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Spatio-temporal Evolution as Bigraph Dynamics

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Abstract. We present a novel approach to modelling the evolution of spatial entities over time by using bigraphs. We use the links in a bigraph to represent the sharing of a common ancestor and the places in a bigraph to represent spatial nesting as usual. We provide bigraphical reaction rules that are able to model situations such as two crowds of people merging together while still keeping track of the resulting crowd's historical links.

Keywords: spatio-temporal change, bigraphs, filiation

1 Introduction

The combined modelling of space and time is a well-established aspect of the theory of spatial information [8, 12, 18, 9, 3, 2]. It also provides particular challenges when dealing with granularity and vagueness [14, 4]. Objects can move, cities and countries can retain their identities while changing their boundaries, new entities can be formed from old ones as in the redistribution of parcels of parcels of land or the more rapid change seen as crowds of demonstrators are divided by police and then re-form and take on new activity. Such examples are recorded in systems having purposes as diverse as tracking the delivery of consumer goods in a postal system, the legal record of land ownership, or the surveillance of crowds of people in public demonstrations.

Despite all these, and many more examples that could be mentioned, the formal description of spatio-temporal change in a way that suits the needs of information systems is still at an early stage. In the purely spatial case, certain basic systems of spatial relationships have been found useful; the 9-intersection model [5] has acquired the status of a standard and systems for qualitative spatial reasoning [1], including the Region-Connection Calculus, have been very widely studied and applied. The spatial relations modelled in such systems include widely accepted notions such as 'overlapping', 'inside but not touching the boundary', and 'disjoint'.

In contrast models of spatio-temporal change, while numerous and containing much valuable work, have not reached any consensus about the atomic concepts they need to provide. We can imagine spatio-temporal scenarios between regions, such as one moving to encircle another, or two regions moving further apart to allow a third to pass between them. The most basic scenarios of single regions splitting and merging have been rigorously analysed in [9, 15], but it is not clear whether more complex behaviours can be treated in a similar way. In order to study such behaviour it is necessary to have a formal framework that is capable of modelling spatio-temporal change without pre-judging the kinds of higher level events and process that will be significant. This means that we should base our study on primitive concepts that appear to be essential and which can be combined to exhibit a variety of different behaviours. In this paper we propose that structures known as bigraphs provide what is needed. In addition to drawing the attention of the spatial information theory community to this area, we also introduce a novel way of using bigraphs to model relationships between entities in terms of shared ancestry.

Bigraphs were introduced by Milner [11] and are so called as they provide a single set of nodes having two distinct kinds of edges between them. The nodes with one kind of edge form a set of trees which allow the nodes to represent spatial nesting. This can model situations such as a person being in a room which is inside a building. The nodes taken together with the other kind of edge constitute a hypergraph where one edge may be incident with a set of nodes (not just one or two). The original motivation for bigraphs uses this hypergraph (called the link graph) as a way of modelling communication between the things represented by the nodes. For example, two nodes representing people might be joined by a link representing their participation in a phone call. In another scenario one of the hyperedges could represent a local area network, and the nodes computers connected by means of it.

The applicability of bigraphs to spatial information theory has already been noted in [16] and [7]. In [16] Walton and Worboys make extensive use of bigraphs to model image schemas. Their work proposes bigraph reaction rules to model dynamic schemas and uses bigraph composition to model change in level of detail.

The spatial relationships modelled in bigraphs are clearly restricted, as even simple overlapping of spatial entities is excluded. However, the interaction of spatial structure and communication even in this simplified case presents challenges to a fully rigorous analysis, and it is appropriate to ensure the simpler setting is fully understood before proceeding to more elaborate models. There has been some work [13] on bigraphs in which a node may be shared between two distinct containing nodes, but we do not make use of this in the present paper.

There are a number of reasons why bigraphs deserve to be studied in the context of spatio-temporal change. One is that they have a sound theoretical basis with a catalogue of results that can be used in any situation to which they are applied. Another is that besides the presence of an explicit spatial component they also come with mechanisms to specify change, that is to specify when and how one bigraph may be modified to another. This is achieved by means of rewrite rules allowing one part of a bigraph to be replaced by another. The process of rewriting is essentially familiar from simplifications such as replacing an instance of $x + 0$ by x in an algebraic expression such as $(2 + 0)y$ to end up with $2y$. The fact that a bigraph can be written as an algebraic expression means that sequences of spatial changes have algebraic counterparts allowing these changes to be analysed in a rigorous way.

The main novelty to which we draw attention in the present work is the way in we are able to use the links in a bigraph (the edges in the hypergraph) to model shared ancestry. The idea behind this is explained in terms of relations and hypergraphs in Section 2. Bigraphs are introduced in Section 3 where, as these structures are not widely known in spatial information theory, we provide an expository account of the basic ideas and refer the reader to [11] for more details. In Section 4 we present a scenario of one kind of situation where spatio-temporal modelling is important. Our case study involves crowds of people moving in a city. The ability of bigraphs to model the essential dynamic features of this case study is demonstrated in Section 5 where we give reaction rules for changes in the location and compostion of the crowds. Finally, Section 6 provides conclusions and outlines directions for further work.

2 Relations and Summaries

2.1 Filiation

Many formal models proposed for spatio-temporal evolution involve the mathematical concept of a relation between two sets. If the sets are X and Y then a relation from X to Y can be visualized as a set of arrows leading from elements of X to elements of Y. These arrows are subject only to the restriction that given $x \in X$ and $y \in Y$ there is at most one arrow from x to y. The suitability of this for modelling the most basic features of change is evident if we take X and Y to be sets of entities at two times, the second coming after the first.

Considering more times than just two we can use a sequence of sets. In Figure 1 there are four sets of entities and three relations between them. Each set represents a snapshot of the entities at a paticular time, and the relations model links between these entities and the ones present at the previous or next time in the sequence. The nature of the links will depend on the particular scenario being modelled.

