ROBUST CONTROL OF UNKNOWN DISCRETE MULTIVARIABLE SYSTEMS

by

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Abstract

Some recent results on controller design for unknown discrete multivariable plant are extended to include quantitative measures of the robustness of the final design.

A number of recent papers (see, for example, [1] - [4]) have considered the general problem of investigating the robustness of stable multivariable feedback systems in terms of topological or quantitative conditions on admissible plant perturbations that retain closed-loop stability. The greatest impact of these ideas will most probably lie in the characterization of the robustness of specific design techniques (see, for example [1], [5], [6]) where identification of design conditions which guarantee a large degree of robustness will be important in providing both computable measures and rules of thumb for applications purposes. This notion takes its extreme form in the construction of controllers for unknown systems [5], [7]-[12] where the controller must be designed to ensure that closed-loop stability is guaranteed and also insensitive to the unknown components of system dynamics. The purpose of this paper is to discuss the robustness of the technique given in [10] and to extend its applications to provide computable measures of permissible data inaccuracies.

Consider an m-input/m-output linear, time-invariant system described by the model

$$\dot{x}(t) = A\,x(t) + B\,u(t)$$
$$y(t) = C\,x(t) \qquad (1)$$

that is to be controlled digitally using synchronous input actuation and output sampling of sample interval $h > o$. The sampled input and output vectors $u_k = u(kh)$ and $y_k = y(kh)$, $k > o$, are related by a model of the form,

$$
\begin{aligned}
x_{k+1} &= \Phi\, x_k + \Delta\, u_k \\
y_k &= C\, x_k
\end{aligned}
\qquad (2)
$$

with z-transfer function matrix

$$
G(z) = C(zI - \Phi)^{-1}\Delta \qquad (3)
$$

The following fundamental result is proved in Owens [10]:

Lemma: Suppose that the controllable and observable mxm invertible, minimum-phase, discrete system with z-transfer function matrix $Q(z)$ is to be approximated by the mxm, invertible, minimum-phase discrete system $Q_A(z)$. Suppose that Q_A is stable in the presence of unity negative feedback and that the poles of the closed-loop system generated by Q lie in the open ball $|z| < R$ where $R > 1$. Then the system Q is stable in the presence of unity negative feedback if the contraction constant

$$
\lambda \triangleq \left\| (I + Q_A^{-1})^{-1} (Q_A^{-1} - Q^{-1}) \right\| < 1 \qquad (4)
$$

where, if $L(z)$ is any mxm matrix of z analytic and bounded in $\{z: 1 < |z| < R\}$,

$$
\|L\| \triangleq \max_{1 \leq j \leq m} \; \max_{\substack{|z| = 1 \\ |z| = R}} \; \sum_{i=1}^{m} |L_{ji}(z)| \qquad (4)
$$

Remark 1: The use of the parameter R is for technical reasons only and can be set to an arbitrarily large value in applications without loss of information if L is proper. Also, no assumption about open-loop stability is required.

The result has a direct robustness interpretation (in fact, with suitable choice of signal spaces, it is embedded in the results of [2]) but our concern here is the application to stable controller design for unknown discrete systems. More precisely, consider the case when the unknown plant G is unknown in the numerical sense but it is known to have the structure

$$G^{-1}(z) = (z-1)\,B_o + B_1 + B_o\,H(z) \tag{6}$$

where $|B_o| \neq 0$ and $H(z)$ is proper and stable. Equivalently, the plant is minimum phase with $C\Delta(= B_o^{-1})$ nonsingular. The minimum-phase requirement needs a model or physical information for verification but, using the techniques of [10], B_o can be deduced from open-loop plant step response tests. Also, with the above definitions, B_1 is not unique so it will be assumed that it is chosen such that $H(1) = 0$ i.e. B_1 represents plant steady state characteristics and can again be deduced from plant tests [10]. Throughout the remainder of the paper we will assume that H is not known.

The control design procedure discussed in [10] is initiated by estimating B_o and B_1 from plant tests and basing controller design on the first-order approximate model G_A of the form

$$G_A^{-1}(z) = (z-1)B_o + B_1 \tag{7}$$

obtained by ignoring the states generating H in (6). The advantage of this approach is that controller design for G_A is a straightforward matter leading to the proportional plus summation form of controller [10],

$$K(z) = B_o\ \mathrm{diag}\left\{1 - k_j c_j + \frac{(1-k_j)(1-c_j)z}{(z-1)}\right\}_{1\leqslant j\leqslant m} - B_1 \tag{8}$$

which generates a stable feedback system if

$$|k_j| < 1 \quad , \quad -1 < c_j \leqslant 1 \quad , \quad 1 \leqslant j \leqslant m \tag{9}$$

and is capable of producing excellent responses from the approximate feedback system [10].

