Structural safety uncertainties in codes of practice for reinforced concrete

Kyriacos Neocleous[1], BEng, PhD

Kypros Pilakoutas[2], BSc, PhD

Peter Waldron[3], BSc, PhD, CEng, MICE, MIStructE

1. Marie-Curie Post-Doctoral Research Fellow, k.neocleous@sheffield.ac.uk
2. Reader of Structural Engineering, k.pilakoutas@sheffield.ac.uk
3. Professor of Structural Engineering, p.waldron@sheffield.ac.uk

Centre for Cement and Concrete, Department of Civil & Structural Engineering, The University of Sheffield, Mappin Street, Sheffield, S1 3JD, Tel 0114 – 2225738, Fax 0114 - 2225700

Synopsis

The limit-state design approach, currently used in codified design of concrete structures reinforced with steel reinforcement, is based on semi-probabilistic procedures. Although modern concrete codes of practice are more sophisticated than older codes based on the permissible stress approach, they still have fundamental uncertainties with regards to structural safety. The work reported in this paper investigates these uncertainties for the BS8110 and Eurocode-2 codes of practice by performing a structural reliability assessment using the Monte-Carlo Simulation method in conjunction with the Latin Hypercube and Conditional Expectation variance reduction techniques. The assessment considers both the flexural and shear failure modes. In the case of BS8110, it is shown that it may be more appropriate to increase the characteristic value of the tensile strength of steel reinforcement rather than to use the reduced partial safety factor of 1.05.

Notation

d_i	effective depth of the beam, determined at each simulation cycle
f_c	concrete compressive strength
n_i	flexural failure factor calculated at each simulation cycle
x	neutral axis of RC beam
x_i	neutral axis depth calculated, assuming ductile failure, at each simulation cycle
z	lever arm of RC beam
F_i	component for the resistance-capacity at each simulation cycle (it denotes the load carrying capacity of the beam for the failure mode under consideration)
F_Q	cumulative distribution function of a variable load
G_i	permanent load evaluated at each simulation cycle, i
G_k	characteristic value of permanent load
N	number of simulation cycles performed

P_f probability of failure

P_{ft} target structural reliability level

$\overline{P}_f$ mean probability of failure, which corresponds to the notional P_f

P_{fi} probability of failure evaluated at each simulation cycle

Q variable load

Q_k characteristic value of variable load

R Resistance component of limit-state function

R_i resistance-capacity evaluated at each simulation cycle

S Action effect component of limit-state function

$\dfrac{G}{Q}$ ratio of permanent to variable load

PVL_{ratio} ratio of permanent to variable load

RCM   resistance-capacity margin

γ_G load factor for permanent load

γ_{ms} partial safety factor for steel reinforcement

γ_Q load factor for variable load

ε_c concrete strain developed in the RC beam

ε_y yield strain of the steel reinforcement

μ_{Fshear} mean shear resistance-capacity

$\mu_{Fflexure}$ mean flexural resistance-capacity

μ_G mean value of permanent load

μ_Q mean value of variable load

ρ ratio of longitudinal reinforcement

Introduction

Modern concrete codes of practice utilise safety level one of structural reliability theory to restrict the nominal probability of failure within specific target levels (CEB-FIB Model Code 1990[1]). However, when the design and safety philosophy of such codes of practice is investigated, a number of fundamental structural safety uncertainties emerge.

One such uncertainty arises from the fact that there is a lack of published records regarding the derivation of the adopted partial safety factors. The British concrete code of practice (BS8110[2]) states that the adopted partial safety factors have been calibrated with pre-existing practice and experience by taking into account the uncertainties relevant to structural loading and strength of materials. An example of this is the reduction in the partial safety factor adopted for the strength of steel reinforcement (γ_{ms}) from 1.15 in the previous code to 1.05[3]. In the case of the Eurocodes, Eurocode 1 (ENV 1991-1[4]) states that the partial safety factors were derived by calibration to historical and empirical design methods, with amendments based on the safety level two method of structural reliability theory. Eurocode 1 has also a clear target structural reliability level (P_{ft}) of 7×10^{-5} for the design working life. In the case of BS8110, although there is no published evidence about the adopted P_{ft}, it seems that a similar value to Eurocode 1 is implicitly adopted by BS8110, since the two codes of practice use similar load and partial safety factors.