To give some examples of the possible meaning of such a link we can consider Figure 1 where in relation Q we see that a_1 is linked, or related, to both b_1 and b_2. This situation of one entity at the earlier time being related to two at the later allows many interpretations. These include a parcel of land divided into two, a mother having a child with her own existence continuing, an island being split into two by rising sea levels, a group of animals separating into two groups, a plant producing an offshoot which develops into a separate individual plant, and so on. The example of the mother and child shows that we need not use exactly the same interpretation for every link. The link between a_1 (the mother at the earlier time) and b_1 (the mother at the later time) can denote the continuing

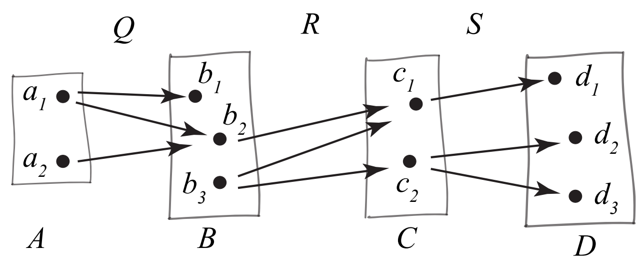


Fig. 1. Three relations between four times

existence of an entity, while the link from a_1 to b_2 can denote the earlier entity giving rise to a separate entity (the child b_2) at the later stage. We use the term *filiation* for a link of any kind connecting entities in this way, and this topic has been studied further in [3].

The way in which identity continues in objects that change is a long-standing issue in philosophy [6, 17]. However, the existence of a filiation link does not necessarily indicate a continuation of identity. A filiation link from a parent to a child could be regarded as the continuing identity of the family, or with equal validity as the creation of a separate personal identity. The choice between these two would depend on the application domain but would be some additional structure beyond the existence of a filiation link. We do annotate the filiations to show different kinds of behaviour with respect to identity in the case study in Section 4, but in the present work this annotation is not modelled by the operations on bigraphs that we describe. The continuation of identity is important in information systems [8, 10]. In [8] Hornsby and Egenhofer study operations for the construction of composite objects based on features of identity which include the creation, continuation and elimination of identity. The incorporation of this type of approach in our use of bigraphs would be an interesting direction for further work.

2.2 Summarizing Evolution

The representation of every known timepoint in the sequence and the filiation links between every successive pair of times is the highest level of detail in the model. For many purposes this level of detail can be unnecessarily complex and a less detailed, or more coarse grained, view is more approriate. In the example involving just four times with sets of entities A, B, C and D illustrated in Figure 1 we might need to summarize the change from the time of A to that of C. The usual way to summarize this change would be to compose the relations Q and R as in Figure 2.

In the summary by relation composition we see that c_1 has both a_1 and a_2 as ancestors. The summary however has lost two pieces of information: that c_1 and c_2 have a common ancestor, and that a_1 was linked to two entities between the

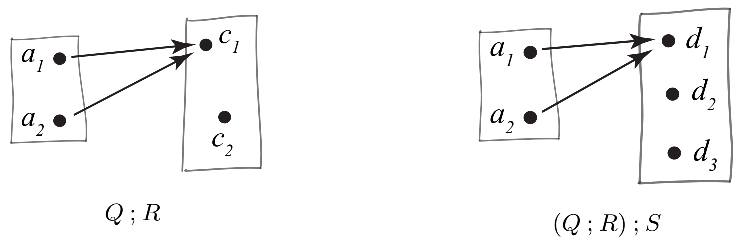


Fig. 2. Composite relations

two times evident in the summary. It is in the nature of a useful summarization technique for information to be lost, but there are practical cases where the fact that two entities shared a common ancestor would be something that it would be useful for a summary to maintain.

It is possible to define a way of summarizing a composable pair of relations that is different to their composition. The idea is that given relations $Q : A \to B$ and $R : B \to C$ we can enlarge A to include any entities in B which are not linked to anything in A. In the next definition $Q(A)$ means $\{b \in B \mid \exists a \in A \; (a \; Q \; b)\}$.

Definition 1 (Cumulative Product) *Let $Q : A \to B$ and $R : B \to C$ be relations, where A and $B - Q(A)$ are disjoint. The cumulative product of Q and R is the relation $Q \star R : A \cup (B - Q(A)) \to C$ where*

$$x \; Q \star R \; c \textit{ iff } \begin{cases} x \in B - Q(A) \textit{ and } x \; R \; c, \textit{ or} \\ x \in A \textit{ and } x \;\; Q \,;\, R \;\; c. \end{cases}$$

Examples of this construction are shown in Figure 3. The assumption that A and $B - Q(A)$ are disjoint in the definition may appear restrictive. However the elements of the sets are not the individuals being modelled in the world, rather they are tokens which can be mapped to the world. This permits distinct tokens to take the same identity, and the issue here can be understood more fully by using an analysis analogous to the idea of support for bigraphs used in [11].

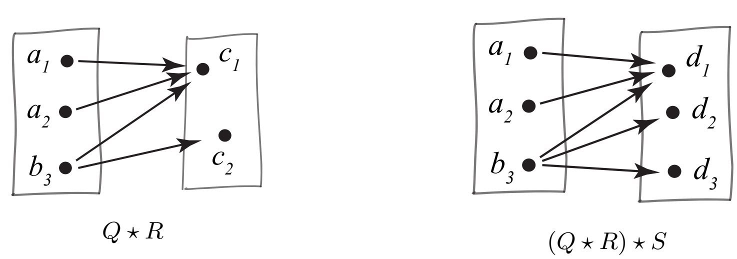


Fig. 3. Examples of the Cumulative Product

Conceptually the cumulative product is quite distinct from composition. The composition of relations describes a process of entities changing from past to present. The cumulative product models the state in the present, looking back. This suggests the idea of a map which shows the present state of the world but also contains evidence of past history indicating how the present state arose through accumulating changes. For this reason we sometimes refer to accumulation instead of the cumulative product.

