumption was made by the present authors in the interests of mathematical tractability of the system T.F.M. The assumption permits the ready diagonalisation of the system matrices and this eases analytical solution very considerably. The model predicts non-minimum-phase behaviour (- a novel result for distillation columns) although most other aspects of behaviour predicted agree closely with the findings of other researchers: approximate - analytical and empirical.

The present report outlines an approach to T.F.M. calculation from the system partial differential equations (p.d.e's) that reduces the labour of calculation for any given system. With the aid of this method, it has been possible to calculate the T.F.M. for zero vapour-capacitance in the column. This is clearly a condition far-removed from the equal capacitance assumption and is closer to the condition at which most columns actually operate. (In tray-type column analysis, vapour-capacitance is usually set to zero). The results show that either assumption leads to substantially similar transient response predictions so demonstrating the excellent robustness of the model previously derived by the authors.

2. General T.F.M. description

Previous reports[1,2] have shown that the composition dynamics of a statically-symmetrical column may be described, after double Laplace transformation, as follows:

$$\underline{\tilde{Q}}^{-1}(s) \begin{pmatrix} \approx \\ y \\ \approx \\ y_e \end{pmatrix} + \begin{pmatrix} \tilde{y}(o) \\ \tilde{y}_e(o) \end{pmatrix} + \frac{1}{s} \begin{pmatrix} 1 & 0 \\ 0 & \alpha \end{pmatrix} \begin{pmatrix} \tilde{z}_1 \\ \tilde{z}_2 \end{pmatrix} = 0 \qquad (1)$$

$$\underline{\tilde{Q}}^{-1}(-s) \begin{pmatrix} \tilde{\tilde{x}}'_e \\ \tilde{\tilde{x}}' \end{pmatrix} - \begin{pmatrix} x'_e(o) \\ x'(o) \end{pmatrix} + \frac{1}{s} \begin{pmatrix} \alpha & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \tilde{z}_1 \\ \tilde{z}_2 \end{pmatrix} = 0 \qquad (2)$$

where $$\underline{\tilde{Q}}^{-1}(s) = \begin{pmatrix} 1 + cp - s , & -1 \\ 1 , & -(1 + p + s) \end{pmatrix} \qquad (3)$$

where y and x' denote small disturbances in vapour and liquid composition in the rectifier and stripper respectively, y_e and x_e' their associated equilibrium values, z_1 and z_2 are related to perturbations v and ℓ in vapour and reflux rates thus

$$\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \frac{G}{V_r} \begin{pmatrix} v \\ \ell \end{pmatrix} \qquad (4)$$

where G is the steady-state composition gradient and V_r the nominal rectifier vapour rate. The initial slope of the linearised equilibrium curve is α, p and s the Laplace variables for transforms w.r.t normalised time τ and distance h, related to real time t and distance h' by:

$$\tau = t\,k/H \text{ and } h = h'k/V_r \qquad (5)$$

where H is the stripper liquid capacitance p.u. length, and k the rectifier evaporation rate p.u. length p.u. composition departure from equilibrium. h is measured from an origin at the ends of the column (bent into a conceptual u-tube) so that y(o), x'(o), $x'_e(o)$ and $y_e(o)$ denote the composition variables entering or leaving the terminating vessels. Superscript ≈ denotes transforms in p and s w.r.t τ and h whilst ~ denotes transforms in p w.r.t. τ only.

Parameter c denotes the ratio of rectifier-vapour to stripper-liquid capacitance (= stripper-vapour/rectifier liquid capacitance) and it is the effect of varying this parameter with which the present report is principally concerned.

The terminal boundary conditions are :

$$\begin{pmatrix} \tilde{y}_e(o) \\ \tilde{x}'_e(o) \end{pmatrix} = \alpha^{-1} h_e(p) \begin{pmatrix} y(o) \\ x'(o) \end{pmatrix} \qquad (6)$$

where $h_e(p) = \dfrac{1}{1 + Tp}$ (7)

where T is the normalised time-constant of the end-vessels so that from (1), (2) and (6) we obtain, upon inversion to the h,p domain :

$$\begin{pmatrix} \tilde{y}(L) \\ \tilde{y}_e(L) \end{pmatrix} + \tilde{y}(o)\underline{Q}(L) \begin{pmatrix} 1 \\ \alpha^{-1}h_e \end{pmatrix} + \underline{R}(L) \begin{pmatrix} 1 & 0 \\ 0 & \alpha \end{pmatrix} \begin{pmatrix} \tilde{z}_1 \\ \tilde{z}_2 \end{pmatrix} = 0 \qquad (8)$$

