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ON THE EFFECT OF NONLINEARITIES IN MULTIVARIABLE FIRST ORDER PROCESS CONTROL

by

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ON THE EFFECT OF NONLINEARITIES IN MULTIVARIABLE FIRST ORDER PROCESS CONTROL

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SUMMARY

The paper considers the use of discrete and continuous, first order, linear, multivariable, approximate models as the basis of controller design for large-scale and/or badly-defined linear engineering systems where the output measurement is the sum of memoryless bounded and finite incremental gain forms. The techniques have the advantage that high performance closed-loop systems can be designed using only simple properties of system matrices or plant transient response data, and bounds for the transient behaviour of the real, nonlinear system can be computed from the responses of the approximate, linear systems.

Taking $r \in E$, assumption (i) ensures that $y_L \in E$ and hence that equation (7) takes the functional form $y = Wy$ where, using (ii) and (iii) $W: E \to E$ is causal and a global contraction. The well-known contraction-mapping theorem (see (5) or (6)) immediately indicates that equation (1) has a unique solution in E obtainable as the strong limit of the sequence $y_{k+1} = Wy_k$, $k \geq 1$, for any initial quess $y_1 \in E$. The nonlinear feedback system is hence stable. Standard results (see (6)) indicate that $||y-y_2|| \leq (\lambda/(1-\lambda))||y_2-y_1||$ and hence equation (5) follows by choosing $y_1 = 0$ (when $y_2 = y_L$) and truncating. This completes the proof of the theorem.

The condition (4) is an 'incremental gain condition' on the nonlinearity error N-F and is the crucial assumption. More precisely it guarantees that the stability of Fig. 3 implies the stability of Fig. 1 and provides the easily computed upper bound of equation (5) for the modelling error expressed entirely in terms of the responses of the linear feedback system. For example, if E is the Banach space of bounded continuous mxl vector functions of time on the interval $0 \leq t < +\infty$ with the normal uniform norm, then equation (5) implies, for any $t > 0$, that

$$\max_{1 \leq i \leq m} |y_i(t) - (y_L(t))_i| \leq \frac{\lambda}{1-\lambda} \max_{\substack{1 \leq i \leq m \\ 0 \leq s \leq t}} |(y_L(s))_i| \qquad (8)$$

which has the obvious interpretation in providing an upper bound on the transient modelling error $y(t) - y_L(t)$.

The following corollary is of great interest in the next section:

Corollary 1: With the assumptions of theorem 1, suppose also that F has a bounded causal in verse on E. Then the nonlinear feedback system is stable if

$$\lambda_1 \triangleq ||L_c F|| \quad \lambda_0 < 1 \qquad (9)$$

where

$$\lambda_0 = \sup_{T>0} \sup_{y,z \in E} ||P_T\{(F^{-1}Ny-y) - (F^{-1}Nz-z)\}|| / ||P_T(y-z)|| \qquad (10)$$

Moreover we can choose $\lambda = \lambda_1$ so that, when $N0 = 0$, equation (5) holds with λ replaced by λ_1.

Proof: Write $L_c = (L_c F)F^{-1}$ and deduce from the properties of norms that (4) holds with $\lambda = \lambda_1$.

Example 1: If the system is m-input/m-output with N a diagonal nonlinearity, $y_i \longmapsto f_i(y_i)$ $1 \leq i \leq m$, satisfying the conditions, $1 \leq i \leq m$,

$$\alpha_i \leq \frac{f_i(y) - f_i(z)}{y - z} \leq \beta_i \qquad (11)$$

for some strictly positive scalars α_i, β_i. Choosing $F = \mathrm{diag}\{(\alpha_i + \beta_i)/2\}_{1 \leq i \leq m}$, then it is easily verified that we can choose

$$\lambda_0 = \max_{1 \leq i \leq m} \left\{ \frac{\beta_i - \alpha_i}{\beta_i + \alpha_i} \right\} < 1 \qquad (12)$$

The validity of equation (9) now depends only upon the value of $||L_c F||$.

A More General Class of Nonlinearities

Consider now the situation when

$$N = N_0 + N_1 \qquad (13)$$

where N_0 satisfies an incremental gain condition of the form of (4) or (10) and $N_1: E_0 \to E$ is bounded nonlinearity (see ref (4)) of the form

$$||N_1 z|| \leq q/2 \quad \forall z \in E_0 \qquad (14)$$

We can now state the following theorem generalizing that used in ref. (4):

Theorem 2: Suppose that

(i) the linear configuration of Fig. 3 is input-output stable

(ii) $L_c(N_0 - F)$ is causal and maps E into itself,

(iii) equation (4) holds with N replaced by N_0 and

(iv) $N_1: E_0 \to E$ satisfies equation (14).

