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HIGH PERFORMANCE CONTROLLERS FOR UNKNOWN MULTIVARIABLE SYSTEMS

by

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Abstract

Recent work on the design of robust proportional plus integral non-adaptive process controllers for unknown minimum-phase (but possibly unstable) multivariable systems is compared and contrasted with recent work, extended to predict permissible data inaccuracies and illustrated by application to an open-loop unstable batch process.

1. Introduction

In recent papers Penttinen and Koivo (1980) and Porter (1981) have considered specific examples of the general problem discussed by Davison (1976) of constructing simple, non-adaptive process controllers for unknown multivariable systems (unknown in the sense that its model is unknown or of too high order to make normal design calculations feasible) using only elementary computations based on inspection of graphical system open-loop step response data. Astrom (1980) has also considered a special case for single-input/single-output systems. When applicable, the techniques are capable (in principle) of generating simple process controllers that are easily tuned on-line and, in this sense, can be regarded as preliminary attempts to extend and generalize classical tuning methods such as that due to Ziegler and Nichols (1942). In the authors opinion, all of these techniques lay an important foundation to help bridge the gap between control theory and engineering practice based on experience and intuition. We should note that the problem could also be approached using sophisticated, microprocessor-implemented adaptive (self-tuning or model-reference adaptive) controllers (see, for example, Billing and Harris (1981)).

This is most probably the preferred approach if the plant is subject to significant parameter variation. We take the view, however, that, in those cases where plant parameters are known to remain sensibly constant, there is still a vital need to retain the simplicity, familiarity and known robustness of fixed-parameter, non-adaptive control elements.

It is the purpose of this paper to provide a discussion of underlying theoretical concepts relevant to the construction of a viable theory of non-adaptive unknown systems control and to note that there are two natural and complementary situations (namely those of 'high' and 'low' controller gains) where useful analysis is possible. The work of Penttinen and Koivo, Porter, Davison and Astrom all require the use of 'low' control gains. The high gain situation is then illustrated by a generalization of controller design as discussed by Edwards and Owens (1977) and Owens (1978,1979b) and applied to control design for the open-loop unstable system previously considered by Munro (1972) and Rosenbrock (1974). The technique is seen to be the natural 'inverse' to that of Penttinen and Koivo and has the advantage that the effect of measurement nonlinearities can be estimated during the design exercize (see Boland and Owens (1980) and Owens (1981a)).

2. Alternative Approaches to Non-adaptive Unknown Systems Control

It would appear to be a general principle that, in any attempt to design a controller for an unknown system with m-inputs and ℓ-outputs described by (say) the (unknown) continuous, linear, time-invariant model in R^n

$$\dot{x}(t) = A\,x(t) + B\,u(t)$$

$$y(t) = C\,x(t) \qquad \ldots(1)$$

it is necessarily true that the structure of the design and the limitations encountered should reflect this uncertainty and generate a closed-loop system that is robust (i.e. insensitive to the unknown dynamics). The details will, of course, vary from situation to situation and will depend upon the physical nature of the plant. It is possible however to generate two distinct situations where general statements can be made about stability as outlined in the following sections. It is important to recognize however that the presence of unknown dynamics in the plant inevitably constrains the form of theoretical result obtainable and clearly precludes, in general, the possibility of making precise predictions about performance in situations of interest. The precision of the results obtainable clearly depends upon the degree to which the plant is unknown! At the most primitive level, the smallest data set allowed seems to consist of some structural information about the plant (eg stability, minimum-phase, rank) that is presumed to be known from physical or other considerations together with a small amount of parametric information (eg steady-state data, rise-time) known from (say) plant tests. Given such a small data set, precise analytic predictions about closed-loop performance cannot even be attempted and the theory must turn its attention to the important but less (numerically) precise questions:

(i) Given the structural information known, does a stabilizing controller of the required structure exist and

(ii) is the parametric information required to construct such a controller available?

If the answer to both questions is in the affirmative the control problem is viable but is not solved unless the theory also provides some indication of a systematic search procedure for finding a controller. We must not preclude the possibility that the procedure takes the form of simple guidelines for use in on-line tuning!

