Modified Null Broadening Adaptive Beamforming: A Constrained Optimisation Approach

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Abstract

A constrained optimisation approach for null broadening adaptive beamforming is proposed. This approach improves the robustness of the traditional MVDR beamformer by broadening nulls for interference direction and the mainlobe for the desired direction. This optimisation is efficiently solved by semidefinite programming. The proposed approach, when applied to high altitude platform (HAP) communications using a vertical linear antenna array, provides significantly better coverage performance than a previously reported null broadening technique.

Introduction: Antenna beampattern optimisation is widely used to improve the system capacity in wireless communications. Adaptive beamformers are well known for their high resolution and sidelobe suppression [1]. Among them, the Minimum Variance Distortionless Response (MVDR) beamformer [2] is one of the most traditional techniques. However, the performance of the MVDR beamformer significantly degrades if the steering information is inaccurate [3]. The most common technique to improve the robustness of the MVDR beamformer is the diagonal loading [3], [4]. The drawback is that there is currently no reliable method to select the optimal loading factor [5]. Null broadening [6], [7] is another robust approach. In [6], a cluster of equal-strength incoherent sources are artificially distributed around each original source in order to generate a trough like pattern. However, the mainlobe is not broadened. This may result in a poor coverage performance when the system is suffering high steering errors while the mainlobe of the beampattern is narrow. Furthermore, arranging equal number of additive sources for each interferer evidently limits the coverage performance.

In this letter, we propose an improved null broadening method. In particular, the following refinements are introduced to improve the robustness of null broadening techniques: 1) a new constrained optimisation problem is formulated to broaden both nulls and mainlobe; 2) the optimisation problem is solved using semidefinite programming (SDP) [8], [9]; 3) non-equal number of additive sources are assigned to different users to reduce the total number of optimisation parameters, which makes the SDP solution more efficient. The proposed approach is compared with the conventional null broadening technique proposed in [6] for downlink communications from high altitude platforms (HAPs) [10]. The performance improvement due to the modified null broadening approach is significant.

Null broadening techniques: MVDR adaptive beamforming places nulls in the direction of interferers whilst assigning unit power to the desired user. The $N \times 1$ MVDR weight vectors for the mth desired user is given by [2]

$$\mathbf{w}_m = \frac{\mathbf{R}^{-1}\mathbf{u}(\theta_m)}{\mathbf{u}^H(\theta_m)\mathbf{R}^{-1}\mathbf{u}(\theta_m)}, \quad m = 1 \cdots M, \tag{1}$$

where: $[\cdot]^H$ denotes the Hermitian transpose; M is the total number of users; $\mathbf{u}(\theta_m)$ is the $N \times 1$ steering vector pointing θ_m degrees from the broadside; N is the number of antenna elements; $\mathbf{R}$ is an $N \times N$ covariance matrix whose entries can be written in an analytical form as [7]

$$R_{qp} = N_p \delta_{qp} + \sum_{m=1}^{M} P_m e^{jk_0(x_q - x_p)\sin\theta_m}, \quad q, p = 1 \cdots N, \tag{2}$$

where N_p is the noise variance, δ_{qp} is the Kronecker delta function, P_m is the received power of the mth user, $j = \sqrt{-1}$, $k_0 = 2\pi/\lambda$ is the wave number, λ the wavelength, and x_q represents the element location. In order to produce a trough pattern in each of the interference direction, Mailloux proposed to distribute U equal-strength incoherent sources around each original interference source [6]. Fig.1 illustrates the null broadening approach in the scenario of HAP communications [10] with a vertical linear antenna array mounted on a HAP at an altitude H, providing the coverage over a circular area of radius r. In this case, the modified covariance matrix $\widetilde{R}$ has entries [6]

$$\widetilde{R}_{qp} = \frac{\sin(U\Lambda_{qp})}{U\sin(\Lambda_{qp})} R_{qp}, \quad q, p = 1 \cdots N, \tag{3}$$

where $\Lambda_{qp} = \pi(x_q - x_p)\xi/\lambda$, $\xi = \varepsilon/(U-1)$ and ε represents the trough width, shown in Fig.1.

An improvement of Mailloux's method can be achieved by broadening the mainlobe in addition to broadening the nulls. Let's denote

$$\Theta = [\Theta_1, \Theta_2, \cdots, \Theta_M] \tag{4}$$

and

$$\Theta_m = [\theta_m^{(-\frac{U-1}{2})}, \theta_m^{(-\frac{U-1}{2}+1)}, \cdots, \theta_m^{(0)}, \cdots, \theta_m^{(\frac{U-1}{2}+1)}, \theta_m^{(\frac{U-1}{2})}], \quad m = 1, \cdots, M, \tag{5}$$

