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SYNCHRONISATION OF THE SUPERIMPOSED TRAINING METHOD FOR CHANNEL ESTIMATION IN THE PRESENCE OF DC-OFFSET

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ABSTRACT

The superimposed training method estimates the channel from the induced first-order cyclostationary statistics exhibited by the received signal. In this paper, using vector space decomposition, we show that the information needed for training sequence synchronisation, and for DC-offset estimation, can be extracted from the first-order cyclostationary statistics as well. Necessary and sufficient conditions for channel computation and equalisation are derived, when training sequence synchronisation and DC-offset removal are required. The computational burden of the practical implementation of the method presented here is much lighter than for existing algorithms. At the same time, simulation results show that the performance, in terms of the MSE of the channel estimates and BER, is not diminished when compared to these existing algorithms.

1. Introduction

In communications, the system estimation problem is often solved by the inclusion of a training sequence, as opposed to the long data record demanding blind-identification techniques. Traditionally, the training sequence and the data sequence were allocated in separate time slots (as in TDM) thus wasting bandwidth. This problem was addressed by the superimposed (implicit) technique (ST/IT) [1,2], where a periodic training sequence is actually added to the data prior to transmission, at the expense of a small data-power loss.

The knowledge of the added training sequence at the receiver is what enables the ST method to estimate the channel; any other sequence received at the receiver (including the data) must be considered as noise. But the negative effects of this 'data noise' can be completely removed. To see how it is done, it is easier to examine the signal in the frequency domain. Given that the training sequence is periodic of period P, its Discrete Fourier Transform (DFT) have non-zero energy at only P equally spaced DFT bins. The goal is to make the energy of the data sequence zero at these bins, thus removing their effect on the training sequence. The details are given in [3], where the data dependent ST (DDST) method is developed.

In both ST and DDST, it is important that the position within the received sequence, that corresponds to the start of a training sequence period, is known at the receiver. We will refer to this kind of synchronisation as 'training sequence synchronisation' (TSS). TSS for ST was first studied in [1] in conjunction with DC-offset estimation. The TSS method presented in [1] was based on higher-order statistics (HOS) and polynomial rooting, and only required that the training sequence period is no smaller that the number of channel taps M—i.e. $P \geq M$. The use of HOS and polynomial rooting was avoided in the TSS method presented in [4], but required $P \geq 2M+1$. These two TSS methods can be applied to DDST as well.

In this paper we present a new TSS method and we will apply it specifically to ST. It is based on the properties of the projections, onto two specifically defined subspaces, of the cyclic permutations of the vector that contains the received sequence's first-order, cyclostationary statistics. This new method for TSS has a much lighter computational burden than the methods in [1,4], while at the same time it shows better or equivalent behaviour —as the included simulations illustrate— in terms of the MSE of the channel estimates and the BER.

2. Problem description and geometrical interpretation

The familiar system set-up required for the ST method is depicted in Fig. 1 [1]. Accordingly, the received data block in the ST method has the following form [1,2]:

$$x(k) = \sum_{l=0}^{M-1} h(l)b(k-l) + \sum_{l=0}^{M-1} h(l)c(k-l) + n(k) + m \tag{1}$$

with $k = 0, 1, \dots, N-1$, where $b(k)$ is the information bearing sequence, $h(k)$ is the channel impulse

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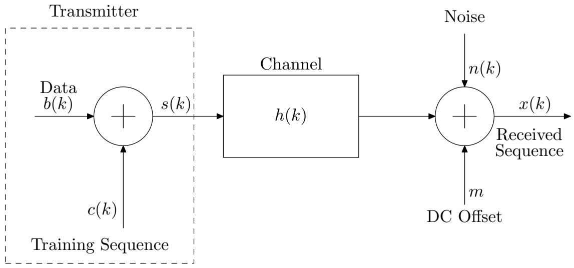

Fig. 1. The mathematical model for ST.

response, $n(k)$ is the noise and m represents an unknown DC-offset term due to using first-order statistics (see (2)) with non-ideal r.f. receivers (see [1]). Furthermore, $c(k)$ is the superimposed training sequence of mean $\bar{c} = \frac{1}{N_P}\sum_{k=0}^{P-1} c(k)$ and power $\sigma_c^2 = \frac{1}{N_P}\sum_{k=0}^{P-1} |c(k)|^2$, periodic with period $P \geq M$.

