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Coalitional Control: Clustering Based on the $\mathcal{H}_2$ Norm

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***Abstract*—This letter presents an original analysis of the coupling sources in a network of dynamical systems. A coalitional controller exploits switching between different groupings of network members to achieve its regulation goals. In this letter, we rigorously analyse a key assumption in robust distributed control methods: the weak coupling assumption. We present an analysis based on Linear Matrix Inequalities to assess this assumption to design Robust Control Invariant sets for all possible network partitions. We also exploit our coupling analysis to propose a novel partition selection optimisation based on the system's $\mathcal{H}_2$ norm. This strategy simplifies the partition selection process, as demonstrated through illustrative examples, leading to enhanced computational efficiency while rigorously preserving the stability and robustness essential for distributed control system applications.**

***Index Terms*—Model predictive control, clustering, distributed systems, plug-n-play events, tube-based MPC.**

I. Introduction

The control of distributed systems involves techniques addressing dimensionality, scalability, and robustness to achieve a shared objective. A common approach is to divide the system into interconnected subsystems managed by a local controller, the so-called *agent*, to simplify and efficiently solve the control task. This decomposition leads to a partitioning problem often studied using graphs [1], which capture the communication topology and coupling strengths between agents. Moreover, partitioning introduces a coordination challenge that requires handling couplings and constraints.

System partition and agent coordination are usually managed by cluster-oriented strategies, such as *coalitional* control [2], which preserve performance and stability under disturbances and changing conditions. Coalitional control lies between distributed and decentralised approaches, dynamically activating communication links as required for performance. Its main idea is to group agents into non-overlapping coalitions, each managed as a single control entity [3]. Coalitional control provides a dynamic control framework that adapts the control policy to time-varying interactions, couplings, and plug-and-play operations such as connecting and disconnecting subsystems from the network [4], [5].

Within the field of coalitional control, our focus is on Model Predictive Control (MPC) [6], an optimisation-based framework well suited to deal with constraints and disturbance information while guaranteeing objectives such as cost efficiency, tracking and stability. Coalitional MPC has been successfully applied in practical domains such as irrigation canals [3], telecommunication networks [7], and solar plants [7]. Several works have also explored coalitional MPC schemes with formal stability and robustness guarantees, *e.g.*, using invariant sets [8] and tube methods [5], [9], [10]. For instance, [9] proposes a decentralised MPC strategy to manage coalitions with high-order dynamics and analyses recursive feasibility under partition switching. Moreover, [10] studies the relation between partitions and recursive feasibility.

The *partitioning* problem in coalitional MPC has been addressed from different perspectives. In general, partitions are determined either via a bottom-up approach, in which agents select their coalitions using local information (*e.g.*, through consensus [11]), or a top-down approach, in which a supervisory controller defines the partition based on global criteria [3], [7]. The latter usually leads to a two-layer coalitional MPC structure [5], in which the high-level MPC periodically solves a centralised problem that accounts for system-wide dynamics, constraints, and objectives. Afterwards, the best partition is selected by balancing performance and communication overhead. Based on this partition, the low-level MPC controllers are clustered into coalitions that solve their corresponding control problems.

A key limitation of high-level MPC-based partitioning is its computational complexity, which grows rapidly with the number of partitions. We propose a lightweight partitioning method based on the $\mathcal{H}_2$ norm to handle this. Our approach builds on the tube-based coalitional MPC framework in [5], which reduces conservatism via shrinking disturbances and online-optimised Robust Control Invariant (RCI) sets. Our contributions are twofold. First, in Section III-A, we analyse inter-agent couplings and derive a set of Linear Matrix Inequalities (LMI) to ensure partition feasibility, integrated

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into the RCI set design. Second, in Section III-B, we propose a partitioning criterion that minimises a cost capturing both coupling strength and network connectivity. Section IV formulates the optimisation problem to select a partition based on the $\mathcal{H}_2$ norm. In Section V, we test our method on an example, showing similar performance to existing approaches while reducing computational burden.

Notation: A *graph* $\mathcal{G} = (\mathcal{M}\ \mathcal{E}, W)$ consists of node set $\mathcal{M} = \{1, \dots, M\}$; an edge set $\mathcal{E} \subseteq \mathcal{M} \times \mathcal{M}$ where each $e = (i, j)$ denotes a connection between two nodes i and j; and a weight matrix $W \in \mathbb{R}^{M \times M}$. A graph is *undirected* if W is symmetric. The neighbour set of node $i \in \mathcal{M}$ is $\mathcal{M}_i = \{j \in \mathcal{M} \colon \exists (i, j) \in \mathcal{E}\}$. The $(\cdot)^C$ denotes the set complement, $(\cdot)^\top$ the transpose, and $\otimes$ the matrix Kronecker product. For a square matrix $A \in \mathbb{R}^{N \times N}$, $\mathrm{Tr}(A) = \sum_{i=1}^{N} A_{ii}$ is the trace and $\lambda(A)$ its eigenvalues. For a compact, convex $\mathcal{A} \subset \mathbb{R}^n$, the support function $h \colon \mathbb{R}^n \to \mathbb{R}$ is $h(u) = \sup\{u^\top x \colon x \in \mathcal{A}\}$. The Minkowski sum of sets $\mathcal{A}, \mathcal{B} \subseteq \mathbb{R}^n$ is $\mathcal{A} \oplus \mathcal{B}$, and the Cartesian product of a finite collection $\{\mathcal{A}_i\}_i$ is $\bigtimes_i \mathcal{A}_i$. A set $\mathcal{R} \subset \mathbb{R}^n$ is RCI for a system $x^+ = f(x, u, w)$ with input constraints $\mathbb{U}$ and compact disturbances $\mathbb{W}$, if $\forall x \in \mathcal{R}$, $\exists \tilde{u} \in \mathbb{U}$, $\forall w \in \mathbb{W}$, $f(x, \tilde{u}, w) \in \mathcal{R}$.

II. Problem Setting

Consider a discrete linear time-invariant system:

$$x^+ = Ax + Bu \tag{1}$$

where $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$ are the state and input, respectively; x^+ denotes the successor state. We consider, additionally, that (1) admits a basic partitioning into M independently actuated but dynamically coupled subsystems. The dynamics of each subsystem are:

$$x_i^+ = A_{ii} x_i + B_{ii} u_i + \sum_{j \in \mathcal{M}_i} A_{ij} x_j + B_{ij} u_j \tag{2}$$

where $x_i \in \mathbb{R}^{n_i}$ and $u_i \in \mathbb{R}^{m_i}$ are the state and input for $i \in \mathcal{M}$ such that $x = (x_1, \dots, x_M)$ and $u = (u_1, \dots, u_M)$.

Assumption 1: For each $\forall i \in \mathcal{M}$, the pair (A_{ii}, B_{ii}) is controllable.