In the following sections attention is focussed on the problem of the design of a proportional plus integral unity negative-feedback controller for an unknown linear plant. If the problem is solvable it will clearly provide a control system ensuring regulation, tracking of step demands and total rejection of step disturbances. Theoretical analysis, as expected, does not produce a precise numerical form for the controller but it does provide parameterizations of classes of controllers in which a suitable controller is known to exist. For example, it is possible to prove that suitable controllers lie in the one-parameter family

$$K(s) = k\ (K_1 + \frac{1}{s} K_2) \qquad \ldots(2)$$

where K_1 and K_2 are computable matrix gains and the parameter k is an 'overall loop gain', and that suitable controllers can be constructed using any k in a non-empty but unknown range

$$k_1 < k < k_2 \qquad \ldots(3)$$

The form of (2) and the known existence of a stabilizing value of k clearly makes possible the systematic search for a suitable value either on-line or by simulations using a complex system model. In those cases where $k_1 = 0$ (resp. $k_2 = +\infty$) the analysis also provides additional 'trend' information indicating that instability found on implementation of a trial value of gain k can be removed by decreasing (resp. increasing) its value on-line.

2.1 The Low Gain Philosophy for Stable Unknown Plant

Intuitively, a stable system of the form of (1) will, in general, retain its stability in the presence of 'low gain' output feedback. The mathematical formulation of these ideas for the case of proportional plus integral unity negative output feedback requires more careful thought but can be resolved (Davison (1976), Pentinnen and Koivo (1980), Porter (1981)) to prove that a controller of the form of (2) exists guaranteeing closed-loop stability for all k in some open interval of the form of (3) with $k_1 = 0$ if the plant has the structural characteristics of stability and the absence of transmission zeros at the origin $s = 0$. In particular, in the case of $m = \ell$, it is then possible to choose $K_1 = G^{-1}(0)$ and $K_2 = \varepsilon G^{-1}(0)$ with $\varepsilon \geq 0$ and $G(0)$ equal to the matrix of d.c. gains deduced from open-loop plant step tests. No other structural information or numerical data is necessary to ensure the successful completion of the design by on-line tuning of the parameter k.

Finally, it must be emphasized that, as the theoretical treatments concentrate on stability predictions, there is no guarantee that closed-loop transient performance will be acceptable. Indeed, if the unknown upper gain bound k_2 is numerically small, we will expect to achieve only sluggish closed-loop performance. If, however, k_2 is large (as will be the case for overdamped plant) improved transient performance will be achieved by the use of the higher gains permitted. Unfortunately, at the level of plant knowledge assumed, the bound k_2 cannot be computed, its value (good or bad) being revealed by the plant at the on-line tuning stage. This is the inevitable price that must be paid for attempting controller design at this level of plant ignorance! Improved predictions can only be obtained if more detail on plant dynamics is included in the design (see Owens and Chotai (1981)).

2.2 The High Gain Philosophy for Minimum Phase Unknown Plant

The design of high gain feedback controllers for uncertain systems originates with the pioneering work of Bode (1945) and Horowitz (1963) for single-input/single-output systems. A corresponding theory is not yet available for the multivariable case despite the promising beginnings in the work of the authors (Edwards and Owens (1977), Owens (1978), Owens and Chotai (1980), Owens (1981a)), the work of Willems (1981) and Kimura (1981) and the important work of Kouvaritakis, Shaked, Owens and others on multivariable root-loci (see, for example, Kouvaritakis and Shaked (1976), Shaked (1976), Owens (1978) or the review paper by Owens (1981b)).

The relevance of high gain feedback to non-adaptive unknown systems control can be illustrated by considering the situation when $m = \ell$ and structural information is available to the designer to indicate that the plant is minimum-phase and possesses only infinite zeros of first and second order. It is then a simple exercize in compensation theory (Owens (1979a)) to demonstrate that there very often exists a unity negative feedback controller of the form of (2) that places all system asymptotes in the open-left-half complex plane and hence stabilizes the plant for all gains k in some range (3) with $k_2 = +\infty$. Moreover the existence and form of such controllers can be deduced from any parametric data set containing the Markov parameters CB, CAB and CA^2B if the plant possesses second order characteristics or simply the single Markov parameter matrix CB if the plant possesses only first order characteristics at high gain. This last case corresponds to the case when CB is nonsingular and it is interesting to note that it can be estimated from open-loop transient step response data (Edwards and Owens (1977)) alone.

