Published paper

Clarke, SD, Smith, CC and Gilbert, M (2010) *Analysis of the stability of sheet pile walls using discontinuity layout optimization*. In: Proceedings of the 7th European Conference in Numerical Methods in Geotechnical Engineering. 7th European Conference in Numerical Methods in Geotechnical Engineering, June, 2010, Trondheim, Norway. CRC Press , 163 - 168. ISBN 9780415592390

http://dx.doi.org/10.1201/b10551-30

Analysis of the stability of sheet pile walls using Discontinuity Layout Optimization

S. D. Clarke, C. C. Smith & M. Gilbert
The University of Sheffield, UK

ABSTRACT: In this paper it is demonstrated that one-dimensional rigid-plastic elements can be used in conjunction with the recently developed Discontinuity Layout Optimization (DLO) procedure Smith & Gilbert (2007a) to permit the modelling of sheet pile walls. The resulting procedure allows identification of a wide variety of failure modes, including those involving wall translation and / or rigid body rotation, and also rigid-plastic bending of the wall due to the formation of one or more plastic hinges. Results from the procedure are compared with those obtained (i) using classical retaining wall theory, and (ii) from other numerical limit analysis procedures described in the literature, demonstrating its efficacy. A series of increasingly complex example problems are then studied, showing the ability of the procedure to treat problems involving water and a variety of wall support arrangements.

1 INTRODUCTION

Sheet pile wall design requires knowledge of how the ground and structure can be made to work together in order to produce a safe design. Sheet pile walls are most often designed to resist the effects of active earth pressures. The Rankine or Coulomb methods of analysis are normally used to estimate these pressures once the mechanical characteristics of the soil to be retained have been suitably approximated. These pressures allow a factor of safety against collapse to be determined for any given wall embedment depth.

This paper aims to demonstrate that the Discontinuity Layout Optimization (DLO) procedure can be used to model sheet pile walls. In the paper computed collapse loads are compared with those calculated using established methods, and subsequently with results obtained using a lower bound finite element limit analysis method (Krabbenhoft et al. 2005). In the latter case, more complex problems, which include a water table and ground anchors, are considered.

2 DISCONTINUITY LAYOUT OPTIMIZATION

At the present time, there are two main numerical limit analysis methods available for geotechnical applications, Finite Element Limit Analysis (FELA) (Lysmer 1970, Sloan 1988, Makrodimopoulos & Martin 2006) and Discontinuity Layout Optimization (DLO) (Smith & Gilbert 2007a, 2007b, 2008). Limit analysis can also be carried out using conventional elasto-plastic finite element analysis, by iterating towards a collapse state; however this paper is concerned only with direct limit analysis approaches. With both FELA and DLO the collapse state can be identified directly using optimization techniques.

A numerical limit analysis problem can be formulated as an upper or lower bound problem. Using an upper bound 'kinematic' formulation, the DLO procedure defines a discontinuous velocity field covering the entire problem domain, utilising a set of problem variables which represent deformations along potential discontinuities (of which there may be many millions). A rigorous mathematical optimization approach (e.g. linear programming) is then used to select the set of variables that minimises the energy dissipated in order to find the critical collapse mechanism.

The DLO method has been implemented into the geotechnical stability software package, LimitState:GEO (LimitState 2009), which has been used to analyse all example wall problems considered in this paper.

3 CANTILEVER WALL ANALYSIS

3.1 *Sliding failure*

Sliding failure may be analysed using a classical Rankine approach assuming a rigid retaining wall. In this example a smooth interface is assumed between

