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SIMPLE MODELS FOR ROBUST CONTROL OF UNKNOWN OR BADLY-DEFINED MULTIVARIABLE SYSTEMS

by

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ABSTRACT

The paper considers the problems of controller design for unknown or badly-defined multivariable systems if the decision is taken to use fixed proportional plus integral controllers with no adaptive or self-tuning mechanism. Attention is focussed primarily on the case of discrete/sampled-data systems with synchronous input and output sampling. It is shown that controller design can very often be undertaken on the basis of a 'rough and ready' plant model that is a multivariable generalization of the classical notion of a first order lag deduced from open-loop plant step response data. The consequent controller is capable, despite possibly large modelling errors, of generating high-performance feedback systems exhibiting fast response speeds, zero steady state errors and small loop interaction effects. The transient effect of bounded nonlinearities such as measurement dead-zone and/or quantization effects is also discussed. Several numerical examples are described.

1. Introduction

Almost without exception frequency domain methods for the design of feedback control schemes for both scalar (Raven (1978)) and multivariable systems (Rosenbrock (1974), Owens (1978), Harris and Owens (1979) and MacFarlane (1980)) rely upon the existence of a model of the process to be controlled (the plant) in a form suitable as a basis for design calculations such as simulation, transfer function matrix or frequency response evaluation, calculation of poles and zeros etc. There are many instances however when a plant model is not known or the available plant model (obtained perhaps from a detailed analytical modelling exercize) is so complex that design calculations other than simulation are not feasible with available computing facilities. In either situation the plant model is (at least partially) unknown for the purposes of controller design yet the problem of constructing the control system still remains!

At the present time, there appear to be three general philosophies providing possible solutions to the problem:

(a) Identification (Eykhoff (1974)) of a low-order approximate system model from off-line analysis of input/output data obtained from plant records or simulation of a more complicated plant model. The resulting model is then used as the basis of controller design and the success of the approach assessed by on-line tuning at the commissioning stage or by extensive simulations of the controller using the real plant model.

(b) Self-tuning control of the unknown system (as described in other chapters) using a control strategy based on an assumed low-order parametric system model and on-line identification of the required controller parameters.

(c) Robust design of the control system in a manner ensuring that closed-loop stability and performance are insensitive to the unknown components of system dynamics.

All three philosophies have their own problems and areas of applicability and it is not the purpose of this chapter to make abstract judgements. We will however restrict attention to the notion of robust controller design for unknown systems and highlight its place in the scheme of things using the following observations:

(i) If a controller designed on the basis of an identified plant model produces satisfactory closed-loop performance from this model, it is not necessarily true that the real plant is even stable and, if stable, the design is not necessarily insensitive to modelling errors, time-variation of parameters or nonlinearities. This is particularly true if high performance specifications are demanded for the closed-loop system!

(ii) Self-tuning controllers are known to be capable of providing useful solutions to practical design problems and a number of stability conditions are known (see other chapters) when the plant model and the identified model have the same order. Little is known, however, of the general effect of order mismatch on the performance of the algorithms or of the effect of nonlinearities!

(iii) Both identification and self-tuning concepts require either access to sophisticated identification software or the use of high-level control hardware and software.

In this chapter it will be shown that it is possible to identify a class of unknown multivariable process plant for which robust proportional plus integral control systems can be designed without encountering the difficulties (i)-(iii) above. Clearly if a given piece of plant belongs to this class, robust design is a powerful alternative to the other strategies. This power is, however, obtained at a price - it is (in theory) necessary to have certain *a priori* information on the system structure! In many cases this structural information may well be self-evident from the physical laws governing dynamic behaviour, but, in other cases, it may be necessary to assume that the structure is correct and assess the validity of the assumption by the success (or failure) of the final design.

The conceptual basis of the ideas, as illustrated by the well-known technique from classical control of approximating plant dynamics by a first-order lag, is described in section two together with its theoretical justification from root-locus ideas. Section three extends the notion of a first-order lag to the case of multivariable process plant (Owens 1978, 1979) and outlines how simple two term controllers ensuring fast system responses with the required overshoot and damping characteristics and small loop interaction effects are easily designed without the need for other than 'back of envelope' computing facilities. In section four the use of the 'first-order controller' for the control of higher order plant is described (Owens 1979, Owens and Chotai 1980a) with emphasis on prediction of stability and transient performance and an evaluation of its sensitivity/robustness (Owens and Chotai 1981) to data errors. Section five describes some illustrative applications of the theory and, in section six, it is noted that the design technique is easily extended to cope with assessment of the effect of measurement nonlinearities (Boland and Owens 1980, Owens 1981a) such as quantization and deadzone. Finally, in section seven, a brief review of current work in the area is given.

Throughout the chapter attention is focussed primarily on the intuitive source of the ideas, the form of the rigorous mathematical results and their interpretation in practice. The proofs (which rely on extensive use of linear systems theory and functional analysis) can be found in the references.

2. The Single-input/Single-output Case: A Motivation

In order to illustrate the basic notion underlying the technical content of this chapter, consider the single-input/single-output system described by an n-dimensional linear, time-invariant model of the state-space form

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) \qquad \ldots(2.1)$$

or the equivalent differential equation

$$P(D)y(t) = Q(D)u(t) \qquad \ldots(2.2)$$

over the range of signal amplitudes of interest. Here A,B,C are real matrices of appropriate dimension and P,Q are polynomials in the 'D-operator' D = d/dt. Denote the system transfer function by

$$G(s) \triangleq C(sI_n-A)^{-1}B \equiv \frac{Q(s)}{P(s)} \qquad \ldots(2.3)$$

and consider the problem of designing a two-term controller K(s) for G(s) to ensure the required stability, transient performance and tracking characteristics from the closed-loop system shown in Fig.1(a).

It is clear that controller design for the plant is possible in the normal manner if G(s) is known or, at least, if the frequency response $G(i\omega)$, $\omega \geq 0$, and the number of closed right-half-plane poles of G is known. Suppose that, for reasons such as those outlined in the introduction, G(s) is not known but that the system response y(t) to a unit step input has been obtained from plant tests or a simulation of an available complex model. A 'rough and ready' approximate model of the process is easily obtained from the well-known graphical construction shown in Fig.2 and can be represented by the first-order transfer function

$$G_A(s) \triangleq \frac{a}{1 + s\tau} \qquad \ldots(2.4)$$

which is defined and non-trivial if $a \neq 0$ and $y(t)$ has non-zero derivative at $t = 0+$. An alternative form is

$$G_A^{-1}(s) = sA_0 + A_1 \quad , \quad A_0 \neq 0 \qquad \ldots(2.5)$$

where, from the initial and final value theorems, we have the useful identities

$$A_0^{-1} = \frac{a}{\tau} = \left.\frac{dy_A(t)}{dt}\right|_{t=0+} = \left.\frac{dy(t)}{dt}\right|_{t=0+} = \lim_{s\to\infty} sG(s) = CB \qquad \ldots(2.6)$$

(from which we note immediately that $CB \neq 0$ and that the system transfer function must have rank unity) and

$$A_1^{-1} = a = y_A(+\infty) = y(+\infty) = \lim_{s\to 0} G(s) \qquad \ldots(2.7)$$

The unit step response $y_A(t)$ of the approximate first-order model may be very similar to $y(t)$, particularly if there are a large number of dipoles present in $G(s)$, but, in general, it should not be anticipated that the approximation is good in open-loop conditions.

Suppose now that the two-term controller

$$K(s) = \frac{p\tau}{a}\left(1 + \frac{1}{sT}\right) \quad , \quad T > 0 \qquad \ldots(2.8)$$

is designed for the approximate plant G_A to ensure the required stability and transient response characteristics from the approximating feedback system shown in Fig.1(b). This is a straightforward exercize but it is of no great value unless the results enable us to make useful predictions about the stability and performance of the real closed-loop system Fig.1(a) incorporating the final design. It is at this stage of the theoretical work that it is necessary to make some assumption concerning the structure of the unknown real plant. More precisely, we will suppose that, by means unknown, it is known that the plant G is minimum-phase! Remembering from the above that we also require that G has rank unity, a simple classical argument yields the observation that the root-locus of the configuration Fig.1(a) for gains $p \geq 0$ has one first order asymptote and n dipoles in the left-half plane at high gains and hence that

(a) the real closed-loop system is stable for all high enough gains p, and

(b) the closed-loop response has a first order character at high gains.

