POLE AND ZERO RETENTION IN THE FACTORIZATION OF MULTIVARIABLE SYSTEMS, WITH APPLICATION TO MODEL REDUCTION

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Abstract

The problem of pole and zero retention in multivariable system reduction is regarded as the generation of a series factorization of the system transfer function matrix. Necessary and sufficient conditions for the existence of the factorization are derived in terms of the decomposition of the state space into A-invariant and {A,B}-invariant subspaces.

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1. Introduction

The facility to retain dominant poles and zeros of an mxm linear time-invariant system $S(A,B,C)$ in R^n as poles and zeros of a reduced model $S(A_r,B_r,C_r)$ in R^{n_r} is an important part of model reduction methodology. In the case of a single-input, single-output system with strictly proper transfer function, $g(s)$, of order n, this can always be achieved by series factorization of $g(s)$ in the form

$$g(s) = g_1(s)h(s) \qquad \ldots(1)$$

where $g_1(s)$ and $h(s)$ are proper transfer functions of order n_1 and n_2 respectively, $n_1+n_2 = n$, and the dominant poles and zeros of $g(s)$ are subsets of the poles and zeros of $g_1(s)$. A reduced model $g_r(s)$ of $g(s)$ of the required form is then obtained by computing a reduced model $h_r(s)$ of $h(s)$ and setting

$$g_r(s) = g_1(s)h_r(s) \qquad \ldots(2)$$

This paper considers the extension of this approach to the multi-input, multi-output case in the natural manner by considering the existence of the series factorization of the system transfer function matrix, $G(s)$, in the form

$$G(s) = G_1(s)H(s) \qquad \ldots(3)$$

where G_1 and H are proper transfer function matrices possessing realizations of order n_1 and n_2 respectively, $n_1+n_2 = n$, and the dominant poles and zeros of G are subsets of the poles and zeros of G_1. The condition $n = n_1+n_2$ is required to ensure that we do not introduce any extra system states.

A general solution of this problem is not presented. A useful solution is obtained however for the case when $G_1(s)$ is strictly proper and $H(s)$ is taken to have the form

$$H(s) = I + G_2(s) \qquad \ldots(4)$$

where $G_2(s)$ is strictly proper. This decomposition is illustrated in Fig.1. It has the advantage that it can be generated by state feedback transformations and enables the problem to be examined using geometric methods.

The geometric formulation of the problem is described in section 2, where necessary and sufficient conditions for the existence of the decomposition are derived in terms of the existence of direct sum decompositions of the system state space. The results are illustrated by an example in section 3 and, in section 4, their application to model reduction is outlined.

2. Factorization of the Transfer Function Matrix

Consider the mxm system S(A,B,C) and the factorization of $G(s) = C(sI-A)^{-1}B$ into the form defined by equations (3) and (4). The following lemmas establish the connection between this problem and geometric feedback theory.

Lemma 1: Let F be an arbitrary mxn matrix, then

$$G(s) \equiv C(sI_n-A+BF)^{-1}B\{I_m + F(sI_n-A)^{-1}B\}$$

Proof: Follows directly from the identity

$$C(sI_n-A+BF)^{-1}B = C(sI_n-A)^{-1}(I_n+BF(sI_n-A)^{-1})^{-1}B$$

$$= C(sI_n-A)^{-1}B(I_m+F(sI_n-A)^{-1}B)^{-1}$$

Lemma 2: G(s) has a factorization of the form $G(s) = G_1(s)(I+G_2(s))$ where each strictly proper transfer function matrix G_i has a realization of dimension n_i $(i = 1,2)$ with $n_1+n_2 = n$ if, and only if, there exists an mxn matrix F_o such that

$$G_1(s) = C(sI-A+BF_o)^{-1}B \quad , \quad G_2(s) = F_o(sI-A)^{-1}B \qquad \ldots(7)$$

have a realization of dimension n_i $(i = 1,2)$ and $n_1+n_2 = n$. Furthermore, if S(A,B,C) is controllable, then for a specific choice of $G_1(s)$ and $G_2(s)$ the matrix F_o is unique.

