Published paper:

Odling, NE, West, LJ, Hartmann, S and Kilpatrick, A (2013) *Fractional flow in fractured chalk; a flow and tracer test revisited.* Journal of Contaminant Hydrology, 147. 96 - 111.

http://dx.doi.org/10.1016/j.jconhyd.2013.02.003

Fractional flow in fractured chalk; a flow and tracer test revisited

N.E. Odling[1], L.J. West[1], S. Hartmann[2], A. Kilpatrick[1]

[1] School of Earth and Environment, University of Leeds, Woodhouse Lane, Leeds LS1 9JT, UK. Emails: n.odling@see.leeds.ac.uk, l.j.west@see.leeds.ac.uk, a.kilpatrick@see.leeds.ac.uk

[2] Environment and Heritage Service (EHS), Water Management Unit Groundwater management team, Lisburn, N. Ireland, BT28 3AL. Email: Silke.Hartmann@doeni.gov.uk

Corresponding author: N.E. Odling, School of Earth and Environment, University of Leeds, Woodhouse Lane, Leeds LS1 9JT, UK. Tel: +44 113 3432806, Fax: +44 113 343 5259, Email n.odling@see.leeds.ac.uk.

Abstract

A multi-borehole pumping and tracer test in fractured chalk is revisited and reinterpreted in the light of fractional flow. Pumping test data analysed using a fractional flow model gives sub-spherical flow dimensions of 2.2-2.4 which are interpreted as due to the partially penetrating nature of the pumped borehole. The fractional flow model offers greater versatility than classical methods for interpreting pumping tests in fractured aquifers but its use has been hampered because the hydraulic parameters derived are hard to interpret. A method is developed to convert apparent transmissivity and storativity (L^{4-n}/T and S^{2-n}) to conventional transmissivity and storativity (L^2/T and dimensionless) for the case where flow dimension, 2<*n*<3. These parameters may then be used in further applications, facilitating application of the fractional flow model. In the case illustrated, improved fits to drawdown data are obtained and resultant transmissivities and storativities are found to be lower by 30% and an order of magnitude respectively, than estimates from classical methods. The revised hydraulic parameters are used in a reinterpretation of a tracer test using an analytical dual porosity model of solute transport incorporating matrix diffusion and modified for fractional flow. Model results show smaller fracture apertures, spacings and dispersivities than those when 2D flow is assumed. The pumping and tracer test results and modelling presented illustrate the importance of recognising the potential fractional nature of flow generated by partially penetrating boreholes in fractured aquifers in estimating aquifer properties and interpreting tracer breakthrough curves.

Keywords: pumping test, tracer test, fractional flow, fractured chalk

1. Introduction

Accurate information on aquifer hydraulic characteristics is needed to aid management of water resources by providing input for regional and local scale flow models, for assessing groundwater vulnerability and, most recently, to estimate the impact of climate change on groundwater resources. The accuracy of assessments and model predictions depends greatly on the accuracy of aquifer hydraulic parameters. One of the most important tools for estimating hydraulic parameters is the pumping test. Data from pumping tests is most commonly interpreted through fitting analytical models to observed drawdown curves. Over the last 80 years since fundamental work by Theis (1935), a wide range of analytical models have been developed for transient flow to a pumped well under a range of conditions (e.g. Kruseman and de Ridder 1990). These methods are designed for either 2 or 3D radial flow to the well in a homogeneous porous medium. The derived hydraulic parameters depend on the choice of an appropriate model and application of an inappropriate model can lead to errors.

Classical methods have been highly successful in analyzing pumping test data in sediments and sedimentary rocks but do not always give good fits for such data in fractured rocks. In response to such poor fits, Barker (1988) introduced a generalized radial flow (GRF) model where the flow dimension is an extra fitting parameter and may take on any integer or non-integer value. The rationale for allowing a non-integer flow dimension is that, in fractured rocks, flow may have a wide variety of geometries which form a continuum from flow in a single channel (1D), to a single fracture (2D) and a fracture network (3D). In this paper, a pumping and tracer test in the fractured Chalk Aquifer of East Yorkshire, UK, previously analysed using classical techniques (Hartmann et al. 2007) is revisited. A method is developed for deriving conventional transmissivity and storativity from the generalized fractional flow model (Barker 1988) and the impact of flow dimensionality on flow and transport parameters are investigated. The implications of revised estimates for transmissivity on the interpretation of a previously conducted tracer test at the site are then explored.

2. The GRF Model

The GRF model (Barker 1988) generalizes the basic solutions of flow to a well to fractional flow dimensions which greatly increasing the range of drawdown type curves that may be fitted to observed data. The drawdown given by the GRF model for the case of constant flow from an infinitesimal source in an infinite region where well bore storage is negligible is (Barker 1988):

$$h(r,t) = \frac{Qr^{2\nu}}{4\pi^{1-\nu} K_f b^{3\text{-n}}} \Gamma(-\nu, u), \quad \nu < 1 \quad , \qquad \text{equation 1}$$

where

$$u = \frac{S_{sf} r^2}{4K_f t} \qquad \text{equation 2}$$

and

$$\nu = 1 - n/2 \quad , \qquad \text{equation 3}$$

where h is drawdown, r is radial distance to the pumped borehole, t is time, K_f is the hydraulic conductivity, S_{sf} is specific storage, Γ is the incomplete gamma function, n is the flow dimension, Q is the pumping rate, and b is 'the extent of the flow region'. The variation in the shape of the drawdown curves with flow dimension, from convex upward ($1<n<2$) to convex downward ($2<n<3$), is shown in Figure 1a. For $n=2$, the GRF model reverts to the Theis (1935) model for transient radial 2D flow where the drawdown curve on a plot of log time versus drawdown tends to a straight line. Variations in flow geometry are illustrated in Figure 1b where, in 1D flow (e.g. a single channel), b is the square root of the ‘flow through area’, in 2D flow, b is the thickness of the flow-through region (e.g. aquifer thickness, fracture aperture) and in 3D flow, the exponent for b is undefined. Despite its versatility, the GRF model has not been widely used because the physical implications of a fractional flow dimension are difficult to conceptualise. Also, the GRF model results in hydraulic parameters that are a function of the parameter b and n. The quantities T_f^{3-n} and S_{sf} b^{3-n} have been termed the ‘generalized’ or ‘apparent’ transmissivity and storativity (e.g. Bangoy et al. 1992, Marechal et al. 2003), with units of L^{4-n}/T and L^{2-n} respectively. It is not clear how such parameters may be incorporated in further applications such as groundwater flow models.

3. Previous Applications of Fractional Flow and the GRF Model

Since Barker’s paper in 1988, a number of authors have investigated the application and physical meaning of fractional flow through numerical modelling investigations and analysis of case studies. Doe (1991) points out that a flow dimension may reflect the power by which either the surface area through which flow is conducted (flow-through area) or aquifer properties (hydraulic conductivity, specific storage) change with distance from the source (borehole). In the case of flow-through area, conduit cross-sectional area changes with distance from source proportional to r^{n-1}. Fractional flow may also be generated by hydraulic conductivity that changes with distance from the source, proportional to r^{n-2} (Doe 1991, Acuna and Yortsos 1995). Whatever the origin of fractional flow, the fractional flow dimension does not necessarily imply a fractal aquifer geometry (Doe 1991).

A number of authors have used numerical modelling approaches to investigate system geometries that give rise to fractional flow. Several authors have demonstrated that fractal fracture networks give rise to fractional flow (Acuna and Yortsos 1995, Doughty and Karasaki 2002, de Dreuzy and Davy 2007, Cello et al. 2009). In non-fractal fracture networks both Euclidean and fractional flow can occur. Jourde et al. (2002) simulated pumping tests in 3D fracture networks based on a stratified system of 'ladder' style fracture patterns that closely mimic some natural systems. For partially penetrating boreholes, the flow dimension was found to be fractional (near spherical) demonstrating that non-fractal, space-filling 3D networks of fractures can give rise to fractional flow and illustrating the potential impact of well geometry on flow dimensionality.

The GRF model has been increasingly used over the last 15 years as an aid to interpreting field pumping tests in fractured rocks. These studies have identified a range of flow dimensions from 0.5 to 3.0 for distances between pumped and observed boreholes of 2 to 400m. A survey of some 12 studies (see Table 1) indicates flow dimensions mostly less than 2 (between linear and radial) linked to flow dominated by single fractures or fracture zones which intersect the pumped well (e.g. Bangoy 1999, Leveinen et al. 1998, Van Tonder et al. 2002, Le Borgne et al. 2004). Flow dimensions larger than 2 are reported from pumping tests in partially penetrating wells (Verbovsek 2009) and packer tests (Kuusela-Lahtinel et al. 2003) in fractured rocks and where a pervasive 3D fracture network is accessed through a single fracture intersecting a pumped well (Marechal et al. 2003, Marechal et al.

2004). In the above field cases, the semi-log plot of drawdown versus time does not converge to a straight line at late times as predicted by the Theis model. The GRF model provides a better fit to the data by allowing flow dimensions other than 2 in which the slope of the drawdown curves evolve with time (see Figure 1).The problem in obtaining conventional hydraulic parameters transmissivity, T and storativity, S, from application of the GRF model has been tackled in a variety of approaches. Some authors simply quote the apparent transmissivity, $K_f\, b^{(3-n)}$, and storativity, $S_{sf}\, b^{(3-n)}$ (e.g. Leveinen et al.1998, Le Borgne et al. 2004, Marechal et al. 2003, 2004). Other authors have attempted to derive K_f and S_{sf} by assuming the parameter b to be equivalent to the aquifer thickness, e.g. thickness of a fracture zone intersecting the pumped well (e.g. Leveinen et al. 1998, Van Tonder et al. 2002). Leveinen et al. (1998) evaluated the parameter b for the 1D and 2D flow as bounding values where the flow dimension lies between 1 and 2. They point out, however, that K values so derived should only be compared for like values of n due to these approximations. Lods and Gouze (2004) derive K and S from the GRF model by defining the Equivalent Cylindrical Transmissivity (ECT) through equating fractional and cylindrical flows. However, they give no details on the derivation of ECT and there is little discussion of the assumptions or validity of this approach in their paper.