Applying the lemma with $Q = GK$ and $Q_A = G_A K$ it is immediately deduced that:

Proposition 1: Suppose that G and K are minimum-phase and that the controller K stabilizes the approximate model G_A in the presence of unity negative feedback, then K also stabilizes the real plant G if the contraction constant

$$\lambda = ||(G_A^{-1} + K)^{-1} B_o H|| < 1 \tag{10}$$

This result has the robustness interpretation that K stabilizes all unknown minimum-phase, invertible systems of the form of (6) generated from (7) with H satisfying (10). Of course, if a given system is unknown, then H is unknown and (10) cannot be checked. A useful result can be obtained however if the sample rate h^{-1} is regarded as a design parameter [10]:

Proposition 2 (see theorem 4.1 in Owens [10]): Let the underlying continuous system (1) be minimum-phase with $|CB| \neq 0$. Then for each choice of parameters k_j, c_j, $1 \leq j \leq m$, satisfying (9) there exists a strictly positive h^* such that, for sampling rates $h^{-1} > (h^*)^{-1}$, the discrete system generated by (1) has the form of (6) and the contraction condition (10) is satisfied.

Remark 2: In the proof given in [10] it is, in fact, shown that $\lim_{h \to o+} \lambda = 0$ by proving the useful relationship

$$\lim_{h \to o+} ||H|| = 0 \tag{11}$$

Remark 3: In general h^* is unknown as it depends upon the unknown dynamics of H.

The theoretical interpretation of Proposition 2 is immediate, namely that, in the presence of the defined continuous plant structure, controller design for the discrete plant based upon the approximate first order model will stabilize the plant (and hence, if integral action is included, generate ideal steady state performance in response to step demands) provided that the chosen sampling rate is fast enough. Morever, bearing in mind Remark 2, it is clearly true that <u>any increase in sampling rate will increase the</u>

robustness of the design to changes in the unknown part of system dynamics H.

In the above analysis, despite the robustness of the design with respect to the unknown dynamics H, the parameter matrices B_o, B_1 are assumed to be known exactly. We will now extend the above results to include an assessment of the robustness of the design to errors in these parameter matrices. More precisely, suppose that $\tilde{B}_o$ and $\tilde{B}_1$ are numerical estimates of B_o and B_1 and that the approximate first order plant model (7) and designed controller (8) are generated using these estimates father than the correct values. Applying the lemma with $Q = GK$ and $G_A = G_A K$ it is easily verified that Proposition 1 remains true if (10) is replaced by

$$\lambda = ||(G_A^{-1} + K)^{-1}(G^{-1} - G_A^{-1})|| < 1 \tag{12}$$

We can now state the following main result of this note.

Theorem: Let the underlying continuous system (1) be minimum-phase with $|CB| \neq 0$ and suppose that the parameters k_j, c_j, $1 \leqslant j \leqslant m$, are specified and satisfy (9). Suppose also that

(i) the chosen procedure for choosing $\tilde{B}_1$ is such that

$$\limsup_{h \to o+} ||\tilde{B}_1 - B_1|| < +\infty \tag{13}$$

(ii) the chosen procedure for choosing $\tilde{B}_o$ is such that $|\tilde{B}_o| \neq 0$ and

$$\lambda_\infty \triangleq \limsup_{h \to o+} \max_{1 \leqslant i \leqslant m} \max_{|z|=1} \left|\frac{(z-1)^2}{(z-k_j)(z-c_j)}\right| \sum_{j=1}^{m} |\tilde{B}_o^{-1}(\tilde{B}_o - B_o))_{ij}|$$

$$< 1 \tag{14}$$

Then there exists a strictly positive number h^* such that the controller K of the form of (8) with B_o and B_1 replaced by $\tilde{B}_o$ and $\tilde{B}_1$ respectively stabilizes the plant G for sample rates $h^{-1} > (h^*)^{-1}$.

(Note: Both B_o and B_1, and hence the estimates $\tilde{B}_o$ and $\tilde{B}_1$, are clearly dependent upon the sample rate h^{-1}).

The proof of the theorem is given at the end of the paper. Before giving this however, it is important to interpret the result in the correct manner. More precisely, it is important to interpret conditions (i) and (ii) correctly. The other conditions can then be given the same interpretation as those in Proposition 2. Both conditions relate to the required accuracy in measurement of B_o and B_1.

Condition (i) is simply an abstract statement of the requirement that errors in the estimate of the steady state performance of the plant should have a guaranteed upper bound that is independent of chosen sampling rate. This is easily achieved as steady state performance (and hence B_1) is independent of sample rate. In fact the simple choice of $\tilde{B}_1 = 0$ (see [10]) will satisfy (i) and provide a partial simplification of the controller structures as an added bonus.

In a similar manner condition (ii) represents an accuracy requirement in the estimation of the 'high frequency' parameter B_o, although, in contrast to B_1, the accuracy required in estimation of B_o depends, partially, upon the chosen controller parameters k_j, c_j, $1 \leq j \leq m$.