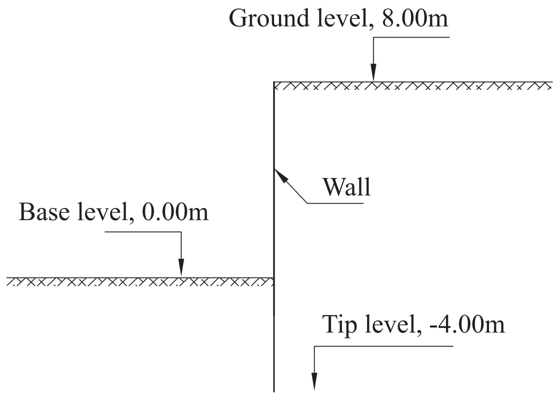

Figure 1: Rankine analysis geometry

Table 1: Soil parameters

Unit weight, γ	18 kN/m^3
Cohesion intercept, c'	0 kPa
Angle of friction, ϕ'	30°

the soil and the retaining wall. The geometry for the basic analysis is shown in Figure 1 and the soil parameters given in Table 1 have been used. A sliding only failure can be modelled in LimitState:GEO by setting *Model rotations* to be false.

Using the Rankine earth pressure coefficients (Eqs. 1, 2), the pressures acting on the active (K_a) and passive (K_p) sides of the wall can be calculated.

$$K_a = \frac{1 - \sin\phi'}{1 + \sin\phi'} \quad (1)$$

$$K_p = \frac{1 + \sin\phi'}{1 - \sin\phi'} \quad (2)$$

The active force is given by $P_a = 0.5H^2\gamma K_a$ where H is the total height of the retaining wall from crest to tip. The passive force is similarly given by $P_p = 0.5H^2\gamma K_p$. By examining horizontal equilibrium, the required depth below the base of the wall to the tip (d_w) can be calculated, which for the geometry in Figure 1 is 4.0m. To show that the DLO method can calculate the same mechanism and critical depth, the problem was set up in LimitState:GEO with the geometry as specified in Figure 1. In LimitState:GEO the sheet pile wall is defined as an *engineered element* (LimitState 2009) with an infinite moment resistance to match the rigid sheet pile modelled in the Rankine analysis. The wall must also have an infinite lateral capacity, N, so that the DLO method treats the wall as an impenetrable barrier as far as the soil is concerned. The interface between the retaining wall and the soil can be defined within LimitState:GEO by adding a second material to the wall. In this case, as a smooth interface is assumed, the second material is set with $\phi' = 0$ to remove frictional effects.

Figure 2 shows the failure mechanism obtained using LimitState:GEO. To find a collapse mechanism

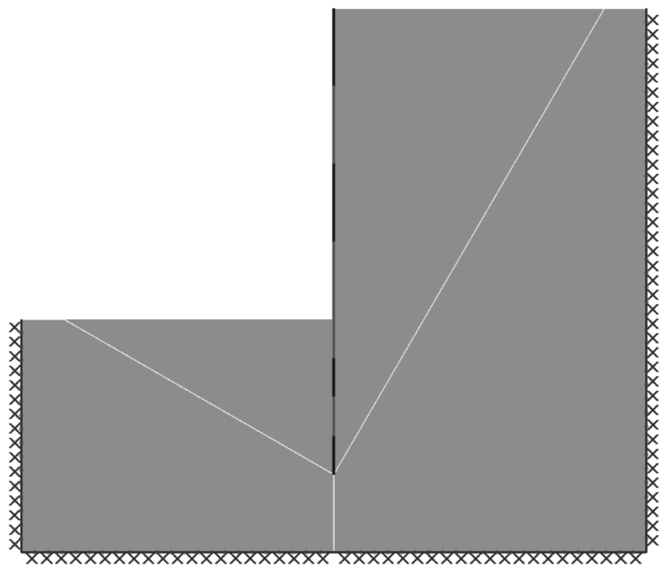
Figure 2: DLO failure mechanism

Table 2: Variation of adequacy factor with nodal density

Nodal density	No. of nodes	Adequacy factor
Course	250	1.012072156
Medium	500	1.003030377
Fine	1000	1.001162663
Custom	3000*	1.000544063

*User defined

an 'adequacy factor' must be applied to a force or self weight in the system to precipitate failure. For this scenario, adequacy was applied to the retained soil weight, and was found to be 1.001162663 for a 'Fine' nodal density. The adequacy factor is dependent on the nodal density of the analysis, and for an increasing number of nodes (which provide the end-points of potential lines of discontinuity) the adequacy factor will converge towards the analytical solution of 1.0. This is shown in Table 2.

