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Signal Detection for Non-Orthogonal Space-Time Block Coding Over Time-Selective Fading Channels

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***Abstract*—In the case of non-quasi-static (i.e., time-selective fast fading) channels, which do exist in practice, the performance of the existing NO-STBC detectors can suffer from an irreducible error floor. To this end, this letter proposes a zero-forcing-based signal detector, which is not only computationally simple but also highly effective in mitigating the impact of channel variation on system performance.**

***Index Terms*—Nonorthogonal space-time block coding (NO-STBC), signal detection, time-selective fading channels, transmit diversity.**

I. Introduction

To IMPROVE the coding rate of orthogonal space-time block coding (O-STBC) in the four transmit antenna (4-Tx) case, several nonorthogonal STBC (NO-STBC) schemes have recently been proposed (see [1]–[4] and the references therein). Although NO-STBC schemes theoretically offer a lower diversity order, they do lead to a full rate transmission. In addition, the decoding complexity of all the NO-STBC schemes is still relatively low, although they have different robustness against channel correlation conditions [3], [4].

Most existing results on NO-STBC (or O-STBC), however, were obtained under the assumption of quasi-static channels. While such an assumption can be met approximately in most cases, time-selective fast fading (i.e., non-quasi-static) channels do exist [5]–[9]. For example, high speed trains in Europe can easily reach 250 kilometers per hour (km/h) even today. In the latter situation, it is imperative to understand how the performance of the standard NO-STBC systems would suffer and more importantly how to mitigate the impact of channel time-selectiveness in a preferably smooth and implicit manner (i.e., without needing to switch coding schemes).

In this letter, we first relax the quasi-static assumption by introducing a time selective channel model. As the existing NO-STBC detectors are in theory no longer applicable to such channels, we derive a computationally simple zero-forcing detector which has the following two features: 1) it is highly effective in mitigating the impact of channel variation on system performance and 2) it offers a similar performance to the quasi-static channel based decoders if the channel is indeed quasi-static (or slow fading). As such, the proposed detector can handle both slow and fast fading channels in a smooth and implicit manner, eliminating the need to switch from one coding scheme (e.g., STBC) to another (e.g., non-STBC).

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II. The Time Selective Fading Channel

Consider a system with 4 transmit (4-Tx) and 1 receive (1-Rx) antennas. 4 complex symbols, s_1, s_2, s_3, and s_4, are first grouped together and then passed through a NO-STBC encoder before being transmitted over $4T_s$ (T_s : symbol period). The output of the encoder is a 4×4 matrix $\mathbf{C} = [c_{ij}]$, where c_{ij} is either $\pm s_k$ or $\pm s_k^*$ (x^* means the conjugate of x), and is transmitted by Tx j at time i. By letting the channel gain from Tx j to the Rx at time $i$be $h_j(i)$, the received signal at time i is

$$r_i = \sum_{j=1}^{4} h_j(i) c_{ij} + n_i \tag{1}$$

where n_i is a complex additive white Gaussian noise (AWGN) with zero mean and a variance of σ_n^2 (therefore $0.5\sigma_n^2$ per dimension). Also, $h_j(i)$ is subject to Rayleigh fading but is normalized, i.e., $\sigma_h^2 = 1$, or $\mathrm{Re}[h_j(i)], \mathrm{Im}[h_j(i) \sim (\mathrm{N}(0, 0.5)$.

In most existing NO-STBC (or O-STBC) systems, a quasi-static channel is assumed, i.e., $h_j(i) = h_j$ over $4T_s$. While such an assumption is normally reasonable for the 2-Tx STBC case, it is more likely to be untrue in the 4-Tx STBC case [9]. As a result, the following channel model is adopted in this letter: from one symbol period (T_s) to the next, the channel is time variant via the well-known AR(1) model [5]–[8]:

$$h_j(i+m) = \alpha_m h_j(i) + w_j(i+m) \tag{2}$$

where $h_j(i)$ is subject to Jakes fading with autocorrelation $\alpha_m = \mathrm{E}[h_j(i) h_j^*(i+m)] = \mathrm{J}_0(2\pi m f_d T_s), f_d$ the Doppler frequency, and $\mathrm{J}_0(\cdot)$ the zeroth-order Bessel function of the first kind. Also, $w_j(i)$ is another independent complex Gaussian random variable having zero mean and variance σ_w^2 (i.e., $\sigma_w^2 + \alpha_m^2 = 1$).