As in section 2.1 the lower gain bound is unknown and hence the analysis produces no guarantee that system performance or control input magnitudes are acceptable. If k_1 is large then stability may only be achieved using large control inputs and/or (if the plant possesses second order infinite zeros) at the expense of severe oscillation in the closed-loop system. The use of such high gains may be physically necessary if the plant is open-loop unstable but, if the plant is stable, excessively high gains must be avoided. This will only be possible if the lower gain bound k_1 turns out to be small. Unfortunately, with the minimal plant knowledge assumed, k_1 cannot be predicted theoretically, its value (good or bad) and the consequent success of the design being revealed only at the on-line tuning stage.

2.3 Low-gain versus High-gain: inverse approaches

In order to highlight the relationships between the high and low gain philosophies, divide the class of all square, invertible linear systems into four subclasses:

(a) Stable and minimum-phase systems,

(b) Stable and non-minimum-phase systems,

(c) Unstable and minimum-phase systems and

(d) Unstable and non-minimum-phase systems.

Subject to the availability of the required structural information, the low-gain philosophy applies to unknown systems in (a) and (b) only whereas the high-gain philosophy applies only to unknown systems in (a) and (c). Clearly the methodologies have distinct areas of applicability with overlap in subclass (a). They must hence be regarded as distinct alternatives to controller design for unknown plant known to lie in (a), (b) or (c). Neither philosophy can cope with case (d) but, intuitively, a plant with the dual complexity of right-half-plane zeros and instability must be modelled accurately if control design is to be successful! The existence of distinct but overlapping areas of applicability is underlined by noting that, although stability, asymptotic tracking and disturbance rejection are fundamentally important design specifications in process control, other applications demand higher performance specifications including fast rise-time, small overshoot, small transient interaction etc. The relevance of the low-gain philosophy to such situations is small as it is generally true that high performance systems require tight (ie high gain) control loops!

Finally, we note that the low and high gain philosophies can be pictured as inverses of each other in the sense that the gains and the required structural information have an inverse relationship:

(a) the inverse of a high gain is a low gain, and

(b) a minimum-phase system has a stable inverse.

3. High Performance Controllers based on Approximate Plant Models

Although root-locus arguments provide a useful starting point for a theory of controller design for minimum-phase unknown multivariable systems, it does not produce a complete picture. For example, specific choices of K_1 and K_2 must be made to ensure, not only stability, but excellent time responses for the closed-loop system. The approach being pursued by the authors is to base controller design upon a very simple model of plant behaviour, to ensure that the derived controller produces the desired stable, high performance responses from the approximate model and to demonstrate that, at high gains, the real plants stability and transient characteristics are arbitrarily close to those predicted by the approximate model. The theory is under development at the present time and is certainly not complete. We will restrict our attention therefore to the special case when $m = \ell$ and $|CB| \neq 0$. This is clearly a restrictive assumption on plant structure that is equivalent to the existence of only first order infinite zeros of the root-locus or, in transfer function terms, it is the multivariable equivalent of the classical situation when the plant has a pole-excess of unity or a high frequency slope of −1 on the amplitude curve of the Bode diagram. In practice, the assumption is not satisfied in the sense that unmodelled fast dynamics almost always increase the 'pole-excess'. There are situations however when it is a reasonable approximation over the bandwidth of interest. We therefore take the view that analyses of this special case will provide a bench mark/starting point for the analysis of more complex cases.It seems to be self-evident that, if this case defies analysis, the analysis of the more general situation will be impossible!

Suppose that structural information is available to indicate that a square, unknown multivariable system is minimum-phase with CB nonsingular. It follows that the plant has transfer function matrix G(s) with inverse of the form

$$G^{-1}(s) = sA_o + A_1 + A_oH(s) \qquad \ldots(4)$$

where $A_o = (CB)^{-1}$ and $H(s)$ is proper and stable. Both A_1 and H are not defined uniquely as, replacing A_1 by $\tilde{A}_1$, (4) is structurally unchanged if we also replace H by $\tilde{H} = H + A_o^{-1}(A_1 - \tilde{A}_1)$. There is no loss of generality therefore in taking $H(o) = 0$ when $A_1 = \lim_{s\to 0} G^{-1}(s)$.