We will assume the following:

H1) All terms in (1) can be complex valued.

H2) The sequences $b(k)$ and $n(k)$ are independent and identically distributed (i.i.d.) random sequences of zero mean.

H3) The channel is of order $M-1$ —i.e. $h(0) \neq 0$ and $h(M-1) \neq 0$.

H4) The channel order is known.

Under H2 and the periodicity of the training sequence, we can see from (1) that the output sequence $x(k)$ is first-order cyclostationary of period P. Thus, we can define its cyclostationary mean

$$y_0(j) := \mathrm{E}[x(iP+j)] = \sum_{l=0}^{M-1} h(l)c(j-l)_P + m \quad (2)$$

with $j = 0, \ldots, P-1$ where $(\cdot)_P$ indicates arithmetic modulo-P, and the subscript '0' indicates that it is a fixed (deterministic) value as opposed to a general variable $y(j)$, and this nomenclature will be used throughout the rest of the paper. Equation (2) can be written in matrix form as

$$\mathbf{y}_0 = \mathbf{C}_{[M]}\mathbf{h}_0 + \mathbf{m}_0 \quad (3)$$

where $\mathbf{C}_{[M]}$ is $P\times M$ and $\mathbf{h}_0 = [h(0),\ h(1), \ldots,\ h(M-1)]^T$ is $M \times 1$; $\mathbf{y}_0 = [y_0(0),\ y_0(1),\ \ldots,\ y_0(P-1)]^T$ and $\mathbf{m}_0 = [m,\ \ldots,\ m]^{\mathrm{T}}$ are both $P \times 1$. Matrix $\mathbf{C}_{[M]}$ corresponds to the first M columns of matrix $\mathbf{C} = \mathrm{circ}(c(0),\ c(P-1),\ c(P-2),\ \ldots,\ c(1))$, where the operation 'circ' produces a circulant matrix [5]. Matrix $\mathbf{C}$ is thus composed of $\mathbf{C}_{[M]}$ in (3) and its 'complement' $\mathbf{C}_{\langle P-M\rangle}$, i.e. the last $P-M$ columns of $\mathbf{C}$, where

$$\mathbf{C} \equiv \left[\mathbf{C}_{[M]}|\mathbf{C}_{\langle P-M\rangle}\right]. \quad (4)$$

To make the subspace interpretation that follows meaningful, we require $\mathbf{C}$ to be full rank. Note that this is not a necessary condition for channel estimation using ST assuming perfect TSS, as was shown in [6]. To make $\mathbf{C}$ full rank, we are going to use optimum channel independent (OCI) training sequences that were introduced in [1]. These also give, $\mathbf{C}^{\mathrm{H}}\mathbf{C} = \mathbf{C}\mathbf{C}^{\mathrm{H}} = P\sigma_c^2\mathbf{I}_{P\times P}$, simplifying the projection operation between subspaces. Thus, from the least squares solution to (3), the filter coefficients are obtained by (noting that $\mathbf{C}_{[M]}^{\mathrm{H}}\mathbf{C}_{[M]} = P\sigma_c^2\mathbf{I}_{M\times M}$)

$$\mathbf{h}_0 = \frac{1}{P\sigma_c^2}\mathbf{C}_{[M]}^{\mathrm{H}}\left(\mathbf{y}_0 - \mathbf{m}_0\right). \quad (5)$$

In the situation where there is no TSS, once the cyclic means in (2) are computed, there is no way to say which of them corresponds to $j = 0$, $j = 1$ and so on. The only thing we know is that they appear sequentially and that the computed cyclostationary mean is an unknown cyclic permutation $\mathbf{P}_0\mathbf{y}_0$ of the true one ($\mathbf{y}_0$), due to the periodicity of $c(k)$. It is important to note that a matrix $\mathbf{P}_0$ that performs a cyclic permutation operation on a vector is a circulant matrix as well. The vector $\mathbf{m}_0$ is not affected by any (cyclic) permutation because all its components are equal and so $\mathbf{m}_0 = \mathbf{P}_0\mathbf{m}_0$.