Note that if relations Q, R, S are composable so that $Q\,;R\,;S$ is defined then we may form the accumulation $(Q \star R) \star S$ but not, in general, the accumulation $Q \star (R \star S)$. This is because the enlarged domain of $R \star S$ by the addition of elements not present in R means that the co-domain of Q may not match the domain of $R \star S$. This behaviour is as one would expect given the way that accumulation is inherently directional, building on the past.
We can visualize a sequence of relations as follows with relations *between* times.

$$\begin{array}{cccccccc} X_0 & \xrightarrow{R_1} & X_1 & \xrightarrow{R_2} & X_2 & \ldots & X_{n-1} \xrightarrow{R_n} & X_n \\ \\ t_0 & & t_1 & & t_2 & \ldots & t_{n-1} & t_n \end{array}$$

For accumulation, the picture naturally places the relations *above* the times. The relation A_i describes the entities at time t_i and (some of) their past history.

$$\text{———} \; A_1 = R_1$$

$$\text{—————} \; A_2 = A_1 \star R_2$$

$$\text{————————} \; \ldots A_{n-1} = A_{n-2} \star R_{n-1}$$

$$\text{————————} \; \ldots \; \text{—————} \; A_n = A_{n-1} \star R_n$$

$$t_0 \qquad\qquad t_1 \qquad\qquad t_2 \qquad \ldots \qquad t_{n-1} \qquad\qquad\qquad t_n$$

2.3 Hypergraphs

A hypergraph can represent a view in which the entities in the present are nodes bearing additional structure (edges) which represent the past state and the way the past has become the present. We explain hypergraphs below, and then show how accumulation is seen as an operation describing the change from one time to the next.

A hypergraph is essentially a generalization of the notion of graph in which an edge may be incident with an arbitrary number of nodes and not just one or two. A relation $R \subseteq X \times Y$ is really just a hypergraph in disguise: the elements of X being the edges, the elements of Y being the nodes, and $x \; R \; y$ holding iff node y is incident with edge x.

Definition 2 *A **Hypergraph** H consists of sets V_H of vertices and E_H of edges (where $V_H \cap E_H = \varnothing$), and an incidence relation $i_H : E_H \to \mathcal{P}(V_H)$.*

A hypergraph differs from an undirected graph in that an edge may be incident with an arbitrary set of nodes and not just one or two. Note that we do allow edges incident with the empty set of nodes, $\varnothing$. A hypergraph with edges E and vertices V is the same as a relation from E to V; an edge is related to the set of vertices with which it is incident. This is illustrated in Figure 4 for the relations Q and R from Figure 1. In the figure the hyperedges appear as loops enclosing their incident nodes.

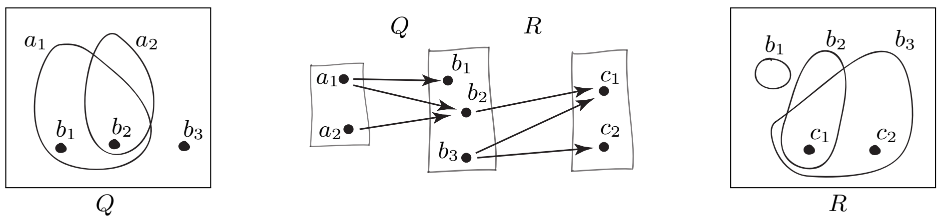


Fig. 4. Relations Q and R from Figure 1 as hypergraphs

The accumulation of two relations can be described in terms of hypergraphs.

Definition 3 *Let G and H be hypergraphs where $V_G = E_H$. We define the hypergraph $G \star H$ to have vertices V_H, and edges $E_G \cup \{v \in V_G \mid i_G^{-1}(v) = \varnothing\}$. The incidence relation j is given by*

$$j(x) = \begin{cases} \bigcup\{i_H(v) \subseteq V_H \mid v \in i_G(x)\} & \textit{if } x \in E_G \\ i_H(x) & \textit{if } x \in \{v \in V_G \mid i_G^{-1}(v) = \varnothing\} \end{cases}$$

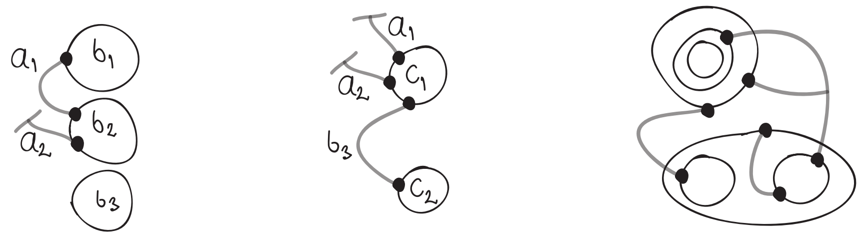
3 Bigraphs: Static Aspects

In the previous section we considered entities subject to change, but without modelling any spatial relationships between these entities. If we introduce spatial structure in addition to the links representing shared ancestry between nodes then we have essentially the bigraphs introduced by Milner [11].

3.1 Bare Bigraphs

To illustrate the basic features of bigraphs we continue with the relations $Q : \{a_1, a_2\} \to \{b_1, b_2, b_3\}$ and $R : \{b_1, b_2, b_3\} \to \{c_1, c_2\}$ used in the earlier examples. These provide us with bare bigraphs which are a simple case of the general notion of a bigraph which has interfaces so that it can be combined with other bigraphs as described in section 3.2 below.

If we assume that all the entities involved $(a_1, a_2, \ldots)$ are spatially disjoint we arrive at Figure 5. This figure shows the usual means of depicting bigraphs with the spatial entities shown as discs in the plane and the links connecting them drawn as lines attached to the discs. This differs from the more usual way of drawing an edge in a hypergraph as a boundary containing those nodes with which it is incident. We used this edge-as-container vizualization in earlier figures with hypergraphs, but this only works when the nodes do not have a spatial extent.



Bare bigraph for Q bare bigraph for $Q \star R$ Bare bigraph with nesting
(assuming $\{b_1, b_2, b_3, c_1, c_2\}$ spatially disjoint)

Fig. 5. Examples of bare bigraphs

The examples in Figure 5 of the bigraphs for the relations Q and $Q \star R$ are particularly simple in that the nodes are spatially disjoint. In general nodes may be nested with each other, as indicated in the example at the right of Figure 5. The place structure (that is the nesting of nodes) is independent of the link structure (that is the edges of the hypergraph part of the bigraph). This means that although it is significant when nodes are drawn inside other nodes, there is no significance attached to where the links cross the boundaries of nodes

The bare bigraph for Q shown in Figure 5 has one edge (a_2) that is incident with just one node. This has been drawn in the diagram as a link which has been terminated, not linking the node to anything. This follows Milner's diagrams [11] but in other contexts such edges are often drawn as loops with both ends attached to the incident node.