These ideas are illustrated in Fig.3. The first observation is reassuring as it indicates that controller design on the basis of a rough and ready first order model can be successful. The second observation provides some intuitive justification of the use of the first order model for prediction of closed-loop transient performance.

The justification of the above ideas can be obtained from the rigorous multivariable arguments outlined in Owens (1978), Edwards and Owens (1977) or, in more detail, in Owens and Chotai (1980a). Our main concern is to consider the generalization of this procedure to the case of multivariable sampled-data systems (Owens 1979, Owens and Chotai 1980a). This is described in the following sections but it is useful at this point to highlight the essential features in the context of the scalar case.

Suppose that the system input and output are actuated and sampled synchronously with sample interval h. Assuming piecewise constant inputs, the input and output $u_k \triangleq u(kh)$, $y_k = y(kh)$ are related by the discrete state-space model

$$x_{k+1} = \Phi x_k + \Delta u_k$$

$$y_k = Cx_k \qquad \ldots(2.9)$$

or by an equivalent difference equation

$$\tilde{P}(z^{-1})y_k = \tilde{Q}(z^{-1})u_k \qquad \ldots(2.10)$$

where $\tilde{P},\tilde{Q}$ are polynomials in z^{-1} and

$$\Phi = e^{Ah} \quad , \quad \Delta = \Phi\int_0^h e^{-At}B\,dt \qquad \ldots(2.11)$$

The system z-transfer function is denoted

$$G(z) \triangleq C(zI_n-\Phi)^{-1}\Delta \equiv \frac{Q(z^{-1})}{P(z^{-1})} \qquad \ldots(2.12)$$

A 'rough and ready' approximate model of the process can now be obtained from a known response from zero initial conditions to a unit step input by the construction illustrated in Fig.4 to yield a first-order z-transfer function

$$G_A(z) = \frac{a}{1 + (z-1)a/b} \qquad \ldots(2.13)$$

which is defined and non-trivial if $a \neq 0$ and $y(h) \neq 0$. An alternative form is

$$G_A^{-1}(z) = (z-1)B_o + B_1 \quad , \quad B_o \neq 0 \qquad \ldots(2.14)$$

where

$$B_o^{-1} = y_A(h) = y(h) = \lim_{|z|\to\infty} zG(z) = C\Delta \qquad \ldots(2.15)$$

and

$$B_1^{-1} = a = y_A(\infty) = y(\infty) = \lim_{z\to 1} G(z) \qquad \ldots(2.16)$$

Clearly the real and approximate plants have identical initial and steady state response characteristics.

Consider now the question of identifying situations when a two-term controller designed using the simple first-order model will produce a stable closed-loop system with the required performance characteristics when applied to the real plant G. This problem is non-trivial (Owens 1979, Owens and Chotai 1980a) but the intuitive basis can be motivated using the results for the continuous case. More precisely, we have seen that the use of 'high' control gains is sufficient to ensure success in the continuous case. Interpreting the use of 'high' gains as the generation of 'fast' closed-loop responses and noting that fast closed-loop responses will require fast sampling conditions in the discrete case, it is intuitively obvious that the use of 'fast' sample rates will form part of a set of sufficient conditions in the discrete case. Noting also that, under fast sampling conditions, a discrete system will tend to behave (roughly speaking) like a continuous system we must also expect that, as in the continuous case, the underlying continuous system will need to be minimum-phase with rank one transfer function. In summary, on intuitive grounds, the following conditions can be expected to be sufficient to guarantee the success of controller design based on the first-order approximate model:

(a) The sample rate h^{-1} must be 'reasonably high', and

(b) The underlying continuous system should be minimum-phase with rank one transfer function.

These statements are in fact correct and carry over to the multivariable case provided that a little care is taken over the choice of control system! The generalization is described in the following sections.

3. Discrete First-order Lags: The Multivariable Case

The generalization of the material of section two to the multi-input/multi-output case relies upon the generalization of the idea of a first-order lag (Owens 1979, 1981b and Owens and Chotai 1980a). By analogy with the scalar case (in the form of equation (2.14)) an m-input/m-output system with synchronous output sampling and control actuation of frequency h^{-1} and with mxm z-transfer function matrix $G_A(z)$ is an mxm discrete first-order lag if, and only if,

$$G_A^{-1}(z) = (z-1)B_o + B_1 \qquad \ldots(3.1)$$

for some mxm matrices B_o, B_1 with $|B_o| \neq 0$. In the case of $m = 1$, this definition clearly reduces to the familiar scalar first-order lag.

The suggested controller for G_A has the multivariable proportional plus summation form

$$K(z) = B_o \operatorname{diag}\{1-k_jc_j + \frac{(1-k_j)(1-c_j)z}{(z-1)}\}_{1\leq j\leq m} - B_1 \qquad \ldots(3.2)$$

parameterized in terms of the parameter matrices B_o, B_1 and scalars $k_1,..,k_m$ and $c_1,..,c_m$. The controller can be realized via the state-variable model

$$q_{k+1} = q_k + e_k$$

$$u_k = B_o \, \text{diag}\{(1-k_j)(1-c_j)\}_{1\leq j\leq m} q_k$$

$$+ (B_o \, \text{diag}\{2-k_j-c_j\}_{1\leq j\leq m} - B_1)e_k \qquad \ldots(3.3)$$

or, in cases where $(1-k_j)(1-c_j) = 0$ for a number of indices j, a minimal realization of this model obtained by deleting the states in q that correspond to these indices.

After a little manipulation, the closed-loop z-transfer function matrix

$$H_c(z) \triangleq (I_m+G(z)K(z))^{-1}G(z)K(z)$$

$$\equiv \text{diag}\left\{\frac{1}{(z-k_j)(z-c_j)}\right\}_{1\leq j\leq m} (z\,(\text{diag}\{2-k_j-c_j\}_{1\leq j\leq m}$$

$$-B_o^{-1}B_1) + (B_o^{-1}B_1 - \text{diag}\{1-k_jc_j\}_{1\leq j\leq m})) \qquad \ldots(3.4)$$

which takes the form

$$H_c(z) = \text{diag}\left\{\frac{z(2-k_j-c_j) - (1-k_jc_j)}{(z-k_j)(z-c_j)}\right\}_{1\leq j\leq m} \qquad \ldots(3.5)$$

in the special (and very important) case of $B_o^{-1}B_1 = 0$. Clearly the closed-loop system is stable if

$$-1 < k_j < 1 \quad , \quad -1 < c_j \leq 1 \quad (1\leq j\leq m) \qquad \ldots(3.6)$$

with rise-times and reset times in loop j obtained by suitable choice of 'tuning' parameters k_j, c_j respectively and zero steady state errors in response to steps if $c_j \neq 1$. In particular, if $B_o^{-1}B_1 \simeq 0$, equation (3.5) indicates that the closed-loop system also possesses small interaction effects between its loops.

In summary it is seen from the above that the given controller (3.2) is capable of producing the required rise and steady state responses from G_A by suitable choice of parameters, and, in a given special case, the closed-loop system is also approximately non-interacting. A more detailed analysis (Owens 1979, Owens and Chotai 1981a) also indicates that the special condition $B_o^{-1}B_1 \simeq 0$ can always be achieved under fast sampling conditions and hence will frequently be encountered in practice.

4. Control Design for Unknown Discrete Multivariable System using a First-order Approximate Model

Consider an m-input/m-output sampled-data system with synchronous input actuation and output sampling of frequency h^{-1} and described by a state-variable model of the form of (2.9) which is supposed to be generated by the underlying continuous model (2.1).