For $P = M$, and no TSS, the solution to (3) is a cyclically permuted version of the true channel coefficient vector, $\mathbf{P}_0\mathbf{h}_0$. This solution is obtained from (5) with $\mathbf{y}_0$ replaced by $\mathbf{P}_0\mathbf{y}_0$, making $\mathbf{m}_0 = \mathbf{P}_0\mathbf{m}_0$, and noting that $\mathbf{C}\mathbf{P}_0 = \mathbf{P}_0\mathbf{C}$ —i.e. circulant matrices commute [5]. Thus, TSS reduces to finding the correct permutation $\mathbf{P}_0$, as was (implicitly) done in [1].

For $P > M$, the channel vector can not be retrieved as in the previous paragraph ($P = M$), because the matrices $\mathbf{C}_{[M]}$ and $\mathbf{P}_0$ do not now commute. One possible option then is to pre-process the available vector $\mathbf{P}_0\mathbf{y}_0$ and select the correct $\mathbf{y}_0$ among a set of candidates, before solving (3).

Important clues to develop a new method for synchronisation can be derived from the previous paragraphs. Recall that the method in [1] ($P = M$) required HOS and polynomial rooting, and so is rather complex. On the other hand, the method in [4] ($P > M$) uses the FFT and is simpler that the former. Furthermore, (5) obtains the channel vector just by projecting on the subspace spanned by the columns of $\mathbf{C}_{[M]}$ (recall that $\mathbf{C}$ is OCI). So we set out to investigate the advantages of using an overdetermined system of equations ($P > M$) and try to interpret the problem resolution as a projection process.

To start with, we study what happens to the cyclostationary mean vector after a cyclic permutation. From the RHS of (3), we can confirm that after a cyclic permutation of $\mathbf{y}_0$, $\mathbf{C}_{[M]}\mathbf{h}_0$ will be the only affected term —recall that $\mathbf{m}_0$ is invariant under permutations. Thus, in the next Lemma we study the effect of a permutation on $\mathbf{C}_{[M]}\mathbf{h}_0$.

Lemma 1 *Let* $\boldsymbol{C}$ *be a full rank circulant* $P\times P$ *matrix,* $\boldsymbol{P}$ *any cyclic permutation* $P\times P$ *matrix, and* $\boldsymbol{h}$ *any* $M\times$

1 *vector. Then,* $\boldsymbol{PC}_{[M]}\boldsymbol{h}$ *can be uniquely decomposed as*

$$\begin{aligned}\boldsymbol{PC}_{[M]}\boldsymbol{h} =& \boldsymbol{C}_{[M]}(\boldsymbol{P}[\boldsymbol{h}^T\ \boldsymbol{0}^T_{P-M}]^T)_{[M]} + \\ &+ \boldsymbol{C}_{\langle P-M\rangle}(\boldsymbol{P}[\boldsymbol{h}^T\ \boldsymbol{0}^T_{P-M}]^T)_{\langle P-M\rangle}\end{aligned} \tag{6}$$

where $\boldsymbol{0}_{P-M}$ *is the column vector* $[\underbrace{0,\ \dots,\ 0}_{P-M}]^T$ *and for a vector* $\boldsymbol{v}$, $\boldsymbol{v}_{[M]}$ ($\boldsymbol{v}_{\langle P-M\rangle}$) *are its first* M *(last* $P-M$*) elements.*

Proof: First note that $\mathbf{C}_{[M]}\mathbf{h} = \mathbf{C}[\mathbf{h}^T\ \mathbf{0}^T_{P-M}]^T \Rightarrow \mathbf{PC}_{[M]}\mathbf{h} = \mathbf{PC}[\mathbf{h}^T\ \mathbf{0}^T_{P-M}]^T$. Now, using the commutativity of circulant matrices, $\mathbf{PC}[\mathbf{h}^T\ \mathbf{0}^T_{P-M}]^T = \mathbf{CP}[\mathbf{h}^T\ \mathbf{0}^T_{P-M}]^T$, and (6) follows from (4). The uniqueness comes because $\mathbf{C}$ is full rank. Q.E.D

The interpretation of Lemma 1 is clear. Consider the vector space spanned by the columns of matrix $\mathbf{C}$, which are a base for this space as well because $\mathbf{C}$ is full rank. In turn, $\mathbf{C}_{[M]}$ and $\mathbf{C}_{\langle P-M\rangle}$ span two subspaces V and $V^{\perp}$ respectively, which are orthogonal because $c(k)$ is OCI. Assume for the moment that $m = 0$ in (3). The true cyclostationary mean vector $\mathbf{y}_0$ lies exactly on V—i.e. it is a linear combination of the columns of $\mathbf{C}_{[M]}$—but any cyclic permutation $\mathbf{P} \neq \mathbf{I}$ of it will have components in $V^{\perp}$ as well. This important property can be used to achieve TSS in the DC-offset free case. The next section proposes a general method to deal with TSS in the presence of a non-zero DC-offset.