Another safety uncertainty arises from the fact that there is no information regarding the resistance-capacity margins (RCM) that exist between the various failure modes (limit states). For example, if there is flexural over-strength, it is impossible to predict the failure mode that will occur (e.g. shear, bond) and at which load level. Hence, it is uncertain whether the application of the adopted partial safety factors would always result in the desired type of failure (i.e. flexural yielding) assumed by the codes of practice.

Finally, it is not known if the structural reliability levels, (P_f, i.e. probability of failure), are uniform for all structural members (beams, slabs, columns, foundations etc). Members designed according to these partial safety factors may be either unsafe or over-conservatively safe as affirmed by Holicky and Vrouwenvelder[5] for the case of RC columns designed according to Eurocode 2 (ENV 1992-1-1[6]). Similar findings were reported in studies by Duprat et al[7] and by Neuenhofer and Zilch[8].

The work reported in this paper, which continues along the lines of an earlier study by Neocleous and Pilakoutas[9] on the use of new materials in concrete construction,

investigates some of these safety concerns for the two codes of practice of relevance to UK engineers (BS8110 and Eurocode 2). Structural reliability theory is utilised to determine the annual, notional P_f of RC beams, designed to resist flexure and shear in accordance with the two codes of practice. The RCM between the flexural and shear failure modes is also evaluated.

Methodology

The structural reliability assessment is performed for the case of simply supported, singly reinforced concrete beams, which are designed in accordance with the BS8110 and Eurocode-2 codes of practice to resist uniformly distributed floor loads. The effect of the reduction of the BS8110 value of γ_{ms} on the notional P_f of the beams is also examined by performing the assessment for γ_{ms} equal to 1.15 and 1.05; this is examined for both codes of practice for comparative reasons.

The BS8110 value of γ_{ms} was reduced from 1.15 to 1.05 mainly due to the evidence of safe designs[3], which were primarily attributed to the fact that the yield stress of reinforcing bars - currently used for RC construction in the UK - is significantly higher than the characteristic value adopted by BS8110. The effect of this assumption on the structural reliability is examined by carrying out a further assessment on the hypothesis that the yield stress of steel reinforcement statistically conforms to the characteristic value of 460 N/mm^2 adopted by the British Standards[10]. The statistical data for a hypothetical yield stress acceptable to BS4449 is shown (as "BS4449" Steel) in Table 3 together with the other data used for the properties of steel reinforcement.

In addition, the effect of a number of design parameters on P_f and RCM is examined by carrying out the examination for forty-eight different beam configurations (summarised in Table 1). The design parameters considered are concrete compressive strength (f_c), ratio of longitudinal reinforcement (ρ) and ratio of permanent to variable load (PVL-ratio). The beams are designed according to the two codes of practice to avoid shear failure and to achieve under-reinforced sections.

The probability of occurrence of brittle failure due to concrete crushing (P_{fc}) is also determined to examine whether RC beams, designed to attain under-reinforced sections, would sustain a brittle failure due to concrete crushing. This will therefore examine, if the code's assumption about the desired mode of failure is valid.

The assessment is performed by applying the Monte Carlo simulation method in conjunction with the joint application of the Latin Hypercube[11] and Conditional Expectation[12] variance reduction techniques. The entire procedure followed in the assessment, illustrated in Figure 1 and summarised in Appendix A, is elaborated by Neocleous[13].

Statistical Data

The statistical data used for the probabilistic modelling of all random variables are either taken from published literature or derived from the analysis of experimental data supplied by manufacturers. In the case of the geometrical variables, the data presented by Mirza and McGregor[14] are adopted (Table 2). A truncated normal probability distribution is used to model the geometrical variation; the tips of the distribution are truncated to avoid generating impossible values, such as values that may result from human errors. The geometric tolerance limits recommended by the CEB FIB Model code 1990[1] are used to derive the minimum and maximum allowable values at which the probability distribution is truncated.

The statistical data used for the modelling of f_c are derived from the analysis of cube strengths from concrete batches with different cement contents[13], provided by British ready mix manufacturers. Figure 2 shows the standard deviation versus the mean compressive strength for over 300 specimens presented in 10 groups according to their cement range. Based on the results of the analysis, a constant standard deviation (6 N/mm^2) is adopted for all concrete strengths, and the normal probability distribution is adopted to model the variation of the concrete compressive strength[15]. The distribution is truncated at both tips to avoid the generation of impossible random values, such as negative strength. Although the analysis of the available test data indicated that different values could be adopted for the minimum and maximum allowable concrete

compressive strength, it is deemed more appropriate to adopt the worst available value. Hence, the minimum and maximum allowable values are taken as 3.8 standard deviations away from the mean value.