III. Zero-Forcing Detector for NO-STBC

As was mentioned earlier, several NO-STBC schemes now exist. For comparison, only the NO-STBC code in [2] is employed here, which in fact is a special case of the codes in [3] and [4]. However, the same methodology can be applied to any other NO-STBC schemes with a 4×4 code matrix (e.g., those in [2]–[4]).

For symbol group $\mathbf{s} = [s_1,\ s_2,\ s_3, s_4]^T$, the NO-STBC code matrix [2]–[4] is

$$\mathbf{C} = \begin{bmatrix} \mathbf{g}_{12} & \mathbf{g}_{34} \\ -\mathbf{g}_{34}^* & \mathbf{g}_{12}^* \end{bmatrix} \tag{3}$$

where

$$\mathbf{g}_{ij} = \begin{bmatrix} s_i & s_j \\ -s_j^* & s_i^* \end{bmatrix}.$$

From (1), the received signal can then be written as

$$\mathbf{r} = \mathbf{H}\mathbf{s}+\mathbf{v} \tag{4}$$

where $\mathbf{r} = [r_1, r_2^*, r_3^*, r_4]^T, \mathbf{v} = [n_1, n_2^*, n_3^*, n_4]^T$, and the channel matrix

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}_{12}(1) & \mathbf{h}_{34}(1) \\ \mathbf{h}_{34}^*(3) & -\mathbf{h}_{12}^*(3) \end{bmatrix}$$

with

$$\mathbf{h}_{ij}(l) = \begin{bmatrix} h_i(l) & h_j(l) \\ h_j^*(l+1) & -h_i^*(l+1) \end{bmatrix}.$$

Owing to the channel's time-selectiveness (i.e., the structure of the above $\mathbf{H}$), the pair-wise maximum likelihood (ML) detection strategy in [2] can in theory no longer be applied here, even under perfect channel state information (CSI, i.e., $\mathbf{H}$). The main reason is that the likelihood function can no longer be decomposed into the sum of two pair-wise functions. In view of this, the following two-step zero-forcing scheme has been developed by taking full advantage of the perfect CSI.

Step 1: Pass $\mathbf{r}$ through a transform $\mathbf{\Theta}$ so that

$$\mathbf{\Theta}\mathbf{r} = \mathbf{\Theta}\mathbf{H}\mathbf{s} + \mathbf{\Theta}\mathbf{v} = \mathbf{D}\mathbf{s}+\mathbf{n}' \tag{5}$$

where $\mathbf{D} = \mathrm{diag}(q_1, q_2, q_3, q_4)$ and q_i's are the diagonal elements of $\mathbf{\Theta}\mathbf{H}$ (also see below). This leads to the following block (or partitioning) structure for $\mathbf{\Theta}$:

$$\mathbf{\Theta} = \begin{bmatrix} \boldsymbol{\theta}_{11} & \boldsymbol{\theta}_{12} \\ \boldsymbol{\theta}_{21} & \boldsymbol{\theta}_{22} \end{bmatrix} \tag{6}$$

where

$$\boldsymbol{\theta}_{11} = \boldsymbol{\theta}_{12}\mathbf{h}_{12}^*(3)\mathbf{h}_{34}^{-1}(1), \tag{7}$$
$$\boldsymbol{\theta}_{22} = -\boldsymbol{\theta}_{21}\mathbf{h}_{12}(1)[\mathbf{h}_{34}^*(3)]^{-1}, \tag{8}$$
$$\boldsymbol{\theta}_{12} = \mathrm{DT}(\mathbf{x}_1), \quad \boldsymbol{\theta}_{21} = \mathrm{DT}(\mathbf{x}_2), \tag{9}$$
$$\mathbf{x}_1 = \mathbf{h}_{34}^*(3) + \mathbf{h}_{12}^*(3)\mathbf{h}_{34}^{-1}(1)\mathbf{h}_{12}(1), \tag{10}$$
$$\mathbf{x}_2 = \mathbf{h}_{34}(1) + \mathbf{h}_{12}(1)[\mathbf{h}_{34}^*(3)]^{-1}\mathbf{h}_{12}^*(3) \tag{11}$$