3.2 Substitution

Bare bigraphs display the key features of linking and placing, but an important aspect of the theory of bigraphs is the way that they can be combined with each other. By means of these combinations, complex bigraphs can be constructed out of simpler components, and there are two kinds of composition which enable this. To define these compositions a bigraph needs to contain not only nodes connected by links and by place connections (nestings) but also to have additional machinery to allow substitution. This means that a bigraph can be inserted into a larger context and it can also act as a context into which more detail is inserted.

It may be helpful to give an analogy with simple algebra in which letters stand for numbers. A complicated expression such as $(3x+2y)^2+2(3x+2y)+1$ can be built out of the simpler expression z^2+2z+1 by replacing z by $(3x+2y)$. Informally, the z in z^2+2z+1 acts as a 'hole' which can be 'filled in' by the expression $(3x+2y)$.

General bigraphs may contain 'holes' of two types called *sites* and *inner names* into which 'fillers' called respectively *roots* and *outer names* may be placed. These additional features are illustrated in Figure 6. The ability to compose bigraphs makes them morphisms in a category where the objects (known as *interfaces*) are pairs $\langle m, X\rangle$ where m is essentially the number of place holes and X is a finite set of names. The number m is treated as a finite ordinal, that is a natural number viewed as a set of smaller ordinals $m = \{0, 1, \ldots, m-1\}$.

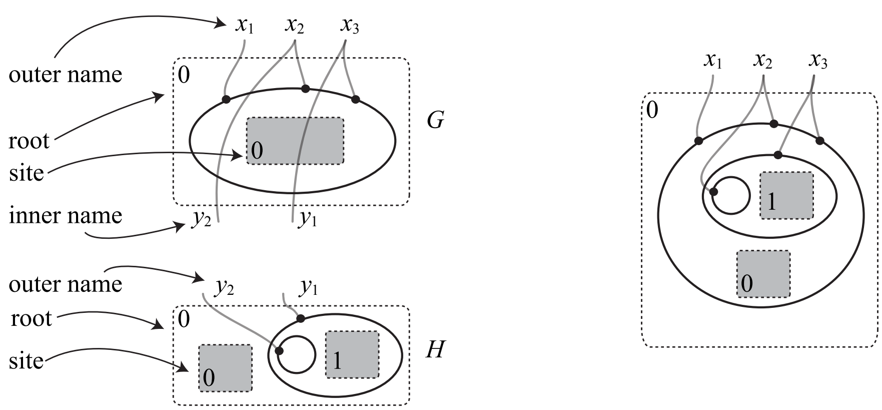


Fig. 6. Bigraphs $G : \langle 1, \{y_1, y_2\}\rangle \to \langle 1, \{x_1, x_2, x_3\}\rangle$ and $H : \langle 2, \varnothing\rangle \to \langle 1, \{y_1, y_2\}\rangle$ and the composite $G \circ H$

Although the operations $\circ$ on bigraphs and ; on relations are both called 'composition' they are unrelated. Relations between spatially nested entities can be modelled by bigraphs and the composition of relations can thus be modelled by an operation on bigraphs. However this operation would only be defined under conditions that would be very different from the conditions under which $\circ$ is defined and the two operations are quite separate.

3.3 Ports

Bigraphs also provide a set $\mathcal{K}$ of types for nodes, called a signature. Each $k \in \mathcal{K}$ has an arity, which gives the number of ports through which attachments to a link (hyperedge) may be made for nodes of type k. These ports are shown as black discs in the diagrams. For example, in the examples we provide later we use different types of node for buildings, suburbs and crowds. In this setting the arity of a particular type of crowd is the number of instances of 'original crowds' whose members are present in it. It should be noted that the formal definition of bigraphs [11, p15] allows a link to be connected to the same node by a number of different ports. This means that the link structure is actually more general than a hypergraph as defined above since each edge may be incident with a multiset (or bag) of nodes and not just a set.

3.4 Tensor product and derived operations

Besides the operation of composition, $\circ$, bigraphs also support an operation $\otimes$ known as the tensor product. This is easy to visualize and corresponds to placing bigraphs with disjoint names alongside each other aligned horizontally.

Further operations that we use in formulating the rules later in the paper can be expressed in terms of composition and tensor product. These are the parallel product, ($\|$), nesting (.), and the merge product ($|$). A full account of these operations would occupy more space than we have available, and [11] should be consulted for details. Briefly, however, $G \parallel H$ is similar to the tensor product except that common outer names are shared. The nesting $G \,.\, H$ places H inside G and allows the outer names of H to be visible. The merge product $G \mid H$ merges roots in addition to sharing links as in the parallel product.

3.5 Modelling Parents and Children

We introduced hypergraphs by showing how edges might represent the sharing of common ancestors between entities. When in addition entities have spatial structure limited to nesting a bigraph can represent both the filiation links and the nesting relationships. The original motivation for bigraphs uses the links to model communication of various types.

An individual person can be represented by a node with three ports where we can attach links to (1) their mother, (2) their father, (3) their children. From this example it is clear that the link structure is independent of the spatial structure: who a person is related to has no bearing on their location. It should also be noted that although the links have no specified direction, we can make use of the signature to use particular ports in particular ways. By this means we can tell for example that a link from port 3 of node a to port 1 of node b means that a is a child of b.

The simple notion of links to represent ancestry can be used also in other situations where there has been some transmission of material, such as one might want to observe in a communication between suspected terrorists in a surveillance operation.

