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Quantum Objects are Vague Objects

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Introduction

Is vagueness a feature of the world or merely of our representations of the world? Of course, one might respond to this question by asserting that insofar as our knowledge of the world is mediated by our representations of it, any attribution of vagueness must attach to the latter. However, this is to trivialize the issue: even granted the point that all knowledge is representational, the question can be re-posed by asking whether vague features of our representations are ultimately eliminable or not. It is the answer to this question which distinguishes those who believe that vagueness is essentially epistemic from those who believe that it is, equally essentially, ontic. The eliminability of vague features according to the epistemic view can be expressed in terms of the supervenience of 'vaguely described facts' on 'precisely describable facts':

> If two possible situations are alike as precisely described in terms of physical measurements, for example, then they are alike as vaguely described with words like 'thin'. It may therefore be concluded that the facts themselves are not vague, for all the facts supervene on precisely describable facts. (Williamson 1994, p. 248; see also pp. 201-204)

It is the putative vagueness of certain identity statements in particular that has been the central focus of claims that there is vagueness 'in' the world (Parfit 1984, pp. 238-241; Kripke 1972, p. 345 n. 18). Thus, it may be vague as to who is identical to whom after a brain-swap, to give a much discussed example. Such claims have been dealt a forceful blow by the famous Evans-Salmon argument which runs as follows: suppose for reductio that it is indeterminate whether $a = b$. Then b definitely possesses the property that it is indeterminate whether it is identical with a, but a definitely does not possess this property since it is surely not indeterminate whether a=a. Therefore, by Leibniz's Law, it cannot be the case that a=b and so the identity cannot be indeterminate (Evans 1978; Salmon 1982).

However, the claims for ontic vagueness have been reasserted in precisely this context by drawing on the famous indistinguishability of particles in quantum mechanics (Lowe 1994). Such particles are indistinguishable in a much stronger sense than mere possession of all intrinsic, or state independent, properties in common, as in classical mechanics, and this is expressed by the Indistinguishability Postulate of quantum statistics which asserts that permutations of particles of the same kind are not observable, unlike the case in classical statistics. The claim then is that given this latter indistinguishably, there is simply no fact of the matter as to whether two particles, two electrons say, are identical or not. The vagueness here is truly ontic (Lowe ibid.).

The force of such claims hinges on how we understand the Indistinguishability Postulate (French and Krause 1995; French, Krause and Maidens forthcoming). One possibility is to regard it in terms of a restriction on the sets of states the particles can occupy (French 1989a). Thus the particles are 'assigned' (perhaps at the moment of creation!) to bosonic or fermionic states and once in such states the dynamics, as represented by Schrödinger's equation, ensures that they remain there. On this interpretation the particles are distinct, albeit indistinguishable, individuals, like their classical counterparts, the difference being that unlike the latter they are constrained as to the kinds of states they can occupy.

Where does vagueness arise on this account? Consider an electron *a*, say, captured by an atom to form a negative ion which then emits electron *b* (Lowe op. cit.). Quantum mechanics, as standardly understood, ascribes 'entangled' states to the systems *a* plus atom and atom plus *b* such that it is not possible to say whether $a=b$ or not. The central issue in the philosophy of quantum mechanics is precisely how to understand such states. Teller understands them in terms of a failure of 'supervenience' in the sense that they represent relational properties which do not supervene on the non-relational properties of the particles (if they did there would be a violation of Bell's Theorem; Teller 1986, 1989; French 1989b). The indeterminacy of identity arises, therefore, because of this 'veil' of non-supervenient relations: there simply is no way of piercing the veil to determine which particle is which (French and Krause op. cit.; French, Krause and Maidens forthcoming).

What about the Evans-Salmon argument in this case? An essential step in the argument is the move from the determinacy of the self-identity of *a*, say, to the claim that *a* definitely lacks the property that it is indeterminate whether it is identical with *a* (which is possessed by *b*). However, the latter property cannot be determinately distinct from the property of being indeterminate whether the object is identical with *b*, since the two properties differ only by a permutation of *a* and *b* and it is indeterminate whether $a=b$ by assumption (Lowe op. cit.). Hence the possession by either *a* or *b* of an identity involving property such as these cannot serve to determinately differentiate the two. Recasting the Evans-Salmon argument in terms identity-free properties (Noonan 1995) then 'forces' the defender of ontic vagueness to accept that vague objects must be strongly indistinguishable in the sense that any identity-free property determinately possessed by either must be determinately possessed by both, but that is precisely what quantum mechanics tells us is the case!