3. Proposed training sequence synchronisation method

Because of the lack of TSS, assume that the cyclic permutation of the cyclostationary mean available at the receiver is $\mathbf{P}_0\mathbf{y}_0$. To work with the most general case possible, a DC-offset will be taken into account as well. Let us now consider the decomposition of $\mathbf{P}_0\mathbf{y}_0$ in V and $V^{\perp}$.

So, applying a cyclic permutation operator to both sides of (3), we need to know the decomposition of the permuted $\mathbf{C}_{[M]}\mathbf{h}$ and the decomposition of (the permuted) $\mathbf{m}_0$. The former is given by Lemma 1 while the decomposition of the DC-offset term is given by

$$\mathbf{m}_0 = \mathbf{C}_{[M]}\tilde{\mathbf{m}}_{0[M]} + \mathbf{C}_{\langle P-M\rangle}\tilde{\mathbf{m}}_{0\langle P-M\rangle} \tag{7}$$

where $\tilde{\mathbf{m}}_0$ is a $P \times 1$ vector of constant elements $\frac{m}{P\bar{c}}$, as can easily be confirmed. So from (3), and using (7) and Lemma 1, then we have,

$$\begin{aligned}\mathbf{P}_0\mathbf{y}_0 =& \mathbf{C}_{[M]}(\mathbf{P}_0[\mathbf{h}_0^T\ \mathbf{0}^T_{P-M}]^T)_{[M]} + \\ &+ \mathbf{C}_{\langle P-M\rangle}(\mathbf{P}_0[\mathbf{h}_0^T\ \mathbf{0}^T_{P-M}]^T)_{\langle P-M\rangle} + \\ &+ \mathbf{C}_{[M]}\tilde{\mathbf{m}}_{0[M]} + \mathbf{C}_{\langle P-M\rangle}\tilde{\mathbf{m}}_{0\langle P-M\rangle}.\end{aligned} \tag{8}$$

Consider now the projection of $\mathbf{P}_0\mathbf{y}_0$ onto the $V^{\perp}$ space. So, multiply both sides of (8) by $\frac{1}{P\sigma_c^2}\mathbf{C}^H_{\langle P-M\rangle}$:

$$\begin{aligned}&\frac{1}{P\sigma_c^2}\mathbf{C}^H_{\langle P-M\rangle}\mathbf{P}_0\mathbf{y}_0 = \\ &\qquad = (\mathbf{P}_0[\mathbf{h}_0^T\ \mathbf{0}^T_{P-M}]^T)_{\langle P-M\rangle} + \tilde{\mathbf{m}}_{0\langle P-M\rangle}.\end{aligned} \tag{9}$$

Now, two different cases are clearly distinguishable:

- C1) For $\mathbf{P}_0 = \mathbf{I}$ the RHS of (9) reduces to $\tilde{\mathbf{m}}_{0\langle P-M\rangle}$—i.e., a vector with all its components of equal value $\frac{m}{P\bar{c}}$.
- C2) For $\mathbf{P}_0 \neq \mathbf{I}$ the first term of the RHS of (9) does not vanish, and thus, we will not have a vector of equal components.

Note that C2 is only valid *in general*. The conditions under which C2 is *always* true will be discussed shortly. The properties of (9) under cases C1 and C2 can be used for TSS, but prior to the formalisation of these properties in form of a useful proposition, it is necessary to develop a measure of how equal are the elements of a vector. So, define the operator $\mathcal{J}\{\mathbf{v}\} = \|\mathbf{v} - \bar{\mathbf{v}}\|^2$, where $\bar{\mathbf{v}} = [\bar{v},\ \dots,\ \bar{v}]^T$ and $\bar{v}$ is the mean of all the elements of $\mathbf{v}$. The desired property of $\mathcal{J}\{\mathbf{v}\}$ is that $\mathcal{J}\{\mathbf{v}\} = 0$ iff all the elements of $\mathbf{v}$ are equal to each other (and thus, equal to the mean).

