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Breaking of Larmor's theorem in quantum Hall states with spin-orbit coupling

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We investigate the effect of spin-orbit (SO) interaction on the long-wavelength collective spin excitation in a two-dimensional electron gas (2DEG) in the fractional quantum Hall regime. The many-body correction to the single-particle electron spin resonance (ESR) energy is found to be non-zero, providing theoretical evidence of a breaking of Larmor's theorem. Such breaking is due to the loss of spin-rotational invariance introduced by the SO-induced structural inversion asymmetry in the system. This effect, whose magnitude is a significant percentage of the single-particle ESR, exhibits remarkable features in a wide range of experimentally relevant parameters and is found to be nearly material-independent.

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As early as in 1930 Bloch demonstrated [1] the inherent interplay between long-wavelength excitations and long-range magnetic order. Both reflect the impact of symmetry on the effect of interaction in a many-body system. For instance, in a two-dimensional electron gas (2DEG) with continuous spin-rotational invariance, Larmor's theorem [2] predicts that the long-wavelength collective spin-wave excitation will occur exactly at the Larmor frequency of the single-particle electron spin resonance (ESR). In such a system, according to Mermin-Wagner's theorem [3], long-range magnetic order is impossible at finite temperature. Conversely, the breaking of Larmor's theorem is associated with a manifestation of long-range order in the discovery of He^3 superfluidity, which was based on the observation [4] of a shift in the nuclear magnetic resonance frequency with respect to the Larmor frequency due to a spontaneous breaking of the spin-orbit (SO) symmetry [5]. Investigating the behaviour of collective spin excitations in a system can therefore provide deep insights into the nature of its long-range magnetic order. Here we provide theoretical evidence of a SO-induced breaking of Larmor's theorem in a 2DEG in the fractional quantum Hall (FQH) state, caused by structural inversion asymmetry, which results in a non-zero many-body contribution to the ESR. Whilst the presence of SO coupling could be expected to perturb the spin invariance in the system, the effect reported here is more subtle in that it originates from the asymmetry introduced in the system by the presence of an external *electric* field perpendicular to the plane of the 2DEG. Such field interacts with the electron spin through the Bychkov-Rashba SO term in the Hamiltonian, leading to a field-dependent many-body shift in the ESR. The calculated magnitude of this effect exhibits remarkable features in a wide range of parameters relevant to experiment: it is found to be rather insensitive to changes in the g-factor, and not to depend sensitively on the specific material properties.

A 2DEG confined in an asymmetric quantum well (QW) of a narrow-gap diamagnetic semiconductor, such as InAs, provides a unique system to investigate the behaviour of collective spin excitations and the nature of their relationship to long range order. The presence of a large SO coupling raises fundamental questions regarding its impact on the spin symmetry. In the single-particle picture without SO coupling, the Larmor precession frequency for a 2DEG in a perpendicular B field can be determined from the Zeeman splitting:

$$\hbar\omega_L = E_Z = |g^*|\mu_B B. \tag{1}$$

Equation (1) was confirmed by ESR measurements in microwave conductivity experiments [6]. In an asymmetrically doped QW, the presence of structural inversion asymmetry (SIA) causes SO coupling, characterised by the zero-field spin splitting energy Δ_{SO} [7]. The single-particle spin precession frequency ω_S is shifted from the Larmor frequency and is given by

$$\omega_S = [(\omega_C + \omega_L)^2 + \omega_{SO}^2]^{1/2} - \omega_C \tag{2}$$

in which we have defined a SO precession frequency $\omega_{SO} = 2\Delta_{SO}/\hbar$ and $\omega_C = eB/m^*$ is the usual cyclotron frequency. This shift from Larmor frequency is not a consequence of many-body (MB) corrections due to electron-electron interaction, but results simply from the inclusion of the additional SO term (it is easy to see that ω_S reduces to ω_L for vanishingly small Δ_{SO}). In fact, Eq. (2) can be visualised in the classical picture of a rotating gyroscope [8], with the rotating angular velocity ω_C parallel to the precession angular velocity ω_L, but perpendicular to the other precession angular velocity ω_{SO}. It can also be derived exactly from a single-particle quantum mechanical model, using the parabolic-band approximation [7]. The two pictures are equivalent inasmuch as band structure details are neglected, i.e., m^* and g^* are considered independent of the 2DEG's kinetic energy. Equation (2) was recently confirmed by the measurement of the combined resonance (CBR) in far-infrared photoconductivity experiments [9].