and

$$\mathrm{DT}(\mathbf{z}) = \begin{bmatrix} -z_{22} & z_{12} \\ z_{21} & -z_{11} \end{bmatrix}$$

for any 2×2 matrix

$$\mathbf{z} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}.$$

Also, it can be shown that $q_1 = q_2 = -\det(\mathbf{x}_1)$, and $q_3 = q_4 = -\det(\mathbf{x}_2)$, where $\det(\mathbf{z})$ is the determinant of $\mathbf{z}$: $\det(\mathbf{z}) = z_{11}z_{22} - z_{12}z_{21}$. It is easy to verify that $\mathbf{\Theta}\mathbf{H} = \mathbf{D}$ indeed. This guarantees the complete removal of the inter element interference (IEI).

Step 2: A simple least square (LS) detection can then follow:

$$\widehat{s}_i = \arg\left\{\min_{s_m \in \mathcal{S}} |[\mathbf{\Theta}]_i\mathbf{r}-q_i s_m|^2\right\} \tag{12}$$

where $i = 1, \ldots 4$, and S is the symbol alphabet and $[\mathbf{\Theta}]_i$ is the ith row of $\mathbf{\Theta}$.

1) Algorithm Complexity: The operations in (6)–(11) involve 2×2 matrices only. Two matrix inversions are required, but these are inversions of 2×2 matrices only, and for any 2×2 matrix $\mathbf{z}, \mathbf{z}^{-1} = \mathrm{DT}(\mathbf{z})/\det(\mathbf{z})$. This makes the algorithm's implementation computationally very affordable. In fact, as a linear processing detector, the complexity of this algorithm for an M-ary constellation is $\mathrm{O}(M)$/symbol, while it is $\mathrm{O}(M^2)$/symbol for the pairwise ML detector in [2]. Also, note that in the case of quasi-static channels, the above detector will reduce to the detector in [3].

2) Algorithm Optimality: In terms of noise properties, we have $\mathrm{E}[\mathbf{n}'\mathbf{n}'^H] = \sigma_n^2\mathbf{\Theta}\mathbf{\Theta}^H$. It is easy to show that $\mathbf{\Theta}\mathbf{\Theta}^H$ in general is not diagonal. As a result, (12) is no longer an ML detector and hence represents a suboptimum approach.

3) Comparison With Existing ZF Procedures: The above two-step ZF (TS-ZF) detector may look "similar" to some existing ZF procedures such as Verdu's decorrelator [10]. As is detailed below, however, they differ from each other in some fundamental aspects. 1) The matrix in [10] is a "cross-correlation matrix", which is normally large (e.g., the number of active users) and has many specific properties (e.g., block tridiagonal, symmetric blocks, and often real). This leads to many different decorrelating structures/implementations [10], [11]. 2) The matrix $\mathbf{H}$ in our TS-ZF is "channel matrix", which is only 4×4 in size but has very different properties (e.g., full, nonsymmetric, and nearly always complex). However, the specific block structure of $\mathbf{H}$ (thanks to the STBC!) allows us to use the STBC specific block (or partitioning) structure for $\mathbf{\Theta}$ in (6) to derive a smooth, effective and robust detection algorithm (e.g., $\mathbf{h}_{34}^{-1}(l)$ will simply reduce to $\mathbf{h}_{34}^H(l)/[\mathbf{h}_3^2(l) + \mathbf{h}_4^2(l)]$ in case of quasi-static channels). This leads to a very low computational complexity: $\mathrm{O}(M)$/symbol, where M is the constellation level. A brute-force matrix inversion or zero forcing would not have these important benefits either. 3) Most parameters for the decorrelator in [10] (e.g., near-far resistance) have no counterparts in our TS-ZF detector, similarly to many other cases (e.g., single user equalizer).