This has obvious implications for the epistemic view. If facts are plausibly taken to involve properties, then the supervenience of vaguely described facts on precisely described facts must be understood in terms of the supervenience of the relevant properties. But as we have just seen, this is denied in the quantum context. Inasmuch as the 'facts' involving entangled states do not supervene on any facts involving the intrinsic properties of the particles or hidden variables (this being ruled out by Bell's Theorem) there is genuine ontic vagueness here.

This latter point needs further emphasis. Of course, our grasp (such as it is) of the quantum domain is mediated via a representation, namely quantum theory itself, but if this is counted as enough to rule out the vagueness involved as 'truly' ontic, then the epistemic complement is utterly trivialised. Indeed, the force of Bell's Theorem lies in its generality and it is this which renders the vagueness ontic in the sense that it is not dependent upon a *particular* representation. Quantum particles are therefore vague irrespective of whether or how they are represented, if the issue of representation is understood in this non-trivial sense.

Returning to the Indistinguishability Postulate, it can be understood alternatively as leading to a kind of non-individuality for quantum particles. The argument goes roughly as follows: In both classical and quantum mechanics particles of the same kind are regarded as indistinguishable in the sense of possessing all 'intrinsic' properties in common. Yet in classical statistical mechanics a particle permutation is counted as observable, whereas in the quantum theory it is not. Since the former result is typically accounted for by appealing to the particles' individuality which goes beyond or 'transcends' their intrinsic properties (Post 1963), the latter is taken to suggest that the particles have lost this individuality and that they are, indeed, 'non-individuals' in some sense (Schrödinger 1952, 1957; Born 1943; Weyl 1949; Post 1963).

Explicating this sense is, of course, metaphysically problematic. A possible ontological 'attractor' for one's spiraling ruminations as to how an entity could be a 'non-individual' is the notion of 'identity'. Inextricably linked with individuality through the history of philosophy, it is precisely a failure of (self-)identity that is attributed to quantum particles by Schrödinger and Hesse, for example, the latter remarking that '[w]e are unable to identify individual electrons, hence it is meaningless to speak of the self-identity of electrons …' (1970, p. 50). Denying identity is a formally tractable way of representing this notion of non-individuality and indeed interesting formal systems can be constructed for doing so. Before we outline these, however, two further points need to be emphasised.

The first is that the above metaphysical package which denies 'transcendental' individuality is typically taken to derive support from the manner in which it meshes with the ontology of quantum field theory, where particle labels are simply not assigned right form the word go (Redhead and Teller 1991, 1992; Teller 1995). There are problems with such claims but insofar as the 'quanta' of this view are non-individual objects, they too can be represented by these non-standard formal frameworks (French and Krause forthcoming). Secondly, here again we have vagueness of an ultimately ontic form — the quanta themselves are vague not in the respect that their intrinsic properties are somehow 'blurred' or 'fuzzy' but in that as non-individuals, their very identity is vague. It is to the formal representation of this kind of vagueness that we now turn.

Vague Predicates, Opaque Predicates and their Extensions

Following Terricabras and Trillas (1989), we may characterize a predicate of a (say) first-order logic as *vague* on the following grounds. Standard ('Fregean') predicates are such that their denotation provides a bipartition in the domain *D* into two disjoint subsets, the *extension* of *D*, denoted *Ext(P)* and its complement relative to *D*. The objects of the domain which belong to *Ext(P)* are said to have the property ascribed by *P*, while those that belong to the complement of *P* don't have the property. Vague predicates are those predicates which do not provide such a bipartion in the domain. That is, there remain in *D* some objects which neither belong to *Ext(P)* nor to its complement. For such objects, it is asserted that it is vague whether they have the considered property or not.

Vagueness in this sense is characterized as a feature of certain linguistic expressions, such as the property ascribed by the predicate *P* in the above example, and not as concerning the objects of the domain, which are supposed to be well-defined. In other words, in considering a vague predicate like 'to be a profound thinker', it may be vague if Ms. X, a philosopher, is a profound thinker or not, but it is generally agreeded that she is a well known person, since we *know* who we are talking about. This way of interpreting vagueness bears a realist view of science, at least according to Putnam, who said that 'On the metaphysical realist view there are vague conceptions, vague ways of talking, but not vague *objects*' (Putnam 1983). But in the real sciences there is vagueness in a truly ontic sense and our discussion above pulls the rug out from under claims such as Putnam's.[1] Let us describe the underlying intuitions concerning semantics in this case.