By modelling the sheet pile wall as a rigid material the stress distribution around the wall can be plot-

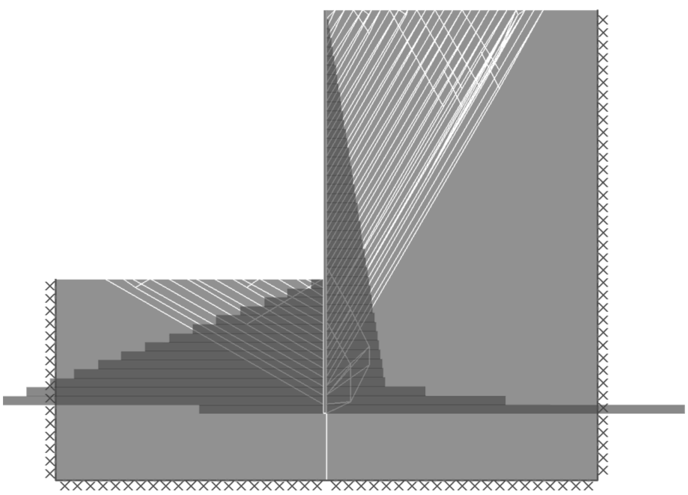
Figure 3: Stress distribution for Rankine DLO analysis (simple rotational mechanism)

ted. In a conventional sliding analysis active and passive earth pressures are assumed to vary linearly with depth. However this is not a strict requirement in plastic limit analysis. Only the resultant of the stress distributions must be the same. The DLO method without model rotations only examines horizontal and vertical equilibrium and generates a mechanism involving a single wedge either side of the wall. This does not necessarily require a linear variation of pressure with depth. However if the wall is allowed to rotate, and fail in the expected mode for a cantilever wall, then the soil must also yield throughout the adjacent 'wedges' and the pressure distribution becomes well-defined. A linear variation with depth above the point of rotation is then predicted, as shown in Figure 3.

3.2 *Rotational and bending failure*

The previous section has shown that the DLO method is capable of accurately analysing a sheet pile retaining wall for failure against sliding. In this section more complex modelling is undertaken with the inclusion of both rotational failure mechanisms, a water table and yielding of the sheet pile wall. The DLO solutions have then been compared with results obtained using a lower bound finite element limit analysis (FELA) method (Krabbenhoft et al. 2005). The problem geometry is defined in Figure 4, with the depth to the wall tip varying depending upon the support conditions and friction on the soil-wall interface.

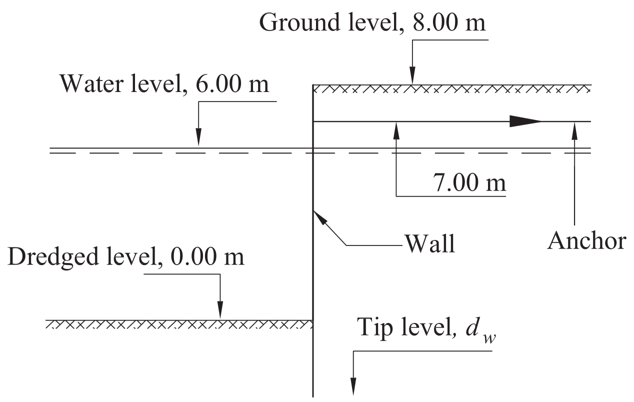

Figure 4: Problem geometry, Krabbenhoft et al. (2005)

Table 3: Cantilever pile wall parameters

Plastic moment resistance, M_p	982 kNm/m
Tip level, d_w	-9.6 m
Angle of wall friction, ϕ'	0°