IV. Simulations and Conclusions

The signal-to-noise ratio (SNR) at the receiver is defined as $\mathrm{SNR} = (4\sigma_h^2 P_T)/\sigma_n^2 = 4P_T/\sigma_n^2(\sigma_h^2 = 1)$, where P_T is the Tx power at each antenna. Also, Gray encoded 4-QAM is used, carrier frequency $f = 2$ GHz, and $T_s = 128/(3.84 \times 10^6)$. Plotted in Fig. 1(a) are the bit error rates (BER) of the new zero forcing (ZF) detector with three vehicle speeds: $\mathrm{V} = 150, 200$, and 250 km/h (corresponding to $f_dT_s = 0.0093, 0.0123$, and 0.0154). For comparison, as shown in Fig. 1(b), the pairwise maximum-likelihood (ML) detector [2] is also simulated under

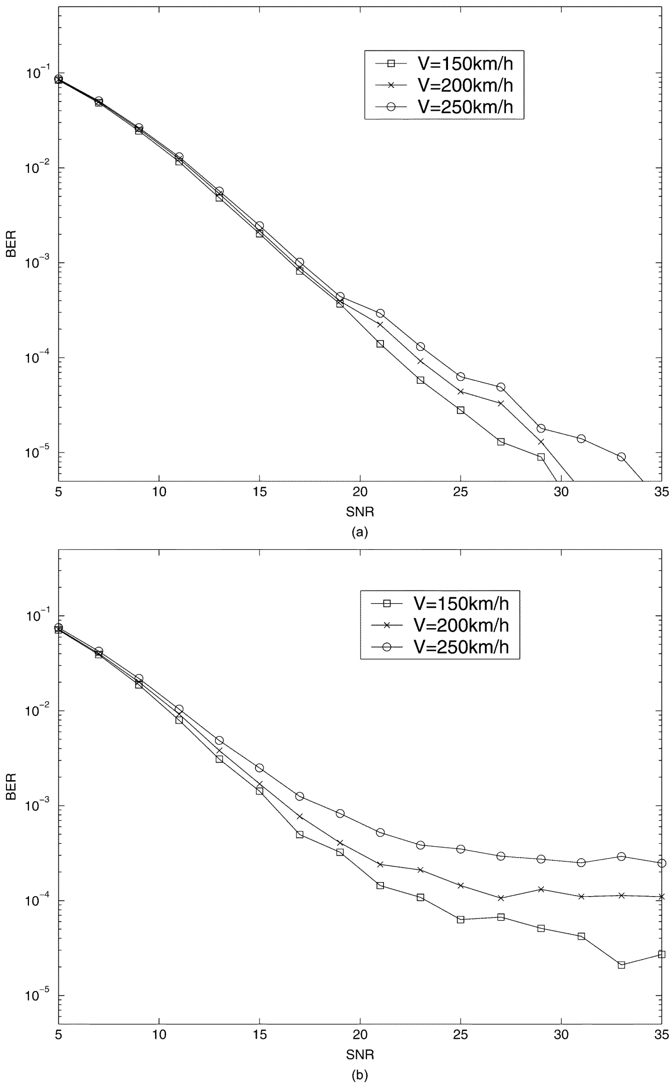

Fig. 1. The BER results of (a) the new TS-ZF detector, and (b) the pairwise ML detector.

the same channel conditions, by assuming that $h_j(i) = h_j(1)$ at the receiver although the true $h_j(i)$ is generated via (2). For low speed or slow fading scenarios, our simulations indicated that the new ZF detector offers a similar performance to the pairwise ML detector (but with a lower computational complexity).

Clearly, the new ZF NO-STBC detector is highly effective in suppressing the impact of channel time-selectiveness and there is no error floor. In contrast, the pairwise ML method suffers from an irreducible error floor in the high SNR cases. In addition, the new ZF detector has a very low (linear) computational complexity.

When the channel changes from slow to fast fading, an obvious alternative is to switch the transmitter to a non-STBC mode. This however involves two drawbacks: 1) the loss of space diversity and 2) the need to set up a "switching threshold," with which comparison must be made at all times (not a trivial task either!). In contrast, the new ZF detector can handle both slow and fast fading channels in a smooth and implicit manner, eliminating any need for switching the coding schemes. Finally, although the channel estimation issue is beyond the scope of this letter, schemes such as the pilot overlay in 3GPP do allow for symbol-by-symbol channel estimation.

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