The studies above have shown that fractional flow dimensions can be generated in both fractal and non-fractal fracture systems. A wide range of fractional flow dimensions have been observed from pumping tests in fractured rocks showing that the flow dimension is controlled by the fracture system in combination with pumped borehole location and completion. The main difficulty that remains is the determination of conventional parameters T and S where the flow dimension is non-integer. In the following, the GRF model is applied to pumping test data from fractured chalk and a method developed by which the conventional parameters T and S may be determined for non-integer flow dimensions between 2 and 3. This facilitates the use of the GRF model where flow is non-integer by providing estimates of transmissivity and storativity that may be used in further applications.

4. The Pumping Test

4.1 The Field Site

The site is located in the Chalk Aquifer at Wilfholme Landing on the Holderness Plain of East Yorkshire, approximately 20 km north of Kingston-upon-Hull, NE England (Figure 2a). The Chalk Aquifer of NE England extends over an area of approximately 1800 km^2 and is confined over most of the Holderness Plain by glacial and alluvial deposits of Quaternary age. The aquifer at Wilfholme is composed of well bedded chalk with thin marl (clay-rich) horizons and is confined by 10 to 13 m of glacial, clay-rich sediments. At the site, three boreholes (M1, M2 and M3) are arranged in an equilateral triangle, each 25 m from a fourth central borehole, P with a fifth located at a distance of 75 m from P (see Figure 2b). Correlations between boreholes from acoustic logs indicate that bedding dips at around 3^o eastwards and that there are no significant faults at the site (Hartmann et al. 2007). Driller's logs, flow logging (Parker et al. 2009) and dilution testing (West and Odling 2007), together with core from borehole P, indicate the presence of some 4 m of low conductivity 'putty' chalk directly beneath the glacial sediments, passing downwards into around 9 m of highly fractured and weathered chalk of high hydraulic conductivity, and finally into more solid fractured chalk of lower conductivity (Hartmann et al. 2007), see Figure 2(c). Fracture orientations in the more solid chalk below the casing from acoustic logs

(Hartmann et al. 2007) show a wide range of trends lacking preferred orientations and a range of dips from 40^o to 90^o. This is consistent with observations from coastal exposures which indicate a well-connected fracture system with no distinct preferred orientations. Acoustic logs are not available from the highly fractured, high conductivity layer but core, although showing poor recovery from this section, suggests it consists of roughly equi-dimensional fracture blocks the majority of which range from 2 to 15 cm in diameter. Many fracture surfaces show iron hydroxide staining indicating that they are natural and not a result of drilling.

The boreholes are cased for 17.4 to 26.2 m through the glacial sediments and most of the highly weathered chalk (which is too weak to sustain an open hole) and thereafter open to depths of 60 to 80m below ground surface, see Figure 2(c). This is typical of borehole construction in the area. Only a small thickness (0.11 to 3.3 m) of the highly conductive layer at the base of the casing is open and thus boreholes partially penetrate the aquifer. Details of borehole completion and layer thicknesses are given in Table 2.

4.2 Pumping Test Data Collection and Pre-processing

The drawdowns recorded at the time of the tracer test (Hartmann et al. 2007) were of somewhat low resolution and compromised by intermittent problems with the pumping rates at late times. A further high resolution, constant rate pumping test was carried out in 2008 (Kilpatrick 2008) with a pumping rate of 423 m^3/day over a period of 47 hours. Water levels and barometric pressure were recorded using pressure transducers (resolution 0.09 cm) at 30 second intervals. Background water level data from all boreholes was also recorded 5 days prior to and 3 days after the pumping test, at 5 minute intervals.

Before analysis, the pumping test drawdown data was corrected for effects of barometric pressure, Earth tides, and recharge trend. In confined aquifers, water levels in open boreholes rise during falling barometric pressure and vice versa (Rasmussen and Crawford 1997) and is characterized by the barometric efficiency of the aquifer (Rasmussen and Crawford 1997, Clark 1967). Water levels recorded by the pressure transducers can be compensated for the effects of barometric pressure using the equation (Rasmussen and Crawford 1997):

$$WL = TP - (1 - BE)BP \qquad \text{equation 4}$$

where WL is the corrected water level, TP is the total pressure recorded by the pressure transducer (water pressure plus barometric pressure), BE is the barometric efficiency of the aquifer and BP is the barometric pressure expressed in equivalent units of water head. The barometric efficiency of the aquifer was estimated using 20 weeks of water level and barometric pressure data to be 0.65 and this value . is used in equation 4 to compensate all data recorded by pressure transducers for barometric pressure.

The data recorded for 5 days prior to the pumping test was used to identify the influence of Earth tides, local pumping and recharge. Earth tides cause a regular cyclic variation (wavelength 24 hours) in borehole water levels with an amplitude of around 2 cm, see Figure 3. The Earth tide influence on borehole water levels over the pumping test period was estimated from the theoretical Earth tide signal at Wilfholm(TSOFT software, van Camp and Vauterin 2005),This signal, ranging from +0.2 to -1.5 cm, was then subtracted from the compensated water level data.

A falling linear trend in water levels of -0.025 cm/hour due to seasonal recharge variations is observed in the background data. This is removed from the water level data over the pumping test period, resulting in a maximum correction of 1.46 cm over the 47 hours of the pumping test. The effects of short term pumping from a nearby domestic borehole some 200 m to the south can be seen as sharp water level drops of 1 to 2 cm over 30 min periods, occurring once or twice a day (see Figure 3). Due their irregular nature, it is not possible to correct for these effects and in the final drawdown curves, these periods of short term pumping appears as small bumps in the drawdown curve. However, drawdowns due to external pumping of 1 to 2 cm are small compared to the total drawdown in monitoring boreholes during the pumping test of around 0.7 m.

Figure 4(a) shows a plot of the corrected pumping test data for all boreholes which shows drawdowns of up to 1.9 m for the pumped borehole and up to 0.8 m for monitoring boreholes. The drawdown curves for the monitoring boreholes at 25 m distance (M1, M2, M3) and the pumped borehole (P) show a similar geometry with an initial steep rise in drawdown followed by a convex downwards curve. The first derivative of drawdown with respect to time (slopes of these curves) was calculated using the Savitsky-Golay method (Press et al. 1992) with a sampling window of 15 data points to minimise the effects of noise. The derivatives, shown in Figure 4(b), show that the maximum slopes (inflexion points on drawdown curves) occur at times between 1.3 and 2.5 minutes after which slopes steadily decrease with time. For the monitoring borehole at 75m (M4) the initial steep increase in drawdown is much less marked. All drawdown curves show small scale kinks at times around 1.3 and 3.8 hours which are due to sporadic pumping from a domestic borehole some 200 m away.

The drawdown curves for all boreholes flatten after 19 hours (Figure 4a). This is thought to be caused by pressure support to the aquifer provided by leakage from the overlying glacial deposits. Although the head in the glacial sediments has not been monitored specifically, it is known that field drains limit head in these sediments to a maximum of around 0.5 m below ground level. Heads in the aquifer were above this level prior to the pumping test, and below this level by the end of pumping at all boreholes except M4. Thus towards the end of the pumping test, heads in the glacial deposits could provide support to the aquifer.

The corrected drawdown curves up to 19 hours were analyzed, firstly using classical techniques and secondly using the GRF model.

5. Pumping test analysis – classical methods

The corrected drawdown curves for up to 19 hours for boreholes M1, M2, M3 and M4, shown in Figure 4(a), were analysed using a number of classical methods including the Theis (1935) method for radial flow, and modifications to include delayed monitoring borehole response (Black and Kipp 1977), effects of partial penetrating wells and leaky aquifers (Hantush 1961, 1964), and impact of dual porosity (Boulton and Streltsova 1977). The results are described below and summarised in Table 3.

5.1 Theis and Hantush methods

Theis (1935) gives the solution for transient radial flow in a homogeneous aquifer to a fully penetrating well and model curves are shown on plots of $1/u$ against the well function $W(u)$ where $u = r^2 S / 4Tt$. Applying the Theis method (using AquiferWin32 software) shows that it is not possible to fit the entire drawdown curves for the boreholes at 25 m to the Theis model curve. The fit is improved by excluding the first 5 minutes of drawdown data (shown by arrows in Figure 5a) giving transmissivities of 460-488 m^2/d and storativities of 1-10 x 10^{-5}, see Table 3. The drawdown curve from borehole M4 at 75m shows a much better fit to the Theis curve (Figure 5a) with a similar transmissivity of 469 m^2/d and storativity of 2 x 10^{-4}.

A possible explanation for the early steep parts curves is a delayed response in monitoring boreholes due to flow resistance between borehole and aquifer and such model type curves (Black and Kipp, 1977) are shown in Figure 5(a). Only the monitoring well at 75m, shows an acceptable fit indicating a small borehole response time of around 23 seconds.

Borehole storage and/or skin effects are known to contribute to early time responses in both pumped and observation boreholes and the potential impact of these effects on the drawdown data presented here are explored below. Borehole storage effects generate an initial drawdown segment in the pumped borehole with a slope of 1 on a log-log plot of drawdown versus (1/time) or $W(u)$ versus time or $1/u$ (Papadopulos and Cooper 1967). Borehole storage effects in the pumped borehole cause a delay in drawdown and a reduction in the slope of the drawdown curve at observation boreholes (Streltsova 1988). The potential significance of borehole storage on the pumped borehole response is tested using the method of Strelsova (1988) where borehole capacity is given by, $F = \pi r_w^2 / \rho g$, where r_w is the radius of the borehole, ρ is the density of water and g is the gravitational constant. Time, t, to which borehole storage effects are insignificant at the pumped well is given by $t = 9.6766 \pi r_w^2 / T$ where T is the transmissivity of the formation. This is the time at which formation flow rate constitutes 99% of the pumping flow rate (Streltsova 1988). Using the lowest value of transmissivity of 450 m^2/d with a borehole (casing) radius of 0.075 m gives a value of t of 33 s. The impact of well bore storage in observation boreholes diminishes with distance from the pumped well and at r/r_w >200 is negligible (Streltsova 1988). With a ratio r/r_w of 333 for the monitoring wells at 25 m, the impact of borehole storage at the pumped borehole is thus predicted to be insignificant at the observation boreholes. Thus, it is unlikely that the initial steep section of the drawdown curves at observation boreholes is due to borehole storage effects. In addition, the early time segment of the drawdown curve for the pumped borehole, P, shows a slope greater than 1 on a log-log plot of drawdown versus time suggesting that significant borehole storage effects are not present.