4.1 Construction of First Order Approximate Model

The approximate model (3.1) is characterized by the matrices B_o and B_1. The method of estimating them depends upon whether or not the large, complex model (2.9) is known. If it is then B_o can be defined by the natural generalization of (2.15) to the multivariable case,

$$B_o^{-1} = \lim_{|z|\to+\infty} zG(z) = C\Delta \qquad \ldots(4.1.1)$$

provided that $C\Delta$ is nonsingular. It turns out (Owens and Chotai 1980a,1981) that the choice of B_1 is not too critical. If we require the approximate model to reproduce the same steady state characteristics as the plant, the natural choice is the generalization of (2.16), namely

$$B_1^{-1} = \lim_{z\to 1} G(z) = C(I_n - \Phi)^{-1}\Delta \qquad \ldots(4.1.2)$$

if $I_n - \Phi$ is nonsingular. Alternatively the choice of $B_1 = O$ can be acceptable (Owens 1979, Owens and Chotai 1980a) and has the advantage of considerably simplifying the form of the approximate model. In more general situations it may be possible (Owens and Chotai 1980a) to choose B_1 to achieve other desirable properties but such considerations are outside the scope of this chapter.

Supopose now that the model (2.9) is not known but that plant tests are undertaken to estimate the output vector sequence $\{y_1^{(i)}, y_2^{(i)}, \ldots\}$ generated by a unit step input in the i^{th} plant input from zero conditions and that these experiments are repeated for all indices, $1 \leq i \leq m$. Defining

$$Y_k = [y_k^{(1)}, y_k^{(2)}, \ldots, y_k^{(m)}] \quad , \quad k \geq 0 \qquad \ldots(4.1.2)$$

then it is clear that

$$C\Delta = Y_1 \qquad \ldots(4.1.3)$$

and, if the plant is stable, that

$$C(I_n - \Phi)^{-1}\Delta = \lim_{k\to\infty} Y_k \qquad \ldots(4.1.4)$$

Relations (4.1.3) and (4.1.4) can then be used in (4.1.1) and (4.1.2) respectively to obtain computed estimates of B_o and B_1.

4.2 Stability and Performance of the Real Feedback System

Consider now the problem of predicting the stability and performance characteristics of the feedback system of Fig.1(a) (with the controller (3.2) deduced from a first order approximate plant model) in terms of the characteristics of the approximating feedback system of Fig.1(b). Several approaches are possible (Owens and Chotai 1980a) based upon refinements of the following result:

Theorem 4.1: An unknown m-input/m-output discrete multivariable plant (2.9), known to be generated from an unknown continuous multivariable plant (2.1) that is both minimum-phase and of uniform rank one (ie CB is nonsingular (Owens 1978)), will be stable in the presence of unity negative feedback with forward path controller (3.2) deduced from a first order approximate model if

(a) the tuning parameters k_j, c_j, $1 \le j \le m$, satisfy (3.6) (ie the approximating feedback system Fig.1(b) is stable),

(b) the sampling rate h^{-1} is sufficiently fast

Moreover, under these simple conditions, we have, for each reference demand sequence $\{r_k\}_{k \ge 0}$, the relation

$$\lim_{h \to 0+} (y_k - (y_A)_k) = 0 \qquad \ldots(4.2.1)$$

holds uniformly on the integers $k \ge 0$.

Remarks: (i) In more general situations (Owens and Chotai 1980a) it is necessary to add in the condition that the procedure for choosing B_1 is such that $B_o^{-1}B_1 \to 0$ as $h \to 0+$. This will always be the case if the procedures of section 4.1 are followed!

(ii) It is not necessary that the real plant is stable, nor that the real and approximate plants have similar stability characteristics.

Despite its simple form, the theorem requires careful interpretation for the purpose of application. More precisely, the result states that, if direct computation (using a model when available) or a combination of physical insight and intelligent guess work suggests that the discrete plant under consideration has underlying continuous dynamics possessing the structural properties of being minimum-phase (Owens 1978) with CB nonsingular, then the procedure of approximating open-loop plant dynamics by a multivariable discrete first order lag using the techniques of section 4.1, designing the controller using this model using the results of section 3 and finally implementing the designed controller on the real plant will be successful if (a) the controller stabilizes the approximate plant and (b) the sampling rate used in the implementation of the control scheme is fast enough. Success is, of course, measured by the consequent stability of the final design implemented on the real plant and, in particular, the fact (deduced from (4.2.1) that, under fast sampling conditions, the closed-loop responses of real and approximating feedback systems will be almost identical.

The application of the result is illustrated in later sections. It is important to point out that, except in certain special cases (Owens and Chotai 1980a), it is not possible to provide computable estimates of the 'slowest' sampling rate $(h^*)^{-1}$ that will guarantee stability. This is a direct consequence of our assumed ignorance of those parts of the dynamics of the plant that cannot be modelled by first order dynamics and cannot be removed unless more detailed information is used. For a given choice of sampling rate it is clear therefore that the success or failure of the approach will only be discovered when the controller is finally tried out on plant or plant model. If the response characteristics obtained are not satisfactory, the theorem at least points out that the use of an increased sample rate will improve matters. If the responses are satisfactory, then ....

4.3 Sensitivity and Robustness (Owens and Chotai 1981)

An important interpretation of the above results is that careful design of the proportional plus summation control system and the use of reasonably fast sampling will yield a closed-loop system that is insensitive to a large class of perturbations to plant dynamics that leave the chosen values of B_o and B_1 unchanged and also do not violate the minimum-phase requirement. The control system is said to be robust with respect to this class of perturbations. The available proof of the results however (Owens and Chotai 1980a) does rely on exact evaluation of the B_o and B_1. Clearly this is not going to be possible in practice either because of computational errors, the effect of noise or errors in plant measurement equipment. Fortunately it can be shown (Owens and Chotai 1981) that stability of the final closed-loop design will be achieved even if large errors are introduced and, in particular, that stability is most sensitive to errors in estimation of B_o.

5. Illustrative Examples

In each of the following examples the 'unknown' system will be represented by a known linear model for the purposes of obtaining comparative responses. This fact will not be used in the controller design however.

5.1 Level Control: A Single-input/single-output Example

Consider the problem of the construction of a proportional plus summation digital controller for the system illustrated in Fig.5 in order to regulate the liquid level in vessel one to a specified equilibrium level. It is assumed that input actuation and output sampling are synchronous.

Assuming linear dynamics the plant has a (assumed unknown) model of the second order form

$$\dot{x}(t) = \begin{bmatrix} -\frac{1}{a_1}\left(\frac{1}{R_1}+\frac{1}{R_{12}}\right) & \frac{1}{a_1 R_{12}} \\ \frac{1}{a_2 R_{12}} & -\frac{1}{a_2}\frac{1}{R_{12}} \end{bmatrix} x(t) + \begin{bmatrix} \frac{1}{a_1} \\ 0 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t) \qquad \ldots(5.1.1)$$

which has $CB = a_1^{-1} \neq 0$ and one zero at the point $-a_2^{-1}R_{12}^{-1}$ in the left-half complex plane ie this system is always of uniform rank one and minimum phase! Taking, for numerical simplicity, the case of $a_1 = a_2 = 1$ and $R_1 = R_{12} = 1$, the system has poles at $s = -0.3$ and $s = -2.7$. Assuming a sampling interval of $h = 0.1$ (which intuitively corresponds to a fairly fast sampling rate for the system), then analysis of the response to a unit step input from zero initial conditions following the procedure suggested in section (4.1) leads to the data $B_0 = 11.02$ and $B_1 = 1.00$ defining the approximating first order plant model. The unit step responses of real plant and first order model are compared in Fig.6. They clearly differ but the first order model captures the 'essential' features of the plant response.

The design of the two-term controller for the approximate model boils down to (dropping subscripts) the choice of k,c. But k and c can be interpreted as governing the rise and reset characteristics of the approximating feedback system in the sense that (equations (3.4) and (3.5)) k can represent the fast pole and c the slow pole of the system. We will therefore choose k = 0.5 and c = 0.95 for illustrative purposes. Theorem 4.1 now tells us that, provided that the chosen sampling rate is fast enough, we can expect that the resulting controller (obtained from (3.2) and the given data) will stabilize the real plant with response characteristics very close to those predicted by the first order model. That this is the case is verified by the responses of real and approximating feedback systems to unit step demands shown in Fig.7. The design procedure has clearly been successful in this case!