Then the nonlinear feedback system of Fig. 1 is stable and, if $N_0 0 = 0$,

$$||P_T(y-y_L)|| \leq \frac{\lambda}{1-\lambda} ||P_T y_L|| + \frac{1}{1-\lambda} ||P_T L_c|| \frac{q}{2} \quad \forall T > 0 \qquad (15)$$

Proof: Noting that $v = r - N_1 y \in E$ whenever $r \in E$ and reconstituting Fig. 1 in the form of Fig. 4, it is clear from theorem 1 that $y \in E$ and hence that the system is stable. Also if y_V is the linear system (Fig. 1) response to input V, then, using equation (5),

$$||P_T(y-y_V)|| < \frac{\lambda}{1-\lambda} ||P_T y_V|| \quad \forall T > 0 \qquad (16)$$

It is trivially verified that $y_V = y_L - L_c N_1 y$ and hence that $||P_T(y_V - y_L)|| \leq ||P_T L_c|| q/2$ and $||P_T y_V|| \leq ||P_T y_L|| + ||P_T L_c|| q/2$. Clearly

$$||P_T(y-y_L)|| \leq ||P_T(y-y_V)|| + ||P_T(y_V-y_L)||$$

$$\leq \frac{\lambda}{1-\lambda} ||P_T y_V|| + ||P_T L_c|| \, q/2$$

$$\leq \frac{\lambda}{1-\lambda} \{||P_T y_L|| + ||P_T L_c|| \tfrac{q}{2}\} + ||P_T L_c|| \frac{q}{2} \qquad (17)$$

which is simply equation (15).

(Note: writing $L_c = L_c F F^{-1}$ leads to a natural analogue of corollary 1, defining λ_0 by (10) with N replaced by N_0 but this is straightforward and omitted here).

MULTIVARIABLE FIRST ORDER CONTROL

A major difficulty in the application of the above results is the need to compute useful bounds for λ and $||P_T L_c||$ and/or $||L_c F||$. This may be particularly difficult if the design of K proceeded by analysis of the approximate configuration Fig. 2. More previously, assuming zero initial conditions and writing the solution of equation (2) in the form $y_{LA} = L_{cA} r$ where L_{cA} is the linear operator in E 'approximating' L_c, we may know the response y_{LA} and the operator L_{cA} but we do not necessarily know the response y_L or the operator L_c. Clearly, in such a situation, the above results cannot be directly applied, nor can the conditions of the theorems be directly checked. It is the purpose of this section to indicate how this problem can be overcome (in a certain sense) in the case of multivariable first order control. For simplicity we consider only the discrete case and note that the results carry over to the continuous case by replacing 'sample rate' h^{-1} by gain k, the space E_o by the continuous m-vector valued functions on $[o, +\infty[$ and E by the linear subspace of E_o of bounded functions. The norm on E is the normal uniform norm.

Let E_o by the natural vector space of infinite sequences $\{y_o, y_1, \ldots\}$ of real m-vectors and let E be the Banach subspace of vectors of finite norm $||y|| = \sup||y_k||_m$ where $||.||_m$ is the normal uniform norm on R^m. Consider the m-input/m-output linear discrete plant G in E_o described by the mxm invertible, minimum-phase discrete model $S(\Phi,\Delta,C)$

$$x_{k+1} = \Phi x_k + \Delta u_k \qquad , \; x_k \in R^n$$
$$y_k = C x_k \qquad , \; k \geq o \qquad (18)$$

with inverse z-transfer function matrix (TFM)

$$G^{-1}(z) = (z-1)B_o + B_1 + B_o H(z) \qquad (19)$$

with $|B_o| \neq 0$ and H(z) proper. We assume that $S(\Phi,\Delta,C)$ is derived from a minimum-phase continuous model S(A,B,C) with $|CB| \neq 0$ and synchronous sampling of period h.