Significant positive skin is thought unlikely for the boreholes at the site as all boreholes have been repeatedly pumped (for flow logging, Parker 2010) which is likely to have cleared blocked fractures. Analysis of recovery data at borehole P, using the method of Matthews and Russel (1967) described in Kruseman and de Ridder (1994) gives only a small positive skin factor of 4.1. In addition, the impact of skin on drawdown is negligible at observation boreholes when borehole storage effects are small (Tongpenyai 1981, Streltsova 1988, Bulter 1990, Pucknell and Clifford 1991), as is the case here. Thus the early steep parts of the drawdown curves for the observation boreholes (Figure 4) are unlikely to be due to skin effects.

Since it is known that the boreholes are cased through significant proportions of the highly conductive layer, the Theis method modified for partially penetrating wells in anisotropic aquifers (Hantush 1961, Reed 1980) was applied (using Aquifer Win32 software). Significantly improved fit to entire drawdown curves for boreholes at 25 m (M1, M2, M3) are obtained if anisotropies (K_z/K_r) of 2 to 3 orders of magnitude are invoked, see Figure 5b and Table 3. Compared to the Theis results, the resulting transmissivities are slightly higher (up to 30%) and storativities one to two orders of magnitude higher for boreholes at 25 m (M1, M2, M3) and unchanged for the borehole at 75 m (M4). However, early time segments of drawdown curves for boreholes at 25 m are still significantly steeper than any of the type curves (Figure 5b). When the first 5 minutes of data were omitted, the Hantush model gives a much improved fit with values of T, S and K_z/K_r within 10% of the previous results, see Table 3. The method of Hantush (1964) for partially penetrating wells in a leaky aquifer gave very similar results with a small reduction in T of 9 to 19%, see Table 3. The similarity of parameters from models with and without leakage suggests that leakage has not significantly modified the drawdown curves for times up to 19 hours. Both Hantush (1961, 1964) methods require significant anisotropy with horizontal:vertical hydraulic conductivity ratios of 2 to 3 orders of magnitude. Examination of core from the borehole, P, suggests that the weathered section of the chalk is intensively fractured generating overall equi-dimensional blocks of 2-15cm across consistent with a lack of any preferred fracture orientation (section 4.1 above). This makes a high degree of anisotropy in the highly conductive layer difficult to justify.If an anisotropy ratio of 1 is enforced in the Hantush method (late data), the results are very similar to those of the Theis method applied to late data.

5.2 Dual porosity effects

In a dual porosity aquifer such as fractured chalk, when heads in fractures are drawn down due to pumping, matrix bocks may provide pressure support to fractures (e.g. Boulton and Streltsova, 1977). This is expressed on a drawdown versus log time plot as a reduction in slope of the drawdown curve at intermediate times. The possible impact of dual porosity effects was investigated using the model of Boulton and Streltsova (1977). Model curves are controlled by the dimensionless parameter r_D where :

$$r_D = \frac{r}{H}\sqrt{\frac{k_m}{k_f}} \quad , \qquad \text{equation 5}$$

where H is the half matrix block width, k_m and k_f are the matrix and fracture permeability, respectively. An r_D of zero reflects no contribution, and large r_D reflects strong support by matrix blocks. The observed drawdown curves show poor fits to dual porosity model curves, as illustrated in Figure 6 for borehole M1, as it is not possible to generate a fit to both the early and late time segments. Also, the observation that the borehole at 75m (M4) shows a tolerably good fit to a Theis curve indicates that dual porosty effects are not significant.

In summary, application of the classical Theis (1935) method for 2D radial flow show a reasonable fit to the observed drawdown data for the more distant borehole (75 m from the pumped borehole) and to the boreholes at 25 m for drawdown data later than 6 minutes. For all boreholes, the Hantush method (partially penetrating boreholes) gives improved fits to drawdown if high hydraulic conductivity anisotropies of 2 to 3 orders of magnitude are invoked, although the validity of this is questionable. Methods that include borehole response time and dual porosity effects do not result in improved fits to the observed drawdown data. Overall, the classical methods suggest transmissivities of around 500 m^2/d and storativities around 10^{-4} (Theis) or $2x10^{-3}$ (Hantush), see Table 3.

6 Pumping Test Analysis – the GRF Model

The drawdown curves up to 19 hours for boreholes M1, M2, M3 and M4 were fitted to the fractional flow GRF model of Barker (1988) using a Fortran program which employs the method described in Le Borgne et al. (2004) and calculates best fit curves using the Levenberg-Marquardt method (Press et al. 1992). The program gives the best fit estimates for apparent transmissivity, T_a, apparent storativity S_a, flow dimension, n, and RMSE (root mean square error) for the fit.

For the boreholes 25 m distant from the pumped borehole (M1, M2 and M3) it was found that, as for the classical methods, the entire drawdown curves cannot be explained in terms of a single flow dimension. However, good fits were obtained for drawdown data from 5 minutes to 19 hours (over two orders of magnitude in time) as shown in Figure 7 and Table 4. The boreholes at 25 m (M1-M3) show a narrow range of flow dimensions from 2.33 to 2.38 and for the borehole at 75 m (M4) the whole data set is well explained by a single flow dimension of 2.2. The fits obtained by the GRF model (RMSE 0.0028-0.0038) are significantly better than those obtained from the classical methods when applied to the whole data set (RMSE 0.008-0.02 m). The fit to the Hantush model is also improved when when the first 5 mins of data are omitted but the RMSEs (0.0055-0.0074 m) are still around double those of the GRF model (0.0028-0.0038 m), see Table 3. The GRF model also has the advantage that there is no need to invoke high degrees of hydraulic conductivity anisotropy which are required by the Hantush model but which are inconsistent with core observations. Thus the GRF model provides the preferred fit to the drawdown curves.

Apparent transmissivities, T_a, from the GRF model lie in the range 45-55 m^{4-n}/d for boreholes at 25 m and 114 m^{4-n}/d for the borehole at 75 m and all apparent storativities, S_a, lie in the range 0.4-1.0 x 10^{-4} m^{2-n}. However, these values cannot be directly compared with values of T and S estimated using classical methods due to a mismatch in units. A method for converting T_a and S_a to T and S is developed below.

6.1 Converting Apparent Transmissivity and Storativity

The flow-through area, A_n, (units L^2) for a flow dimension n, as defined by Barker (1988) is given by the product of the surface area $S(n)$ (units of L^{n-1}) and b^{3-n} where b is the extent of the flow-through area (e.g. van Tonder 2002):

$$A_n(r) = \frac{2\pi^{n/2} r^{n-1} b^{3-n}}{\Gamma(n/2)} \quad . \qquad \text{equation 6}$$

For integer flow dimensions 1, 2 and 3, equation 6 reduces to surface areas of a 1D channel ($2b^2$, $n=1$), a cylinder ($2\pi rb$, $n=2$) and a sphere ($4\pi r^2$, $n=3$). In 1D flow, b therefore represents square root of half the flow-through area, in 2D flow ($n=2$) b represents the vertical extent of the flowing region (thickness of the aquifer) and in 3D flow ($n=3$), the parameter b reduces to 1 (Barker 1988). Thus for flow dimensions between 1 and 2, the b must lie between the square root of the flow-through region and the thickness of the aquifer. Leveinen et al. (1998) and Van Tonder et al. (2002) use values of b for flow dimensions of 1 and 2 to obtain bounding values of T and S. However, for non-integer flow dimensions between 2 and 3, only one bound to the parameter b (that for 2D flow) exists.

The drawdown curves from the monitoring boreholes at 25 m distance from the pumped borehole (M1, M2, M3) suggest sub-spherical flow, most likely caused by the partially penetrating nature of the boreholes casings. From the model of Hantush

(1961) for partially penetrating wells, Reed (1980) states that flow occupies less that the full thickness of the aquifer when $r < 1.5B\sqrt{K_r / K_z}$ where r is the distance between pumped and observation boreholes and B is the thickness of the aquifer. In the case of an isotropic aquifer (suggested by the nature of fracturing observed in core), this suggests that flow is likely to occur throughout the full thickness of the aquifer at distances of more than around 14 m from the pumped borehole. Thus at the distance of the monitoring boreholes (25 m), flow is likely to occupy the entire thickness of the aquifer. The GRF model assumes constant hydraulic conductivity, K_f, and specific storage, S_s, and a flow-through area that evolves according to r^{n-1}. This is however, not compatible with the case where flow is restricted to the height of the aquifer. One way to reconcile these observations, is to consider a model where the flow-through area is confined to the thickness of the aquifer (as in 2D radial flow) while transmissivity increases with distance from the pumped borehole, as described in Doe (1991).

Consider a cylinder, radius r, with the same surface area as the flow-through area, A_n, (equation 6) where $2<n<3.$ The transmissivity of the aquifer at distance r from the pumped borehole is $T= K_f\ F,$ where F is the height of the cylinder, so:

$$T = K_f F = \frac{A_n}{2\pi r} K_f \quad , \qquad \text{equation 7}$$

Now assume that flow is restricted to cylinder height B (thickness of the aquifer) so that to preserve transmissivity, hydraulic conductivity becomes dependent on distance, r, from the pumped borehole. Substituting for F (equation 7) and using $K_f\ b^{3-n} = T_a$ then gives:

$$T = T_a \pi^{(n/2-1)} r^{n-2} / \Gamma(n/2) \quad . \qquad \text{equation 8}$$

Storativity, S, may be derived from apparent storativity, S_a , by similar arguments to give:

$$S = S_a \pi^{(n/2-1)} r^{n-2} / \Gamma(n/2) \quad . \qquad \text{equation 9}$$

Thus equations 8 and 9 can be used to estimate T and S from T_a and S_a given the flow dimension n, for flow dimensions, $2< n<3$. The ratio T/T_a (from equation 8), increases with increasing flow dimension, n, and distance, r. At r=25 m and n=2.4 (boreholes M1, M2 and M3), the ratio of T/T_a is around 5 while for r=75 m and n=2.2 (borehole M4), it is 2.8.