Proof: Given G_1 and G_2 satisfying equation (7) with $n_1+n_2 = n$ then sufficiency follows from Lemma 1 with $F = F_o$. Conversely, if G(s) has a decomposition of the required form, let $G_i(s)$ have a realization of dimension n_i described by the triple (A_i, B_i, C_i) $(i = 1,2)$. A state representation of the system is hence

$$\dot{x}_1(t) = A_1x_1(t) + B_1v(t)$$
$$v(t) = u(t) + z(t)$$
$$\dot{x}_2(t) = A_2x_2(t) + B_2u(t)$$
$$y(t) = C_1x_1(t) \quad , \quad z(t) = C_2x_2(t) \qquad \ldots(8)$$

or, using the composite state $x(t) = (x_1^T(t), x_2^T(t))^T$,

$$\dot{x}(t) = \begin{pmatrix} A_1 & B_1C_2 \\ O & A_2 \end{pmatrix} x(t) + \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} u(t)$$

$$y(t) = [C_1 \ , \ 0] \ x(t) \qquad \ldots(9)$$

Then the choice of $F_o = [0, C_2]$ satisfies the desired conditions.

Finally if $S(A,B,C)$ is controllable and $G_2(s) = F_o(sI-A)^{-1}B = \tilde{F}_o(sI-A)^{-1}B$ for some matrices $F_o, \tilde{F}_o$, it follows that

$$F_oA^{i-1}B = \tilde{F}_oA^{i-1}B \qquad (i = 1,2,\ldots n)$$

whence

$$(F_o-\tilde{F}_o) \ [B, AB, \ldots, A^{n-1}B] = 0$$

and controllability implies that $F_o = \tilde{F}_o$.

The immediate interpretation of Lemma 2 is that if we drop (for the moment) the condition that the pole and zero sets of G_1 contain specified subsets of the pole and zero sets of G, then the factorization problem reduces to the generation of conditions for the existence of F_o. These are described in the following theorem.

Theorem 1:

Suppose that S(A,B,C) is both controllable and observable. Then there exists an F_o satisfying the conditions of Lemma 2 if, and only if, there exist subspaces η_1 and η_2 in R^n of dimension n_1 and n_2 such that

$$A\eta_2 \subset \eta_2 + R(B) \quad ; \quad C\eta_2 = \{0\} \qquad \ldots(10)$$

$$A\eta_1 \subset \eta_1 \qquad \ldots(11)$$

$$\eta_1 \oplus \eta_2 = R^n \qquad \ldots(12)$$

Proof

Given (10)-(12), there exists F_o such that

$$(A-BF_o)\eta_2 \subset \eta_2 \quad , \quad F_o\eta_1 = \{0\} \qquad \ldots(13)$$

so that η_2 is just the unobservable subspace of $S_1 = S(A-BF_o,B,C)$ and η_1 is just the unobservable subspace of $S_2 = S(A,B,F_o)$. It follows directly that G_1 and G_2 (as given by (7)) have realizations of the required dimensions.

Conversely, suppose there exists F_o satisfying the conditions of Lemma 2. Note that S_1 and S_2 are controllable but not observable. Let η_1 (resp. η_2) be the unobservable subspace of S_2 (resp. S_1), then controllability and observability imply that η_i has dimension n_i, i = 1,2. It is easily verified that equations (10) and (11) are satisfied by this choice of η_1 and η_2. Equation (12) is proved by using the definitions to show that $\eta_1 \cap \eta_2$ is an A-invariant subspace in the kernel of C. Observability then implies that $\eta_1 \cap \eta_2 = \{0\}$ which proves (12).

Our original problem now reduces to the problem of choosing subspaces η_1 and η_2 which satisfy (10)-(12) and such that the pole and zero sets of G_1 contain specified subsets of the pole and zero sets of G. Note that the conditions of Theorem 1 take a natural form for the analysis of this problem, as

(a) The A-invariant subspace η_1 can be associated with poles $p_1, p_2, \ldots p_{n_1}$ of S(A,B,C) obtained by computing the characteristic polynomial $\rho_1(s) = (s-p_1)\ldots(s-p_{n_1})$ of the restriction $A|\eta_1$ of A to η_1.

(b) The {A,B}-invariant subspace η_2 in the kernel of C can be associated (Wonham, 1974) with invariant zeros $z_1, z_2, \ldots z_{n_2}$ of S(A,B,C) obtained by computing the characteristic polynomial $\rho_2(s) = (s-z_1)\ldots(s-z_{n_2})$ of $(A-BF_o)|\eta_2$.