Proposition 1 *Let* $P \geq 2M + 1$*, hereafter known as the strong constraint, then* $\mathcal{J}\left\{\boldsymbol{C}^H_{\langle P-M\rangle}\boldsymbol{P}_0\boldsymbol{y}_0\right\} = 0$ *iff* $\boldsymbol{P}_0 = \boldsymbol{I}$.

Proof: The necessary condition ($\Leftarrow$) is proved by C1. For the sufficient condition ($\Rightarrow$), we need to find the conditions under which C2 is always true for all $\mathbf{P}_0$, $\mathbf{y}_0$ and OCI $c(k)$. Thus, let us work with the worst case scenario—i.e. when all the M components of $\mathbf{h}_0$ are equal. So, if we require $(\mathbf{P}_0[\mathbf{h}_0^T\ \mathbf{0}_{P-M}]^T)_{\langle P-M\rangle}$ not to be a vector of equal components for any $\mathbf{P}_0 \neq \mathbf{I}$ and $\mathbf{h}_0 \neq \mathbf{0}_M$, then we require that its length is larger that M—i.e. $P - M > M$. Q.E.D

TSS is finally achieved as follows. The available cyclic permutation of the cyclostationary mean vector, $\mathbf{P}_0\mathbf{y}_0$, is cyclically permuted by all the cyclic permutations of P elements. The cyclic permutation $\mathbf{PP}_0\mathbf{y}_0$ of $\mathbf{P}_0\mathbf{y}_0$ minimising the operator $\mathcal{J}\left\{\mathbf{C}^H_{\langle P-M\rangle}\mathbf{PP}_0\mathbf{y}_0\right\}$ is the true cyclostationary mean vector $\mathbf{y}_0$. This follows because by proposition 1 $\mathbf{PP}_0 = \mathbf{I}$, and thus $\mathbf{PP}_0\mathbf{y}_0 = \mathbf{y}_0$.

Once $\mathbf{y}_0$ is known, the DC-offset m can be computed, using (9) under case C1, from any of the elements of $\mathbf{C}^H_{\langle P-M\rangle}\mathbf{y}_0$. Nevertheless, we propose to perform an average of the elements of $\mathbf{C}^H_{\langle P-M\rangle}\mathbf{y}_0$ because when it comes to the practical implementation of the method, i.e. estimation, the average will of course give a smaller variance. So,

$$m = \frac{\bar{c}}{\sigma_c^2}\frac{1}{P-M}[\underbrace{1,\ \dots,\ 1}_{P-M}]\mathbf{C}^H_{\langle P-M\rangle}\mathbf{y}_0 \tag{10}$$

and m is the mean just mentioned normalised by the quotient between the mean ($\bar{c}$) and the power (σ_c^2) of the training sequence.

Finally, once $\mathbf{y}_0$ and m are known, the channel coefficients can then be computed from (5).

3.1 Relaxing assumption H4 —conditions for equalisation

The assumption H4 is required in order to apply Proposition 1, which is the basis of the TSS method presented here. The channel order is needed twice in Proposition 1. Firstly, so that the *strong* constraint can be enforced; secondly, it appears in the argument of the operator $\mathcal{J}$. If H4 is not fulfilled, the channel cannot be estimated. Nevertheless, when it comes to equalisation what is needed is just an upper bound for the channel order, as it will be shown next.

The effects of using an upper bound are then twofold. Firstly, the range of values of P satisfying the strong constraint is included in the range of values obtained if the actual channel order is used. So, no problem is encountered here. Secondly, if the channel order is assumed to be bigger than what it actually is, then $\mathbf{h}_0$ will have extra zero taps at the tails. This will allow the operator $\mathcal{J}$ to give more than one possible solution following proposition 1. Anyway, all the allow solutions obtained will be related by a linear shift and the only effect on equalisation is a delay. This delay can also appear in a practical implementation of the proposed TSS method, if the first or last of the channel taps is very close to zero.

4. Actual application of the method

In an actual application, the elements of the cyclostationary mean vector $\mathbf{y}_0$ have to be estimated using, as usual, time averages: $\hat{y}_0(j) = \frac{1}{N_P}\sum_{i=0}^{N_P-1} x(iP+j)$, $j = 0,\ 1,\ \ldots,\ P-1$. But because of lack of TSS, this estimate will correspond to an unknown cyclic permutation of $\mathbf{y}_0$, i.e. $\mathbf{P}_0\hat{\mathbf{y}}_0$. To simplify notation, replace $\mathbf{P}_0\hat{\mathbf{y}}_0$ by $\hat{\mathbf{y}}_0^{(\mathbf{P}_0)}$. Based on this estimate, the TSS, the DC-offset estimation and the channel estimation are sequentially obtained as shown in Table 1.