Figure 5 shows soil displacement vectors generated by the FELA method (Krabbenhoft et al. 2005) for the cantilevered sheet pile wall with no anchor. The soil and wall parameters are given in Tables 1 & 3. Specifying this problem in LimitState:GEO was done as for the sliding analysis but with the addition of

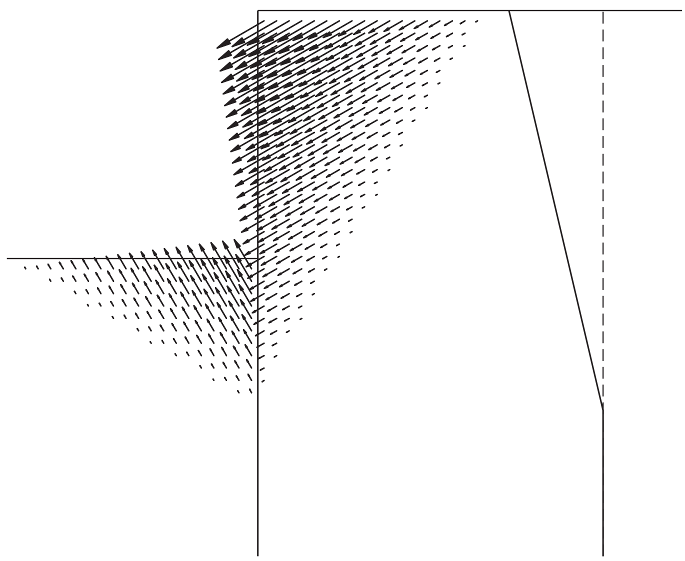

Figure 5: FELA vector plot for a smooth cantilever wall, Krabbenhoft et al. (2005)

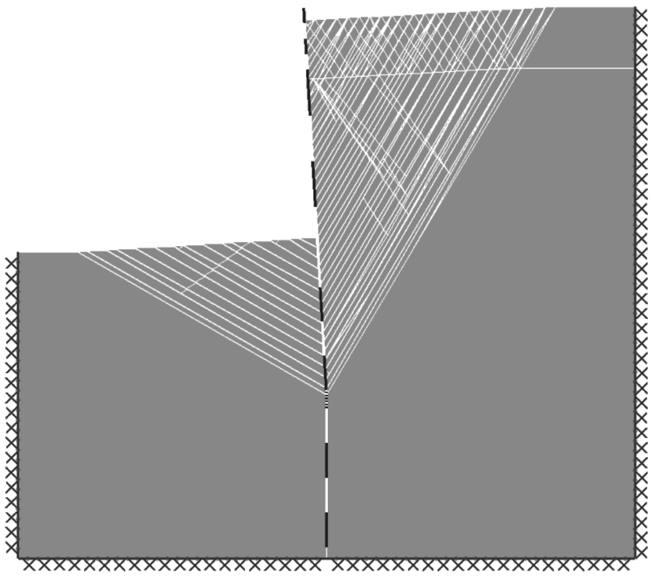

Figure 6: DLO failure mechanism for a smooth cantilever wall

vertices being placed on the sheet pile wall around the expected point of yield (from Figure 5). LimitState:GEO 2.0 is able to model rotations at vertices. By manually adding vertices the sheet pile wall is allowed to yield and bend at these locations. Figure 6 shows the failure mechanism predicted by the DLO method, indicating that the sheet pile wall yields at the same depth as predicted by the FELA analysis (Krabbenhoft et al. 2005). As the DLO method computes upper bound solutions, it would be expected to give a higher predicted factor of safety than a lower bound limit analysis method. Factor of safety is normally defined on soil strength rather than self weight for retaining walls, thus partial factors are applied to $\tan\phi'$ until the adequacy factor on the retained soil self weight becomes 1. For the cantilever wall analysis the required factor was 1.035, meaning that the DLO method gave a result 3.5% higher than the lower bound FELA analysis result given in Krabbenhoft et al. (2005). The true solution will lie somewhere between the two results.

