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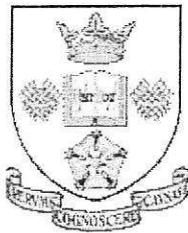


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# Modelling and Characterisation of a Mobile Robot's Operation

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# Modelling and characterisation of a mobile robot's operation

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**Abstract.** The investigation of robot-environment interaction is the main aim of the RobotMODIC project at the Universities of Essex and Sheffield. The methods developed under this project model and characterise all aspects relevant to the robot's operation: modelling of sensor perception ("environment identification" or simulation), sensor modelling, and task modelling.

In this paper we describe a new procedure to obtain the control code for a mobile robot: Initially, a robot is controlled by a human operator who manually guides the robot through a desired sensor-motor task. The robot's motion is then "identified" using the NARMAX system identification technique. The resulting transparent model is then used to control the movement of the robot. Using a transparent model for robot control has the advantage that the robot's motion can be analysed and characterised quantitatively, resulting in a better understanding of robot-environment interaction.

## 1 Introduction

There are two independent objectives of robotics research: on the one hand, to create control programs that are capable of making the robot carry out useful tasks in the real world ("robot engineering") and, on the other hand, to obtain a theoretical understanding of the design issues involved in making those programs ("robot science").

Currently there is a strong tendency to work in the field of mobile robotics from an engineering point of view. Nevertheless, the development of a control program written with a specific task in mind almost never produces the desired behaviour straight away. Instead, iterative refinement is used: a good first guess at a feasible control strategy is implemented, then tested in the target environment. The drawback of this approach is that even if a solution is achieved through this process, it represents an "existence proof": it is proven that a particular robot can achieve a particular task under a particular set of environmental conditions, but this existence proof doesn't imply that the robot can behave in a similar way anywhere else.

Fundamentally, the behaviour of a robot is influenced by three components: i) the robot's hardware, ii) the program it is executing, and iii) the environment.

the robot is operating in. This results in a highly complex system whose fundamental properties are only partially understood. Our aim is to quantify and model this robot-environment interaction. This would allow the investigation of, for instance, i) the effects of modifications of the robot, ii) the effect of modifications of the environment on the overall behaviour of the robot, and iii) the influence of the robot control program on robot behaviour.

The development of a theory for robot-environment interaction is one of the main issues addressed by the RobotMODIC project conducted at the universities of Essex and Sheffield (see also section 2). The work described in this article, part of the RobotMODIC process, is mostly concerned with a novel procedure to program robots through behaviour identification (section 3), rather than an empirical trial-and-error process of iterative refinement. The application of this novel procedure to solve a particular task in mobile robotics is described in section 4. As we'll see in section 5, transparent modelling of a robot's behaviour allows the analysis of important factors involved in robot environment interaction and also the formulation of new and testable hypotheses which lead to new approaches in experimentation and design of robot controllers.

## 2 The RobotMODIC project

The RobotMODIC project at the universities of Essex and Sheffield investigates the underlying phenomena governing robot-environment interaction. The project aims to "identify" - in the sense of mathematical modelling - both a mobile robot's motor responses to perceptual stimuli (task identification), and the perceptual properties of the robot's environment (environment identification). Models are represented as either linear or non linear polynomials, and are obtained using ARMAX (Auto-Regressive Moving Average model with eXogenous inputs), and NARMAX (Non-linear Auto-Regressive Moving Average model with eXogenous inputs) system identification [1].

The techniques developed under this project represent a step towards a science of mobile robotics, because they reveal fundamental properties of the sensor-motor couplings underlying the robot's behaviour. In fact, the transparent and analysable modelling methods (ARMAX and NARMAX), have been already applied to model and characterise a robot controller [2], to achieve platform independent programming [3, 4], to analyse the relationship between a mobile robot's perception, motion and position (i.e. addressing the problem of sensor-based self-localisation) [5], and to "translate" one sensor modality into another. Furthermore, the RobotMODIC procedure has also been applied to model the sensor perceptions (publication in preparation), in such a way that generic simulation programs could be replaced by specific models of robot-environment interaction, derived from real-world data obtained in robotics experiments.