Equations 8 and 9 were used to determine transmissivity (T) and storativity (S) from the apparent transmissivity (T_a) and storativity (S_a), and flow dimensions, n, assuming an aquifer thickness (B) of 9.5 m. The resultant transmissivities lie in the range 196 to 337 m^2/d and storativities 2 to 5 x 10^{-4} (Table 4). In comparison with classical methods results, the GRF model transmissivities are smaller by a factor of 2 to 3 and storativities around one order of magnitude smaller than those from the Hantush method. As a check on the values of T and S given by equations 8 and 9, upper and lower bound may be calculated by using the aquifer thickness of 9.5 m for b as in 2D flow giving a lower bound and by assuming $b^{3-n} = 1$, as in 3D flow, giving an upper bound. The values of T and S from equations 8 and 9 are seen to lie between these bounding values, see Table 5.

7 Analysis of the Multi-borehole Tracer Test

The nature of flow has potentially important implications for the analysis of radially convergent tracer tests. A multi-borehole tracer test was previously conducted at the site in 2001 in which borehole P was pumped and tracers injected into the observation boreholes M1, M2 and M3. The aquifer transmisisivty, estimated using classical techniques, was used as an input parameter in modelling of breakthrough curves to derive fracture characteristics. In the following section, the modelling method is adapted for fractional flow and the tracer test re-interpreted to investigate the implications of fractional flow for the derivation of fracture characteristics.

There have been few studies that have incorporated fractional flow in the interpretation of tracer tests in fractured rocks. Van Tonder et al. (2002) developed generalized equations incorporating fractional flow dimensions into expressions for Darcy and seepage velocities from dilution and tracer tests which were applied to tracer and dilution tests in fractured sandstones by Riemann et al. (2002). Flow dimensions for a fracture zone of 1.75 to1.85 were determined and the estimated Darcy and seepage velocities and kinematic porosity showed sensitivity to flow dimension. Kurtzman et al. (2005) estimated dilution factors in forced gradient tracer tests in fractured chalks based on fractional flow. They obtained flow dimensions of 1.8 to 2.0 and inferred that flow was dominated by a network of channels comprising 1 to 3% of the fracture planes.

7.1 The tracer test and previous interpretation

A multi-borehole radial tracer test was conducted at the Wilfholme site in 2001 (Hartmann et al. 2007). Tracer dyes were injected into the three monitoring boreholes at 25 m from borehole P (Eosine in M1, Amino G Acid in M2 and Rhodamine WT in M3) while borehole P was pumped at a rate of 330 m^3/day for 8 days. The tracers were injected as single slugs (Dirac injection) after 7 hours of pumping to achieve a near steady state flow field and injection boreholes M1, M2 and M3 were re-circulated throughout the tracer test. The concentration of tracers in injection boreholes showed a sharp decline with 10% or less remaining after 24 hours (Figure 8). The breakthrough curves showed similar shapes with initial breakthrough from 6 to 11 hours, a sharp initial rise with peak concentrations at 31 to 49 hours, and a long tail (Figure 8). However, recoveries varied greatly from Eosine (M1) at 9%, to Rhodamine WT (M3) at 17%, and Amino G (M2) at 57%. Aquifer transmissivity and storativity were estimated from drawdowns, manually dipped during the tracer test, using the Theis (1935) and Hantush (1964) leaky aquifer methods.

The tracer break-through curves were modelling using the 1D analytical dual porosity model of Barker and Foster (1981) for fracture flow with matrix diffusion modified for radial flow. Input parameters supplied by laboratory tests on core samples of chalk matrix from the site (Hartmann et al. 2007) are the matrix porosity (30%), matrix hydraulic conductivity (0.07 to 0.68 x 10^{-4} m/d), and effective diffusion coefficient (1.4 to 5 x 10^{-11} m^2/s). The injection borehole tracer concentrations were used as source functions and the estimated transmissivities were used as an additional fitting condition. Best fits models to the observed breakthrough curves, obtained through trial and error, showed reasonably good fits and resulted in estimates of fracture aperture of 363 to 384 μm, fracture spacing of 6 to 9 cm and fracture dispersivities of 1 to 5 m.

The tracer test breakthrough curves from the 2001 tracer test are here re-analysed incorporating transmissivities derived from the 2008 pumping test, assuming i) 2D flow and ii) fractional flow.

7.2 Modelling the Tracer Breakthrough Curves

The tracer breakthrough curves are modelled using an analytical, dual porosity model described in Barker and Foster (1981) for 1D flow in parallel, equally spaced fractures, modified for 2D and fractional radial flow. The model of Barker and Foster (1981) considers advection of water along fractures with velocity, v, and longitudinal dispersion, α, coupled with exchange by diffusion between the mobile fracture water and immobile matrix pore water. The matrix of porosity, ϕ_m, is divided into regular blocks by fractures of aperture, a, and constant spacing, d, see Figure 9. Water flows along the fractures with a constant velocity, v_f. Solute concentration in the fractures and matrix pores is initially zero and at time, $t=0$, solute of concentration c_0 is introduced into the fractures at $x=0$ (Figure 9). The evolution of the solute concentration in the fractures and matrix is described by equations 10-14 (Barker and Foster 1981, Hartmann et al. 2007):

$$\frac{\partial c_f}{\partial t} + v_f \frac{\partial c_f}{\partial x} = \alpha \left| v_f \right| \frac{\partial^2 c_f}{\partial x^2} - \frac{2D_e}{a} \frac{\partial c_m}{\partial z}\bigg|_{z=b} = 0 \quad , \qquad \text{equation 10}$$

$$\phi_m \frac{\partial c_m}{\partial t} = D_e \frac{\partial^2 c_m}{\partial z^2} \quad ; 0 < z < d/2 \quad , \qquad \text{equation 11}$$

with boundary conditions:

$$\frac{\partial c_m}{\partial z}\bigg|_{z=d/2} = 0 \quad ,$$

$$\lim_{x \to \infty} c_m(x, z, t) = 0 \quad ,$$

$$c_f(x = 0, t) = c_0(t) \quad , \qquad \text{equations 12}$$

and initial conditions:

$$c_f(x, t=0) = 0 \quad ,$$

$$c_m(x, z, t=0) = 0 \quad , \qquad \text{equations 13}$$

where c_f and c_m are the concentrations of solute in the fracture and matrix respectively, $c_0(t)$ is the source function (solute concentration introduced at $x=0$, the in-flow end of the fracture, Figure 9) and D_e is the effective diffusion coefficient for the matrix.

The model is evaluated through five parameters; advection time, t_a, characteristic time for diffusion from fracture to matrix, t_{cf}, matrix to fracture porosity ratio, σ, relative dispersivity, α_{rel}, and a scaling parameter, C :

$$t_{cf} = \frac{a^2}{4.D_e\phi_m} \; , \quad t_{cf} = \frac{a^2}{4.D_e\phi_m} \quad \text{and} \quad \alpha_{rel} = \frac{\alpha}{R} \qquad \text{equations 14}$$

where R is the distance between injection and pumped boreholes.

7.3 Travel Times to a Borehole in Fractional Radial Flow

In the case of radial flow in an aquifer of constant thickness confined by impermeable layers above and below, the flow-through area at distance r from the pumped borehole is constrained to be the surface area of a cylinder of radius r and height B (the aquifer thickness). Consider a thin shell of thickness Δr which is bounded by two cylinders of radius r and $r+\Delta r$. Assume that flow within this shell is conducted by vertical fractures oriented radially to the pumped borehole with spacing d, see Figure 10a. This is a reasonable representation of fracture flow in rocks with fracture systems composed of two or more orthogonal sets (as is the case at this site) where the fractures that are most closely oriented radially to the pumped borehole will carry most flow.

In the case of 2D flow, the number of flowing fractures, N_f, with spacing, d, at distance, r, from the pumped borehole is:

$$N_f = \frac{2\pi r}{d} \quad . \qquad \text{equation 15}$$

Flow in a single fracture, Q_f, is given by the pumping rate, Q_p, divided by the number of fractures, N_f . The average flow velocity in each fracture is then fracture flow, Q_f, divided by fracture cross-sectional area ($a.B$) and thus the average travel time, Δt_a, over distance Δr, is :

$$\Delta t_a = \frac{2\pi r\,(a.B)\Delta r}{Q_p\, d} \quad . \qquad \text{equation 16}$$

Following the previous development, it is assumed that flow is confined to the thickness of the aquifer, B, and that hydraulic conductivity depends on r. T can be related to fracture aperture, a, spacing, d, and aquifer thickness B by the Cubic Law:

$$T = \frac{a^3 B}{12d}\frac{\rho g}{\mu} \quad . \qquad \text{equation 17}$$

where ρ is the density of water, g is the gravitational constant and μ is the dynamic viscosity of water. If fracture aperture is assumed to be constant, fracture spacing, d, depends on r , see Figure 10b. An expression for spacing, $d(r)$, can then be derived from equations 17 and equation 8, for the case of fractional flow:

$$d(r) = \frac{a^3 \Gamma(n/2) B}{12 T_a \pi^{(n/2-1)} r^{n-1}}\frac{\rho g}{\mu} \quad , \qquad \text{equation 18}$$

and thus travel time for distance, Δr (substituting for d in equation 16) is:

$$\Delta t_a = \frac{24\pi^{n/2} T_a\, r^{n-1}}{Q_p a^2 \Gamma(n/2)} \frac{\mu}{\rho g} \Delta r \qquad . \qquad \text{equation 19}$$

Travel time from distance r to the pumped borehole at r_w (borehole radius) is given by integrating the above equation between the limits of r and r_w:

$$t_a = \frac{24\pi^{n/2} T_a}{Q_p a^2 \Gamma(n/2) n} \frac{\mu}{\rho g} \left(r^n - r_w^n\right) \qquad . \qquad \text{equation 20}$$

When $n=2$, T is equivalent to T_a, and equation 20 reduces to $t_a = \left(\pi\, a\, B / Q_p d\right)\left(r^2 - r_w^2\right)$ which is the equation used by Hartmann et al. (2007) for travel time in 2D radial flow.

