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A Method to Assess Demand Growth Vulnerability of Travel Times on Road Network Links

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ABSTRACT

Many national governments around the world have turned their recent focus to monitoring the actual reliability of their road networks. In parallel there have been major research efforts aimed at developing modelling approaches for predicting the potential vulnerability of such networks, and in forecasting the future impact of any mitigating actions. In practice—whether monitoring the past or planning for the future—a confounding factor may arise, namely the potential for systematic growth in demand over a period of years. As this growth occurs the networks will operate in a regime closer to capacity, in which they are more sensitive to any variation in flow or capacity. Such growth will be partially an explanation for trends observed in historic data, and it will have an impact in forecasting too, where we can interpret this as implying that the networks are vulnerable to demand growth. This fact is not reflected in current vulnerability methods which focus almost exclusively on vulnerability to loss in capacity. In the paper, a simple, moment-based method is developed to separate out this effect of demand growth on the distribution of travel times on a network link, the aim being to develop a simple, tractable, analytic method for medium-term planning applications. Thus the impact of demand growth on the mean, variance and skewness in travel times may be isolated. For given critical changes in these summary measures, we are thus able to identify what (location-specific) level of demand growth would cause these critical values to be exceeded, and this level is referred to as Demand Growth Reliability Vulnerability (DGRV). Computing the DGRV index for each link of a network also allows the planner to identify the most vulnerable locations, in terms of their ability to accommodate growth in demand. Numerical examples are used to illustrate the principles and computation of the DGRV measure.

1. INTRODUCTION

In the last decade, two complementary considerations have emerged in the analysis of road transport networks:

Monitoring reliability: Empirically, typically based on measurements of travel times, how reliable has a transport network been in the past in delivering repeatable levels-of-service to its users? Such a consideration has particularly attracted the attention of local and national government agencies, who wish to assess changes over time (days, months, years) to such level-of-service repeatability, and to gauge the impact of any past measures that have been taken.

Forecasting reliability: How reliable is a transport network likely to be in the future, taking into account its connectivity (availability of alternatives), potential failures (e.g. in capacity), and any proposed remedial measures to improve its reliability? This has largely attracted the attention of the research community in developing suitable network modelling methods, but clearly holds significant interest for transport planners who wish to select among potential measures and policies based on their potential for reliability improvements.

If our focus is relatively short-term, then in monitoring a network we may be interested to know in detail what has happened to reliability over the past few hours, days or weeks, and our forecasts may pertain to only the immediate future. In the longer-term, on the other hand, we may be interested in monitoring reliability more widely in order to 'benchmark' it against the levels seen in previous years, or to forecast how it might change over future horizons in the order of years. While the practical division between 'short-term' and 'longer term' is arbitrary, a key element that distinguishes them is that in the longer term we have the possibility of systematic, confounding changes to the underlying transport system, which may have major consequences for any comparisons made between states of reliability observed or predicted many years apart from one another. This kind of consideration serves as the context in which the present paper is written, with a focus beyond the short-term, and motivated by both the monitoring and forecasting issues described above.

Looking to the literature, it is apparent that this emerging research field of transport network reliability has provided a new focus for transport planning (e.g. Bell & Cassir, 2000). Rather than only focusing on 'typical' or 'average' performance of a transport system, in studies of network reliability we are able to focus on the impact of *variability* in the factors that affect the system, be they frequently-occurring or abnormal events. While the origins of the field drew from the study of major natural disasters such as earthquakes, the remit has broadened to encompass a range of factors affecting both the capacity of the transport system (from accidents and man-made disasters to weather conditions), the demand for travel on the system (from normal daily fluctuations in commuting patterns to special events), and the behaviour of the actors involved in the system (from individual travellers adapting their behaviour to agencies providing information, planning remedial measures or designing robust transport networks). For various recent reviews of special issues in this field, the interested reader is referred, for example, to the literature cited in Clark & Watling (2005) and Sumalee & Kurauchi (2006).

Within this field, we may identify two complimentary perspectives that are developing. The first of these perspectives begins by representing the variability in the system components, typically through some probability distribution, and aims to compute some measure of reliability that the system would attain. For example, in studying Travel Time Reliability we may assume link capacities or origin-destination demands to be stochastic, then based on some behavioural response of travellers to such stochasticity we may infer a probability distribution for the travel times on the network, and thereby obtain a measure such as the probability that the OD travel time will exceed some value (for examples, see Clark & Watling, 2005; Shao et al, 2006; Sumalee & Watling, 2008). Many variants on this approach exist, ranging from the initial work on Connectivity Reliability, where our interest is in the probability of a given OD pair remaining connected even when some links are unavailable in a degraded network (Bell and Iida 1997), to the work on Capacity Reliability by Yang et al (2000) and Chen et al (2002) whereby the probability of catering for a given level of demand is computed in the face of stochastic network capacities. In spite of the many variations on this theme, the key common element is the premise that we have sufficient information to be able to specify both (a) the component probability distributions and (b) the responses of travellers to the variable conditions.

The second perspective starts from an alternative viewpoint that, instead of trying to estimate *how unreliable* is the system under assumptions about the unreliability of its components, the focus should be on identifying potential weak-points; this is the area of Vulnerability Analysis. Such works aim to identify vulnerable elements of a network based on what could happen under pessimistic assumptions (e.g. Bell, 2000; D'Este & Taylor, 2001; Berdica, 2002; Nicholson & Dalziell, 2003; Taylor et al, 2006; Jenelius et al, 2006; Szeto et al, 2006; Jenelius, 2008). As in the first earlier theme, this grouping of papers comprises quite a diverse range of approaches, though commonly focused on the problem from a supply (capacity) point of view. For example, D'Este &

Taylor (2001) and Taylor et al (2006) analysed network-related socio-economic impacts based on reduced accessibility in a degraded network, whereas Nicholson & Dalziell (2003) considered the impacts of network degradability in the event of natural disasters such as volcanic eruptions. Berdica (2002) considered vulnerability from the viewpoint of how well the transport system performs when exposed to interruptions to critical links, general capacity reductions and variability of the demand, using equilibrium models to assess the sensitivity of networks to such potential interruptions. Bell (2000) and Szeto et al (2006), on the other hand, directly represented an evil entity that aims to degrade system performance, suggesting that we might evaluate worst-case scenarios by examining a game played about between the evil entity and the travellers in the network. The common theme in these papers is that we do not aim to evaluate how reliable is the transport system, and therefore do not need to provide as input the probability distributions of the degraded components. This is particularly attractive in cases of rare events, where it is extremely difficult as a modeller to give a reliable estimate of the input probability distribution.

In spite of the differing perspectives of reliability and vulnerability analyses, a common theme running through these works is the examination of a transport system under conditions of variability or potential failure, but in a context where the transport system essentially stays constant. That is to say, the laws governing variability or failure (and their impacts) are constant, be they probability laws, behavioural responses or the background network/demands against which potential failures are explored. In practice, however, there are likely to be underlying systematic changes to a transport system over the longer term, and these changes may not be neutral to an assessment of reliability/vulnerability. As we shall discuss later (section 2), this has particular practical relevance for organisations wishing to benchmark against previous years' performance, or to plan for robust systems in the longer term.

The focus of the present paper is from the perspective of vulnerability, yet we shall make several departures in our methodology from approaches proposed previously, especially motivated by the issue of longer-term systematic change mentioned above:

- Firstly, our focus will be on the role of *demand* in network vulnerability, whereas previous vulnerability analyses have almost exclusively focussed on link capacities.
- Secondly, we shall explore the impact of systematic growth in demand (that might occur over a period of years) on vulnerability. To our knowledge, this is not an issue that has previously been addressed in either reliability or vulnerability analyses. In essence, the thesis is that demand growth will lead our transportation system to operate in a regime where it is more vulnerable to fluctuations (in either flows or capacities). Therefore, identifying the links of a network that are most vulnerable to demand growth should allow us to direct policy interventions to design more robust networks. For example, we may choose to divert traffic away from an area that has little capacity to accommodate demand growth, even if conventional economic analysis of current conditions directs us otherwise.
- Thirdly, our assessment of vulnerability will be a stochastic one, in the sense that we shall assess vulnerability not just with respect to mean performance, but also with respect to higher order moments. In this respect, it might be said that we aim to assess Vulnerability of Reliability.
- Fourthly, our focus will be on a component-wise, link-by-link assessment of network vulnerability. An alternative approach, more in line with previous studies, would have been to integrate this link-by-link assessment with a network assessment through use of some network equilibrium model, e.g. a kind of vulnerability counterpart of the approach in Clark & Watling (2005). The reason for not adopting this approach is that we wished to devise a method that made minimal assumptions. As noted earlier, techniques such as Travel Time Reliability, Connectivity or Capacity Reliability all need to make two kinds of assumptions: not only assumptions about the probability distributions of the input components (capacities, demands) but also assumptions about how the system/travellers respond (typically assuming self-

organisation through a network equilibrium approach). Past vulnerability analyses have typically only removed the need for the first kind of assumption.

In essence the thinking behind the approach is that it will be a simple means of providing transport planners with a way to assess the vulnerability of travel time reliability on a link to growth in demand for using that link. It is anticipated that this could be the first stage in an assessment, which may subsequently lead to a network assessment of vulnerability.