The predicates to be considered here, which we refer to as *opaque*, resemble the above case but are distinct in the following sense: the 'vagueness' lies in the objects of the domain, and not in the predicate itself. To exemplify this idea, let us consider the property 'to have spin up in the direction *x*', which can be meaningfully ascribed to a certain collection of (say) electrons. Note that the predicate is well-defined, since physicists know perfectly well the requirements an electron must satisfy on order to have spin up in the *x*-direction (such details are not important here). So, by making use of an adequate intrumental apparatus, they can find a certain number, say *n* of electrons which satisfy the property, and the same number is obtained if the experiment is repeated. However there is no way to assert either *which* are the electrons of the collection that have such a property or if the *n* electrons of the first measurement *are the same* as those of the second experiment. This, of course, is not a distinctive feature of the *x*-direction or of the electrons, but constitutes one of the fundamental pressupositions of quantum mechanics.

The underlying idea is that, roughly speaking, the electrons are absolutely indistinguishable in the 'strong' sense indicated above, so we cannot pick out one of them from the collection in order to verify if it has spin up in the *x*-direction or not. Electrons, like the other elementary particles, have no names, have no identity, and cannot be distinguished from one another.[2] In other words, the 'vagueness' now concerns the *objects* of the domain.

The reason we call these predicates *opaque* is that the part of the domain to which they should be related (by the usual semantical techniques) is seen to be concealed by a kind of veil, which prevents us from seeing its elements clearly. So, in order to provide an adequate semantical analysis of a logic involving opaque predicates in this sense, there is the necessity of not only characterizing the predicates as opaque, but also of explaining what kind of entity is to be considered as the domain. A standard *set* (as in standard semantics), does not serve for the purposes, since a set is, according to Cantor's well known 'definition' «… a gathering into a whole of objects which are quite *distinct* in our intuition or our thought» (cf. Bourbaki 1993, p. 25, our emphasis). In other words,

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[1] Indeed we have suggested that one way to maintain a form of realism in the quantum context is to take this vagueness seriously (French, Krause and Maidens forthcoming).

[2] It should be recalled that it is consistent with the formalism of quantum mechanics to treat elementary particles as 'individuals' subjected to certain restrictions in their possible states (French and Redhead 1988).

in considering the domain as a *set* as described by the theories of sets, we cannot approach the idea of *opacity* in the sense just mentioned. The question then is: what is to be considered as the extension of an opaque predicate?

Before considering a possible answer to this question, let us comment in brief on standard set theories. It is important to note that although no axiomatic system *defines* its primitive concepts, in the sense observed by Skolem, and this is so in particular with respect to the concept of *set*.. Thus, standard set theories like Zermelo-Fraenkel, von Neumann-Bernays-Gödel, Kelley-Morse or others, do not modify Cantor's intuition as expressed in his 'definition'. This point is clear in Zermelo's paper from 1908, in which he introduces the first axiomatic set theory. Zermelo acknowledges that Cantor's definition may be restricted, but even so applies the axiomatic method to a «historically existing 'theory of sets' ' (Zermelo 1908), that is, in preserving Cantor's intuitions.[3] All other set theories derive from Zermelo's, and so they also maintain the idea of collections of *distinguishable objects*, and this is so also with regard to intensional set theories, which emphasise the manner in which the mathematical objects are given to us (cf. Feferman 1985), but do not question the underlying ontology.

Concerning the question mentioned above, it is important to note that we are not trying to provide a mathematical trick by means of which opacity in our sense could be semantically characterized. What we are trying to explain here is a much more profound insight intrinsically related with the very nature of *quanta*, to use a word which does not compromise us with the intuitive idea of 'particle' or an object with individuality.[4] Following Schrödinger's suggestion of regarding them as entities to which the concept of identity cannot be applied (cf. Schrödinger op. cit.; see da Costa and Krause 1994), we have developed logico-mathematical systems in which this intuition can be formally developed (da Costa and Krause 1994; da Costa and Krause forthcoming a, b; Krause 1992; Krause forthcoming; French and Krause op. cit.; Dalla Chiara, Giuntini and Krause forthcoming.).