and

$$\begin{pmatrix} \tilde{x}'_e(L) \\ \tilde{x}'(L) \end{pmatrix} - \tilde{x}'(o)\underline{Q}^*(L) \begin{pmatrix} \alpha^{-1}h_e \\ 1 \end{pmatrix} - \underline{R}^*(L) \begin{pmatrix} \alpha & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \tilde{z}_1 \\ \tilde{z}_2 \end{pmatrix} = 0 \qquad (9)$$

where L is the normalised length of rectifier and stripper and

$$\tilde{\underline{Q}}(s) = \mathcal{L}_h \underline{Q}(h) \quad , \quad \tilde{\underline{R}}(s) = \mathcal{L}_h \underline{R}(h)$$
$$\tilde{\underline{Q}}(-s) = \mathcal{L}_h \underline{Q}^*(h) \quad , \quad \tilde{\underline{R}}(-s) = \mathcal{L}_h \underline{R}^*(h) \qquad (10)$$

where

$$\tilde{\underline{R}}(s) = s^{-1}\tilde{\underline{Q}}(s) \text{ and } \tilde{\underline{R}}(-s) = -s^{-1}\tilde{\underline{Q}}(-s) \qquad (11)$$

Now the feed boundary conditions are

$$y(L) = x'_e(L) - (\varepsilon/2)z_1 \qquad (12)$$

and $$x'(L) = y_e(L) + (\varepsilon/2)z_2 \qquad (13)$$

where $$\varepsilon = \alpha - 1 \qquad (14)$$

and on eliminating the feedpoint variables from (8) and (9) using (12) and (13) we finally obtain the important result

$$\left[\underline{Q}(L)\begin{pmatrix} 1 \\ \alpha^{-1}h_e \end{pmatrix}, \underline{Q}^*(L)\begin{pmatrix} \alpha^{-1}h_e \\ 1 \end{pmatrix}\right]\begin{pmatrix} y(o) \\ x'(o) \end{pmatrix} = -\left[\underline{R}(L)\begin{pmatrix} 1 & o \\ o & \alpha \end{pmatrix} + \underline{R}^*L\begin{pmatrix} \alpha & o \\ o & 1 \end{pmatrix} - \frac{\varepsilon}{2}\begin{pmatrix} 1 & o \\ o & 1 \end{pmatrix}\right]\begin{pmatrix} \tilde{z}_1 \\ \tilde{z}_2 \end{pmatrix} \qquad (15)$$

In principle therefore a T.F.M. between $[\tilde{y}(o), x'(o)]^T$ and $[\tilde{z}_1, \tilde{z}_2]^T$ can be obtained, for any given c, by calculation of $\underline{Q}(L)$, $\underline{Q}^*(L)$, $\underline{R}(L)$ and $\underline{R}^*(L)$. (via (3) and (10)), forming the p-dependent coefficient matrices on the left-and right-hand sides of (15) and cross multiplying the L.H.S matrix.

The task is reasonably straightforward for c = 1 since, because of their symmetry, both left-and right-hand-side coefficient matrices readily diagonalise by choosing output and input vectors $[\tilde{y}(o) - x'(o), \tilde{y}(o) + \tilde{x}'(o)]^T$ and $[\tilde{z}_1+\tilde{z}_2, \tilde{z}_1-\tilde{z}_2]^T$ leading to the same diagonal T.F.M. produced somewhat more laboriously in previous reports.

3. Special Case of Zero Vapour Capacitance (c=o)

If we define complex variable q such that

$$q^2 = p^2/4 + p \qquad (16)^*$$

then we obtain after transform inversion and lengthy but elementary algebra

*Note the different definition of $q^2 (=p^2+2p)$ in previous reports based on c = 1.0.