The structure of the paper is as follows. In section 2, we briefly discuss the practical context of our work, introducing the measures of reliability of travel time as used in practice in various countries such as the USA, the Netherlands and England. We then develop our analytical approach in two stages. Our focus initially will apparently not be on vulnerability at all, but rather on a retrospective analysis of past, observed conditions. This is the calibration-phase of the model, where we historical data being used to estimate the parameters of the model. Thus sections 2-7 all take this retrospective, historical perspective. It is not until section 8 that we then use the same techniques in a forward-looking context; effectively the methods are the same, but are instead used to estimate the (future) vulnerability to demand growth of the network links. Preceding this, then, in section 3 a theoretical model is proposed to explain flow-related variations in travel times and the role of systematic traffic growth, alongside residual, unexplained sources of travel time variation. It is then shown, in sections 4-6, how this method may be applied to retrospectively adjust observed travel times for growth in traffic. The essential simplicity of the method, which can be lost in the algebraic manipulations required for the most general case, is illustrated by reference to a special case in section 7, where travel times follow a second order relationship in flows, and where very simple explicit relationships can be derived. Finally, section 8 moves on to the final objective of assessing vulnerability to demand growth, including a numerical illustration

2. CONTEXT: MEASURES OF TRAVEL TIME RELIABILITY IN PRACTICE

In this section, we shall briefly set out the practical motivation for our work, by considering the measures and approaches to reliability presently proposed by several governments around the world, and in particular will examine the extent to which a longer-term temporal dimension plays a part in such measures and policy objectives.

In the USA, the FHWA (2005) suggests several statistical measures to be used in the monitoring of travel time reliability, including both simple statistical summary measures such as the standard deviation, coefficient of variation, or 95th percentile, as well as measures in which a traveller is assumed to compare their experience against an expectation, such as the buffer index and planning time index, both of which consider the time cushion needed by travellers to ensure on-time arrival. In making the case for such measures, Figure 2 of this paper (the Figure entitled ‘Reliability measures capture the benefits of traffic management’) illustrates how the tracking of such reliability measures over a period of two years, before and after some network improvement, allows such reliability benefits to be estimated. However, implicit in the comparison made is that the changes are only attributable to the measure introduced; the comparison not be appropriate, for example, if there had been significant systematic change in travel demand levels over the period monitored. This is a perfect example of the issue of interest to the present paper, namely that simple monitoring of reliability may not be sufficient to isolate the effects of scheme impacts, if other confounding factors are at work at the same time.

The Rijkswaterstaat (2005) in the Netherlands interestingly start with a policy target that has an explicit time dimension with respect to reliability, namely ‘a specific ambition to have 95% of rush-hour movements arriving on time by 2020’. Given such a relatively long-term (15 year) target, there is clearly considerable potential for systematic changes in several factors to confound any monitoring exercise of journey time repeatability. The measure suggested for monitoring reliability over this period, simply the standard deviation of travel times, will clearly be sensitive to systematic changes in demand that may make the network more or less sensitive to day-to-day and within-day fluctuations in demands and capacities. While it may be that policies aimed at reducing the overall level of demand on the road network could indeed be said to fall within the remit of a measure to achieve this ambition, and so be a quite legitimate factor to capture in the monitoring, it is not clear that any counter-effect of other exogenous factors which may increase demand are also intended to be part of this assessment.

In England, the Highways Agency is responsible for monitoring and delivering an agreed level of service on the strategic road network in England comprising Motorways, A roads and trunk roads. The level of service provided is measured by a certain PSA (Public Service agreement) target (known as PSA target1) for the strategic network. This target is defined as effectively that the slowest 10% journeys in the reference year 2007/8 should not be worse off when compared to the slowest 10% journeys in the base year 2004/5 (DfT 2007). PSA target1 requires computing what is referred to as 'Average Vehicle Delay' in both the reference year and the base year. AVD is measured in minutes per 10 vehicle-miles travelled, which is equal to the sum of the *total delay* on the slowest 10% of the daytime journeys divided by the sum of the *vehicle-miles*, on each of the identified routes on the strategic network over each 15-minute departure period between 6am and 8pm for each day of the week (DfT, 2007).In order to account for the day-to-day variability in travel time, the slowest 10% journeys are identified based on a distribution of the average journey times for each route within each 15-minute window on each week day. For every route in each route direction, for a given week day within each 15-minute departure period, the *total delay* is computed as the product of the average flow and the difference between the experienced journey time in the reference year and the journey time in the base year (August 2004 – July 2005). *Vehicle-miles* is equal to the product of the average flow (for which the travel time exceeds the 90^{th} percentile travel time) and the route length in miles. Experienced journey time is the average journey time in each 15-minute departure period, whereas the journey time in the base year is based on the weekday off-peak median speed. Reference journey times are set at lower than the speed limits because on an average, vehicle speeds are less than the speed limits. Finally, average vehicle delay is obtained by dividing the total delay by the vehicle-miles. The reliability index on which PSA target1 for England is based holds special interest for the present paper, since it includes within it an explicit baseline timeframe, of the reliability measured some several years previously.

In summary, from these three examples we see diverse reasons as to why a consideration of systematic change in demand over time is relevant in a reliability context: i) identifying from a data monitoring exercise the impact of a past intervention on reliability (as in the FHWA example); ii) allowing for future demand changes as part of a longer term policy objective to improve reliability (as in the Netherlands example); and iii) as part of a more complex measure of reliability itself which incorporates an idea of a baseline level of reliability in some past year (as in the example of the Highways Agency in England). The purpose of our research, having identified this gap, is to address it by developing a practically useable methodology based on a sound mathematical footing, as we shall describe in the subsequent sections.

3. MODEL ASSUMPTIONS

In monitoring the performance of a highway facility over a course of years, it is natural to examine observations of travel times, recorded at similar times of day for similar types of day, compared between some base year and a current year. Since travel times, even when distinguished by time-of-day and day-type, are rather variable from day-to-day, then this comparison naturally takes the form of comparing elements of the *distribution* of travel times, e.g. the mean, variance and percentiles between the base- and current-year. In observing any differences in such distributions, a natural objective is to attempt to attribute them to some underlying factors, and this is the topic of the present paper. In particular, the primary aim of the paper is to develop a methodology to adjust for the confounding effect of systematic traffic growth over time, as is observed in many developed and developing countries around the world.

We consider a particular stretch of road, a 'link', where it is supposed that repeated observations of travel times are made over a number of days. These observations are grouped according to time periods within the day, and day-types, the latter to account for systematic patterns within the week, e.g. differences of Saturdays to Mondays. We shall henceforth focus on one particular time period within the day (representing, say, a period of 5-15 minutes duration), and a particular day type, and suppose that the observations for each particular day are used to form a mean for that day for that particular period of the day (intra-period mean). For example, if a particular day-type were 'Friday', and we observed all Fridays for a year between the times of 8:00-8:15, then we would obtain 52 observations (one for each Friday in the year) of the intra-period mean travel time for 8:00-8:15. When we refer to 'travel time observations' below, then, we are referring to the observations of such intra-period mean travel times over a sequence of days of the same type, for the same period of the day.

The day-to-day variability in these intra-period mean travel times is modelled by a random variable T with two components of variation:

1. Flow–related sources of day-to-day travel time variability t(F), where F is a random variable denoting day-to-day variability in flow for the period of the day being modelled, and t(.) is some deterministic (generally, non-linear) function.

2. Residual sources of travel time variability:
$$T\,|\,(F = f) = t(f) + \varepsilon \qquad (1)$$
where ε is a random variable denoting the additional day-to-day variability in travel time that arises, even given knowledge of a given day's flow level f.

Based on this basic model, several specific assumptions are now made for the purposes of subsequent analysis.

Assumption 1
The function t(.), known as the *travel time function*, is of a polynomial form:
$$t(f) = b_0 + \sum_{i=1}^{n} b_i f^{\,i} \qquad (2)$$
for some constants $b_0, b_1, ..., b_n$.

Assumption 2
The flow f denotes total flow measured in Passenger Car equivalent Units (PCUs) per hour[1]:
$$f = \sum_{k=1}^{K} \lambda_k g_k \qquad (3)$$

[1] The precise definition of flow can be varied depending on the circumstances, providing that a consistent definition is adopted throughout. Thus, for example, we could define flow as PCUs per hour *per lane* for a multi-lane highway, which may be a convenient definition when reference is made to standard speed-flow relationships in these units. However, when any transformation is made to the base units of flow, care needs to be taken to transform both the flow mean and variance required as part of (2.4).

where g_k denotes the flow of vehicle type k and λ_k denotes the passenger car equivalent of vehicle type k.

Assumption 3
Day-to-day variation in flow may be approximated by a Normal distribution:

$$F \sim \mathrm{Nor}(\mu, \sigma^2) \tag{4}$$

where μ and σ respectively denote the mean and standard deviation in traffic demand levels.

Assumption 4
ε is a zero-mean[2] random variable.

The assumptions above apply to a particular link, for a particular time period of the day; any of the parameters in the assumptions may vary by link and time period of the day, given sufficient data.

4 STATISTICAL FOUNDATION: TRAVEL TIME MOMENTS

The method that will be developed for understanding the role of traffic growth will be based on two stages. Firstly we aim to understand how the theoretical travel time distribution, arising from the assumptions in section 3, depends on various parameters of the model (sections 4 and 5). We do not specifically mention a base year and reference year, but it is through these latter parameters that we will be able to reflect changes between the observation years, e.g. traffic growth. This leads to the second stage, in section 6, where we are able to compare changes in the theoretical distribution with changes in the observed data between the reference and current year, and thereby derive a method for adjusting the current-year observations for traffic growth. In the present section, we focus on the first part of the first stage of this process, monitoring changes to the theoretical distributions.

In practice, we can specify a probability distribution in many ways, not just as is standard through the probability density function; one such specification is through its *moments*, which has an advantage of allowing a more parsimonious description of given families of distribution. For example, the Normal distribution is fully specified by its first two moments, the mean and variance, and for arbitrary distributions we can fit the parameters of the distribution to observed moments (by the 'method of moments'), provided that we have as many moments as parameters. For our subsequent analysis, we shall propose to make use of a three parameter, asymmetric distribution, namely the lognormal, for representing the distribution of travel times, and this motivates the later parts of the discussion below to focus attention on the first three moments of the travel time distribution (as the fitted distribution has three parameters). However, the methods presented can be extended to any other desired distribution with any number of parameters, by using the general formulae below to generate as many moments as there are parameters.