Let $\tilde{A}_o$ and $\tilde{A}_1$ be numerical estimates of A_o and A_1 obtained from a complex model or, if open-loop plant step response data is available, from initial rate and (if the plant is stable) steady state data (see Edwards and Owens (1977), Owens (1978)). This numerical information can be used to construct an approximate first order model (Owens (1978)) of plant dynamics of the form

$$G_A^{-1}(s) = s\tilde{A}_o + \tilde{A}_1 \qquad \ldots(5)$$

which approximates the high and low frequency plant characteristics only.

Consider now the two-term parametric controller

$$K(s) = \tilde{A}_o \operatorname{diag}\{k_j + c_j + \frac{k_j c_j}{s}\} - \tilde{A}_1 \qquad \ldots(6)$$

generalising previous work of Owens (1978 p.120). A simple calculation yields the identity

$$(I_m + G_A(s)K(s))^{-1}G_A(s)K(s) \equiv (G_A^{-1}(s) + K(s))^{-1}K(s)$$

$$\equiv \operatorname{diag}\{\frac{1}{(s+k_j)(s+c_j)}\}_{1\le j\le m} \quad (\operatorname{diag}\{(k_j+c_j)s+k_jc_j\}_{1\le j\le m} - s\tilde{A}_o^{-1}\tilde{A}_1) \qquad \ldots(7)$$

and hence the approximate system is stable in the closed-loop situation iff $k_j>0$ and $c_j\ge 0$, $1\le j\le m$. If we identify the k_j's with fast modes and c_j's with slower modes then a simple pole-residue calculation yields the results

(a) responses in loop j have time-constants of the order of k_j^{-1}, zero steady state errors in response to unit step demands if $c_j \neq 0$ and reset times of the order of c_j^{-1}, and

(b) defining $k = \min_j k_j$, then the system response speeds in response to unit step demand can be made to be arbitrarily fast and transient interaction effects arbitrarily small as the 'gain' k becomes large.

Clearly the controller (6) generates a high performance closed-loop system for the approximate plant (5) if loop gains are high! Suppose that we now apply the controller to the real (unknown) system (4): The following result is a natural inverse to those of Davison and Penttinen and Koivo.

<u>Theorem</u>: Using the above assumptions and constructions, there exists a real 'overall-gain' $k^* \geq 0$ such that the unknown multivariable plant considered will be stable in the presence of unity negative feedback with forward path proportional plus integral controller given in (6), if

(i) the tuning parameters $k_j > 0$, $c_j \geq 0$ $(1 \leq j \leq m)$

(ii) $\lambda_\infty \triangleq \max_{1 \leq i \leq m} \sum_{j=1}^{m} |(\tilde{A}_o^{-1}(\tilde{A}_o - A_o))_{ij}| < 1$

(iii) the gain parameter $k \triangleq \min_{1 \leq j \leq m} k_j > k^*$

<u>Proof</u>: Using the results of Edwards and Owens (1977), <u>sufficient</u> conditions for closed-loop stability are that K stabilizes G_A and that $\lambda \triangleq \max_{1 \leq i \leq m} \sup_{s \in D} \sum_{j=1}^{m} |F_{ij}(s)| < 1$ where $F = (I + Q_A^{-1})^{-1}(Q_A^{-1} - Q^{-1})$, $Q = GK$ and $Q_A \equiv G_A K$ and D is the usual Nyquist contour. A little manipulation indicates that

$$F(s) \equiv \mathrm{diag}\{\frac{s}{(s+k_j)(s+c_j)}\}_{1 \leq j \leq m}(s\tilde{A}_o^{-1}(\tilde{A}_o - A_o) + \tilde{A}_o^{-1}(\tilde{A}_1 - A_1) - \tilde{A}_o^{-1}A_o H(s) \quad \ldots(8)$$

Condition (i) is required to ensure that K stabilizes G_A. The theorem follows as conditions (ii) and (iii) together ensure the existence of k^* such that $\lambda < 1$ for $k > k^*$. Simply note that H is proper and stable and use (8) to prove that $\lim_{k \to +\infty} \lambda \leq \lambda_\infty < 1$.

The application of the result is illustrated in the next section. Before continuing we note that conditions (i) and (iii) together provide a guaranteed range of tuning parameters for stability. They can be tuned on-line or using simulations of a complex model and using the approximate model predictions (observation (a) preceding the theorem) to provide some correlation with expected transient performance.

