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A systematic review of the use of ICTs in developing pupils' understanding of algebraic ideas

Report written by Maria Goulding and Chris Kyriacou

EPPI-Centre
Social Science Research Unit
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REPORT

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The results of this systematic review are available in four formats. See over page for details.

The results of this systematic review are available in four formats:

SUMMARY	Explains the purpose of the review and the main messages from the research evidence
REPORT	**Describes the background and the findings of the review(s) but without full technical details of the methods used**
TECHNICAL REPORT	Includes the background, main findings, and full technical details of the review
DATABASES	Access to codings describing each research study included in the review

These can be downloaded or accessed at **http://eppi.ioe.ac.uk/reel/**

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CONTENTS

List of abbreviations

ABI	Autograph-based instruction
AEI	Australian Education Index
BEI	British Education Index
BSRLM	British Society for Research into Learning Mathematics
CAS	Computer algebra systems
DfES	Department for Education and Skills
DG	Dynagraph
ERIC	Educational Resources Information Center
GC	Graphics calculator
ICTs	Information and communication technologies
ILP	Individual / integrated learning programme
ILS	Independent/Individual Learning Systems
IWB	Interactive white board
KS2	Key stage 2 (ages 7-11 years)
KS3	Key stage 3 (ages 11-14 years)
KS4	Key stage 4 (ages 14-16 years)
MAT	Mathematics achievement test
NC	National Curriculum for England
NCETM	National Centre for Excellence in Teaching Mathematics
NS for KS3	National strategy for key stage 3
OHP	Overhead projector
REEL	Research Evidence in Education Library
SBI	Spreadsheet-based instruction
TBI	Traditional-based instruction
TDA	Training and Development Agency for Schools
WoE	Weight of evidence
WWC	What Works Clearing House

Preface

The review set out to answer the following review question:

> *How have different information and communication technologies (ICTs) contributed to the development of understanding of algebra for pupils up to the age of 16?*

After keywording, the question was narrowed down to the following:

> *How have different information and communication technologies (ICTs) contributed to the development of understanding of functions for pupils up to the age of 16 (with particular reference to the relationships between different representations and the interpretation of graphical representations)?*

Who wants to know and why?

This review is set in the context of the National Strategies for primary and secondary education in England and Wales, which are both part of the drive to raise standards in schools. The use of a range of ICTs is encouraged by these strategies in the expectation that effective use of ICTs, which requires substantial funding, will raise standards. The review has been commissioned by the Training and Development Agency for Schools, which has already commissioned reviews into the use of ICTs in English and Science. This review will focus upon Mathematics and, in particular, a crucial part of the algebra curriculum, that of functions. The TDA were particularly interested, not just in whether ICTs could contribute to the development of understanding of functions, but also under what conditions that understanding developed. Others involved in policy, practice and research in mathematics education in England and Wales also need to know what the best quality international research can offer to inform teaching with ICT in this aspect of the mathematics curriculum.

Methods of the review

Identifying relevant studies involved carrying out an electronic search using keywords with bibliographic databases, handsearching conference proceedings, citations and publications recommended by contacts. This resulted in 33 studies being identified for the systematic map and 14 for the in-depth review.

Results

The studies in the in-depth review give us statistical evidence of gains in understanding as a result of interventions incorporating ICTs, evidence of the nature of these understandings, evidence of some common difficulties experienced when using graphical calculators, and detail of ways of working in the interventions.

Gains in understanding: Three studies give evidence of general gains in interventions, each using one type of ICT. One study indicates that pupils working in the computer medium performed better than those in the paper and pencil medium, although both made gains in graphical interpretation. One study evidences differences in gains according to the type of software, and, importantly, that an intervention not incorporating technology was more effective than one of the interventions incorporating use of a spreadsheet. In this case, the pupils had been taught how to use the spreadsheet but not in a mathematical context.

Nature of understanding: There is evidence of some students successfully using visualisation with graphing software to fit graphs to datasets, to solve equations and to transform functions. In terms of interpreting graphs of rates of change, there is evidence that pupils working in a computer environment reached higher levels of thinking and were able to explain their thinking better than pupils working in a paper and pencil medium. There is also some evidence of lower attaining students preferring to work arithmetically with tables of values and only later moving to integrate the tables of values with computer generated graphs. There is also some evidence of pupils having difficulty with moving between symbolic, tabular and graphical forms when solving equations. Some of these differences may be accounted for by differences in the tasks and whether the tasks were context free or contextualised.

Difficulties of working with graphics calculators: There is evidence that students do not always know how to use the technology, interpret ambiguities in the output and exercise critical judgment when using some of the facilities of advanced calculators. These studies are of relevance to our review question, because they show that the learner has to learn how to use the tool critically before it can be used effectively and also that difficulties in using the tool effectively may be exposing conceptual difficulties.

Ways of working: There is evidence that students working together in small groups and also working interactively with their teachers in whole classes provided a learning environment in which the ICTs were harnessed effectively. The individual or small group use of the technology gave pupils a valuable opportunity for inquiry and experimentation. However, unless the teacher pulled this together and orchestrated whole class plenaries, each individual student could develop their own idiosyncratic knowledge which might or might not accord with the common knowledge the teacher was intending to develop in the lesson.

There is evidence from one study that students can work with several different ICT tools and evaluate their respective advantages. There is evidence from three studies that students who use ICT out of school were better able to use it effectively within school.

Implications

(a) Teachers need to help pupils to use the technology critically so that they understand how to interpret the output and in particular how changing scales and windows can change the visual image produced by graphing software. They also need to know how the resolution of the screen image may be constraining and needs to be augmented by alternative information.

(b) Teachers need to make links between functions represented symbolically, in tables and in graphs. Symbolic representations give insights into the structure of functions but require some algebraic fluency to produce. Tables of values, whether produced manually or by technology, are an accessible way into the function idea and give an insight into the effect of inputs on outputs. They emphasise a discrete point-wise view of functions, rather than a continuous idea. Graphs produced by technology give a visual image of a function as an object which can be manipulated in its own right but they also give information about particular points on the functions which is of use in solving equations, and in investigating rates of change.