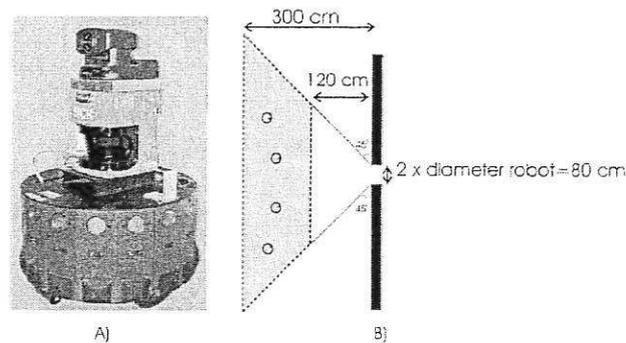


Fig. 1. RADIX, THE MAGELLAN PRO ROBOT USED IN OUR EXPERIMENTS. B) EXPERIMENTAL SCENARIO FOR THE DOOR TRAVERSAL BEHAVIOUR, THE INITIAL POSITIONS OF THE ROBOT WERE WITHIN THE SHADED AREA.

### 3 Task Identification and Robot Training

#### 3.1 Motivation

Our recent efforts have been oriented towards the development of a novel procedure to program a robot controller, based on system identification techniques. Instead of refining an initial approximation of the desired control code through a process of trial and error, we identify the motion of a manually, “perfectly” driven robot, and subsequently use the result of the identification process to achieve autonomous robot operation.

The process works in two stages: first the robot is driven under manual control (robot training), demonstrating the behaviour we want to achieve. While the robot is being moved we log enough information to model the relationship between the robot’s sensor perceptions and motor responses. After this first stage, a Non-linear Auto-Regressive Moving Average model with eXogenous inputs (NARMAX) is estimated (see next section). This model relates robot sensor values to actuator signals, and it can be analysed and subsequently used to control the movement of the robot.

There are alternatives to the approach we used here. For instance, artificial neural networks or genetic programming could be applied to model the robot’s behaviour, but have the disadvantage that the models produced are opaque. Through our proposal the behaviour of the robot is modelled through a polynomial representation that can be analysed to understand the main aspects involved in robot behaviour. Furthermore, a polynomial model is easily and accurately transferable to any robot platform with a similar sensor configuration. As this polynomial can be used to control the robot’s movement directly, the program code is very compact (which is useful for applications where memory and processing speed matter).

The robot we use for our experiments is a Magellan Pro mobile robot (figure 1), equipped with front-facing laser, sonar, infrared, tactile and vision sensors. In the experiments reported here we only used the laser, which covers the semi-circle in front of the robot, and the sixteen omni-directional sonar sensors of the robot.

### 3.2 NARMAX Modelling

The NARMAX modelling approach is a parameter estimation methodology for identifying both the important model terms and the parameters of unknown non-linear dynamic systems. For multiple input, single output noiseless systems this model takes the form:

$$\begin{aligned}
y(n) = & f(u_1(n), u_1(n-1), u_1(n-2), \dots, u_1(n-N_u), u_1(n)^2, u_1(n-1)^2, u_1(n-2)^2 \\
& , \dots, u_1(n-N_u)^2, \dots, u_1(n)^l, u_1(n-1)^l, u_1(n-2)^l, \dots, u_1(n-N_u)^l, u_2(n) \\
& , u_2(n-1), u_2(n-2), \dots, u_2(n-N_u), u_2(n)^2, u_2(n-1)^2, u_2(n-2)^2, \dots, \\
& , u_2(n-N_u)^2, \dots, u_2(n)^l, u_2(n-1)^l, u_2(n-2)^l, \dots, u_2(n-N_u)^l, \dots, \\
& u_d(n), u_d(n-1), u_d(n-2), \dots, u_d(n-N_u), u_d(n)^2, u_d(n-1)^2, u_d(n-2)^2, \dots, \\
& u_d(n-N_u)^2, \dots, u_d(n)^l, u_d(n-1)^l, u_d(n-2)^l, \dots, u_d(n-N_u)^l, y(n-1), \\
& y(n-2), \dots, y(n-N_y), y(n-1)^2, y(n-2)^2, \dots, y(n-N_y)^2, \dots, y(n-1)^l, \\
& y(n-2)^l, \dots, y(n-N_y)^l)
\end{aligned}$$

where  $y(n)$  and  $u(n)$  are the sampled output and input signals at time  $n$  respectively,  $N_y$  and  $N_u$  are the regression orders of the output and input respectively and  $d$  is the input dimension.  $f()$  is a non-linear function and it is typically taken to be a polynomial or wavelet multi-resolution expansion of the arguments. The degree  $l$  of the polynomial is the highest sum of powers in any of its terms.

