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IDENTIFICATION OF NONLINEAR SYSTEMS

USING CORRELATION ANALYSIS AND

PSEUDORANDOM INPUTS

by

S. A. BILLINGS, B.Eng.,Ph.D.,M.Inst.M.C.,AMIEE

S. Y. FAKHOURI, B.Sc.,M.Sc.,AMIEE

University of Sheffield
Department of Control Engineering
Mappin Street
Sheffield S1 3JD

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ABSTRACT

Algorithms for the identification of open and closed-loop
nonlinear systems composed of linear dynamic and static nonlinear
elements are developed. It is shown that correlation analysis
based upon compound pseudorandom inputs provides estimates of
the individual component subsystems. The selection of pseudo-
random inputs is discussed and simulated examples are included.

1. INTRODUCTION

Although a number of authors have considered the analysis and synthesis of simple feedback connections of linear dynamic and static nonlinear elements[1,2,3,4], the identification of such systems in terms of the component subsystems appears to have been largely neglected. Characterization using the functional Volterra series provides an elegant mathematical representation, but even if the kernels can be identified, the physical interpretation of the final model in terms of the elements of the original system is usually difficult to achieve. Since identification is often motivated by the need to gain further information about a process and to design an efficient regulator, identification should wherever possible preserve the structure of the system under investigation. For systems which are composed of linear dynamic and static nonlinear elements this can often be achieved by considering extensions of established linear identification techniques.

The large class of systems which can be represented by combinations of linear dynamic and static nonlinear elements have been studied by numerous authors. A transform representation and rules for the algebraic manipulation of systems in this class have been developed by George[5]. A structure theory for cascade systems was developed by Smith and Rugh[6], and Shanmugan and Lal[7], and Smets[8] considered the analysis and synthesis of nonlinear systems using canonic structures. Nonlinear feedback systems have been studied by Barrett and Coales[1], and Zames[4] and Narayanan[9] analysed the distortion in feedback amplifiers. Identification algorithms have been developed for cascade[10,11,12,13,14,15,16,17], feedforward[18], multiplicative[19] and unity feedback connections of linear dynamic and static nonlinear elements[20].

In the present study nonlinear feedback systems are analysed and an identification algorithm based upon pseudorandom excitation, which provides estimates of the individual component subsystems from measurements of the input and noise corrupted output is developed. Initially, the open-loop general model consisting of a linear system in cascade with a nonlinear element followed by a second linear system is analysed. Previous results[16,17] based upon the theory of separable processes have shown that this system can be identified in terms of the component subsystems when the input has the properties of a white Gaussian process. As an alternative to this procedure an algorithm which is based upon a compound pseudo-random input sequence is derived. Although it can readily be shown that a binary pseudorandom sequence is a separable process, it is not separable under linear transformation and hence previous results[16,17] cannot be applied for these inputs. However by simultaneously injecting the sum of two pseudorandom sequences the components of the general model can be identified using the algorithm presented in section 2. The problem of anomalies[21,22] normally associated with the multidimensional autocorrelation functions of such sequences when applied to the identification of nonlinear systems are avoided.

By considering the structural form of the first two kernels in the Volterra series expansion of a general class of nonlinear feedback systems the results for the open-loop model are generalised to the feedback case. Simulated examples are included to demonstrate the validity of the algorithms.

2. THE OPEN-LOOP GENERAL MODEL

Before nonlinear feedback systems can be considered it is necessary to develop an identification algorithm for the open-loop general model illustrated in Fig.1. The system consists of a linear system $h_1(t)$ in cascade with a zero memory nonlinear element followed by a second linear system $h_2(t)$. For generality it is assumed that the measured output contains an unknown additive noise component $v(t)$ and that the nonlinear element can, in theory, be represented by a polynomial of the form

$$y(t) = \gamma_1 q(t) + \gamma_2 q^2(t) + \ldots + \gamma_k q^k(t) \tag{1}$$

Note that the Hammerstein[12,18] and Wiener[14] models are special cases of the general model.

The identification problem can now be defined as identification of the individual component $h_1(t)$, $h_2(t)$ and a suitable representation of the static nonlinear element $F[\cdot]$ from measurements of the input $u(t)$ and noise corrupted output $z(t)$.

2.1 Identification using pseudorandom sequences

From Fig.1 the measured system output $z(t)$ can be represented by the functional series[16]

$$z(t) = \gamma_1 \iint\limits_{-\infty}^{\infty} h_1(\tau_1) h_2(\tau_2) u(t-\tau_1-\tau_2) d\tau_1 d\tau_2$$

$$+ \gamma_2 \iiint\limits_{-\infty}^{\infty} h_1(\tau_1) h_1(\tau_2) h_2(\tau_3) u(t-\tau_1-\tau_3) u(t-\tau_2-\tau_3) d\tau_1 d\tau_2 d\tau_3$$

$$+ \quad \vdots$$

$$+ \gamma_k \int\limits_{-\infty}^{\infty} \cdots \int\limits_{-\infty}^{\infty} h_1(\tau_1) \ldots h_1(\tau_k) h_2(\tau_{k+1}) u(t-\tau_1-\tau_{k+1})$$

$$\cdots u(t-\tau_k-\tau_{k+1}) d\tau_1 \ldots d\tau_{k+1} + v(t)$$

$$= w_1(t) + w_2(t) + \ldots + w_k(t) + v(t) \tag{2}$$

where the Volterra kernels $g_m(\tau_1 \ldots \tau_m)$ have the form

$$g_m(\tau_1, \ldots \tau_m) = \gamma_m \int_{-\infty}^{\infty} h_2(\sigma) \prod_{p=1}^{m} h_1(\tau_p - \sigma) d\sigma \tag{3}$$

and $w_i(t)$ represents the output of the isolated i'th order kernel.

When the input u(t) is a compound input defined as

$$u(t) = x_1(t) + x_2(t) \tag{4}$$

where $x_1(t)$ and $x_2(t)$ are pseudorandom sequences, the output z(t) can be expressed as

$$z(t) = \sum_{i=1}^{k} \{\gamma_i \int \ldots \int_{-\infty}^{\infty} h_1(\tau_1) \ldots h_1(\tau_i) h_2(\theta) \{$$

$$\prod_{j=1}^{i} (x_1(t-\tau_j-\theta) + x_2(t-\tau_j-\theta)) d\tau_j\} d\theta\} + v(t) \tag{5}$$

The first order correlation function for the compound input can then be defined as

$$\phi_{x_1'z'}(\sigma) = \overline{(z(t)-\overline{z(t)})(x_1(t-\sigma)-\overline{x_1(t-\sigma)})} = \overline{z'(t)x_1'(t-\sigma)}$$

$$= \sum_{i=1}^{k} \{\gamma_1 \int \ldots \int h_1(\tau_1) \ldots h_1(\tau_i) h_2(\theta)\{ \overline{\prod_{j=1}^{i} (x_1(t-\tau_j-\theta)}$$

$$\overline{+x_2(t-\tau_j-\theta)) d\tau_j - \prod_{j=1}^{i} (x_1(t-\tau_j-\theta) + x_2(t-\tau_j-\theta))} d\tau_j\}(x_1(t-\sigma)-\overline{x_1(t-\sigma)}) d\theta$$

$$\overline{+(x_1(t-\sigma)-\overline{x_1(t-\sigma)})(v(t)-\overline{v(t)})}$$

$$= \sum_{i=1}^{k} \phi_{w_i'x_1'}(\sigma) + \phi_{x_1'v'}(\sigma) \tag{6}$$

where the superscript ' is used throughout to indicate a zero mean process.

If $\phi_{x_1'z'}(\sigma)$ is evaluated directly as defined above the terms $\sum\limits_{i=2}^{k} \phi_{w_i'x_1'}(\sigma)$ give rise to anomalies[21,22,31] associated with multi-dimensional autocorrelations of the pseudorandom sequences $x_1(t)$ and $x_2(t)$. This problem can be overcome by isolating $\phi_{x_1'w_1'}$, the first order correlation function of the output of the first order kernel.