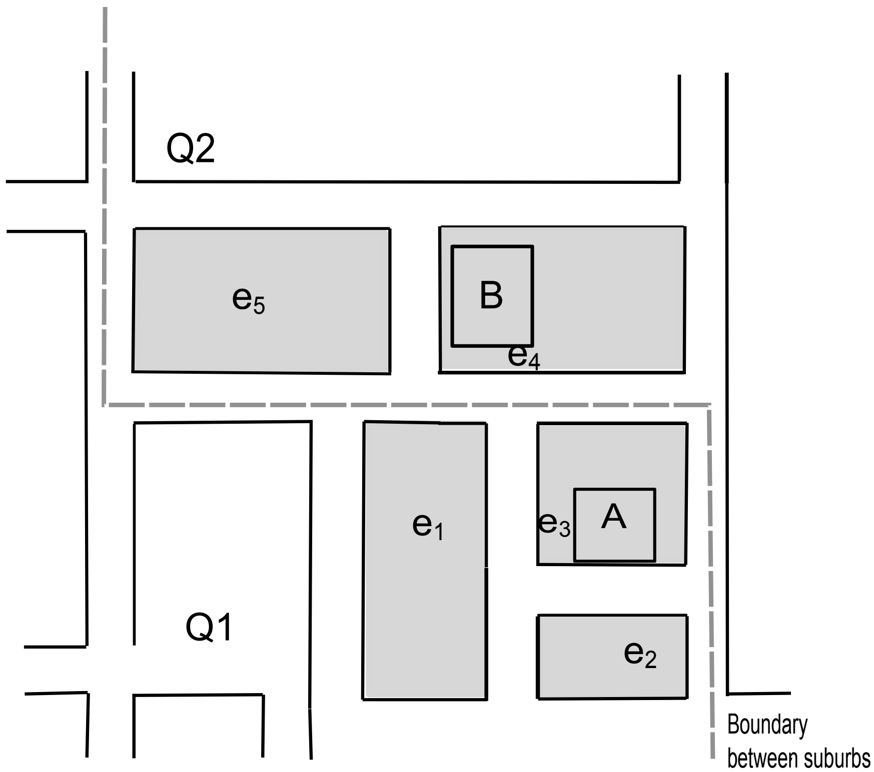


Fig. 7. Two suburbs Q_1 and Q_2 and their areas and specific buildings. The dashed line shows the boundary between suburbs.

4 Case study

We now present a case study that involves crowds of people that move in and between suburbs of a city and where the crowds can split and merge over time. Figure 7 shows a portion of the city where the action takes place. This example reflects the evolution of groups of people in a city during a demonstration. We assume that the entities to be modelled are groups of people, and that the identity and filiations of an entity are determined by the people that compose this group.

4.1 Overall Scenario

We consider the following four types of group which vary according to their behaviour and the distinction between these types would be significant in a surveillance operation. Different instances of these types arise for different values of i.

- C_i: Demonstrators
- W_i: Pedestrians not involved in the demonstration
- O_i: Observers
- G_i: Unidentified people

We can record the filiation using the technique presented in [3] which distinguishes between derivation and continuation. The case of continuation models the preservation of identity (such as an individual persisting throughout their life), and derivation models a new entity which depends in some manner on an earlier but distinct entity (for example a child could be modelled as a derived entity from each of their parents). We assume that filiation in the present scenario is determined as follows:

(1) If one or more people leave a group X and join another group Z, and/or if person(s) from another group Y join a group X, then there are filiation links of type derivation δ between X and Y, and X and Z. (Figure 8(a) and 8(b))
(2) Between two times, if a group X remains the same without any addition/deletion of people, it is considered in continuation relation γ. (Figure 8(c))

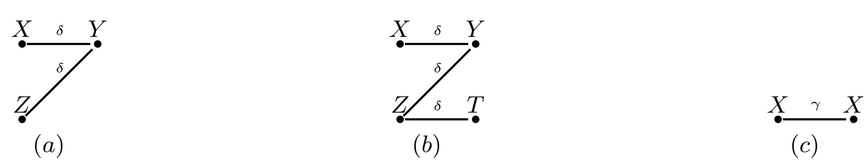


Fig. 8. Filiations: (a) Derivation: persons from Z and X join and create Y, (b) Derivation: some persons from Z and all persons from X create Y, the rest of the people from Z create T (c) Continuation: X remains the same

In the course of the demonstration, the different groups move around in the city. We suppose that the largest spatial unit we consider in the city is a suburb, and that there are two of these: Q_1 and Q_2 (Figure 7). Within suburbs there are areas determined by the street pattern and within some of these areas particular buildings have been identified as significant.

- Q_1 contains three areas (e_1, e_2 and e_3) and a building A located in e_3.
- Q_2 contains two areas (e_4 and e_5) and a building B located at e_4.

4.2 Filiation Relations

Groups of people can may combine with each other, and they may divide into pieces. For example, the filiation relations shown in Figure 9 shows that a part of group C_2 of demonstrators joins the group C_1 to become G_1. Groups are renamed when we consider that there is a change of their identity.

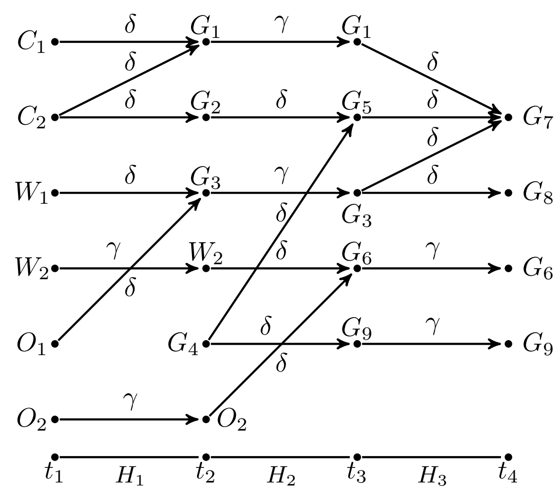


Fig. 9. Filiation relations $H_1 - H_3$ between times $t_1 - t_4$

Between the four times $t_1 - t_4$ there are three relations $H_1 - H_3$. Using the cumulative product as described in section 2 we can compute $H_1 \star H_2$, and this is illustrated in Figure 10. As the group G_4 appears only at time t_2 and is not present at t_1 the cumulative product is able to record the fact that G_5 and G_9 have a common ancestor group. If we use the conventional composition $H_1 ; H_2$ then this information, which could well be significant in a surveillance application, would not be available.