The NARMAX methodology breaks the modelling problem into the following steps: i) Structure detection, ii) parameter estimation, iii) model validation, iv) prediction, and v) analysis. A detailed procedure of how these steps are done is presented in [1, 6, 7].

Any data set that we intend to model is first split in two sets (usually of equal size). The first, referred to as the estimation data set, is used to calculate the model parameters. The remaining data set, referred to as the validation set, is used to test and evaluate the model.

The number of terms of the NARMAX model polynomial can be very large depending on the number of inputs and the values of  $N_u$ ,  $N_y$  and  $l$ . Nevertheless not all the terms are significant contributors to the computation of the output, in fact often most terms can be safely removed from the model equation without this introducing any significant errors. In order to do this, the so-called Error Reduction Ratio (ERR) [1] is computed for each term. The ERR of a term in the model is the percentage reduction in the total mean-squared error (i.e.

the difference between model predicted and true system output) as a result of including (in the model equation) the term under consideration. The bigger the ERR is, the more significant the term. Model terms with ERR under a certain threshold are removed from the model polynomial.

#### 4 Robot "Programming" Through Task Identification: Door Traversal

The example presented in this section demonstrates how robot programming through system identification works in practice. The task we want to solve with the robot is "door traversal", under the experimental conditions shown in figure 1. The trajectories of the robot after crossing 39 times the same door under manual control can be seen in figure 2 (a). The translational velocity was kept constant at 0.07 m/s, so that the human operator only controlled the rotational velocity at every instant. As we can see this is an episodic task, where each episode comprises the movement of the robot from the starting position to the final position once the door has been crossed.

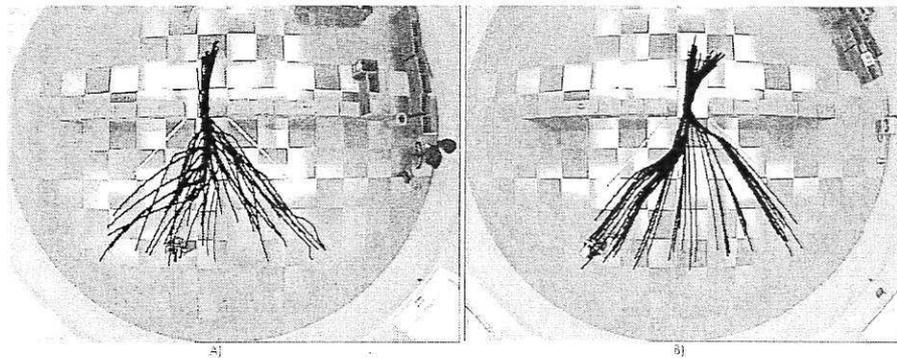


Fig. 2. A) ROBOT TRAJECTORIES UNDER MANUAL CONTROL (39 RUNS, TRAINING DATA). B) TRAJECTORIES TAKEN UNDER MODEL CONTROL (41 RUNS, TEST DATA). THE WHITE LINES ON THE FLOOR WERE USED TO AID THE HUMAN OPERATOR IN SELECTING START LOCATIONS, THEY WERE INVISIBLE TO THE ROBOT.

We then identified the door traversal task, using a NARMAX process, and obtained the model given in table 1. In order to avoid making assumptions about the relevance of specific sensor signals, *all* ultrasound and laser measurements were taken into account. The values delivered by the laser scanner were averaged in twelve sectors of 15 degrees each (laser bins), to obtain a twelve dimensional vector of laser-distances. These laser bins as well as the 16 sonar sensor values, were inverted before they were used to obtain the model, so that large readings indicate close-by objects.