(c) Teachers need to negotiate a balance between the individual constructions which may develop when pupils work alone or in small groups with the technology, and common knowledge developed within the whole class. Although this is a consideration in any teaching situation, technology may be particularly fruitful in encouraging individual experimentation. This is desirable but needs to be tempered by teachers encouraging sharing within the whole class. The last point is also relevant when considering the use of electronic whiteboards and computers connected to data projectors. If this is completely within the control of the teacher, then pupils may not have the opportunity to experiment with the technology themselves.

In order for teachers to address these issues, they need to be supported by policy-makers and those involved in continued professional development.

CHAPTER ONE

Background

Aims and rationale for current review

The Training and Development Agency for Schools (TDA) identified a number of key areas in which systematic reviews of research literature should be carried out over a three-year period from 2003-2006. One of these is the effectiveness of information and communications technology (ICT) in teaching and learning the core curriculum subjects of English, Science and Mathematics. This review focuses on Mathematics.

Although the UK has invested heavily in ICT in schools, it is now clear that simply providing ICT equipment and promoting its use is not enough to produce more than weak gains in attainment. A key finding from one professional user review is that it is the way in which pupils and teachers use ICT that can make a difference (Higgins, 2003). Targeted research-based interventions, which are planned, structured and well integrated, do produce gains in attainment, but even these may not have as much effect as other non-ICT interventions.

In mathematics, despite a considerable literature on ways in which ICT can be used to enhance learning, Ofsted (2004, pp 4-5) reported that 'the use of ICT to promote progress in mathematics remains a relatively weak and underdeveloped aspect of provision…[and] is not as effective as in many other subjects…'. The picture is not entirely negative, however. Sutherland (2004), writing about the InterActive project across subjects and age phases, found that the mathematics teachers in the project had a legacy of ICT use which enabled them to incorporate it more smoothly into their practice and transform their teaching.

One of the ways in which some mathematics teachers have been able to develop this 'legacy of use' has been through reading articles in journals such as Micromath, which has now been amalgamated with Mathematics Teaching, the other journal of the Association of Teachers of Mathematics (ATM). This kind of reporting may be very small scale and localised, but it is accessible to teachers. One of the main aims of this review is to make the best quality evidence available and accessible to teachers, teacher educators and others involved in continued professional development.

Against this background, there were many possible areas in mathematics for the subject of the review. These included focusing on pedagogical issues, specific technologies, software and/or applications, or looking at a specific area of the mathematics curriculum. Given the importance of how teachers use ICT and the decisions involved in terms of choice of technology and software, it seemed important to find evidence of how these factors come together to contribute to teaching in a particular area of mathematics. Algebra is an appropriate focus because it is a crucial aspect for much of secondary phase mathematics, with roots in pre-algebraic activity in the primary phase. In the current version of the Key Stage 3 National Strategy: Framework for Teaching Mathematics, years 7, 8 and 9 (Department for Education and Employment, 2001), there is guidance on the use of ICT and on the teaching of algebra. This review will provide international evidence which may inform future versions of this important policy document for England and Wales.

Definitional and conceptual issues

Algebra

> *Algebraic symbolism should be introduced from the very beginning in situations in which students can appreciate how empowering symbols can be in expressing generalities and justifications of arithmetical phenomena…in tasks of this nature, manipulations are at the service of structure and meanings. (Arcavi, 1994, p 33)*

This statement highlights the fact that an emphasis on superficial aspects of algebra conceals the true essence of its power. Along with many writers, Arcavi identifies the importance of being able to express **generality** in symbolic terms. This generality may apply to relationships between a variety of mathematical objects, but most pupils will first encounter algebraic ideas in a numerical context. They may first explore ideas of pattern in numbers and express generality in words without recourse to any symbols, but later on they will be introduced to the concise and consistent **symbol system** which gives us the ability to form **expressions** (e.g. **formulae, equations, identities**), which can be used in a variety of problem-solving and reasoning contexts. The National Curriculum makes a distinction between the meaning of these words in terms of the contexts and purposes for which they are used, and the National Strategy suggests that work on relationships between **variables** expressed as formulae, equations, inequalities and identities precedes work on **functions** and **graphs**. It is helpful to think of **functions** to be the overarching concept. Indeed, French (2002, p 3) states that 'one could say that algebra is the study of functions and their application to a wide range of phenomena both within mathematics and from the 'real' world'.

The language and grammar of algebra is not studied for its own sake. It is fundamental to the process of **modelling**, where situations are represented by mathematical models in order to explain, predict, solve problems and prove results. For example, the flow of traffic in a city centre may be modelled by expressing relationships between variables as functions. In the process of trying to solve problems of congestion and keep traffic flowing, equations derived from these functions can be solved to find values of **unknowns**. These values can then be input into devices used to control traffic (e.g. timing in traffic light systems). Modelling involves not only deriving algebraic expressions, but also manipulating and operating upon them.

ICTs

Different ICTs are used to refer to both software and hardware. Within software, the following are included:

- small programs, related to specific aspects of algebra
- programming languages, such as Logo
- spreadsheets and graph-plotting software
- independent / individual learning systems (ILS)
- computer algebra systems

Within hardware, the following are included:

- interactive whiteboards (IWBs) and other projection equipment
- stand-alone computers
- graphical calculators, including those with symbolic capabilities
- Tablet PCs and other personal devices
- Data-loggers

This allows comparisons to be made between the ways in which algebraic ideas are developed using *different software* (e.g. variables using Logo and graph-plotting software). In doing this, there is a need to recognise that the 'algebras' involved in classical algebra, Logo, spreadsheets, graph-plotters and other ICT environments were different from each other, and address questions about transfer between these environments. Comparisons are also made between similar algebraic ideas being developed using *different hardware*: for example, the opportunities offered when graph-plotting is being taught, using devices with the whole class, stand-alone computers or graphical calculators. This provides an opportunity to come to judgments about the relative merits of these different ICTs.