In 1993, Longo and Kallin [2] investigated the MB corrections to spin-flip excitations, in which an electron is

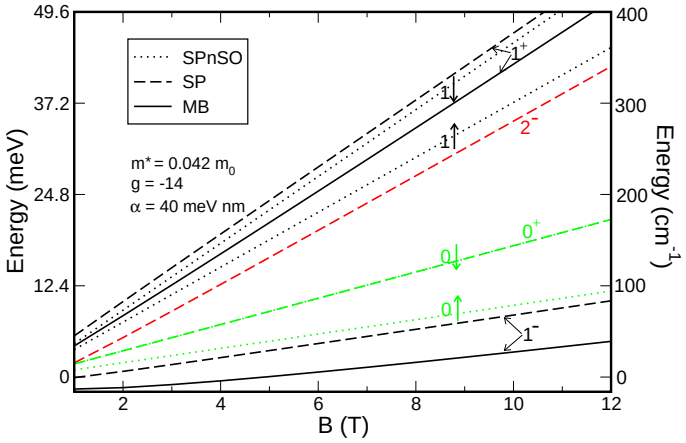

FIG. 1: (Color online) Calculated energy levels of a 2DEG in the $\nu = 1/3$ FQH state as a function of the applied magnetic field in the Single-Particle [both with (SP) and without (SPnSO) SO interaction] and Many-Body (MB) approximation. States with the same "principal" quantum number m are displayed with the same color. The solid line "0^+" coincides with the dashed line labeled "0^+" and the dotted line labeled "$0\downarrow$" and is not shown for clarity.

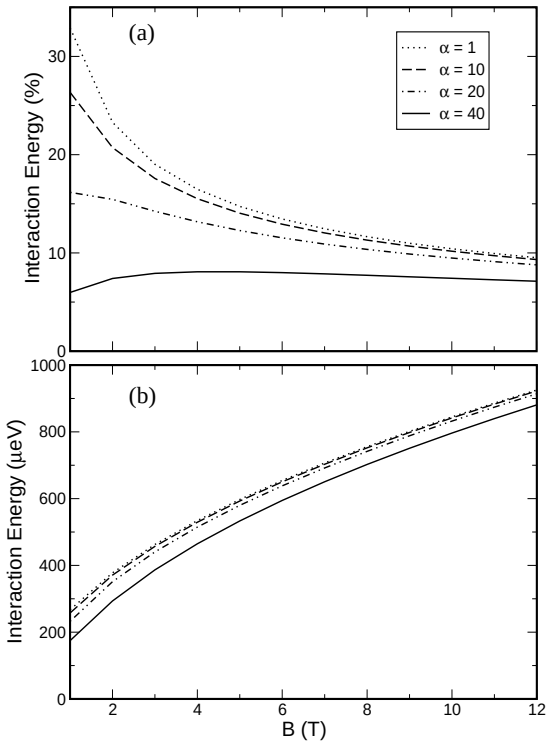

FIG. 2: Calculated electron-electron interaction energy, defined as the difference between the values of $\hbar\omega_S$ in the MB and SP approaches, as a function of the applied magnetic field, for α = 1, 10, 20, 40 nm·meV: (a) percentage, (b) absolute value.

both promoted from one Landau level to the next and its spin reversed, in the highly correlated electron fluid described by Laughlin's wave function [10]. In the single-particle picture the frequency of this transition is exactly $\omega_{SF} = \omega_C + \omega_L$. No work has been published as yet for the MB counterpart of Eq. (2), which should shed light on the nature of broken spin symmetry in a MB system with strong SO coupling, and could form the basis for investigating interplays between long-wavelength excitations and long-range magnetic order in 2DEGs. This is what we aim to do in this report.

In the absence of SO interaction, Kohn's theorem and Larmor's theorem predict MB corrections to be identically zero for both cyclotron and ESR collective excitations in a 2DEG with translational- and spin-rotational invariance. The introduction of SO brakes the spin-rotational invariance of the system, and leads to a non-zero many-body contribution to the ESR excitation energy. In this situation, the energy levels can no longer be characterised using the Landau level index $n = 0, 1, 2, \ldots$ and the spin projection $s_z = -1/2, 1/2$, as in the case for single-particle levels in the absence of SO. Each new level is now a mixture of Landau levels with indices n and $(n+1)$ and also of spin-up and spin-down states. The new quantum numbers are $j = 1, 2, \ldots$ and $\tilde{s}_z = -, +$. The relationship between the two sets of quantum numbers is visualised in Fig. 1, that shows the lowest calculated energy levels in the single-particle and MB pictures. We see that the SP levels labelled as "-" ("+") are very close in energy to those that, in the absence of SO, were characterised as spin-up (-down). This is because the main spin component of "-" ("+") levels is up (down), and the contribution of the opposite-spin, minority, component is proportional to the SO coupling constant α ($\propto \Delta_{SO}$). Although electron-electron interactions introduce further level mixing, the MB levels can still be labelled with the same quantum numbers used for SP levels, as the main contribution to the MB level labelled as $j, \tilde{s}_z$ comes from the SP level with quantum numbers $j, \tilde{s}_z$. The equivalent of Eq. (2) in the MB approach can therefore still be formally written as

