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Design Philosophy Issues of FRP RC Structures

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Abstract: The conventional design philosophy for reinforced concrete (RC) relies heavily on the ductile properties of steel. These ductile properties are used as a 'fuse' and conceal the large uncertainty in the determination of modes of failure caused by concrete. Current design guidelines for FRP RC structures have inappropriately adopted the same design philosophy used for steel RC, leading either to the adoption of large safety factors or reduced structural reliability. A reliability-based analysis of FRP RC beams shows that the partial safety factors for FRP reinforcement on their own do not influence the structural safety of over-reinforced concrete elements. Proposals are made for the modification of the material partial safety factors to achieve target safety levels.

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Introduction

The widespread adoption of any new type of reinforcement, such as fiber reinforced polymers (FRP), requires the development of product specification, testing standards and codes of design practice, a process that can take many years to be completed.

Though research in this field is very recent, the first generation of design guidelines has already been developed in Japan, Canada, America and Europe (JSCE 1997, CHBDC 1996, ACI 440 2001, IGDRCS 1999). All of these guidelines are based on modifications of existing codes of practice for steel reinforced concrete (RC) structures, which, though they have the clear objective of achieving flexural failures through steel yielding, do not have an identifiable design safety philosophy (DSP). It should be pointed out, that the codes have different DSP, with the ACI code relying on material reduction factors, whilst the other codes using partial material safety factors. In addition, even when these guidelines are used for the design of conventional RC structures:

1. the actual safety, or reliability, levels of the structures are unknown, (even though standards such as the Eurocode 1 require reliability levels of 10^{-6})
2. the actual reliability levels vary for different structural elements (depending on the actions and resistance mechanisms), and
3. the capacity margin between each failure mode is unknown (for example the capacity margin between flexure and shear, making seismic design codes such as Eurocode 8 to adopt radical solutions in order to avoid shear failures).

The above problems are worse when dealing with FRP RC elements, since the predominant mode of failure is likely to be dependent on the concrete rather than on the reinforcement.

The first generation design guidelines for FRP RC structures are mainly provided in the form of modifications to existing steel RC codes of practice, which are predominantly using the limit state design approach. The modifications consist of basic principles, which are heavily influenced by the unconventional mechanical properties of FRP reinforcement and empirical equations that are based on insufficient experimental work on FRP RC elements. Though the brittle linear-elastic behavior of FRP reinforcement is an influencing factor behind all of the existing design guidelines, the impact of the change of failure mode is not addressed in detail.

When dealing with FRP reinforcement, the amount of reinforcement to be used has to be determined by a different approach due to the lower stiffness and the high strength of composite materials. In fact, for FRP reinforcement, the strength to stiffness ratio is an order of magnitude greater than that of steel and this affects the distribution of stresses along the section. Hence, when considering a balanced section, a condition desired for steel RC design, the neutral axis depth for FRP RC sections would be very close to the compressive end as shown in Fig. 1.

This implies that for such a section a larger amount of the cross-section is subjected to tensile stresses and the compressive zone is subjected to a greater strain gradient. Hence, for similar cross sections to that of steel, much larger deflections and less shear strength are expected (Fig. 2).

If all of the other modes of failure are avoided, flexural failure can be reached either by crushing of the concrete in compression or by rupture of the FRP reinforcement in tension. Both modes are brittle and undesirable. Whichever is the desired mode of flexural failure, this is attained primarily by the application of specific material partial safety factors for FRP reinforcement (γ_{FRP}) or member strength reduction factors. The application of γ_{FRP} implies that there is a target structural reliability level (P_{ft}). Recent investigations, however, have shown that the application of specific γ_{FRP} would neither lead to the desired mode of flexural failure nor would attain the target P_{ft} (Neocleous et al.1999).

This paper initially presents and discusses the variety of safety factors currently adopted by the current first generation design guidelines and then goes on to examine the above issues further for over-reinforced beams designed to resist uniformly distributed loads in accordance to the preliminary design guidelines developed in Europe (IGDRCS 1999). The work has been carried out as part of the European Union sponsored TMR network "ConFibreCrete" and task group 9.3 of the International Federation of Concrete (*fib*), whose aim is the development of design guidelines for concrete structures reinforced, prestressed or strengthened with advanced composites.

The probability of flexural failure occurring due to concrete crushing (P_{fc}) and the flexural notional structural reliability level (P_f) of 48 rectangular beam configurations, reinforced with Eurocrete FRP reinforcement (Eurocrete Project 1997), are determined for a number of γ_{FRP}. The investigation is carried out for two cases: a) simply supported beams reinforced with carbon FRP

(CFRP) reinforcement and b) simply supported beams reinforced with glass FRP (GFRP) reinforcement.

From the results of the above research the paper goes on to identify the DSP problems that need to be addressed before the emergence of a new generation of design guidelines for FRP RC structures.

Current Safety factors

The most recent Japanese design guidelines for FRP RC (JSCE 1997), which are based on modifications of the Japanese steel RC code of practice, provide a set of material partial safety factors for the FRP reinforcement as indicated in Table 1. These guidelines, however, do not provide any information regarding the predominant mode of failure that would result from the application of the proposed partial safety factors, nor do they cover product specification. Hence the small safety margins given may not adequately cover reinforcement materials which have large variability in their properties.

The Canadian design guidelines (CHBDC 1996) provide only general information about FRP reinforcement.

The American design guidelines (ACI 440 2001) are based on modifications of the ACI 318-99 (ACI 318 1999) RC code of practice. These guidelines propose that the predominant mode of failure is flexural concrete crushing rather than flexural re-bar fracture. Thus, a minimum limit is imposed on the amount of FRP reinforcement in order to attain the desired predominant failure mode. The guidelines also adopt a conservative approach for the derivation of strength reduction factors, ϕ. This is due to the fact that there is very little data regarding the service and long-term behavior of FRP RC structures. Thus, a ϕ of 0.7 is recommended for flexure, whereas for shear, it is recommended that the value of ϕ be the same as the value adopted by ACI 318-99.