5.2 Digital Control of an Open-loop Unstable Multivariable Plant

Consider the digital control of the two-input/two-output unstable batch process discussed by Rosenbrock (1974) with continuous model defined by the matrices

$$A = \begin{pmatrix} 1.38 & -0.2077 & 6.715 & -5.676 \\ -0.5814 & -4.29 & 0 & 0.675 \\ 1.067 & 4.273 & -6.654 & 5.893 \\ 0.048 & 4.273 & 1.343 & -2.104 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & 0 \\ 5.679 & 0 \\ 1.136 & -3.146 \\ 1.136 & 0 \end{pmatrix} , \quad C = \begin{pmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \qquad \ldots(5.2.1)$$

This model is assumed to be known but we will suppose that the designer wishes to attempt controller design using only simple calculations with verifying simulations. It is easily verified that the system is minimum-phase with CB nonsingular and that it is open-loop unstable. Assuming a sampling interval of h = 0.02, simulations of the open-loop responses to unit step demands in each input in the manner described in section 4.1 leads to the data (equation (4.1.3))

$$B_o = \begin{pmatrix} 0.0018 & 9.17 \\ -16.07 & 0.0057 \end{pmatrix} \qquad \ldots(5.2.1)$$

but, as the system is open-loop unstable, (4.1.4) does not hold and we must use (4.1.2) to give

$$B_1 = \begin{pmatrix} 0.141 & 0.296 \\ 0.995 & 2.455 \end{pmatrix} \qquad \ldots(5.2.2)$$

The responses of the first output of the real and approximate model to a unit step input in the first channel are given in Fig.8 for comparative purposes.

The design of the two-term controller for the approximate model relies purely on the choice of tuning parameters. Remembering that k_j and c_j are poles corresponding to rise-time and reset characteristics in the j^{th} loop, choose $k_1 = k_2 = 0.5$ and $c_1 = c_2 = 0.95$ for illustrative purposes and note that the elements of $B_o^{-1}B_1$ are small so that we can expect a high performance controller with little interaction between the two loops of the approximating feedback system. Again theorem (4.1) tells us that, if our choice of sample rate is high enough, the resulting controller (obtained from (3.2) and the given data) will stabilize the real plant with response characteristics close to those predicted by the first order model. This is indeed the case as illustrated by the closed-loop responses of the real and approximating feedback systems given in Fig.9.

6. Effect of Measurement Nonlinearities (Boland and Owens 1980, Owens 1981)

The assumption of linear plant dynamics is very often justifiable in engineering applications, particularly if the primary function of the controller is to act as a regulator about a fixed operating condition with only occasional and small deviations from this point. Also the assumption of control linearity is reasonable as the control calculations are performed 'inside' a computer. It is frequently the case however that the control actuators and, in particular, output sensors possess simple nonlinear characteristics such as dead-zone or quantization. We will restrict our attention to the case of measurement nonlinearity illustrated in Fig.10, where the element N is a time-invariant, memoryless nonlinearity satisfying the constraint (Boland and Owens 1980)

$$||N\eta - \eta||_m \leq \frac{q}{2} \qquad \ldots(6.1)$$

for all mx1 real vectors η. Here $||x||_m = \max_{1\leq j\leq m} |x_j|$ is the normal uniform norm on m-vectors. This class of nonlinearity covers the case of quantization and deadzone and can be extended to cover more general cases (Owens 1981).

The precise problem that must be considered if the plant model is unknown and controller design is undertaken in the manner outlined in section four is the relationship between the dynamics of the approximating linear feedback system of Fig.1(b) and those of the real nonlinear feedback system of Fig.10. The level of ignorance of system dynamics assumed prevents a precise answer to this question that covers all possible conditions. Following the general ideas illustrated in theorem 4.1, it is possible, however, to prove the following nonlinear analogue (Boland and Owens 1980, Owens 1981):

Theorem 6.1: Under the conditions of theorem 4.1, for each reference demand sequence $\{r_k\}_{k\geq 0}$, the responses of the approximating linear feedback system and the real nonlinear feedback system satisfy the relation

$$\lim_{h\to 0+} \| (y_A)_k - (y_{NL})_k \|_m \leq \frac{q}{2} \qquad \ldots(6.2)$$

for each $k\geq 0$, the limit being uniform on this interval.

The practical interpretation of this abstract result follows from the definition of the uniform norm $\|\cdot\|_m$ ie, under the stated conditions, the maximum modulus of the transient error involved in the use of the approximating linear feedback system to predict the dynamics of the real nonlinear feedback system is less than the non-linearity constant $q/2$ under fast sampling conditions. The result can, of course only be applied in an approximate manner in real (finite sampling rate) conditions, but it can be a good working approximation (Boland and Owens 1980) and, at minimum, provides a reassuring guarantee of stability (in the bounded-input/bounded-output sense) and an estimated upper bound on the transient magnitude of any possible limit cycles.

7. Conclusions and a Review of Related Work

The chapter has reviewed the conceptual basis and some formal mathematical results available for the design of stabilizing two-term controllers for a well-defined class of multivariable process plant for which detailed knowledge of process dynamics is not available and illustrated the application of the results by two non-trivial examples. The power of the techniques lie in the guaranteed ability to design high performance, small interaction feedback systems using only a small amount of system data deduced from plant step responses, an assumed knowledge of some elementary structural properties of the underlying continuous system and the implementation of a reasonably fast sampling rate. The ideas do in fact apply in the case of analogue control (Edwards and Owens 1977, Owens 1978, Owens and Chotai 1980a, 1980b, 1981 and Owens 1981) and can be regarded as the natural generalization of the well-known classical notion of fitting a first order model to plant dynamics. In this case, it is a partial generalization (in the delay-free case) of the well-known 'tuning' technique of Ziegler and Nichols (1942) as the designed controller has a simple form, being defined in terms of elementary plant parameters together with a number of 'tuning parameters' that can either be adjusted on-line to produce the required performance or estimated from analysis of the approximate first-order plant model.

The material can be extended in a number of directions to increase its applicability and the information available. Some indication of the effect of nonlinearities has already been obtained, with encouraging results, and the use of more complex approximate models is envisaged to cover cases where a first order model carries insufficient information to make it a useful vehicle for design. This is the topic of present study.

Finally we note that there are clear connections between the ideas expressed here and those expressed in chapter six and the work of Davison (1976), Astrom (1980) and Pentinnen and Koivo (1980) which provide distinct and viable approaches to cover other aspects of unknown systems control. All of these techniques do provide information on stability of the final design but the ideas expressed here (when applicable) have the added bonus of providing information on detailed transient performance in both the linear and nonlinear cases.