$$\begin{aligned}
\hat{\theta}(t) = & 0.272 + 0.189 * (1/d_1(t)) - 0.587 * (1/d_3(t)) - 0.088 * (1/d_4(t)) - 0.463 * (1/d_6(t)) \\
& + 0.196 * (1/d_8(t)) + 0.113 * (1/d_9(t)) - 1.070 * (1/s_9(t)) - 0.115 * (1/s_{12}(t)) \\
& + 0.203 * (1/d_3(t))^2 - 0.260 * (1/d_5(t))^2 + 0.183 * (1/s_9(t))^2 + 0.134 * (1/(d_1(t) * d_3(t))) \\
& - 0.163 * (1/(d_1(t) * d_4(t))) - 0.637 * (1/(d_1(t) * d_5(t))) - 0.340 * (1/(d_1(t) * d_6(t))) \\
& - 0.0815 * (1/(d_1(t) * d_8(t))) - 0.104 * (1/(d_1(t) * s_9(t))) + 0.075 * (1/(d_2(t) * s_7(t))) \\
& + 0.468 * (1/(d_3(t) * d_5(t))) + 0.046 * (1/(d_3(t) * s_5(t))) + 0.261 * (1/(d_3(t) * s_{12})) \\
& + 1.584 * (1/(d_4(t) * d_6(t))) + 0.076 * (1/(d_4(t) * s_4(t))) + 0.341 * (1/(d_4(t) * s_{12}(t))) \\
& - 0.837 * (1/(d_5(t) * d_6(t))) + 0.360 * (1/(d_5(t) * d_7(t))) - 0.787 * (1/(d_6(t) * d_9(t))) \\
& + 3.145 * (1/(d_6(t) * s_9(t))) - 0.084 * (1/(d_6(t) * s_{13}(t))) - 0.012 * (1/(d_7(t) * s_{15}(t))) \\
& + 0.108 * (1/(d_8(t) * s_3(t))) - 0.048 * (1/(d_8(t) * s_6(t))) - 0.075 * (1/(d_9(t) * s_4(t))) \\
& - 0.105 * (1/(d_{10}(t) * d_{12}(t))) - 0.051 * (1/(d_{10}(t) * s_{12}(t))) + 0.074 * (1/(d_{11}(t) * s_1(t))) \\
& - 0.056 * (1/(d_{12}(t) * s_7(t)))
\end{aligned}$$

Table 1. NARMAX MODEL OF THE ANGULAR VELOCITY  $\hat{\theta}$  FOR THE DOOR TRAVERSAL BEHAVIOUR. THE SONAR READINGS ARE REPRESENTED AS  $s_1, \dots, s_{16}$ , AND THE 12 LASER BINS ARE  $d_1, \dots, d_{12}$ .

Figure 2 (b), shows the trajectories of the robot under NARMAX model control. Door traversal was performed 41 times. The initial positions of the robot during testing were located in the same area as those used for training (see figure 1).

Figure 2 reveals some interesting phenomena: In the first door traversal under human control, the human operator moved the robot towards the centre of the door when the robot was still far from the opening. As the human operator gained experience, he was able to execute more efficient motions, nearer the door. Figure 2 (b) shows how the NARMAX model controlled the robot in a manner that was smooth in all trajectories.

To determine the degree of similarity between the trajectories achieved under human control and those observed under automatic control, we analysed the trajectories through the door, comparing the  $x$  positions the robot occupied when it was at the centre of the opening. There is a statistically significant difference between these distributions (U-test,  $p < 0.05$ ): the model-driven robot traverses the door more centrally than the human-driven robot.

## 5 Quantitative Task Characterisation

### 5.1 Sensitivity Analysis Through Modulation with Noise

This section shows several ways of how NARMAX models can be used to understand the main factors involved in the robot's operation in the environment.

Once a model is obtained, one would, for instance, want to estimate the influence of individual sensor readings upon the robot's global behaviour [8]. We have therefore carried out some initial experiments in order to have an idea of the sensitivity of our door traversal model.

To check the sensitivity of our model we selected randomly one of the testing episodes shown in figure 2. To analyse the sensitivity of the steering velocity with

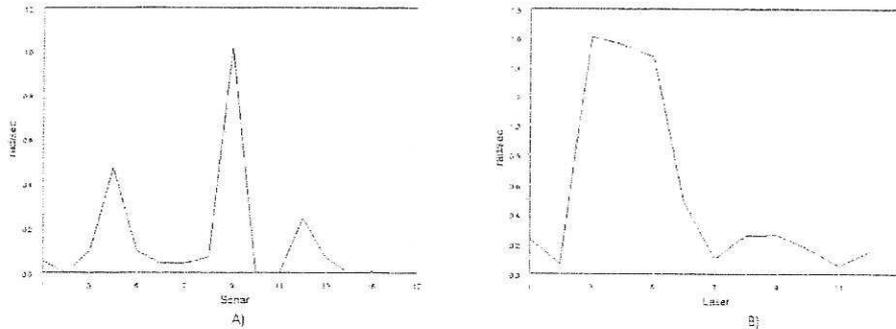


Fig. 3. MAXIMUM CHANGE IN THE ANGULAR VELOCITY WHEN EACH SENSOR IS MODULATED WITH NOISE ALONG ONE OF THE ROBOT'S TRAJECTORIES SHOWN IN FIGURE 2(B). IN A) THE SENSORS MODULATED ARE THE ULTRASOUND ONES, WHILE IN B) WE MODIFIED THE LASER BINS.

respect to one particular sensor, we use the model to recalculate the angular velocity the robot should attain along the trajectory at every instant,  $t$ , when the chosen sensor's readings,  $s(t)$ , take values which go from  $0.4s(t)$  to  $1.6s(t)$ . It is important to notice that only one sensor is modulated, all other sensors remain unaltered. Figure 3 shows one example of such an analysis.