These considerations enable us to characterize the pole-zero structure of G_1 and H explicitly in terms of the choice of η_1 and η_2. Let $\rho(M)$ denote the characteristic polynomial of a square matrix M. Then it is trivially verified from (10)-(13) that

$$\rho(A-BF_o) = \rho(A|\eta_1)\rho(A-BF_o|\eta_2) = \rho_1(s).\rho_2(s) \qquad \ldots(14)$$

Using this expression we can prove the following corollary to Theorem 1.

<u>Corollary</u>:

With the preceding notation, if the conditions of Theorem 1 are satisfied then $\rho_1(s)$ is the characteristic polynomial of any minimal realization of $G_1(s)$, and $\rho_2(s)$ is the zero polynomial of any minimal realization of H(s).

Proof:

Note first that a minimal realization of a controllable system is obtained by factoring out the unobservable subspace, or unobservable modes corresponding to cancelling poles and zeros.

If S(A,B,C) is controllable and observable then (c.f. proof of Thm. 1) η_2 is the unobservable subspace of $S(A-BF_o,B,C)$. Hence by (14), $\rho_1(s)$ is the characteristic polynomial of any minimal realization of $G_1(s)$.

The zero polynomial (Owens, 1978) of $S(A,B,F_o,I)$ is given by

$$Z_H(s) \triangleq \begin{vmatrix} sI_n-A & -B \\ F_o & I_m \end{vmatrix} = |sI_n-A+BF_o| = \rho(A-BF_o)$$

by application of Schur's lemma. As η_1 is the unobservable subspace of $S(A,B,F_o)$, (14) implies that the zero polynomial of any minimal realization of H(s) is just $\rho_2(s)$.

This result could also be obtained by direct transformation to the basis (η_1,η_2) for R^n and using the system state equations in the form (9).

We are now in a position to solve our original problem. The following theorem summarizes the development so far and provides necessary and sufficient conditions.

Theorem 3:

Let the m-input,m-output system S(A,B,C) be invertible, controllable and observable and have n_z zeros. Let

$P_o = \{p_1, p_2, \ldots, p_\ell\}$ be a subset of the poles of $S(A,B,C)$ and let $Z_o = \{z_{n_z-r+1}, \ldots, z_{n_z}\}$ be a subset of its zeros. Then the system transfer function matrix $G(s)$ has a factorization of the required form where P_o and Z_o are subsets of the poles and zeros of a minimal realization of $G_1(s)$ if, and only if, there exists subspaces η_1 and η_2 satisfying the conditions of theorem one and such that

(a) $(s-p_1)(s-p_2)\ldots(s-p_\ell)$ divides $\rho_1(s)$

(b) $\rho_2(s)$ divides $(s-z_1)(s-z_2)\ldots(s-z_{n_z-r})$

(Note: in particular it is necessary that $n_1 \geq \ell$ and $n_2 \leq n_z - r$)

Proof

The proof follows from the previous development noting that, if $G_1(s)$ is to have Z_o as a subset of its zeros, then the zeros of $H(s)$ are a subset of $\{z_1, z_2, \ldots, z_{n_z-r}\}$.

Given the subspaces η_1 and η_2 satisfying the conditions of theorem 3, the matrix F_o can, in principle, be computed from equation (13). The procedure for choosing η_1 and η_2 is particularly simple if $S(A,B,C)$ has distinct eigenvalues and zeros. In this case let $\{w_1, w_2, \ldots, w_n\}$ be the linearly independent eigenvectors corresponding to the poles $\{p_1, p_2, \ldots, p_n\}$ of A and let $\{v_1, v_2, \ldots, v_{n_z}\}$ be the linearly-independent zero-directions corresponding to the zeros $\{z_1, z_2, \ldots, z_{n_z}\}$. It is trivially verified that a subspace is A-invariant if, and only if, it is spanned by a finite subset of $\{w_1, w_2, \ldots, w_n\}$, and {A,B}-invariant if, and only if, it is spanned by a finite subset of $\{v_1, v_2, \ldots, v_{n_z}\}$. There are hence only a finite number of candidates for η_1 and η_2. The number of candidates is further reduced by noting that we must have

$$\hat{\eta}_1 \triangleq \text{span}\{w_1, w_2, \ldots, w_\ell\} \subset \eta_1 \qquad \ldots(15)$$

and

$$\eta_2 \subset \text{span}\{v_1, v_2, \ldots, v_{n_z - r}\} \triangleq \hat{\eta}_2 \qquad \ldots(16)$$

(If the system state-space is transformed to the basis $\{w_1, \ldots w_n\}$, and the zero directions recalculated, selection of suitable η_1 and η_2 becomes particularly simple as the eigenvectors are now just unit vectors $e_1, e_2, \ldots, e_n$).