Table 3 shows the statistical data adopted to model the variation in the yield stress of steel reinforcement. The statistical data is mainly based on data published by UK CARES[16] (over 1000 samples) and incorporates the recommendations by Mirza and McGregor[17] on the probability distributions. It is noted that the minimum value provided by UK CARES is not used to truncate the lower tip of the distribution. It is deemed more appropriate instead to adopt the UK characteristic value. The variation in the cross sectional area of the reinforcement is modelled as a separate random variable, since the supplied yield stress was determined using the gross cross-sectional area of the reinforcement. Similarly to the other random variables, the adopted probability distribution for the cross-sectional area is truncated at both tips to account for the quality control procedures applied during the manufacture of the reinforcement and at various stages of RC construction.

The variation of the permanent and variable loads is modelled based on data obtained from published literature (Table 4). In the case of the intensity variation of the permanent floor loads, following the recommendations by Östlund[18], a coefficient of variation of 5% is adopted. Whereas for the variable loads, a coefficient of variation of 40% is used for the annual maximum floor loads. These values are similar to the ones recommended by the CEB-FIB Model Code 1990[1].

Discussion of results:

Flexural Design

Flexural design of the examined beams is based on the assumption that under-reinforced sections will be attained and hence, brittle failure would be avoided. The results obtained for P_{fc} indicate that, for some of the beams considered in this study, there is a very small probability (ranging from 10^{-3} to 10^{-6}) that the beams will sustain brittle failure due to concrete crushing. This is due to the fact that a higher γ_m is adopted for

concrete than for steel reinforcement. Hence, this confirms that the limit state approach for flexural design is sound.

Partial Safety Factors for Steel Reinforcement

Analysis of the flexural and shear results shows that the target P_f (7×10^{-5}), adopted by Eurocode 1 for the design working life of a structure, is generally satisfied for γ_{ms} equal to 1.05 and 1.15 (for both BS8110 and Eurocode 2).

Figure 3 shows the effects of γ_{ms} on the flexural P_f of beams. As expected, the results obtained for γ_{ms} equal to 1.15 (red bar) and 1.05 (grey bar) indicate that the flexural P_f increases as γ_{ms} decreases. Analysis of the results also shows that the reduction in γ_{ms} to 1.05 (adopted by BS8110 since 1997) affects the flexural structural reliability, in particular, when the yield stress of the reinforcement complies strictly with the British Standards (blue bar in Figure 3). It is obvious that, since the use of steel statistically complying with British Standards can result in a high P_f, the reduction in γ_{ms} to 1.05 was not the best solution. It is proposed that, rather than decrease the value of γ_{ms}, it is more appropriate to increase the characteristic yield stress of steel reinforcement to 500 N/mm^2. This proposal would also put the British Standards in line with the new European Standards.

Reliability Differentiation

Figures 4 to 7 show the effects of the main design parameters on the P_f for flexure and shear, both for BS8110 and Eurocode 2. These figures show that the flexural and shear values of P_f are not uniform across the range of design configurations considered. This is especially true for the shear failure mode, where P_f varies from 10^{-5} to 10^{-22} (for γ_{ms} 1.05) and 10^{-6} to 10^{-20} (for γ_{ms} 1.15) for BS8110 and Eurocode 2, respectively.

Figures 4 to 7 also show, for both failure modes examined, that P_f is greatly influenced by the PVL-ratio. Structural reliability improves as this ratio increases. This is because the permanent load, whose variation is much lower than that for the variable load, becomes the dominant parameter for the action effect component (*S*) of the limit-state function and hence, the variation of *S* reduces accordingly. This results in an increase in

the margin between S and the resistance component (R) and a subsequent reduction in P_f.

Different PVL-ratios are implicitly used in different types of structures (due to their geometry and intended application). Hence, the above effect implies that the structural reliability of different structures would not be the same, if the same values of γ_m (or load factors) are used for the design of these structures. It is therefore recommended that future codes of practice should consider the use of different load factors for different types of structure (buildings, bridges, etc) in order to attain a more uniform P_f. However, reliability differentiation may be desirable for certain types of safety critical structures, such as bridges and hospitals.

For the flexural failure mode, the results show that ρ and f_c also influence structural reliability, with the effect of ρ being greater than that of f_c. In the first instance, the variation in flexural P_f as a result of ρ and f_c is surprising, since these design parameters are included in flexural design equations. From further examination, this variation is found to be caused by the influence of the two parameters on the ratios of the mean to design value of the neutral axis depth (x) and lever arm (z). As ρ increases, x increases proportionally, whereas z decreases non-linearly (Figure 8). Consequently, the ratio of mean to design value of x does not change with ρ, whereas the corresponding ratio for z increases with ρ (Figure 9). Hence, this increase causes a decrease in P_f (Figure 10). The opposite effect is observed for f_c since the ratio of z decreases as f_c increases (Figure 11).