Suppose now that G is to be approximated, for the purposes of controller-design, by the (multivariable!) first order approximate model G_A in E_o with inverse mxm z-TFM

$$G_A^{-1}(z) = (z-1)B_o + B_1 \qquad (20)$$

obtained from G by neglecting H or deduced from plant step responses. Consider now the unity-feedback system $L_{cA}F$ realised in the form of Fig. 5 where F is realized by an mxm constant, invertible matrix. Choosing the <u>proportional</u> controller K such that KF has the suggested 'first-order' form (see ref(3))

$$K(z)F = B_o \operatorname{diag}\{1-k_j\}_{1\leq j\leq m} - B_1 \qquad (21)$$

with $|k_j| < 1$, $1 \leq j \leq m$, then it follows (ref (3)) that Fig. 5 (and hence Fig. 2) is stable, that

$$\lim_{h\to o+} ||L_{cA}F|| = \max_{1\leq j\leq m} \frac{(1-k_j)}{(1-|k_j|)} \qquad (22)$$

and that

$$\lim_{h\to o+} ||L_c - L_{cA}|| = 0 \qquad (23)$$

We are lead therefore to the result:

Theorem 3: With the above construction, suppose that N has the 'summation' form of equation (15), is independent of sampling rate and

(i) N_o is causal and maps E into itself

(ii) $N_1 : E_o \to E$ satisfies equation (14), and

(iii) $\lambda_2 = \lambda_o \max_{1\leq j\leq m} \frac{(1-k_j)}{(1-|k_j|)} < 1$ (with λ_o

defined by equation (10) with N replaced by N_o). Then the nonlinear feedback system of Fig. 1 is stable for all <u>fast enough</u> sampling rates. If also $N_o 0 = 0$, then, assuming zero initial conditions

$$\limsup_{h\to o+} ||P_T(y-y_{LA})|| \leq \frac{\lambda_2}{1-\lambda_2} \lim_{h\to o+} ||P_T y_{LA}||$$
$$+ \frac{1}{1-\lambda_2} \max_{1\leq j\leq m} \frac{(1-k_j)}{(1-|k_j|)} ||F^{-1}||_m \frac{q}{2} , \forall T > 0 \qquad (24)$$

In essence, the result states that a proportional controller K designed on the basis of the simple, approximate, first-order G_A of the complex (possibly partially unknown) plant G will not only produce excellent approximate responses y_{LA} (see ref (3)) under fast sampling conditions but will also ensure the stability of the real system G in the presence of a large class of nonlinearities N. The error $y - y_{LA}$ in the prediction of the output response can also be accurately bounded (equation (24)) under fast sampling conditions using only control data $F, k_1, \ldots, k_m$ and the (known!) response y_{LA}.

Example 2: If, $o \leq k_j < 1$, $1 \leq j \leq m$, and N_o has the form defined by example 1 with $\alpha_i > 0$, $1 \leq i \leq m$, then it is clear that $\lambda_2 = \lambda_o < 1$ and the nonlinear feedback system with the defined controller is stable under fast sampling conditions with

$$\limsup_{h\to o+} ||P_T(y-y_{LA})|| \leq \frac{\lambda_o}{1-\lambda_o} \lim_{h\to o+} ||P_T y_{LA}||$$
$$+ \frac{1}{1-\lambda_o} \max_{1\leq j\leq m} \frac{2}{(\alpha_j+\beta_j)} \frac{q}{2} , \forall T > 0 \qquad (25)$$

Proof of Theorem 3: We verify that the conditions of theorem 2 are valid for $h\to o+$. Certainly L_{cA} and hence (equation (23)) L_c are causal and bounded in E and hence stable. Also $L_c(N_o - F)$ is causal and maps E into itself, and, using the definitions,

$$\limsup_{h\to o+} ||P_T L_c\{(Ny - Fy) - (Nz - Fz)\}||$$
$$\leq \lim_{h\to o+} ||P_T L_c F|| \; \lambda_o \; ||y-z|| \leq \lambda_2 ||y-z|| \qquad (26)$$

as $\lim_{h\to o+} ||P_T L_c F|| \leq \lim_{h\to o+} ||L_c F|| = \lim_{h\to o+} ||L_{cA}F||$ by equation (23). Finally, if $N_o 0 = 0$, then (24) follows from (15) noting that $||y_L - y_{LA}|| = ||(L_c - L_{cA})r|| \to o$ as $h \to o+$ by (23) and $\lim y_{LA}$ exists (see ref (3))