$$\left(\underline{Q}(L) \begin{pmatrix} 1 \\ \alpha^{-1} h_e \end{pmatrix} , \underline{Q}^*(L) \begin{pmatrix} \alpha^{-1} h_e \\ 1 \end{pmatrix} \right)$$

$$= \frac{1}{2q} \begin{pmatrix} a\, e^{(q-p/2)L} + b\, e^{-(q+p/2)L} & ; & -c\, e^{(q+p/2)L} - d\, e^{-(q-p/2)L} \\ c\, e^{(q-p/2)L} + d\, e^{-(q+p/2)L} & ; & -a\, e^{(q+p/2)L} - b\, e^{-(q-p/2)L} \end{pmatrix} \tag{17}$$

$$\left.\begin{aligned} \text{where } a &= -1 + \alpha^{-1} h_e - p/2 - q \\ b &= 1 - \alpha^{-1} h_e + p/2 - q \\ c &= \alpha^{-1} h_e (1 - \alpha h_e^{-1} + p/2 - q) \\ d &= \alpha^{-1} h_e (-1 + \alpha h_e^{-1} - p/2 - q) \end{aligned}\right\} \tag{18}$$

and because of the symmetry of the R.H.S of (17) this diagonalises fairly readily to give the expression

$$\begin{pmatrix} 1 , & 1 \\ 1 , & -1 \end{pmatrix} \begin{pmatrix} (\alpha^{-1} h_e - 1)(q/p) \sinh q L - (1/2)(\alpha^{-1} h_e + 1) \cosh q L , & 0 \\ 0 , & -(\alpha^{-1} h_e + 1)(p/4q) \sinh q L + (1/2)(\alpha^{-1} h_e - 1) \cosh q L \end{pmatrix}$$

$$\begin{pmatrix} 1 , & -1 \\ 1 , & 1 \end{pmatrix} \begin{pmatrix} e^{-pL/2} , & 0 \\ 0 , & e^{pL/2} \end{pmatrix} \quad (= \text{R.H.S. } (17)) \tag{19}$$

The similarity of the expressions in the diagonal matrix to the denominator expressions for the c = 1 T.F.M. in previous reports[1,2] is striking but the extraction of the diagonal, post-multiplying delay-matrix must not be ignored in this case (c = o). The diagonalisation achieved clearly leads to a great saving in effort in finding the final T.F.M.

The R.H.S of general equation (15) is somewhat more tedious in its evaluation and does not diagonalise by choosing simple input output combinations. After careful inversion of their constituent Laplace transforms, expressions given in equations (20) and (21) are obtained for $\underline{R}(L)$ and $\underline{R}^*(L)$

$$\underline{R}(L) = \frac{1}{2qp} \begin{pmatrix} 2q(1+p) - (q+p/2)(q+p/2+1)e^{(q-p/2)L} + (q-p/2)(q-p/2-1)e^{-(q+p/2)L} , & -2q + (q-p/2)e^{-(q+p/2)L} + (q+p/2)e^{(q-p/2)L} \\ 2q - (q-p/2)e^{-(q+p/2)L} - (q+p/2)e^{(q-p/2)L} , & -2q + (q-p/2)e^{(q-p/2)L} + (q+p/2)e^{-(q+p/2)L} \end{pmatrix} \tag{20}$$

$$\underline{R}^{*}(L) = \frac{1}{2qp} \begin{pmatrix} -2q(1+p) - (q-p/2)(q-p/2-1)e^{(q+p/2)L} + (q+p/2)(q+p/2+1)e^{-(q-p/2)L} , & 2q - (q+p/2)e^{-(q-p/2)L} - (q-p/2)e^{(q+p/2)L} \\ -2q + (q+p/2)e^{-(q-p/2)L} + (q-p/2)e^{(q+p/2)L} , & 2q - (q-p/2)(q+p/2+1)e^{(q+p/2)L} + (q+p/2)(q-p/2-1)e^{-(q-p/2)L} \end{pmatrix} \tag{21}$$

Now from equations (17) and (19) it is clear that for $c = o$, equation (15) may be expressed in the form

$$\begin{pmatrix} d_1 & 0 \\ 0 & d_2 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} e^{-pL/2} & , \\ 0 & ,e^{pL/2} \end{pmatrix} \begin{pmatrix} \tilde{y}(o) \\ \tilde{x}'(o) \end{pmatrix} = \underline{S} \begin{pmatrix} \tilde{z}_1 + \tilde{z}_2 \\ \tilde{z}_1 - \tilde{z}_1 \end{pmatrix} \qquad (22)$$

$$\text{where } \underline{S} = -\frac{1}{4} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \left(\underline{R}(L) \begin{pmatrix} 1 & 0 \\ 0 & \alpha \end{pmatrix} + \underline{R}^{*}(L) \begin{pmatrix} \alpha & 0 \\ 0 & 1 \end{pmatrix} - \frac{\varepsilon}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right) \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \qquad (23)$$

$$d_1 = (\alpha^{-1}h_e - 1)(q/p)\sinh q\,L - (1/2)(\alpha^{-1}h_e + 1)\cosh q\,L \qquad (24)$$

and

$$d_2 = -(\alpha^{-1}h_e + 1)(p/4q)\sinh q\,L + (1/2)(\alpha^{-1}h_e - 1)\cosh q\,L \qquad (25)$$