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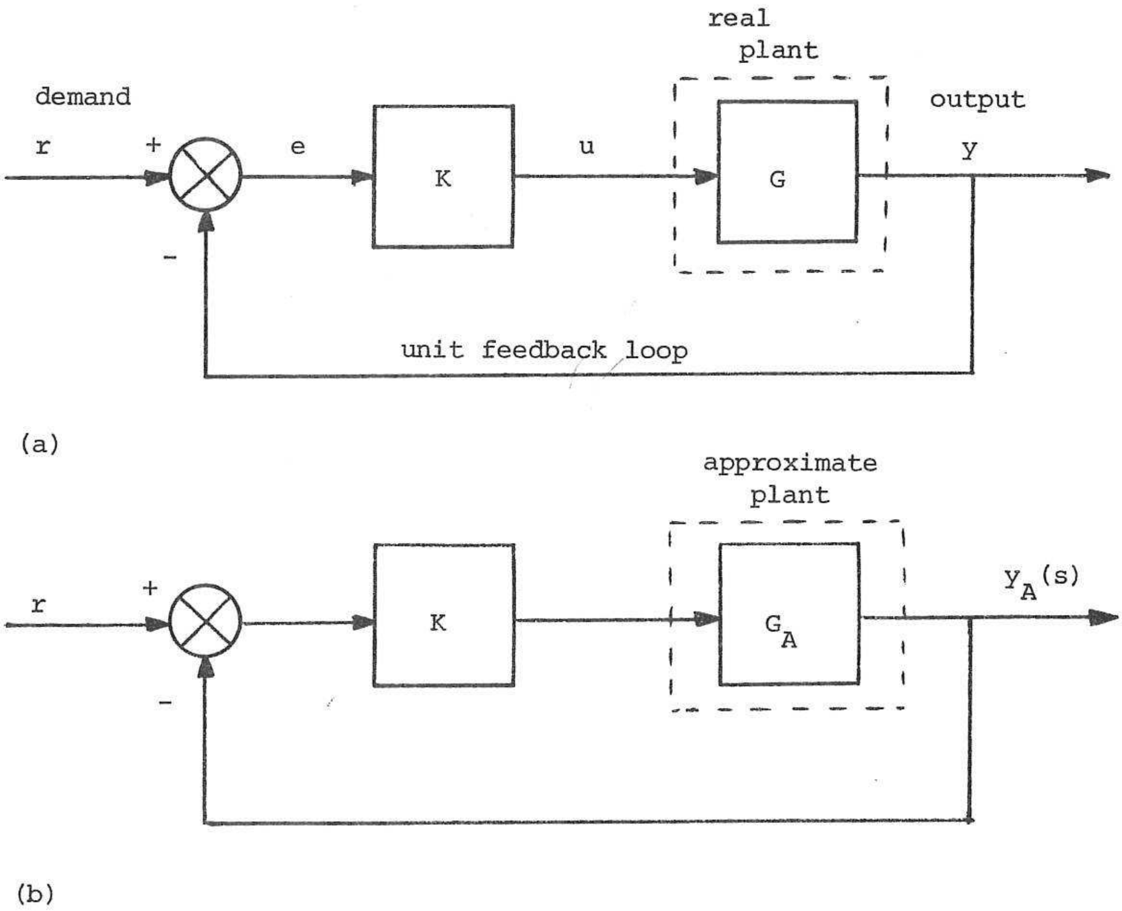

Fig. 1. Real (a) and Approximating (b) Feedback Control Schemes

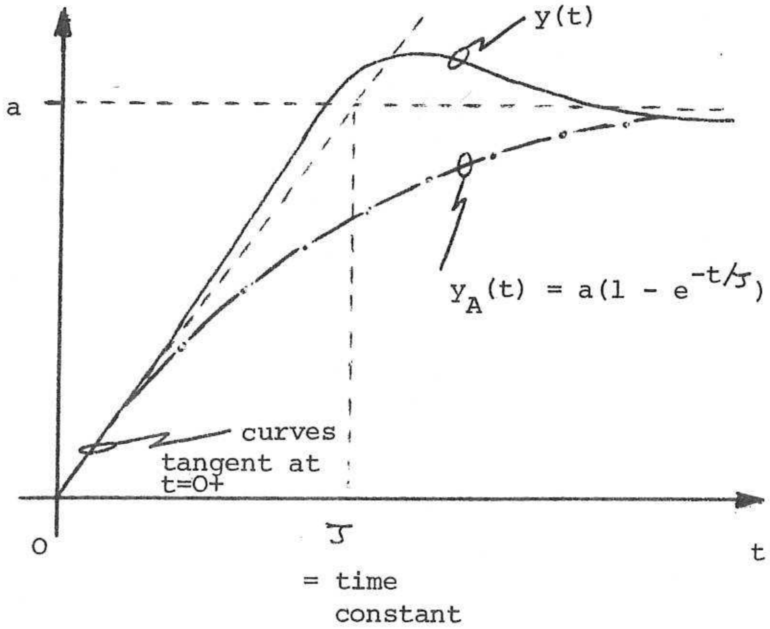

Fig. 2. Construction of Approximate First Order Model from Time Constant and Steady State Data