Consider a series of experiments with multilevel inputs $\alpha_i u(t)$ where $\alpha_i \neq \alpha_\ell \; \forall \; i \neq \ell$, then the output correlation function $\phi_{x_1'z_{\alpha_i}'}(\sigma)$ can be expressed as

$$\phi_{x_1'z_{\alpha_i}'}(\sigma) = \sum_{j=1}^{n} \alpha_i^{\,j} \phi_{x_1'w_j'}(\sigma) \;,\quad i = 1,2\ldots n \tag{7}$$

assuming that the input signal $x_1(t)$ and noise process $v(t)$ are statistically independent, $\overline{x_1'(t-\sigma)v'(t)} = 0 \; \forall \; \sigma$, and where z_{α_i} is the response of the system to the input $\alpha_i u(t)$. In matrix formulation

$$
\begin{pmatrix}
\phi_{x_1'z_{\alpha_1}'}(\sigma) \\
\cdot \\
\cdot \\
\cdot \\
\cdot \\
\phi_{x_1'z_{\alpha_n}'}(\sigma)
\end{pmatrix}
=
\begin{pmatrix}
\alpha_1 & 0 & \cdot & \cdot & \cdot & \cdot & 0 \\
0 & \alpha_2 & \cdot & \cdot & \cdot & \cdot & 0 \\
\cdot & \cdot & & & & & \\
\cdot & \cdot & & & & & \\
\cdot & \cdot & & & & & \\
\cdot & & & & & & \\
0 & \cdot & & & & & \alpha_n
\end{pmatrix}
\begin{pmatrix}
1 & \alpha_1 & \cdots & \alpha_1^{n-1} \\
1 & \alpha_2 & & \cdot \\
\cdot & & & \cdot \\
\cdot & & & \cdot \\
\cdot & & & \cdot \\
1 & \alpha_n & & \alpha_n^{n-1}
\end{pmatrix}
\begin{pmatrix}
\phi_{x_1'w_1'}(\sigma) \\
\cdot \\
\cdot \\
\cdot \\
\cdot \\
\phi_{x_1'w_n'}(\sigma)
\end{pmatrix}
$$

$$\tag{8}$$

The first matrix on the rhs of eqn (8) is clearly nonsingular for all $\alpha_i \neq 0$ and the second is the transpose of the Vandermonde matrix which is nonsingular for $\alpha_i \neq \alpha_\ell$. Thus for every value of σ, eqn (8) has a unique solution for $\phi_{x_1'w_j'}(\sigma)$, $j = 1,2\ldots n$.

Isolating $\phi_{x_1'w_1'}(\sigma)$ using the above procedure gives the result

$$\phi_{x_1'w_1'}(\sigma) = \gamma_1 \iint\limits_{-\infty}^{\infty} h_1(\tau_1) h_2(\theta)(\phi_{x_1 x_1}(\tau_1+\theta-\sigma)-\bar{x}_1^{\,2})\,d\tau_1\,d\theta \tag{9}$$

If $x_1(t)$ and $x_2(t)$ are independent zero mean pseudorandom sequences, $\phi_{x_1 x_2}(\lambda) = 0 \;\forall\; \lambda$, $\bar{x}_1 = \bar{x}_2 = 0$, with autocorrelation functions

$$\phi_{x_i x_i}(\lambda) = \beta_i \delta_i(\lambda) \quad, \qquad i = 1,2$$

where
$$\delta_i(\lambda) = \begin{cases} 1/\Delta t_i & \lambda = 0 \\ 0 & \lambda \neq 0 \end{cases} \tag{10}$$

Δt_i is the clock interval and $\int \delta_i(\lambda)\,d\lambda = 1.0$, equation (9) reduces to

$$\phi_{x_1' w_1'}(\sigma) = \beta_1 \gamma_1 \int_{-\infty}^{\infty} h_1(\sigma-\theta) h_2(\theta)\,d\theta \tag{11}$$

If the system illustrated in Fig.1 is to be identified in terms of the component subsystems $h_1(t)$, $h_2(t)$ and $F[\cdot]$ a second relationship similar to eqn (11) must be derived[16,17] such that $h_1(t)$ and $h_2(t)$ can be isolated. This can be achieved by defining the second order correlation function

$$\phi_{x_1' x_2' z'}(\sigma) = \overline{(z(t)-\overline{z(t)})(x_1(t-\sigma)-\overline{x_1(t-\sigma)})(x_2(t-\sigma)-\overline{x_2(t-\sigma)})}$$

$$= \sum_{i=1}^{k} \gamma_i \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} h_1(\tau_1)\ldots h_1(\tau_i) h_2(\theta)$$

$$\{ \prod_{j=1}^{i} \overline{(x_1(t-\tau_j-\theta)+x_2(t-\tau_j-\theta))\,d\tau_j}$$

$$- \prod_{j=1}^{i} \overline{(x_1(t-\tau_j-\theta)+x_2(t-\tau_j-\theta))\,d\tau_j}\}$$

$$\overline{(x_1(t-\sigma)-\overline{x_1(t-\sigma)})(x_2(t-\sigma)-\overline{x_2(t-\sigma)})}\,d\theta$$

$$+\overline{(x_1(t-\sigma)-\overline{x_1(t-\sigma)})(x_2(t-\sigma)-\overline{x_2(t-\sigma)}(v(t)-\overline{v(t)})}$$

$$= \sum_{i=1}^{k} \phi_{x_1' x_2' w_i'}(\sigma) + \phi_{x_1' x_2' v'}(\sigma) \tag{12}$$

Providing $x_1(t)$, $x_2(t)$ and $v(t)$ are mutually independent $\phi_{x_1'x_2'v'}(\sigma) = 0 \ \forall \ \sigma$ and the estimate of eqn (12) will be unbiased. However the presence of undesirable spikes in the high even order (>2) autocorrelation functions[21,22,31] of $x_1(t)$ and $x_2(t)$ precludes the evaluation of eqn (12) directly and the procedure of eqn (8) must be employed to isolate the second order correlation function of the output of the second order kernel

$$\phi_{x_1'x_2'w_2'}(\sigma) = \gamma_2 \int\limits_{-\infty}^{\infty}\!\!\!\int\!\!\int h_1(\tau_1)h_1(\tau_2)h_2(\theta)\{ \overline{\prod_{j=1}^{2}(x_1(t-\tau_j-\theta)+x_2(t-\tau_j-\theta))d\tau_j}$$

$$- \prod_{j=1}^{2}\overline{(x_1(t-\tau_j-\theta)+x_2(t-\tau_j-\theta))d\tau_j}$$

$$\overline{(x_1(t-\sigma)-\overline{x_1(t-\sigma)})(x_2(t-\sigma)-\overline{x_2(t-\sigma)})}d\theta \qquad (13)$$

If $x_1(t)$ and $x_2(t)$ are independent pseudorandom sequences eqn (13) can be expressed as

$$\phi_{x_1'x_2'w_2'}(\sigma) = \gamma_2 \int\!\!\int\!\!\int h_2(\tau_1)h_1(\tau_2)h_2(\theta)\{\phi_{x_1x_1}(\tau_1+\theta-\sigma)$$

$$\phi_{x_2x_2}(\tau_2+\theta-\sigma)+\phi_{x_1x_1}(\tau_2+\theta-\sigma)\phi_{x_2x_2}(\tau_1+\theta-\sigma)$$

$$-\overline{x}_1^2(\phi_{x_2x_2}(\tau_2+\theta-\sigma)+\phi_{x_2x_2}(\tau_1+\theta-\sigma))$$

$$-\overline{x}_2^2(\phi_{x_1x_1}(\tau_2+\theta-\sigma)+\phi_{x_1x_1}(\tau_1+\theta-\sigma))+2\overline{x}_1^2\overline{x}_2^2\}d\tau_1 d\tau_2 d\theta$$

which reduces to

$$\phi_{x_1'x_2'w_2'}(\sigma) = 2\beta_1\beta_2\gamma_2 \int h_1^2(\sigma-\theta)h_2(\theta)d\theta \qquad (14)$$

when $x_1(t)$ and $x_2(t)$ are zero mean processes with the properties defined in eqn (10).