4 ANCHORED WALL ANALYSIS

The previous examples have looked at embedded cantilever sheet pile walls where the failure mechanism involves a combined failure of the sheet pile and surrounding soil. To reduce the required embedded depth and required moment resistance of the pile, ground anchors can be used to help stabilize the retaining wall. The location of the anchor is given in Figure 4, being 1m down from the crest of the wall. Generally ground anchors can be modelled in two distinct ways:

- by an equivalent prop force acting on the face of the wall or,
- as a discrete soil reinforcement element.

Both approaches can be specified within LimitState:GEO but to correctly assess the interaction between the ground anchor and the failure mechanism the anchor should be modelled as a discrete soil reinforcement element.

4.1 *Anchor capacity*

If the anchor is located too close to the wall, then the failure mechanism directly affects the pull-out resistance of the soil reinforcement element as the slip surface may cut through the position of the anchor. In this situation the capacity of the anchor is reduced to that equal to only the embedded end, thus giving a realistic failure mechanism. The pull-out capacity of the anchor T can be assumed to vary proportionally with the vertical effective stress:

$$T = \alpha(c' + \sigma'_v \tan \phi'_{mob})a \quad (3)$$

where α = interaction coefficient; c' = drained cohesion intercept; σ'_v = vertical effective stress; and ϕ'_{mob} = mobilized angle of friction between the anchor and soil mass. For essentially 1D reinforcement such as soil nails, $a = n\pi D$ where D is the diameter of the soil nail, and n is the number of soil nails per unit width.

In certain situations the capacity of the anchor per metre length is unknown, but an ultimate pull-out resistance for a given anchor is known. This happens most frequently when benchmarking the DLO method against work done by other authors, where the exact location of the generated slip surface is unknown (Krabbenhoft et al. 2005). Modelling a single, ultimate, pull-out resistance in LimitState:GEO which is independent of the failure mechanism can be achieved in several ways. Here it will be implemented by using an *engineered element* to tie the sheet pile to an anchor at the boundary of the model, as shown in Figure 7.

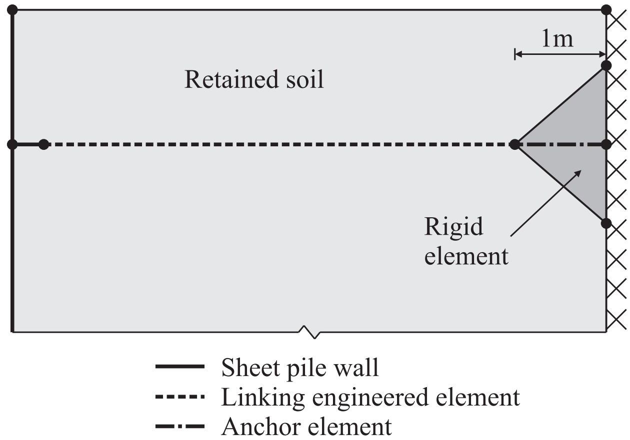

Figure 7: Specifying an ultimate anchor capacity in LimitState:GEO

There are three key components to this implementation in LimitState:GEO, which are:

- the requirements of the linking engineered element,
- the interface between the sheet pile wall and linking engineered element and
- the specification of the anchor element.

The linking engineered element (LEE) is required to attach the sheet pile wall to the anchor that provides the pull-out resistance. To have no impact on the failure mechanism the LEE must have zero bending moment, lateral and pull-out resistances, thus solely acting as a inextensible tie (or debonded tendon) between the sheet pile wall and the anchor.

Figure 7 shows that the point at which the linking engineered element attaches to the sheet pile wall is not located directly on the wall. If the LEE (with a bending moment resistance of zero) is attached directly to the sheet pile wall then this results in LimitState:GEO modelling a zero-bending moment also at this point in the sheet pile wall. By attaching the LEE to an extended section of sheet pile wall this problem is eliminated.

Finally the capacity of the tie-back system is defined by the pull-out resistance of the anchor element within the rigid zone on the boundary, as seen at the right of Figure 7. This anchor element is modelled as a distinct engineered element with a specified pull-out capacity T. The anchor capacity is thus $1 \times T$.