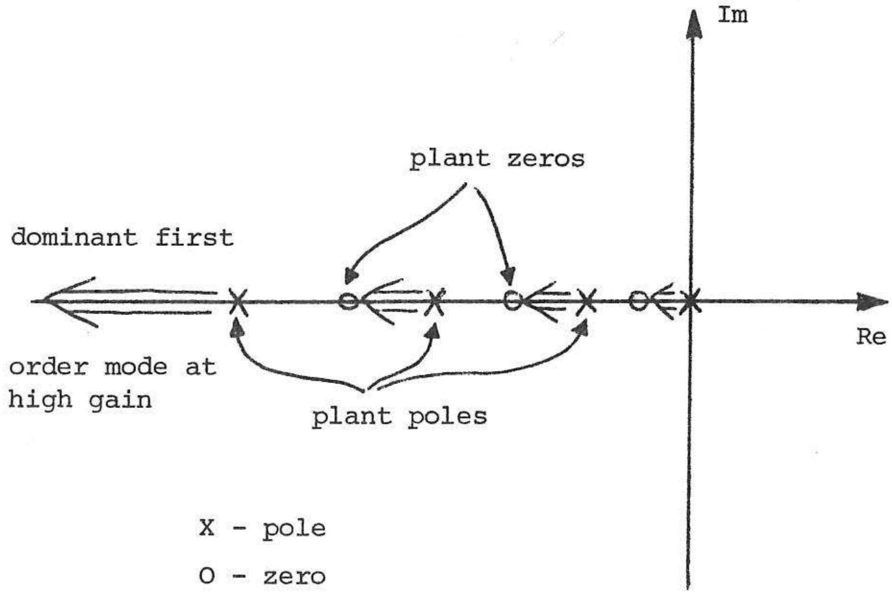

Fig. 3. Typical Root-locus Plot

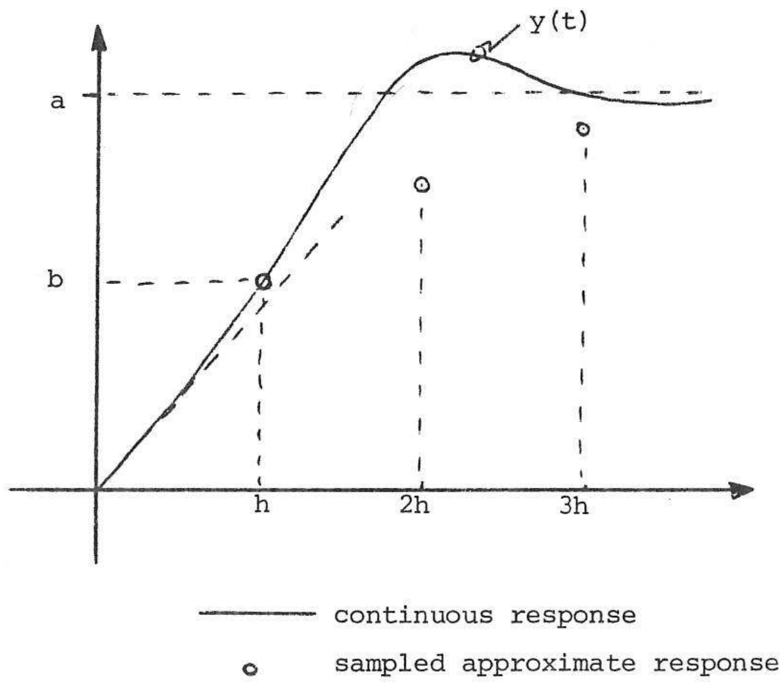

Fig. 4. Graphical Construction for Approximate First-order Discrete System

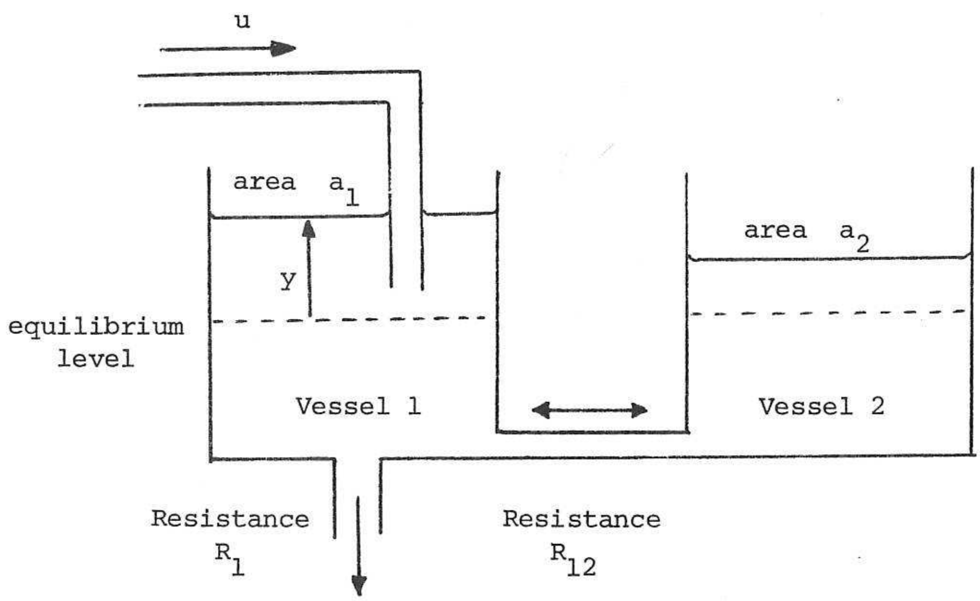

Fig. 5. Liquid Level System

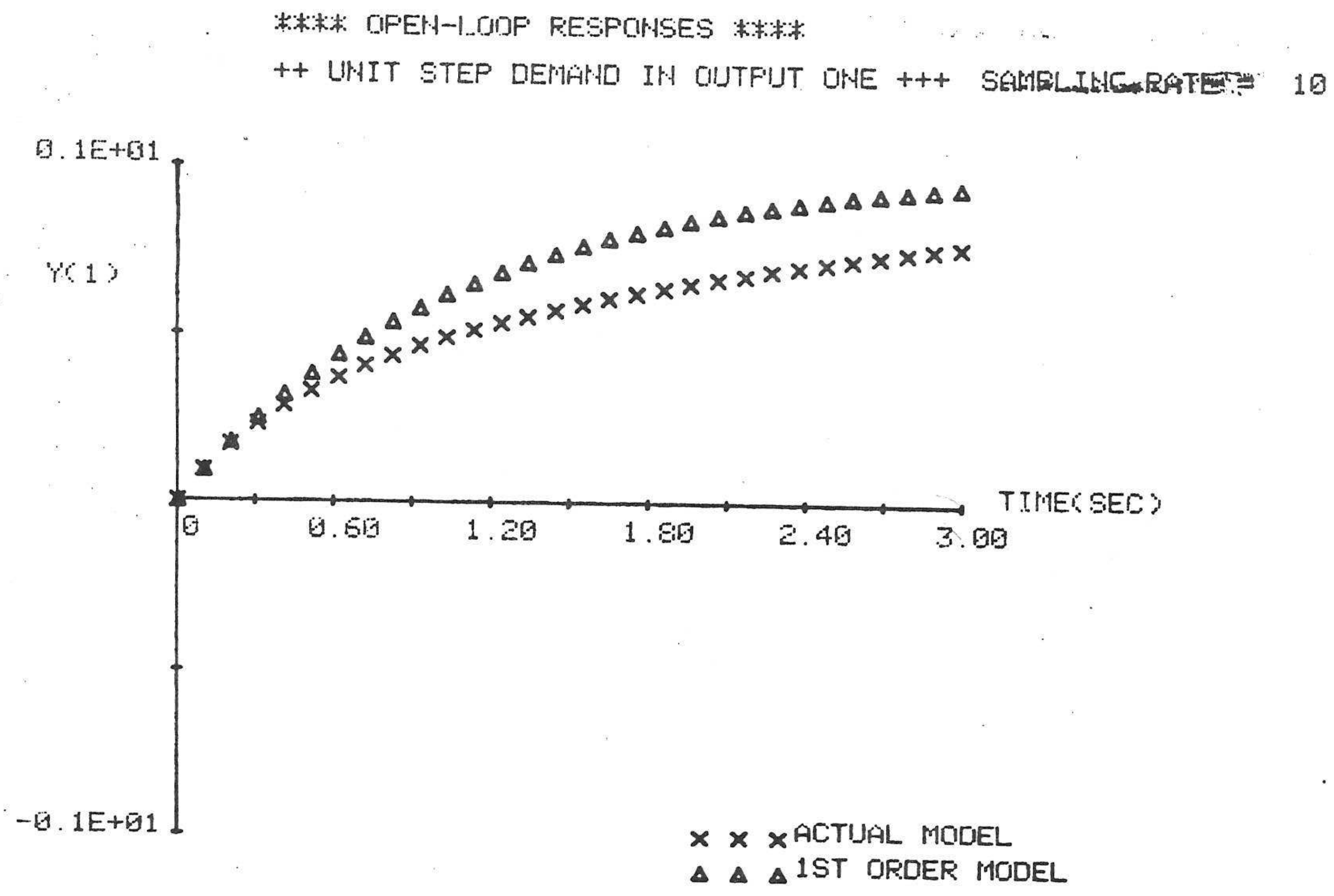

Fig. 6. Unit Step Responses of Real and Approximating Liquid Level Models

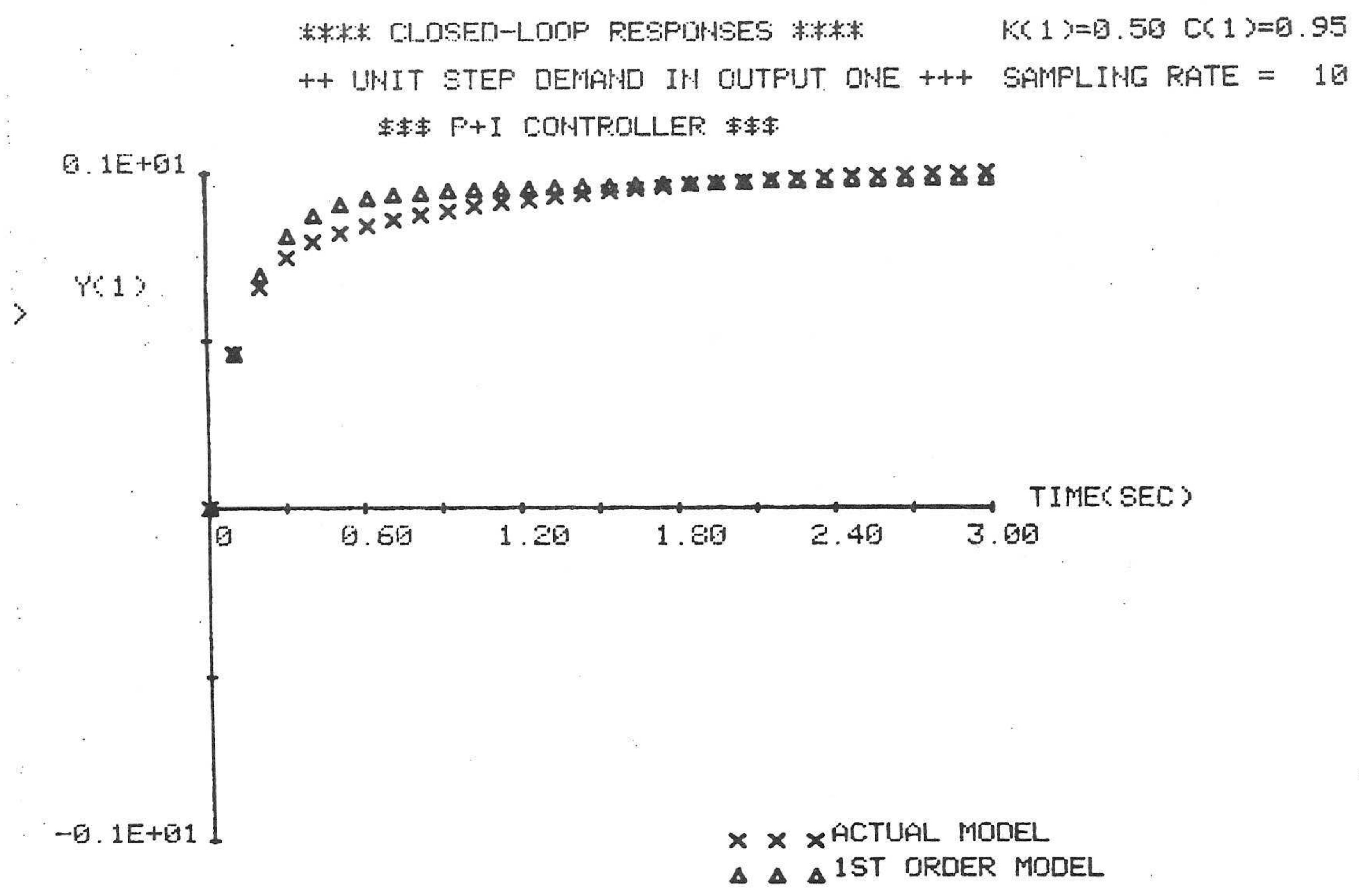

Fig. 7. Closed-loop Unit Step Responses of Real and Approximating Level Control Configurations