From (22) therefore we obtain the T.F.M. relationship

$$\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} e^{-pL/2} & 0 \\ 0 & e^{pL/2} \end{pmatrix} \begin{pmatrix} \tilde{y}(o) \\ \tilde{x}'(o) \end{pmatrix} = \underline{G}'(o,p) \begin{pmatrix} \tilde{z}_1 + \tilde{z}_2 \\ \tilde{z}_1 - \tilde{z}_2 \end{pmatrix} \qquad (26)$$

where T.F.M. $\underline{G}'(o,p)$ is given by

$$\underline{G}'(o,p) = \begin{pmatrix} d_1^{-1} & 0 \\ 0 & d_2^{-1} \end{pmatrix} \underline{S} \qquad (27)$$

and in terms of 'tilt' and 'total' composition changes we obtain

$$\begin{pmatrix} y(o) - x'(o) \\ y(o) + x'(o) \end{pmatrix} = \underline{G}(o,p) \begin{pmatrix} \tilde{z}_1 + \tilde{z}_2 \\ \tilde{z}_1 - \tilde{z}_2 \end{pmatrix} \qquad (28)$$

$$\text{where } \underline{G}(o,p) = \begin{pmatrix} \cosh pL/2 & \sinh pL/2 \\ \sinh pL/2 & \cosh pL/2 \end{pmatrix} \underline{G}'(o,p) \qquad (29)$$

The problem therefore reduces to that of calculating matrix $\underline{S}$ (eqn. 23) from $\underline{R}(L)$ and $\underline{R}^{*}(L)$ (eqns. 20 and 21) from which T.F.M's $\underline{G}'(o,p)$ and then $\underline{G}(o,p)$ may be quickly determined. It is convenient to express $\underline{S}$ thus

$$S = -\frac{1}{4}\left\{\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\left\{\underline{R}(L)\begin{pmatrix}1 & 0\\ 0 & \alpha\end{pmatrix} + R^{*}(L)\begin{pmatrix}\alpha & 0\\ 0 & 1\end{pmatrix}\right\}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} - \begin{pmatrix}\varepsilon & 0\\ 0 & \varepsilon\end{pmatrix}\right\} \qquad (30)$$

and as a stepping stone towards finding $\underline{S}$ from eqn. (30) its non-diagonal constituent matrices, derived from (20) and (21) are given in equations (31) and (32). {In simplifying the coefficients of equations (20),(21),(31) and (32) considerable use is made of equation (16) relating q and p.}

The elements of T.F.M. $\underline{G}'(o,p)$ may be finally obtained from $\underline{S}$, d_1 and d_2 in the form given in equations (33) to (35):

$g'_{11}(o,p) =$

$$\frac{\cosh q\, L\{(\varepsilon-p/2)e^{-pL/2}+(\varepsilon+\alpha p/2)e^{pL/2}\} - q\sinh q\, L(e^{-pL/2}+\alpha e^{pL/2}) - \varepsilon p(1+2/p)}{2\, q(1-\alpha^{-1}h_e)\sinh q\, L + p(1+h_e\alpha^{-1})\cosh q\, L} \qquad (33)$$

$g'_{21}(o,p) =$

$$\frac{-\varepsilon q + e^{-pL/2}\{(\varepsilon-p/2)\sinh q\, L - q\cosh q\, L\} + e^{pL/2}\{-(\varepsilon+\alpha p/2)\sinh q\, L + \alpha q\cosh q L\}}{p(\alpha^{-1}h_e+1)\sinh q\, L - 2q(\alpha^{-1}h_e-1)\cosh q\, L} \qquad (34)$$