Each subsystem's state and input are constrained within sets $\mathbb{X}_i$ and $\mathbb{U}_i$ satisfying the following assumption:

Assumption 2: For each $i \in \mathcal{M}$, the set $\mathbb{X}_i \subset \mathbb{R}^{n_i}$ is a *C-set*, *i.e.,* a compact and convex set containing the origin, and $\mathbb{U}_i \subset \mathbb{R}^{m_i}$ is a *C-set* with the origin in its interior.

A. Clustering and Further Partitions of the System

The partitioning of (1) into a set $\mathcal{M}$ of subsystems is not the only possible subdivision of the system. Considering the elements of $\mathcal{M}$ as the system's atoms, we can define further partitions based on clusters or coalitions of these atoms; see Fig. 1. We formalise this notion as follows:

Definition 1 (Partition of the System): A *partition* of the system is an arrangement of the M subsystems into $C \leq M$ coalitions: the partition of $\mathcal{M} = \{1, \dots, M\}$ is the set $\mathcal{C}$, satisfying the following properties:

i) Coalition $c \in \mathcal{C}$ fits $c \subseteq \mathcal{M}$ and its cardinality is M_c.

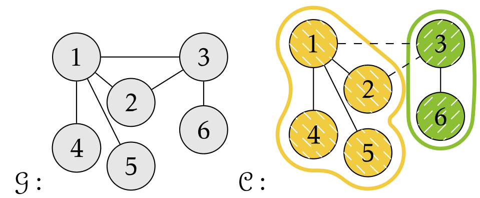

Fig. 1. Example of a coupled system $\mathcal{M} = \{1, \dots, 6\}$ that is partitioned into $\mathcal{C} = \{c_1, c_2\}$ with two coalitions $c_1 = \{1, 2, 4, 5\}$ and $c_2 = \{3, 6\}$.

ii) Coalitions are non-overlapping: $c \cap d = \emptyset$ for all $c \neq d$ and $c, d \in \mathcal{C}$.

iii) Coalitions cover the set of subsystems: $\bigcup_{c \in \mathcal{C}} c = \mathcal{M}$.

The set of all possible partitions $\Pi_{\mathcal{M}} = \{\mathcal{C} \colon \mathcal{C}$ is a partition of $\mathcal{M}\}$ contains both trivial cases, *i.e.,* the grand coalition $\mathcal{C}_{\text{cen}} = \{c\}$ where $c = \mathcal{M}$ and the decentralised coalition $\mathcal{C}_{\text{dec}} = \{\{1\}, \dots, \{M\}\}$. The set $\Pi_{\mathcal{M}}$ is also a partially ordered set concerning partition refinement, denoted by $'\preceq'$. Given $\mathcal{C}, \mathcal{D} \in \Pi_{\mathcal{M}}$, the partition $\mathcal{D} \preceq \mathcal{C}$ ($\mathcal{D}$ *refines* $\mathcal{C}$, or $\mathcal{C}$ *coarsens* $\mathcal{D}$) if every member of $\mathcal{D}$ is contained in some member of $\mathcal{C}$. Given a partition $\mathcal{C}$, the state and input of coalition c are, respectively, $x_c = (x_i)_{i \in c}$ and $u_c = (u_i)_{i \in c}$. The dynamics of coalition c are

$$x_c^+ = A_{cc} x_c + B_{cc} u_c + w_c \tag{3}$$

where the matrices A_{cc} and B_{cc} contain, as sub-blocks, the matrices of subsystems within the coalition: $A_{cc} = [A_{ij}]_{i,j \in c}$ and $B_{cc} = [B_{ij}]_{i,j \in c}$. Similar to the system basic partition, coalitions remain coupled via their dynamics: coalition c is coupled with coalition d via the matrices $A_{cd} = [A_{ij}]_{i \in c, j \in d, d \neq c}$ and $B_{cd} = [B_{ij}]_{i \in c, j \in d, d \neq c}$ such that $w_c \triangleq \sum_{d \in \mathcal{M}_c} A_{cd} x_d + B_{cd} u_d$ where $\mathcal{M}_c \triangleq \{d \in \mathcal{C} \setminus \{c\} \colon [A_{cd}\ B_{cd}] \neq 0\}$ is the set of neighbours of coalition $c \in \mathcal{C}$. The structure of constraints for all possible partitions of set $\mathcal{M}$ is the following:

Assumption 3: For any partition $\mathcal{C} \in \Pi_{\mathcal{M}}$ and $\forall c \in \mathcal{C}$, the input and state constraint sets are $\mathbb{U}_c = \bigtimes_{i \in c} \mathbb{U}_i$ and $\mathbb{X}_c = \bigtimes_{i \in c} \mathbb{X}_i$.

Assumption 3 implies the absence of coupled constraints in our setting; this is clear by taking $\mathcal{C}_{\text{cen}} \in \Pi_{\mathcal{M}}$. A consequence of Assumptions 2 and 3 is that the interactions are bounded within $\mathbb{W}_c = \bigoplus_{d \in \mathcal{M}_c} A_{cd} \mathbb{X}_d \oplus B_{cd} \mathbb{U}_d$.

1) Graph-Theoretic Point of View: From the decentralised partition, we can define a coupling graph $\mathcal{E} \subset \mathcal{M} \times \mathcal{M}$ based on the coupling structure, as shown in Fig. 1. When adopting a different partition $\mathcal{C} \in \Pi_{\mathcal{M}}$, there are two types of coupling: the *inter*-coalition and the *intra*-coalition. The graph theoretical approach allows us to characterise these couplings as follows.

Definition 2: For a basic partition set $\mathcal{M}$ with coupling graph $\mathcal{E}$, for each $\mathcal{C} \in \Pi_{\mathcal{M}}$, there are:

i) intra-coalition coupling: $\mathcal{E}_{\mathcal{C}} = \mathcal{E} \cap \bigcup_{c \in \mathcal{C}} c \times c$, and

ii) inter-coalition coupling: $\mathcal{E}_{\mathcal{C}}^C = \mathcal{E} \cap \bigcap_{c \in \mathcal{C}} (c \times c)^C$.

Proposition 1: Given a set $\mathcal{M} = \{1, \dots, M\}$ and a graph $(\mathcal{M}, \mathcal{E})$. For any element $\mathcal{C} \in \Pi_{\mathcal{M}}$, the *intra*-coalition graph $\mathcal{E}_{\mathcal{C}}$ has at least $|\mathcal{C}|$ connected components.

Proof: The proof follows from Definitions 1 and 2. By construction, $\nexists (i, j) \in \mathcal{E}_{\mathcal{C}}$ such that $i \in c$ and $j \in d$ for any $c, d \in \mathcal{C}$, so the graph $\mathcal{E}_{\mathcal{C}}$ contains $|\mathcal{C}|$ connected components.