The results of eqn's (11) and (14) can be readily decomposed using a least squares algorithm[16] to provide estimates of the individual linear subsystems $\mu_1 h_1(t)$ and $\mu_2 h_2(t)$ where μ_1 and μ_2 are constants. Thus computation of the first and second order correlation functions $\phi_{x_1'w_1'}(\sigma)$ and $\phi_{x_1'x_2'w_2'}(\sigma)$ respectively provides estimates of the linear subsystems which apart from a constant scale factor are quite independent of the non-linear characteristic. The nonlinear element can then be identified by minimising the sum of squares of the error

$$J = \sum_{j=1}^{N} (z(i) - \hat{z}(i))^2 \tag{15}$$

where

$$\hat{z}(i) = \mu_2 \sum_{j=1}^{p} \hat{h}_2(j) F\left[\hat{q}(i-j)\right] \tag{16}$$

$$\hat{q}(i) = \mu_1 \sum_{j=1}^{\ell\ell} h_1(j) u(i-j) \tag{17}$$

If a polynomial representation is required the coefficients can be estimated using a simple least squares algorithm[16,17]. Alternatively, an algorithm by Peckham[23] can be employed to fit a series of straight line segments[24] or any other suitable function if this is appropriate.

Providing $x_1(t)$ and $x_2(t)$ are pseudorandom sequences with properties as defined in eqn (10), the results of eqn's (11) and (14) are exact and the errors normally associated with the identification of this class of systems using pseudorandom inputs and correlation analysis are avoided. The only problem remaining is the selection of the pseudorandom inputs to satisfy eqn (10).

Independent binary pseudorandom sequences with the same bit interval can be generated either by multiplying by the rows of a

Hadamard matrix or correlating over the product of sequence lengths but independent ternary sequences can only be generalised using the latter approach[25]. Since the results of eqn's (11) and (14) are dependent upon $x_1(t)$ and $x_2(t)$ having zero mean values an obvious choice of input would be a compound ternary sequence with the properties[26]

$$\overline{x_i(t)} = 0$$

$$\phi_{x_i x_i}(\lambda) = \begin{cases} \dfrac{2(N+1)a_i^2}{3N_i} & \lambda = 0 \bmod N_i \\[3ex] \dfrac{-2(N+1)a_i^2}{3N_i} & \lambda = N_i/2 \bmod N_i \\[3ex] 0 & \forall \text{ other } \lambda \end{cases}$$

$$\beta_i = \frac{2(N_i+1)a_i^2 \Delta t_i}{3N_i} \quad , \quad i = 1,2 \tag{18}$$

where $N = 3^n-1$ is the sequence length, a the amplitude and n the order.

Alternatively, a compound input composed of the sum of an inverse repeat or antisymmetric pseudorandom input $x_1(t)$ and a pseudorandom binary sequence $x_2(t)$ could be employed. The inverse repeat sequence[27,28] can be generated by multiplying the elements of the pseudorandom binary sequence $x_2(t)$ alternatively by -1 and $+1$ and has the properties

$$\overline{x_1(t)} = 0$$

$$\phi_{x_i x_i}(\lambda) = \begin{cases} a_i^2 & \lambda = 0 \bmod 2N_i \\[1.5ex] -a_i^2 & \lambda = N_i \bmod 2N_i \\[1.5ex] -a_i^2/N_i & \lambda = 2,4,6\ldots \neq 0 \bmod 2N_i \\[1.5ex] a_i^2/N_i & \lambda = 1,3,5\ldots \neq N_i \bmod 2N_i \end{cases}$$

$$\beta_i = a_i^2 \Delta t_i \tag{19}$$

Although correlation over the period of the inverse repeat sequence ensures that this is independent of the prbs the ripple in the auto correlation function, eqn (19), will introduce a bias into the results which will only become small as N_i becomes large.

It would however be far more convenient if pseudorandom binary inputs[26] could be employed in this application. The only problem associated with this choice is the non-zero mean level of these sequences, and an analysis of the errors incurred is necessary.

When the input u(t) is a compound pseudo random maximal length binary sequence $u(t) = x_1(t)+x_2(t)$, where $x_1(t)$ and $x_2(t)$ are independent with properties

$$\overline{x_i(t)} = a_i/N_i$$

$$\phi_{x_i x_i}(\lambda) = \begin{cases} a_i^2 & \lambda = 0 \bmod N_i \\ -a_i^2/N_i & \lambda \neq 0 \bmod N_i \end{cases} \tag{20}$$

$$\beta_i = \frac{a_i^2(N_i+1)\Delta t_i}{N_i}$$

$\phi_{x_1' w_1'}(\sigma)$ defined in eqn (9) reduces to

$$\phi_{x_1' w_1'}(\sigma) = \beta_1 \gamma_1 \int h_1(\sigma-\theta)h_2(\theta)d\theta$$

$$-\gamma_1 \frac{a_1^2}{N_1^2}(N_1+1)\iint h_1(\tau_1)h_2(\theta)d\tau_1 d\theta \tag{21}$$

Providing the systems $h_1(t)$ and $h_2(t)$ are stable, bounded-inputs bounded outputs, the last term on the rhs of eqn (21) is a constant bias which can be readily removed to yield

$$\phi_{x_1'w_1'}(\sigma) = \beta_1\gamma_1 \int h_1(\sigma-\theta)h_2(\theta)d\theta \tag{22}$$

Evaluation of $\phi_{x_1'x_2'w_2'}(\sigma)$, defined in eqn (14), when the input consists of the sum of two independent pseudo random maximal length binary sequences yields

$$\phi_{x_1'x_2'w_2'}(\sigma) = 2\beta_1\beta_2\gamma_2\int h_1^2(\sigma-\theta)h_2(\theta)d\theta+e(\sigma) \tag{23}$$

where

$$e(\sigma) = -2\gamma_2(a_1\bar{x}_1\beta_2+a_2\bar{x}_2\beta_1+\bar{x}_1^2\beta_2+\bar{x}_2^2\beta_1)\int\int h_1(\tau_1)h_1(\sigma-\theta)h_2(\theta)d\tau_1 d\theta$$

$$+2\gamma_2\bar{x}_1\bar{x}_2(a_1a_2+\bar{x}_1a_2+\bar{x}_2a_1+\bar{x}_1\bar{x}_2)\int\int\int h_1(\tau_1)h_1(\tau_2)h_2(\theta)d\tau_1 d\tau_2 d\theta \tag{24}$$

Providing the systems $h_1(t)$ and $h_2(t)$ are stable bounded-inputs bounded-outputs the last term on the rhs of eqn (24) is a constant dc bias which can readily be removed. The first term on the rhs of eqn (24) represent a time varying bias in the estimate of eqn (23) which tends to zero as N_1 and N_2 become large. Thus if the constant bias is removed and N_1 and N_2 are large eqn (23) reduces to

$$\phi_{x_1'x_2'w_2'}(\sigma) \doteq 2\beta_1\beta_2\gamma_2\int h_1^2(\sigma-\theta)h_2(\theta)d\theta \tag{24}$$

3.   NONLINEAR FEEDBACK SYSTEMS

Although previous results[20] have shown that the general model under unity feedback can be identified by applying the theory of separable processes based upon white Gaussian excitation, these results cannot be applied for pseudorandom inputs. However, the algorithm derived in the previous section for the open-loop general model can be applied to a general class of nonlinear feedback systems composed

of linear dynamic and static nonlinear elements in both the forward
and feedback paths if the information in the first two terms of
the Volterra series expansion can be decomposed to provide estimates
of the component subsystems.   The class of feedback systems which
can be identified using this approach can be defined by analysing
the Volterra series expansion for such systems.

3.1   Volterra Series Expansions for a Class of Nonlinear Feedback Systems

Consider the general nonlinear feedback system and the equivalent
open-loop system $\underline{G}$ illustrated in Fig.2, where $\underline{A}$ and $\underline{B}$ are nonlinear
systems with known functional expansions.   Applying the operator
calculus developed by Brilliant[29] and George[5] gives

$$\underline{G} = \underline{A} * (\underline{I} - \underline{B} * \underline{G})$$

$$= \underline{A} * \underline{K} \tag{25}$$

where $\underline{I}$ is the identity operator.   Since $\underline{K}$, $\underline{B}$ and $\underline{G}$ are the sums
of homogeneous functional operators, $\underline{K}$, $\underline{G}$ defined in eqn (25) can
be expressed as

$$\sum_{\ell=1}^{\infty} \underline{K}_\ell = \underline{I} - (\sum_{i=1}^{\infty} \underline{B}_i) * (\sum_{j=1}^{\infty} \underline{G}_j)$$

$$= \underline{I} - \sum_{\ell=1}^{\infty} \sum_{n=1}^{\infty} \sum_{Q} \underline{B}_n \circ (\underline{G}_{i_1} . \underline{G}_{i_2} \ldots \underline{G}_{i_n}) \tag{26}$$

$$\sum_{\ell=1}^{\infty} \underline{G}_\ell = (\sum_{i=1}^{\infty} \underline{A}_i) * (\sum_{j=1}^{\infty} \underline{K}_j)$$

$$= \sum_{\ell=1}^{\infty} \sum_{n=1}^{\ell} \sum_{Q} \underline{A}_n \circ (\underline{K}_{i_1} . \underline{K}_{i_2} \ldots \underline{K}_{i_n}) \tag{27}$$

where the summation $\sum_Q$ is taken over all the integers $i_1, i_2 \ldots i_n$ such
that $i_1 + i_2 + \ldots i_n = \ell$ and $1 \leq i_q \leq \ell$.