4.2 *Failure mechanism independent analysis*

The above method of specifying the ground anchor has been tested by comparing the results obtained with those from Krabbenhoft et al. (2005), where a constant anchor yield force, T, of 112 kN was used to stabilize the retaining wall in Figure 4, reducing the

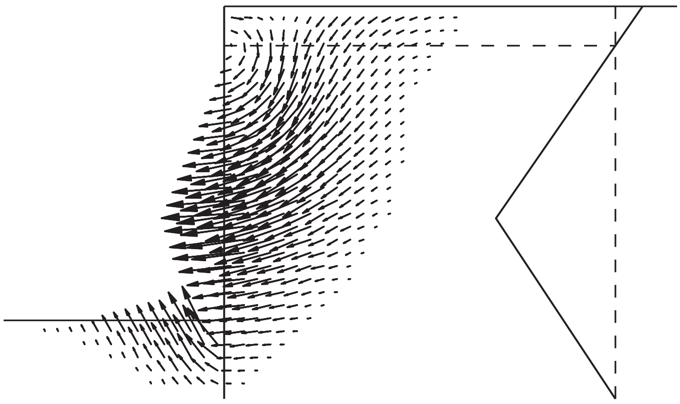

Figure 8: FELA vector plot for a rough anchored wall, Krabbenhoft et al. (2005)

Table 4: Anchored wall parameters

Plastic moment resistance, M_p	115 kNm/m
Tip level, d_w	-2.0 m
Angle of wall friction, ϕ'	30°
Anchor force, T	112 kN

embedded depth to 2 m and the required moment resistance to 115 kNm/m. This analysis was done to mobilize full soil friction on the pile. The problem was set up in LimitState:GEO using the geometry given in Figure 4 and the soil and wall parameters given in Table 4. As was the case for the cantilevered wall, the factor of safety is defined in terms of a partial factor on soil strength. For an adequacy factor of 1, a partial factor of 1.032 was required, showing that the difference between the lower and upper bound methods is 3.2%. From a comparsion between the vector plot and failure mechanism, it can be seen that the mode of failure is the very similar in both models.

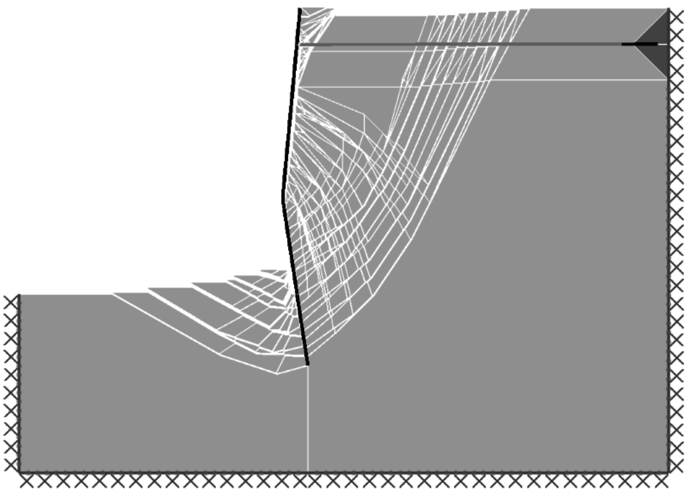

Figure 9: DLO failure mechanism for a rough anchored wall

4.3 *Failure mechanism dependent analysis*

One of the main strengths of the DLO method is its ability to evaluate the stability of geotechnical problems of any geometry, always finding the lowest energy solution. The anchored retaining wall analysis in the last section showed that DLO method is capable of closely matching the failure mechanism and factor of safety obtained using a lower bound FELA analysis of a problem involving an anchor whose capacity is independent of the failure mechanism. As discussed earlier, the DLO method can also model discrete reinforcement, modelling the interaction of an anchor with the surrounding soil. An example of this is shown in Figure 10 where the LEE was replaced by an engineered element with a capacity T of 20 kN/m along its full length, and the single anchor adjacent to the boundary was removed. Now the factor of safety becomes a function of the length of the anchor and its influence on the failure mechanism. The parameters for the sheet pile wall remain unchanged from those given in Table 4. The analysis indicated that an anchor length of 11.45m was required to ensure wall stability.