<u>ILLUSTRATIVE EXAMPLE</u>

Consider the open-loop unstable continuous system (ref (4)) described by

$$A = \begin{pmatrix} 2 & 1 & 0.1 \\ 1 & -1 & 0.2 \\ 0.5 & 0.8 & -1 \end{pmatrix} , \; B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$
$$C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \qquad (27)$$

and note that $|CB| \neq 0$ and the system is minimum-phase. The design problem considered is the choice of proportional controller K in the configuration of Fig. 1 with a diagonal feedback nonlinearity $y_i \to f_i(y_i)$, i=1,2 of the form

$$f_1(y_1) = \frac{(1+1.05\,|y_1|)}{(1+0.95\,|y_1|)}\, y_1$$

$$f_2(y_2) = \begin{cases} 0 \quad , & |y_2| \leq 0.05 \\ y_2 - 0.05 \operatorname{sgn} y_2 & \end{cases} \tag{28}$$

describing small deviations from linearity in measurement of y_1 and a deadzone in the measurement of y_2. Discretizing the system with a sample interval h = 1/20, the approximating first-order lag matching the high frequency and steady state plant characteristics is defined by

$$B_o = \begin{pmatrix} 19.52 & -21.0 \\ -0.5 & 20.51 \end{pmatrix} , \quad B_1 = \begin{pmatrix} -0.95 & -1.92 \\ -1.1 & 0.84 \end{pmatrix} \tag{29}$$

and we attempt the design of K by analysis of Fig. 2, with $F = I_2$ (2x2 identity matrix). More precisely, using the controller K defined by equation (21) with $k_1 = k_2 = 0.0$ leads to the excellent, deadbeat response y_{LA} to a unit step demand in y_1 of the form indicated in Fig. 6. The remaining problem is to estimate the degree to which the calculated response y_{LA} deviates from the real system response y. If the plant model is known exactly one need only simulate. If, however, the plant model is not known (i.e. B_o and B_1 were computed from plant transient tests) we can use the results of theorem 3 and the observation that the plant sampling rate is high to predict (a) that the nonlinear feedback system is stable if $\lambda_2 = \lambda_o < 1$ and (b) that the peak transient error can then be estimated from equation (24) by deleting the limits i.e. the relation

$$||P_T(y-y_{LA})|| \leq \frac{\lambda_2}{1-\lambda_2} ||P_T y_{LA}|| + \frac{1}{1-\lambda_2} \max_{j=1,2} \frac{(1-k_j)}{(1-|k_j|)} ||F^{-1}||_2 \frac{q}{2} \tag{30}$$

holds to high accuracy for all T > 0. Both of these predictions are verified in Fig. 7 where the responses y_{LA} and y of the approximate and real feedback configurations are plotted together with the estimated error bounds implied by equation (30) with the data $\lambda_2 = \lambda_o = 0.105$ and q = 0.1. It is clear that $\lambda_2 < 1$ and that the estimated error bounds are highly pessimistic.

CONCLUSIONS

Previous work (1) - (3) has demonstrated the viability of using very simple models of linear plant dynamics as the basis of linear controller design for a well-defined class of discrete or continuous multivariable plant. The (first-order) approximate models used have the advantage thay they are easily computed from a complex plant model (if available) or, if the system is open-loop stable, from rise-time and steady-state properties obtained from plant step response tests. In the second case it is clear that a detailed plant model is not required!

The work described in this paper has extended that of ref (4) by demonstrating that the effect of measurement nonlinearities on closed-loop system stability and performance can also be assessed using elementary calculation based on the nonlinearity and the responses of the approximate linear feedback system. The results obtained are only strictly correct under high gain/fast sampling conditions but examples show that they provide useful working estimates under a relaxation of these conditions.

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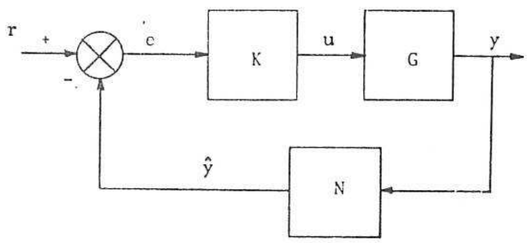