Given suitably chosen η_1 and η_2, transformation to a basis matrix $[E_1, E_2]$ for $\eta_1 \oplus \eta_2$ yields the system $S(\hat{A}, \hat{B}, \hat{C})$ in the form of equation (9), i.e.

$$\hat{A} = \begin{pmatrix} A_1 & B_1C_2 \\ O & A_2 \end{pmatrix} \; ; \; \hat{B} = \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} \; ; \; \hat{C} = [C_1 \quad O] \qquad \ldots(17)$$

In this representation, B_1 is full rank (for $B_1x = 0$ implies that $Bx \in \eta_2 \subset \nu^*$, and invertibility then implies that $x = 0$), so that C_2 may be calculated uniquely. Hence minimal realizations $S(A_i, B_i, C_i)$ of $G_i(s)$ $(i = 1,2)$ can be derived directly from the transformed system of eqn. (17).

The following simple numerical example has been constructed to illustrate the application of the preceding theory.

3. Example of Transfer Function Matrix Factorization

Consider the invertible system S(A,B,C) defined by

$$A = \begin{bmatrix} 2 & 0 & 1 & 0 & 0 & 1 \\ 2 & 1 & 3 & 1 & -3 & 1 \\ 3 & 5 & 4 & 1 & -7 & 1 \\ -2 & -2 & -6 & -2 & 6 & 0 \\ 2 & 4 & 3 & 1 & -6 & 1 \\ -6 & 0 & -1 & 0 & 0 & -5 \end{bmatrix} \; ; \; B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 2 \\ 1 & 0 \\ 0 & 2 \\ -0.9 & 0.1 \end{bmatrix} \; ; \; C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

which is controllable and observable, with poles at $\{0,0,3,-2,-3,-4\}$, zeros at $\{-1.378,-3.387,-4.4351,1\}$, and transfer function matrix

$$G(s) = \begin{bmatrix} \dfrac{s^3+4.1s^2-2.9s-12.2}{s(s+2)(s-3)(s+4)} & \dfrac{2.1s^2+9.2s+3.7}{s(s-3)(s+3)(s+4)} \\[2ex] \dfrac{2.1s^2+8.1s-0.2}{s(s+2)(s-3)(s+4)} & \dfrac{s^3+5.1s^2-0.8s-20.3}{s(s-3)(s+3)(s+4)} \end{bmatrix}$$

We wish to factor G(s) into the form of Fig.1 such that $G_1(s)$ has $\{0,0,3\}$ as a subset of its poles, and $\{1\}$ as a subset of its zeros.

With the eigenvalues $\{0,0,3\}$ and the zeros $\{-1.378,-3.387,-4.435,1\}$ are associated the eigenvectors

$$w_1 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ -2 \\ -1 \\ 0 \end{bmatrix} , \quad w_2 = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 0 \\ -1 \\ -1 \end{bmatrix} , \quad w_3 = \begin{bmatrix} 1 \\ 1 \\ 2 \\ -2 \\ 1 \\ -1 \end{bmatrix}$$

and the zero directions

$$v_1 = \begin{bmatrix} 0 \\ 0 \\ -0.608 \\ 1 \\ -0.608 \\ -0.14 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 0 \\ 1.721 \\ 1 \\ 1.721 \\ 0.334 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 0 \\ 0 \\ 1.41 \\ 1 \\ 1.41 \\ 1.025 \end{bmatrix}, \quad v_4 = \begin{bmatrix} 0 \\ 0 \\ -3.78 \\ 1 \\ .667 \\ .113 \end{bmatrix}$$

respectively. The subspaces $\eta_1 \triangleq \mathrm{span}\{w_1, w_2, w_3\}$ and $\eta_2 \triangleq \mathrm{span}\{v_1, v_2, v_3\}$ do not intersect, and hence satisfy the conditions of Theorem 3. Defining the more convenient basis $\{v_1', v_2', v_3'\}$ for η_2 as