Moreover, if within a coalition $c \in \mathcal{C}$, $\exists j \in c$ such that $(j, i) \notin \mathcal{E} \cap (c \times c)$ for all $i \in c$, then the graph $\mathcal{E} \cap (c \times c)$ has more than one connected component. ■

B. Control Problem

The control objective is to regulate the state of the networked system from $x(0)$ with a control policy that minimises the following cost function while respecting constraints $\mathbb{X} = \bigtimes_{i\in\mathcal{M}} \mathbb{X}_i$ and $\mathbb{U} = \bigtimes_{i\in\mathcal{M}} \mathbb{U}_i$:

$$J_\infty = \sum_{k=0}^{\infty}\Big(\ell\big(x(k), u(k)\big) + g\big(\mathcal{C}(k)\big)\Big) \tag{4}$$

where $\ell(x(k), u(k)) = \sum_{i\in\mathcal{M}} x_i(k)^\top Q_i x_i(k) + u_i(k)^\top R_i u_i(k)$ is the stage cost with positive definite weights Q_i and R_i, and $g(\mathcal{C}(k))$ is the cost of clustering subsystems. In this letter, we seek to characterise the term $g(\mathcal{C}(k))$ concerning the coupling strength of the network. Our construction exploits intrinsic properties of the network in contrast to existing approaches, such as [11], which rely on coupling measures via the states.

C. Design of RCI Sets

Following the RCI formulation in [12], an implicit representation of an RCI for coalition c in (3) can be parameterised by sets $\mathbb{W}_c$, $\mathbb{X}_c$, and $\mathbb{U}_c$. The resulting RCI set $\mathcal{R}_c(\mathbb{W}_c) \subset \mathbb{X}_c$ with induced invariant control law $\mu_c\colon \mathcal{R}_c(\mathbb{W}_c) \to \mathbb{U}_c$, as used in [5] and [11], can be found by solving a linear program:

$$\min\{\epsilon_c : \phi = \big(\mathbf{S}_{c,H}, a_c, b_c, \epsilon_c\big) \in \Phi_c\} \tag{5}$$

with $\Phi_c = \big\{\phi\colon \mathbf{S}_{c,H} \in \mathcal{S}_{c,H}, \mathcal{R}_c(\mathbb{W}_c) \subseteq a_c\mathbb{X}_c, \mu_c\big(\mathcal{R}_c(\mathbb{W}_c)\big) \subseteq b_c\mathbb{U}_c,\ (a_c, b_c) \in [0, 1]\times[0, 1],\ s_x a_c + s_u b_c \le \epsilon_c\big\}$ with weights $s_x > 0$ and $s_u > 0$, and $\mathbf{S}_{c,H} = (S_{c,0}, \dots, S_{c,H-1})$, with $S_{c,h} \in \mathbb{R}^{m_c\times n_c}$ and $h = 0, \dots, H-1$, characterising the RCI set as:

$$\mathcal{R}_c(\mathbb{W}_c) = \bigoplus_{h=0}^{H-1} D_h\big(\mathbf{S}_{c,H}\big)\mathbb{W}_c \quad \text{and} \quad \mu_c(\mathcal{R}_c(\mathbb{W}_c)) = \bigoplus_{h=0}^{H-1} S_{c,h}\mathbb{W}_c$$

with $D_h(\mathbf{S}_{c,H}) = A_{cc}^h + \sum_{l=0}^{h-1} A_{cc}^{h-1-l}\ B_{cc}\ S_{c,l}$ for $h \ge 1$ and $D_0(\mathbf{S}_{c,H}) = I$ for $h = 0$, such that $D_H(\mathbf{S}_{c,H}) = 0$ provided that H is greater than or equal to the controllability index of (A_{cc}, B_{cc}), *i.e.,* $\mathcal{S}_{c,H} \triangleq \{\mathbf{S}_{c,H} : D_H(\mathbf{S}_{c,H}) = 0\}$.

D. Shrinking Disturbances and Tube MPC

From [5], the constraint sets evolve as $\alpha_c(k)\mathbb{X}_c$ and $\beta_c(k)\mathbb{U}_c$ for time-varying $\alpha_c(k), \beta_c(k) \in (0, 1]$ and $\alpha_c(0) = \beta_c(0) = 1$. Therefore, the disturbance sets also depend on these scaling factors $\mathbb{W}_c(\alpha, \beta) = \bigoplus_{d\in\mathcal{M}_c} A_{cd}\alpha_d\mathbb{X}_d \oplus B_{cd}\beta_d\mathbb{U}_d$. These scaling factors evolve as [5, Lemma 1]:

$$\alpha_c^+ = \alpha_c(a_c + \bar{\alpha}_c(1 - a_c)), \quad \beta_c^+ = \beta_c(b_c + \bar{\beta}_c(1 - b_c)) \tag{6}$$

The values of $(\bar{\alpha}_c, \bar{\beta}_c)$ are obtained as part of the solution of the following optimisation problem:

$$\begin{aligned} \mathbb{P}_c(\bar{x}_c,\ \alpha_c, \beta_c, a_c, b_c)\colon \ & \min_{\bar{u}_c,\ \bar{\alpha}_c,\ \bar{\beta}_c} \tau_\alpha\ \bar{\alpha}_c + \tau_\beta\ \bar{\beta}_c + \bar{J}_{N,c} \\ \text{s.t.}\quad & (\bar{\alpha}_c, \bar{\beta}_c) \in [0, 1]^2,\ \bar{x}_c(0) = \bar{x}_c,\ t = 0, \dots, N-1, \\ & \bar{x}_c(t+1) = A_{cc}\bar{x}_c(t) + B_{cc}\bar{u}_c(t), \\ & \bar{x}_c(t) \in \bar{\alpha}_c(1 - a_c)\alpha_c\mathbb{X}_c,\ \bar{u}_c(t) \in \bar{\beta}_c(1 - b_c)\beta_c\mathbb{U}_c \\ & \bar{x}_c(N) \in \bar{\mathbb{X}}_c^{\mathrm{f}}(\alpha_c, \beta_c, a_c, b_c), \end{aligned} \tag{7}$$

where $\bar{J}_{N,c} = V_c^{\mathrm{f}}(\bar{x}_c(N)) + \sum_{t=0}^{N-1} \ell_c(\bar{x}_c(t), \bar{u}_c(t))$ is an MPC cost, $\tau_\alpha, \tau_\beta > 0$, $\bar{\mathbb{X}}_c^{\mathrm{f}}(\alpha_c, \beta_c, a_c, b_c)$ is a positive invariant terminal set. This optimisation problem is parameterised by the current values of (α_c, β_c), the RCI scaling constants (a_c, b_c), and the current nominal state $\bar{x}_c$.

III. Coupling Analysis

In this section, we analyse and study the different sources of coupling in the distributed system network.