Figure 1 Nonlinear Feedback System

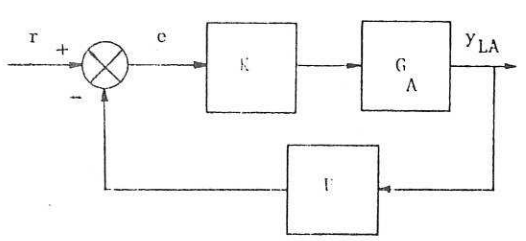

Figure 2 Linear feedback system with linear plant and measurement approximations

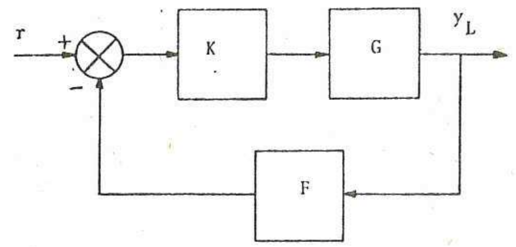

Figure 3 Linear feedback system with linear measurement approximation.

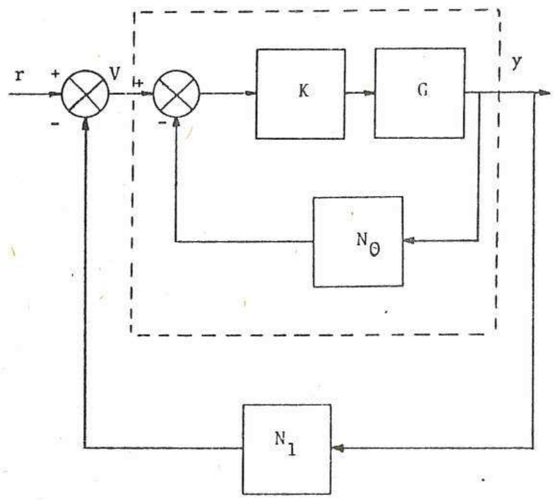

Figure 4 A system decomposition

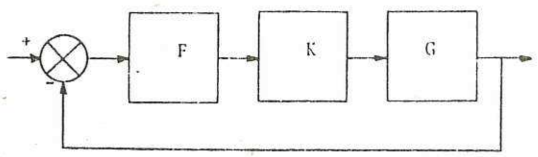

Figure 5 Linear feedback system used for Design of K.

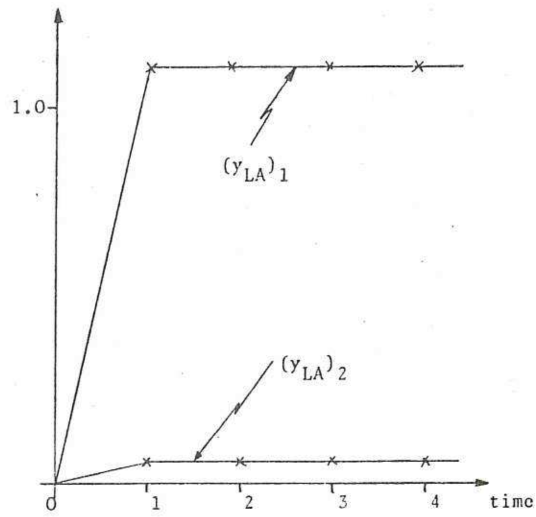

Figure 6 Response y_{LA} of approximate linear feedback system to a unit step demand in output one.

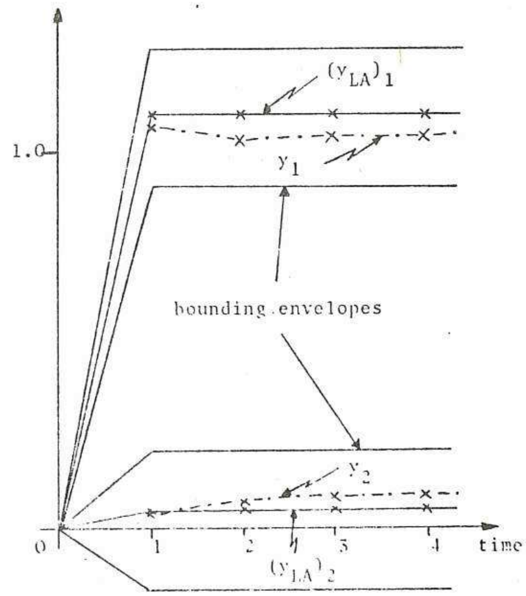

Figure 7 Closed-loop of real, nonlinear and approximate, linear feedback systems to a unit step demand in output one.