For the purposes of this review, the focus is on learning algebra up to the age of 16, and the use of the internet or videoconferencing are not considered.

Understanding

Understanding is a complex term, but one which is often used in education. It is taken here to be more than the knowledge of definitions or procedures, involving making meaningful connections and relationships with previous knowledge. In algebra, it would involve being able to extend ideas of relationships expressed numerically, and to identify, describe and use generality, functions and graphical representations. Faced with a problem for which algebra could be used, understanding would involve knowing what to do and when to do it, as well as how to do it. The importance of having technical skills is not downplayed, as these skills could be a basis for making connections and are a necessary part of problem-solving. An important part of the Review Group's view of understanding is the ability to operate appropriately in different contexts, and to choose between alternative procedures and representations.

The Review Group does not see understanding as a once and for all state, and would expect pupils to develop more complex webs of connections and representations over time. Since the seeds of algebra may be sown in the primary phase, a lower age limit was not used for the question, but a restriction was made to algebraic ideas involving symbolism.

Policy and practice background

There is a requirement in the National Curriculum for England (NC) that ICT is incorporated into the teaching of all subjects, and teachers have been required to undertake training under the New Opportunities Fund to improve their ICT competence.

In the NC programmes of study for mathematics (1999), pupils are expected to 'use a variety of resources and materials, including ICT'. The Key Stage 3 National Strategy (Department for Education and Employment, 2001) is explicit both about introducing and developing algebra and the use of ICT. In the strategy, 'ICT includes calculators and extends to the whole range of audiovisual aids, including broadcasts and video film'. Algebra for this age phase is taken to include 'equations, formulae and identities and sequences, functions and graphs', with links made between these topics and with arithmetic. Within the supplement of examples, calculators, spreadsheets, data-loggers, graph-plotters and graphical calculators are all explicitly mentioned.

Despite this inclusion in the written mathematics curriculum, there are still concerns about the use of ICT to promote learning and progress in mathematics. There is an unquestioned assumption (Ofsted, 2004) that ICT is beneficial to learning: 'the most significant impact of ICT is when it is used to enable pupils to model, explore, analyse and refine mathematical ideas and reasoning' (Ofsted, 2004, p 4).

It is also assumed that the problem of ICT use in mathematics teaching is one of implementation. Ofsted argues that there needs to be better distribution of materials, ideas and resources; that schools need better guidance on selecting and using software; and that all schools need to write ICT activities into their schemes of work.

One of the challenges of determining the role of ICTs in the learning of mathematics is that curriculum and mathematical methods may be influenced by the tools available. This is as true for digital technologies today as it was, for instance, when the Greeks used compasses and straight edges in geometry. So, ideas about functions and variables may have subtly different meanings and manifestations within and without an ICT environment. Trying to judge the effectiveness of the ICTs in developing understanding in algebra will have to take this into consideration.

There is also the question of how teachers incorporate ICTs into their existing practices and if they then transform those practices in response to new ways of seeing and doing mathematics. Sutherland (2004) describes how communities of practice - social networks arising out of a desire to teach differently using ICT and to share knowledge, expertise and experience - were created in the InterActive Education Project. The curricular and working context of the studies in this review were examined and included within the synthesis. It is not possible to look at the effectiveness of ICTs without examining the conditions in which they are used.

Research background

Understanding of algebra

For many pupils, the deeper meanings and purposes for algebra are hidden and they see it as a meaningless activity in which they have to memorise rules and methods for manipulating symbolic expressions (Kieran, 1994). Moreover, although algebra has its roots in arithmetic, pupils often find the transition from the one to the other problematic (Nickson, 2004) as it involves using structural, rather than procedural, features of arithmetic. For this reason, recent work with elementary children in the United States has focused on generalised arithmetic, and has enabled children to progress to the use of algebraic symbolism (Carpenter et al., 2003).

A range of barriers to progress in algebra has been found in the secondary phase (see French, 2002 for a useful summary), including the following:

- Pupils interpreting letters as objects (e.g. a for apples) rather than as unknowns with a specific value or values (e.g. $x + 3 = 10$) or variables which can vary across a range of values (e.g. the x and y variables in the function $y = x - 1$)

- Pupils interpreting expressions simply as processes rather than both processes and objects. For example, pupils may only see $y = x - 1$ as a rule used to draw a straight-line graph, but not also as an object which can be transformed in its own right (e.g. by manipulating constants to produce a set of parallel lines without recalculating values for x and y).

- The isolated practice of skills and routines, which tend to be forgotten

- The lack of meaningful, but not necessarily ‘real life’ contexts

- The lack of connections between ideas and representations (e.g. between a table of values for a function, its symbolic representation and its graph)

Working with a small group of teachers as part of a larger Teacher Training Agency (TTA) project, Brown (2005, Developing algebraic activity in a ‘community of inquirers’) helped to develop classroom cultures in which year 7 pupils had a personal need to use algebra. Looking for distinctions – that is, exploring what was the same and what was different in situations - enabled pupils to find structural or algebraic representations useful to them. Teachers also found it helpful for pupils to use writing, both when doing mathematics and also when reflecting on what they had learned. This project also highlighted the advantages of teachers, researchers and teacher researchers working collaboratively.

ICTs and algebra

Much of the background for this section draws on a set of research bibliographies from Micromath (Jones, 2004, 2005a, 2005b, 2005c).

Arguments for the potential of ICT to enhance the teaching of algebra abound, but evidence for its effectiveness is more mixed.