Now, moments of the travel time distribution *about the mean* $(\phi_m ; m = 2,3,\ldots)$ can be obtained from the binomial expansion of its definition:

$$\phi_m = E[(T - E[T])^m] = \sum_{j=0}^{m} (-1)^{m-j} \frac{m!}{j!(m-j)!} E[T^j] \left(E[T]\right)^{m-j} \qquad (m = 2,3,\ldots). \tag{5}$$

The right-hand side of (5) therefore requires knowledge of the travel time moments *about the origin*:

[2] The assumption that ε has zero mean is not restrictive, since if the additional sources of variation (unrelated to flow) have an impact on mean travel time, then this may be accommodated in the constant term b_0 of t(f).

$$\phi'_m = E[T^m] \qquad (m = 1,2,...).. \tag{6}$$

Below, it is important to keep track of these two kinds of moments, those about the mean and those about the origin, as we shall move between them several times. Note that from (6), the mean is ϕ'_1 and from (5) the variance is ϕ_2, and for higher values of m these moments describe other features of a distribution such as its skewness. Using (6), and taking the convention that $\phi'_0 = 1$, then we can write (5) as:

$$\phi_m = \sum_{j=0}^{m} (-1)^{m-j} \frac{m!}{j!(m-j)!} \phi'_j \left(\phi'_1\right)^{m-j} \qquad (m = 2,3,...). \tag{7}$$

Now, let us turn to the specification of the distribution of travel time T under the assumptions specified in section 3, comprising a component related to flow and a residual component. Using a standard statistical identity and by taking expectations of powers of $T \mid F = t(F) + \varepsilon$ with respect to ε and then F, we obtain:

$$\phi'_m = E[T^m] = E[E[T^m \mid F]] = E[E[(t(F) + \varepsilon)^m \mid F]] \ . \tag{8}$$

It should be noted that, while the notation in (8) is standard, it hides a number of subtle points, in particular the meaning of the expectation operator, where the meaning is taken to be implicit from the context. Thus in writing $E[T^m]$ the expectation is clearly the mean of travel times raised to the mth power, taken across the distribution of the random variable T. However, $E[T^m \mid F]$ might seem less obvious in meaning, since it involves two random variables; however, since we condition on F then this variation cannot be the subject of the expectation, and so the expectation must represent the mean across the conditional distribution of T given F. The result of this expectation is itself a random variable, depending on the distribution of F, and therefore the outer expectation of this term must implicitly refer to the mean across the distribution of F.

Now, with a further expansion inside the last expectation in (8) we have:

$$\phi'_m = E\left[E\left[\sum_{k=0}^{m} \frac{m!}{k!(m-k)!} (t(F))^k \varepsilon^{m-k} \middle| F \right] \right] = \sum_{k=0}^{m} \frac{m!}{k!(m-k)!} E\left[(t(F))^k E\left[\varepsilon^{m-k} \middle| F \right] \right]. \tag{9}$$

For example, let us assume that $\varepsilon \mid F \sim Nor(0, w(F))$ for some function w(.), with this latter function intended to reflect potential heteroscedasticity. Since, as mentioned in the opening paragraph, our interest will be in travel time moments up to order m = 3, then we immediately know from standard properties of the Normal distribution that:

$$E[\varepsilon \mid F] = E[\varepsilon^3 \mid F] = 0 \qquad E[\varepsilon^2 \mid F] = var(\varepsilon \mid F) + (E[\varepsilon \mid F])^2 = w(F) \tag{10}$$

leading to some considerable simplification in (9). If we follow this assumption through to compute the first three travel time moments about the mean from the combination of expressions (9) and (10) above, then firstly for the mean travel time ϕ'_1 we have:

$$\phi'_1 = E[\varepsilon \mid F] + E[t(F)] = E[t(F)]. \tag{11}$$

For the second moment about the mean (variance) ϕ_2 we first have from (7):

$$\phi_2 = (\phi_1')^2 - 2(\phi_1')^2 + \phi_2' = \phi_2' - (\phi_1')^2 \tag{12}$$

and in parallel from (9) and (10) we have:

$$\phi_2' = E[E[\varepsilon^2 \mid F]] + 2E[t(F)E[\varepsilon \mid F]] + E[(t(F))^2] = E[w(F)] + E[(t(F))^2]. \tag{13}$$

Hence, combining (12) and (13):

$$\phi_2 = E[w(F)] + E[(t(F))^2] - (\phi_1')^2 . \tag{14}$$

Finally, for the third moment about the mean ϕ_3, we first have from (7), after some simplification:

$$\phi_3 = \phi_3' - 3\phi_2'\phi_1' + 2(\phi_1')^3 . \tag{15}$$

The only term not yet computed in (15), namely ϕ_3', is then given by (from (9) and (10)):

$$\begin{aligned} \phi_3' &= E[E[\varepsilon^3 \mid F]] + 3E[t(F)E[\varepsilon^2 \mid F]] + 3E[(t(F))^2 E[\varepsilon \mid F]] + E[(t(F))^3] \\ &= 3E[t(F)w(F)] + E[(t(F))^3] \end{aligned} \tag{16}$$

and substituting (16) into (15):

$$\phi_3 = 3E[t(F)w(F)] + E[(t(F))^3] - 3\phi_2'\phi_1' + 2(\phi_1')^3 \tag{17}$$

or writing $\phi_2 + (\phi_1')^2$ in place of ϕ_2' (from (12)):

$$\begin{aligned} \phi_3 &= 3E[t(F)w(F)] + E[(t(F))^3] - 3(\phi_2 + (\phi_1')^2)\phi_1' + 2(\phi_1')^3 \\ &= 3E[t(F)w(F)] + E[(t(F))^3] - 3\phi_1'\phi_2 - (\phi_1')^3 \end{aligned} \tag{18}$$

Thus we can collect together the results above into three sequentially defined expressions:

$$\phi_1' = E[t(F)] \tag{19}$$

$$\phi_2 = E[w(F)] + E[(t(F))^2] - (\phi_1')^2 \tag{20}$$

$$\phi_3 = 3E[t(F)w(F)] + E[(t(F))^3] - 3\phi_1'\phi_2 - (\phi_1')^3 . \tag{21}$$

While w(F) reflects the possible presence of heteroscedasticity, we may also in cases accept a simplifying assumption of a homoscedastic error variance; in this special case, if we let

$$w(F) = \beta^2 \tag{22}$$

for some constant $\beta > 0$, then the moment expressions (19)-(21) reduce to:

$$\phi_1' = E[t(F)] \tag{23}$$

$$\phi_2 = \beta^2 + E[(t(F))^2] - (\phi_1')^2 \tag{24}$$

$$\phi_3 = 3\beta^2 E[t(F)] + E[(t(F))^3] - 3\phi_1'\phi_2 - (\phi_1')^3 .$$

This expression for ϕ_3 may be further simplified, by substituting for $E[t(F)]$ from (23) and substituting for ϕ_2 from (24), yielding:

$$\phi_3 = 3\beta^2\phi_1' + E[(t(F))^3] - 3\phi_1'(\beta^2 + E[(t(F))^2] - (\phi_1')^2) - (\phi_1')^3 .$$

which simplifies to a form that shows that in this case the effect of β on the third moment is eliminated:

$$\phi_3 = E[(t(F))^3] - 3\phi_1' E[(t(F))^2] + 2(\phi_1')^3 . \tag{25}$$

Now, as we have assumed t(.) to be of polynomial form, all the expectations of powers of t(F) in (23)-(25) above, when expanded, can be expressed as expectations of polynomials in F. Since:

$$E[(t(F))] = E\left[b_0 + \sum_{i=1}^{n} b_i F^i\right] = b_0 + \sum_{i=1}^{n} b_i E[F^i] \tag{26}$$

and for m = 2, 3:

$$E[(t(F))^m] = E\left[\left(b_0 + \sum_{i=1}^{n} b_i F^i\right)^m\right] = E\left[\overline{b}_{m0} + \sum_{i=1}^{mn} \overline{b}_{mi} F^i\right] = \overline{b}_{m0} + \sum_{i=1}^{mn} \overline{b}_{mi} E[F^i] \tag{27}$$

for some constants $\overline{b}_{mk}$ $(k = 0,1,2,...,mn)$ that for each m are functions of $b_0, b_1, ..., b_n$. Therefore, all required expectations may be evaluated if the *flow* moments about the origin, up to the mnth power, may be evaluated. The same is also true of the heteroscedastic expressions (19)-(21), if we assume w(.) is also of polynomial form, since the product w(F)t(F) will also be polynomial in F.

While writing the expansions of (27) to identify the constants is a little tedious, for special forms of the kind we shall subsequently examine, their form is not so onerous. Specifically, if we assume a power-law, BPR form of travel time function (for which we simplify the notation a little, by setting $b_0 = a$ and $b_n = b$, with all other $b_i = 0$ $(i = 1,2,...,n-1)$):

$$t(f) = a + bf^n \quad \text{(n a given positive integer)} \tag{28}$$

then:

$$E[t(F)] = E[a + bF^n] = a + bE[F^n] \tag{29}$$

$$E[(t(F))^2] = E[(a + bF^n)^2] = a^2 + 2ab\,E[F^n] + b^2\,E[F^{2n}] \tag{30}$$

$$E[(t(F))^3] = E[(a + bF^n)^3] = a^3 + 3a^2 b\,E[F^n] + 3ab^2\,E[F^{2n}] + b^3\,E[F^{3n}] \ . \tag{31}$$

Thus, drawing together the results of this section we have shown that in the case of a homoscedastic residual variance and a power law, BPR travel time function, then the first three travel time moments (mean, and second and third moments about the mean) are given by (23)-(25), and further that the expectations in these expressions may in turn be expressed in terms of *flow* moments about the origin through (29)-(31).

We have also shown that this method is extensible to more complex cases, with general n^{th} order polynomial travel time functions, whereby the travel time moments are given by (26) and (27), and heteroscedastic residual variances whereby (19)-(21) apply.