A. Coupling and RCI Set Design

Given a partition $\mathcal{C} \in \Pi_\mathcal{M}$ with coalition dynamics given by (3). The RCI design problem requires a *weak coupling assumption* in the form $\mathbb{W}_c \subset \mathbb{X}_c$. This is a necessary condition for $\mathcal{R}_c(\mathbb{W}_c) \subset \mathbb{X}_c$ to exist. However, the disturbance is not arbitrary but contains the dynamic couplings of the neighbours. Using the property, $\mathcal{A} \oplus \mathcal{B} = [\mathbb{I}_n\ \mathbb{I}_n]\mathcal{A} \times \mathcal{B}$ for $\mathcal{A}, \mathcal{B} \subset \mathbb{R}^n$, we can write the expression for the disturbance set as $\mathbb{W}_c = [\hat{A}_{c,*}\ \hat{B}_{c,*}]\,\Gamma_\mathcal{C}\,\mathbb{X} \times \mathbb{U}$ where $\Gamma_\mathcal{C} = \operatorname{diag}\big(\operatorname{diag}\{\alpha_c\mathbb{I}_{n_c}\}_{c\in\mathcal{C}}, \operatorname{diag}\{\beta_c\mathbb{I}_{m_c}\}_{c\in\mathcal{C}}\big)$, and $\hat{A}_{c,*}$, $\hat{B}_{c,*}$ are rows corresponding to coalition c of the matrices $\hat{A}_\mathcal{C} = A_\mathcal{C} - \operatorname{diag}\{A_{cc}\}_{c\in\mathcal{C}}$ and $\hat{B}_\mathcal{C} = B_\mathcal{C} - \operatorname{diag}\{B_{cc}\}_{c\in\mathcal{C}}$, with $A_\mathcal{C} = [A_{cd}]_{c,d\in\mathcal{C}}$ (similarly for $B_\mathcal{C}$). The weak coupling assumption from a network perspective is

$$\Big[\hat{A}_\mathcal{C}\ \hat{B}_\mathcal{C}\Big]\,\Gamma_\mathcal{C}\,\mathbb{X} \times \mathbb{U} \subset \mathbb{X} \tag{8}$$

With the help of the respective support functions, we can turn (8) into an inequality for any $p \in \mathbb{R}^n$:

$$h(\Big[\hat{A}_\mathcal{C}\ \hat{B}_\mathcal{C}\Big]\,\Gamma_\mathcal{C}\,\mathbb{X} \times \mathbb{U}, p)\ \le h(\mathbb{X}, p) \tag{9}$$

The following lemma provides tools to assess whether the weak coupling assumption holds across the network.

Lemma 1 (LMI for Feasibility): Suppose Assumption 2 holds, $\mathbb{X} = \{x \in \mathbb{R}^n\colon P_x^\top x \le q_x\}$ with $P_x \in \mathbb{R}^{n\times n_x}$ and $q_x \in \mathbb{R}^{n_x}$, $\mathbb{U} = \{u \in \mathbb{R}^m\colon P_u^\top u \le q_u\}$ with $P_u \in \mathbb{R}^{n\times m_u}$ and $q_u \in \mathbb{R}^{n_u}$, and $\mathbb{X} \times \mathbb{U} = G\mathbb{B}_\infty^d$ (1) with $G \in \mathbb{R}^{n+m\times d}$. The weak coupling assumption for a network under partition $\mathcal{C}$ is equivalent to $\exists L \in \mathbb{R}^{n_x\times d}$ satisfying the inequalities

$$L\mathbb{1}_d \le q_x, \quad -L \le P_x^\top\Big[\hat{A}_\mathcal{C}\ \hat{B}_\mathcal{C}\Big]\,\Gamma_\mathcal{C} G \le L \tag{10}$$

Proof: Using the properties of the support function, if $p \in \mathbb{R}^n$ is a normal direction of $\mathbb{X}$, then $h(\mathbb{X}, p) = q$ where $q > 0$ defines a supporting hyperplane of $\mathbb{X}$. Using this property, we can rewrite (9) for each column $p_{x,i}$ of P_x as $h([\hat{A}_\mathcal{C}\ \hat{B}_\mathcal{C}]\,\Gamma_\mathcal{C}\,\mathbb{X}\times\mathbb{U},\ p_{x,i}) \le q_{x,i}$ for $i \in \{1, \dots, n_x\}$. From the definition of support function, we obtain $h(\mathbb{X}\times\mathbb{U},\ \Gamma_\mathcal{C}[\hat{A}_\mathcal{C}\ \hat{B}_\mathcal{C}]^\top p_{x,i}) \le q_{x,i}$. Using the property $h(\mathbb{B}_q^d(1),\ p) = \|p\|_{\tilde{q}}$ such that $q^{-1}+\tilde{q}^{-1} = 1$ with $p \in \mathbb{R}^d$, we obtain $\left\|G^\top\Gamma_\mathcal{C}[\hat{A}_\mathcal{C}\ \hat{B}_\mathcal{C}]^\top p_{x,i}\right\|_1 \le q_{x,i}$ for all $i \in \{1, \dots, n_x\}$. There exists a matrix $L \in \mathbb{R}^{n_x\times d}$ such that (10) holds. ■

The following theorem proposes an RCI design methodology for a given partition $\mathcal{C}$ of (1).

Theorem 1: Suppose Assumptions 1, 2, and 3 hold. The problem of finding a family $\{\mathcal{R}_c\}_{c\in\mathcal{C}}$ of RCI sets for the network (1) and a partition $\mathcal{C}$ is equivalent to the existence of $Z_{x,\mathcal{C}} \in \mathbb{R}^{\sum_{c\in\mathcal{C}} n_{x,c}\times Hd}$, $Z_{u,\mathcal{C}} \in \mathbb{R}^{\sum_{c\in\mathcal{C}} n_{u,c}\times Hd}$, $\Xi_{\mathcal{C}} \in \mathbb{R}^{\sum_{c\in\mathcal{C}} m_c\times(n+m)H}$, and $E_{\mathcal{C}} \in \mathbb{R}^{\sum_{c\in\mathcal{C}} n_c\times(n+m)H}$ such that the following LMIs admit a solution:

$$(\alpha_{\mathcal{C}}, \beta_{\mathcal{C}}, a_{\mathcal{C}}, b_{\mathcal{C}}) \in [0,1]^{4\cdot|\mathcal{C}|} \tag{11a}$$

$$Z_{x,\mathcal{C}}\mathbb{1}_{Hd} \le \hat{q}_{x,\mathcal{C}}\, a_{\mathcal{C}}, \quad Z_{u,\mathcal{C}}\mathbb{1}_{Hd} \le \hat{q}_{u,\mathcal{C}}\, b_{\mathcal{C}} \tag{11b}$$

$$-Z_{u,\mathcal{C}} \le \hat{P}_{u,\mathcal{C}}\, \Xi_{\mathcal{C}}\, \mathbb{1}_H \otimes G \le Z_{u,\mathcal{C}} \tag{11c}$$