Equating operators of equal order

$$
\underline{K}_\ell = \begin{cases} \underline{I} - \underline{B}_1 * \underline{G}_1 & \ell = 1 \\[2mm] -\sum_{n=1}^{\ell} \sum_Q \underline{B}_n \circ (\underline{G}_{i_1} \cdot \underline{G}_{i_2} \dots \underline{G}_{i_n}) & \ell \geq 2 \end{cases}
\tag{28}
$$

and

$$
\underline{G}_\ell = \begin{cases} \left[\underline{I} + \underline{A}_1 * \underline{B}_1\right]^{-1} * \underline{A}_1 & \ell = 1 \\[2mm] \left[\underline{I} + \underline{A}_1 * \underline{B}_1\right]^{-1} * \{-\underline{A}_1 \circ \left[\sum_{n=2}^{\ell} \sum_Q \underline{B}_n \circ (\underline{G}_{i_1} \dots \underline{G}_{i_n})\right] \end{cases}
$$

$$
+ \sum_{n=2}^{\ell} \sum_Q \underline{A}_n \circ (\underline{K}_{i_1} \dots \underline{K}_{i_n})\} \qquad \ell \geq 2
\tag{29}
$$

Combining eqn's (28) and (29), the first two Volterra kernels of the equivalent open loop system, Fig.2, can be expressed as

$$
\underline{G}_1 = \left[\underline{I} + \underline{A}_1 * \underline{B}_1\right]^{-1} * \underline{A}_1
\tag{30}
$$

$$
\underline{G}_2 = \left[\underline{I} + \underline{A}_1 * \underline{B}_1\right]^{-1} * \{-\underline{A}_1 * \underline{B}_2 \circ (\underline{G}_1^{\,2}) + \underline{A}_2 \circ ((\underline{I} - \underline{B}_1 * \underline{G}_1)^2)\}
\tag{31}
$$

When $\underline{A}$ and $\underline{B}$ both have the structure of a general model as illustrated in Fig.3, where $C(t)$, $H(t)$, $P(t)$ and $V(t)$ are stable, bounded-input bounded-output, linear subsystems then

$$
\underline{A}_\ell = \gamma_\ell \, \underline{H} \circ (\underline{C}^\ell)
\tag{32}
$$

$$
\underline{B}_\ell = \lambda_\ell \, \underline{V} \circ (\underline{P}^\ell)
\tag{33}
$$

where $\gamma_\ell = 0 \; \forall \ell > k$ and $\lambda_\ell = 0 \; \forall \ell > i$. Substituting these results in eqn's (30) and (31) yields

$$
\underline{G}_1 = \left[\underline{I} + \gamma_1 \lambda_1 \underline{H} * \underline{C} * \underline{V} * \underline{P}\right]^{-1} * \gamma_1 \underline{H} * \underline{C}
\tag{34}
$$

$$
\underline{G}_2 = \left[\underline{I} + \gamma_1 \lambda_1 \underline{H} * \underline{C} * \underline{V} * \underline{P}\right]^{-1} * \{-\gamma_1 \underline{H} * \underline{C} * \lambda_2 \underline{V}
$$

$$
\circ (\underline{P}^2) \circ (\underline{G}_1^{\,2}) + \gamma_2 \underline{H} \circ \underline{C}^2) \circ ((\underline{I} - \lambda_1 \underline{V} * P)^2)\}
\tag{35}
$$

The complexity of the Volterra kernels associated with the nonlinear feedback system illustrated in Fig.3 often dictates that such systems must be characterized by a finite Volterra series. This approach inevitably introduces a truncation error and destroys the structure of the process under investigation. However, reduced forms of the system illustrated in Fig.3 can often be identified in terms of the component subsystems in a manner which preserves the system structure. Classes of nonlinear feedback systems which can be identified using this approach are considered in the following sections.

3.2 Identification of Nonlinear Feedback Systems

3.2.1 Unity feedback systems

Consider the unity feedback general model illustrated in Fig.4. Notice that in general the system output will be corrupted by noise and hence the feedback signal cannot be computed and the problem cannot be reduced to one of open-loop identification.

From eqn's (34) and (35) the first two Volterra kernels for this system can be derived by setting

$$\underline{B}_\ell = \begin{cases} \underline{I} & \ell = 1 \\ 0 & \ell \geqslant 2 \end{cases}$$

to yield

$$\underline{G}_1 = [\underline{I} + \gamma_1 \underline{H} * \underline{C}]^{-1} * \gamma_1 \underline{H} * \underline{C} \tag{36}$$

$$\underline{G}_2 = [\underline{I} + \gamma_1 \underline{H} * \underline{C}]^{-1} * [\gamma_2 \underline{H} \circ (\underline{C}^2) \circ ((\underline{I} - \underline{G}_1)^2)] \tag{37}$$

The outputs $w_1(t)$ and $w_2(t)$ of the Volterra kernels $\underline{G}_1$ and $\underline{G}_2$ can be isolated using the results of eqn (8). Inspection of eqn's (9) and (11) shows that if the input $u(t)$ consists of the sum of

two independent zero mean pseudorandom sequences with the properties
defined in eqn (10), then

$$\phi_{x_1'w_1'}(\sigma) = \beta_1 G_1(\sigma) \tag{38}$$

where β_1 is a known constant, defined in eqn (10). Taking the
Z-transform of eqn (38), a pulse transfer function model can be
fitted to $1/\beta_1 \phi_{x_1'w_1'}(\sigma)$ to yield

$$Z\{\frac{1}{\beta_1} \hat{\phi}_{x_1'w'}(\sigma)\} = \frac{\hat{N}_{g_1}(z^{-1})}{\hat{D}_{g_1}(z^{-1})} = \frac{\gamma_1 H(z^{-1})C(z^{-1})}{1+\gamma_1 H(z^{-1})C(z^{-1})} \tag{39}$$

and estimates of the numerator and denominator can be obtained from

$$\gamma_1 H(z^{-1})C(z^{-1}) = \frac{\hat{N}_{g_1}(z^{-1})}{\hat{D}_{g_1}(z^{-1})-\hat{N}_{g_1}(z^{-1})} \tag{40}$$

$$1+\gamma_1 H(z^{-1})C(z^{-1}) = \frac{\hat{D}_{g_1}(z^{-1})}{\hat{D}_{g_1}(z^{-1})-\hat{N}_{g_1}(z^{-1})} \tag{41}$$

The output data $Z_{\alpha_j}(t)$, $j = 1,2...n$ can now be filtered using
the estimate $\{1+\gamma_1 H(z^{-1})C(z^{-1})\}$, such that the kernel relating the
filtered outputs $w_{r_1}(t)$, $w_{r_2}(t)$ to the input reduce to

$$\underline{G}_1 = \gamma_1 \underline{H}*\underline{C} \tag{42}$$

$$\underline{G}_2 = \left[\gamma_2 \underline{H} o(\underline{C}^2) o((\underline{I}-\underline{G}_1)^2)\right] \tag{43}$$

The second order correlation function $\phi_{x_1'x_2'w_{r_2}'}(\sigma)$ can then be
evaluated using the procedure of eqn's (8), (12) and (14) to yield

$$\phi_{x_1'x_2'w_{r_2}}{}'(\sigma) = 2\beta_1\beta_2\gamma_2 \int H(\theta)T_1^2(\sigma-\theta)d\theta \tag{44}$$

where $\quad \underline{T}_1 = \underline{C} * [\underline{I}-\underline{G}_1].$

Taking the Z-transform of eqn (44) a pulse transfer function model can be fitted to $\phi_{x_1'x_2'w_{r_2}}{}'(\sigma)$ to yield

$$Z\{\phi_{x_1'x_2'w_{r_2}}{}'(\sigma)\} = 2\beta_1\beta_2\gamma_2 H(z^{-1})TT(z^{-1}) \tag{45}$$

The results of eqn's (40) and (45) can be decomposed using a multistage least squares algorithm[16] to provide estimates of the pulse transfer functions $\mu_1 C(z^{-1})$, $\mu_2 H(z^{-1})$ of the individual linear subsystems in Fig.4 where μ_1 and μ_2 are constants. A polynomial[12], a series of straight line segments[24] or any other suitable function can then be fitted to the nonlinear element $F[\cdot]$ by minimising the sum of squares of the error

$$J = \sum_{j=1}^{m} (z(j)-\hat{z}(j))^2 \tag{46}$$

where $\quad \hat{z}(j) = \mu_2 \sum_{i=1}^{m} \{H(i)F[\hat{q}(j-i)]\} \tag{47}$

using an algorithm by Peckham[23]. When there is no instantaneous transmission through the forward path and the input commences at time $t = 0$, $\hat{q}(j-i)$ can be computed directly, otherwise the closed-loop response must be computed iteratively from the open-loop input-output relation using an algorithm by Zames[4].