As mentioned at the start of this section, the reason for wishing to compute the theoretical travel time moments, and relating them to flows, is that such expressions will form the basis of the method (described in section 6) for correcting observed travel times for growth in traffic flows. Therefore, with the expressions derived so far, we are part of the way to achieving this goal. The remaining element to address is how to evaluate the require flow moments. Specifically, focusing on the homoscedastic/power-law case, how do we relate the computation of $E[F^n], E[F^{2n}]$ and $E[F^{3n}]$ required in (29)-(31) to the model assumptions made? This is addressed in section 5.

5 COMPUTATION OF FLOW MOMENTS

The result[3] of section 4 is that we can relate the travel time moments to the flow moments about the origin, $E[F^n], E[F^{2n}]$ and $E[F^{3n}]$. In this section, we consider the computation of these flow moments, which is quite straightforward given the model assumptions made, and uses several similar processes of expansion to those used in section 4.

Now, define the m^{th} order flow moment about the origin as:

$$\theta'_m = E[F^m] \quad (m = 1,2,...) . \tag{32}$$

The moments in (32) can be calculated using a series of results. Firstly, we can relate these moments to yet another kind of moment, namely the flow moments about the *mean* given by:

$$\theta_m = E[(F - E[F])^m] \quad (m = 2,3,...) . \tag{33}$$

A Binomial expansion inside the outer expectation in (33) yields:

$$(F - E[F])^m = \sum_{j=0}^{m} (-1)^{m-j} \frac{m!}{j!\,(m-j)!} F^j \ (E[F])^{m-j} \qquad (m = 2,3,...) \tag{34}$$

and then taking expectations of (34) to give (33), and using substitution (32), gives:

$$\theta_m = \sum_{j=0}^{m} (-1)^{m-j} \frac{m!}{j!\,(m-j)!} \theta'_j (\theta'_1)^{m-j} \qquad (m = 2,3,...) . \tag{35}$$

[3] We restrict attention, for ease of illustration, to the case of a power-law travel time function and homoscedastic residual variance; the principles are just the same for the case of polynomial travel time function and heteroscedastic residual variance, and still the method presented in this section applies.

where we extend the definition of (32) naturally to the case m = 0, by assuming

$$\theta'_0 = 1. \tag{36}$$

A second result we now use is that for a variable $F \sim Nor(\mu, \sigma^2)$, it can be shown that the m^{th} order moment about the mean is given by:

$$\theta_m = \begin{cases} 0 & \text{if } m \text{ odd} \\ \dfrac{m!}{2^{m/2}\,(m/2)!}\sigma^m & \text{if } m \text{ even} \end{cases} \qquad (m = 2,3,...). \tag{37}$$

Taking these results together, since we can compute all moments about the mean directly from (37), then we can relate them to the moments about the origin by (35). Examining (35), then for a given value of m, then θ_m is a linear combination of $\theta'_1, \theta'_2, ..., \theta'_m$; looked at in reverse, if we already knew the values of $\theta'_1, \theta'_2, ..., \theta'_{m-1}$ (i.e. up to the $(m-1)^{th}$ moment about the origin), then we could calculate the next moment about the origin θ'_m explicitly from re-arranging (35) and using expression (37) for θ_m. This suggests a recursive method for calculating the desired moments, whereby $\theta'_1, \theta'_2, ...$ are calculated in order. From (29)-(31) we know that the highest such moment we shall need is $\theta'_{3n} = E[F^{3n}]$, where n is the power of the travel time function (28), so that in fact we generate *all* moments up to this point, namely $\theta'_1, \theta'_2, ..., \theta'_{3n}$, and then at the end select those that we need for substitution into (29)-(31).

In particular, we re-express (35) slightly, by taking out the term corresponding to j = m:

$$\theta_m = \theta'_m + \sum_{j=0}^{m-1} (-1)^{m-j} \frac{m!}{j!\,(m-j)!} \theta'_j (\theta'_1)^{m-j} \qquad (m = 2,3,...) \tag{38}$$

and a slight re-arrangement of (38) yields:

$$\theta'_m = \theta_m - \sum_{j=0}^{m-1} (-1)^{m-j} \frac{m!}{j!\,(m-j)!} \theta'_j (\theta'_1)^{m-j} \qquad (m = 2,3,...). \tag{39}$$

The recursion suggested by (39) thus proceeds as follows:

1. *Initialisation step*. Set $\theta'_0 = 1$ (by convention) and $\theta'_1 = \mu$ (the assumed mean). Initialise iteration counter m to 1.
2. *Iteration counter*. Increment m by 1.
3. *Flow moment about mean*. Calculate the m^{th} moment about the mean θ_m from (37).
4. *Flow moment about origin.* Calculate the m^{th} moment about the origin θ'_m from (39).
5. *Next step.* If m < 3n, then return to step 2.

For small values of n, we might derive explicit expressions from this recursion; for example:

$$\theta'_2 = \theta_2 - (\theta'_1)^2 + 2(\theta'_1)^2 = \sigma^2 + (\theta'_1)^2 = \sigma^2 + \mu^2$$

$$\theta'_3 = \theta_3 + \mu^3 - 3\mu^3 + 3\theta'_2\mu = 0 - 2\mu^3 + 3\mu(\sigma^2 + \mu^2) = \mu^3 + 3\mu\sigma^2 \quad .$$

However, the real point of the recursion is to provide an implicit definition for large values of n, without having to derive explicit expressions for all moments up to order 3n. By this method we are therefore implicitly expressing all such θ'_m as functions of μ and σ. Thus, the recursion becomes a very simple numerical procedure for successively generating all required flow moments, which is applicable whatever value of n is used.

Having completed the recursion, we then select the appropriate flow moments for substitution into (29)-(31), and in turn substitute these expressions into (23)-(25). This completes the process of expressing the travel time mean, variance and third moment about the mean in terms of the assumed distribution of traffic flows. Comparing the travel time distributions derived in the base/reference year and in the current year, we will then obtain a basis for correcting the currently-observed travel times for traffic growth: this process is explained in the following section.

6 METHOD OF CORRECTING FOR TRAFFIC GROWTH

Drawing together the results derived in sections 4 and 5, in the present section we present a method that may be used to make 'corrections' to observed travel time data, to take out the effect of traffic growth that has occurred since some reference year. Effectively, by analogy to re-basing of monetary values, we re-base the current year travel time observations to be in comparable units to the base year observations, if traffic growth had not occurred.

Taking the key results of sections 4 and 5 together, we have shown how the travel time moments $(\phi'_1, \phi_2, \phi_3)$ may—through a combination of (23)-(25), (29)-(31) and the recursive method exploiting (37) and (39)—be related to the travel time function t(.), the probability distribution of flows F, and the residual travel time variability. In terms of the parameters of these latter entities, we have thus related $(\phi'_1, \phi_2, \phi_3)$ to: the parameters of the travel time function (28), namely, a and b; the parameters of the flow distribution μ and σ; and the residual variance as measured by β. Although we do not derive explicit expressions relating these quantities, the combination of relationships and recursive method together define an *implicit* relationship, and in order to reflect this implicit dependence we may write (for some functions h_1, h_2 and h_3):

$$\begin{pmatrix} \phi'_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} = \begin{pmatrix} h_1(\mu,\sigma,a,b,\beta) \\ h_2(\mu,\sigma,a,b,\beta) \\ h_3(\mu,\sigma,a,b,\beta) \end{pmatrix} . \qquad (40)$$

The implicit relationships in (40) provide the opportunity to estimate how much the travel time distribution (as measured by its moments ϕ'_1, ϕ_2, ϕ_3) may be expected to change when any of the parameters μ, σ, a, b, β change.

This approach therefore has rather wide application. For example, it would be natural to reflect capacity improvements relative to some base year through a reduction to the parameter b, and therefore (40) provides the opportunity, say, to examine how the travel time distribution would be expected to change from a base year to a current year, if growth in traffic occurs but the effect of the capacity improvement is taken out (i.e. how bad would conditions have become if the capacity improvements had not taken place). We explore this interpretation further in sections 7.2 and 7.3. There are clearly many variants on this kind of interpretation.

However, our focus in the present section will be on separating out the effect of growth in traffic levels. Specifically, we shall write the mean flow level μ relative to some flow level $\tilde{\mu}$ in the reference year:

$$\mu = k\tilde{\mu}$$

where $k \geq 1$ may be used to reflect growth in mean flow since the reference year. While the mean traffic level is our parameter of interest, we may also postulate whether (and if so, how) the other parameters in (40) might have changed between the reference year and the current year, specifically σ, a, b and β. For example, it might be argued that an increase in mean traffic levels would likely lead to an increase in the variability in traffic levels through σ. Alternatively, there may be evidence that the amount of residual travel time variation, as measured by β, has changed since the reference year. On the other hand, there may have been policy measures that have affected the free-flow travel time a (such as enforcement policies on speed limiters in heavy goods vehicles) or capacity-related parameter b. A natural first approach followed here, however, is that given the lack of strong evidence to the contrary, it is sensible to hold all other parameters except the mean traffic level constant, and aim only to separate out the effect of this change from the observed travel time variability[4]. In this way, we may use (40) to decompose each of the current year travel time moments into two components, namely the value of the moments in the reference year (when $\mu = \tilde{\mu}$ in (40)), and the amount to which they are expected to change in the current year due to growth in mean traffic flow (when $\mu = k\tilde{\mu}$ in (40)). Thus the travel time moments in the current year may be written:

$$\begin{pmatrix} \phi_1' \\ \phi_2 \\ \phi_3 \end{pmatrix} = \begin{pmatrix} h_1(k\tilde{\mu},\sigma,a,b,\beta) \\ h_2(k\tilde{\mu},\sigma,a,b,\beta) \\ h_3(k\tilde{\mu},\sigma,a,b,\beta) \end{pmatrix} = \begin{pmatrix} h_1(\tilde{\mu},\sigma,a,b,\beta) + \left(h_1(k\tilde{\mu},\sigma,a,b,\beta) - h_1(\tilde{\mu},\sigma,a,b,\beta)\right) \\ h_2(\tilde{\mu},\sigma,a,b,\beta) + \left(h_2(k\tilde{\mu},\sigma,a,b,\beta) - h_2(\tilde{\mu},\sigma,a,b,\beta)\right) \\ \underbrace{h_3(\tilde{\mu},\sigma,a,b,\beta)}_{\text{Value in reference year}} + \underbrace{\left(h_3(k\tilde{\mu},\sigma,a,b,\beta) - h_3(\tilde{\mu},\sigma,a,b,\beta)\right)}_{\text{Growth effect}} \end{pmatrix}. \quad (41)$$