$$-Z_{x,\mathcal{C}} \le \hat{P}_{x,\mathcal{C}}\, E_{\mathcal{C}}\, \mathbb{1}_H \otimes G \le Z_{x,\mathcal{C}} \tag{11d}$$

$$A_{cc}^{H}\Big[\hat{A}_{c,*}\ \hat{B}_{c,*}\Big]\Gamma_{\mathcal{C}} + \sum_{h=0}^{H-1} A_{cc}^{H-1-h} B_{cc}\,\Xi_{c,h} = 0 \tag{11e}$$

where $a_{\mathcal{C}} = (a_1,\ldots,a_{|\mathcal{C}|})$ (similarly for $b_{\mathcal{C}}$, $\alpha_{\mathcal{C}}$, and $\beta_{\mathcal{C}}$), $\Xi_{c,h} \in \mathbb{R}^{n_c\times(n+m)}$ is the $(c,h)^{\text{th}}$ block of $\Xi_{\mathcal{C}}$, the blocks of $E_{\mathcal{C}}$ are $[E_{\mathcal{C}}]_{0,c} = [\hat{A}_{c,*}\ \hat{B}_{c,*}]\,\Gamma_{\mathcal{C}}$ and $[E_{\mathcal{C}}]_{h,c} = A_{cc}[\hat{A}_{c,*}\ \hat{B}_{c,*}]\,\Gamma_{\mathcal{C}} + \sum_{l=0}^{h-1} A_{cc}^{h-1-l}B_{cc}\Xi_{c,h}$ for $h \neq 0$, $\hat{P}_{s,\mathcal{C}} = \operatorname{diag}(P_{s,c})_{c\in\mathcal{C}}$, and $\hat{q}_{s,\mathcal{C}} = \operatorname{diag}(q_{s,c})_{c\in\mathcal{C}}$ with $s \in \{x,u\}$.

Proof: From Lemma 1 and (5), the RCI sets satisfy $\bigoplus_{h=0}^{H-1} D_h(\mathbf{S}_{c,H})[\hat{A}_{c,*}\ \ \hat{B}_{c,*}]\,\Gamma_{\mathcal{C}}\mathbb{X}\times\mathbb{U} \subset a_c\mathbb{X}_c$, and $\bigoplus_{h=0}^{H-1} S_{c,h}[\hat{A}_{c,*}\ \ \hat{B}_{c,*}]\,\Gamma_{\mathcal{C}}\mathbb{X}\times\mathbb{U} \subset b_c\mathbb{U}_c$ for all $c \in \mathcal{C}$. Using support functions, we obtain the following component-wise inequalities: $\sum_{h=0}^{H-1}\Big\|P_{x,c}D_h(\mathbf{S}_{c,H})[\hat{A}_{c,*}\ \hat{B}_{c,*}]\,\Gamma_{\mathcal{C}}G\Big\|_1 \le a_c\, q_{x,c}$, and $\sum_{h=0}^{H-1}\Big\|P_{u,c}S_{c,h}[\hat{A}_{c,*}\ \hat{B}_{c,*}]\,\Gamma_{\mathcal{C}}G\Big\|_1 \le b_c q_{u,c}$. Using the technique of Lemma 1 involving the support functions, for all $c\in\mathcal{C}$, $h \in \{0,\ldots,H-1\}$, there exists $Z_{x,c,h} \in \mathbb{R}^{n_{x,c}\times d}$ and $Z_{u,c,h} \in \mathbb{R}^{n_{u,c}\times d}$ such that:

$$\sum_{h=0}^{H-1} Z_{x,c,h}\mathbb{1}_d \le a_c\, q_{x,c}, \quad \sum_{h=0}^{H-1} Z_{u,c,h}\mathbb{1}_d \le b_c\, q_{u,c}$$

$$-Z_{x,c,h} \le P_{x,c}D_h(\mathbf{S}_{c,H})\Big[\hat{A}_{c,*}\ \hat{B}_{c,*}\Big]\Gamma_{\mathcal{C}}G \le Z_{x,c,h}$$

$$-Z_{u,c,h} \le P_{u,c}S_{c,h}\Big[\hat{A}_{c,*}\ \hat{B}_{c,*}\Big]\Gamma_{\mathcal{C}}G \le Z_{u,c,h}$$

These set of inequalities involve bilinear terms which can be removed with the change of variables $\Xi_{c,h} = S_{c,h}[\hat{A}_{c,*}\ \hat{B}_{c,*}]\,\Gamma_{\mathcal{C}}$ for all $h = 0,\ldots,H-1$. This is possible because, for $h = 0$, we obtain only a variable in terms of the scaling, and we can undo the transformation. ■

B. The Nominal System and $\mathcal{H}_2$ Norm

The nominal system $\Sigma_{\mathcal{C}}$ for partition $\mathcal{C}$ is

$$\bar{x}^+ = \bar{A}_{\mathcal{C}}\,\bar{x} + \bar{B}_{\mathcal{C}}\,\bar{u},\ \ \bar{y} = \bar{x} \tag{12}$$

with $\bar{A}_{\mathcal{C}} = \operatorname{diag}\{A_{cc}\}_{c\in\mathcal{C}}$ and $\bar{B}_{\mathcal{C}} = \operatorname{diag}\{B_{cc}\}_{c\in\mathcal{C}}$. From an input-output context, we assume that the state can be measured and that feed-through does not exist. The $\mathcal{H}_2$ norm of system (12) is defined by $\|\Sigma_{\mathcal{C}}\|^2_{\mathcal{H}_2} = \frac{1}{2\pi}\int_{-\pi}^{\pi}\operatorname{Tr}(H_{\mathcal{C}}(e^{j\omega})H_{\mathcal{C}}(e^{j\omega})^H)d\omega$ where $H_{\mathcal{C}}(\cdot)$ is the Laplace transform of $\Sigma_{\mathcal{C}}$. Furthermore, the computation of this norm follows from the following lemma, which we state without proof.

Lemma 2: Given a discrete linear time-invariant system Σ with dynamics $x^+ = Ax + Bu$ with $\lambda_{\max}(A) < 1$ and (A, B) is a controllable pair, its $\mathcal{H}_2$ norm is $\|\Sigma\|_{\mathcal{H}_2} = \operatorname{Tr}(W)$ where W is the solution of $AWA^\top - W = -BB^\top$.

The above lemma allows us to conclude that as long as the open-loop system is stable, the $\mathcal{H}_2$ norm can be computed using only the powers of the matrix A. The following result allows us to relate the $\mathcal{H}_2$ norm of each partition to the connectivity of the underlying graphs.