Though the impact of a strength reduction factor is easier to understand than the impact of partial safety factors, its use can lead to very different levels of safety, depending on the concrete strength and reinforcement ratio. Furthermore, the use of a member reduction factor does not help the engineer understand the overall stress levels in the constituent materials, which means that the level of stress in concrete may be higher than for conventional reinforcement. Since strength reduction factors are not supposed to be derived on the basis of reliability, they will not be addressed further in this paper.

In the case of the European design guidelines (Clarke et al. 1996, Eurocrete report 1997), the recommendations are based on modifications to British and other European RC codes of practice such as Eurocode 2 (ENV 1991-1 1994, ENV 1992-1-1 1992). These guidelines were published in the UK as an interim guidance on the design of FRP RC structures by the Institution of Structural Engineers (IGDRCS 1999). These guidelines include a set of partial safety factors for the material strength and stiffness (Table 2) that take into consideration both the short and long-term structural behavior. They do not, however, provide clear indications about the predominant failure mode that would result from the application of these partial safety factors. The composite action of the reduction factors on the strength and stiffness leads to very conservative results, in particular when using shear reinforcement.

The initial approach of developing design guidelines such as those described above may seem reasonable, but it is not entirely appropriate. The conventional steel RC codes of practice assume that the predominant failure mode is always ductile due to yielding of the flexural reinforcement. This is not the case, however, for the above FRP RC design guidelines, which seem to accept a brittle flexural failure due to concrete crushing. Furthermore, the steel RC codes of practice, which form the basis of these guidelines, have fundamental structural safety uncertainties. These include the derivation of the partial safety factors, the actual structural reliability levels and the resistance capacity margins (RCMs) between the various failure modes.

In order to determine the actual structural reliability levels, it is necessary to have accurate predictive resistance capacity models, a good understanding of the variability of materials and an accurate assessment procedure. With these concerns in mind, both steel and FRP RC elements were analyzed in a comprehensive research program at the University of Sheffield (Neocleous, 1999) and some of the results are given below.

Resistance-capacity prediction models

Structural reliability assessment requires the formulation of a model (limit state function) that represents the structural behavior for the limit state for which the assessment is performed. The limit state function, $G(R, S)$, is represented in terms of a structural resistance component, R, and an action-effect component, S. Both R and S are modelled by mathematical relationships of random basic variables, R_i and S_i, which represent structural material properties and actions, respectively. Due to the complexity of the problem, this paper will try and deal only with the issue of flexural failures, even though work has also been completed on shear and bond failures.

The structural reliability of steel RC beams is assessed in terms of the BS8110 (1997) and Eurocode 2 (ENV1992-1-1, 1992) codes. Hence, the $G(R, S)$ is formulated using the resistance-capacity prediction models adopted for the above failure modes by the two codes of practice. The Eurocode 2 models are elaborated in the appendix and since the BS8110 models are very similar, they are not given in this paper.

In the case of the FPR RC beams, the structural reliability assessment is performed in terms of the European design guidelines for FRP RC structures (Clarke et al, 1996). These guidelines conform to existing European RC codes of practice and thus, the proposed models are based on the models adopted by these RC codes of practice. The model for the flexural failure mode, elaborated in Appendix II, is based on the control of strain of the FRP reinforcement and utilizes the principles of the corresponding Eurocode 2 model.

Statistical data for basic variables

The statistical data used in earlier investigations by the authors are utilized for the statistical modeling of all random variables (Neocleous et al. 1999, Neocleous 1999). Tables 3 and 4 summarize the statistical data used for the geometric and loading variables. In the case of the material strength of concrete and FRP reinforcement, the adopted statistical data are derived from the analysis of experimental results provided by manufacturers. A constant standard deviation of 6 N/mm^2 is used for the concrete compressive strength and the probability distribution is truncated at 3.16 standard deviations from the mean value. In the case of CFRP and GFRP (Eurocrete) reinforcement (Table 5), the strength obtained experimentally in RC beams was adopted, since existing direct tensile tests fail to simulate the in-service load-transfer mechanisms for FRP reinforcing bars.

Assessment Procedure

A numerical simulation approach is adopted to determine the P_{fc} and flexural P_f; the simulations are performed using MATLAB (MATLAB 1999). The procedure followed in the assessment is illustrated in Fig. 3, and is further elaborated in Neocleous (1999). It is noted that the resistance-capacity models adopted by the European design guidelines, are modified accordingly to account for flexural failure occurring either due to concrete crushing or reinforcement fracture.

The flexural P_f and P_{fc} are evaluated by equations 1 and 3, respectively.

$$\overline{P}_f = \frac{\sum_{i=1}^{N} P_{f\,i}}{N} \qquad (1)$$

$$P_{fi} = P(Q > R_i - G_i) = 1 - F_Q(R_i - G_i) \qquad (2)$$

$$P_{fc} = \frac{\sum_{i=1} \varepsilon_c \geq 0.0035}{N} \qquad (3)$$

ANALYSIS OF SIMULATIONS

The flexural design of CFRP RC beams was based on the assumption that a brittle failure would occur due to concrete crushing. In the case of beams with low reinforcement ratio (ρ) and high concrete compressive strength (f_c), however, it was assumed that a brittle failure due to fracture of the reinforcement would be possible. This is a consequence of the fact that the actual tensile strain developed in the reinforcement at the design stage (ε_{FRP}) exceeds the design limit (ε_{FRPd}) imposed by the γ_{FRP}. It is noted that this was only observed in six of the examined cases of CFRP RC beams designed using a γ_{FRP} of 1.8 (Fig. 4).

In the case of the GFPR RC beams designed using a γ_{FRP} of 3.6, it was assumed that brittle failure would occur due to fracture of the longitudinal reinforcement, due to the low strength of the reinforcement . Whereas, in the case of GFRP RC beams designed using a γ_{FRP} of 1.3, only three beams had to be designed for fracture of the reinforcement (Fig. 5).