Suppose now that we observe sample estimates of the moments ϕ_1', ϕ_2, ϕ_3 in the current year, and denote these estimates by $\hat{\phi}_1', \hat{\phi}_2, \hat{\phi}_3$. If we wish to correct these sample estimates for growth, then we need to subtract the growth effects in (41) above. Thus we obtain corrected estimates, with the effect of growth taken out, which we denote $\hat{\phi}_1'^{CORR}, \hat{\phi}_2^{CORR}, \hat{\phi}_3^{CORR}$ and which are given by:

$$\begin{pmatrix} \hat{\phi}_1'^{CORR} \\ \hat{\phi}_2^{CORR} \\ \hat{\phi}_3^{CORR} \end{pmatrix} = \begin{pmatrix} \hat{\phi}_1' - \left(h_1(k\tilde{\mu},\sigma,a,b,\beta) - h_1(\tilde{\mu},\sigma,a,b,\beta)\right) \\ \hat{\phi}_2 - \left(h_2(k\tilde{\mu},\sigma,a,b,\beta) - h_2(\tilde{\mu},\sigma,a,b,\beta)\right) \\ \hat{\phi}_3 - \left(h_3(k\tilde{\mu},\sigma,a,b,\beta) - h_3(\tilde{\mu},\sigma,a,b,\beta)\right) \end{pmatrix}. \quad (42)$$

To apply (42), we therefore go through the process of deriving the theoretical travel time moments twice, once for the case where the mean traffic level is $\tilde{\mu}$ and once for the case where it is $k\tilde{\mu}$, and

[4] If traffic growth really does lead to increased variance in flows, not just increased mean flow, then by assuming a constant variance we will systematically underestimate the true impact of growth on the travel time distribution, so this can be claimed to be a rather conservative assumption.

this provides the implicit evaluation of $h_1(\tilde{\mu},\sigma,a,b,\beta)$ and $h_1(k\tilde{\mu},\sigma,a,b,\beta)$, as required in (42). The extent to which these two quantities differ tell us the extent to which growth since the reference year has affected the current observed travel times, and therefore provides the basis for subtracting the impact of growth from the moments of the empirically observed distribution.

As a result, we end up with new estimates of the mean, variance and skewness that have the effect of growth taken out. However, these summary measures are not exactly what is typically required of an evaluation such as a reliability assessment, which would normally be based on *percentiles* of the travel time distribution, such as the 90^{th} percentile travel time. Therefore, the final stage of the correction process will use these corrected moments to return to the original data, and correct the individual data points in the light of growth[5].

Thus, based on the corrected moment estimates, we begin by fitting a parametric family of probability distributions to the travel time moments. While one could fit some quite flexible family of curves, such as Johnson curves (see Clark & Watling, 2005) which may need an additional fourth moment, it should be adequate for our purposes to fit something relatively easy such as a lognormal. As described in this reference we can fit a three-parameter lognormal of the form:

$$\alpha + \delta \ln(X - \xi) \sim \mathrm{Nor}(0,1) \tag{43}$$

by the method of moments. The measures that we fit to are the mean $\hat{\phi}_1'^{\mathrm{CORR}}$, the variance $\hat{\phi}_2^{\mathrm{CORR}}$, and a measure of skewness derived from the third moment:

$$\hat{\gamma}_3^{\mathrm{CORR}} = \frac{\hat{\phi}_3^{\mathrm{CORR}}}{(\hat{\phi}_2^{\mathrm{CORR}})^{3/2}} . \tag{44}$$

To apply the method of moments, we first solve for ω the equation:

$$(\omega - 1)(\omega + 2) = \hat{\gamma}_3^{\mathrm{CORR}} \tag{45}$$

and if the solution is denoted by $\hat{\omega}$, we can then estimate the parameters of the lognormal distribution (43) from:

$$\hat{\delta} = (\ln \hat{\omega})^{-1/2} \tag{46}$$

$$\hat{\alpha} = \tfrac{1}{2}\hat{\delta} \ln\left\{ \frac{\hat{\omega}(\hat{\omega}-1)}{\hat{\phi}_2^{\mathrm{CORR}}} \right\} \tag{47}$$

$$\hat{\xi} = \hat{\phi}_1^{\mathrm{CORR}} - \exp\left(\frac{1}{\hat{\delta}} \left(\frac{1}{2\hat{\delta}} - \hat{\alpha} \right) \right) . \tag{48}$$

[5] An alternative to the process described would be, after fitting the parametric distribution, to compute the desired percentile theoretically, without resort to the original data; however, we believe there to be an advantage in correcting the individual data points, since the analyst may then be in a better position to judge the reasonableness of the correction applied, and in practice it will make hardly any difference to the final computed percentile whichever method is used.

From this fitted distribution, the final step is then to revise the observed intra-period mean journey times *for each day*, through a normalisation method as follows:

1. Suppose that the *uncorrected* travel times for each of the N days are:
   $\{x_1, x_2, x_3, ..., x_N\}$.

2. For these *uncorrected* travel times, calculate sample estimates of the three moments, $\hat{\phi}'_1$, $\hat{\phi}_2$, $\hat{\phi}_3$.

3. Fit a lognormal distribution to the uncorrected travel times by the method of moments, resulting in parameter estimates of (δ, α, ξ), denoted $(\hat{\delta}_0, \hat{\alpha}_0, \hat{\xi}_0)$.

4. For the uncorrected travel times, form a sample of *normalised* uncorrected travel times $\{z_1, z_2, z_3, ..., z_N\}$, where:
$$z_i = \hat{\alpha}_0 + \hat{\delta}_0 \ln(x_i - \hat{\xi}_0) \qquad (i = 1,2,..., N). \tag{49}$$

5. Correct the sample moments for growth according to (42), yielding $\hat{\phi}_1'^{CORR}$, $\hat{\phi}_2^{CORR}$, $\hat{\phi}_3^{CORR}$.

6. Fit a lognormal distribution to the corrected moments from step 5. by the method of moments, resulting in new parameter estimates of (δ, α, ξ), denoted $(\hat{\delta}, \hat{\alpha}, \hat{\xi})$.

7. From the normalised uncorrected travel times, generate a sample of corresponding travel times corrected for growth, denoted $\{\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, ..., \tilde{x}_N\}$ and given by:
$$\tilde{x}_i = \hat{\xi} + \exp\left(\frac{z_i - \hat{\alpha}}{\hat{\delta}}\right) \qquad (i = 1,2,..., N). \tag{50}$$

Note that the 'normalisations' correspond to the transformation of the assumed lognormal variable that gives a standard Normal(0,1) distribution. The process below is probably the most useful for implementing the method, though we could of course combine steps 1, 4 and 7, by which we can see the relationship between each of the uncorrected data points x_i and the corresponding corrected data point $\tilde{x}_i$, which after simplification can be expressed as:

$$\tilde{x}_i = \hat{\xi} + (x_i - \hat{\xi}_0)^{\hat{\delta}_0/\hat{\delta}} \exp\left(\frac{\hat{\alpha}_0 - \hat{\alpha}}{\hat{\delta}}\right). \tag{51}$$

However, the process is probably more transparent as a series of steps. Once the corrected travel times have been computed, it is then straightforward to calculate, say, percentiles or any other measure from the corrected travel times, as if they were the observed data. The procedure of adjusting the fitted distribution is illustrated in Figure 1.

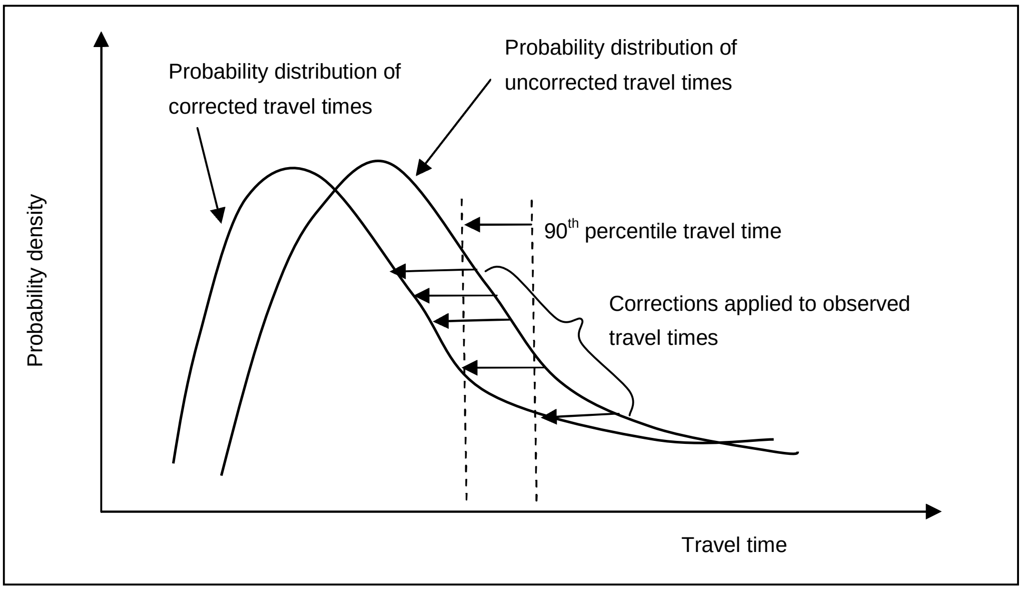


Figure 1: Adjustment of travel time distribution for growth

7 EXPLICIT FORMULAE FOR SECOND ORDER TRAVEL TIME FUNCTION

7.1 Decomposition of growth effects

The basis of the correction procedure (42) is made somewhat difficult to appreciate by the fact that, according to the process described, the functions involved in (42) are only implicit, derived from the application of a recursive process and several other steps from sections 4 and 5. If, however, we are prepared to assume a second-order travel time function, then some considerable simplification is process. While such a function may not be a good fit to real data, the purpose of this section is more as a means of explaining the principles of the more general process, by deriving explicit formulae for this special case.