Because the unity feedback Wiener and Hammerstein models are subclasses of the unity feedback general model the identification procedure is applicable to systems with these structures.

3.2.2 Precascaded feedback systems

Consider the precascaded feedback system illustrated in Fig.5. From eqn's (34) and (35) the first two kernels for this system can be derived as

$$\underline{G}_1 = \underline{V}_1 * \underline{P} = \{[\underline{I} + \lambda_1 \underline{A}_1]^{-1} * \underline{A}_1\} * \underline{P} \tag{48}$$

$$\underline{G}_2 = \underline{V}_2 * \underline{P} = \{-\underline{V}_1 \circ \lambda_2 (\underline{V}_1^2)\} * \underline{P} \tag{49}$$

Following the procedure of the previous section, it can readily be shown that computation of the first order correlation function $\phi_{x_1'w_1'}(\sigma)$ for the system illustrated in Fig.5, yields

$$\phi_{x_1'w_1'}(\sigma) = \beta_1 \int V_1(\sigma - \theta) P(\theta) d\theta \tag{50}$$

Similarly, computation of the second order correlation function yields

$$\phi_{x_1'x_2'w_2'}(\sigma) = 2\beta_1\beta_2\lambda_2 \int G_1^2(\sigma - \theta) V_1(\theta) d\theta \tag{51}$$

Pulse transfer function models can be fitted to the Z-transforms of eqn's (50) and (51) and estimates of $\mu_2 V_1(z^{-1})$ and $\mu_1 P(z^{-1})$ can be obtained using a multistage least squares algorithm[16], where μ_1 and μ_2 are constants. The identified system must be synthesised as illustrated in Fig.6 because a unique estimate of $A_1(z^{-1})$ can only be obtained when $\lambda_1 = 0$. A suitable function can be fitted to the nonlinear element by minimising the sum of squares[16,24], eqn (46), and the identification is complete.

Providing any noise $v(t)$ corrupting the system output is independent of the input, estimates of $\phi_{x_1'w_1'}(\sigma)$ and $\phi_{x_1'x_2'\dot{w}_2'}(\sigma)$ for the feedback systems will be unbiased. The selection of pseudorandom inputs and the error analysis for binary sequences is exactly the same as the open-loop case.

4. SIMULATION RESULTS

The identification algorithms presented above have been used to identify both open and closed loop nonlinear systems.   A compound input $u(t) = x_1(t) + x_2(t)$ was used in all the simulations where $x_1(t)$ and $x_2(t)$ were defined as either

(i) pseudorandom binary sequences of order seven and six respectively.   The two sequences are independent when correlation is performed over $N_1 N_2 = (2^7-1)(2^6-1) = 8001$ points.

(ii) ternary sequences of order four and five respectively. The sequences are independent when the correlation is performed over $(3^4-1)(3^5-1)/2 = 9680$ points.

Initially an open-loop general model consisting of a linear system

$$H_1(z^{-1}) = \frac{0.2z^{-1}}{1 - 0.77z^{-1}} \tag{52}$$

in cascade with the nonlinear element

$$F[\cdot] = 1.0(\cdot) + 0.8(\cdot)^2 + 0.6(\cdot)^3 \tag{53}$$

followed by a second linear system

$$H_2(z^{-1}) = \frac{0.2z^{-1}}{1-1.48z^{-1}+0.548z^{-2}} \tag{54}$$

was simulated using both the compound pseudorandom binary and ternary sequences.   The system response was recorded for three amplitude levels of input $\alpha_i u(t)$, $i = 1,2,3$ where $\alpha_i = \alpha_{i-1} - 0.1$, $\alpha_1 = 1.0$. A comparison of the estimated and theoretical weighting sequences $h_1(t)$ and $h_2(t)$ are illustrated in Fig.7 (a) and (b) respectively for prbs inputs.   The bias due to the nonzero mean level of the

compound prbs input is extremely small and has little effect upon the accuracy of the estimated parameters summarised in Table 1.

A unity feedback system with a forward path consisting of a linear system

$$H_1(z^{-1}) = \frac{0.2z^{-1}}{1-0.88z^{-1}} \tag{55}$$

in cascade with the non-linear element

$$F[\cdot] = (\cdot) + 0.4(\cdot)^2 + 0.2(\cdot)^3 \tag{56}$$

followed by a second linear system

$$H_2(z^{-1}) = \frac{0.3z^{-1}}{1-0.7z^{-1}} \tag{57}$$

was simulated by recording the response to four input levels $\alpha_i u(t)$, $\alpha_1 = 1.0$, $\alpha_2 = -0.96$, $\alpha_3 = 0.92$, $\alpha_4 = -0.88$, of the compound ternary sequence defined in (ii) above. A comparison of the estimated and theoretical weighting sequences of the linear subsystems are illustrated in Fig.8 and the estimated parameters are summarised in Table 2.

The precascaded feedback system, illustrated in Fig.5, consisting of a linear system

$$P(z^{-1}) = \frac{0.5z^{-1}}{1-0.8z^{-1}} \tag{58}$$

in cascade with a closed-loop system composed of a linear subsystem

$$A_1(z^{-1}) = \frac{0.1z^{-1}}{1-1.56z^{-1}+0.7z^2} \tag{59}$$

in the forward path and a nonlinear element

$$E[\cdot] = 1.0(\cdot)^2 + 0.5(\cdot)^3 \qquad\qquad (60)$$

in the feedback path, was simulated for an eight level compound ternary input. The estimated parameters are summarised in Table 3 and a comparison of the estimated and theoretical weighting sequences for the linear subsystems is illustrated in Fig.9.

5. CONCLUSIONS

Identification algorithms for open and closed-loop nonlinear systems based upon pseudorandom input sequences have been derived. Anomalies associated with multidimensional autocorrelation functions of pseudorandom sequences which normally introduce significant errors in the identification of nonlinear systems using correlation analysis are avoided by defining a compound input. Providing the pseudorandom sequences which define the compound input are independent, zero mean and have autocorrelation functions of the form $\phi_{x_i x_i}(\lambda) = \beta\delta_i(\lambda)$, $i = 1,2$, unbiased estimates of the component subsystems can be obtained by computing the first and second order correlation functions. Although the non-zero mean level of pseudorandom binary sequences introduces a small bias in the results, this tends to zero as the sequence lengths are increased and will be negligible in most applications as indicated by the simulation results. Error free estimates can be obtained by using a compound ternary input or other combinations of pseudorandom sequences with the properties defined above.

Although the output of the first two kernels in the Volterra series expansion must be isolated by a multilevel pseudorandom input, characterisation using a finite Volterra series is avoided

and truncation errors are not incurred. Isolation of kernel
outputs is often necessary in nonlinear system identification
although several authors avoid this problem by considering the
identification of systems which are defined by a single kernel.
This constraint can be avoided by using the technique of Lee and
Schetzen[30], but this is based upon white Gaussian inputs and
involves the computation of multidimensional correlation functions
which necessitate excessive computations for even simple systems.
Although the present algorithms are based upon the calculation of
first and second order correlation functions, both these functions
are unit dimensional and estimates of the component subsystems
for a class of open and closed-loop nonlinear systems can be
obtained using simple extensions of established linear techniques.