Condition (ii) provides explicit lower bounds on the maximum permissible error between A_0 (resp. $CB = A_0^{-1}$) and its estimate $\tilde{A}_0$ (resp. $\tilde{C}\tilde{B} = \tilde{A}_0^{-1}$) or, equivalently, lower bounds on the maximum permissible error in rate measurements on open-loop plant step response data. For example, if $m = 1, \lambda_\infty = |1 - \tilde{C}\tilde{B}(CB)^{-1}| < 1$ iff $0 < \tilde{C}\tilde{B} < 2CB$ ie iff the rate measurement has less than 100% error. The design is clearly very insensitive to measurement errors and condition (ii) is a theoretical condition for this inherent robustness at high gain.

Finally, note that A_1 and $\tilde{A}_1$ play no role in the theorem and hence the results are insensitive to the choice of $\tilde{A}_1$ e.g. choosing $\tilde{A}_1 = 0$ the result is still valid and the controller structure considerably simplified.

4. Numerical Example

To illustrate the application of the high gain theorem, consider the unstable batch process discussed by Munro (1972) and Rosenbrock (1974) and defined by the matrices

$$A = \begin{pmatrix} 1.38 & -0.2077 & 6.715 & -5.676 \\ -0.5814 & -4.29 & 0 & 0.675 \\ 1.067 & 4.273 & -6.654 & 5.893 \\ 0.048 & 4.273 & 1.343 & -2.104 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & 0 \\ 5.679 & 0 \\ 1.136 & -3.146 \\ 1.136 & 0 \end{pmatrix}, \qquad C = \begin{pmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \qquad \ldots(9)$$

which is known to be minimum phase and open-loop unstable. We will assume that it is required to design a high performance controller generating a closed-loop system with fast rise-times, zero steady state errors and small interaction effects in response to unit step demands. Also, although the system model is known, we will assume that the controller design must be undertaken without the aid of an interactive computing facility!

Following the procedure defined in section 3, we could use the model to compute CB (which is nonsingular) and hence A_o exactly ie

$$A_o = (CB)^{-1} = \begin{pmatrix} 0 & 0.176 \\ -0.318 & 0 \end{pmatrix} \qquad \ldots(10)$$

or estimate it from model simulations. Suppose that such a procedure or simple rounding of the elements in (10) suggests the choice

$$\tilde{A}_o = \begin{pmatrix} 0 & 0.2 \\ -0.3 & 0 \end{pmatrix} \qquad \ldots(11)$$

with the consequence that $\lambda_\infty = 0.12 < 1$. As the plant is unstable we cannot deduce A_1 from steady state simulation data. It is possible to calculate its exact value $A_1 = -CA^{-1}B$ from the model, but, for illustrative purposes and to simplify the form of the controller, we will take $\tilde{A}_1 = 0$. The resulting approximate model is the 2x2 pure integration $G_A = s^{-1}\tilde{A}_o^{-1}$ which clearly has time responses differing greatly from those of the real plant.

Controller design proceeds by the choice of tuning parameters k_1, k_2, c_1 and c_2 based upon the approximate model predictions that the closed-loop system will have time constants and reset times in loop j of the order of k_j^{-1} and c_j^{-1} resp. The preliminary choices are hence based upon required performance taking into account the achievable response speeds expected from plant experience. The theorem indicates that the resultant controller will certainly stabilize the unstable plant if $k = \min(k_1,k_2)$ is greater than the unknown bound k^*. In this case, this can be checked by simulation of the real plant model with the trial controller and tuning attempted by increasing k if instability is found and reducing k if control input magnitudes are unacceptable.

Suppose that closed-loop time-constants in the range 0.1 to 0.15 are required in both loops with reset times 0.25. This suggests the choice of $k_1 = k_2 = 8.0$ with $c_1 = c_2 = 4.0$. The trial controller is uniquely defined immediately and can be assessed by simulation of the closed-loop system. That $k = \min(k_1,k_2) = 8$ is a large enough gain to stabilize the system is immediately revealed by Fig.1 which shows the closed-loop unit step responses generated by both the real and approximate feedback systems. Note that the plant responses are close to those predicted by the approximate model. Fig.2 shows the corresponding input signals and indicates that the 'high gains' required to generate stability are not unacceptable.

Finally, we note that the final design above generates responses comparable with those obtained by Munro (1972) using sophisticated systematic multivariable frequency response techniques and an interactive computing facility. In contrast, the above design was achieved very rapidly and with only minimal computational requirements! Clearly the considerations outlined in this paper have provided a substantial 'short-cut' to controller design in this case.