We should acknowledge that the idea that the ontology of quantum mechanics does not reduce to that of sets was anticipated by Dalla Chiara and Toraldo di Francia in several works (Dalla Chiara and Toraldo di Francia 1993, 1995, forthcoming; Dalla Chiara 1987, 1987a.). As they

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[3] Wang has also noted that in Cantor's writings there are implicit axioms for sets, such as those concerning extensionality, power-set, sub-sets and others, which were not explicated by Cantor since, according to Wang, they were 'too obvious' (see Wang 1991).

[4] As did Paul Teller in his 1995.

have shown, in order to obtain a adequate semantical analysis for the languages of microphysics, a more suitable (meta)mathematical apparatus should be erected, and they have proposed a theory of *quasets* for this purpose. Having noted that standard sets are not adequate for expressing the extensions of opaque predicates, we may ask: could we use quasets as extensions of opaque predicates? In order to answer this question, let us first of all mention in brief the nature of these mathematical entities.

Roughly speaking, a quaset is a collection of objects which have a well-defined cardinal, but there is no way to tell (with certainty) which are the elements that belong to the quaset. A suitable distinction between two primitive predicates [and { (which is not the negation of the former), meaning ‘certainly belongs to’ and ‘certainly does not belong to’ respectively, is provided by the axiomatics, and so the theory allows situations in which *z* [*y* entails ~(*z* { *y*), but not the converse. Consequently, it may be the case that it is false that *z* certainly does not belong to *y*, but this does not entail that *z* (certainly) belongs to *y*. The elements *z* to which it may be said that ‘it is false that they certainly do not belong to *y*’ are ‘potential members’ of *y*. Furthermore, since the cardinal of the quaset is fixed, there is a kind of ‘epistemic’ indeterminacy with respect to its elements in the sense that we don’t know exactly which objects belong to a quaset.

We could use quasets as the extensions of opaque predicates, but this does not constitute a ‘legitimate’ solution for the problem we have proposed. In fact, it should be noted that a theory of identity continues to hold in the underlying logic of quaset theory (which should be regarded as being the first-order predicate calculus with identity), and so the elements of a quaset are still *distinct objects*, to use Cantor’s words, despite the epistemic indeterminacy that exists in regarding their elements. In other words, they remain *individuals*.[5]

Let us emphasise this point. Quaset theory is a beautiful theory founded on original insights. But in regarding its use for providing mathematical constructs which can conveniently be used as the extensions of opaque predicates, it provides no advantages to other set theories, since none of them achieve any better solution than Weyl’s way of treating aggregates of individuals. In his 1949 work, Weyl simply takes a *set S* (whose cardinal is *n*, for example), together with an equivalence relation *R* on *S* and considers the equivalence classes of the quotient set *S/R*. Then, by ‘forgetting’ the ‘nature’ of the elements of *S* and paying attention

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5 This is of course another source of philosophical controversy, but let us regard an ‘individual’ as an entity for which there exists a reasonable theory of identity which applies to it, and this is the case with the elements of a quaset, as we have remarked.

exclusively to the cardinality $n(i)$ (i=1, …, k) of the equivalence classes, he obtains the 'ordered decomposition' $n(1) + \ldots + n(k)=n$ which, as Weyl emphasises, is precisely what is considered in quantum mechanics (these numbers resemble the occupation numbers of quantum field theory). However, this is a trick, since the elements of a *set* still remain as individuals in our sense, and to 'forget' their individuality may provide a mathematical way of justifying the intuitions here, but of course it does not solve the philosophical problem regarding indistinguishability. With quasets something similar occurs, since it was by modifying the meaning of the membership relation that we arrived at the 'epistemic' indeterminacy of their elements, which despite this can still be regarded as individuals. A more adequate way of providing extensions of opaque predicates is, according to us, by using *quasi-sets*.

In quasi-set theory,[6] the presence of two sorts of atoms (*Urelemente*), termed *m*-atoms and *M*-atoms is allowed, but the concept of identity (on the standard grounds) is restricted to the *M*-atoms only. Concerning the *m*-atoms, there is a weaker 'relation of indistinguishability', which is postulated to have the properties of an equivalence relation, and this relation is used among the *m*-atoms instead of identity. Since the latter (that is, the predicate of equality) cannot be applied to the *m*-atoms, there is a precise sense in saying that they can be indistinguishable without being identical. So, contrary to the case of quasets, the lack of sense in applying the concept of identity to the *m*-atoms produces in quasi-set theory a kind of 'ontic' indeterminacy. That is, the *m*-atoms have their individuality intrinsically undermined.