7.4 Application to Tracer Breakthrough Curves

The dual porosity transport model is evaluated using an Excel spreadsheet originally written by John Barker and modified for radial and fractional flow by the authors. Input parameters that are held constant include aquifer thickness (B), pumping rate (Q_p), the inter-borehole distance (R), borehole radius (r_w) and matrix porosity (ϕ_m), see Table 6. Two values of effective matrix diffusion coefficient (D_e) were used. The value of 5 x 10^{-7} m^2/s measured from core samples of un-weathered, un-fractured chalk from approximately 70 m depth at the site was used as a lower bound. It is likely that the porosity and diffusion coefficient of the weathered chalk matrix from the upper conductive zone is higher and thus a value of 2x10^{-6} m^2/s, representative of less compacted chalk (Hill 1984, Witthüser et al. 2000, Polak et al. 2002, Gooddy et al. 2007), was used as an upper bound. The observed variation in tracer concentration with time in the injection boreholes were used as source functions for M1 (Eosine) and M3 (Rhodamine WT) boreholes (see Figure 8). Due to sampling problems in M4 (Hartmann et al. 2007), a simple box-shaped source function was constructed consisting of initial tracer concentration for a duration equal to the time needed to wash all tracer from the borehole. The flow-through rate of the borehole, Q_{inj}, was estimated from the equation $Q_{inj} = [d\ln(c')/dt]V_{bh}$ (Freeze and Cherry, 1979, equation 9.27) where c' is the observed injection borehole concentration normalized by initial concentration, c_0, and V_{bh} is the volume of the flowing section of the borehole, as in Hartmann et al. (2007). The term $[d\ln(c')/dt]$ was estimated using the first few concentration data points from M2, when concentration fell rapidly, and the volume of the recirculated borehole as V_{bh}, giving Q_{inj} as 0.49 m^3/hr. The time required to wash the injected mass from the borehole is then given by $t_{inj} = M/(Q_{inj}c_0)$, where M is the mass of tracer injected, which gives a source function duration of 1.86 hours.

Travel time (t_a), fracture spacing (d) and fracture dispersivity (α) are constrained to lie within acceptable upper and lower bounds (Hartmann et al. 2007), see Table 6. Travel time was restricted to smaller than the timing of the breakthrough curve peak and fracture dispersivity was allowed to vary between zero and the distance between monitoring and pumped boreholes (25 m). The scaling factor, C, representing the dilution of the tracer due to radial flow, was allowed the maximum range of [0-1]. With the input listed in Table 6, a range of models with similarly good fits to the observed tracer breakthrough curves, comprising a range of fracture aperture and spacing combinations, are possible. In order to further restrict the solution, fracture aperture and spacing were constrained to be consistent with the observed transmissivity through equation 17. Atkinson et al. (2000) has previously used transmissivity, through the Cubic Law, as a check on the validity of different transport models of a tracer test in chalk.

The model was run for each of the three breakthrough curves assuming i) 2D radial flow ($n=2$) using transmissivities determined by the Hantush (1964) method (Table 2) and ii) fractional flow using flow dimensions and transmissivities from the GRF model (Table 3). The spreadsheet determines the best fit values of the parameters t_a, t_{cf}, σ, α_{rel} and C to the observed breakthrough curves using the Solver function in Excel and the RMSE between best fit model and observed data is calculated. From these best fit parameters, fracture aperture, spacing and dispersivity for each model are derived. The results are shown in Figure 11 and listed in Table 7.

Generally good fits between model and observed breakthrough curves were obtained (Figure 11) with RMSEs ranging from 0.0006 to 0.003 m (Table 7). The models for 2D and fractional flow show equally good fits apart from the breakthrough curve for borehole M1 (tracer Amino G Acid) where a significantly better fit was found using a fractional flow dimension of 2.36 with a lower transmissivity of 196 m^2/d. Comparison of the models results assuming 2D and fractional flow shows that the fractional flow models result in smaller fracture apertures (26-34% reduction), smaller fracture spacings (7-16% reduction) and smaller dispersivities (20-35% reduction). Increasing the effective diffusion coefficient by a factor of 4 results in an increase in apertures by a factor of around 1.25 and spacings by up to a factor of 2, while scaling factors and RMSE values are not significantly changed. Resulting apertures from the fractional flow models range from 220 to 300 μm and spacings from 3 to 8 cm, while dispersivities are in the range 1 to 2.2 m. The fracture spacings are consistent with observations from core at borehole P which indicates fracture block sizes of 2 to 15 cm. Travel times, representing time to travel distance from injection to pumped boreholes without matrix diffusion or dispersivity, range from 4.4 to 8.3 hours.

8 Discussion

8.1 A Conceptual Model for Flow Evolution with Time and Distance from the Pumped Borehole

Analysis of the drawdown curves using the GRF model has shown that flow dimensions range from 2.3-2.4 at 25 m to 2.2 at 75 m from the pumped borehole, for times greater than 5 minutes. For times less than 5 minutes, the drawdown curves for pumped borehole and boreholes at 25 m show steep initial sections that are concave upwards (see Figure 7), suggesting flow dimensions between 1 and 2 (sub-radial flow, see Figure 1a). This is followed by a transition period as the flow dimension evolves to a stable value greater than 2 (sub-spherical flow) at times later than 5 minutes. By contrast, the drawdown curve for the borehole at 75 m (M4) shows a dimension of 2.2 for all times.

These changes in flow dimension with time and distance from the pumped borehole suggests a conceptual model for the evolution of flow during the pumping test. At a distance of 25 m, flow is restricted during the first few minutes to a small number of channel ways within fractures giving a flow dimension of less than 2. Flow during this time is likely to be concentrated within a thin zone at the base of the highly conductive layer where the pumped borehole is open to the aquifer (see Figure 2c). That this behaviour is not observed at 75 m suggests flow is more evenly distributed throughout the aquifer thickness at this distance. At around 5 minutes, the flow field stabilizes with respect to time but the flow dimension remains dependent on distance from the pumped borehole from 2.3-2.4 at 25 m to 2.2 at 75m. This suggests that with increasing distance, flow accesses an increasing proportion of the fracture network spreading up from the initial pathways close to the base of the aquifer to encompass the full thickness of the highly conductive layer, see Figure 12. The change in flow dimension with distance suggests that at distances somewhat greater 75 m, a dimension of 2 would be reached where all available fracture flow pathways in the aquifer are exploited, i.e. fully 2D radial flow is reached. Thus the flow dimension evolves at early times from near linear ($1<n<2$) to sub-spherical ($2<n<3$) close to the pumped borehole. At later times, when the flow regime has stabilized, flow dimension evolves from subspherical ($2<n<3$) to radial ($n=2$) with increasing distance from the pumped borehole, illustrated for times greater than 5 minutes in Figure 12. These non-integer flow dimensions are consistent with effects generated by the partially penetrating geometry of the borehole casing and therefore do not reflect intrinsic properties of the aquifer.

The analyses suggest that the effects of partial penetration extend further from the pumped borehole than would be expected in an isotropic porous media aquifer. In the present case, these effects are seen at 75 m compared to 14 m predicted by the Hantush method (see section 6.1) as the maximum extent of the effects of partial penetration in an homogeneous, isotropic porous media aquifer. In the Hantush method, this is explained in terms of anisotropy with values of K_z/K_r of 2 to 3 orders of magnitude, which is not compatible with observations of an essentially isotropic fracture system from core and acoustic logs. This suggests that the effects of partial penetration are not just a matter of restricting flow to a part of the aquifer thickness but also of restricting flow to the subsets of fracture network. Flow may occupy the full thickness of the aquifer at around 14 m from the pumped borehole (as predicted by the Hantush method for an isotropic aquifer) but may be dominated by the larger apertures in fractures that are well connected with the borehole. In this conceptual model it is only at much greater distances from the pumped borehole that flow is likely to occur throughout the entire connected fracture system, as illustrated in Figure 12. This conceptual model suggests that the effects of partial penetration extend much further from the pumped borehole in fractured aquifers than in homogeneous, isotropic porous media. Thus to avoid such effects, monitoring boreholes at larger distances from the pumped borehole should be used (in the present study > 75m). The highly conductive layer of the aquifer in this study is highly fractured with a very small fracture spacing (2-15 cm) compared to many fractured aquifers. It is possible that where fracture spacing is larger, fractional flow/partial penetration effects could extend to much greater distances from the pumped borehole than observed here (around 75 m).

8.2 Re-interpretation of the tracer test

Interpretation of drawdown curves using the GRF fractional flow model results in smaller transmissivities which are reflected in the smaller fracture apertures and spacings from the fractional flow transport models. Model fracture spacings of 3–8 cm are consistent with observations from the core at borehole P which shows fracture block sizes of 2 to 15 cm. For any given model and diffusion coefficient, results from the three injection boreholes show a relatively narrow range of fracture apertures (10-20%) and spacings (25-30%), reflecting the narrow range (10-20%) in transmissivities. There is, however, a much wider variation (by a factor of 6) in the observed tracer recovery (Amino G Acid 57%, Rhodamine WT 17% and Eosine 9%). The question then arises of why such a wide range in recoveries should occur when the dual porosity model predicts narrow ranges of parameters for all boreholes.

Recovery in the dual porosity model is controlled dominantly by the scaling factor, C, which was allowed to vary widely in the interval [0-1] to find the best fit model curve and reproduce the observed recoveries. The dual porosity model uses the shape of the observed breakthrough curve to determine model parameters (fracture aperture, spacing and dispersivity) while the scaling parameter is used to adjust the magnitudes of model break through curve concentrations to reproduce observed recoveries. The dual porosity model therefore does not give a physical explanation for observed recoveries. Scaling factors may also be estimated from the change in injection well concentrations with time (source functions) and injection borehole flow-through rates, Q_{inj}. Flow-through rates for boreholes M1 and M3 were calculated (using the method described in section 7.4) as the rate required to wash the injected tracer mass, M, from the borehole in the time taken for injection well concentrations to fall to less than 1% of initial values. From these flow-through rates and that already estimated for M2 (section7.4), predicted scaling factors (C') were estimated from the ratio Q_{inj}/Q_p (Table 8).

Comparing the predicted scaling factors, C', with those from best fit model curves, C, shows that predicted scaling factors are significantly larger for M1 (Eosine) and M3 (Rhodamine WT) by a factor of 2 and 4 respectively, while C' for M2 (Amino G Acid) lies closer to C. By replacing the best fit scaling factors, C, in the model by predicted scaling factors, C', recoveries predicted by Q_{inj} are obtained (Table 8). Predicted recoveries show a much narrower range (32 - 44%) than the observed recoveries (9-57%) which is consistent with the relatively narrow ranges of the other model output parameters (Table 7). The predicted recoveries, C', are larger than observed, C, for M1 (by 27%) and M3 (14%) and smaller than observed for M2 (13%). Since the dual porosity model does not provide a physical explanation for the observed recoveries, other explanations for these discrepancies must be sought.