Perhaps the most important interpretation of the theorem however is in terms of the robustness of the final design. It is clear that, under fast sampling conditions, the stability of the closed-loop system is robust with respect to the unknown dynamics in H as these dynamics play no part in the stability conditions. It is also clear from (i) that, provided errors in estimation of B_1 are uniformly bounded, any destabilizing effect of a bad choice of $\tilde{B}_1$ can be offset by increased sample rates. Finally, if we arrange our experiments [10] for estimation of B_o to an accuracy specified by (14) any destabilizing effect of such errors can again be removed by increasing the sampling rate.

Clearly the use of fast sampling rates is an important tool in producing a robust design and, under these conditions, the tolerable errors in estimation of B_o and B_1 can be quantifiably large. Although the asymptotic methods used in the proof of the result formally require 'fast sampling rates', experience with the application of the ideas indicates that h^* can in practice be quite large (corresponding to slow sampling conditions). A numerical estimate of the h^* cannot be obtained, however, without numerical information concerning the unknown dynamics in H.

Proof of Theorem: The proof reduces to a demonstration that, under the stated conditions, (12) holds for all fast enough sampling rates. It is easily verified that

$$(G_A^{-1} + K)^{-1} (G^{-1} - G_A^{-1}) =$$

$$\operatorname{diag}\left\{\frac{(z-1)}{(z-k_j)(z-c_j)}\right\}_{1\leq j\leq m} \left((z-1)(\tilde{B}_o^{-1}B_o - I_m) + \tilde{B}_o^{-1}(B_1-\tilde{B}_1) + \tilde{B}_o^{-1}B_o H(z)\right) \tag{15}$$

Noting that $\lambda_\infty < 1$, it follows from (14) that $\limsup_{h\to o+} ||\tilde{B}_o^{-1}(B_o-\tilde{B}_o)|| < 1$ and hence that

$$\limsup_{h\to o+} ||\tilde{B}_o^{-1} B_o|| \leq 2 \tag{16}$$

Using (11), it then follows that

$$\limsup_{h\to o+} ||\tilde{B}_o^{-1}B_o H|| \leq 2 \lim_{h\to o+} ||H|| = 0 \tag{17}$$

Finally, writing $\tilde{B}_o^{-1} = (\tilde{B}_o^{-1}B_o)B_o^{-1}$ and using (16) it is seen that

$$\limsup_{h\to o+} ||\tilde{B}_o^{-1}(B_1-\tilde{B}_1)|| \leq 2 \limsup_{h\to o+} ||B_1 - \tilde{B}_1|| \limsup_{h\to o+} ||B_o^{-1}|| = 0 \tag{18}$$

by (13) and the fact that $B_o^{-1} = C\Delta \to 0$ as $h \to o+$. This completes the proof of the theorem as, using (17) and (18) in (15) it follows from (14) and (12) that

$$\limsup_{h\to o+} \lambda = \lambda_\infty < 1 \tag{19}$$

and hence that $h^* > 0$ exists with the required property.

(Note: the parameter R disappears from the analysis as, taking $R >> 0$, it is trivially verified that the maximum occurs on $|z| = 1$).

Remark 4: Taking $\tilde{B}_o = B_o$ and $\tilde{B}_1 = B_1$ (i.e. exact measurements) the above result reduces to Proposition 2 as (13) and (14) are trivially satisfield.

References

[1] M.G. Safanov, 'Stability and Robustness of Multivariable Feedback Systems', Cambridge Massachusetts, The MIT Press, 1980

[2] N.R. Sandell, 'Robust Stability of Systems with Application to Singular Perturbations', Automatica, 15, pp. 467-470, 1979.

[3] B.A. Francis, 'Robustness of the Stability of Feedback Systems', IEEE Trans. Automat. Contr. vol AC-25, pp. 817-818, 1980.

[4] J.C. Doyle, 'Robustness of Multiloop Linear Feedback Systems', 17th IEEE Conf. Decision and Control, pp. 12-18, San Diego, 1978.

[5] K.J. Astrom. 'A robust Sampled Regulator for Stable Systems with Monotone Step Responses', Automatica, 16, pp. 313-315, 1980.

[6] K.J. Astrom, 'Robustness of a Design Technique Based on Assignment of Poles and Zeros', IEEE Trans. Automat. Contr., vol AC-25, pp. 588-590, 1980.

[7] J. Penttinen, H.N. Koivo, 'Multivariable Tuning Regulators for Unknown Systems', Automatica, 16, pp. 393-398, 1980.

[8] E.J. Davison, 'Multivariable Tuning Regulators: the feedforward and robust control of a general servomechanism problem', IEEE Trans. Automat. Contr., AC-21, pp. 35-47, 1976.

[9] J.B. Edwards, D.H. Owens, 'First-order-type Models for Multivariable Process Control', Proc. IEE, 124, pp. 1083-1088, 1977.

[10] D.H. Owens, 'Discrete First-order Models for Multivariable Process Control', Proc. IEE, 126, pp. 525-530, 1979.

[11] D.H. Owens, A. Chotai, 'Robust Control of Unknown or Large-scale Multivariable Systems using Transient Data only', Dept. Control Engineering, University of Sheffield, UK, Research Report No. 134, 1980.