Theorem 2: Suppose Assumption 1 holds. For a given partition $\mathcal{C}$, assume in addition that $|\lambda(A_{\mathcal{C}})| < 1$ and $\forall c \in \mathcal{C}$, $n, m > 0$, $n_c = n$, and $m_c = m$. The dynamic matrix admits a decomposition $\bar{A}_{\mathcal{C}} = \bar{A}^0_{\mathcal{C}} + M_{\mathcal{C}}\sum_{s=1}^{n}\mathcal{L}_{\mathcal{C},s}\otimes H_s$ for $M_{\mathcal{C}} \in \mathbb{R}^{|\mathcal{C}|n\times|\mathcal{C}|n}$ and some $H_s \in \mathbb{R}^{n\times n}$ with $s \in \{1,\ldots,n\}$. The $\mathcal{H}_2$ norm of the nominal system $\Sigma_{\mathcal{C}}\colon \bar{x}^+ = \bar{A}_{\mathcal{C}}\bar{x} + \bar{B}_{\mathcal{C}}\bar{u}$ satisfies up to first order for some constants $C_0 > 0$ and $C_{s,i} \in \mathbb{R}$:

$$\|\Sigma_{\mathcal{C}}\|_{\mathcal{H}_2} \approx C_0 + \sum_{i\in\mathcal{M}}\sum_{s=1}^{n} C_{s,i}\lambda_i(\mathcal{L}_{\mathcal{C},s}). \tag{13}$$

Proof: The matrix $\bar{A}_{\mathcal{C}}$ is a block diagonal by construction. This matrix contains the links defined in the intra-coalition graph $(\mathcal{M}, \mathcal{E}_{\mathcal{C}})$ with $|\mathcal{C}|$ connected components, see Proposition 1. For each $s \in \{1,\ldots,n\}$, this graph determines a Laplacian matrix $\mathcal{L}_{\mathcal{C},s} \in \mathbb{R}^{M\times M}$; the coupling strength of state $x_{c,i}$ and $x_{c,j}$ of coalition c can be described by $H_s \in \mathbb{R}^{n\times n}$ where $[H_s]_{ij} > 0$. The dynamic matrix can be rewritten as $\bar{A}^0_{\mathcal{C}} = \bar{A}_{\mathcal{C}} - M_{\mathcal{C}}\sum_{s=1}^{n}\mathcal{L}_{\mathcal{C},s}\otimes H_s$ with $M_{\mathcal{C}}$ diagonal and represents a scaling. This last matrix captures the effect of the network on the coalitions defined by partition $\mathcal{C}$.

We drop the subscript $\mathcal{C}$ in the following. The $\mathcal{H}_2$ norm, from Lemma 2, can be computed as the solution of the Lyapunov equation $\bar{A}W\bar{A}^\top - W = -\bar{B}\bar{B}^\top$. For small perturbations M, we have $W = W^0 + M\sum_{s=1}^{n}W_s + \mathcal{O}(M^2)$ where W^0 is the controllability Gramian for $\bar{A}^0$, and each W_s is the first-order correction due to $\mathcal{L}_s$ which corresponds to the expression for the matrix $\bar{A}_{\mathcal{C}}$. The matrix M of the factorisation can be chosen to be small. Substituting into the Lyapunov equation: $\big(\bar{A}^0 + M\sum_{s=1}^{n}\mathcal{L}_s\otimes H_s\big)W\big(\bar{A}^0 + M\sum_{s=1}^{n}\mathcal{L}_s\otimes H_s\big)^\top - W + \bar{B}\bar{B}^\top = 0$. The zeroth order equation is $\bar{A}^0W^0(\bar{A}^0)^\top - W^0 + \bar{B}\bar{B}^\top = 0$. Expanding to first order for all $s = 1,\ldots,n$:

$$\begin{aligned}\bar{A}^0W_s\Big(\bar{A}^0\Big)^\top &+ (\mathcal{L}_s\otimes H_s)W^0\Big(\bar{A}^0\Big)^\top\\ &+\bar{A}^0W^0\Big(\mathcal{L}_s^\top\otimes H_s^\top\Big) - W_s = 0\end{aligned} \tag{14}$$

Each of the Laplacian matrices admits a diagonalisation $\mathcal{L}_s = \sum_{i\in\mathcal{M}}\lambda_{s,i}\xi_{s,i}\xi_{s,i}^\top$ where $\xi_{s,i} \in \mathbb{R}^{|\mathcal{M}|}$ is an eigenvector of $\mathcal{L}_s$. Rewriting (14) in terms of the diagonalisation:

$$\begin{aligned}\bar{A}^0W_s\Big(\bar{A}^0\Big)^\top - W_s = &-\sum_{i\in\mathcal{M}}\lambda_{s,i}\Big((\xi_{s,i}\xi_{s,i}^\top\otimes H_s)W^0(\bar{A}^0)^\top\\ &+\bar{A}^0W^0(\xi_{s,i}\xi_{s,i}^\top\otimes H_s^\top)\Big).\end{aligned}$$

Defining the linear operator $\mathcal{L}(X) = \bar{A}^0X(\bar{A}^0)^\top - X$, therefore $W_s = -\sum_{i\in\mathcal{M}}\lambda_{s,i}\mathcal{L}^{-1}\Big((\xi_{s,i}\xi_{s,i}^\top\otimes H_s)W^0(\bar{A}^0)^\top + \bar{A}^0W^0(\xi_{s,i}\xi_{s,i}^\top\otimes H_s^\top)\Big)$. The $\mathcal{H}_2$ norm of the nominal system can be approximated by first order to $\|\Sigma_{\mathcal{C}}\|_{\mathcal{H}_2} \approx \operatorname{Tr}(W_0) -$

$M \sum_{i \in \mathcal{M}} \sum_{s=1}^{n} \lambda_{s,i} C_{s,i}$ where $C_{s,i} = \mathrm{Tr}\Big(\mathcal{L}^{-1}\big((\xi_{s,i}\xi_{s,i}^\top \otimes H_s) W^0 (\bar{A}^0)^\top + \bar{A}^0 W^0 (\xi_{s,i}\xi_{s,i}^\top \otimes H_s^\top)\big)\Big)$ and $C_0 = \mathrm{Tr}(W_0)$. ■

This norm can be used to gauge the connectivity within each partition. The following result shows how to use this norm to compare partitions.