$$v_1' \triangleq \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad v_2' \triangleq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad v_3' \triangleq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

then transformation of the system $S(A,B,C)$ to the basis $\{w_1, w_2, w_3, v_1', v_2', v_3'\}$ yields $S(\hat{A},\hat{B},\hat{C})$ of the form (17) where

$$\hat{A} = \begin{bmatrix} 0 & 0 & 0 & -\frac{1}{3} & -\frac{1}{3} & -\frac{2}{3} \\ 0 & 0 & 0 & -\frac{1}{3} & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 3 & \frac{1}{3} & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 & -4 \end{bmatrix} \quad ; \quad \hat{B} = \begin{bmatrix} -\frac{1}{3} & -\frac{1}{3} \\ \frac{2}{3} & -\frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} \\ 1 & 0 \\ 0 & 1 \\ 0.1 & 0.1 \end{bmatrix}$$

$$\hat{C} = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ -1 & -1 & 1 & 0 & 0 & 0 \end{pmatrix}$$

The matrices A_1, B_1, C_1, A_2, B_2 are obtained by inspection, and a simple calculation yields C_2. Thus

$$A_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{pmatrix} \quad ; \quad B_1 = \begin{pmatrix} -\frac{1}{3} & -\frac{1}{3} \\ \frac{2}{3} & -\frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad ; \quad C_1 = \begin{pmatrix} 0 & 1 & 1 \\ -1 & -1 & 1 \end{pmatrix}$$

and

$$A_2 = \begin{pmatrix} -2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -4 \end{pmatrix} \quad ; \quad B_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0.1 & 0.1 \end{pmatrix} \quad ; \quad C_2 = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

It is simple to verify that $S(A_1, B_1, C_1)$ does indeed have a zero $z_1 = 1$, and the desired poles. We find that

$$G_1(s) = \frac{1}{s(s-3)} \begin{pmatrix} s-2 & 1 \\ 1 & s-2 \end{pmatrix} \quad ,$$

$$G_2(s) = \frac{1}{(s+4)} \begin{pmatrix} 0.1 & \frac{1.1s+4.3}{(s+3)} \\ \frac{1.1s+4.2}{(s+2)} & 0.1 \end{pmatrix}$$

and $\quad G_1(s)(I+G_2(s)) = G(s)$.

The desired decomposition has thus been achieved!

4. Application to Model Reduction

Again, consider an mxm invertible system $S(A,B,C)$ with transfer function matrix $G(s)$, and suppose that the decomposition $G(s) = G_1(s)(I+G_2(s))$, as described above, exists, where $G_1(s)$ contains specified subsets of the poles and zeros of $G(s)$. These subsets are assumed to be of considerable importance in characterizing the system dynamics, and we wish to retain them in a reduced model. If $G_2^*(s)$ is some reduced order model of $G_2(s)$, then

$$G^*(s) \triangleq G_1(s)(I + G_2^*(s)) \qquad \ldots(18)$$

will be a reduced order model of $G(s)$ which preserves the desired subsets of the poles and zeros of $G(s)$.

Equivalently, in state-space form, if $S(A,B,C)$ is in the form (17) (or (9)) corresponding to the transfer function matrix factorization (18), and $S(A_2^*,B_2^*,C_2^*)$ is a state-space representation of $G_2^*(s)$, then $G^*(s)$ is described by $S(A^*,B^*,C^*)$ where

$$A^* = \begin{pmatrix} A_1 & B_1C_2^* \\ 0 & A_2^* \end{pmatrix} \quad ; \quad B^* = \begin{pmatrix} B_1 \\ B_2^* \end{pmatrix} \quad ; \quad C^* = [C_1 \quad 0] \qquad \ldots(19)$$

Assuming $S(A,B,C)$ and $S(A_2^*,B_2^*,C_2^*)$ are controllable and observable, $S(A^*,B^*,C^*)$ will be a minimal realization of $G^*(s)$, provided that the choice of $G_2^*(s)$ has not produced any pole-zero cancellation between $G_1(s)$ and $(I+G_2^*(s))$.