The results obtained for P_{fc} indicate that the majority of beams would actually fail due to concrete crushing, as intended at the design stage. In some cases, however, there is a large probability (up to 0.2 for CFRP RC beams and 0.73 for GFRP RC beams) of failure due to fracture of the reinforcement. This was observed when: a) the design assumed that failure due to fracture of the reinforcement would occur, and b) the value of ε_{FRP} was relatively close to ε_{FRPd}. In addition, it is observed that the P_{fc} decreases as f_c increases. This can be attributed to the fact that the tensile strength of FRP reinforcement is further utilized. For instance, in the case of the CFPR RC beams, P_{fc} decreases from 0.97 to 0.27 as f_c increases from 33 to 50 N/mm^2.

Fig. 6 shows that the flexural P_f is not affected by γ_{FRP}, as long as the type of failure assumed at the design stage is concrete crushing. In such cases, the flexural resistance capacity remains constant since the ε_{FRP} does not change with γ_{FRP}. Thus, if concrete crushing is chosen as the desired mode of flexural failure, it may be seem sensible to discard γ_{FRP} and incorporate the uncertainties relevant to flexural reinforcement in the partial safety factor adopted for f_c. As Fig. 7 indicates, the P_f is affected by γ_{FRP} if flexural failure occurs due to reinforcement fracture.

In Fig. 6 and 7, it is also observed that the structural reliability generally satisfies the target value of 10^{-6} adopted by Eurocode 1 (ENV 1991-1 1994), however, the figures indicate that the calculated P_f is highly variable. This is due to the effect of various design parameters as illustrated in Fig. 8 and 9 for CFRP and GFRP RC beams, respectively. These figures suggest that the ratio of permanent to variable load greatly influences the flexural P_f. This implies that, for the same γ_{FRP}, the structural reliability varies for different types of structures and hence, in order to avoid reliability differentiation, it is recommended to use different γ_{FRP} (or more appropriately, load factors) for different types of structures. In addition, Fig. 8 and 9 indicate that the ratio of ρ and f_c influence the P_f of both CFRP and GFPR RC beams. The results of the assessment show that the effect of these two parameters is greatly affected by the type of flexural failure assumed at the design stage. If the flexural design assumes reinforcement fracture, the P_f is influenced by both f_c and ρ (P_f increases with ρ, while it decreases as f_c increases) (Fig. 9). If the flexural design assumes concrete crushing, however, the P_f is only affected by f_c (Fig. 8). It is noted that in this case the P_f is more uniform across the range of beam configurations examined.

DESIGN RECOMMENDATIONS

Based on the findings of the analysis, design recommendations are proposed for the flexural (short-term) design of over-reinforced FRP RC beams, reinforced with (Eurocrete) FPR reinforcement. Since the results show that concrete crushing is the most probable type of flexural failure, it is recommended that flexural design be carried out to attain concrete crushing. This can be achieved by ensuring that minimum amounts of flexural reinforcement are provided, which will also protect the structural elements from large cracks developing as soon as the concrete tensile stress is exceeded.

Provided that flexural failure occurs due to concrete crushing, it was determined from the analysis that the use of γ_{FRP} to account for the uncertainties in the mechanical characteristics of the FRP reinforcement is not vital, since the flexural P_f is not affected by γ_{FRP}. Based on this finding, it is proposed that the uncertainties relevant to mechanical characteristics of the flexural reinforcement should be incorporated into the γ_m adopted for f_c, which would involve the modification of concrete γ_m used currently in flexural limit state design.

The mechanical behavior of FRP reinforcement in flexure is not established thoroughly, since accepted standards for the determination of the mechanical characteristics that take into account the behavior of FRP bars in concrete are not yet available. In addition, since FRP reinforcement is primarily intended for use in aggressive environments, its long-term characteristics in concrete should also be taken into account. It will not be prudent, therefore, to abolish the use of γ_{FPR} based on the existing knowledge. Consequently, it is recommended to adopt the smallest γ_{FRP}

examined during the assessment of the CFRP and GFRP RC beams. In the case of CFRP reinforcement, a value of 1.15 is recommended and a γ_{FRP} of 1.3 is selected for GFRP reinforcement. It should be noted that the different γ_{FRP} - recommended for Eurocrete CFRP and GFRP reinforcement - reflect the different material characteristics of the two reinforcements. In addition, it is noted that these γ_{FRP} are recommended for the short-term design of FRP RC beams and hence, they do not take into account the long-term behavior of FRP reinforcement. A limit is also imposed on ρ, as shown in equation 4, in order to diminish the possibility of flexural failure occurring due to reinforcement fracture.

$$\rho_{min} = \frac{0.81(f_{ck} + 8)\,\varepsilon_c}{f_{FPR\,k}\left(\frac{f_{FRPk}}{E_{FRP\,k}} + \varepsilon_c\right)} \tag{4}$$

Finally, the use of safety factors on the stiffness of the FRP only makes sense if that safety factor is used just in the determination of deflections and cracking, and not in conjunction with strength safety factors when determining flexural capacity. It is proposed that the stiffness safety factors be discarded and the effect of stiffness uncertainty taken into account directly in the equations dealing with deformations.

Further research at Sheffield (Neocleous, 1999) has taken into account the effect of other modes of failure as well, and it has led to the development of a more comprehensive DSP based on targeted failure mode hierarchy. This approach allows the designer to choose different failure mode hierarchies, depending on the materials used, by selecting appropriate material safety factors.

CONCLUSIONS

This study has examined the effect of design parameters and γ_{FRP}, adopted for FRP reinforcement, on the flexural behavior of over-reinforced FPR RC beams.

One of the main findings of the assessment is that the desired mode of flexural failure is not attained by the application of γ_{FRP} alone. Thus, in order to attain the desired mode of failure, it is necessary to apply limits on the design parameters considered by the models adopted to predict the resistance-capacity.

A minimum amount of reinforcement is proposed, which will ensure flexural failure due to concrete crushing.

The flexural structural reliability is not uniform due to the effect of various design parameters. The ratio of permanent to variable load is one of the most influencing parameters and hence, it is recommended to adopt different load factors for different types of structures.