5. Conclusions

It has been demonstrated that the low-gain philosophy inherent in the work of Astrom, Koivo and Davison on the control of stable unknown plant has a natural 'inverse' namely, the high gain philosophy for the control of minimum-phase unknown plant. This second approach has been explored by constructing a generalization of the first authors previous work in the form of a robust parametric controller structure capable of stabilizing unknown multivariable plant at high gain. The applicability of the results has been verified by the design of a high performance regulator for an unstable multivariable process that compares favourably with a previous design obtained using model-based multivariable frequency response methods.

An important conclusion is that a meaningful theory of unknown systems control appears to need the availability of both parametric and structural information. Structural information could present a problem unless physical insight into plant behaviour can be invoked. It is present both in the low gain case where the absence of zeros at the origin is needed and in the high gain case where the minimum phase property is required. In the scalar case, open-loop step responses could yield an answer by examination of 'reverse'kick' and steady-state phenomena but, in the multivariable case, the problem is more severe. These problems are however present in other areas such as, model-reference adaptive control and will merit further attention.

Finally, we point out that the principles outlined in this paper will extend naturally to the discrete case with 'high gains' and 'low gains' replaced by 'fast sampling' and 'slow sampling' respectively. The details of this extension are currently under consideration.

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References

Astrom, K.J. (1980). A robust sampled regulator for stable systems with monotone step responses. Automatica, 16, 313-315.

Billings, S.A., Harris, C.J. (Eds) (1981). Self-tuning and Adaptive Control, Peter Peregrinus.

Bode, H.W. (1945). Network Analysis and Feedback Amplifier Design, Van Nostrand.

Boland, F.M., Owens, D.H. (1980). Quantization and other bounded non-linearities in first-order discrete multivariable control. Proc. Inst. Electrical Engrs., Pt.D, 127, 38-44.

Davison, E.J. (1976). Multivariable tuning regulators: the feedforward and robust control of a general servomechanism problem. IEEE Trans. Aut. Control AC-21, 35-47.

Edwards, J.B., Owens, D.H. (1977). First-order-models for multivariable process control. Proc. Inst. Electrical Engrs., 124, 1083-1088.

Horowitz, I.M. (1963). Synthesis of Feedback Systems. Academic Press.

Kimura, H. (1981). A new approach to perfect regulation in multivariable control systems. IEEE Trans. Aut. Contr., AC-26, 253-271.

Kouvaritakis, B., Shaked, U. (1976). Asymptotic behaviour of the root-loci of linear multivariable systems. Int. Jrnl. Contr., 23, 297-340.

Munro, N. (1972). Design of controllers for open-loop unstable system using the inverse Nyquist array. Proc. Inst. Electrical Engrs., 119, 1377-1382.

Owens, D.H. (1974). Feedback stability of open-loop unstable systems: contraction mapping approach. Electron. Lett., 10, 238-9.

Owens, D.H. (1978). Feedback and Multivariable Systems. Peter Peregrinus, Stevenage.

Owens, D.H. (1979a). Compensation theory for multivariable root-loci. Proc. Inst. Elec. Engrs., 126, 538-542.

Owens, D.H. (1979b). Discrete first-order-models for multivariable process control. Proc. Inst. Electrical Engrs., 126, 525-530.

Owens, D.H. (1981a). On the effect of non-linearities in multivariable first order process control. Proceedings Inst. Electrical Engrs. International Conference 'Control and Its Applications', Univ. Warwick (UK), March 23-25.

Owens, D.H. (1981b). Multivariable root-loci: an emerging design tool? Proceedings of the Inst. Electrical Engrs. International Conference, 'Control and Its Applications', Univ. Warwick (UK), March 23-25, 2-7.

Owens, D.H., Chotai, A. (1980). Robust control of unknown or large-scale systems using transient data only. Dept. Control Eng., Univ. Sheffield, Research Report No.134.

Owens, D.H., Chotai, A. (1981). Controller design for unknown multivariable systems using monotone modelling errors. Dept. Control Eng., Univ. Sheffield, Research Report No. 144.

Penttinen, J., Koivo, H.N. (1980). Multivariable tuning regulators for unknown systems. Automatica, 16, 393-398.