$$\omega_S = (E_1^+ - E_1^-)/\hbar - \omega_C \qquad (3)$$

even though there is now no analytical expression for E_1^+, E_1^-. As it was the case for Eq. (2), $\omega_S \to \omega_L$ for $\Delta_{SO} \to 0$.

The energies in Eq. (3) are obtained by solving the MB Schrödinger equation for four electrons per supercell [11] via the exact diagonalisation technique. The MB wavefunctions are expanded in terms of a complete basis expressed as a superposition of solutions of the single-particle Hamiltonian that includes the effects of Rashba coupling in a $\nu = 1/3$ FQH state (for more details on this technique see Ref. [12, 13] and references therein).

We define the interaction energy Δ^{e-e} as the difference between the values of $\hbar\omega_S$ calculated in the MB and SP approaches. Figure 2 shows this difference as a function of the applied magnetic field for four different values

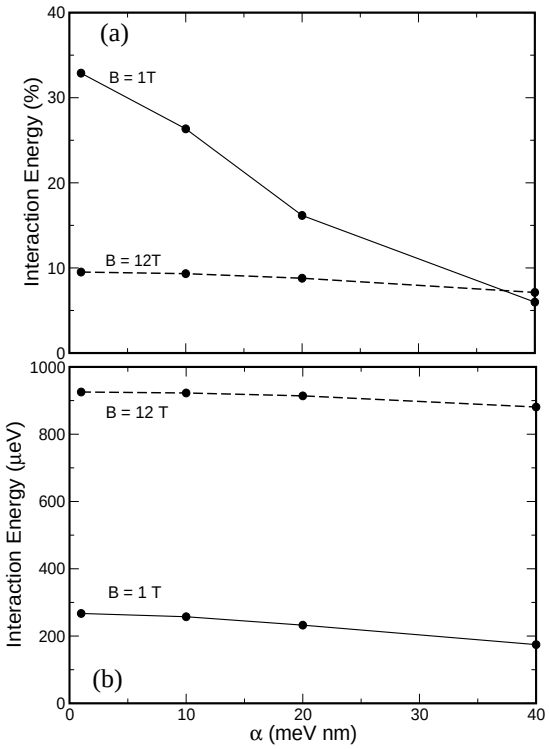

FIG. 3: Calculated electron-electron interaction energy, [(a) percentage, (b) absolute value of the difference between the values of $\hbar\omega_S$ in the MB and SP approaches] as a function of α for two values of the applied magnetic field $B = 1$, 12 T.

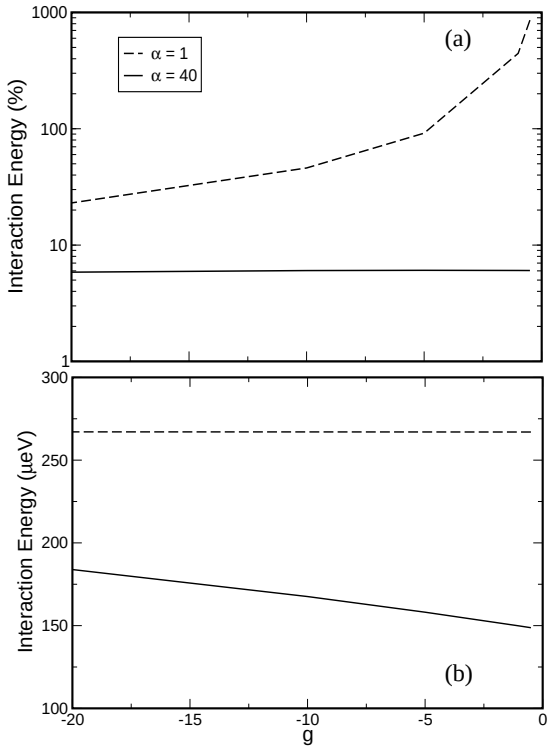

FIG. 4: Electron-electron interaction energy [(a) percentage, (b) absolute value], defined as the difference between the values of $\hbar\omega_S$ in the MB and SP approaches, calculated as a function of the g factor, for $\alpha = 1$, 40 nm·meV and $B = 1$T.