The use of large values for γ_{FRP} for flexural reinforcement is not necessary, if the design is devised to achieve flexural failure due to concrete crushing. Minimum values for γ_{FRP} are proposed and these values should be extended by further research to take into account the long-term behavior of FRP reinforcement in concrete.

The use of safety factors for the stiffness of FRP is not necessary and the effect of stiffness uncertainty should be taken into account by the equations dealing with deformations.

A more comprehensive DSP is required that integrates all of the failure modes and takes into account the properties of different types or reinforcing materials.

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APPENDIX I. Eurocode 2 (ENV1992-1-1, 1992) models for steel Design Moment Resistance RC beams

The design rules of Eurocode 2 utilize the simplified stress block approach (Fig. A.1) to determine the design moment resistance. The following assumptions are made by the design rules.

1. The strains in the concrete and the reinforcement are directly proportional to their distance from the neutral axis.
2. The strain in bonded reinforcement is the same as in the surrounding concrete.
3. The concrete tensile strength is ignored.
4. The concrete compressive stresses and the reinforcement stresses are derived from idealized design stress-strain curves.
5. The concrete compressive strain is limited to 0.002, if the RC section is subjected to pure longitudinal compression. Otherwise, the strain is limited to 0.0035.

The following algorithm is adopted for the evaluation of the design moment resistance.

Initially, it is assumed that plastic failure would occur and the effective depth of the RC beam is evaluated from equation A.1. Expressions for the design force in the tensile reinforcement, $F_{Sd,}$ and the design compressive force of concrete, F_{Cd}, are afterwards derived (equations A.3 and A.1.1 respectively).

$$F_{Cd} = \frac{0.85\, f_{ck}\ 0.8\, x\, b}{\gamma_c} = \frac{0.68\, f_{ck}\ x\, b}{\gamma_c} \qquad (A.1.1)$$

By considering the force equilibrium, $F_{Cd} = F_{Sd}$, x is calculated:

$$x = \frac{A_s\ f_{yk}\ \gamma_c}{0.68\, f_{ck}\ b\, \gamma_s} \qquad (A.1.2)$$

Before proceeding to the calculation of the lever arm, z, of the force couple, the same procedure as in section A.1 is utilized to check if the above value of x corresponds to plastic failure. Thus, the neutral axis limit and ε_y are determined from the section strain diagram and the stress-strain diagram, respectively. If x/d exceeds the neutral axis limit, x is recalculated on the basis that elastic failure occurs due to concrete crushing. Then the value of ε_s is derived from the strain diagram and is used together with the material safety factor to determine F_{Sd}. By considering force equilibrium and substituting in A.1.1, the following expression is obtained:

$$x = \frac{A_s\ E_s\ \varepsilon_u\, (\frac{d - x}{x})\, \gamma_c}{0.68\, f_{ck}\ b\, \gamma_s} \qquad (A.1.3)$$

x is determined by solving the resulting quadratic equation:

$$\frac{0.68\, f_{ck}\ b\, \gamma_s}{A_s\ E_s\ \varepsilon_u\ \gamma_c}\, x^2 + x - d = 0 \qquad (A.1.4)$$

Then, z is calculated by using the appropriate value of x:

$$z = d - 0.4\ x \tag{A.1.5}$$

Finally, the design moment of resistance is obtained:

$$M_u = \frac{0.68\, f_{ck}\ b\ x\ z}{\gamma_c} \tag{A.1.6}$$

APPENDIX II. Models for Design Moment Resistance of FRP RC beams

The model adopted for the design moment resistance of FRP RC beams is based on the design rules of Eurocode 2, and hence the same assumptions apply for the current model. The compression strength of FRP reinforcement is also ignored due to the anisotropic nature of the reinforcement.

The use of FRP reinforcement in RC construction would generally lead to over-reinforced sections since the high strength of FRP is not fully utilized. Thus, there is a change in the failure mode (from ductile to brittle). To accommodate this, the model is modified accordingly and the design is based on the control of the strain in the FRP reinforcement (ACI 440-98, 1998; JSCE, 1997). The following algorithm is applied for the evaluation of the design moment resistance.

Initially the effective depth of the RC is calculated based on an assumed bar diameter. Then, it is assumed that flexural failure occurs due to concrete crushing. Assuming that the concrete compressive strain at failure, ε_c, is equal to 0.0035, the design concrete compressive force, F_{Cd}, is derived (equation A.2.1). Since Eurocode 2 is the basis for the design rules, a specially derived equation (Neocleous, 1999) is used to determine the mean stress factor, $\alpha = -68711\,\varepsilon_c^2 + 464.79\,\varepsilon_c + 0.01$, which is used in the simplified stress block for concrete.

$$F_{Cd} = \frac{\alpha f_{ck}\ x\ b}{\gamma_c} \qquad \text{(A.2.1)}$$

Since it is assumed that failure occurs due to concrete crushing, the actual stress in the reinforcement (equation A.2.2) is deemed to be less than the design stress (equation A.2.3) at which fracture of the reinforcement occurs. The design force of the reinforcement is derived based on this assumption (equation A.2.4).

$$f_{FRP} = \varepsilon_{FRP}\, E_{FRP} \qquad \text{(A.2.2)}$$

$$f_{FRPd} = \frac{f_{FRPk}}{\gamma_{FRP}} \qquad \text{(A.2.3)}$$

$$F_{Sd} = A_{FRP}\, f_{FRP} = A_{FRP}\, \varepsilon_{FRP}\, E_{FRP} \qquad \text{(A.2.4)}$$

By considering a simple strain diagram, the neutral axis depth, x, is derived:

$$x = \frac{\varepsilon_c \, d}{\varepsilon_{FRP} + \varepsilon_c} \tag{A.2.5}$$

Then by considering force equilibrium between F_{Cd} and F_{Sd}, equations A.2.1, A.2.4 and A.2.5 are solved simultaneously to determine the actual tensile strain of the reinforcement, ε_{FRP}:

$$\frac{\alpha \, f_{ck} \left(\dfrac{\varepsilon_c \, d}{\varepsilon_{FRP} + \varepsilon_c} \right) b}{\gamma_c} = A_{FRP} \, \varepsilon_{FRP} \, E_{FRP} \tag{A.2.6}$$