where $\theta_m^{(0)} = \theta_m$ represents the original user position. The set Θ contains all user positions including the additive sources around each original user. We denote $\overline{\Theta}_m$ as the complementary set of Θ_m with respect to Θ, that is $\overline{\Theta}_m \cup \Theta_m = \Theta$. The improved null broadening approach is based on minimising the total power of all original interferers and their respective additive sources, while assigning unity received power to the desired user and its additive sources. Then, the beamformer weight vector $\mathbf{w}_m = [w_m^{(1)}, \cdots, w_m^{(N)}]^T$ providing a beampattern for the mth user can be found by solving the following constrained optimisation problem

$$\mathbf{w}_m = \arg\min_{\mathbf{w}} \sum_{\theta_i \in \overline{\Theta}_m} \left|\mathbf{w}^H \mathbf{u}(\theta_i)\right|^2 \text{ subject to } \sum_{\theta_n \in \Theta_m} \left|\mathbf{w}^H \mathbf{u}(\theta_n)\right|^2 = 1, \tag{6}$$

where $[\cdot]^T$ denotes the transpose, $i = 1, \cdots, (M-1)U, \quad n = 1, \cdots, U$. Different from assigning a unit power to the desired user, as in the case of the MVDR beamformer [2] and the null broadening method in [6], the problem (6) sets a constraint to the total received power of a cluster of 'desired users' in order to broaden the mainlobe. The problem (6) can be described as a minimisation problem, consisting of an objective function and one equality constraint function. Such optimisation problem is especially appropriate for an SDP solver (by using the interior point method) [11]. SDP creates a smooth convex nonlinear barrier function for the constraints and makes it practical to solve convex problems of a large size [8], [9]. However, the performance of SDP is affected by the total number of optimisation parameters, that is $M \times U$ in the problem (6). Therefore, in order to obtain more efficient SDP solutions, the total number of optimisation parameters should be reduced and this can be achieved by assigning non-equal number of additive sources for different users. In particular, the number of additive sources required can be made inversely proportional to the beamwidth of the beampattern projected to the desired user. For a vertical linear array, it can be found that the number of additive sources for the mth original user U_m can be made proportional to $\sin(\theta_m)$. We now rewrite (4) as

$$\Theta = [\Theta_1^{(U_1)}, \Theta_2^{(U_2)}, \cdots, \Theta_M^{(U_M)}], \tag{7}$$

and

$$U_m = [a\sin(\theta_m) + b], \tag{8}$$

where, $[\cdot]$ denotes the rounding integer. For a given number of antenna elements, the coefficients a and b can be found by computer simulations to maximize the signal to interference ratio (SIR), which is defined below.

Numerical results: Assume that M users are uniformly distributed over the coverage area. The mth user receives signals of powers $\{P_1, \cdots, P_M\}$ from all M beams of the HAP antenna. Downlink SIR for the mth user is given by [10]

$$\eta_m = \frac{P_m}{\sum_{u=1,u\neq m}^{M} P_u}. \tag{9}$$

The coverage performance is represented by the probability distribution function $C(\gamma) = Pr\{\eta_m > \gamma\}$. Fig.2 compares the coverage performance of the following four adaptive beamforming methods in the downlink HAPs communication scenario: 1) MVDR beamformer (1); 2) null broadening method in [6]; 3) proposed null broadening method using equal number of additive sources (6) for all users; and 4) proposed null broadening method using non-equal number of additive sources.

The communication scenario is defined as: $r = 32.7$ km; $H = 20$ km; $N = 171$; $M = 60$. The users position errors are random with uniform distribution within the interval [-60m, +60m]. Coefficients in (8) are set to $a = 5$ and $b = 3$. The number of additive sources in method 2 is $\widehat{U} = \frac{1}{M}\sum_{m=1}^{M} U_m$. We assume a noise free scenario for simplicity and use SIR (9) to estimate the coverage performance. Fig.2 shows the comparison results. It can be seen that the MVDR beamformer has a poor performance in such a scenario. The null broadening method in [6] significantly improves the performance when compared to the MVDR beamformer. Method 3, using constrained optimisation, however, can achieve better coverage performance. Further improvement is obtained by method 4, using non-equal number of additive sources; in total, a 4.5 dB SIR improvement is achieved for 95% coverage , compared with the Mailloux's null broadening method proposed in [6].

Conclusions: A constrained optimisation approach is proposed to improve the performance of null broadening adaptive beamforming. This approach broadens the pattern for the desired user as well as the interferers. The constrained optimisation problem is solved by semidefinite programming and its efficiency and performance are further improved when non-equal number of additive sources are distributed among users to reduce the total number of optimisation parameters. Simulation results show that the proposed approach achieves significant improvement in the coverage performance, compared with the previous null broadening method and the MVDR beamformer.