Although we shall not provide all the technical details here (but see Krause forthcoming), we may justify the claim that there is a certain quantity of elements in a quasi-set whose elements are all indistinguishable from one another. The theory encompasses a primitive concept of *quasi-cardinal*, which reduces to the concept of cardinal in the standard sense when there are no *m*-atoms involved (this is due to the fact that when we restrict the axioms to exclude *m*-atoms, they turn out to be exactly the axioms of Zermelo-Fraenkel with *Urelemente*, and in this 'copy' of Zermelo-Fraenkel we can define the standard concepts of set theory). Furthermore, the concept of subquasi-set is like the classical one and the quasi-cardinal of the power quasi-set of a certain quasi-set x (that is, the collection of its subquasi-sets) is greater that the quasi-cardinal of x (let us suppose that it is $2^{qc(x)}$). So the theory *is compatible with* the existence of 'singleton' subquasi-sets of x, although we cannot prove that these 'singletons' are distinct from each other as in the usual extensional

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6 We will make reference to the quasi-set theory presented in Krause forthcoming.

contexts, since this would entail a distinction between their elements, which is precisely what the theory tries to avoid. These 'singletons' are merely *indistinguishable* in the sense that they have the same quantity (ascribed by their quasi-cardinal) of elements of the 'same sort' (that is, they belong to the same equivalence class of indistinguishable objects). The concept of indistinguishability between quasi-sets is captured by the *weak* axiom of extensionality, used instead of the standard axiom of extensionality, and which precisely asserts that quasi-sets with the same quantity of elements of the same sort share the primitive relation of indistinguishability.

This departure from classical set theories with regard to extensionality is necessary in this context, as also remarked by Dalla Chiara and Toraldo di Francia, who proposed an *intensional* semantics for the languages of microphysics (see their papers mentioned above). Quasi-sets of indistinguishable objects of course cannot be extensionaly comparable on standard grounds, but this is not sufficient: we must go further in departing from the classical ontology pressuposed by classical set theories, and the possibility of considering the lack of identity for certain elements seems to enable us to consider a completely new situation concerning collections of objects.

Collections of absolutely indistinguishable *m*-atoms were termed *veiled sets* in Krause and French forthcoming, and such entities are 'natural' objects to be used as the extensions of opaque predicates. In this latter paper, we presented a logic encompassing such predicates, whose semantics is founded in quasi-set theories. In this way, we think we have approached in a more adequate manner the semantics of certain entities, namely the opaque predicates, which are inherent to quantum mechanics. In a certain sense, Bohr was completely right when he said that we cannot approach this subject without the help of a cluster of completely new concepts, including at the logical level, we might add.

Conclusion

We have suggested here that quantum objects are vague objects and, further, that how that vagueness is understood depends on the metaphysical package adopted with regard to their individuality. If quantum objects are taken to be individuals, as Lowe considers them, then the vagueness arises because of the existence of relations which do not supervene on monadic properties of the relata; it is because of such relations that we cannot tell which particle is which in an entangled state. How one represents such relations, both metaphysically and formally, is an interesting question and one possibility, with regard to the latter at least, is to employ *quaset* theory; we leave this as a suggestion for future elaboration.

The alternative package characterises quanta as non-individuals, where this is understood in terms of a lack of identity. The appropriate formal framework is then that of *quasi-set* theory, which provides a semantics for ‘opaque’ predicates as indicated above. There are still some interesting questions to be addressed here, such as how it is that one can refer to objects for which one cannot even say that identity holds. On this point we take our lead from Barcan Marcus who, in discussion with Kripke and Quine, distinguished ‘object-reference’ from ‘thing-reference’, where the former is given in terms of quantification, and the latter is bound up with identity (Barcan Marcus 1993, p. 25).[7] We may thus ‘refer’ to objects for which identity cannot be said to hold, although how we do this in the quantum context is again an issue which requires further discussion (see French and Krause forthcoming).

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[7] And also with other restrictiosn such as spatio-temporal location.

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