Figures 4 and 5 show how the sensitivity varies along the trajectory of the robot for those sensors in which, according to figure 3, the model seems to be more sensitive. In the case of the laser information, figure 3 (b), the sensitivity seems to be higher for those laser bins which comprise the laser readings in the interval  $[45^\circ, 105^\circ]$ . The change in the angular velocity when one of these laser bins is perturbed is also illustrated in figure 6 (a).

As we can see in all these graphs the perturbation of just one sensor doesn't affect the behaviour of the robot, the graphs are mostly flat in all the covered area, indicating that our NARMAX model of door-traversal is stable and insensitive to noise. Only when the robot is close to the door a perturbation can alter the behaviour of the robot significantly. This is reasonable, given that crucial part of the robot's operation is motion near the door, when the accuracy of *every* sensor matters.

We also applied the Monte Carlo mechanism proposed by I. M. Sobol [8] to estimate the sensitivity of the function given in table 1 with respect to individual sensor signals. This confirmed a high sensitivity of the model with respect to the rear sonar sensor 9, and also with respect to the laser bins and sonar sensors located on the right side of the robot.

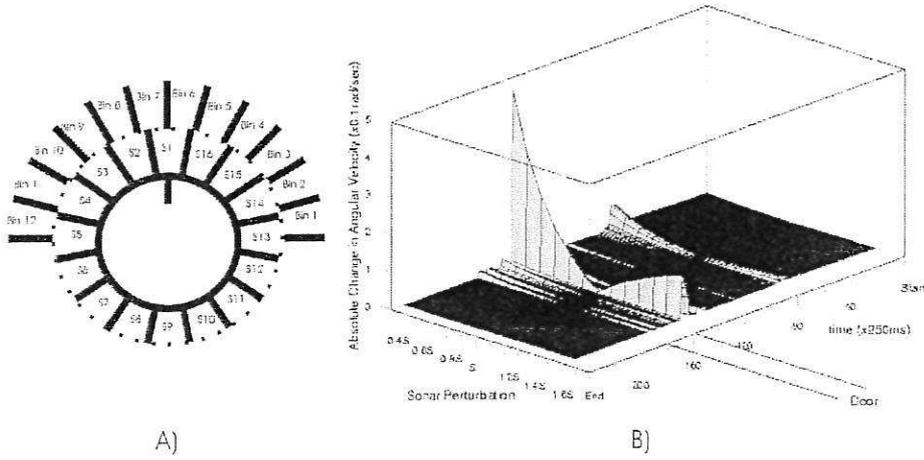


Fig. 4. A) LOCATION OF EACH SONAR SENSOR AND LASER BIN. B) CHANGE IN THE ANGULAR VELOCITY WHEN THE SONAR 4 IS MODIFIED.

## 5.2 Sensitivity Analysis Through Partial Derivation

Partial derivatives can also be used for sensitivity analysis. In the case of sonars 4 and 9, for example, we get:

$$\frac{\partial \dot{\theta}}{\partial s_4} = -0.076d_4^{-1}s_4^{-2} + 0.075d_9^{-1}s_4^{-2} \quad (1)$$

$$\frac{\partial \dot{\theta}}{\partial s_9} = 1.070s_9^{-2} - 0.366s_9^{-3} - 3.145d_9^{-1}s_9^{-2} \quad (2)$$

The presence of terms  $1.070s_9^{-2}$  and  $0.366s_9^{-3}$  in equation 2, explain why sonar 9 readings may alter the angular velocity even far from the door.

To understand these terms, we went back to the training data and checked the values of this sensor especially far from the door. We then saw that in some of the trajectories the human operator moved the robot towards the centre of the door even when it was still far from the opening (figure 2). We therefore removed the first part of those episodes where this experimental artifact took place and re-obtained the NARMAX model. The result was that the sensitivity on sensor 9 far from the door almost disappeared. This result allows us to make the hypothesis that it might be better not to consider sensor 9 as part of the input information in the modelling process.