Porter, B. (1981). Design of error-actuated controllers for unknown multivariable plants. Electron. Lett., 17, 106-7.

Rosenbrock, H.H. (1974). Computer-aided-design of control systems. Academic Press.

Shaked, U. (1976). Design techniques for high gain feedback stability. Int. Jrnl. Contr., 24, 137-144.

Willems, J.C. (1981). Almost invariant subspaces: an approach to high gain feedback design. IEEE Trans. Aut. Contr., AC-26, 235-253.

Ziegler, J.G., Nichols, J.G. (1942). Optimum settings for automatic controllers. Trans. ASME, 64, 75-91.

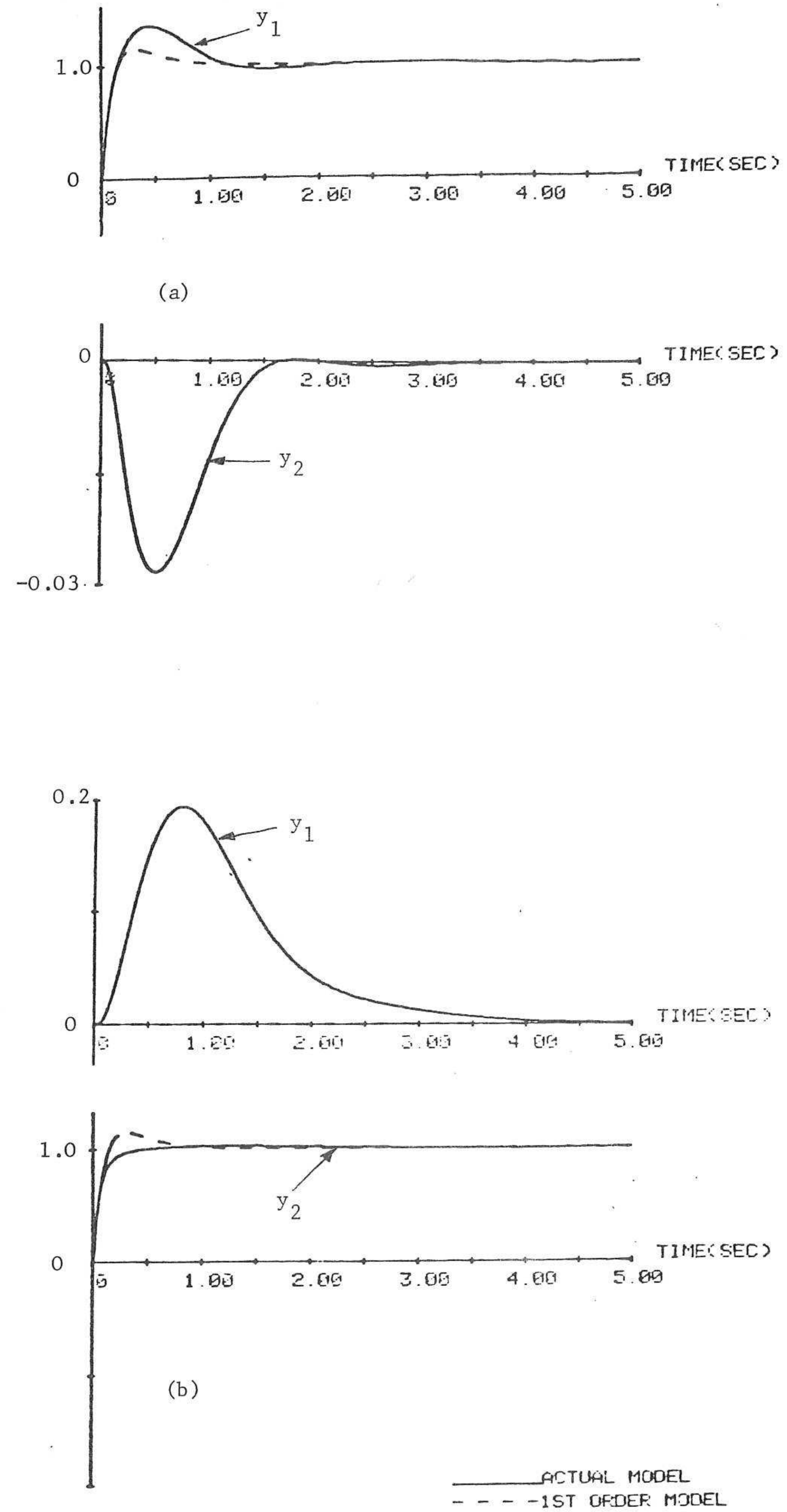

Fig.1. Closed-loop responses to a unit step demand in (a) y_1 and (b) y_2

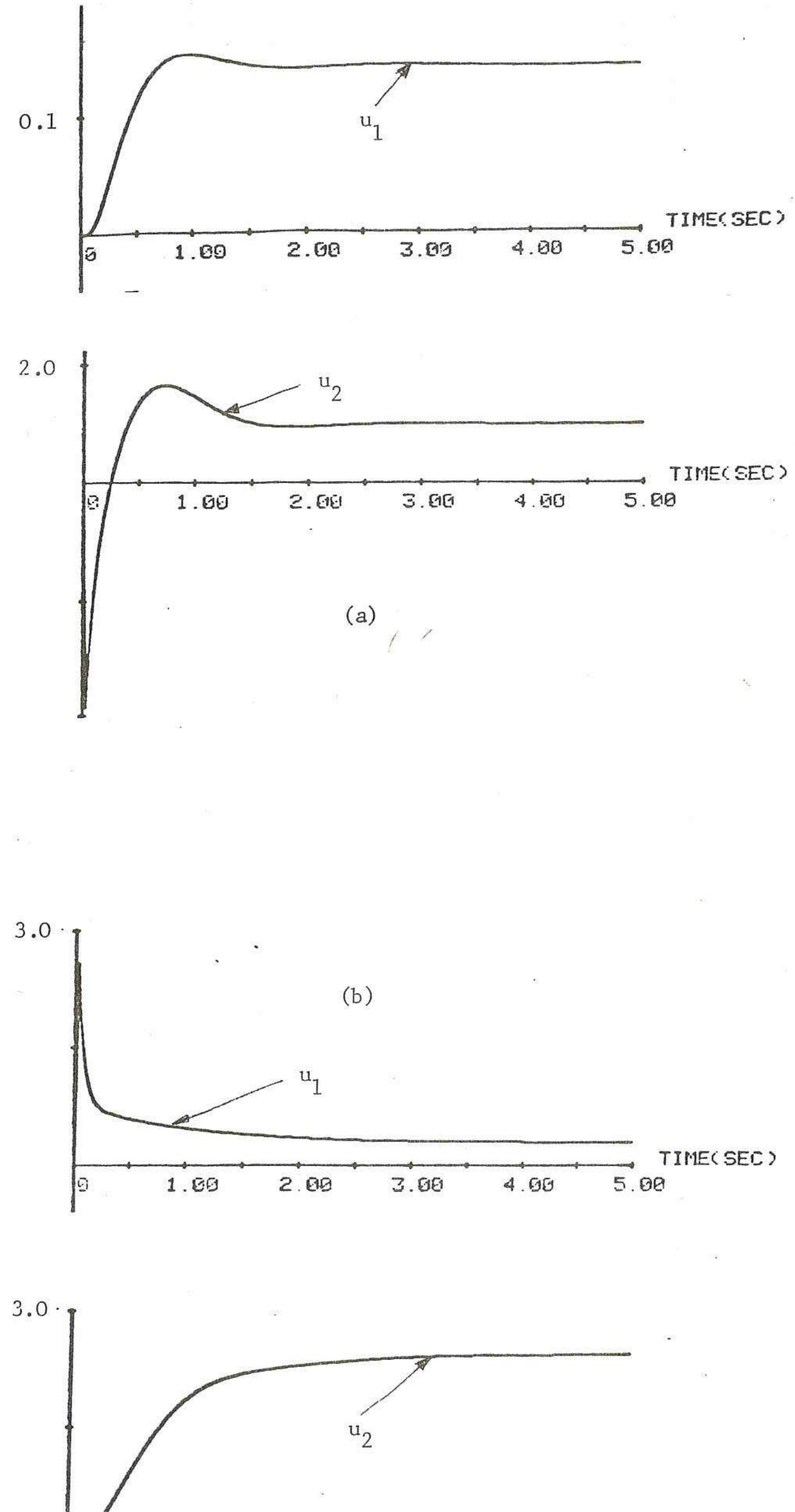

Fig.2. Form of input functions