By solving the following quadratic equation, the actual reinforcement strain, ε_{FRP}, is calculated:

$$\varepsilon_{FRP}^2 + \varepsilon_c \, \varepsilon_{FRP} - \frac{\alpha \, f_{ck} \, b \, d \, \varepsilon_c}{\gamma_c \, A_{FRP} \, E_{FRP}} = 0 \tag{A.2.7}$$

Before proceeding into the calculation of the lever arm, z, and design moment resistance, M_u, it is checked if ε_{FRP} has exceeded the design limit, ε_{FRPd}, which is defined by equation A.2.8.

$$\varepsilon_{FRPd} = \frac{f_{FRPd}}{E_{FRP}} \tag{A.2.8}$$

If ε_{FRPd} is exceeded by ε_{FRP}, then flexural failure occurs due to fracture of the FRP reinforcement. In this case, F_{Sd} and x are determined from equation A.2.9 and A.2.10 respectively. F_{Cd} is also re-derived (equation A.2.11) by substituting equation A.2.10 to A.2.1. The concrete compressive strain is iteratively reduced until the force equilibrium between F_{Cd} (equation A.2.11) and F_{Sd} (equation A.2.9) is satisfied.

$$F_{Sd} = A_{FRP}\, f_{FRPd} \qquad \text{(A.2.9)}$$

$$x = \frac{\varepsilon_c\, d}{\varepsilon_{FRPd} + \varepsilon_c} \qquad \text{(A.2.10)}$$

$$F_{Cd} = \frac{\alpha\, f_{ck} \left(\dfrac{\varepsilon_c\, d}{\varepsilon_{FRPd} + \varepsilon_c} \right) b}{\gamma_c} \qquad \text{(A.2.11)}$$

Using the appropriate value of x, the centroid factor, γ, and the lever arm, z, are then determined from equation A.2.12.

$$z = d - \gamma\, x \quad \text{where, } \gamma = 1962.6\, \varepsilon_c^2 + 17.89\, \varepsilon_c + 0.33 \qquad \text{(A.2.12)}$$

Finally, the design moment resistance, M_u, is calculated, depending on the mode of flexural failure. If failure occurs due to concrete crushing, equation A.2.13 is applied. It should be noted that F_{Cd} is determined from equation A.2.1. Otherwise, if failure is due to fracture of the re-bar, equation A.2.14 is determined by using the appropriate value of F_{Sd}.

$$M_u = F_{Cd}\, z \qquad \text{(A.2.13)}$$

$$M_u = F_{Sd}\, z \qquad \text{(A.2.14)}$$

APPENDIX III. REFERENCES

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APPENDIX IV. NOTATION

E_{FRPk} characteristic value of elastic modulus of FRP reinforcement

$F_i()$ cumulative distribution function of a variable i

f_c concrete compressive strength

f_{FRPk} characteristic value of tensile strength of FRP reinforcement

G_i permanent load evaluated at each simulation cycle, i

N amount of simulation cycles performed

$\overline{P}_f$ mean probability of failure, which corresponds to the notional P_f

P_f notional structural reliability level

P_{fc} probability of flexural failure occurring due to concrete crushing

P_{fi} probability of failure evaluated at each simulation cycle

P_{ft} target structural reliability level

Q variable load

R_i resistance-capacity evaluated at each simulation cycle

ε_c concrete strain

ε_{FRP} actual tensile strain developed in the FPR reinforcement at the design stage

ε_{FRPd} design limit imposed on the tensile strain of FRP reinforcement by γ_{FRP}

γ_{FRP} material partial safety factor for FRP reinforcement

Table 1 Partial safety factors proposed by JSCE (1997)

	Material Factor γ_m			Member Factor	Structural Analysis Factor	Load Factor	Structural Factor
	Concrete γ_c	FRP γ_{mf}	Steel γ_s	γ_b	γ_a	γ_f	γ_i
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Ultimate Limit State	1.3* or 1.5	1.15* * to 1.3	1.0 or 1.05	1.15 to 1.3	1.0	1.0 to 1.2	1.0 to 1.2
Serviceability Limit State	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Fatigue Limit State	1.3* or 1.5	1.15** to 1.3	1.05	1.0 to 1.1	1.0	1.0	1.0 to 1.1

Notes: * 1.3 when characteristic strength of concrete is less than 50 N/mm^2

** 1.15 for FRP with carbon or Aramid fibres

Table 2 Partial safety factors proposed for FRP RC structures by Clarke et al (1996)

(1)	Material (2)	Partial Safety Factor, γ_{FRP} (Short and Long Term) (3)
Strength	E-Glass reinforced	3.6
	Aramid reinforced	2.2
	Carbon reinforced	1.8
Stiffness	E-Glass reinforced	1.8
	Aramid reinforced	1.1
	Carbon reinforced	1.1

Table 3 Statistical data for geometrical basic variables

Dimension Description (1)	Mean Value μ_i (mm) (2)	Standard Deviation σ_i (mm) (3)	Probability Distribution (4)
Width	Nominal + 2.4	4.8	Normal
Overall Depth	Nominal – 3.2	6.4	Normal
Concrete Cover	Nominal + 1.6	11.6	Normal
Beam Spacing and Span	Nominal	17.5	Normal

Table 4 Statistical data for loading

Load Description (1)	Coefficient of variation cov_i (2)	Characteristic i_k (3)	Probability Distribution (4)
Permanent Load G	0.05	$\mu_G + 0.082$	Normal
Variable Load Q	0.4	$\mu_Q \cdot 1.98$	Gamma