6.   REFERENCES

1.   BARRETT, J.F., COALES, J.F.: An introduction to the analysis
         of nonlinear control systems with random inputs; Proc.IEE,
         1955, 103C, pp.190-199.

2.   BARRETT, J.F.: The use of functionals in the analysis of
         nonlinear physical systems; Statistical Advisory Unit
         Report 1/57, Ministry of Supply, 1957.

3.   WEST, J.C., DOUCE, J.L., LEARY, B.G.: Frequency spectrum
         distortion of random signals in nonlinear feedback systems;
         Proc.IEE, 1960, 108, pp.259-264.

4.   ZAMES, G.: Functional analysis applied to nonlinear feedback
         systems; IEEE Trans. Circuit Th., CT-10, 1963, pp.392-404.

5.   GEORGE, D.A.: Continuous nonlinear systems; Tech. Rep. 355,
         Res. Lab. Electron., MIT, 1959.

6. SMITH, W.W., RUGH, W.J.: On the structure of a class of
    nonlinear systems; IEEE Trans. Auto. Control, AC-19,
    1974, pp.701-706.

7. SHANMUGAN, K.S., LAL, M.: Analysis and synthesis of a class
    of nonlinear systems; IEEE Trans. Cirts. and Systems,
    CAS-23, 1976, pp.17-25.

8. SMETS, H.B.: Analysis and synthesis of nonlinear systems;
    IRE Trans. Ct. Th., 1960, pp.459-469.

9. NARAYANAN, S.: Application of Volterra series to intermodulation
    distortion analysis of transistor feedback amplifiers;
    IEEE Trans. Ct. Th., CT-17, 1970, pp.518-527.

10. GARDINER, A.B.: Identification of processes containing single-
    valued non-linearities; Int. J. Control, 18, 5, 1973,
    pp.1029-1039.

11. COOPER, B., FALKNER, A.H.: Identification of simple nonlinear
    systems; Proc.IEE, 1975, 122, No.7, pp.753-755.

12. NARENDRA, K.S., GALLMAN, P.: An iterative method for the
    identification of nonlinear systems using a Hammerstein
    model; IEEE Trans. Auto. Cont., AC-11, 1966, pp.546-550.

13. DOUCE, J.L.: Identification of a class of nonlinear systems;
    Proc.IFAC Symp. on System Identification, Tbilisi, 1976.

14. BILLINGS, S.A., FAKHOURI, S.Y.: Identification of nonlinear
    systems using the Wiener model; Elect. Lett., 13, 1977,
    pp.502-504.

15. BILLINGS, S.A., FAKHOURI, S.Y.: Identification of a class of
    nonlinear systems; IEE Colloquium, Nonlinear System
    Ident., May 1978, Digest No.1978/31.

16. BILLINGS, S.A., FAKHOURI, S.Y.: Identification of nonlinear
    systems using correlation analysis; Proc.IEE, 125, 7,
    1978, pp.691-697.

17. BILLINGS, S.A., FAKHOURI, S.Y.: The theory of separable processes
    with applications to the identification of nonlinear systems;
    Proc.IEE, 125, 9, 1978, pp.1051-1058.

18. BILLINGS, S.A., FAKHOURI, S.Y.: Nonlinear system identification
    using the Hammerstein model; Int. J. Systems Sci. (to
    appear).

19. BILLINGS, S.A., FAKHOURI, S.Y.: Identification of factorable
    Volterra systems; (submitted for publication).

20. BILLINGS, S.A., FAKHOURI, S.Y.: Identification of nonlinear
    unity feedback systems; 17th IEEE Conf. Decision and
    Control, San Diego, Jan. 1979.

21. BARKER, H.A., OBIDEGWU, S.N.: Effects of nonlinearities on the
    measurement of weighting functions by cross correlation
    using pseudorandom signals; Proc.IEE, 120, 7, 1973,
    pp.1293-1300.

22. BARKER, H.A., PRADISTHAYON, T.: High-order autocorrelation
    functions of pseudorandom signals based on m sequences;
    Proc.IEE, 117, 9, 1970, pp.1857-1863.

23. PECKHAM, G.: A new method for minimising a sum of squares
    without calculating gradients; Computer Journal, 13, 1970,
    pp.418-420.

24. BILLINGS, S.A., FAKHOURI, S.Y.: Identification of a class of
    nonlinear systems using correlation analysis; Proc.IEE
    Correspondence, 125, 10, 1978.

25. BRIGGS, P.A.N., GODFREY, K.R.: Pseudorandom signals for the dynamic analysis of multivariable systems; Proc.IEE, 113, 7, 1966, pp.1259-1267.

26. DAVIES, W.D.T.: System Identification for Self-Adaptive Control; Wiley, 1970.

27. SIMPSON, H.R.: Statistical properties of a class of pseudo-random sequences; Proc.IEE, 113, 13, 1966, pp.2075-2080.

28. REAM, N.: Nonlinear identification using inverse repeat in-sequences; Proc.IEE, 117, 1, 1970, pp.213-218.

29. BRILLIANT, M.B.: Theory of the analysis of nonlinear systems; Tech. Rep. 304, Res. Lab. Electron., M.I.T., 1958.

30. LEE, Y.W., SCHETZEN, M.: Measurement of the Wiener kernels of a nonlinear system by cross-correlation; Int. J. Control, 2, 1965, pp.237-259.

31. MARMARELIS, V.Z.: Random versus pseudorandom test signals in nonlinear system identification; Proc.IEE, 125, 5, 1978.

FIGURE CAPTIONS

FIG 1.   The open-loop general model

———————————————

FIG 2.   Nonlinear feedback system

———————————————

FIG 3.   The feedback general model

———————————————

FIG 4.   The unity feedback general model

———————————————

FIG 5.   Precascaded nonlinear feedback system

———————————————

FIG 6.   Identified model of the precascaded feedback system

———————————————

        o o o   Theoretical response $h_1(k)$

        — —   Estimated values $\hat{h}_1(k)$

        x x x   Theoretical response $h_2(k)$

        — - —Estimated values $\hat{h}_2(k)$

FIG 7.   A comparison of impulse responses for the open-loop
          general model

———————————————

        o o o   Theoretical response $h_1(k)$

        — —   Estimated values $\hat{h}_1(k)$

        x x x   Theoretical response $h_2(k)$

        — - —Estimated values $\hat{h}_2(k)$

FIG 8.   A comparison of impulse responses for the unity feedback
          general model

———————————————

        o o o   Theoretical response $P(k)$

        — —   Estimated values $\hat{P}(k)$

        x x x   Theoretical response $A_1(k)$

        — - —Estimated values $\hat{A}_1(k)$

FIG 9.   A comparison of impulse responses for the precascaded
          nonlinear feedback system

———————————————

Parameters		LINEAR SYSTEMS						NONLINEAR ELEMENT			
		$n_{1,1}$	$d_{1,1}$	$n_{2,1}$	$n_{2,2}$	$d_{2,1}$	$d_{2,2}$	γ_1	γ_2	γ_3	MSE
Theoretical values		0.2	-0.77	0.2	0.0	-1.48	0.548	1.0	0.8	0.6	
Estimates	prbs	0.19	-0.79	0.21	0.00	-1.42	0.534	0.972	0.833	0.530	2.28
	ternary	0.19	-0.79	0.21	0.00	-1.46	0.53	1.00	0.801	0.599	0.032

Table 1. A summary of the identification results for the open loop general model

Parameters	Linear Systems				Nonlinear Element			MSE
	$n_{1,1}$	$d_{1,1}$	$n_{2,1}$	$d_{2,1}$	Y_1	Y_2	Y_3	
Theoretical values	0.2	-0.88	0.3	-0.7	1.0	0.4	0.2	-
Estimates	0.204	-0.875	0.298	-0.705	0.990	0.456	-0.074	0.518×10^{-6}

Table 2. Identification results for the unity feedback general model

Parameters	Linear subsystems						Nonlinear element			MSE
	$n_{p,1}$	$d_{p,1}$	$n_{A,1}$	$n_{A,2}$	$d_{A,1}$	$d_{A,2}$	λ_1	λ_2	λ_3	
Theoretical values	0.5	-0.8	0.1	0.0	-1.56	0.7	0.0	1.0	0.5	—
Estimates	0.499	-0.801	0.103	-0.002	-1.55	0.69	0.0	1.23	0.328	0.526×10^{-4}