Capponi and Balacheff (1989) found that there was no easy transfer of algebraic knowledge into the spreadsheet context, while Ainley (1996) found evidence that children's understanding of variables was assisted by the use of spreadsheets. More recently, Ainley et al. (2004) operated a spreadsheet-based teaching programme, using the technology as a tool within purposeful tasks. Pupils had a need to use algebraic symbolism and the affordances of the technology stimulated some, but not all pupils, to engage with expressing generality symbolically.

There is some evidence from a case study of Logo use (Harries and Sutherland, 1995) that the computer environment allowed a greater emphasis on the language and structure of algebra, although some difficulties with equivalence and variables were still found.

Much of the research on graphing - which can be done using interactive whiteboards, stand-alone computers or graphics calculators - has tended to focus on graphics calculators. There is evidence from independent experimental studies (e.g. Graham and Thomas, 2000) that 13-14 year-old students using graphics calculators improved their understanding of variables, and that regular users (Ruthven, 1990) employed graphical strategies to solve problems. A review of research published by Texas Instruments (e.g. Burrill, 2002) concluded that the use of graphics calculators helped students improve their understanding of algebra concepts, and encouraged problem-solving in applied contexts and the interpretation of graphs. A subsequent review (Interactive Educational Systems Design, 2003), drawn from the same database but focusing only on those studies with an experimental or quasi-experimental design, found that graphing calculator use led to higher achievement.
As well as potential benefits, there is some evidence of difficulties with graphical calculator use. For instance, Wilson and Krapfl (1994) identified problems with scaling, and Mitchelmore and Cavanagh (2000) found that uncritical acceptance of the graphical image on the calculator led students into error.

Interactive whiteboards (IWBs) are a relatively recent introduction to mathematics classrooms in the UK and the research tends to focus on general pedagogical issues, such as pupil participation. Glover and Miller (2001) found that they can be effective, depending on the quality of the teaching, but that the novelty effect of IWBs could wear off. Another study (Godwin and Sutherland, 2004) found the potential for increased understanding of functions and graphs with IWBs within inquiry-based teaching, but also point out the potential of ordinary whiteboards for encouraging interactivity.

Software, such as DERIVE and MAPLE, allows for the symbolic manipulation of algebraic functions and so present similar issues for advanced mathematics as do calculators for arithmetic. This software typically operates on computers (hence the generic name computer algebra systems (CASs)), but more recently, complex calculators, with symbolic as well as numeric and graphical capabilities, have also been introduced. In France, there has been a considerable body of research into teaching and learning mathematics with these tools. Much of this has focused on the complexity of instrumentation, learning to use the new technology so that it becomes a tool for use (Lagrange, 1999). Using CAS places technical and conceptual demands on students as they require mastery of the formal ways of interacting with the software, and the ability to interpret the results of operations (Artigue, 2002). This requires time and carefully designed activities. Ruthven and Hennessy (2002) point out the difficulty of fully realising the potential of CAS if they are not given status within secondary school mathematics.

ICT use in context

The quality of teaching has already been mentioned in the context of IWBs. In the spreadsheet context, Rojano (1996) found evidence that judicious use of spreadsheets led to algebraic understanding. A review of graphic calculator use (Penglase and Arnold, 1996) warns that many research studies do not clarify the relationship between the use of the graphic calculator and the context in which it is being used. Rodd and Monaghan (2002) found a range of teacher factors in determining graphical calculator use, including their positive regard for calculators as a learning aid and their perceptions that computers were a higher resource priority. Teachers clearly mediate the use of ICT in their classrooms and have views on the features of successful ICT use, together with concerns and qualifications (Ruthven and Hennessy, 2002). These views and constraints will clearly affect how teachers integrate ICT into their teaching. As well as the salience of the nature of tasks and the role, knowledge and beliefs of the teacher, Doerr and Zangor (2000) found that student communication was sometimes inhibited by the use of the graphics calculator as a personal device, but that, when shared, whole class learning was supported.

The review therefore aims to clarify the conditions under which ICTs can be used to develop understanding of algebraic ideas. From the above, it is clear that the teacher's role is crucial. Some evidence was found about the ways in which the teachers worked with the technology, the tasks they used, the pedagogical practices they adopted, and, in some cases, how they used different technologies in complementary ways. This last issue is important in the context of England and Wales as electronic whiteboards become much more common.

Authors, funders and other users of the review

The Review Group consists of key groups involved in mathematics education from universities, schools and local education authorities. All have a professional interest in both the substance of the review and the methodological approach to systematic reviewing. For this review, the existing EPPI-Centre Review Group for Mathematics was enlarged to include members with particular expertise in ICTs and Mathematics. This review was led by Maria Goulding, who has worked as co-investigator with Chris Kyriacou on two previous EPPI-Centre reviews. They have both have published substantive and methodologically focused papers in academic journals based on previous reviews, and both are involved in the professional preparation of secondary mathematics teachers, for whom the outcomes of this review are particularly important.

The project has been funded by the Training and Development Agency for Schools (TTA), which is concerned with bringing reviews of research literature to bear on the training and continued professional development of teachers. It is hoped that the results of this review will inform beginning and continuing teachers about the impact of ICT on a crucial aspect of the mathematics curriculum. The review not only identified studies in which the use of ICT was shown to be effective in the teaching of algebra, but also the conditions under which this effectiveness occurred.

As well as those involved in mathematics curriculum research and policy-making, the principal audiences for the review are likely to be teacher educators, researchers and policy-makers involved in the initial and continuing preparation of mathematics teachers. The recent setting up of the National Centre for Excellence in Teaching Mathematics (NCETM), and the appointment of regional advisers provides a forum and mechanism for dissemination, as well as the existing academic and professional networks and conferences.

As with previous mathematics reviews, dissemination will take place through internet access to the review report, conference papers and publication in refereed journals. Conference presentation planned for 2007 are at a one-day conference for the British Society for Learning Mathematics and the annual conference of the British Educational Research Association.