4.2 Computational burden

Consider the overall computational burden of the practical implementation, in terms of total products and divisions. For the proposed method in Table 1, the computational burden is $P^3 + (1-M)P^2 + 2P + 3$, i.e. $\mathcal{O}(P^3)$; for the method in [4], the computational burden is $MNP + 2P^3 + (M+1)P^2 - (M+2)P + 1$, i.e. $\mathcal{O}(MNP)$. The filtering steps required in [4], contributing toward the MNP term, can be a significant part of the computational burden of [4]. For example, let $N = \mathcal{O}(P^3)$ and $M = \mathcal{O}(P)$ as in [4] and as in the following simulation, then $\mathcal{O}(MNP)$ becomes $\mathcal{O}(P^5)$.

Cyclostationary mean estimation: $\hat{\mathbf{y}}_0^{(\mathbf{P}_0)}(j) = \frac{1}{N_P}\sum_{i=0}^{N_P-1} x(iP+j+k_0)$ $j = 0,\ 1,\ \ldots,\ P-1$ where k_0 is an unknown synchronisation offset.
Training sequence synchronisation: $\{\mathbf{P}_l\}_{l=1}^{P}$ =set of all $P \times P$ cyclic permutation matrices. Compute $\mathbf{P}_{\text{opt}} = \arg\min_{\mathbf{P}_l} \left\{ \mathcal{J}\left\{ \mathbf{C}_{\langle P-M\rangle}^{\mathrm{H}} \mathbf{P}_l \hat{\mathbf{y}}_0^{(\mathbf{P}_0)} \right\} \right\}$
DC-offset estimation: $\hat{m} = \frac{\bar{c}}{\sigma_c^2}\frac{1}{P-M}\underbrace{[1,\ \ldots,\ 1]}_{P-M}\mathbf{C}_{\langle P-M\rangle}^{\mathrm{H}}\mathbf{P}_{\text{opt}}\hat{\mathbf{y}}_0^{(\mathbf{P}_0)}$
Channel estimation: From (5), $\hat{\mathbf{h}}_0 = \frac{1}{P\sigma_c^2}\mathbf{C}_{[M]}^{\mathrm{H}}\left(\mathbf{P}_{\text{opt}}\hat{\mathbf{y}}_0^{(\mathbf{P}_0)} - \underbrace{[\hat{m},\ \ldots,\ \hat{m}]}_{P}\right)$.

Table 1. Proposed method for training sequence synchronisation of the ST method for channel estimation in the presence of DC-offset.

Comparing $\mathcal{O}(P^3)$ of the proposed method with $\mathcal{O}(P^5)$ of the method in [4], it is evident that the computational burden reduction achieved with the new method is very significant. And this reduction could be even bigger if the number of samples N used in the estimation is increased.

5. Simulation

Three-tap Rayleigh fading channels were simulated. The channel coefficients were complex Gaussian, i.i.d. with unit variance. The average energy of the channel was set to unity. The data was a BPSK sequence, to which an OCI training sequence (see [1]) was added before transmission. The training to information power ratio $\left(\text{TIR} = \frac{\sigma_c^2}{\sigma_b^2}\right)$ was set to -6.9798 dB, the training sequence period to $P = 7$ and the number of samples to $N = 399$ —the same values that were used in [4]. We generated $N_B = 300$ blocks at the transmitter, where (1) represents just one of these blocks. Note that only N samples were used for channel estimation, but all the blocks were used for BER computation. A deterministic DC-offset (m) was added at the channel output, together with a zero-mean white Gaussian noise. The value of the DC-offset was determined by the DC-offset to signal AC-component (DCAC) power ratio as defined in [2]