4.3 Bigraph modelling

Figure 11 represents the state of the region of study for t_1 and t_4 with groups defined in Figure 9. This spatial information, togther with the filiations, is trans-

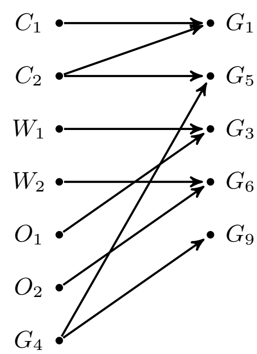


Fig. 10. Cumulative product $H_1 \star H_2$ showing that common ancestry of G_9 and G_5 is recorded whereas in $H_1\,;H_2$ it is forgotten

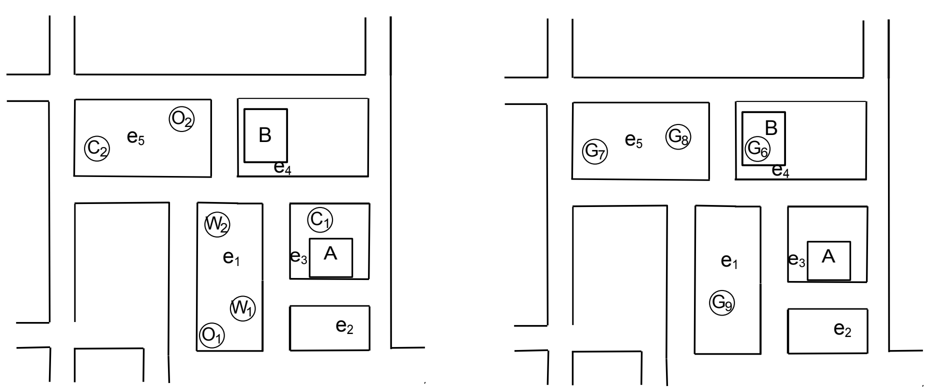


Fig. 11. Location of entities at t_1 (left) and t_4 (right)

lated into the bigraph setting in Figures 12 and 13. In the first of these figures we provide the bigraph for the whole spatial area by presenting it as composite $S \circ K_1$. This demonstrates another valuable feature of bigraphs as a spatial modelling tool: their ability to deal with spatial granularity. The bigraph S represents a low level of detail in which the two suburbs are distinguished but nothing is said about what may be found within them. The action of adding this detail and passing to the fully detailed description $S \circ K_1$ corresponds precisely to the operation of composition for bigraphs.

As the change only affects a level of detail more specific than that modelled by S, we are able to show the changes at times $t_2 - t_4$ by just showing the bigraph which is composed with S. The three bigraphs we need are given in Figure 13. In these a link between two groups appears if there is a filiation between their ancestors. For example, at time t_3, there is a filiation link between G_9 and G_5 because each contains some part of G_4. Similarly, the link between G_1 and G_2 at t_2 leads to a link between G_1 and G_5 at t_3. Here G_1 remains the same between these two times, and G_2 is only changed by the addition of people.

We have derived the bigraphs in Figures 12 and 13 by using the filiation data from Figure 9 and adding hypothetical spatial information. This approach means that we have treated each bigraph as a static snapshot of the situation at a given time, albeit a snapshot that contains some additional information about the ancestry of the groups present. This is certainly useful, but the full power of bigraphs only becomes apparent once we include rules as part of our modelling that specify how a bigraph at one stage may evolve into one at a subsequent stage. The introduction of rules is also significant in that by permitting only changes that are possible by given rules we can enforce integrity constraints in the model and ensure that semantically invalid changes are prohibited, such as moving one suburb inside another. In the next section we show how such rules can be introduced.

5 Bigraphs: Dynamic Aspects

5.1 Rewriting and Composition

We have seen that bigraphs represent spatial nesting and links. These links may be given several different interpretations, including channels of communication and records of communication having ocurred in the past. Both the place graph and the link graph can be subject to change, and in general these two features can change independently. To understand the mechanism of reaction rules, as they are called in the bigraph context, it may be helpful to consider the basic algebraic idea of rewrite rules.

An equation $-(-x) = x$ may be seen as a rewrite rule $-(-x) \rightarrowtriangle x$ allowing the left hand side to be replaced by the right hand side. Such a rule can be used in a context larger than the left hand side, allowing for example $42 + (-(-x))$ to be replaced by $42 + x$. The rule can also be used when some expression is substituted for x, for example allowing $-(-(y + 3))$ to be rewritten to $y + 3$.