ON THE EFFECT OF NONLINEARITIES IN MULTIVARIABLE FIRST ORDER PROCESS CONTROL

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INTRODUCTION

The viability of using both continuous and discrete first order linear approximate models as the basis for controller design for large-scale and/or badly defined linear multivariable engineering systems has been investigated recently by Owens (1), Edwards and Owens (2) and Owens (3). The techniques have the advantage that high performance closed-loop systems can be designed in a straightforward manner using only simple properties of the system matrices or graphical analysis of transient response data obtained from plant trials or simulations. More recently Boland and Owens (4) have demonstrated that the approximate first order model can be used to obtain easily computed estimates of the effect of a common class of measurement nonlinearities on the transient response of the closed-loop system. The nonlinearities were restricted to bounded nonlinearities such as quantization and deadzone and the results take the form of upper bounds on the difference between the transient responses of the linear and nonlinear closed-loop system under comparable conditions.

The results presented in this paper extend those of reference (4) to include the use of approximate first order multivariable plant models for the estimation of (a) the effect of measurement nonlinearities of finite incremental gain on stability and transient response and (b) the effect of nonlinearities written as the sum of bounded and finite incremental gain forms.

NONLINEAR FEEDBACK RELATIONS

Consider the (discrete or continuous) feedback system illustrated in Fig. 1 and regard the reference signal r, error e, input signal u and output signal y as elements of a common vector space E_o large enough to contain 'all possible'. The plant and controller are assumed to be linear and characterized by causal, linear operators G and K respectively mapping E_o into itself. It is also assumed that the measured output signal $\tilde{y}$ depends on the real output y in a manner described by the causal, memoryless, nonlinear map $N: E_o \rightarrow E_o$ and hence that the system is described by the functional equations

$$y = G K e + y_o \quad , \quad e = r - Ny \tag{1}$$

where y_o is an initial condition term. Let E be a linear vector subspace of E_o, regarded as a Banach space with norm $||.||$. Then the feedback system of Fig. 1 is said to be input/output stable if, for every demand $r \in E$ and for all initial conditions, the system equations (1) have a unique solution y that lies in E.

Now let F and G_A be causal, linear, approximations to N and G (respectively) to be used for the purposes of the design of the forward path controller K, and suppose that the resulting approximate, linear feedback system (Fig. 2) described by the relations

$$y_{LA} = G_A K e + y_{oA} \quad , \quad e = r - Fy_{LA} \tag{2}$$

is stable and possesses 'satisfactory' performance characteristics. We ask the following questions:

(a) When does the stability of the linear, approximate configuration of Fig. 2. imply the stability of the real configuration of Fig. 1?

(b) If P_T is the truncation operator (see, for example, Cook (5) or Holtzman (6)), then can we find computable upper bounds for the modelling error $||P_T(y-y_{LA})||$, $T > 0$, in terms of y_{LA}? The solution of these problems is of vital importance if F and G_A are to be useful approximations for design purposes. Some partial solutions are outlined below in the special case when the plant approximation G_A is identical to the plant i.e. $G = G_A$ when Fig. 2 takes the form of Fig. 3. The application of the results to the situation when G_A is a first-order approximate model of the plant G is left to the following section.

Nonlinearities of Finite Incremental Gain

It is trivially verified that the configuration of Fig. 3 is described by the linear relations

$$y_L = GKe + y_o \quad , \quad e = r - F y_L \tag{3}$$

and that, in the case of zero initial conditions ($y_o = 0$), we can write $y_L = L_c r$ where L_c is a linear operator in E_o. We now state the following theorem:

Theorem 1: Suppose that

(i) the linear configuration of Fig. 3 is input-output stable,
(ii) $L_c(N-F)$ is causal and maps E into itself and
(iii) there exists a real scalar $\lambda \in [0,1[$ such that

$$||P_T L_c\{(Ny - Fy) - (Nz - Fz)\}|| \leq \lambda \, ||P_T(y-z)|| \; \forall T > 0 \; \forall y, z \in E \tag{4}$$

Then the nonlinear feedback system of Fig. 1 is input/output stable and, if $N0 = 0$,

$$||P_T(y-y_L)|| \leq \frac{\lambda}{1-\lambda} ||P_T y_L|| \; \forall T > 0 \tag{5}$$

Proof: Writing equation (1) in the form

$$y = GKe + y_o \quad , \quad e = \{r + Fy - Ny\} - Fy \tag{6}$$

and using equation (3) yields

$$y = y_L + L_c \{Fy - Ny\} \tag{7}$$