Table 5 Statistical data for CFRP and GFRP reinforcement

	Tensile Strength (N/mm^2)		Young's Modulus (N/mm^2)	
	CFRP	GFRP	CFRP	GFRP
(1)	(2)	(3)	(4)	(5)
Mean μ_i	1380	810	115000	45000
Standard Deviation σ_i	69	40.5	5750	2250
Coefficient of Variation cov_i	0.05	0.05	0.05	0.05
Minimum i_{min}	1235.1	725	105800	41400
Maximum i_{max}	1524.9	895.1	124200	48600
Characteristic i_k	1272.2	746.7	106012.8	41483.3
Probability Distribution	Normal	Normal	Normal	Normal

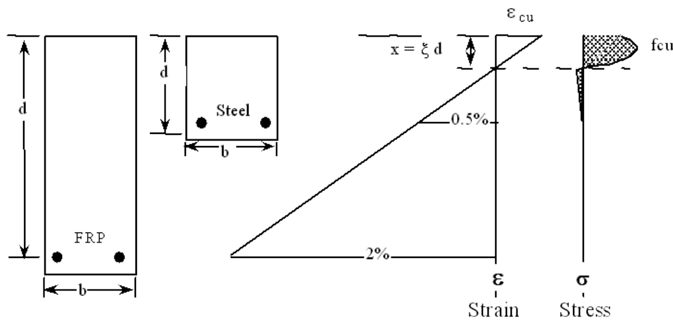

Figure 1 Strain distribution for a GFRP RC section

Figure 2 Deflection and cracking in FRP RC beams

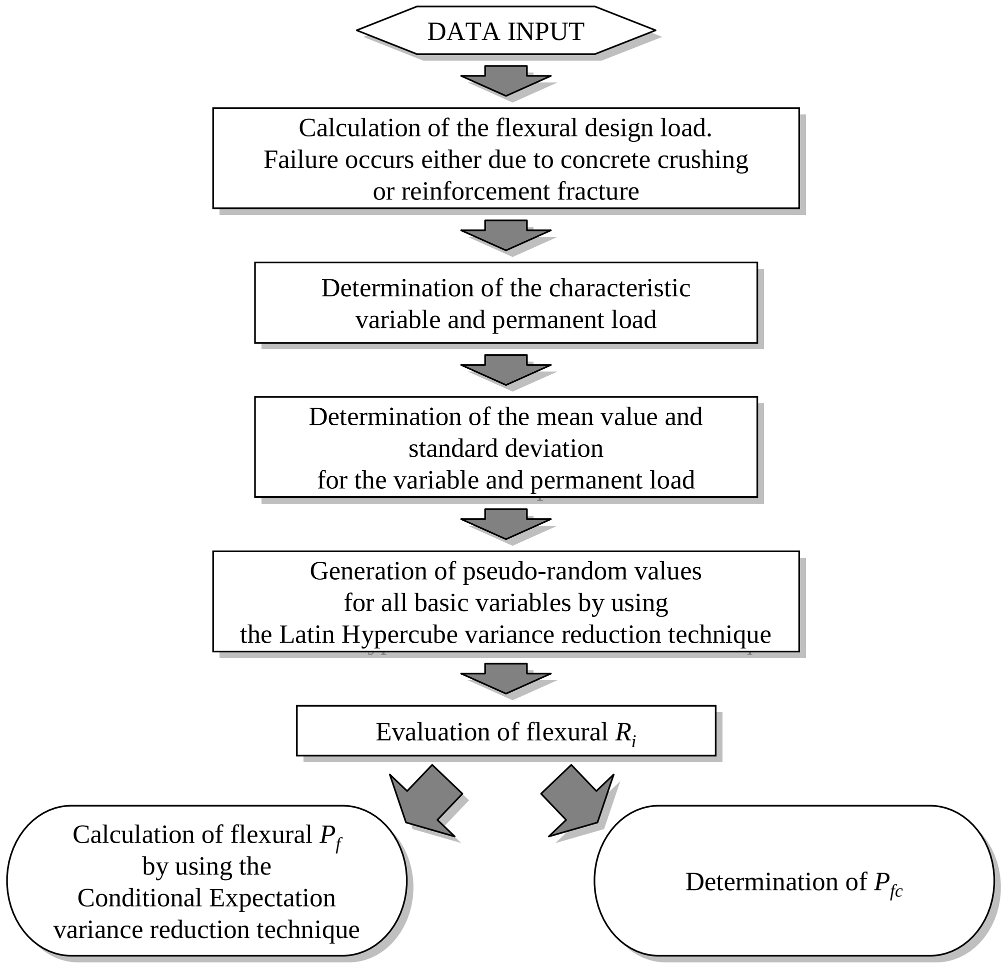

Figure 3. Assessment procedure

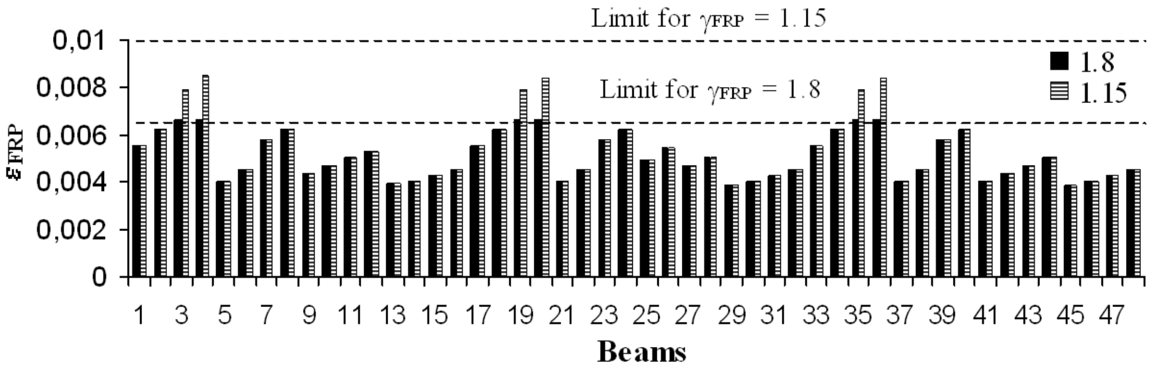

Figure 4. Actual tensile strain developed in the CFPR reinforcement at the design stage