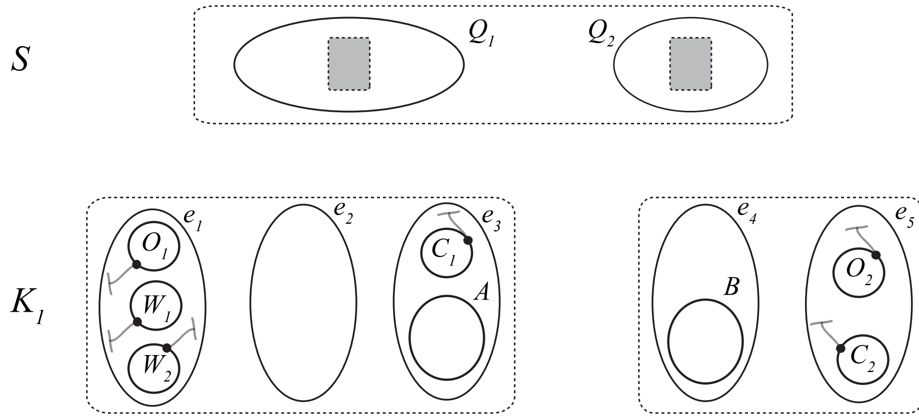


Fig. 12. Entire bigraph at t_1 is the Composite $S \circ K_1$

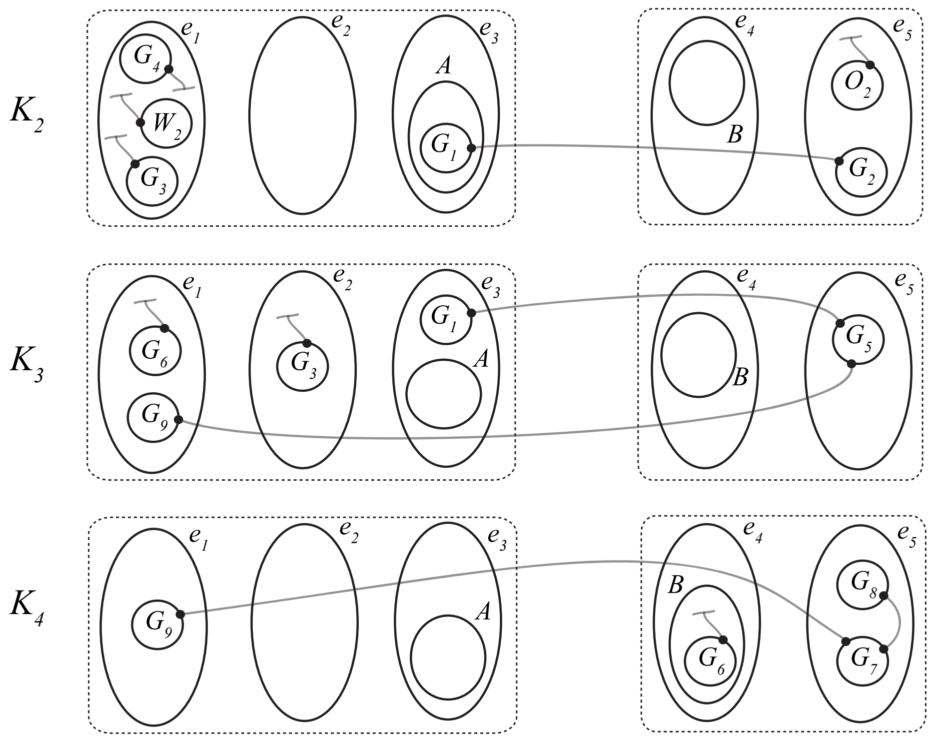


Fig. 13. Bigraphs K_2, K_3, K_4 corresponding to times t_2, t_3, t_4

In expressing spatial dynamics with bigraphs a particular kind of change will consist of replacing one bigraph by another, just as we can replace $-(-x)$ by x in the above example. The same kind of change may take place in many different contexts. In bigraphs this corresponds to the fact that if $H \rightarrowtriangle K$ then $G \circ H \rightarrowtriangle G \circ K$ (assuming the composition is defined). Also the same kind of change may be made more specific in many different ways, just as essentially the same change is happening in rewriting $-(-(y+3))$ to $y+3$ as in rewriting $-(-(2y+2))$ to $2y+2$. This situation correponds to the fact that for bigraphs if $H \rightarrowtriangle K$ then $H \circ G' \rightarrowtriangle K \circ G'$ again assuming the composition is defined.

5.2 Rules for the Case Study

Here we present rules that enable the features of the case study to be modelled. To extract the most significant features of the case-study, we assume that there are just crowds of people without distinguishing different types. This restriction could be lifted by introducing a more elaborate signature, but would not involve any essentially different features of rules.

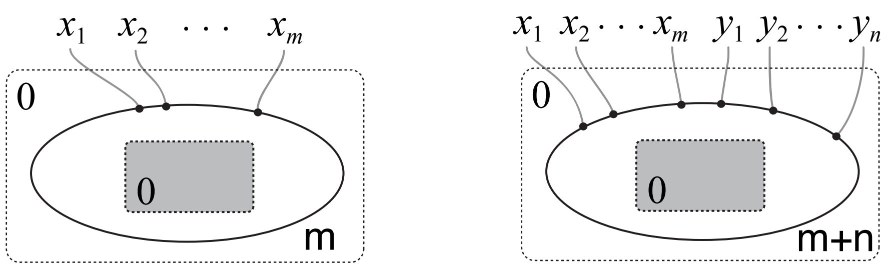


Fig. 14. The discrete ions m_x and $(\mathsf{m}+\mathsf{n})_{xy}$

For each crowd we are interested in what earlier groupings constitute the crowd. To formulate the rules we need to introduce some additional technicalities that were not necessary to convey the main ideas of the case study in the previous section. Milner [11, p30] uses the term *discrete ion* for a bigraph having a single node containing the single site of the bigraph, where also there are no inner names and the outer names are linked bijectively to the ports of the single node. Our signature has one type of node for each possible arity, where each port is capable of modelling a particular ancestor crowd. Thus a crowd constituted from three earlier ones needs three ports. This is illustrated in Figure 14 showing a discrete ion of arity m and type m. When the inner names are $x_1, x_2, \ldots, x_m$ we denote the ion by m_x. To model the merging of two crowds, given for example by nodes of types m_x and n_y, we need to refer to a node having arity $m+n$ with inner names $x_1, x_2, \ldots, x_m, y_1, y_2, \ldots, y_n$. We denote the type of such a node by $(\mathsf{m}+\mathsf{n})_{xy}$.

In addition to the crowds, we need to model the buildings, areas and suburbs introduced in the case study. For these we use nodes of arity 0, since we do not model the historical development of these entities. The signature includes types A for areas, B for buildings and S for subsurbs.

As a first example, the rules need to permit a crowd to enter, say, a building and to leave the building. This is straightforward, and is illustrated in Figure 15. The idea of entering or leaving a building is already well-known and appears as one of the motivating examples in [11]. However, the rules we present in Figures 16, and 17 represent a novel use of bigraphs in their use of links to model shared ancestry.

The two rules shown in Figure 16 allow one crowd to divide into two, and allow two crowds to merge into one. These can be used in cases such as the splitting of G_4 into G_5 and G_9 in our case study.

The rules shown in Figure 17 provide additional capabilities. These permit one crowd to surround another but to remain distinct. This could arise when a group of police surrounds a small crowd of deomstrators and forces them to move to another location, keeping them surrounded while moving. Although this behaviour is not illustrated by the case study, we include these rules as evidence of the power of bigraph rules to model more elaborate kinds of change.