Thus, in this section we assume a special case of a second order, power-law travel time function:

$$t(f) = a + bf^2 \ . \qquad (52)$$

Then simple, explicit formulae for the travel time moments of the mean ϕ_1', variance ϕ_2, and third moment about the mean ϕ_3 can be readily deduced. From (23)-(25), we thus need to compute the travel time moments about the origin (29)-(31), which for n = 2 require knowledge of $E[F^2]$, $E[F^4]$, $E[F^6]$. These flow moments can be shown to be:

$$E[F^2] = \mu^2 + \sigma^2 \qquad (53)$$

$$E[F^4] = \mu^4 + 6\mu^2\sigma^2 + 3\sigma^4 . \qquad (54)$$

$$E[F^6] = \mu^6 + 15\mu^4\sigma^2 + 45\mu^2\sigma^4 + 15\sigma^6 \ . \qquad (55)$$

Substitution into (29)-(31) yields:

$$E[t(F)] = a + b(\mu^2 + \sigma^2) \tag{56}$$

$$E[(t(F))^2] = a^2 + 2ab(\mu^2 + \sigma^2) + b^2(\mu^4 + 6\mu^2\sigma^2 + 3\sigma^4) \tag{57}$$

$$\begin{aligned} E[(t(F))^3] = a^3 + 3a^2b(\mu^2 + \sigma^2) + 3ab^2(\mu^4 + 6\mu^2\sigma^2 + 3\sigma^4) \\ + b^3(\mu^6 + 15\mu^4\sigma^2 + 45\mu^2\sigma^4 + 15\sigma^6) \end{aligned} . \tag{58}$$

and substitution into (23)-(25) gives, after simplification:

$$\phi_1' = a + b(\mu^2 + \sigma^2) \tag{59}$$

$$\phi_2 = \beta^2 + 2b^2\sigma^2(2\mu^2 + \sigma^2) \tag{60}$$

$$\phi_3 = 8b^3\sigma^4(3\mu^2 + \sigma^2) . \tag{61}$$

Expressions (59)-(61) are the explicit form of the system (40), meaning that we can combine all required expressions in sections 4 and 5 (including avoiding the need to implement the recursion given by (37)/(39)). In principle, we could anticipate all values of the power n that might occur, and repeat this process for all such values, giving explicit formulae like (59)-(61) for each such value of n; however, the algebraic manipulation required would be enormous, and the implicit definition, including the recursive definition of the flow moments, is then both algebraically and computationally attractive. Returning attention to the special case of n = 2 we consider in this section, the relatively simplicity of the expressions (59)-(61) is striking, making it possible to see many plausible, logical features. For example, we may see the way in which an increase in flow variability σ would increase all of the travel time mean, variance and third moment (and therefore the upper 90th percentile, for example).

We now examine the impact of traffic growth; as in section 6, we achieve this by assuming that $\tilde{\mu}$ denotes the mean traffic flow level in some base/reference year, and that for given $k \geq 1$ the mean traffic flow in the current year is $k\tilde{\mu}$. Since (59)-(61) define the functions in (40), we may apply (41) to give the decomposition of the moments into growth-related and growth-independent parts, which after some simplification gives:

Mean travel time: $$\phi_1' = \underbrace{a + b(\tilde{\mu}^2 + \sigma^2)}_{\text{Value in reference year}} + \underbrace{b(k^2 - 1)\tilde{\mu}^2}_{\text{Growth effect}} \tag{62}$$

Variance in travel time: $$\phi_2 = \underbrace{\beta^2 + 2b^2\sigma^2(2\tilde{\mu}^2 + \sigma^2)}_{\text{Value in reference year}} + \underbrace{4b^2\sigma^2(k^2 - 1)\tilde{\mu}^2}_{\text{Growth effect}} \tag{63}$$

Skewness numerator: $$\phi_3 = \underbrace{8b^3\sigma^4(3\tilde{\mu}^2 + \sigma^2)}_{\text{Value in reference year}} + \underbrace{24b^3\sigma^4(k^2 - 1)\tilde{\mu}^2}_{\text{Growth effect}} . \tag{64}$$

We may then apply (42) to yield the method of correcting observed estimates of the moments for traffic growth, by subtracting the growth effects, which just comes down to the simple equations:

$$\begin{pmatrix} \hat{\phi}_1'^{CORR} \\ \hat{\phi}_2^{CORR} \\ \hat{\phi}_3^{CORR} \end{pmatrix} = \begin{pmatrix} \hat{\phi}_1' - b(k^2 - 1)\tilde{\mu}^2 \\ \hat{\phi}_2 - 4b^2\sigma^2(k^2 - 1)\tilde{\mu}^2 \\ \hat{\phi}_3 - 24b^3\sigma^4(k^2 - 1)\tilde{\mu}^2 \end{pmatrix}. \tag{65}$$

Given these corrected estimates, we may then apply the remainder of the procedure described in section 6 to correct the individual observations, and thus compute percentiles from the corrected observations as required.

7.2 Decomposition of capacity interventions in the light of traffic growth

Although the focus of this note has been on ultimately subtracting the effect of traffic growth from observed travel time data, the methodology presented has wider applicability, as has been suggested on a number of occasions. To illustrate this point, we consider again the second order form (52) in order to derive explicit expressions, even though the principle is quite general. In particular, we will now aim for a different kind of decomposition. We suppose now that capacity improvements have been made since the reference year, and that this may be reflected by writing:

$$b = p\tilde{b} \tag{66}$$

where $\tilde{b}$ is the value in the reference year of the parameter multiplying f^2 in (52), and $p\tilde{b}$ is the value of this parameter in the current year, for some $p \leq 1$. That is to say, an increase in capacity is naturally associated with a decrease in the b parameter in (52).

Our aim will now be to examine the effect of traffic growth since the base year (as in section 7.1) with and without the capacity improvements; so that ultimately we can see what the travel time distribution would have looked like (in the face of traffic growth), if the improvements actually made had not been made. The general approach is to modify (41) to examine the effects with and without the capacity improvements:

$$\begin{pmatrix} \phi_1' \\ \phi_2 \\ \phi_3 \end{pmatrix} = \begin{pmatrix} h_1(k\tilde{\mu},\sigma,a,p\tilde{b},\beta) \\ h_2(k\tilde{\mu},\sigma,a,p\tilde{b},\beta) \\ h_3(k\tilde{\mu},\sigma,a,p\tilde{b},\beta) \end{pmatrix} = \begin{pmatrix} h_1(k\tilde{\mu},\sigma,a,\tilde{b},\beta) + \left(h_1(k\tilde{\mu},\sigma,a,p\tilde{b},\beta) - h_1(k\tilde{\mu},\sigma,a,\tilde{b},\beta)\right) \\ h_2(k\tilde{\mu},\sigma,a,\tilde{b},\beta) + \left(h_2(k\tilde{\mu},\sigma,a,p\tilde{b},\beta) - h_2(k\tilde{\mu},\sigma,a,\tilde{b},\beta)\right) \\ \underbrace{h_3(k\tilde{\mu},\sigma,a,\tilde{b},\beta)}_{\text{Value at ref year capacity}} + \underbrace{\left(h_3(k\tilde{\mu},\sigma,a,p\tilde{b},\beta) - h_3(k\tilde{\mu},\sigma,a,\tilde{b},\beta)\right)}_{\text{Capacity improvement effect}} \end{pmatrix}. \tag{67}$$

Note that in (67), all flows are evaluated at their current (growth) values, as we are now accepting that growth occurs but factoring out the impact on the current year travel time distribution of the capacity improvements. Following the same *rationale* as for the case of factoring out traffic growth, it follows that in order to correct any currently-observed travel time moments for the effect of capacity improvements (i.e. how bad would they have been without the capacity improvements), then we need to subtract the 'capacity improvement effects' in (67) from the observed moment estimates, to get a 'capacity-corrected' set of moments (denoted CC):

$$\begin{pmatrix} \hat{\phi}_1'^{CC} \\ \hat{\phi}_2^{CC} \\ \hat{\phi}_3^{CC} \end{pmatrix} = \begin{pmatrix} \hat{\phi}_1' - \left(h_1(k\tilde{\mu},\sigma,a,p\tilde{b},\beta) - h_1(k\tilde{\mu},\sigma,a,\tilde{b},\beta)\right) \\ \hat{\phi}_2 - \left(h_2(k\tilde{\mu},\sigma,a,p\tilde{b},\beta) - h_2(k\tilde{\mu},\sigma,a,\tilde{b},\beta)\right) \\ \hat{\phi}_3 - \left(h_3(k\tilde{\mu},\sigma,a,p\tilde{b},\beta) - h_3(k\tilde{\mu},\sigma,a,\tilde{b},\beta)\right) \end{pmatrix}. \tag{68}$$

Turning attention to the special case of second-order travel time functions (52), then we may again use expressions (59)-(61), to obtain in this case an explicit version of (67):

Mean travel time:
$$\phi_1' = \underbrace{a + \tilde{b}((k\tilde{\mu})^2 + \sigma^2)}_{\text{Value at ref. year capacity}} - \underbrace{(1-p)\tilde{b}((k\tilde{\mu})^2 + \sigma^2)}_{\text{Capacity improvement effect}} \quad (69)$$

Variance in travel time:
$$\phi_2 = \underbrace{\beta^2 + 2\tilde{b}^2\sigma^2(2(k\tilde{\mu})^2 + \sigma^2)}_{\text{Value at ref. year capacity}} - \underbrace{2(1-p^2)\tilde{b}^2\sigma^2(2(k\tilde{\mu})^2 + \sigma^2)}_{\text{Capacity improvement effect}} \quad (70)$$

Skewness numerator:
$$\phi_3 = \underbrace{8\tilde{b}^3\sigma^4(3(k\tilde{\mu})^2 + \sigma^2)}_{\text{Value at ref. year capacity}} - \underbrace{8(1-p^3)\tilde{b}^3\sigma^4(3(k\tilde{\mu})^2 + \sigma^2)}_{\text{Capacity improvement effect}}. \quad (71)$$

Therefore for a second order travel time function, the correction in (68) takes the form:

$$\begin{pmatrix} \hat{\phi}_1'^{CC} \\ \hat{\phi}_2^{CC} \\ \hat{\phi}_3^{CC} \end{pmatrix} = \begin{pmatrix} \hat{\phi}_1' + (1-p)\tilde{b}((k\tilde{\mu})^2 + \sigma^2) \\ \hat{\phi}_2 + 2(1-p^2)\tilde{b}^2\sigma^2(2(k\tilde{\mu})^2 + \sigma^2) \\ \hat{\phi}_3 + 8(1-p^3)\tilde{b}^3\sigma^4(3(k\tilde{\mu})^2 + \sigma^2) \end{pmatrix}. \quad (72)$$

Note that in this case (in contrast to section 7.1) the correction adds something positive to all moments rather than subtracts, since the correction is aiming to show how much worse things would have become in the current year, without the capacity improvements.