$g'_{12}(o,p) =$

$$\frac{-\varepsilon p/2+e^{-pL/2}\{-q\sinh qL-(1+\alpha+p/2)\cosh qL\}+e^{pL/2}\{-\alpha q\sinh qL+(1+\alpha+\alpha p/2)\cosh qL\}}{2q(1-\alpha^{-1}h_e)\sinh q\, L + p(1+h_e\alpha^{-1})\cosh q\, L} \qquad (35)$$

and

$g'_{22}(o,p) =$

$$\frac{2\varepsilon q+e^{pL/2}\{(1+\alpha+\alpha p/2)\sinh qL-\alpha q\cosh qL\}+e^{-pL/2}\{(1+\alpha+p/2)\sinh qL+q\cosh qL\}}{-(\alpha^{-1}h_e+1)p\sinh q\, L + 2q(\alpha^{-1}h_e-1)\cosh q\, L} \qquad (36)$$

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \underline{R}(L) \begin{pmatrix} 1 & 0 \\ 0 & \alpha \end{pmatrix}$$

$$= \frac{1}{2q\,p} \begin{pmatrix} 2q(2+p) - 2q(q+p/2+1)e^{(q-p/2)L} + 2q(q-p/2-1)e^{-(q+p/2)L} & , & \alpha\{-4q + 2qe^{-(q+p/2)L} + 2q\,e^{(q-p/2)L}\} \\ 2qp - (q+p/2)^2\, e^{(q-p/2)L} + (q-p/2)^2 e^{-(q+p/2)L} & , & \alpha\{-p\,e^{-(q+p/2)L} + p\,e^{(q-p/2)L}\} \end{pmatrix} \qquad (31)$$

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \underline{R}^*(L) \begin{pmatrix} \alpha & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \frac{1}{2q\,p} \begin{pmatrix} \alpha\{-2q(2+p) - 2q(q-p/2-1)e^{(q+p/2)L} + 2q(q+p/2+1)e^{-(q-p/2)L}\} & , & 4q - 2q\,e^{-(q-p/2)L} - 2q\,e^{(q+p/2)L} \\ \alpha\{-2qp - (q-p/2)^2 e^{(q+p/2)L} + (q+p/2)^2 e^{-(q-p/2)L}\} & , & -p\,e^{-(q-p/2)L} + p\,e^{(q+p/2)L} \end{pmatrix} \qquad (32)$$

4. Zero- and High-Frequency Checks

There is of course enormous scope for error in the algebraic manipulation leading to the results given above and it is necessary to apply whatever cross checks are available to guard against such errors. Fortunately, the results for zero frequency will be independent of capacitance ratio c and this result has already been obtained and tested for the case of c = 1.

4.1 Zero-frequency T.F.M.

The limit

$$\lim_{p \to o} G'(o,p) = \lim_{p \to o} G(o,p) = \begin{bmatrix} g_{11}(o,o) , & g_{12}(o,o) \\ g_{21}(o,o) , & g_{21}(o,o) \end{bmatrix} \quad (37)$$

and on taking the limits* of the R.H.S's of equations (33) to (36) it is found that

$$g_{11}(o,o) = \frac{\alpha\{L^2 \varepsilon - L(\alpha+1) - \varepsilon/2\}}{2\varepsilon L + \alpha + 1} \quad (38)$$

$$g_{12}(o,o) = g_{21}(o,o) = 0 \quad (39)$$

and

$$g_{22}(o,o) = -\frac{\alpha\{\varepsilon/2 + (\alpha+1)L\}}{\varepsilon} \quad (40)$$

these results being identical to the c = 1 case previously derived, so giving some confidence in the model derivation. A high frequency check provides the additional corroboration needed for full confidence.

4.2 High-frequency T.F.M

Now from (16) we deduce that

$$q = (p/2)(1+4/p)^{0.5} \quad (41)$$

so that $q \to (p/2)(1 + 2/p) = p/2 + 1 \quad , \quad \text{as } |p| \to \infty$ (42)

Setting $p = j\omega$ therefore, from (29) we obtain

$$\underline{G}(j\omega) = \begin{bmatrix} \cos\frac{\omega L}{2} , & j \sin\frac{\omega L}{2} \\ j \sin\frac{\omega L}{2} , & \cos\frac{\omega L}{2} \end{bmatrix} \underline{G}'(j\omega) \quad (43)$$

*Note that $q \to p$ as $p \to o$

and using results (42) and (43) we quickly deduce from (33) to (36) that

$$\lim_{\omega \to \infty} \underline{G}(j\omega) = \frac{(\alpha e^{-2L} - 1 - 1.5\varepsilon e^{-L})}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + \frac{\varepsilon e^{-L}}{4} e^{-j\omega L} \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} \qquad (44)$$

The four Nyquist loci thus converge to small orbits of radius $\varepsilon\, e^{-L}/4$ about the real point $-(1+1.5\varepsilon e^{-L} - \alpha\, e^{-2L})/2$.