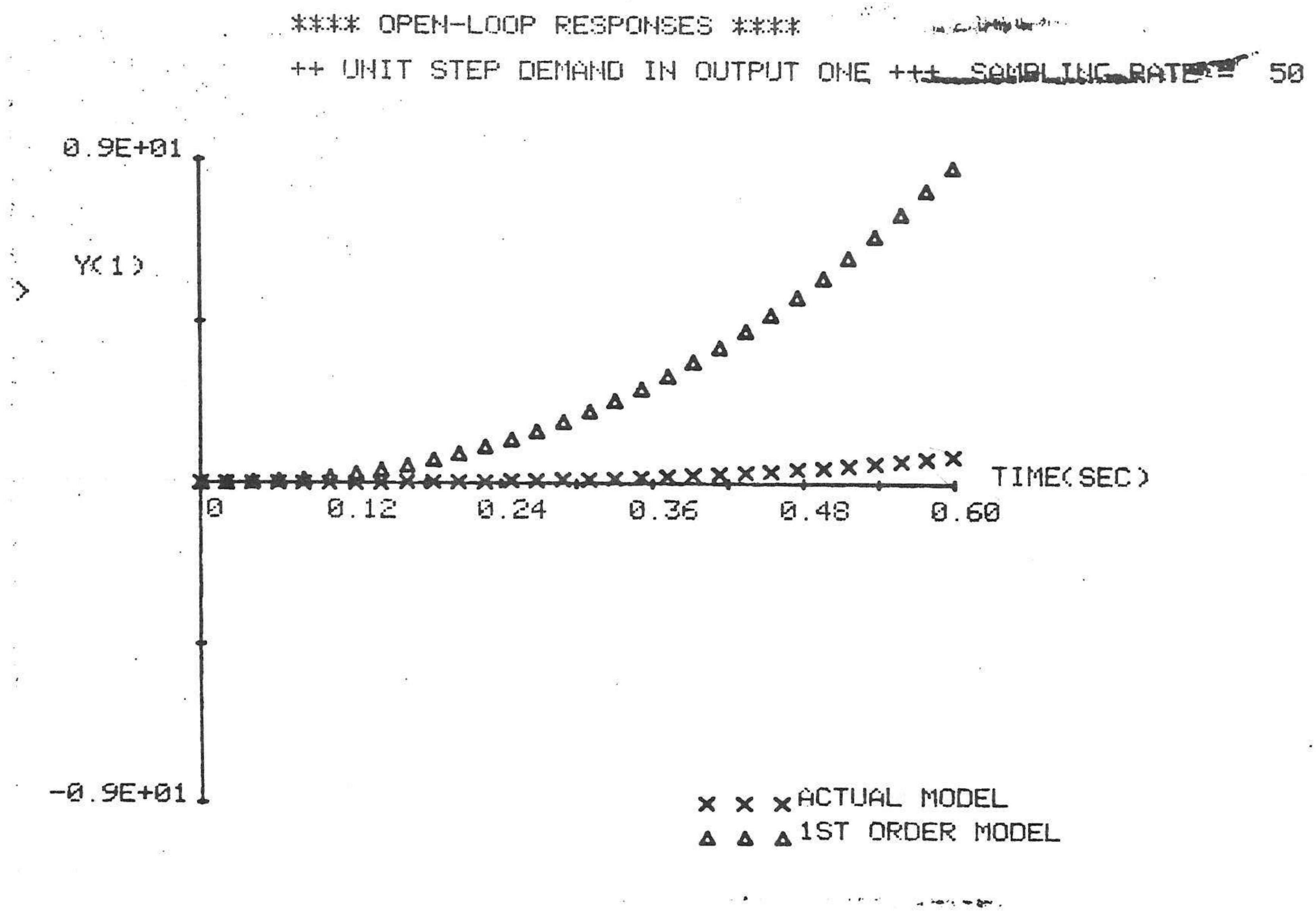

Fig. 8. Responses of Real and Approximating Batch Process

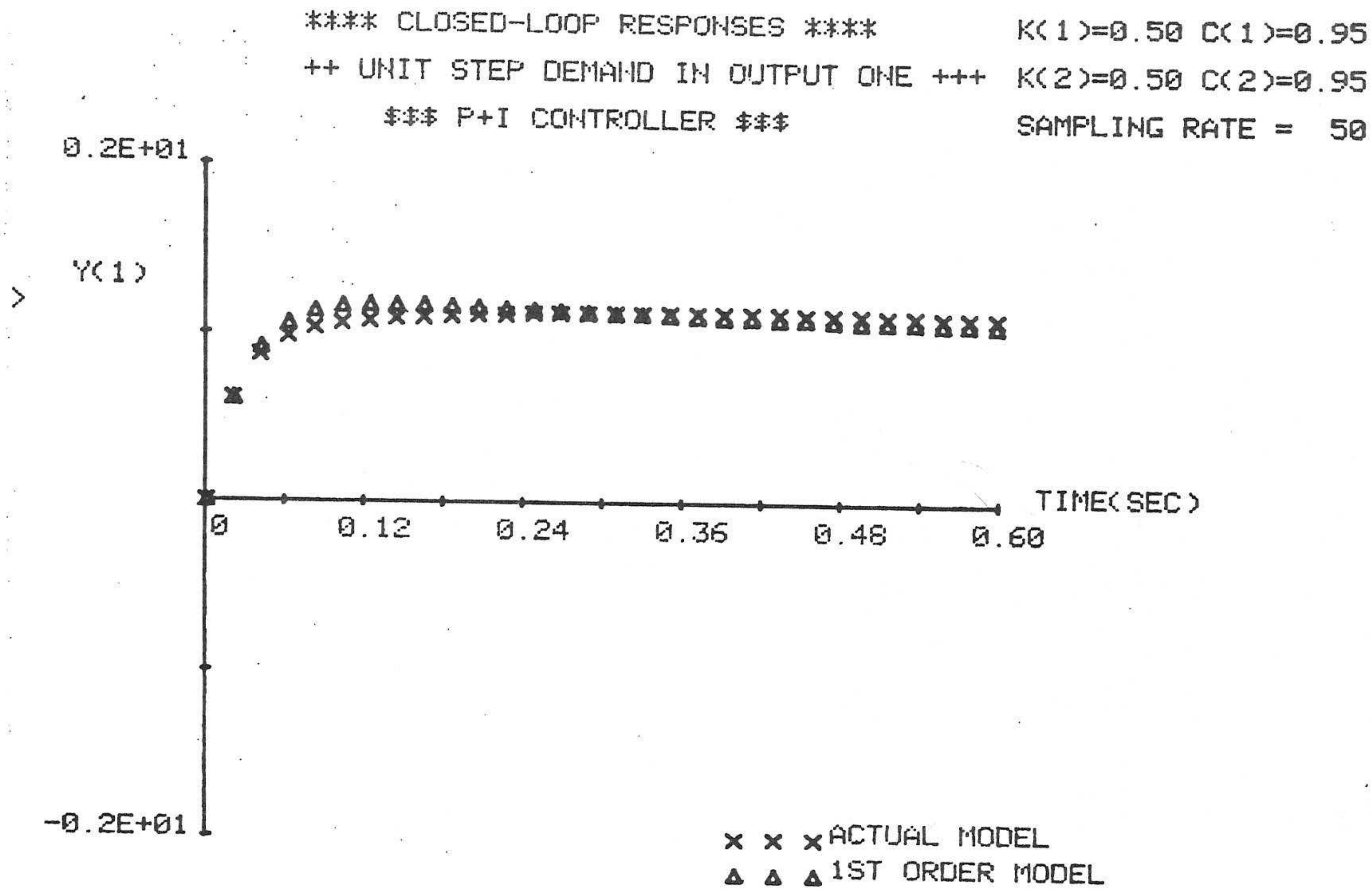

Fig. 9(a). Closed-loop Responses of Real and Approximating Batch Processes to a Unit Step Demand in First Output