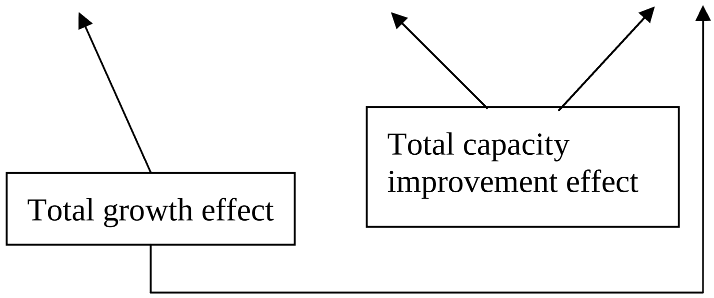
7.3 Decomposition of variance components: capacity interventions and traffic growth

While we have dealt separately with traffic growth and capacity improvements in sections 7.1 and 7.2, it is only natural to ask whether we may deal with them simultaneously in any sensible way. The answer is 'yes'. While we could aim to make a correction to the observed data to show what would have happened had growth not occurred and had the capacity improvements not occurred (and such a correction naturally follows from the decomposition below), it is probably more instructive to finish step before, by examining the decomposition of the moments in the light of these factors.

The approach for making this decomposition follows a natural extension of the ideas already presented in sections 7.1 and 7.2, and here we simply present and comment on the explicit results (See Table 1) that can be readily derived from (59)-(61) for the case of a second order travel time function:

Table 1 Impact of Capacity Improvements and Traffic Growth

Moment	No growth, no capacity improvement	Growth, but no capacity improvement	No growth, but capacity improvement	Growth and capacity improvement together
Mean	$a + b(\tilde{\mu}^2 + \sigma^2)$	$+ b(k^2 - 1)\tilde{\mu}^2$	$-(1-p)\tilde{b}\sigma^2$	$-(1-p)\tilde{b}(k\tilde{\mu})^2$
Variance	$\beta^2 + 2b^2\sigma^2(2\tilde{\mu}^2 + \sigma^2)$	$+ 4b^2\sigma^2(k^2 - 1)\tilde{\mu}^2$	$-2(1-p^2)\tilde{b}^2\sigma^4$	$-4(1-p^2)\tilde{b}^2\sigma^2(k\tilde{\mu})^2$
Skewness numerator	$8b^3\sigma^4(3\tilde{\mu}^2 + \sigma^2)$	$24b^3\sigma^4(k^2 - 1)\tilde{\mu}^2$	$-8(1-p^3)\tilde{b}^3\sigma^6$	$-24(1-p^3)\tilde{b}^3\sigma^4(k\tilde{\mu})^2$



Thus, depending on which correction terms are applied to the observed travel time moments, we may choose to correct (a) for growth only, (b) for capacity improvements only, or (c) for both growth and capacity improvements. Note, however, that these effects are not purely additive due to the last column in the table above that involves both growth and capacity improvement effects. That is to say, it is not correct to first adjust the moments for growth, and then subsequently to adjust those corrected moments for capacity improvements, since then we will effectively double-count the fourth term. Correcting for both growth and capacity improvements together is best done by returning to the original, uncorrected observed moments, and then subtracting all three of the correction terms in columns 2-4 of the table above.

8. NUMERICAL APPLICATION OF THE MODEL TO ASSESSING VULNERABILITY OF TRAVEL TIMES TO NETWORK GROWTH

This section illustrates the principles described in section 6 of this paper. As described in section 1, the numerical illustrations take a retrospective view initially to obtain the model parameters and then assess the vulnerability of road links for growth in demand in the future. In order to illustrate the method, we now need to consider specific types of roads with known speed flow relationships. In transport modelling, commonly, we use standard speed-flow relationships which can be easily converted to travel time – flow relationships, provided the lengths of road sections are known. The procedure for converting standard speed-flow curves from the NTM FORGE (DfT 2005) is described in the next section.

8.1 Fitting Flow-Delay Curves to NTM's Speed-Flow Curves

In this research we have fitted flow-delay curves to standard speed–flow relationships for UK roads which are used in various transport modelling applications, and notably the UK National Transport Model (NTM; DfT, 2005). This approach facilitates us illustrating the travel time adjustment method in application to various common types of roads found in practical planning studies (e.g., A-roads, single, dual carriageways). Table 2 shows the traffic flows and speeds used to derive speed-flow curves for motorways by the NTM. A complete set of values for different types of roads in England can be found in Table 9 of the NTM Report (DfT 2005) but in the following table, we show the value for a Motorway.

Table 2: Motorway Speed – Flow Values Used to Derive Standard NTM Relation

Speed, kmph	Flow, PCU/hour/lane
115.8	0
112.6	1398
55.6	2330
20	2913

Source: Table 9, NTM FORGE model (DfT, 2005)

The standard NTM relationship is derived by assuming a linearly decreasing speed between the corresponding values of flow indicated in Table 2. In order to convert this speed – flow relationship into a travel time – flow relationship, we need to identify the characteristics of each link, e.g. its length. The travel time relationship to calibrate is an n^{th} order power law of flow:

$$t(f) = a + b f^{n}$$

where t(f) is the travel time in seconds at a flow f , a is the free flow time in seconds, f is the flow in PCUs/hour/lane, and n is a positive integer. The constant b is calibrated from the knowledge of the capacity (in PCUs/hour/lane) and the capacity time (travel time at capacity, in seconds) by rearranging the travel time expression at a flow = capacity:

$$b = \frac{\text{Capacity Time} - a}{\text{Capacity}^{n}} \ .$$

From Table 2 and for a given stretch of a Motorway, we are able to deduce the free flow time, i.e. the travel time at no (or low) flow, and also the capacity expressed in PCUs/hour/lane and the corresponding time at that capacity. Therefore, we know all other information to fit a travel time function except the parameter n; for any given value of n, we can calculate the appropriate values of a and b from the data given. The selection of an appropriate value of n was achieved by trial and error method. Hence, the calibration is all about obtaining the values of a, b and n for any given road link. It is easy to imagine that the value of b is likely to be very small (as we see in the results below) for any given value of n, in fact, the larger the value of n, the smaller the value of b. A typical travel time – flow relationship fitted to the NTM speed – delay curve is shown in Figure 2.