Dilution tests of the boreholes show significant variations in flow with depth (Hartmann 2004, West and Odling 2007) with M1 showing one, and M3 two, distinct flow horizons in addition to that at the base casing. By contrast, flow logging at M2 (Parker 2010), shows that flow is very strongly dominated by the zone just below the base casing. In boreholes M1 and M3, it is therefore likely that significant amounts of tracer were drawn into aquifer at subdominant flowing horizons for which breakthrough may not be observed. The slight increase in tracer concentration at 110-150 hours observed for M1 (Eosine) in Figure 11 could be interpreted as a secondary breakthrough resulting from tracer in the lower flow horizon.

Another cause of reduced tracer recovery may be the nature of connection between the borehole and the fracture network. While transmissivities determined from the GRF model reflect averages over the thickness of the aquifer and distances from pumped to observation boreholes, tracers are introduced over much smaller volumes constrained by the borehole radius and the narrow intervals where the boreholes are open to the highly conductive aquifer layer (0.11 to 3.3 m). The injection borehole flow-through rates, Q_{inj} , are likely indicators of the 'goodness' of borehole connection with high flow pathways of the fracture network. The highest observed recovery (57%) and flow-through rate (0.49 m^3/hr) at borehole M2 (Amino G Acid) suggests that it is well connected to high flow pathways in the fracture network. This is supported by the early drawdown curve slope for this borehole (Figure 7) which is thought to reflect a flow dimension close to 1, consistent with the presence of a high flow pathway. By contrast, the lower flow-through rates (0.144 and 0.055 m^3/hr) of

M2 (Rhodamine WT) and M1 (Eosine) which correlate with lower recoveries (17 and 9% respectively) may indicate poorer connections with the fracture network.

This suggests that while the dual porosity model estimates fracture apertures, spacing and dispersivities averaged over the distance between injection and pumped boreholes, recoveries are sensitive to other influences not explicitly included in the model and may vary widely. High recoveries and borehole flow-through rates may be indicative of flow dominated by a single horizon and good connection between borehole and fracture network. Low recoveries and borehole flow-through rates may indicate the presence of multiple flowing horizons and poor connection with the higher conductive parts of the fracture network.

8.3 Implications for determination of aquifer properties

The fractional nature of flow in the vicinity of pumped, partially penetrating boreholes becomes important in the interpretation of pumping and tracer tests where nearby observation boreholes are used. Application of classical methods when flow is fractional will tend to overestimate T for flow dimensions greater than 2 and underestimate T for flow dimensions less than 2. For flow dimensions of 2.3-2.4, the classical methods overestimate T by factor of up to 3.

The flow and tracer test results analysed here illustrate the impact of borehole construction on flow dimension. The simple borehole construction at Wilfholme of a solid casing extending partially into the highly conductive layer is typical throughout this region. The analysis here shows that pumping from such partially penetrating boreholes in the Chalk Aquifer can have an impact on the flow dimension for at least 75 m distant from the pumped borehole. Similar drawdown curve shapes to those observed at Wilfholme are reported from pumping tests in the confined Chalk aquifer in Yorkshire and Lincolnshire (MacDonald 1977). Reported transmissivities derived from these pumping tests using classical methods (Theis) for the confined Chalk Aquifer range from around 30 to over 3000 m^2/day (MacDonald 1977, MacDonald and Allen 2001). If the boreholes in these pumping tests partially penetrate the highly conductive part of the aquifer, as they do at Wilfholme, the upper end of this range may significantly overestimate transmissivity. Similarly, where forced gradient tracer tests in dual porosity media such as chalk are carried out over relatively short distances (e.g. to ensure tracer breakthrough within a reasonable time period), ignoring the potential impact of partially penetrating borehole construction on flow dimension in such fracture aquifers may result in overestimation of both fracture aperture and spacing.

9 Conclusions

The GRF model (Barker 1988) which incorporates non-integer flow dimensions, offers greater versatility than classical methods but has not been widely used as it gives apparent transmissivity and storativity with dimensions that are dependent on the flow dimension. A method is developed to convert apparent transmissivity and storativity (L^{4-n}/T and S^{2-n}) to conventional transmissivity and storativity (L^2/T and dimensionless) for the case where flow dimension, $2<n<3$. These parameters may then be interpreted in the usual way and used in further applications, thus facilitating application of the GRF model.

Classical methods tend to overestimate T and S where flow dimensions are greater than 2, and underestimate T and S where flow dimensions less than 2. In the case illustrated here, flow dimensions are 2.3-2.4 and transmissivities and storativities from the GRF model are lower by 30% and an order of magnitude respectively, than estimates from classical methods. These non-integer flow dimensions are interpreted to be the result of the partially penetrating nature of the pumped borehole.

Partially penetrating boreholes in fractured rocks create flow regimes in which flow dimension varies with time and distance from the pumped borehole. At early times flow dimensions are less than 2 (sub-radial). At later times, flow dimensions are greater than 2 (sub-spherical) around the pumped borehole, reducing to 2 (radial) with distance as the influence of borehole construction diminishes. For the case illustrated here, the transition from sub-radial to sub-spherical flow occurs at around 5 minutes and 2D radial flow is reached at distances larger than 75 m.

A previous multi-well tracer test at the site (Hartmann et al. 2007) is reinterpreted in the light of fractional flow using an analytical model of solute transport in fractured media that incorporates matrix diffusion (Barker and Foster 1981), modified for fractional radial flow. Application of the model shows that incorporating fractional flow reduces predicted fracture apertures and dispersivities by around 30%, and fracture spacing by around 12%. The dual porosity, solute transport model provides fracture apertures, spacing and dispersivity averaged over the volume from injection to pumped borehole. Tracer recovery is, by contrast, highly variable and strongly dependent on conditions in the immediate vicinity of the injection borehole such as presence of subordinate flowing horizons and local heterogeneities in the fracture network. From the tracer test presented here, recoveries correlate with flow-through rates in injection boreholes (Q_{inj}), with high recovery and flow-through rate reflecting good connection between the injection borehole and high conductive pathways in the fracture network.

The pumping and tracer test results and modelling presented here illustrate the importance of recognising the potential fractional nature of flow generated by borehole construction in estimating aquifer properties and interpreting tracer breakthrough curves. In the case of partially penetrating boreholes, failure to recognize the fractional flow regimes can lead to overestimation of transmissivity, storativity, and fracture apertures, spacing and dispersivity.

Acknowledgements

We thank John Barker for discussion of GRF model development, for providing an initial version of the analytical dual porosity transport model and for commenting on an earlier version of the manuscript. The research was supported by EU FW4 collaborative project SALTRANS (EVK1-CT-2000-00062). The UK Environment Agency, particularly Rolf Farrell, is thanked for supplying the field site and for their assistance in drilling boreholes. Rolf Farrell is also thanked for comments on the research during the project.

References

Acuna, J.A., Yortsos, Y.C.,1995. Application of fractal geometry to the study of networks of fractures and their pressure transient. Water Resources Research. 31, 527-540. DOI: 10.1029/94WR02260

Bangoy, L.M., Bidaux, P., Drogue, C., Plégat, R., Pistre S., 1992. A new method of characterizing fissured media by pumping tests with observation wells. Journal of Hydrology, 138, 77-88. DOI: 10.1016/0022-1694(92)90156-P

Bangoy, L.M., Richard D., 1999. Determination of fractured transmissivity trend and its contribution to a fractured media study. Bulletin de la Societe Geologique de France 170, 391-399.

Barker, J.A., 1988. A generalized radial flow model for hydraulic tests in fractured rock. Water Resources Research 24, 1796-1804.

Barker, J.A., Foster, S.S.D., 1981. A diffusion exchange model for solute movement in fissured porous rock. Q.J.eng.Geol.London 14, 17-24.

Bernard, S., Delay, F., Porel, G., 2006. A new method of data inversion for the identification of fractal characteristics and homogenization scale from hydraulic pumping tests in fractured aquifers. Journal of Hydrology 328, 647-658. DOI: 10.1016/j.jhydrol.2006.01.008

Black, J.H., Kipp, K.L., 1977. Observation well response time and its effect upon aquifer test results. Journal of Hydrology 34, 297-306.

Boulton, N.S., Streltsova, T.D., 1977. Unsteady flow to a pumped well in a fissured water-bearing formation. Journal of Hydrology 35, 257-269.

Butler J.J. 1990. The role of pumping tests in site characterization: some theoretical considerations. Groundwater, 28(3), 394-402.

Cello, P., A., Walker, D.D., Valocchi A.J., Loftis, B., 2009. Flow Dimension and Anomalous Diffusion of Aquifer Tests in Fracture Networks. Vadose Zone Journal 8, 258-268. DOI: 10.2136/vzj2008.0040

Chang Y-C., Yeh H-D., Kuo M-C.T. 2011. Scale dependency of fractional flow dimension in a fractured formation. Hydrology and Earth Systems Science, 15, 2165-2178, DOI: 10.5194/hess-15-2165-2011 .

Clark, E., 1967. Computing the barometric efficiency of a well. Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers. 93(HY4), 93-98.

de Dreuzy, J-R., Davy, P. 2007. Relation between fractional flow models and fractal or long-range 2-D permeability fields. Water Resources Research 43, Article Number: W04431. DOI: 10.1029/2006WR005236

Delay, F., Porel, G., Bernard, S., 2004. Analytical 2D model to invert hydraulic pumping tests in fractured rocks with fractal behavior. Geophysical Research Letters 31, Article Number: L16501. DOI: 10.1029/2004GL020500

Doe T.W., 1991. Fractional dimension analysis if constant-pressure well tests. 66th Annual Technical Conference and Exhibition SPE, 461-467. SPE 22702

Doughty, C., Karasaki, K., 2002.Flow and transport in hierarchically fractured rock. Journal of Hydrology, 263, 1-22. DOI: 10.1016/S0022-1694(02)00032-X

Freeze, R.A., Cherry, J.A., 1979. Groundwater, Prentice-Hall, Englewood Cliffs, N.J., USA, 604pp.