Regarding sensor 4, we know that the possible change in the angular velocity due to *only* this sensor can be locally estimated as:

$$\Delta \dot{\theta} = \Delta S_4 \frac{\partial \dot{\theta}}{\partial s_4} \quad (3)$$

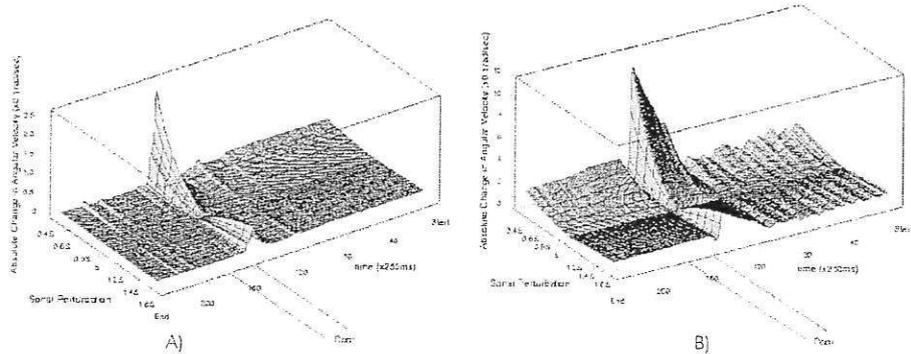


Fig. 5. CHANGE IN ANGULAR VELOCITY WHEN SONAR 12 IS MODULATED WITH NOISE (A), AND WHEN SONAR 9 IS MODIFIED (B).

$|\frac{\partial \dot{\theta}}{\partial s_4}|$  increases very significantly if one of the opposite laser bins, 4 or 9, (figure 4 (a)), has a reading clearly bigger than the other (figure 6). According to equation 3) this should cause an important change in the angular velocity. Only when the two laser bins are similar (the robot is aligned with the centre of the door), is  $|\frac{\partial \dot{\theta}}{\partial s_4}| = 0$ .

## 6 Summary and Conclusion

The development of a theory of robot-environment interaction, one of the main goals of the RobotMODIC project, would allow the formulation of hypothesis for testing, make predictions, and thus serve as safeguard against unfounded or weakly supported assumptions. The determination of transparent computer models of robot-environment interaction, as presented in this paper, is one element of such a theory.

In this article, we describe a novel mechanism to program robots through system identification, rather than an empirical trial-an-error process of iterative refinement. To achieve sensor-motor tasks, we first operate the robot under human supervision, making it follow a desired trajectory. Once data is acquired in this way, we use the NARMAX modelling approach to obtain a model which identifies the coupling between sensor perception and motor responses. This model is then used to control the robot when it moves autonomously.

As one of the main contributions of this paper, we have shown how a NARMAX model can be used to identify the main factors involved in the robot's execution of a particular task. In particular it allows not only the analysis of the stability and sensitivity of robot's operation, but also the formulation of new and testable hypotheses which lead to a new stage in the experimentation and design of a controller.

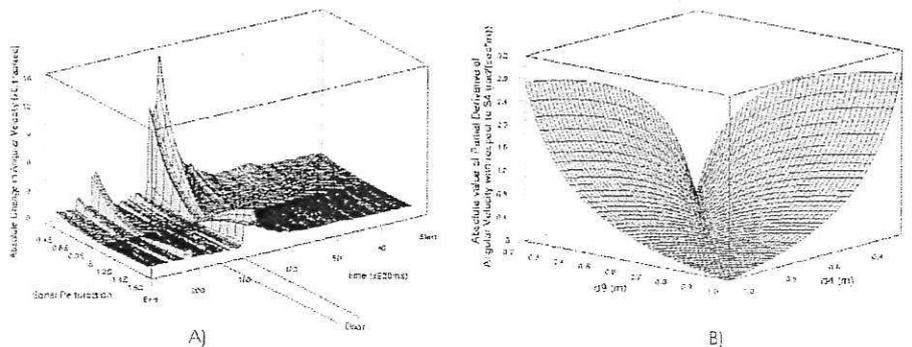


Fig. 6. A) CHANGE IN THE ANGULAR VELOCITY WHEN THE LASER BIN 4 IS MODIFIED. B) REPRESENTATIONS OF THE ABSOLUTE VALUE OF  $\partial\dot{\theta}/\partial s_4$  WHEN  $s_4 = 0.33m$ .

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