CHAPTER TWO

Methods of the review

Systematic review methods were followed, using the EPPI-Centre guidelines and tools for conducting systematic reviews. Detailed explanations of the methods are included in the technical report

Initial discussions for this review were held with the TDA, the English and Science Review Teams at York, the Mathematics Review Group, and other teachers in schools who were not in the Review Group. The Mathematics Review Group – which includes teachers, teacher trainers, educational researchers and a local education authority adviser – met and discussed several possible foci before deciding on the question of the review. The experiences of trainee teachers in schools were taken into account, following discussions after observed lessons.

These groups represent the main users of the review and were consulted at later stages. The Mathematics Review Group met twice during the progress of the review, once to decide on the research question and once during the key-wording process. The British Society for Research into the Teaching and Learning of Mathematics (BSRLM) was also consulted and emergent findings will be presented at a one-day conference, as for previous Mathematics Education EPPI reviews. When the final review has been approved by peer referees, it will go out to three users to provide user perspectives which can be published on REEL, alongside the final review (as for the first EPPI Mathematics Education Review).

For a paper to be included in the systematic map, it had to report a study on the effectiveness of different ICTs on the development of understanding in algebra for pupils up to the age of 16. As the focus of the study is on the effects of ICT, papers using methods to identify such effects are required. Thus the focus is on evaluations, either naturally occurring or researcher-manipulated.

The review is limited to the period between 1996 and 2006. This is quite a generous timeframe, given rapid developments in the field.

Inclusion criteria

- Must be an empirical study of the effects of ICTs, as defined for this review, in mathematics teaching
- Must be a study of the effects of using different ICTs, as defined for this review, on understanding in algebra, as defined for this review
- Must focus on students up to the age of 16
- Must be in a mainstream school setting
- Must be an evaluation study
- Must be in English and published in a professional or academic journal, or presented at an academic conference between 1996 and 2006

Papers were identified from the following sources:

- Searching the electronic bibliographic databases: Educational Resources Information Center (ERIC), British Educational Index (BEI), Australian Education Index (AEI)

- Handsearching proceedings of recent conferences and handbooks of the British Society for Research into Learning Mathematics (BSRLM), the International Group for the Psychology of Mathematics Education (PME), the International Conference on Technology in Mathematics Teaching (ICTMT). This identified studies which were too recent to have been published in academic journals.

- Handsearching key academic and professional journals

  Educational Studies in Mathematics

  International Journal for Technology in Mathematics Education (formerly International Journal for Computer Algebra in Mathematics Education)

  International Journal of Computers for Mathematics Learning

  Journal of Computer Assisted Learning

  Journal of Mathematical Behaviour

  Journal for Research in Mathematics Education

  For the Learning of Mathematics

  Mathematics Teaching

  Mathematics in Schools

  Micromath

After mapping, all the included studies, they were categorised as focusing on the following:

1. the development of algebraic symbolism

2. multi-representations of functions

3. graphical representations of functions

4. operations on symbolic expressions

Most studies could be placed into one or more of these categories, but, in two studies, it was not clear what aspect of algebra was being addressed by the ICTs. The research question and the inclusion / exclusion criteria were narrowed and refined for the in-depth review.

The narrowed research question was as follows:

How have different information and communication technologies (ICTs) contributed to the development of understanding of functions for pupils up to the age of 16 (with particular reference to the relationships between different representations and the interpretation of graphical representations)?

CHAPTER THREE

What research was found?

The 33 studies in the systematic map fell into four categories: the development of symbolism; the relationship between different ways of representing functions; the interpretation of graphical representations of functions; and operations on symbolic expressions.

The development of symbolism (10 studies)

Clark and Redden, 2000; Drijvers 2004; Gage, 2002; Graham and Thomas, 2000; Healy and Hoyles, 1996; Hegedus and Kaput, 2003; Hershkowitz and Kieran, 2001; Tynan and Asp, 1998; Wilson and Ainley, 2006; and Yerushalmy, 2000.

The relationship between different ways of representing functions (8 studies)

Doerr and Zangor, 2000; Friedlander and Stein, 2001; Gomes-Ferreira, 1998; Godwin and Beswetherick, 2002; Gray and Thomas, 2001; Mitchemore and Cavanagh, 2000; Ninness, Rumph, McCuller, Harrison, Ford, and Ninness, 2005; and Yerushalmy, 2000.

The interpretation of graphical representations of functions (14 studies)

Borba and Confrey, 1996; Doerr and Zangor, 2000; Friedlander and Stein, 2001; Gomes-Ferreira, 1998; Godwin and Beswetheric, 2002; Godwin and Sutherland, 2004; Gray and Thomas, 2001; Hegedus and Kaput, 2003; Hershkowitz and Kieran, 2001; Isiksal and Askar, 2005; Mitchelmore and Cavanagh, 2000; Ninness et al., 2005; Sivasubramaniam, 2000; and Yerushalmy, 2000.

Operations on symbolic expressions (18 studies)

Aczel, 1998; Bills et al., 2005; Cedillo, 2001; Doerr and Zangor, 2000; Drijvers, 2004; Drijvers and van Herwaarden, 2001; Friedlander and Stein, 2001; Gray and Thomas, 2001; Hershkowitz and Kieran, 2001; Isiksal and Askar, 2005; Kramarski and Hirsch, 2003; Merriweather and Tharp, 1999; Norton and Cooper, 2001; Norton et al., 2002; Strickland and Al-Jumeily, 1999; Tynan and Asp, 1998; Yerushalmy, 2000; and Zehavi, 1997.

Two studies did not report what the ICT tool does and did not give any details of activities undertaken by students: Carter and Smith (2001), and Morgan and Ritter (2002).

CHAPTER FOUR

What were the findings of the studies?