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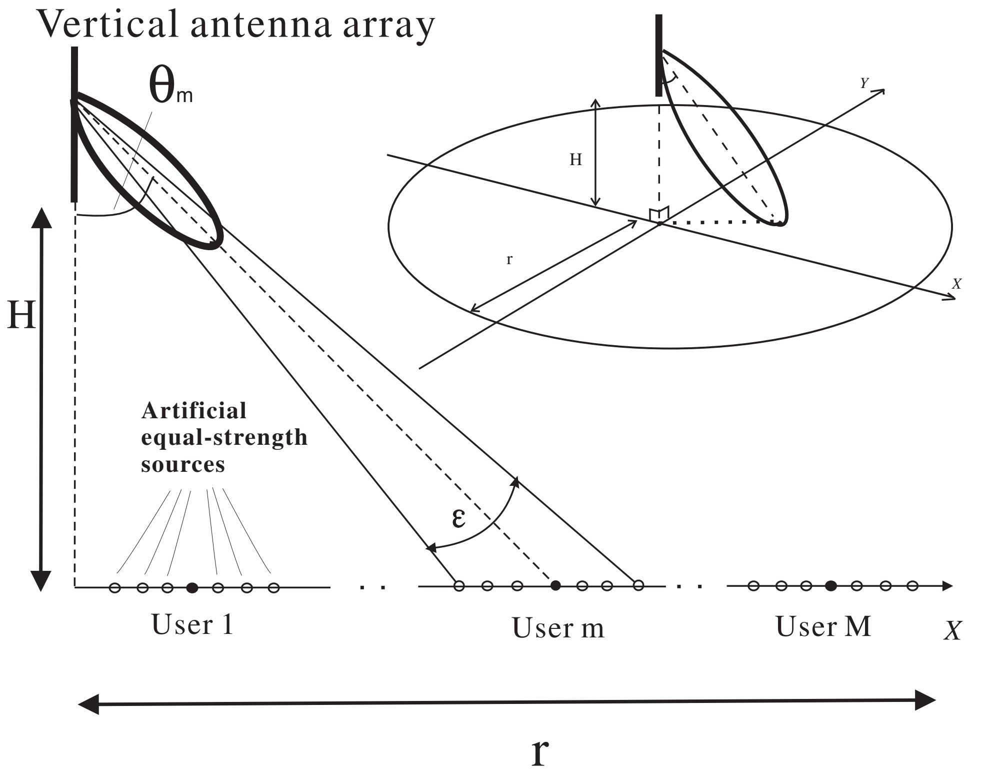
Figure captions

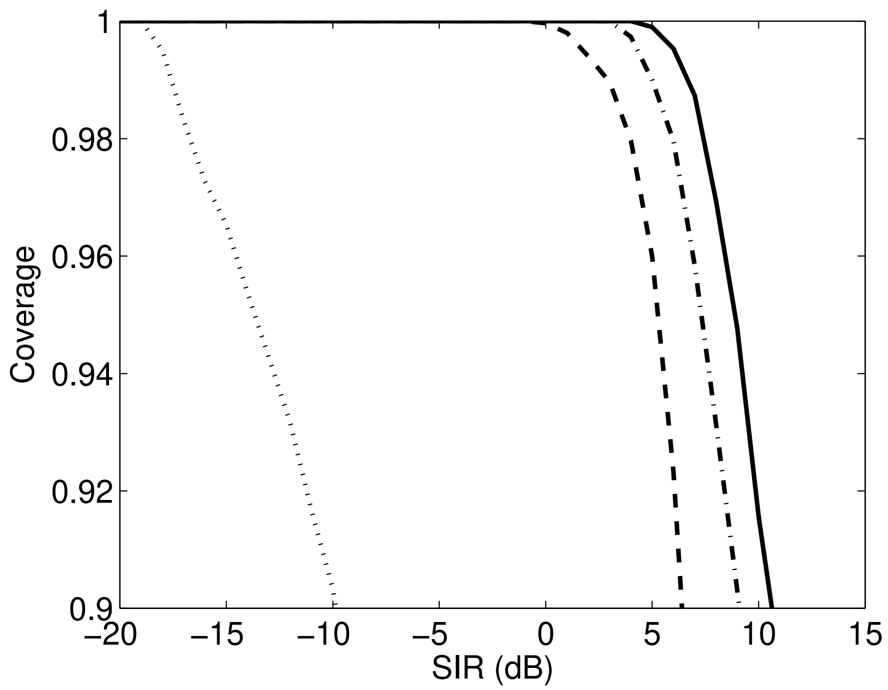
Fig.1 The use of null broadening approach for HAPs communications.

Fig.2 Coverage performance comparison of MVDR beamformer, null broadening method [6] and the improved null broadening method in the HAPs communications scenario: $H = 20$ km; $r = 32.7$ km; 0.06 km maximum user position error; $M = 60$.
dotted: MVDR beamformer;
dashed: null broadening approach proposed in [6];
dash-dotted: improved null broadening approach using constrained optimisation, uniform distribution for the number of additive sources;
solid: improved null broadening method using constrained optimisation, non-uniform distribution for the number of additive sources.



Vertical antenna array
θm
H
Artificial
equal-strength
sources
ε
User 1
User m
User M
X
r
H
r
Y
X

Fig. 1.



Coverage
SIR (dB)
1
0.98
0.96
0.94
0.92
0.9
–20
–15
–10
–5
0
5
10
15

Fig. 2.