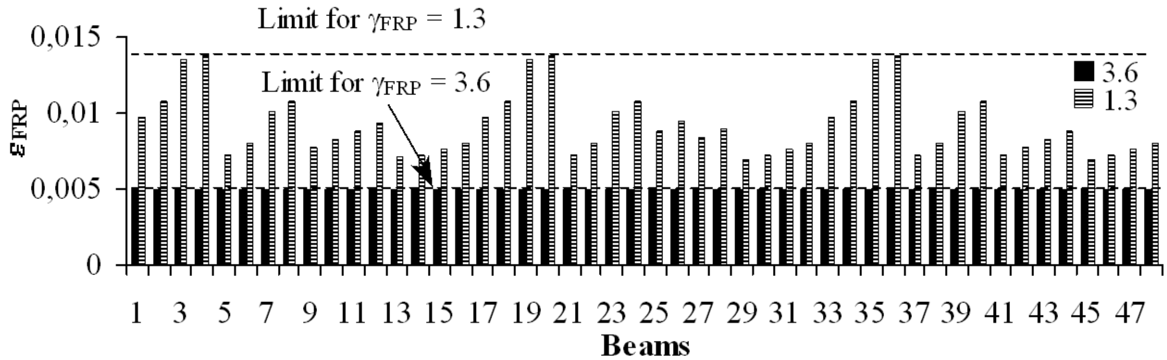

Figure 5. Actual tensile strain developed in the GFPR reinforcement at the design stage