It is also found that the shear P_f is affected by ρ and f_c; however, further examination could not identify a clear pattern. This is because shear resistance is the sum of concrete shear resistance (influenced by both ρ and f_c) and the additional shear resistance from shear links. The ratio between the two resistances, for the beams examined, is not constant.

Shear-Flexure Resistance Capacity Margins (RCMs)

Figure 12 shows that the shear-flexure RCMs, for Eurocode 2 and BS8110, are not uniform for all the beams examined. The values determined for BS8110 varied from 0.9 to 2.1, whereas for Eurocode 2, the range of values was 1.1 to 2.8. Further analysis of the results indicates that the RCMs are variable due to the effect of the main design parameters, such as ρ and f_c, on the flexural and shear resistance capacities of the beams. This is illustrated in Figure 13, where the flexural μ_F increases proportionally with ρ, whereas the shear μ_F increases at a lower rate and, hence, the RCMs decrease as ρ increases.

The influence of f_c on the RCMs was found to be less important than that of ρ. In the case of BS8110, the RCMs in general decrease as f_c increases. Whereas, for Eurocode 2, it was observed that the RCMs increase with f_c.

Code Comparison

Analysis of the results shows that there is a difference in the flexural and shear values of P_f determined from BS8110 and Eurocode 2. In the case of the flexural P_f, the values obtained by Eurocode 2 are lower than those obtained by BS8110 due to the different load characteristics of each code. In the case of the shear P_f, it is determined that the values obtained by Eurocode 2 are again lower than the ones determined by BS8110. This can be partly attributed to the higher concrete γ_m adopted for shear design by Eurocode 2.

Conclusions

The various safety uncertainties that are relevant to BS8110 and Eurocode 2 have been examined by assessing the structural reliability of concrete beams singly reinforced with steel reinforcement.

One of the main findings of the study is that the calculated flexural and shear structural reliability is not uniform across the range of beams examined due to the effect of the different design parameters. It is shown that the ratio of permanent to variable load is one of the most influential parameters on structural reliability.

The reduction in the partial safety factor for steel reinforcement to 1.05, introduced in BS8110 in 1997, may reduce the notional structural reliability of RC beams, if the yield stress of the steel reinforcement used complies strictly with the characteristic value used by British Standards. Hence, to overcome this possibility, it is recommended that the characteristic value of the yield stress of steel reinforcement is increased to 500 N/mm2 to reflect the over-strength currently provided by the manufacturers of reinforcement.

The resistance-capacity margins are found to vary between different beams due to the effect of the concrete compressive strength and ratio of longitudinal reinforcement. These margins range from 0.9 to 2.8. This highlights the need for a more consistent and economic design, but this can only be achieved if a different design philosophy is adopted.

Overall, a difference is observed between the results obtained for BS8110 and Eurocode 2. This is attributed to the different load characteristics and partial safety factors adopted by each code of practice.

Further work is required to examine the structural reliability for other modes of failure, such as bond (anchorage and splice) and other structural elements, such as continuous beams and columns.

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Appendix A - Description of Assessment

The structural reliability assessment is performed by utilising the procedure illustrated in Figure 1. The assessment is based on the philosophy that the RC beams are designed to fail in flexure. It is noted that the assessment is performed separately for each RC beam considered in the study.

The first step in the procedure is to define the data relevant to the RC beam under consideration. This includes the data used in code based design and the statistical data for all basic variables considered in the assessment.

The design flexural resistance of the RC beam is calculated at the second step. The flexural resistance is determined by applying the prediction models adopted by the code of practice under examination. It is noted that the characteristic value adopted by BS8110 (i.e. 460 N/mm^2) is only used for the evaluation of the design flexural resistance.

The calculation of the design flexural resistance is followed by the evaluation of the nominal transverse reinforcement required by the beam to resist a design (shear) load equal to the design flexural resistance of the RC beam. It is assumed that the transverse reinforcement is provided in the form of vertical shear links.