The in-depth review addressed the narrower research question. To be included in the in-depth review, the studies had to make explicit which aspect of algebra was being addressed with the ICTs and this had to have to focus on the different ways of representing functions, including the interpretation of graphical representations. The following 14 studies comprise the in-depth review:

Borba and Confrey (1996)

Doerr and Zangor (2000)

Freidlander and Stein (2001)

Godwin and Beswetherick (2002)

Godwin and Sutherland (2004)

Gomes and Ferreira (1998)

Gray and Thomas (2001)

Hegedus and Kaput (2003)

Hershkowitz and Keiran (2001)

Isiksal and Askar (2005)

Mitchelmore and Cavanagh (2000)

Ninness et al. (2005)

Sivasubramaniam (2000)

Yerushalmy (2000)

Summary of results of the synthesis

Gains in understanding: Three studies give evidence of general gains in interventions each using one type of ICT (Godwin and Sutherland, 2004; Gray and Thomas, 2001; Hegedus and Kaput, 2003). One study indicates that pupils working in the computer medium performed better than those in the paper and pencil medium, although both made gains in graphical interpretation (Sivasubramaniam, 2000). One study evidences differences in gains according to the type of software, and importantly that an intervention not incorporating technology was more effective than the intervention using a spreadsheet (Isiksal and Askar, 2005). In this case, the pupils had been taught how to use the spreadsheet but not in a mathematical context. This points to the importance of the design of the particular software and the way in which it is introduced to the pupils.

Nature of understanding: There is evidence of some students successfully using visualisation with graphing software to fit graphs to datasets, to solve equations and to transform functions (Borba and Confrey, 1996; Doerr and Zangor, 2000; Friedlander and Stein, 2001; Godwin and Beswetherick, 2002; Godwin and Sutherland, 2004). In terms of interpreting graphs of rates of change, there is evidence that pupils working in a computer environment reached higher levels of thinking and were able to explain their thinking better than pupils working in a paper and pencil medium (Sivasubramaniam, 2000). There is also some evidence of lower attaining students preferring to work arithmetically with tables of values and only later moving to integrate the tables of values with computer generated graphs (Yerushalmy, 2000). There is also some evidence of pupils having difficulty with moving between symbolic, tabular and graphical forms when solving equations (Gray and Thomas, 2001). Some of these differences may be accounted for by differences in the tasks and whether the tasks were context free or contextualised.

Difficulties of working with graphics calculators: There is evidence that students do not always know how to use the technology, interpret ambiguities in the output and exercise critical judgment when using some of the facilities of advanced calculators (Hershkowitz and Kieran, 2001; Mitchelmore and Cavanagh 2000). These studies are of relevance to the review question, because they show that the learner has to learn how to use the tool critically before it can be used effectively and also that difficulties in using the tool effectively may be exposing conceptual difficulties.

Ways of working: There is evidence that students working together in small groups, and also working interactively with their teachers in whole classes, provided a learning environment in which the ICTs were harnessed effectively (Doeer and Zangor, 2000; Godwin and Beswetherick, 2002; Godwin and Sutherland, 2004). The individual or small group use of the technology gave pupils a valuable opportunity for inquiry and experimentation (Gray and Thomas, 2001; Hershkowitz and Kieran, 2001; Yerushalmy, 2000). However, unless the teacher pulled this together and orchestrated whole class plenaries, each individual student could develop their own idiosyncratic knowledge which may or may not accord with the common knowledge the teacher was intending to develop in the lesson. In one study, the connectivity of the computers allowed the teacher to demonstrate the work of individual pupils and build up collective knowledge in this way (Hegedus and Kaput, 2003). In the study in which one student worked with a researcher, the ability to listen carefully to the student was seen to be crucial (Borba and Confrey, 1996).

There is evidence from one study (Freidlander and Stein, 2001) that students can work with several different ICT tools and evaluate their respective advantages. There is evidence from three studies that students who use ICT out of school are better able to use it effectively within school (Godwin and Beswetherick, 2002; Gray and Thomas, 2001; Mitchelmore and Cavanagh, 2000).

CHAPTER FIVE

Implications, or ‘What does this mean?’

One strength of this review is the publicly visible nature of the review procedure, and the collaboration of the Review Group, the EPPI-Centre and many other individuals who offered help and advice. Another strength is the way in which it has focused on a specific area of the mathematics curriculum and so can give very precise details about the ways in which ICTs can develop understanding of functions.

The main limitations of the review are that the constraints involved in terms of time and cost inevitably mean that decisions about the focus of the review question and the review process have to be made to keep the review manageable. This meant that the Review Group did an in-depth study on just two of the areas identified in the systematic map:

- the relationship between different ways of representing functions
- the interpretation of graphical representations of functions

The following two other areas have not been subject to in-depth analysis:

- the development of algebraic symbolism
- operations on symbolic expressions

Another limitation of any review of this type is that the individual studies did not set out to answer the review question. They all have different designs and instruments. This is particularly relevant in terms of the tasks used to assess understanding where small differences may make a noticeable difference to the students’ responses. Although all the studies in the in depth review were considered to be evaluations, not all used control groups and not all compared different kinds of software and hardware. So there is evidence of gains, but it is not always known if those gains could have been achieved without the use of ICT. Another limitation is the amount of evidence of the nature and quality of the teacher input. Most studies in this review concentrated on pupils and did not give detailed evidence of how the teachers supported their pupils in developing knowledge of the functional concept and knowledge of how to use the ICT tools. Any conclusions must therefore remain tentative.

The What Works Clearing (WWC) House reviews were not screened. Subsequently, the middle school curriculum review has been found to contain titles which may report potentially relevant interventions. Interpretation and application of the results of this review requires further work by different users of research, but initial implications are outlined below.