Proposition 2: The pair $(\Pi_{\mathcal{M}}, \mathfrak{d}_{\mathcal{H}_2})$ is a pseudo-metric space with $\mathfrak{d}_{\mathcal{H}_2} \colon \Pi_{\mathcal{M}} \times \Pi_{\mathcal{M}} \to \mathbb{R}^+$ defined as $\mathfrak{d}(\mathcal{C}, \mathcal{D}) = \big|\|\Sigma_{\mathcal{C}}\|_{\mathcal{H}_2} - \|\Sigma_{\mathcal{D}}\|_{\mathcal{H}_2}\big|$

Proof: The statements to proof are the following: *i)* $\mathfrak{d}(\mathcal{C}, \mathcal{C}) = 0$, *ii)* $\mathfrak{d}(\mathcal{C}, \mathcal{D}) = \mathfrak{d}(\mathcal{D}, \mathcal{C})$, and *iii)* $\mathfrak{d}(\mathcal{C}, \mathcal{B}) \leq \mathfrak{d}(\mathcal{C}, \mathcal{D}) + \mathfrak{d}(\mathcal{D}, \mathcal{B})$ for all $\mathcal{B}\mathcal{C}\mathcal{D} \in \Pi_{\mathcal{M}}$. The statements follow from the definition of absolute value and Lemma 2. ■

The function $\mathfrak{d}(\cdot, \cdot)$ is a pseudo metric because two partitions may have the same $\mathcal{H}_2$ norm when $(i, j) \notin \mathcal{E}$.

Remark 1: Selecting elements from the partition set $\Pi_{\mathcal{M}}$ may be computationally challenging due to the combinatorial explosion of the number of partitions. This problem is not trivial, and existing works have focused on reducing the number of partitions to evaluate, as in [5] and [11].

IV. Coalitional Controller

Given a partition $\mathcal{C}$, the N–step feasible set is given by the product of individual feasible sets $\mathcal{X}_{\mathcal{C}}^N = \bigtimes_{c \in \mathcal{C}} \mathcal{X}_c^N$ where each $\mathcal{X}_c^N = \bar{\mathcal{X}}_c \oplus \mathcal{R}_c(\mathbb{W}_c)$ and $\bar{\mathcal{X}}_c$ is the set where the optimisation problem (7) admits a solution. At a given state $x \in \mathcal{X}_{\mathcal{C}}^N$, a partition $\mathcal{D}$ is *recursively feasible* if $x^+ \in \mathcal{X}_{\mathcal{D}}^N$ and it is *strongly recursively feasible* if $x^+ \in \bar{\mathcal{X}}_{\mathcal{D}}^N$, see [11, Proposition 5.5] for more details of recursive feasibility becoming strong as the coupling weakens. The main contribution of this letter is the optimisation problem for selecting a partition:

$$\mathcal{C}^*(x, \bar{x}) = \arg \min_{\mathcal{C} \in \Pi_{\mathcal{M}}} \gamma \|\Sigma_{\mathcal{C}}\|_{\mathcal{H}_2} + \|\mathcal{L}_{\mathcal{C}}(x)\| + d\big(\bar{x}, \bar{\mathbb{X}}_{\mathcal{C}}\big) \quad (15)$$

with weight $\gamma > 0$, $\bar{\mathbb{X}}_{\mathcal{C}} = \bigtimes_{c \in \mathcal{C}} \bar{\alpha}_c (1 - a_c) \alpha_c \mathbb{X}_c$, and $d(\bar{x}, \bar{\mathbb{X}}_{\mathcal{C}})$ representing the distance of state $\bar{x}$ to set $\bar{\mathbb{X}}_{\mathcal{C}}$. Note that Problem (15) considers two types of couplings: one structural arising from the nominal system of partition $\mathcal{C}$ and measured using the $\mathcal{H}_2$ norm and another measuring the instantaneous coupling that exists in the network given by the term $\|\mathcal{L}_{\mathcal{C}}(x)\|$ at current state x. In our setting, the associated Laplacian matrix $\mathcal{L}_{\mathcal{C}} \in \mathbb{R}^{M \times M}$ of the inter-coalition graph, following Proposition 1, is block diagonal containing $|\mathcal{C}|$ blocks $\mathcal{L}_{\mathcal{C}}(x) = \mathrm{diag}\{\mathcal{L}_c(x) \colon c \in \{1, \ldots, |\mathcal{C}|\}\}$. For each $c \in \mathcal{C}$, the components $\mathcal{L}_c(x)$ determine the strength of the coupling $[\mathcal{L}_c]_{ij}(x) = -\frac{1}{2}\Big(\|A_{ij}x_j\| + \|A_{ji}x_i\|\Big)$ for each $i, j \in c$. This Laplacian measures the amount of coupling "absorbed" by partition $\mathcal{C}$ at state x. The last term in (15) penalises the loss of strong feasibility. The weight γ is used to make both norms comparable. We are now in a position to state the following theorem:

Theorem 3: Suppose Assumptions 1–3 hold. Given a partition $\mathcal{C}$ and a state $x \in \mathcal{X}_{\mathcal{C}}^N$, the optimal solution $\mathcal{C}^*(x, \bar{x})$ of (15) is strongly feasible.

Proof: Given a partition $\mathcal{C}$, scaling factors $(\alpha_{\mathcal{C}}, \beta_{\mathcal{C}})$, and a state $x \in \mathcal{X}_{\mathcal{C}}^N$. The solution of (11) yields the necessary scaling for a partition to admit a family of RCI sets, which will determine the size of $\bar{\mathcal{X}}_{\mathcal{C}}^N$. The term $d(\bar{x}, \bar{\mathbb{X}}_{\mathcal{D}}) \geq 0$ for a feasible partition $\mathcal{D} \in \Pi_{\mathcal{M}} \setminus \mathcal{C}$. The condition $d(\bar{x}, \bar{\mathbb{X}}_{\mathcal{D}}) = 0$ implies the state evolution lies within the definition of the nominal state set of equation (7). The partition, therefore, can be chosen according to the partition with the lowest $\mathcal{H}_2$ norm, *i.e.,* the partition, $\mathcal{C}^*$, minimising the coupling as shown in Theorem 2 such that $\mathfrak{d}(\mathcal{C}, \mathcal{C}^*)$ is minimal. ■

The proposed approach is summarised in Algorithm 1, where a central coordinator decides the system partition, and then each coalition computes its control action.

Algorithm 1 Norm-Based Coalitional MPC Algorithm

Upper control layer: Every T_{up} time steps, select the new system partition $\mathcal{C}$ by solving (15) according to the current values $\alpha = (\alpha_c)_{c \in \mathcal{C}}$ and $\beta = (\beta_c)_{c \in \mathcal{C}}$.

Lower control layer: Given the current partition, at each time step k, each coalition $c \in \mathcal{C}$:

1: Updates $\mathbb{W}_c$ according to the current factors α_c, β_c, calculate its RCI set based on a_c and b_c by (5), and compute the terminal set $\bar{\mathbb{X}}_c^{\mathrm{f}}$.

2: Solves problem (7) to obtain $\bar{u}_c^*$, $\bar{\alpha}_c^*$ and $\bar{\beta}_c^*$.

3: Applies $\bar{u}_c = \bar{u}_c^*(1)$ to get $\bar{x}_c^+$ and define $u_c = \bar{u}_c + \mu_c(x_c - \bar{x}_c)$, with the RCI control law $\mu_c(\cdot)$, to get x_c^+.