The use of the factorization $G(s) = G_1(s)(I+G_2(s))$ as a means of preserving poles and zeros in model-reduction has the bonus that suitable choice of reduced model, $G_2^*(s)$, of $G_2(s)$ will ensure that $G^*(s)$, as described by equation (18), matches a desired number of moments of $G(s)$ both about $s = 0$ and $s = \infty$. To state this precisely (without proof, which is obvious):

<u>Proposition</u>:

If $G_2^*(s)$ is a reduced-order model of $G_2(s)$ which matches the first m_o terms of the series expansion about $s = \infty$,

$$G_2(s) = \sum_{i=1}^{\infty} M_i s^{-i}$$

and the first n_o terms of the series expansion about $s = 0$,

$$G_2(s) = \sum_{i=0}^{\infty} N_i s^{i}$$

then $G^*(s)$, as defined by equation (18), will match the first m_o+1 and n_o terms respectively of the series expansions of $G(s)$ about $s = \infty$ and $s = 0$.

The application of the above ideas can be illustrated by considering again the simple example of section (3). Here it is clear that the unstable poles 0,0,3 and the R.H.P. zero at $z = 1$ are of such importance that they must be retained explicitly in a reduced order model. We make use of the given factorization $G(s) = G_1(s)(I+G_2(s))$.

It can be seen that the residue of the pole of $G_2(s)$ at -4 is small, and, applying a well-established model reduction principle (Davison 1966, Marshall 1966), we neglect this mode to obtain

$$A_2^* = \begin{pmatrix} -2 & 0 \\ 0 & -3 \end{pmatrix} \quad ; \quad B_2^* = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad ; \quad C_2^* = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

The reduced system has transfer function matrix

$$G^*(s) = \begin{pmatrix} \frac{s^2-3}{s(s-3)(s+2)} & \frac{2s+1}{s(s-3)(s+3)} \\ \frac{2s}{s(s-3)(s+2)} & \frac{s^2+s-5}{s(s-3)(s+3)} \end{pmatrix}$$

which again has poles at 0,0,3, and a zero at 1.

5. Conclusions

This paper considers one possible approach to the problem of simultaneous retention of poles and zeros in the derivation of reduced-order models of linear multivariable systems. It is based on a series factorization of the system transfer function matrix into proper and strictly proper parts. Because of its demonstrated connection with state feedback, the factorization can be shown to exist precisely when the system state-space admits a direct sum decomposition in terms of A- and {A,B}-invariant subspaces. These invariant subspaces lend themselves naturally to the problem of ensuring that one component of the factorization retains desired pole and zero subsets of the original system; reduction of the remaining subsystem will then yield a reduced-order model with the desired properties.

Although consideration has only been given to the particular case in which $G_1(s)$ is strictly proper and $H(s)$ takes the form $I+G_2(s)$, with $G_2(s)$ strictly proper, it may be useful to examine conditions under which the system admits a more general factorization, e.g. $G_1(s), H_1(s)$ both strictly proper, or both proper but not strictly proper. This problem together with the computational problem presented by the result of this paper are the subject of future work.

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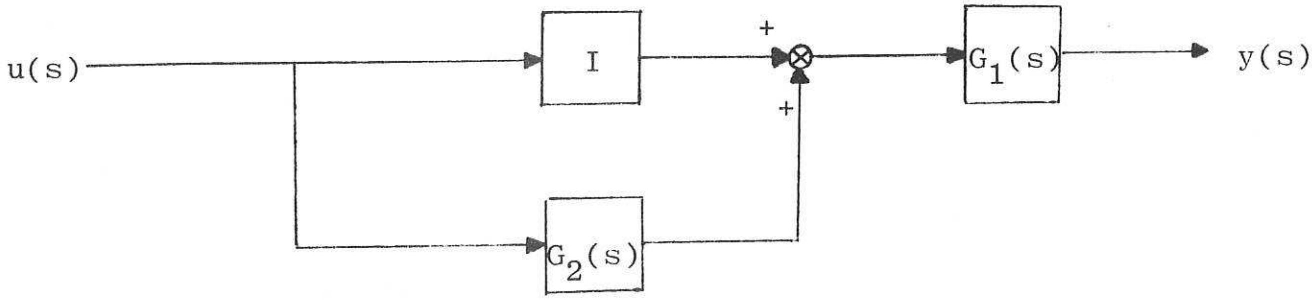

Figure 1