Now a result for the initial response of the c = o system to input steps, v and ℓ, is quickly obtainable directly from the system p.d.e's (1) and (2) and equation (3):

In response to a sudden step in v and ℓ the liquid composition changes, y_e and x', will be zero initially because of the liquid capacitances in the rectifier and stripping sections. Furthermore, output changes $y_e(o)$ and $x'_e(o)$ from the accumulator and reboiler capacitances will likewise be zero initially. Insertion of these results in (1), (2) and (3) gives simply the vapour-change equations:

$$(1 - s)\tilde{\tilde{y}} = -\tilde{y}(o) - \tilde{z}_1/s \qquad (45)$$

and

$$(1 + s)\tilde{\tilde{x}}'_e = -\tilde{z}_1/s \qquad (46)$$

These may be quickly solved in conjunction with the feed boundary equation (21) to give

$$\tilde{y}(o) = -(1.5\,\varepsilon\, e^{-L} + 1 - \alpha\, e^{-2L})\tilde{z}_1 \qquad (47)$$

which may be expressed, knowing that $\tilde{x}'(o) = 0$ initially, in the matrix form

$$\begin{pmatrix} \tilde{y}(o) - \tilde{x}'(o) \\ \tilde{y}(o) + \tilde{x}'(o) \end{pmatrix} = \frac{-(1.5\varepsilon^{-L} + 1 - \alpha\, e^{-2L}}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} z_1 + z_2 \\ z_1 - z_2 \end{pmatrix} \qquad (48)$$

Apart from the small orbital terms of equations (44) therefore the initial unit step response obtained from (48) is identical to $\lim \underline{G}(j\omega)$ as $\omega \to \infty$, as would be expected. This evidence, together with that of the zero-frequency analysis of Section 4.1, thus validates the accuracy of the T.F.M. expressions (33) - (36) very thoroughly indeed.

5. Computed results and discussion

Fig. 1 shows transient responses of $y(o,\tau) - x'(o,\tau)$ for a unit step input in $z_1 + z_2$ (with $z_1 - z_2 = 0$) computed from the system p.d.e'2 and boundary conditions for various values of c in the range $o < c < 1.0$. The system parameters are $L \doteq 5$, $\varepsilon = 1$, $(\alpha = 2)$, $T = 5$. It is interesting to note the persistence of the non-minimum phase behaviour of the system with changing c and that the initial value of the response for $c \simeq o$ is identical to that predicted by equation (44) {obtained from the T.F.M. $\underline{G}(o,p)$} and equation (48) (obtained analytically from the p.d.e's directly). The final value of $y(o,\tau) - x'(o,\tau)$ is found to accord with the equation (38) for $g_{11}(o,o)$ {also obtained from $\underline{G}(o,p)$}, as does the response of $y(o,\tau) + x'(o,\tau)$ (not given here) for a unit step in $z_1 - z_2$ (with $z_1 + z_2 = 0$).

The similarity of the $c = o$ and $c = 1$ cases is demonstrated by the inverse Nyquist loci* $g_{11}^{-1}(o,j\omega)$ for the two cases given for $L = 5$, $\varepsilon = 1$, $T = 5$ in Fig. 2. The high-frequency tail on the $c = 1.0$ case, not present with $c = o$ merely inidicates the rapid but finite (negative) rate-of-rise of $y(o,\tau) - x'(o,\tau)$ in the presence of vapour capacitance. The finite negative destination of the locus for $c = o$ merely indicates the abrupt negative nature of the system's step response in this case. The general features of the loci for these two extreme cases are otherwise very similar.

We may therefore conclude that setting $c=1.0$ is not too important a limitation on the applicability and robustness of the T.F.M. model previously derived by the authors[1,2,3] and the advantage of its relative simplicity may be safely exploited in control system studies.

*Obtained from equation (33) and reference 1 respectively.