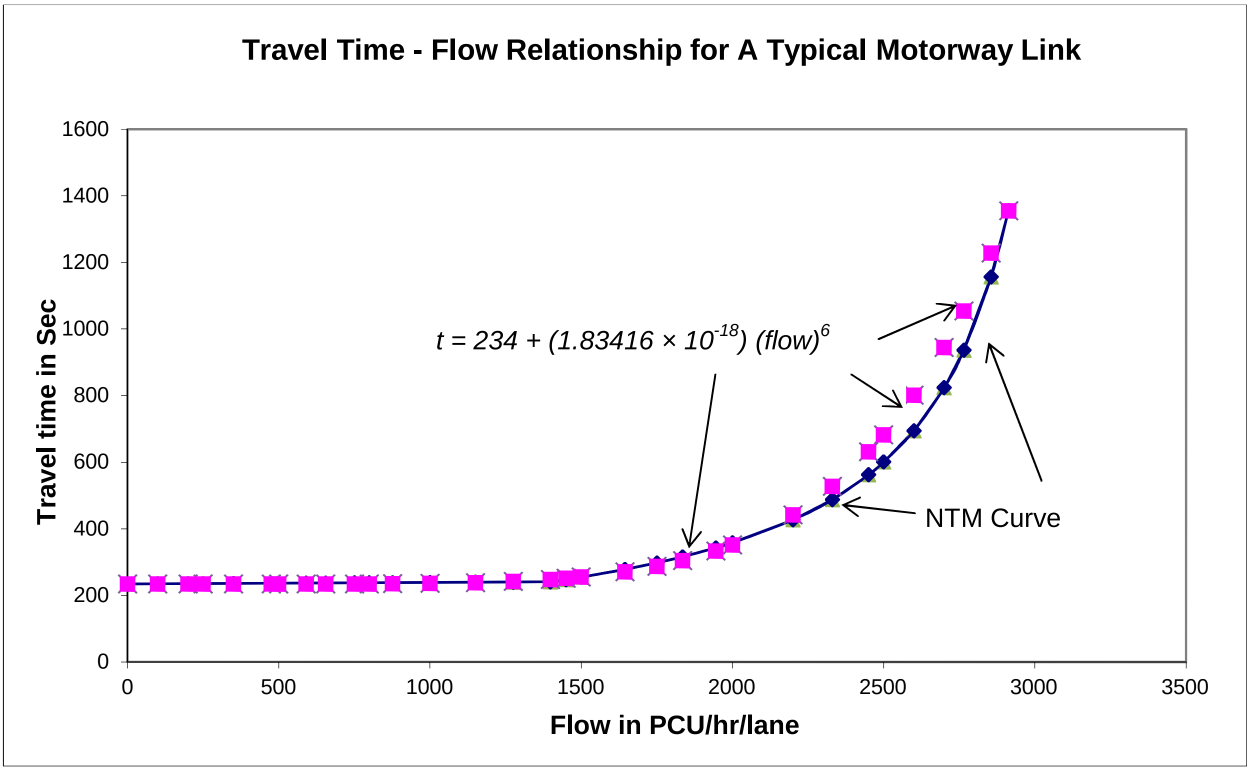


Figure 2 Fitted Travel Time-Flow Relationship

8.2 Numerical Results

In this section, we consider some typical road stretches purely for illustration purpose, however, they do not necessarily represent any existing roads. Numerical illustrations given below assume road stretches of specific types with known characteristics such as road length, summaries of flow (viz., mean flow, $\tilde{\mu}$ in pcu/hr/lane and its variance, σ^2) and travel time (viz., mean travel time $\hat{\phi}'_1$ sec, its variance, $\hat{\phi}_2$ and skewness, $\hat{\phi}_3$). Each numerical example set out below considers various scenarios of growth in traffic demand and assesses the vulnerability of a road stretch for demand growth. For the purpose of this illustration, we define the Travel Time Reliability (TTR) as a percentage change in mean, Standard Deviation (SD) and skewness, relative to the base. More formally, TTR measures can be defined as below using the notation previously introduced.

% Change in Mean Travel Time, $\Psi_1 = (\hat{\phi}_1'^{CORR} - \hat{\phi}'_1) \times 100 / \hat{\phi}'_1$

% Change in SD of Travel Time, $\Psi_2 = (\hat{\phi}_2^{CORR} - \phi_2) \times 100 / \hat{\phi}_2$

% Change in Skewness of Travel Time, $\Psi_3 = (\hat{\phi}_3^{CORR} - \hat{\phi}_3) \times 100 / \hat{\phi}_3$

Let the critical values of TTR for mean, SD and skewness be 1%, 10% and 1% respectively, which means that in the future if the TTR exceeds any of the critical values, then the road stretch is said to be vulnerable for growth in demand. Finally, let us define the term, Demand Growth Reliability Vulnerability (DGRV) as the maximum growth rate κ% pa which can result in TTR being greater than the critical values identified earlier. For the purpose of illustration, we assume a horizon period of three years, denoted by j (= 3 in all cases, unless stated otherwise) over which the vulnerability of a road stretch being assessed. In the methodology section, we have defined a parameter k to indicate the growth in demand which can be related to growth rate κ% pa by using a simple relation, $k = (1 + \kappa)^j$.

Numerical Example 1: (B-road)

Firstly, let us consider a busy B-type road[6] of length 3km (1.864 miles) connecting two A-type roads. UK Department for Transport's NTM FORGE Model (DfT 2005) specifies the speed-flow relationships for a hierarchy of roads including B-type roads which can be readily converted into a travel time - flow relationship of type (28) for the road stretch being considered. The parameters of the relationship are given below. Note that the value of b depends on the units in which the flow has been measured and hence due care should be taken to ensure their consistency with each other.

$a = 140\text{sec}$

$\tilde{b} = 9.81023 \times 10^{-5}$ (up to 5 d.p.)

$n = 2$

Let us assume that the mean flow on this road in the current year, $\tilde{\mu}$ = 1000 pcu/hr/lane with a variance, σ^2 = 8000. Let us also assume that the 'observed' mean travel time $\hat{\phi}_1'$ = 200 sec with a variance $\hat{\phi}_2$ = 12000 sec^2 and skewness $\hat{\phi}_3$ = 10. Considering various growth scenarios over the horizon period of three years, TTR measures have been worked out and summarised in Table 3. TTR measures were then compared to the corresponding critical values and vulnerability of the road stretch is readily assessed for each growth scenario. The mean travel time marginally increases with growth in demand, but the SD increases significantly over the base situation. The skewness reduces slightly with increase in demand reflecting the significant increase in SD. When the traffic grows at κ = 3% pa, Ψ_2 exceeds the critical TTR value and hence the road stretch is considered to be vulnerable to traffic growth at that level. A simple maximisation of κ has yielded the vulnerability of the road stretch, DGRV = 2.487 which means that the road is vulnerable to demand growth rates above this value.

Table 3 Vulnerability Analysis of a B-road Stretch

TTR Measure	Growth Rate of Demand κ%pa		
	1%	2%	3%
Ψ_1	0.00144	0.00283	0.00416
Ψ_2	4.52259	8.34861	11.61321
Ψ_3	-0.02664	-0.05453	-0.08372
Assessment	Not Vulnerable	Not Vulnerable	Vulnerable

Numerical Example 2: (Motorway Stretch)

A second numerical example has been constructed in much the same way as before, but this time for a high capacity motorway stretch of 44 km in length. Using DfT's NTM information, travel time – flow relationship has been derived and the parameters for this stretch of motorway are as indicated below.

$a = 170\text{sec}$

$\tilde{b} = 1.39537 \times 10^{-18}$ (up to 5 d.p.)

$n = 6$

[6] B-roads in England are usually regional in nature connecting non-primary A-type roads. Essentially, they are low capacity roads with undivided carriageways with at-grade junctions which may be signal controlled. A-type roads are relatively higher capacity roads and primary A-roads may even have dual carriageways and grade separations.

Let us assume that the rush hour flow and travel time summaries in the base year are known which are given below:

$\tilde{\mu}$ = 1800 pcu/hr/lane
σ^2 = 5000.
$\hat{\phi}'_1$ = 200 sec
$\hat{\phi}_2$ = 15000 sec^2
$\hat{\phi}_3$ = 8.5.

Note that the traffic flow is very high relative to the previous example, and the variance in travel time is also significantly high reflecting a large spread of travel times on this busy motorway stretch. A slightly lower, but positive skewness indicates that some drivers have experienced extremely long travel times on this road. TTR measures were then computed using identical procedures as before considering various growth scenarios. However, the growth rate has been kept to much lower levels compared to the previous example as the motorway stretch under consideration cannot simply accommodate that high degree of growth. Table 4 shows the assessment of vulnerability. Again, by maximising κ satisfying the condition that TTR measure is less than the critical TTR value yielded DGRV = 0.2357. This stretch of motorway can be classified as vulnerable to growth in demand above the value of DGRV.

Table 4 Vulnerability Analysis of a Motorway Stretch

TTR Measure	Growth Rate of Demand, κ% pa		
	0.1%	0.25%	0.5%
Ψ_1	0.42967	1.05915	2.06952
Ψ_2	0.01282	0.03133	0.06034
Ψ_3	-0.02134	-0.05344	-0.10719
Assessment	Not Vulnerable	Vulnerable	Vulnerable

Motorway stretch, being extremely busy, seems to operate steadily during the rush hour with very little spread of travel times. Any increase in traffic flow levels will significantly increase the mean travel time on the road stretch rather than the SD as in the previous case. In a situation like this, perhaps, a small increase in road capacity, rather than an addition of a complete lane, may help. Modern management of highway operations involves increasing the throughput by adopting some measures such as deploying traffic officers on motorways to dampen the unstable flow conditions. Such measures can be assimilated to small increases in capacity of the roads. Let us denote the %increase in capacity pa by ρ which can be related to the parameter p defined earlier following the simple relationship, $p = [(1+\rho)^j - 1]$. A 1% increase in motorway capacity, i.e. ρ = 1%, (together with assuming 0.5% increase in demand) would ease the situation by reducing the change in mean travel time Ψ_1 to 1.39647%. Corresponding changes in SD and skewness worked out as, Ψ_2 = 0.03709 and Ψ_3 = -0.10718. A further increase in capacity to 2% will make the road stretch less vulnerable to growth in demand by brining $\Psi_1$to below 1% which is the critical TTR value. In this later case with capacity expansion, DGRV works out to 0.5696 above which the road is vulnerable to growth in demand. This analysis clearly shows that the interventions made by the authorities could help beating the inflation in traffic flows, provided the interventions are implemented at a suitable scale. Otherwise, any interventions may simply seem like little or no value for money.

9. CONCLUDING REMARKS

In this paper, we have argued that growth in demand is a relevant factor to consider when assessing the vulnerability of a road network, since it will tend to mean the network will operate in a regime closer to capacity, where small variations in flow or capacity may have major impacts on travel times (and thus on the reliability of travel times). Based on this observation, a quite simple methodology has been proposed, which effectively tracks moments of the flow and travel time distribution, as a means of separating out the effect of traffic growth on link travel time moments. Such a method can be applied to historic data, to identify the contribution of past growth in demand on observed travel time moments. However, the focus of the present paper has been to utilise this technique in a forward-looking manner, whereby the planner specifies critical changes in the travel time moments, and the model is used to identify the vulnerability of links in terms of the demand growth they can accommodate before breaching these critical levels. The resulting measure, termed Demand Growth Reliability Vulnerability, may be attributed to each link of a transport network in order to identify levels of flow growth to which each link is vulnerable. An example application of this method has been given to two prototypical road links.

The method presented may be extended in a number of ways in the future. As noted earlier, the link-level analysis of vulnerability may be integrated into a network-level analysis, by assessing now the vulnerability to OD demand growth, following an adaptation of the methods in Clark & Watling (2005) – developed for the case of Travel Time Reliability – to apply to the vulnerability analysis. The method may further be developed by assuming within-day dynamic link travel time functions, for which a Taylor series approximation may be used to derive polynomial relationships between travel time and flow, utilising the calculus techniques developed in Balijepalli & Watling (2005). Finally, we may bring together the analysis of capacity and demand growth vulnerability into one combined framework, whereby both sources are simultaneously analysed.

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