By comparing Figures 9 & 10 the effect of the discrete anchor on the failure mechanism can clearly be seen, with the failure mechanism in Figure 10 attempting to circumnavigate the anchor.

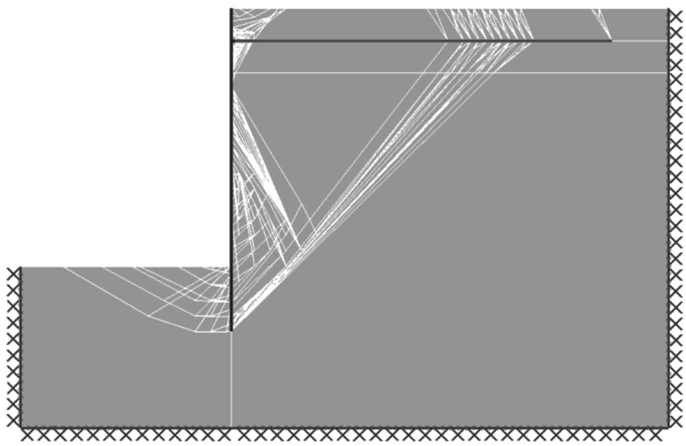

Figure 10: DLO failure mechanism for a rough anchored wall with a discrete anchor

5 CONCLUSIONS

This paper has demonstrated that the discontinuity layout optimization (DLO) procedure can be used to model sheet pile walls, and also, using a variety of methods, connected ground anchors.

A classical Rankine analysis was first used to demonstrate that the DLO procedure is capable of making accurate predictions when only sliding failures are involved. More complex sheet pile wall systems were then modelled, and the results were benchmarked against those presented by Krabbenhoft et al. (2005). It was found that the upper bound DLO results were within 3.2-3.5% of the results obtained using a lower bound FELA method. It was also found that the DLO method is capable of correctly identifying the failure mechanisms for a variety of sheet pile retaining wall problems. The problems are easily specified in the LimitState:GEO software, allowing rapid analysis of retaining wall problems (run times were dependent on the nodal density specified, ranging from seconds to minutes on a modern desktop PC).

REFERENCES

Krabbenhoft, K., Damkilde, L. & Krabbenhoft, S. (2005). Ultimate limit state design of sheet pile walls by finite elements and nonlinear programming, *Computers and Structures 83* pp. 383–393.

LimitState (2009). *LimitState:GEO Manual VERSION 2.0*, sept 3 edn, LimitState Ltd.

Lysmer, J. (1970). Limit analysis of plane problems in soil mechanics, *Journal of the Soil Mechanics and Foundations Division ASCE 96* , 4: 1311–1334.

Makrodimopoulos, A. & Martin, C. (2006). Lower bound limit analysis of cohesive-frictional materials using second-order cone programming, *Int. J. Num. Meth. in Eng. 6* , 4: 604–634.

Sloan, S. (1988). Lower bound limit analysis using finite elements and linear programming, *Int. J. Num. Anal. Meth. in Geomech. 12* , 4: 61–77.

Smith, C. & Gilbert, M. (2008). Limit analysis of the stability of foundations on inclined ground, *2nd BGA International Conference on Foundations, Dundee*, pp. 1683–1692.

Smith, C. C. & Gilbert, M. (2007a). Application of discontinuity layout optimization to plane plasticity problems, *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences 463, 2086* pp. 2461–2484.

Smith, C. C. & Gilbert, M. (2007b). New upper bound solutions for layered soil bearing capacity problems using discontinuity layout optimization, *10th Australia New Zealand Conference on Geomechanics, Brisbane*, pp. 250–255.