Gooddy D.C., Kinniburgh D.G., Barker J.A. 2007. A rapid method for determining apparent diffusion coefficients in Chalk and other consolidated porous media. J. Hydrology, 343, 97-103.

Hamm, S.Y., Bidaux, P., 1996. Dual-porosity fractal models for transient flow analysis in fissured rocks. Water Resources Research, 32, 2733-2745. DOI: 10.1029/96WR01464

Hantush, M.S., 1961. Drawdown around a partially penetrating well. J. Hydraul.Div., Proc. Amer. Soc. Civil. Engrs. 87(HY4), 83-98.

Hantush, M.S., 1964. Hydaulics of wells, in: Chow, V.T., (Ed.), Advances in hydroscience volume 1, Academic Press New York, pp. 281-432.

Hartmann, S., 2004. Flow and transport in the confined Chalk aquifer of East Yorkshire. PhD thesis, University of Leeds, 240pp.

Hartmann, S., Odling, N.E., West, L.J., 2007. A multi-directional tracer test in the fractured Chalk aquifer of E. Yorkshire, UK., *J. Contam. Hydrol.,* 94**,** 315-331. doi:10.1016/j.jconhyd.2007.07.009

Hill D. 1984. Diffusion coefficients of nitrate, chloride, sulphate and water in cracked and uncracked Chalk. Journal of Soil Sci., 35, 27-33.

Jourde, H., Pistre, S., Perrochet, P., Drogue, C., 2002. Origin of fractional flow dimension to a partially penetrating well in stratified fractured reservoirs. New results based on the study of synthetic fracture networks. Advances in Water Resources 25, 371-387. DOI: 10.1016/S0309-1708(02)00010-6

Kilpatrick A. 2008. A high resolution pumping test in the confined Chalk aquifer in East Yorkshire. MSc dissertation, University of Leeds, 100pp.

Kruseman, G.P., de Ridder, N.A. 1990. Analysis and evaluation of pumping test data. International Institute for Land Reclamation and Improvement, Wageningen, Netherlands, 377pp.

Kurtzman, D., Nativ, R., Adar, E.M., 2005. The conceptualization of a channel network through macroscopic analysis of pumping and tracer tests in fractured chalk. Journal of Hydrology 309, 241-257. DOI: 10.1016/j.jhydrol.2004.11.023

Kuusela-Lahtinen, A., Niemi, A., Luukkonen, A., 2003. Flow dimension as an indicator of hydraulic behavior in site characterization of fractured rock. Ground Water 41, 333-341. DOI: 10.1111/j.1745-6584.2003.tb02602.x

Le Borgne, T., Bour, O., de Dreuzy, J.R. Davy, P., Touchard, F., 2004. Equivalent mean flow models for fractured aquifers: Insights from a pumping tests scaling interpretation. Water Resources Research 40, W03512. DOI: 10.1029/2003WR002436

Leveinen, J., Ronka, E., Tikkanen, J., Karro, E., 1998. Fractional flow dimensions and hydraulic properties of a fracture-zone aquifer, Leppavirta, Finland. Hydrogeology Journal 6, 327-340. DOI: 10.1007/s100400050156

Lods, G., Gouze, P., 2004. WTFM, software for well test analysis in fractured media combining fractional flow with double porosity and leakance approaches. Computers & Geosciences 30, 937-947. DOI: 10.1016/j.cageo.2004.06.003

MacDonald A.M., 1977. Comparative analysis of pumping tests in the Chalk aquifer in Yorkshire and Lincolnshire. NERC Technical Report Hydrogeology Series WD/97/25, 20pp.

MacDonald A.M., Allen A.J., 2001. Aquifer properties of the Chalk of England. Q.J.eng.Geol.Hydrogeol., 34, 371-384.

Marechal, J.C., Dewandel, B., Subrahmanyam, K., Torri, R., 2003.Specific methods for the evaluation of hydraulic properties in fractured hard-rock aquifers. Current Science 85, 511-516.

Marechal, J.C., Dewandel, B., Subrahmanyam, K., 2004. Use of hydraulic tests at different scales to characterize fracture network properties in the weathered-fractured layer of a hard rock aquifer. Water Resources Research 40, W11508. DOI: 10.1029/2004WR003137

Matthews C.S., Russel D.G. 1967. Pressure buildup and flow tests in wells. SPE AIME, Monograph 1, 167pp.

Papadopulos, I.S., Cooper, H.H., 1967. Drawdown in a well of large diameter. Water Resources Research 3, 241-244

Parker, A.H., West, L.J., Odling, N.E., Bown, R.T., 2009. A forward modeling approach for interpreting impeller flow logs. *Ground Water*, 47, 79-91. doi:10.1111/j.1745-6584.2009.00600.x

Parker, A.H., 2010. The distribution of permeability in the Chalk Aquifer of East Yorkshire. University of Leeds PhD thesis, 295pp.

Polak A., Nativ R., Wallach R. 2002. Matrix diffusion in northern Negev fractured chalk and its correlation with porosity. Journal of Hydrology, 268, 203-213.

Press, W.H., Teukolsky, S.A., Vetterling, W.T., and Flannery, B.P., 1992. Numerical Recipes in FORTRAN. 2nd ed. Cambridge: University Press.

Pucknell J.K., Clifford P.J. 1991. Calculation of total skin factors. SPE 231100.

Rasmussen, TC., Crawford, L.A., 1997. Identifying and removing barometric pressure effects in confined and unconfined aquifers. Ground Water 35, 502-511.

Reed, J.E., 1980. Type Curves for selected problems of flow to wells in confined aquifers. Techniques of Water-Resources Investigations of the United States Geological Survey. Book 3 Applications of Hydraulics, 106 pp.

Riemann, K., van Tonder, G., Dzanga, P., 2002. Interpretation of single-well tracer tests using fractional-flow dimensions. Part 2: A case study. Hydrogeology Journal 10, 357-367. DOI: 10.1007/s10040-002-0197-5

Streltsova T.D., 1988. Well testing in heterogeneous Formations. John Wiley & Sons, 413pp.

Theis, C.V., 1935. The relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using groundwater storage. Trans. Amer. Geophys. Union, 16, 519-524.

Tongpenyai Y. 1981. The effect of wellbore storage and skin on interference test data. SPE AIME January 1981, 151-160.

Van Camp, M., Vauterin, P., 2005. Tsoft: graphical and interactive software for the analysis of time series and Earth tides, Computer & Geosciences, 31, pp 631-640.

Van Tonder, G., Riemann, K., Dennis, I., 2002. Interpretation of single-well tracer tests using fractional-flow dimensions. Part 1: Theory and mathematical models. Hydrogeology Journal 10, 351-356. DOI: 10.1007/s10040-002-0198-4

Verbovsek, T., 2009. Influences of Aquifer Properties on Flow Dimensions in Dolomites. Ground Water 47, 660-668. DOI: 10.1111/j.1745-6584.2009.00577.x

West, L.J., Odling, N.E., 2007. Characterisation of a Multi-Layer Aquifer Using Open Well Dilution Tests, *Ground Water*, 45**,** 74-84.

Witthüser K., Hötzl H., Reichert B., Sticher W., Nativ R. 2000. Laboratory experiments for diffusion transport processes in fractured chalk. In: Tracers and Modelling in Hydrology (TraM'2000 conference), May 2000, Liege, Belgium, IAHS Publication 262, 303-308.

Zarrouk, S., O'Sullivan, M., Croucher, A., Mannington, W., 2007. Numerical modelling of production from the Poihipi dry steam zone: Wairakei geothermal system, New Zealand. Geothermics 36, 289-303.DOI: 10.1016/j.geothermics.2007.03.006

Figure Captions

Figure 1 a) Drawdown curves showing development of drawdown with time and fractional flow dimension, n. The integer flow type curves (1, 2 and 3 D flow) are shown in heavy lines. The curve for $n=2$ is the Theis curve for 2D radial flow to a well in a homogeneous medium. b) Diagram illustrating 1D, 2D and 3D flow to a pumped borehole.

Figure 2 a) Map of East Yorkshire showing the location of the study site within the confined Chalk Aquifer. b) Outline of the borehole layout at the study site showing the pumped borehole (P) and 4 monitoring boreholes (M1, M2, M3 and M4). c) Cross-section showing confining glacial sediments overlying putty chalk, densely fractured chalk and less densely fractured chalk. The aquifer comprises the densely fractured chalk layer of high hydraulic conductivity. The boreholes (P – pumped borehole, M – monitoring borehole) are cased (thick line) to near the base of the densely fractured chalk layer and thus partially penetrate the aquifer. Black arrows show the impact of partial penetration on flow geometry during pumping from borehole P.

Figure 3 Background water level data from monitoring boreholes at the study site showing the impact of Earth tides (Et) which cause variations in water level of around 1.5 cm with a wavelength of 24 hours. Sharp, short duration drops in water level of 2-3 cm (labelled P) are caused by pumping of a nearby domestic borehole. The dotted line shows the best fit Earth tide model to the data.

Figure 4 a) Drawdown versus time curves for the pumped borehole (P) and 4 monitoring boreholes (M1, M2, M3, M4). The boreholes at 25 m from the pumped borehole all show an initial steep segment. b) Slope (dh/dt) versus time of the drawdown curves in (a) for the five boreholes, showing large slopes at early times. Slopes become somewhat noisy at larger times but show a general trend of decreasing slope with time.

Figure 5. a) Type curves for $W(u,\beta)$ versus $1/u$ for 2D radial flow to a pumped well in a homogeneous media incorporating delayed monitoring well response (Theis 1935, Black and Kipp 1977). The drawdown curve for M4 at 75 m from the pumped borehole shows a small effect suggesting a small delayed response (around 23 seconds). Drawdown curves for monitoring boreholes at 25 m from the pumped borehole (M1, M2 and M3) show much steeper slopes than type curves at early times. b) Type curves for $W(u,\beta)+f$ versus $1/u$ for flow to a partially penetrating well (Hantush, 1961). Improved fits to the observed drawdown data are obtained in comparison to (a) but slopes are steeper than all type curves at early times.

Figure 6. a) and b) Type curves from the dual porosity model of Bolton and Streltsova (1977). T – bulk rock transmissivity, S – bulk rock storativity, rs - ratio of matrix to fracture storativity. The drawdown curve for monitoring borehole M1 at 25 m does not show a good fit to any type curves, the slopes being either too shallow at late times (a) or too steep at early times (b).

