CASCADE CANONICAL FORM FOR LINEAR

MULTIVARIABLE SYSTEMS

by

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Abstract

A canonical form is derived for systems described by an $m \times \ell$ transfer function matrix $G(s)$ and applied to the calculation of system transmission zeros, the feedback control of multivariable second-order type systems and pole allocation using output feedback.

List of Symbols

A^+	conjugate transpose of the matrix A
dim N	dimension of a linear subspace N
span $\{x_j\}_{1\leqslant j\leqslant n}$	subspace generated by linear combinations of the vectors $x_1,\dots,x_n$
A^T	transpose of the matrix A
$\|A\|$	determinant of the matrix A
$\delta_{j\ell}$	Kronecker delta function
$\{e_j\}_{1\leqslant j\leqslant n}$	natural basis in R^n

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1. Introduction

In a recent paper (Owens, 1975) the concept of the classical first order lag has been extended to the multivariable case by defining an mxm multivariable first order system of the form

$$G(s) = \sum_{j=1}^{m} \frac{b_j}{s+b_j} \alpha_j \beta_j^+ \quad , \qquad |G(s)| \not\equiv 0 \qquad \ldots(1)$$

where $\{\alpha_j \beta_j^+\}_{1 \leqslant j \leqslant m}$ is a set of dyads satisfying the constraint that $\bar{b}_j = b_\ell$ implies $\overline{\alpha_j \beta_j^+} = \alpha_\ell \beta_\ell^+$. Closed-form solutions have been derived (Owens, 1975) for proportional and proportional plus integral unity negative feedback controllers which are direct multivariable generalizations of the equivalent classical controllers. The proposed controllers are capable of producing a feedback system with arbitrarily fast response speeds and small interaction effects.

In classical theory, first order lags can be regarded as fundamental building blocks for the construction of the system dynamic behaviour. For example, if g(s) is a non-zero scalar transfer function with pole set $\{p_j\}_{1 \leqslant j \leqslant n}$ and zeros $\{z_j\}_{1 \leqslant j \leqslant m}$ $(m<n)$, then if $k = n-m$,

$$g_\infty^{(k)} \triangleq \lim_{s \to \infty} s^k g(s) \neq 0 \qquad \ldots(2)$$

and,

$$g(s) = \{ \prod_{j=1}^{k} (s-p_j)^{-1} \} \{ g_\infty^{(k)} + h(s) \} \qquad \ldots(3)$$

where h(s) is a strictly proper transfer function with pole set $\{p_j\}_{k<j \leqslant n}$. Quite obviously the decomposition can be extended by application of the same approach to h(s). Such decompositions are useful in the classification of linear systems according to rank and type, the characterization of zeros as due to parallel branches in the system, and in many cases can be used as the basis of a simple model reduction procedure.