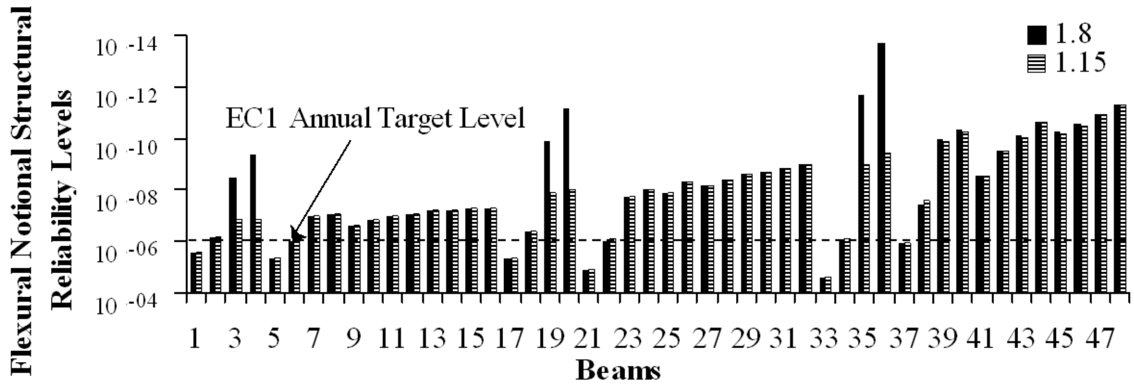

Figure 6. Effect of γ_{FRP} on the flexural P_f of CFRP RC beams

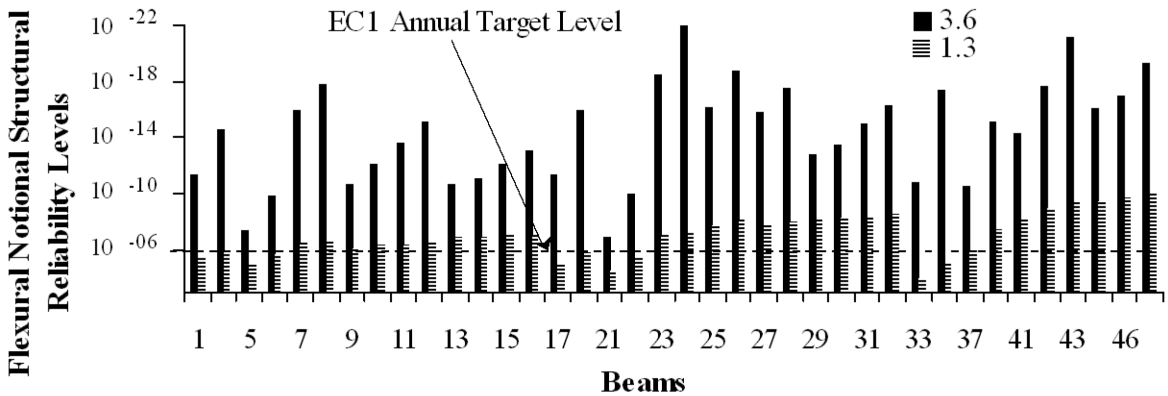

Figure 7. Effect of γ_{FRP} on the flexural P_f of GFRP RC beams

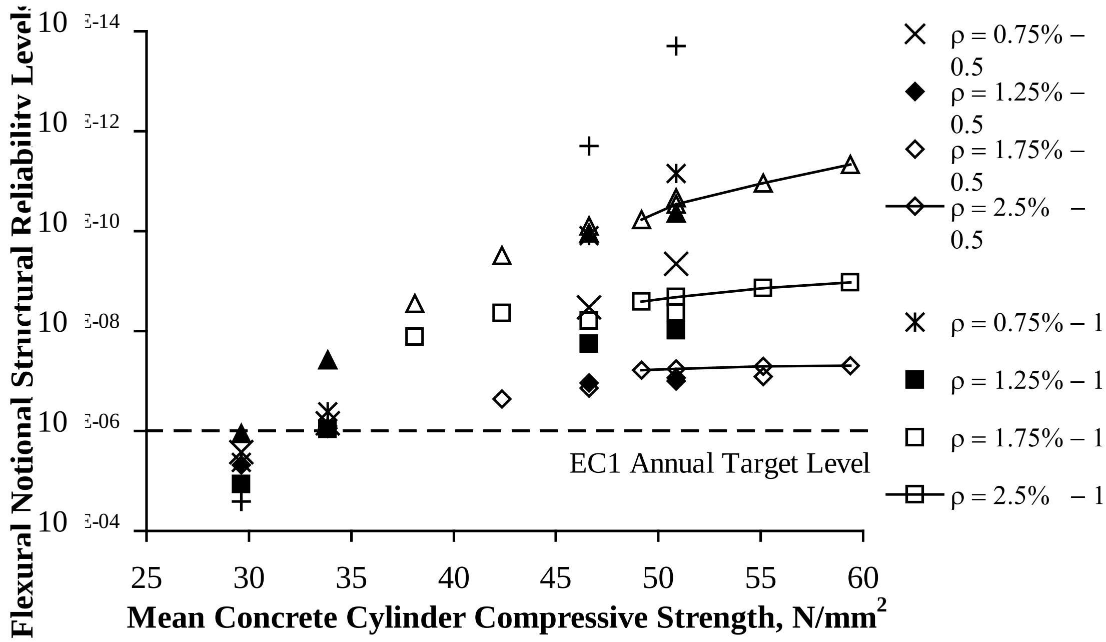

Figure 8. Effect of f_c, ρ and load ratio on the flexural P_f (CFRP RC beams – γ_{FRP} = 1.8)

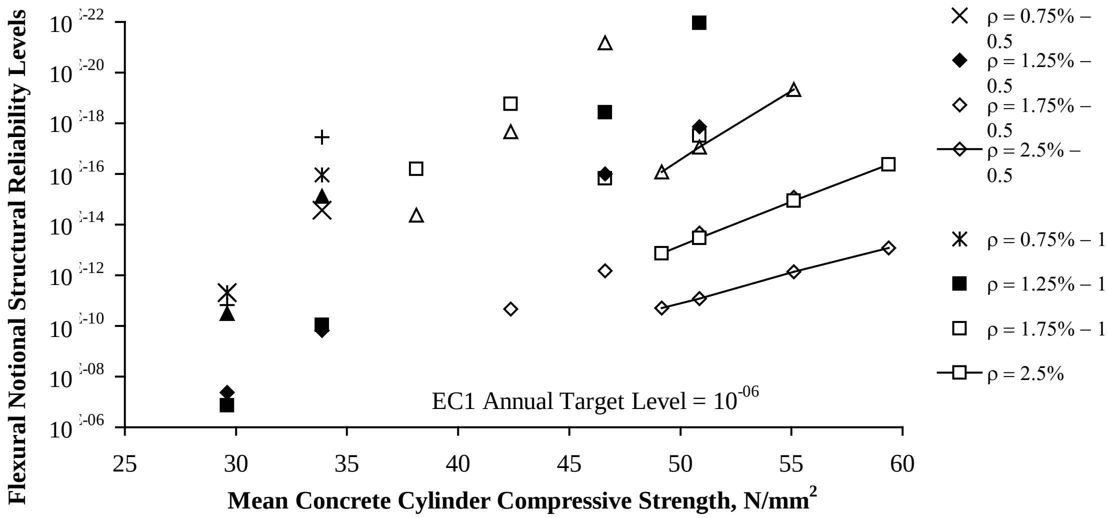

Figure 9. Effect of f_c, ρ and load ratio on the flexural P_f (GFRP RC beams – γ_{FRP} = 3.6)

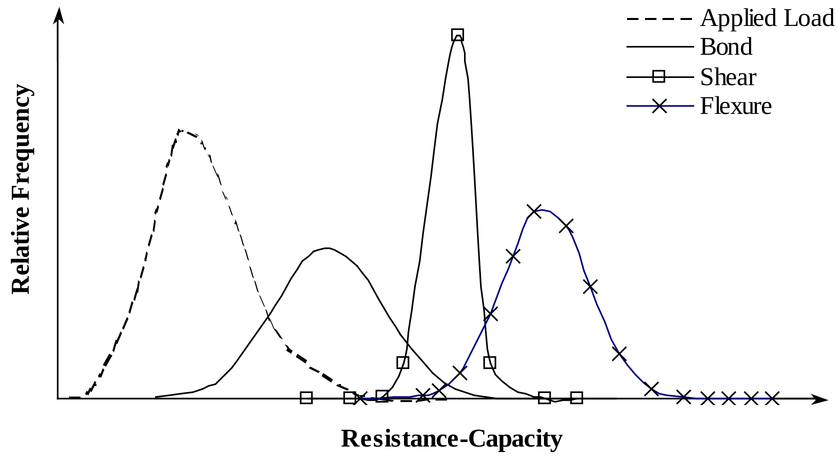

Figure 10 Concept of failure mode hierarchy

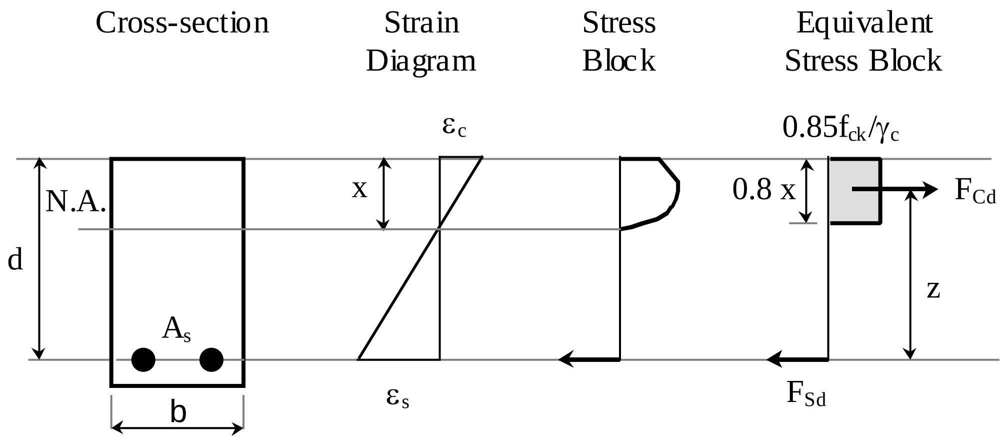

Figure A.1 Simplified stress block used by Eurocode 2

List of Captions