of the SO coupling constant α (all within the experimental range [14]), both in absolute value (in μeV) and as percentage relative to the single-particle ESR energy. If not otherwise specified, all calculations are performed for InAs, using $m^*/m_0 = 0.042$ and $g^* = -14$. We see that although percent-wise the largest MB corrections are found at low magnetic fields, the most sizeable energy shifts in absolute value take place at high magnetic fields where the MB corrections are a factor of 3–4 larger than for $B = 1$ T. In both cases the interaction energy decreases when α is increased from 1 to 40 nm·meV (see Fig. 3). This is rather expected since for larger α there is greater mixing of the higher Landau level [13] with the spinor components. This provides more degrees of freedom for the Coulomb interaction and hence the decrease of the Coulomb energy.

We tested the accuracy of our method by calculating the MB correction to the spin-flip excitation in the absence of SO coupling ($\alpha = 0$) and compared our results with those obtained by Longo and Kallin [2] using the generalised single-mode approximation. This approach was first applied [15] to find the dispersion relations for intra-Landau-level excitations for filling factors less than 1. It has since then been generalised [2, 16] to treat inter-Landau-level transitions and has proved to be most accurate for small values of the wave vector ($k = 0$ in the present case). For GaAs we calculated a MB shift of 0.075 (in units of $e^2/\epsilon l_0$) which is in excellent agreement with the value of ~ 0.07 obtained in Ref. [2]. Furthermore energies of the same order of magnitude were recently obtained [17] in inelastic light scattering measurements. Also very close is our calculated shift in InAs (0.069), showing that the correction does not depend sensitively on the material properties.

This is confirmed further by Fig. 4, that shows the effect on the interaction energy of the variation of the g factor in the range $-0.25 \div -20$, for $\alpha = 1, 40$ nm·meV and $B = 1$ T. The sensitivity of Δ^{e-e} on the actual value of g^* is an important factor as in a 2DEG g^* can vary significantly from its value in the bulk, depending on the experimental conditions. We find that the MB correction is rather insensitive to changes in g^* in this range. The large percentual value of the ESR correction for small g^* and $\alpha = 1$ in Fig. 4a is due to the very small value of Eq. (2) in this parameter range. In fact the absolute value of the MB correction is constant for all values of g^* considered, as shown in Fig. 4b, whereas, in the SP approximation, $\hbar\omega_S$ spans almost 2 orders of magnitude, ranging from 0.03 meV, for $g^* = -0.5$, to 1.16 meV, for $g^* = -20$.

We find therefore that the most important parameters that determine the magnitude of the MB shift are: (i)

the intensity of the applied magnetic field B and (ii) the strength of the SO coupling constant α. Both can be adjusted externally: the former in an obvious way and the latter by tuning the intensity of an electric field applied perpendicularly to the 2DEG plane, obtained either by controlling an applied gate voltage, or by specific design of the heterostructure (e.g., by asymmetrically doping the QW). This field, "felt" in the electron rest frame as an effective in-plane magnetic field (a purely relativistic effect), is the cause of the breaking of the spatial inversion symmetry in the 2DEG that gives rise to the zero-field spin splitting Δ_{SO}. A different kind of spatial inversion asymmetry (not considered in the present work), that is also a source of an effective magnetic field, can be caused by the bulk inversion asymmetry (BIA) of the underlying (zinc-blende) crystal structure. However, whereas the effective magnetic field due to SIA is always in the 2DEG plane, the direction of the field due to BIA depends on the crystallographic orientation of the QW. In a (110)-oriented QW, the BIA effective field is along the external magnetic field and its direction varies from parallel to antiparallel to B, depending on the position. Its effect can be modelled as a modulation of the g factor which makes it position-dependent.

As shown in Fig. 4, however, the interaction energy is not very sensitive to variations of g^*. In (001)-grown QWs, where the effective field due to BIA is in the 2DEG plane, its effects are small compared to those due to SIA in InAs-based heterostructures at low applied magnetic fields [18].

In summary we provided theoretical evidence of a SO-induced breaking of Larmor's theorem in 2DEGs. The magnitude of the calculated many-body correction to the single-particle ESR frequency is found to be rather insensitive to changes in the g-factor in a wide range, and not to depend sensitively on the specific material properties. In contrast we find that the MB shift is influenced mainly by externally-tunable parameters such as the intensity of the applied electric field. As the latter is precisely the source of the loss of spin-rotational invariance (through the SO-induced structural inversion asymmetry) in the system, we conclude that investigating the long-wavelength collective spin excitations in a 2DEG can be a very powerful tool to gain deep insights into the nature of its long-range magnetic order.

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