Figure 7. Best fit type curves for the GRF model (Barker 1988) to drawdown curves for monitoring boreholes. Good fits are obtained for drawdown data later than 5 minutes (0.8 hours) for boreholes M1, M2, and M3 and for the whole drawdown curve in the case of borehole M4. The curves indicate flow dimensions of 2.33 to 2.38 for the monitoring boreholes at 25 m (M1, M2 and M3) and 2.21 for the borehole at 75 m from the pumped borehole.

Figure 8. Tracer breakthrough and injection well concentration curves (adapted from Hartmann et al. 2007). Breakthrough curves show similar shapes but greatly varying recoveries from 9 to 57%.

Figure 9. Conceptual model for the analytical 1D dual porosity model for solute transport in fractured rock with matrix diffusion (Barker and Foster 1981). Water with solute travels in the x direction along parallel fractures with aperture, a, and spacing, d. Solute also diffuses between fractures and matrix blocks in the z direction.

Figure 10. Conceptual models for flowing fracture geometry around the pumped borehole. a) In 2D radial flow, fracture spacing and bulk hydraulic conductivity are constant with increasing distance from the pumped borehole. b) In fractional flow with dimension $2<n<3$, fracture spacing decreases and thus bulk rock hydraulic conductivity increases with increasing distance from the pumped borehole.

Figure 11. Tracer breakthrough curves and best fit model curves for 2D flow and fractional flow. Model fits are equally good for boreholes M1 and M3 but the fractional flow model shows a better fit than the 2D flow model in the case of M2.

Figure 12. Conceptual model for flowing fractures in the fractured chalk aquifer during the pumping and tracer tests. Arrows indicate general direction of flow during pumping from borehole P. At the start of pumping, flow is initiated at the base of the aquifer where the boreholes are open to the aquifer. At later times, progressively more fractures conduct flow with increasing distance from the pumped borehole giving a flow dimension close to 1 in the immediate vicinity of the pumped borehole, > 2 at 25 m and ≅2 at distances greater than 75 m.

Table Captions

Table 1. Overview of case studies on fractional flow from the literature, PP - partial penetrating wells, FP – fully penetrating wells.

Table 2. Details of borehole completion, estimated thickness of the high hydraulic conductivity layer and thickness of open section at the base of the borehole casing.

Table 3. Results of monitoring borehole drawdown curve analysis using Theis and Hantush (1961, 1967) methods for monitoring boreholes. Theis – Theis (1935) method for radial flow to a well, Hant. PP – Hantush (1961) method for partially

penetrating wells, Hant. PP+leak – Hantush (1967) method for partially penetrating wells with leakage, Hant. PP Late – Hantush (1961) method applied to data later than 5 minutes. r – horizontal distance between pumping and observation wells, T – transmissivity, S – storativity, K_z/K_r – vertical to horizontal conductivity ratio, RMSE – root mean square error. $r/\beta = r\sqrt{K'/(Tb')}$ where K' and b' are the hydraulic conductiviy and thickness of the confining layer.

Table 4. Results of fractional flow modelling applied to drawdown curves at times greater than 5 minutes (0.0035 d) for boreholes M1, M2 and M3 and all data for borehole M4. T_a - apparent transmissity, S_a - apparent storativity, T - transmissivity, S - storativity, n - flow dimension, RMSE - root mean square error. Upper and lower bounds to T and S are calculated assuming 2D and 3D flow.

Table 5. Values and allowable ranges of input parameters for the 1D dual porosity model of solute transport in fractured rock with matrix diffusion (Barker and Foster 1981).

Table 6. Best fit dual porosity transport model results to breakthrough curves for tracers in boreholes M1, M2 and M3 and for two values of effective diffusion coefficients (D_e), 5.0E-11 m^2/s (first value) and 2.0E-10 m^2/s (second value).

Table 7. Recoveries (observed and calculated), flow-through rates, Q_{inj}, and scaling factors, C (best fit) and C' (estimated) for breakthrough curves from boreholes M1, M2 and M3.

Table 1.

Authors	**Date**	**Lithology**	**Fractional flow dimension**
Hamm and Bidaux	1996	granite	1.45
Bangoy and Richard	1999	granite and limstone	0.76-1.34 0.5-1.14
Leveinen et al.	1998	gneisses	1.5, 1.2, 2.0-2.25
Van Tonder et al.	2002	sandstones, mudstones	1.75-1.85
Kuusela-Lahtinen et al.	2003	gneisses	1.5-2.5
Le Borgne et al.	2004	schists, gneisses	1.4-1.7
Marechal et al.	2003, 2004	granite	1.2-2.5
Delay et al.	2004	limestones	1.9-2.0
Bernard et al.	2006	limestones	1.85-2.0
Verbovsek	2009	dolomites	PP, average 2.19 FP, average 1.73
Chang et al.	2011	slates and metasandstones	1.3-2.27
Zarrouk et al.	2007	volcanics	2.4-2.75

Table 2

borehole	depth to base casing (m)	depth to base glacial sediments (m)	thickness below base casing (m)	estimated thickness high K layer (m)
P	25.77	13	0.11	8.58
M1	26.15	11	0.11	9.26
M2	26.19	13	0.14	11.33
M3	22.55	10	3.3	8.85
M4	17.35	13.3	0.18	3.53

Table 3

BH	r (m)	Theis			Hant. PP				Hant. PP+ leak					Hant. PP Late			
		T (m^2/d)	S x 10^{-3}	RMSE (m)	T (m^2/d)	S x 10^{-3}	K_z/K_r	RMSE (m)	T (m^2/d)	S x 10^{-3}	K_z/K_r	r/β	RMSE (m)	T (m^2/d)	S x 10^{-3}	K_z/K_r	RMSE (m)
M1	25	460	0.119	0.078	564.4	4.55	0.006	0.008	501	4.60	0.006	0.053	0.008	548.8	4.17	0.006	0,0062
M2	25	482	0.017	0.128	646.8	3.89	0.003	0.0179	585	3.50	0.004	0.044	0.017	563.9	2.53	0.004	0.0055
M3	25	488	0.022	0.098	560.3	1.02	0.002	0.0194	457	1.07	0.003	0.045	0.016	611.9	0.86	0.001	0.0074
M4	75	469	0.209	0.007	504.2	0.189	1.0	0.01	445	0.225	1.0	0.051	0.0051				

Table 4

BH	Time range (d)	S_a ($S_s.b^{3-n}$)	T_a ($K.b^{3-n}$)	n	RMSE (m)	T = KB (m^2/d)	T bounds (m^2/d)	$S=S_s.B$	S bounds
M1	0.0035-0.8	1.170E-04	45.0	2.38	0.0035	206.4	106, 428	5.36E-04	2.75E-04, 1.11E-03
M2	0.0035-0.8	4.760E-05	46.1	2.36	0.0031	195.6	104, 438	2.02E-04	1.07E-04, 4.52E-04
M3	0.0035-0.8	5.062E-05	54.5	2.33	0.0028	205.1	115, 518	1.91E-04	1.06E-04, 4.81E-04
M4	0.0-0.8	7.708E-05	114.6	2.21	0.0038	337.0	184, 1089	2.27E-04	1.24E-04, 7.32E-04

Table 5

parameter	**range**
effective diffusion coefficient (D_e), m^2/s	5.0E-11, 2.0E-10
matrix porosity (ϕ_m)	0.3
travel time (t_a), hr	0 – 31.0
scaling factor, C	0 – 1.0
fracture spacing (d), m	0.01 - 1.0
dispersivity (α), m	0.1 - 25.0
aquifer thickness (B), m	9.5
pumping rate (Q_p), m^3/hr	13.8
borehole distance (R), m	25.0
borehole radius (r_w), m	0.075

Table 6

Borehole/ Parameter	2D radial flow	Fractional radial flow
M1: Eosine		
Flow dimension (*n*)	2.0	2.38
Transmissivity (*T*), m^2/d	501	206
Aperture (*a*), μm	332 – 420	232 - 297
Spacing (*d*), m	4.32E-02 – 8.73E-02	3.63E-02 – 7.59E-02
Travel time (t_a), hr	10.4 – 6.51	7.26 – 4.44
Dispersivity (α), m	2.16 – 1.53	1.72 – 1.06
Scaling factor (*C*)	1.47E-03 – 1.46E-03	1.45E-03 – 1.40E-03
RMSE, m	6.74E-03 – 6.89E-04	6.91E-04 – 6.97E-04
M2: Amino G acid		
Flow dimension (*n*)	2.0	2.36
Transmissivity (*T*), m^2/d	585	196
Aperture (*a*), μm	358 - 417	219 – 276
Spacing (*d*), m	4.60E-02 – 7.29E-02	3.23E-02 – 6.48E-02
Travel time (t_a), hr	10.5 – 7.73	7.77 – 4.89
Dispersivity (α), m	0.67– 1.30	1.14 – 0.84
Scaling factor (*C*)	3.29E-02 – 4.29E-02	4.01E-02 – 4.00E-02
RMSE, m	3.78E-03 – 1.72E-03	2.03E-03 – 2.01E-03
M3: Rhodamine WT		
Flow dimension (*n*)	2.0	2.33
Transmissivity (*T*), m^2/d	457	205
Aperture (*a*), μm	295 - 375	219 – 278
Spacing (*d*), m	3.30E-02 – 6.79E-02	3.08E-02 – 6.27E-02
Travel time (t_a), hr	12.1 – 7.5	8.26 – 5.14
Dispersivity (α), m	3.07 – 2.37	2.22 – 1.75
Scaling factor (*C*)	4.77E-03 – 4.70E-03	4.36E-03 – 4.32E-03
RMSE, m	3.82E-03 – 4.17E-03	3.70E-03 – 3.94E-03

Table 7

Borehole /tracer	**M1 Eosine**	**M2 Amino G Acid**	**M3 Rhodamine WT**
Observed recovery (%)	9	57	17
Flow-through rate, Q_{inj} (m^3/hr)	0.082	0.492	0.108
Scaling factor from best fit model, C	0.00142	0.040	0.00434
Recovery from best model fit (%)	8.8	56.27	17.2
Predicted scaling factor from Q_{inj}, C'	0.00594	0.0311	0.00783
Predicted recovery from C' (%)	36.5	43.8	31.0