Table 3. Estimated parameters for the precascaded nonlinear feedback system

Fig. 1. The Open-loop General Model

Fig. 2. Nonlinear Feedback System

Fig. 3. The Feedback General Model

Fig. 4.   The unity feedback general model

Fig. 5.   Precascaded nonlinear feedback system

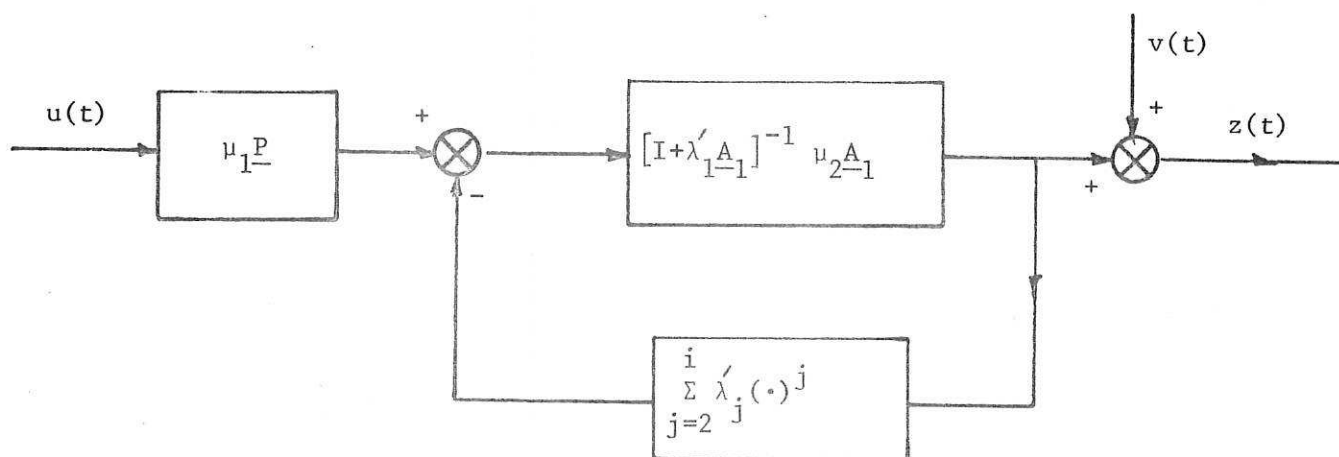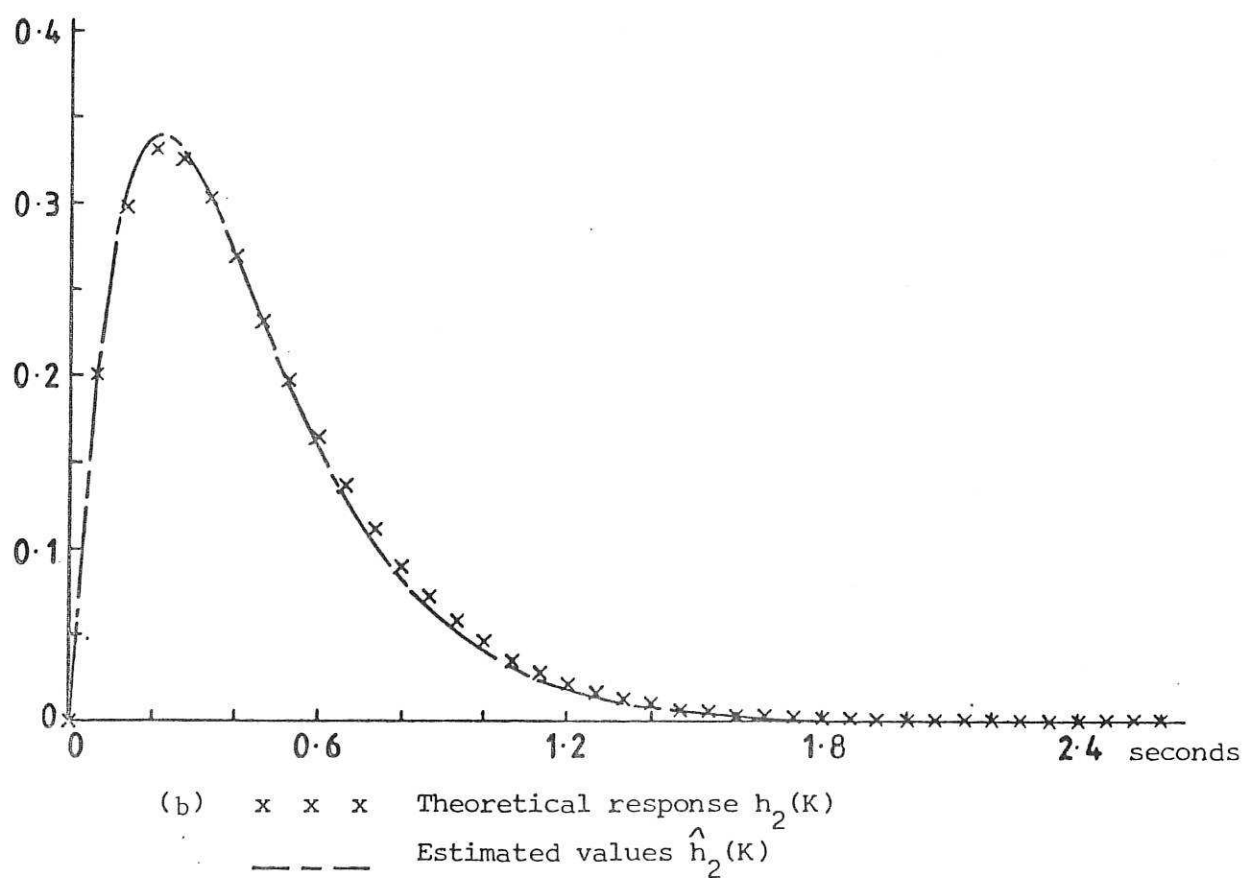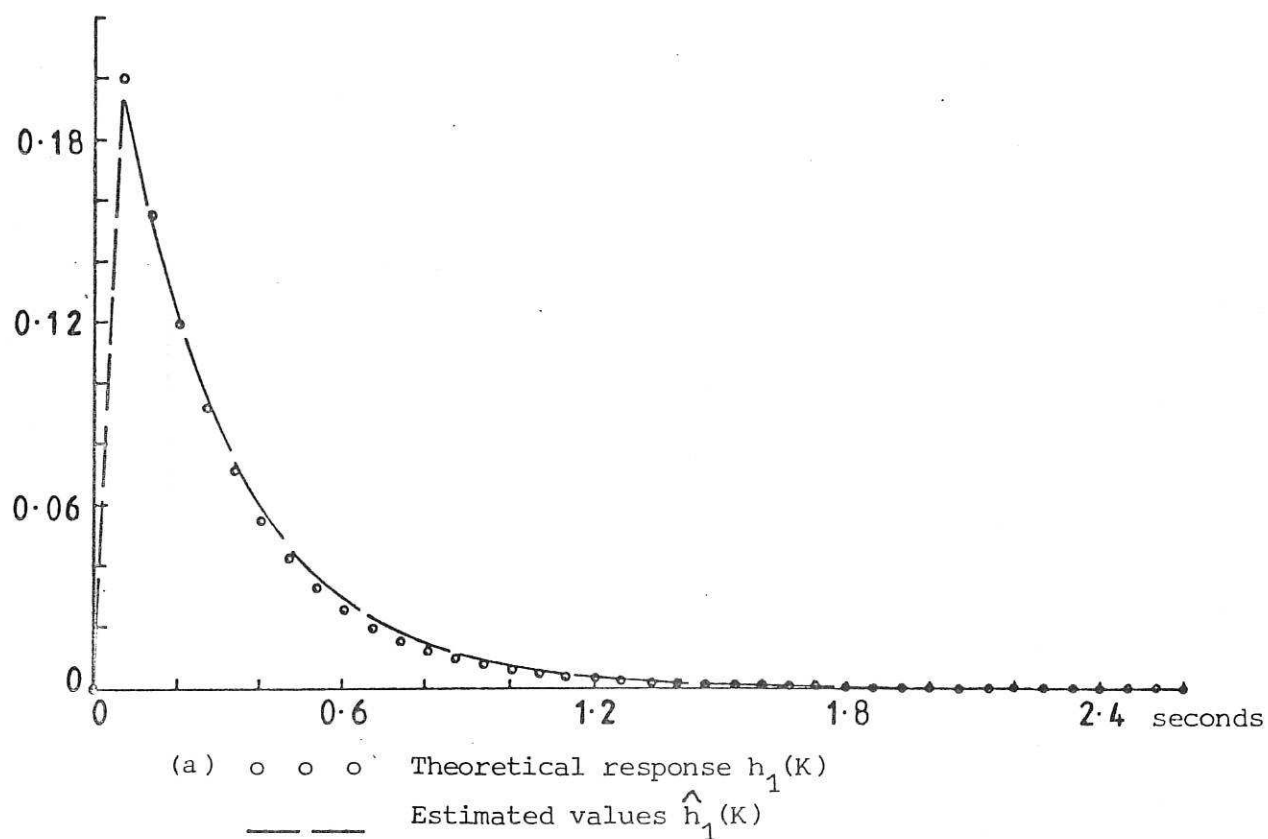Fig. 6.   Identified model of the precascaded feedback
system