4: Updates the scaling factors as (6) and share them with neighbours $\mathcal{M}_c$. Check if $\mathcal{R}_c(\mathbb{W}_c^+)$ exists; otherwise, hold the previous factors: $\alpha^+ = \alpha$ and $\beta^+ = \beta$.

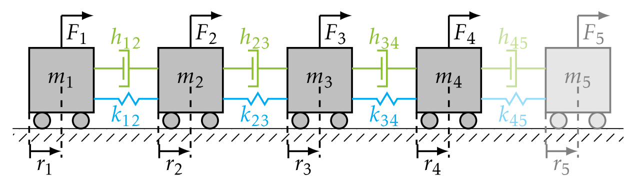

Fig. 2. Academic example comprising four trucks connected in series via springs and dampers. At time $k_{\mathrm{plug}} = 16$, an extra truck is plugged into the system.

V. Case Study

Consider a multi-truck system composed of $M = 4$ subsystems coupled by a damper and a spring with their neighbours and, at time step $k_{\mathrm{plug}} = 16$, a fifth truck is plugged, as illustrated in Fig. 2. More details on plug-and-play operations can be found in [5, Section 5]. The continuous second-order dynamics model of each truck:

$$\dot{r}_i = v_i, \quad m_i \dot{v}_i = \sum_{j \in \mathcal{M}_i} k_{ij}\big(x_j - x_i\big) + h_{ij}\big(v_j - v_i\big) + 100 m_i u_i,$$

where its state is given by displacement r_i and velocity v_i, its control input u_i represents the horizontal force. The masses (kg) are: $m_1, m_3 = 3$, $m_2, m_4 = 2$ and $m_5 = 6$; damping factors (N·s/m): $h_{12}, h_{34} = 0.3$, $h_{23} = 0.4$, and $h_{45} = 1$; and string constants (N/m): $k_{12} = 0.5$, $k_{23} = 0.7$, $k_{34} = 0.6$, and $k_{45} = 1.5$. Its continuous dynamics is discretised with sampling time $T_s = 0.2\,\mathrm{s}$. The prediction horizon is $N = 10$, the weights are $Q_i = I$ and $R_i = 100$, and $\tau_\alpha = 5 \cdot 10^{-5}$ and $\tau_\beta = 0$.

The control objective is to steer the state to the origin while satisfying the constraints $|r_i| \leq 2\,\mathrm{m}$, $|v_i| \leq 1\,\mathrm{m/s}$,

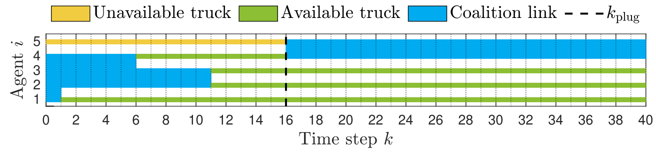

Fig. 3. Evolution of the partition $\mathcal{C}(k)$. During transients, the system requires more cooperation, *i.e.,* $\mathcal{C}(0) = \mathcal{C}_{\text{cen}}$; as transients wane, cooperation decreases, leading to decentralised operation, *i.e.,* $\mathcal{C}(11) = \mathcal{C}_{\text{dec}}$.

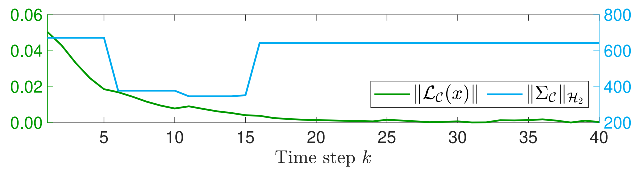

Fig. 4. Norm values in our coalitional MPC: the $\mathcal{H}_2$-norm decreases with more decentralised partitions, then rises when a new truck joins. The Laplacian norm drops as coupling weakens near equilibrium.

TABLE I
COMPARISON OF NUMERICAL RESULTS

MPC methods	$\bar{t}_{\mathcal{C}}$ $[s]$	$\bar{t}_c$ $[s]$	J_{stage}	J_{clust}	J_{total}
Centralised MPC	–	0.88	28.49	14.5	43.00
MPC-based Coalitional	24.78	0.52	28.94	4	32.94
Norm-based Coalitional	4.73	0.53	28.94	4	32.94

$|u_i| \leq 1$ N/kg and dealing with a plug-in. Initially, there are eight possible partitions: $\mathcal{C}_{\text{dec}} = \{\{1\}, \{2\}, \{3\}, \{4\}\}, \mathcal{C}_2 = \{\{1\}, \{2\}, \{3, 4\}\}, \ldots, \mathcal{C}_{\text{cen}} = \{\{1, 2, 3, 4\}\}$ and, after the plug-in, the number of possible partitions increases to sixteen. A new partition is selected every $T_{\text{up}} = 5$ steps, based on two alternative clustering criteria:

- *MPC-based partitioning criterion*, proposed in [5, Algorithm 1], solves an MPC problem for each $\mathcal{C} \in \Pi_{\mathcal{M}}$. We select the partition with the lowest cost J_N (4), where $g(\mathcal{C}(k))$ is the cooperation cost, *i.e.,* the number of links in $\mathcal{C}(k)$ times $c_{\text{link}} = 0.1$.
- *Norm-based partitioning criterion*, here proposed, selects the partition minimising (15) with $\gamma = 10^{-3}$.

Both approaches share the same lower control layer and objective as Algorithm 1. Fig. 3 illustrates the partition evolution, identical across both clustering criteria. Fig. 4 presents the $\mathcal{H}_2$ norm of $\Sigma_{\mathcal{C}}$ and the Laplacian 2-norm, with a spike in $\mathcal{H}_2$ at k_{plug} when the 4th truck forms a coalition with the plugged truck. Our approach selects the partition that ensures recursive feasibility and stability. Table I reports the accumulated stage cost $J_{\text{stage}} = \sum_{k=1}^{40} \ell(x(k), u(k))$, the clustering cost $J_{\text{clust}} = \sum_{k=1}^{40} g(\mathcal{C}(k))$, their sum J_{total}, and the average times for partition selection $\bar{t}_{\mathcal{C}}$ and for solving the coalitional problem $\bar{t}_c$. Our norm-based approach is 80.91% faster than the baseline, offering a more efficient partitioning.

VI. CONCLUSION

This letter studies the coupling structure in networks of dynamical systems. We proposed a set of LMIs to verify the weak coupling assumption and to synthesize RCI sets across the network. We also analysed the internal coupling of a partition via its *intra-coalitional* graph. To quantify coupling, we introduced the $\mathcal{H}_2$ norm in terms of the eigenvalues of the *intra-coalitional* Laplacian, and used it to compare partitions. We illustrated our findings through an academic example, comparing our approach to the existing ones in the literature. In future work, we aim to explore the scalability of the proposed method in more challenging scenarios.

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