Implications for policy and practice

The findings of this review offered some support for the use of ICTs in the teaching and learning of functions, an important part of learning algebra. This confirms the stance on the use of ICTs in the current National Curriculum Mathematics and Key Stage 3 National Strategy Framework for teaching Mathematics: years 7, 8 and 9, which will presumably be continued in future policy guidance. Some of the detail in the supplement of examples in the last document is very helpful in including the output of graphics calculators and spreadsheets to illustrate the following:

- how they can be used to generate sequences
- drawing out the meanings involved in interpreting the graphical output of functions
- making links between the graph and the coordinate pairs on the graphics calculator display as the trace function is used

There could be more on the following:

- the links between tables of values, symbolic representation and graphical representation, which could provide the bridge between functions and the solution of equations (apparently not included in the section on graphs of functions)
- critical use of graph-plotters, including how changing the scale can alter the appearance of the graph, how to use the zoom function, how to change windows, and how to interpret pixel displays

The National Strategy presently advocates the use of a three-part lesson, incorporating interactive whole class teaching. The research here shows that this structure could provide the framework for a mix of individual / group work and whole class plenaries to allow the experimentation, direction and sharing which seems to maximise the potential of the ICTs. Time spent on constructing meanings in this way would seem to be particularly important in algebra, given the problems already outlined in the background.

Policy-makers have an important role in giving direction on the judicious use of different tools. Graphics calculators and computers can be used by pupils in individual or group activity; interactive whiteboards or computers with projectors can be used for whole class work. The evidence on ways of working in this review suggests that both have a place. However, there is also evidence of a teacher using a non-digital whiteboard with an overhead projector (OHP) to draw together effectively aspects of the pupils' work with graphics calculators. Digital and non-digital technologies can be used together to enhance learning. With increased use of the interactive whiteboard, it will be important to ensure that pupils still have the opportunities for autonomy and experimentation afforded by graphics calculators or class computers, and that personal constructions are shared with the whole class.

This review supports the use of ICTs in developing understanding of functions but the teacher has a pivotal role in structuring and supporting the learning, so any recommendations have to take account of the teacher's role in mediating the learning, and the teaching and learning context. Simply using ICT will not guarantee that students make more learning gains than using traditional paper and pencil methods.

Teachers need to be confident users of the technology themselves, although relatively straightforward starting points can stimulate rich activity. The teacher needs to be aware of how the scale, window and resolution may present misleading images. One way of overcoming these difficulties is to smooth the path for students by setting the scale and window for them. Another way is to use cognitive conflict, to present students with a puzzling image (e.g. part of a parabola which looks like a straight line because of the choice of scale, two lines which do not cross within the set window) and encourage them to work through their misconceptions. Students need to be alert to these potential sources of confusion and given good access to the technology so that they can develop familiarity.

An effective method for studying families of functions and exploring transformations is to start with a prototype function expressed symbolically, generate similar examples and also non-examples, relate the symbolic expression to the graph, and find a way of describing the family or transformation, using mathematical language. Giving students some room to experiment with more open questions in this process can be productive.

Teachers need to help students make links between symbolic, tabular and graphical output, by making these links explicit. A common approach to graphing functions is to start with a symbolic expression, make a table of values and plot these by hand. This can give students a point-wise view of a function, a process to be done rather than an object in its own right. Graphical software, on the other hand, takes the plotting away from the learner and presents the graph as an object which can be explored. This is very important when investigating families and transformations, and checking whether functions are equivalent; however, when solving equations, a point-wise view is also important, as the coordinates of specific points on the graphs will give solutions. It is essential then that these links are made explicit and reinforced when working within any one of the representations. The review indicates that a full understanding of the links between different representations may take time and may be facilitated by regular access to the technology.

One message that comes out of the review for teachers is to encourage meaningful activity by moving between representations, discussing their methods, and explaining their thinking and interpretations.

It is not possible to conclude from this review what degree of emphasis teachers should place on the use of ICT in lessons.

Interpreting the curriculum in all its detail and developing the pedagogical practices clearly has implications for those involved in continued professional development policy.

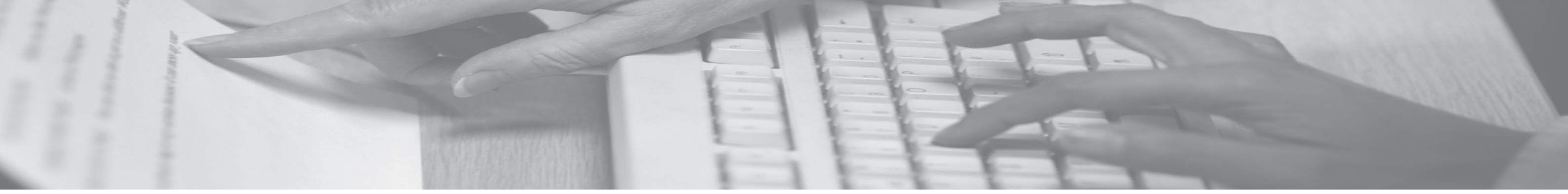
Implications for research

This review can contribute in two main ways to the research community: first in terms of methodology and secondly in terms of substance. Although most of the studies in the in-depth review were judged to be of high quality, there tended to be little justification given for the choice of sample, and little attention to issues of reliability and validity at the data- collection and analysis stages. It would also be helpful to declare what counts as success in an intervention. Some interventions, although taking place with a whole class, select single students or pairs for report. While this gives a valuable in-depth picture of the potential of ICT, it is not known how typical these responses were or why these pupils were selected for report. This is not to say that small sample studies are not valuable. Small sample studies can give a valuable in-depth picture. However, more detail about the participants in the sample would enable the reader to gauge the limits on generalisability, and provide a useful starting point for large scale evaluation. One of the studies, while reporting statistical gains overall, was cautious in claiming too much for the intervention because only a minority of students attained multi-representational fluency. Other researchers, however, may have claimed this as a success.