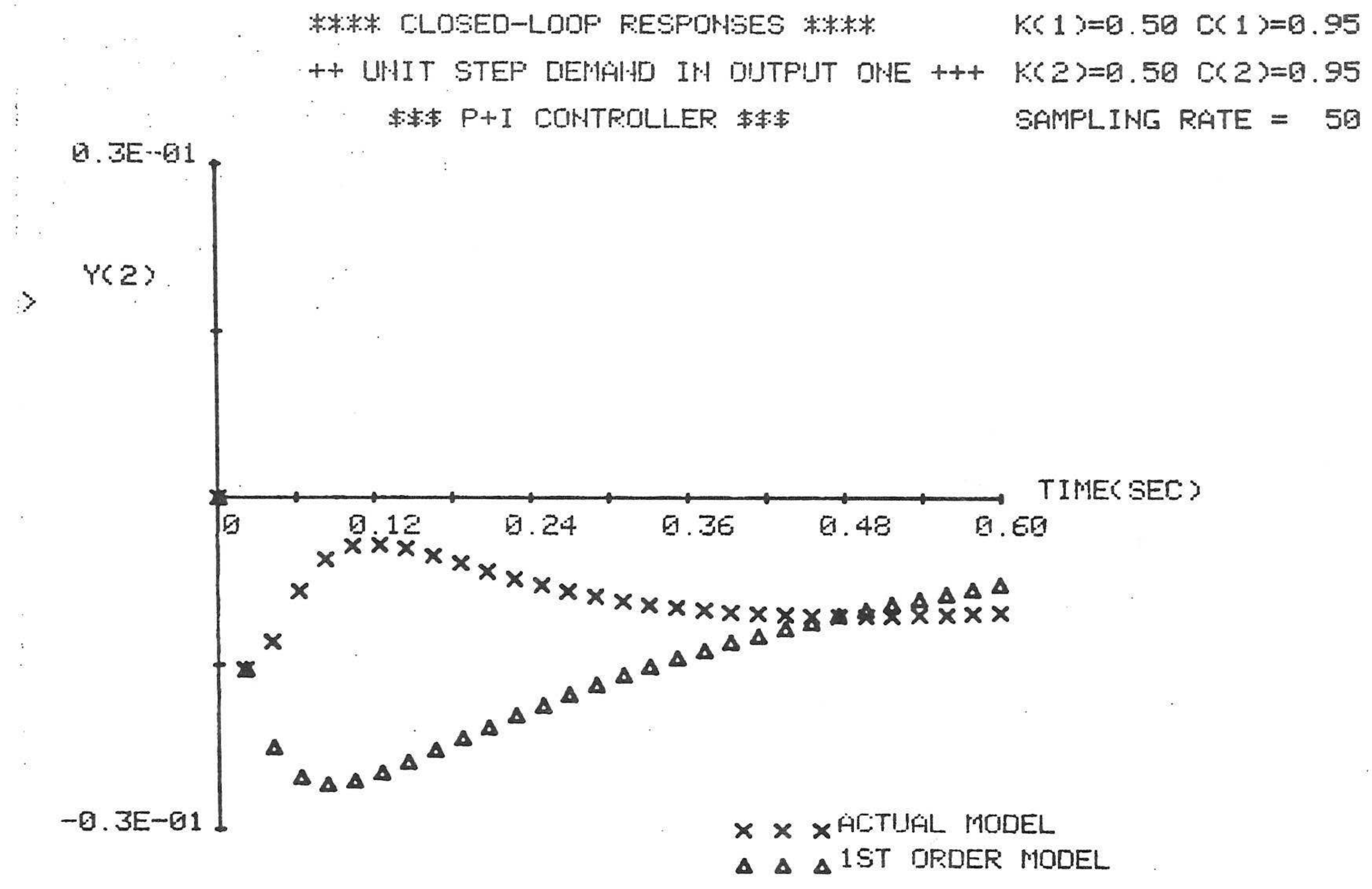

Fig. 9(b). Closed-loop Responses of Real and Approximating Batch Processes to a Unit Step Demand in First Output

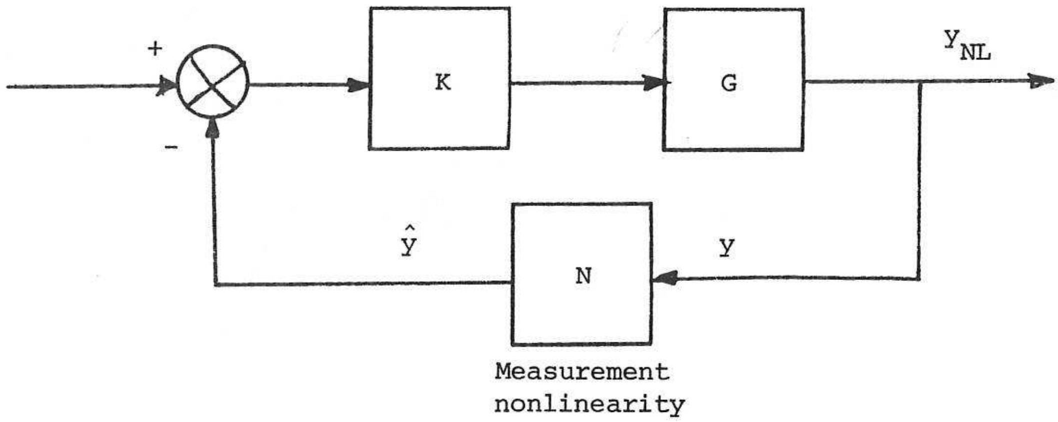

Fig. 10. Multivariable Feedback System with Measurement Nonlinearity

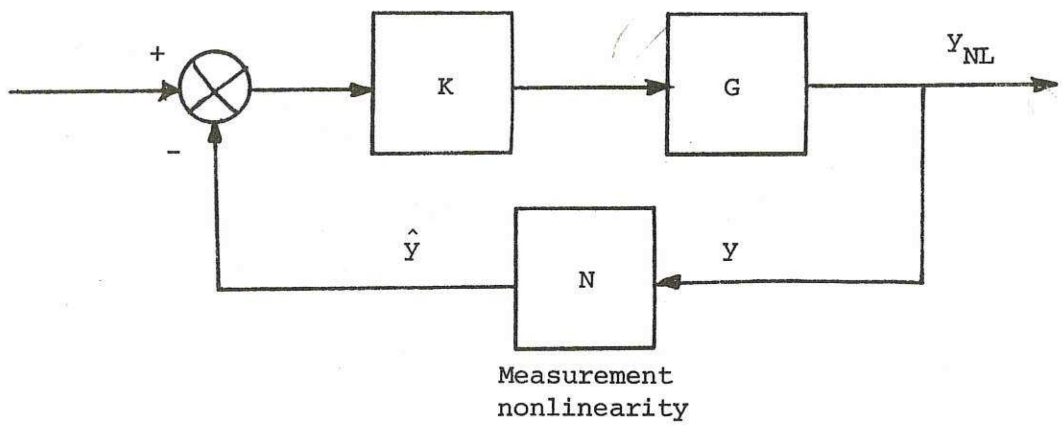

Fig. 10. Multivariable Feedback System with Measurement Nonlinearity