The next step is to evaluate the characteristic value, mean value and standard deviation for both the permanent and variable loads. Equations A.1 and A.2 are used to calculate the characteristic values for the variable and permanent loads respectively. The mean value and standard deviation for the variable and permanent load are then determined by utilising the data tabulated in table 4.

$$Q_k = \frac{\text{Design Resistance}}{\gamma_Q + \gamma_G \dfrac{G}{Q}} \tag{A.1}$$

$$G_k = Q_k \frac{G}{Q} \tag{A.2}$$

At step 5, the Latin Hypercube variance reduction technique is applied to generate pseudo-random values (at each simulation cycle) for all conditioned basic variables (such as depth, width, length, concrete strength, reinforcement strength, and permanent load). It is assumed that all basic variables are un-correlated. Furthermore, the variable load is selected as the control variable and thus it is kept constant at its mean value.

Step 6 involves the formulation of the limit state functions for both the flexural and shear failure modes. The limit state function ($G(R_i, S_i)$), which is evaluated at each simulation cycle (equation A.3), represents the structural behaviour for the limit state (failure mode) for which the assessment is performed. $G(R_i, S_i)$ is represented in terms of the structural resistance component (R_i) and action effect component (S_i). Both R_i and S_i are modelled by mathematical relationships of conditioned basic variables, which represent structural material properties and actions respectively. Some conditioned basic variables are common for both components, for instance the variables representing the structural geometry. This step also calculates the probability of occurrence of brittle failure due to concrete crushing, P_{fc}, (equation A.4).

$$G(R_i, S_i) = F_i - G_i \tag{A.3}$$

$$P_{fc} = \frac{\sum_{i=1}^{N} n_i}{N} \quad \text{Where} \quad \begin{array}{l} n_i = 1 \ \ \text{if} \ \dfrac{x_i}{d_i} > \dfrac{\varepsilon_c}{\varepsilon_c + \varepsilon_y} \\[4mm] n_i = 0 \ \ \text{if} \ \dfrac{x_i}{d_i} \le \dfrac{\varepsilon_c}{\varepsilon_c + \varepsilon_y} \end{array} \tag{A.4}$$

In addition, the Conditional Expectation technique is used to evaluate the flexural and shear P_f. Initially, equation A.5 is used to calculate the P_{fi} at each simulation cycle. Once all simulation cycles are performed, equation A.6 is used to evaluate the average probability of failure. This corresponds to the notional (or theoretical) structural reliability level, since the effect of human errors is not included in the assessment. At the end of the assessment, the shear-flexure RCM is calculated by equation A.7.

$$P_{fi} = P(Q > G(R_i, S_i)) = 1 - F_Q(F_i - G_i) \tag{A.5}$$

$$\overline{P}_f = \frac{\sum\limits_{i=1}^{N} P_{f_i}}{N} \qquad\qquad (A.6)$$

$$\mathrm{RCM} = \frac{\mu_{\mathrm{Fshear}}}{\mu_{\mathrm{Fflexure}}} \qquad\qquad (A.7)$$

Table 1 Data for the RC beams considered in the reliability assessment

| Beam | Width | Overall depth | Longitudinal reinforcement. ratio | Length | f_{cu} | Ratio of permanent to variable load |
	(mm)	(mm)		(m)	(N/mm^2)	
1	180	260	0.75%	4.3	25	0.5
2	180	260	0.75%	4.3	30	0.5
3	180	260	0.75%	4.3	45	0.5
4	180	260	0.75%	4.3	50	0.5
5	355	480	1.25%	8.7	25	0.5
6	355	480	1.25%	8.7	30	0.5
7	355	480	1.25%	8.7	45	0.5
8	355	480	1.25%	8.7	50	0.5
9	530	700	1.75%	13.1	40	0.5
10	530	700	1.75%	13.1	45	0.5
11	530	700	1.75%	13.1	50	0.5
12	530	700	1.75%	13.1	55	0.5
13	180	260	2.50%	4.3	48	0.5
14	180	260	2.50%	4.3	50	0.5
15	180	260	2.50%	4.3	55	0.5
16	180	260	2.50%	4.3	60	0.5
17	355	480	0.75%	8.7	25	1
18	355	480	0.75%	8.7	30	1
19	355	480	0.75%	8.7	45	1
20	355	480	0.75%	8.7	50	1
21	530	700	1.25%	13.1	25	1
22	530	700	1.25%	13.1	30	1
23	530	700	1.25%	13.1	45	1
24	530	700	1.25%	13.1	50	1

Notation:

f_{cu} is the characteristic concrete cube compressive strength used in BS8110,

All geometrical dimensions are nominal, concrete cover is 30mm

Table 1 (continued) Data for the RC beams considered in the reliability assessment