In terms of substance, there is a need for more studies of different types probing students' understanding of functions within an ICT environment. In particular, teachers need to know more about the areas in which they need to provide carefully structured support in order to make full use of the ICT tools. Studies could include more comparative work, with larger samples, investigating the relative merits of different software and ICT tools. While there is some evidence of difficulties with graphics calculators, there is no comparative evidence of difficulties with graphing software used by individuals / small groups on computers, or graphing software used on inter-active whiteboards and/or computers with projectors used with the whole class.

More in-depth studies are needed of teachers and pupils working in the naturalistic setting of the classroom setting, and more in-depth probing of students' understanding using similar tasks in clinical interviews.

Researchers could also follow up the potentially relevant interventions in ICT and algebra contained in the middle school curriculum review of the What Works Clearing (WWC) House (http: //www.w-w-c.org).



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(i) Studies included the systematic map

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Appendix 1.1: Authorship of this report

This work is a report of a systematic review conducted by the Mathematics Education Review Group

The authors of this report are:

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They conducted the review with the benefit of active participation from the members of the review group.

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Dr Sue Sanders (Department of Education, University of Swansea)
John Sharpe (City of York LEA Advisory Service)
Professor Peter Tymms (Centre for Evaluation and Monitoring, University of Durham)

Advisory group membership

The membership of the Advisory Group is the same as the Review Group. There was an initial meeting in January 2006 to discuss the background and possible focus of the review, followed by email discussions to decide upon the review question. However, other individuals (teachers, researchers, policy-makers) with an interest in the review question were also invited to comment on the work of the Review Group at appropriate times. This was largely done through email and through discussions at conferences. In particular, the membership of British Society for Research into Learning Mathematics (BSRLM) were contacted at regular intervals and were considered to be an expert group which provided additional input.

Thanks are given for additional advice from the following:

Professor Janet Ainley (University of Leicester)
John Bibby (QED of York)
Dr Liz Bills (University of East Anglia)
Laurinda Brown (University of Bristol)
Dr Paul Drijvers (Freudenthal Institute, Utretcht University, The Netherlands)
Jennny Gage (University of Cambridge)
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Professor Michael Shayer (King's College, University of London)
Alison Clark Wilson (West Sussex Institute of Education)
Professor Dylan Wiliam (Board of Science Education, Washington, USA)

The Review Group met again in July 2006 to discuss work in progress, in particular the keywording process. After keywording, the Review Group was consulted in the process of narrowing down the review question. A further meeting in November was cancelled because it was difficult to find a meeting date for a sufficiently large group to attend a viable meeting.

Conflict of interest

There were no conflicts of interest for any members of the Review Group.

Acknowledgements

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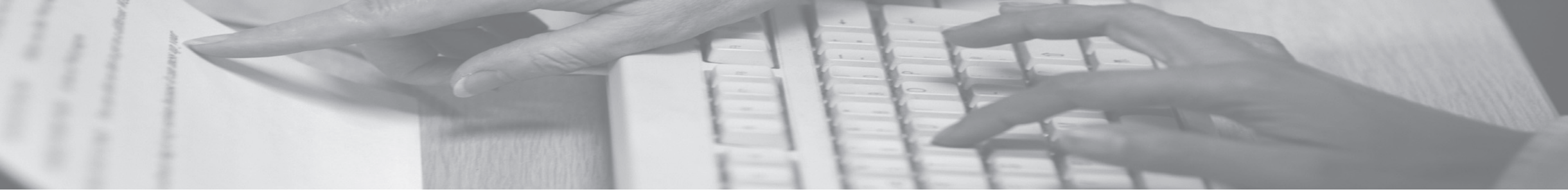

Appendix 2: The standard EPPI-Centre systematic review process

What is a systematic review?

A systematic review is a piece of research following standard methods and stages (see figure 1). A review seeks to bring together and 'pool' the findings of primary research to answer a particular review question, taking steps to reduce hidden bias and 'error' at all stages of the review. The review process is designed to ensure that the product is accountable, replicable, updateable and sustainable. The systematic review approach can be used to answer any kind of review question. Clarity is needed about the question, why it is being asked and by whom, and how it will be answered. The review is carried out by a review team/group. EPPI-Centre staff provide training, support and quality assurance to the review team.

Stages and procedures in a standard EPPI-Centre Review

- Formulate review question and develop protocol
- Define studies to be included with inclusion criteria
- Search for studies – a systematic search strategy including multiple sources is used
- Screen studies for inclusion
    - o Inclusion criteria should be specified in the review protocol
    - o All identified studies should be screened against the inclusion criteria
    - o The results of screening (number of studies excluded under each criterion) should be reported
- Describe studies (keywording and/or in-depth data extraction)
    - o Bibliographic and review management data on individual studies
    - o Descriptive information on each study
    - o The results or findings of each study
    - o Information necessary to assess the quality of the individual studies

At this stage the review question may be further focused and additional inclusion criteria applied to select studies for an ‘in-depth’ review.

- Assess study quality (and relevance)
  - A judgement is made by the review team about the quality and relevance of studies included in the review
  - The criteria used to make such judgements should be transparent and systematically applied
- Synthesise findings
  - The results of individual studies are brought together to answer the review question(s)
  - A variety of approaches can be used to synthesise the results. The approach used should be appropriate to the review question and studies in the review
  - The review team interpret the findings and draw conclusions implications from them

Quality assurance (QA) can check the execution of the methods of the review, just as in primary research, such as:

- Internal QA: individual reviewer competence; moderation; double coding
- External QA: audit/editorial process; moderation; double coding
- Peer referee of: protocol; draft report; published report feedback
- Editorial function for report: by review specialist; peer review; non-peer review

The results of this systematic review are available in four formats:

SUMMARY	Explains the purpose of the review and the main messages from the research evidence
REPORT	**Describes the background and the findings of the review(s) but without full technical details of the methods used**
TECHNICAL REPORT	Includes the background, main findings, and full technical details of the review
DATABASES	Access to codings describing each research study included in the review

These can be downloaded or accessed at **http://eppi.ioe.ac.uk/reel/**

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