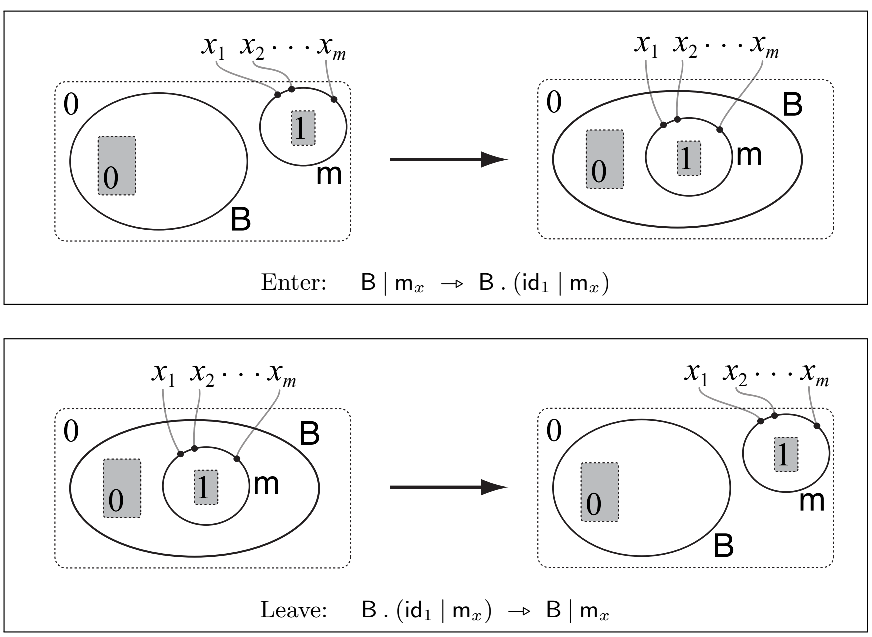


Fig. 15. Entering and leaving a building

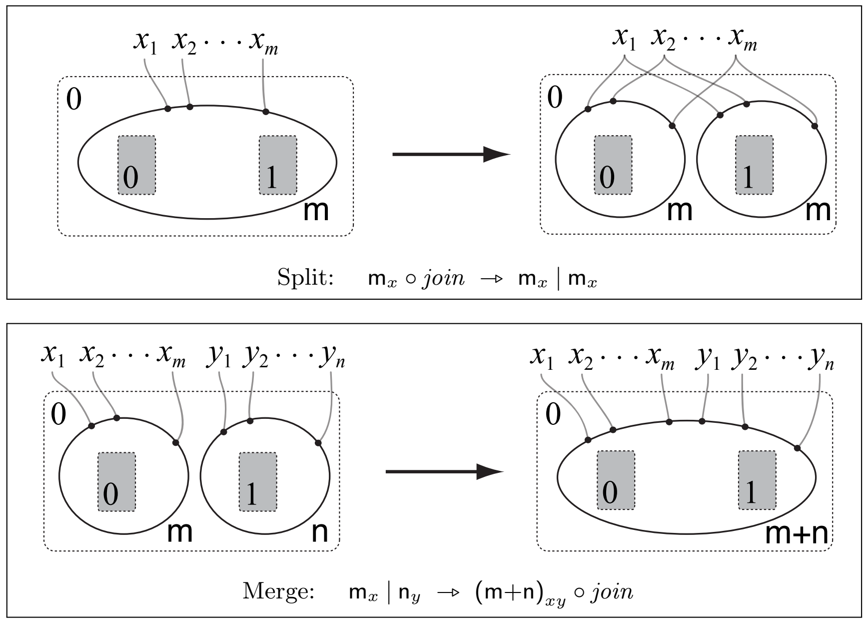


Fig. 16. Crowd Split and Merge Rules

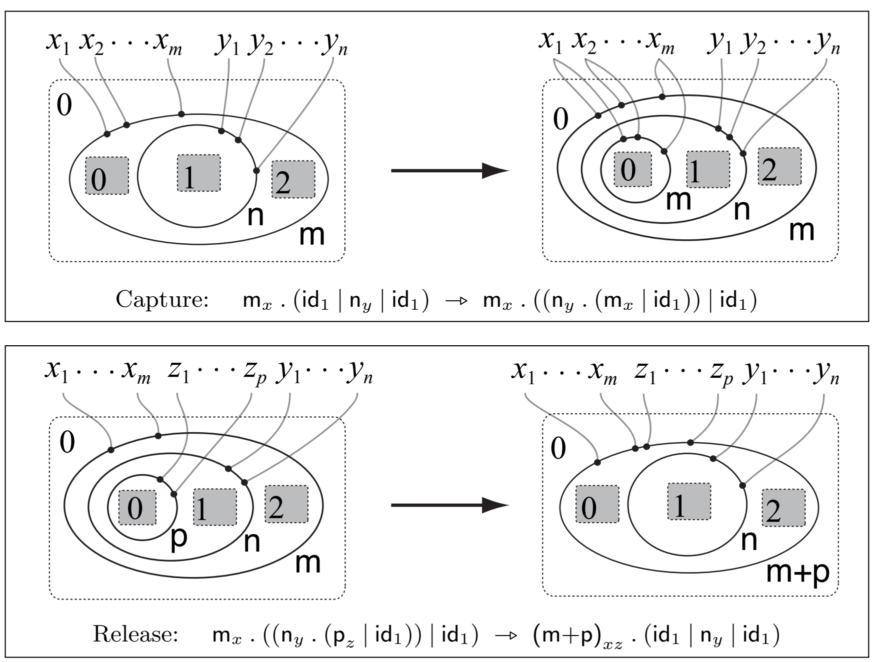


Fig. 17. Crowd Capture and Release Rules

6 Conclusions and Further Work

We have given an expository account of the basic features of bigraphs and we have shown how a novel interpretation of the communication links as shared ancestry can be incorporated into models of spatio-temporal change. This interpretation is based on a way of combining relations, the cumulative product, that has advantages over the conventional composition operation. While this product is unlikely to be mathematically novel, we are not aware that it has been used before in the context of monitoring change in applications such as our case study.

We have formulated bigraph reaction rules which can be used to model the splitting and merging of crowds of people and we have given further rules that model more elaborate behaviour including one group surrounding another so as to contain it.

Further work is necessary to analyse the theoretical properties of particular systems of rules. This could establish what kinds of spatio-temporal change are possible from particular rules. There will be close connections between the behaviour of the split and merge rules for bigraphs and the splitting and merging studied in [9, 15]. There are also many possible application problems for spatio-temporal analysis described in the literature cited in the introduction. Further evaluation of the value of bigraphs needs to take place using some of these problems. However, the evidence we have presented here demonstrates that bigraphs have several capabilities that are valuable in the modelling of spatio-temporal change.

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