Beam	Width (mm)	Overall depth (mm)	Longitudinal reinforcement. ratio	Length (m)	f_{cu} (N/mm^2)	Ratio of permanent to variable load
25	180	260	1.75%	4.3	35	1
26	180	260	1.75%	4.3	40	1
27	180	260	1.75%	4.3	45	1
28	180	260	1.75%	4.3	50	1
29	355	480	2.50%	8.7	48	1
30	355	480	2.50%	8.7	50	1
31	355	480	2.50%	8.7	55	1
32	355	480	2.50%	8.7	60	1
33	530	700	0.75%	13.1	25	2
34	530	700	0.75%	13.1	30	2
35	530	700	0.75%	13.1	45	2
36	530	700	0.75%	13.1	50	2
37	180	260	1.25%	4.3	25	2
38	180	260	1.25%	4.3	30	2
39	180	260	1.25%	4.3	45	2
40	180	260	1.25%	4.3	50	2
41	355	480	1.75%	8.7	35	2
42	355	480	1.75%	8.7	40	2
43	355	480	1.75%	8.7	45	2
44	355	480	1.75%	8.7	50	2
45	530	700	2.50%	13.1	48	2
46	530	700	2.50%	13.1	50	2
47	530	700	2.50%	13.1	55	2
48	530	700	2.50%	13.1	60	2

Notation:

f_{cu} is the characteristic concrete cube compressive strength used in BS8110,

All geometrical dimensions are nominal, concrete cover is 30mm

Table 2 Statistical data adopted for geometrical basic-variables [14]

Dimension Description	Mean Value μ_i (mm)	Standard Deviation σ_i (mm)	Probability Distribution
Width	Nominal + 2.4	4.8	Normal
Overall Depth	Nominal – 3.2	6.4	Normal
Concrete Cover	Nominal + 1.6	11.6	Normal
Beam Spacing and Span	Nominal	17.5	Normal

Table 3. Statistical data adopted for steel reinforcing bars used in the UK[16, 17]

	Yield stress at 0.43% total strain (N/mm^2) UK CARES steel "BS4449 steel"		Cross-sectional Area, A$_s$/Nominal	Young's Modulus, E$_s$ (N/mm^2)
Mean μ$_i$	530	507	0.982	201000^b
Standard Deviation σ$_i$	32.1	30.9	0.01	6633^b
Minimum i$_{min}$	474 460^a	440	0.90^b 0.81^c	-
Maximum i$_{max}$	630	602	1.21^b 1.44^c	-
Probability Distribution	Log-normal	Log-normal	Normal	Normal

Notation:

a: value modified by the authors and used in the assessment

b: value adopted for longitudinal reinforcing bars

c: value adopted for transverse reinforcing bars

d: value adopted by BS8110 and used to determine the design flexural resistance

Table 4 Statistical data adopted for loading[18]

	Permanent Load G	Variable Load Q
Coefficient of variation cov_i	0.05	0.4
Characteristic i_k	$\mu_G + 0.082$	$\mu_Q \cdot 1.74^a$ $\mu_Q \cdot 1.98^b$
Probability distribution	Normal	Gamma

[a]: BS8110 (1997) value, corresponds to the 95th percentile

[b]: Eurocode 2 (ENV 1992-1-1, 1992) value, corresponds to the 98th percentile

List of captions

Figure 1 Procedure adopted for the structural reliability assessment

Figure 2 Standard deviation versus the mean compressive strength of concrete

Figure 3 Effect of γ_{ms} on the flexural P_f of beams designed according to BS8110

Figure 4 Flexural P_f obtained for BS8110 (γ_{ms} = 1.15)

Figure 5 Flexural P_f obtained for Eurocode 2 (γ_{ms} = 1.15)

Figure 6 Shear P_f obtained for BS8110 (γ_{ms} = 1.05)

Figure 7 Shear P_f obtained for Eurocode 2 (γ_{ms} = 1.15)

Figure 8 Effect of ρ on the margin between the design and mean values of x and z (BS8110)

Figure 9 Effect of ρ on ratio of the mean to design values of neutral axis depth and lever arm (BS8110)

Figure 10 Effect of ratio of mean to design lever arm on flexural P_f (BS8110)

Figure 11 Effect of mean f_c on the ratio of the mean to design values of neutral axis depth and lever arm (BS8110)

Figure 12 Shear – flexure RCMs for BS8110 and Eurocode 2

Figure 13 Effect of ρ on mean resistance-capacity (BS8110)