(a) o o o   Theoretical response $h_1(K)$

    ————   Estimated values $\hat{h}_1(K)$

(b)   x x x   Theoretical response $h_2(K)$

    — — —   Estimated values $\hat{h}_2(K)$

FIG 7   A comparison of impulse responses for the open-loop
General Model

(a)   o   o   o   Theoretical response $h_1(K)$

— — —   Estimated values $\hat{h}_1(K)$

(b)   x   x   x   Theoretical response $h_2(K)$

— — —   Estimated values $\hat{h}_2(K)$

FIG 8   A comparison of impulse responses for the

unity feedback general model

(a)　　o　o　o　　Theoretical response P(K)

　　　———　———　　Estimated values $\hat{P}(K)$

(b)　　x　x　x　　Theoretical response $A_1(K)$
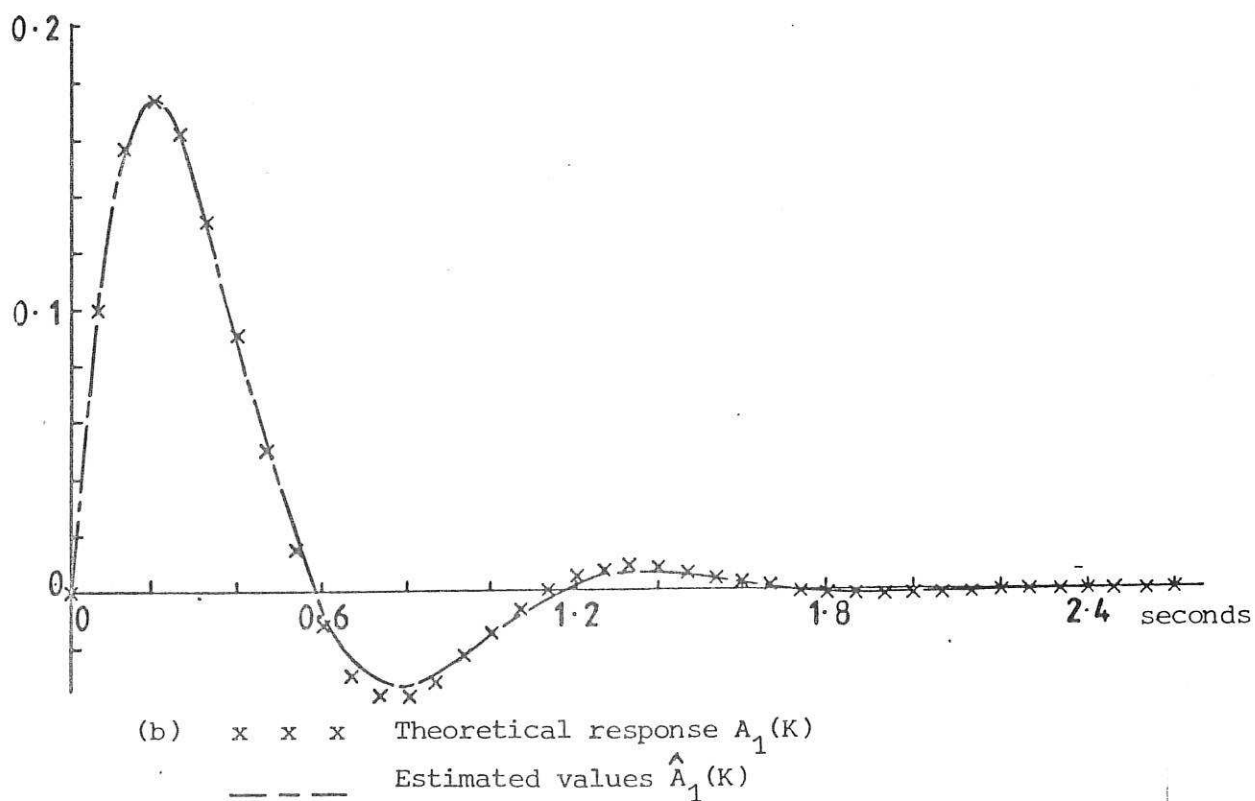
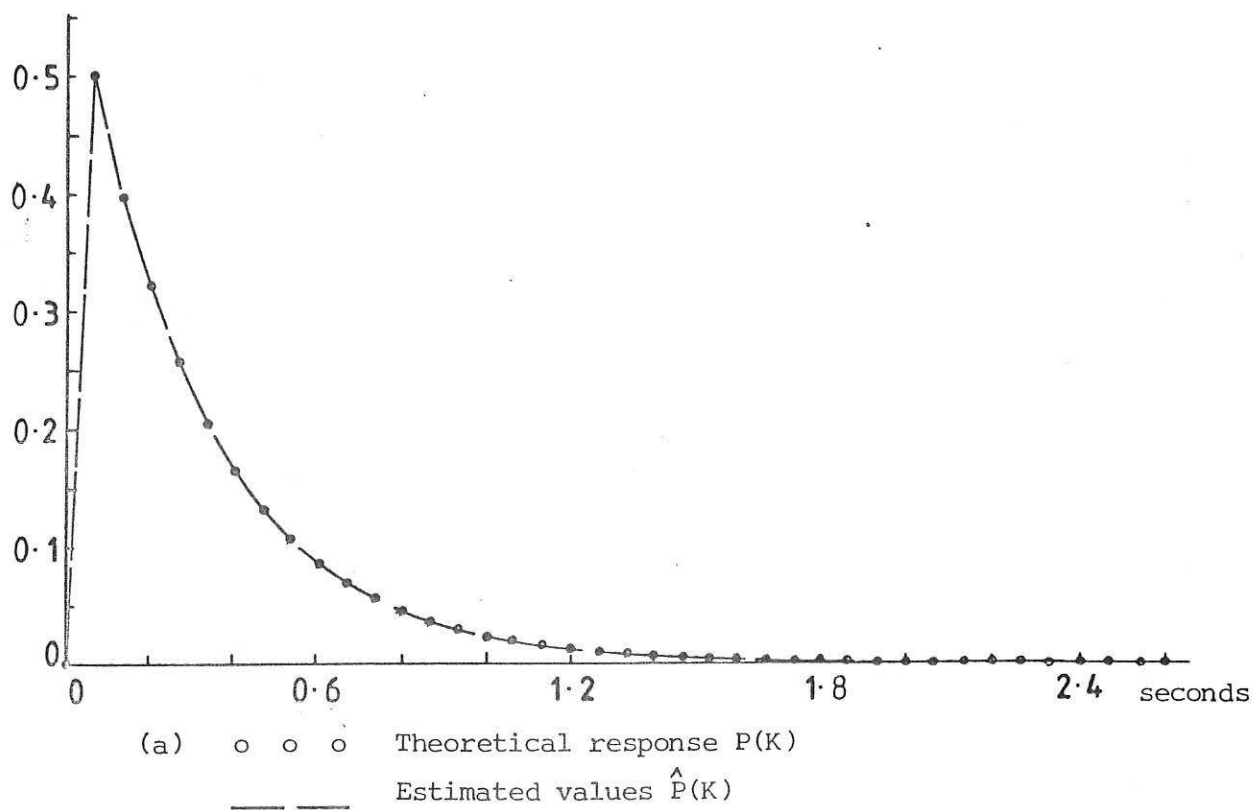　　　———　———　　Estimated values $\hat{A}_1(K)$

FIG 9　　　A comparison of impulse responses for the precascaded

non-linear feedback system