$$\text{DCAC} = m^2/\text{E}\left[|x(k) - n(k) - m|^2\right].$$

In these simulations this was set to DCAC= 0.1. At each realisation, a random synchronisation offset between 0 and $N + P - 1$ was introduced between transmitter and receiver, so we could be at *any* sample index within the first *block*. After channel estimation, an MMSE equaliser, based on the channel estimates, of length 11 and optimum delay was used to compute the BER; 1000 realisations were averaged. As already mentioned at the end of subsection , we may have an unknown delay of the estimated channel with respect to the true one. In this particular case, it may happen because of the randomness of the channel taps, so the first and last channel tap could be close to, or even, zero. This *identification delay*, which in practice has no major consequences, can worsen the simulated BER, misleading the performance analysis of the method. To avoid this, the identification delay was computed by comparing the equalised symbols with the true ones, for different delays, and choosing the delay giving the smallest BER—this problem was reported in [4] as well. For a comparison between the TSS methods in [1] and [4], please refer to [4], where simulations show that the later clearly outperforms the former.

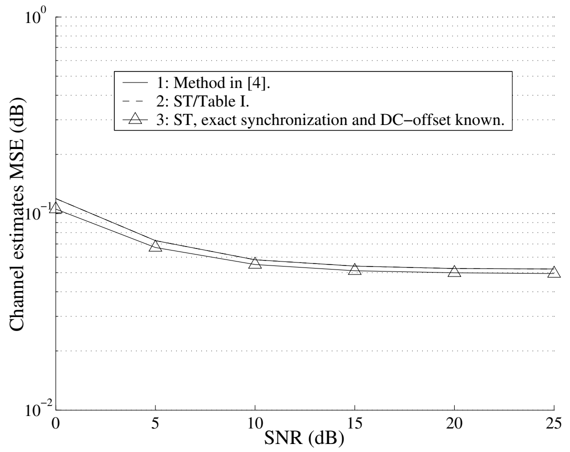

Fig. 2. MSE of channel estimates, as a function of the SNR, computed following Table 1. The identification delay has been considered. The estimates assuming known DC-offset and perfect TSS are included, together with the TSS method in [4], for comparison purposes. Note that methods 1 and 2 are indistinguishable on the graph.

Figure 2 shows the MSE of the channel estimate obtained with the method presented in Table 1. The MSE obtained with the ST method assuming perfect TSS is plotted as well as a benchmark. The proposed method (with no TSS) is not very much different. Finally, compared with the method in [4], the best published TSS method for ST so far, we can see that our

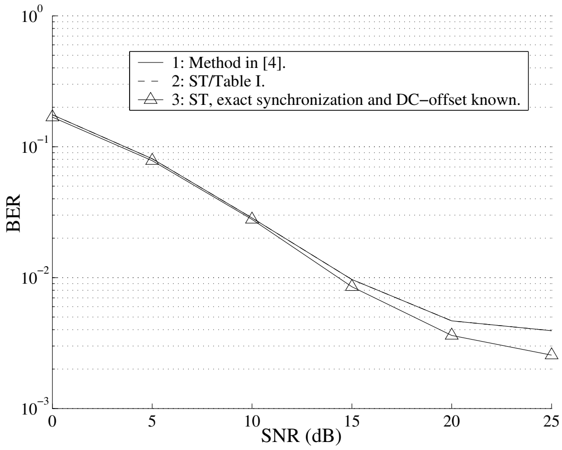

Fig. 3. BER versus SNR obtained using the algorithm of Table 1. Note that methods 1 and 2 are indistinguishable on the graph.

proposed method gives identical results, but with the already mentioned, huge reduction in computational burden.

Figure 3 shows the BER of the proposed method and that of the ST with perfect TSS. The method in [4] is included as well for comparison. The conclusions drawn in the previous paragraph are equally applicable here too.

6. Conclusions

In this work, the channel estimation and equalisation problem has been addressed under the superimposed training scheme. No training sequence synchronisation was provided, and a DC-offset could be present at the output as well. The proposed method for channel estimation relies on the decomposition of the permutations of the cyclostationary mean vector into two vectors: one vector contains information for synchronisation and DC-offset estimation while the second contains information for channel estimation. Both these vectors are projections of the cyclostationary mean vector onto two particular subspaces of the space spanned by all the cyclic permutations of the training sequence. With this geometrical interpretation of the problem, sufficient and necessary conditions for the method to work are easily derived, involving the training sequence period and the channel order. Based on the channel estimate, a MMSE equaliser was constructed. For equalisation, the exact channel order does not need to be known, but just an upper bound. Simulations show that this method performs as well —in terms of the MSE of the channel estimates and the BER— as existing methods, but with greatly reduced computational burden.

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