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Factorial-Decay Lag Weights for Adaptive LASSO in High-Dimensional ADL models

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ABSTRACT: {Monte Carlo simulation is employed to evaluate a factorial-decay lag-adaptive penalization scheme for high-dimensional autoregressive models.}

This paper proposes the Factorial-Decay Lag Adaptive LASSO (FLAdaLASSO), a penalization approach for variable selection in high-dimensional autoregressive time-series models. The method extends existing lag-weighted LASSO procedures by introducing a factorial-decay structure that imposes stronger shrinkage on distant lags, thereby improving the separation between relevant and irrelevant predictors in persistent and multicollinear settings. Monte Carlo simulation experiments conducted across a range of sample sizes, lag dimensions, and dependence structures show that FLAdaLASSO improves lag-selection accuracy, reduces overselection, and stabilizes parameter estimation while delivering forecasting performance comparable to that of existing lag-weighted adaptive methods. An empirical application to Thailand's inflation series from 1975 to 2024 is provided as an illustration of practical relevance. The results indicate that FLAdaLASSO achieves near-minimal forecasting error while maintaining a parsimonious model specification. Overall, the findings suggest that factorial-decay lag weighting provides an effective and computationally attractive strategy for high-dimensional time-series forecasting.

Keywords: High-dimensional time series; Adaptive LASSO; Lag-dependent regularization; Monte Carlo simulation; Variable selection; Forecasting

INTRODUCTION

Recent advances in data storage and computational technologies have facilitated the collection of increasingly large and complex datasets. In many modern applications, the number of potential predictors can easily exceed the sample size, resulting in high-dimensional data structures. Such settings commonly arise in fields such as genomics, finance, and macroeconomics, where incorporating a wide range of variables can potentially improve forecasting accuracy. However, conventional estimation methods such as Ordinary Least Squares (OLS) perform poorly when the number of predictors exceeds the sample size or when predictors are highly collinear, leading to unstable estimates and poor predictive performance (*3, 5*). These challenges motivate the use of regularization techniques that simultaneously perform variable selection and shrinkage (*14*). Among such methods, the Least Absolute Shrinkage and Selection Operator (LASSO), introduced by Tibshirani (*10*), has become a cornerstone of high-dimensional modeling. By penalizing the absolute values of coefficients, LASSO enforces sparsity and retains only the most relevant predictors. Nevertheless, its uniform penalization can induce bias in coefficient estimation, particularly when true signal strengths vary substantially. To address this limitation, Zou (*13*) proposed the Adaptive LASSO (AdaLASSO), which assigns data-driven weights to individual coefficients, yielding improved variable selection consistency and reduced estimation bias.

Extending these ideas to dynamic settings, Park and Sakaori (*9*) introduced lag-dependent penalties, emphasizing that coefficients associated with more distant lags should generally be penalized more strongly. Building on this framework, Konzen and Ziegelmann (*6*) developed the Weighted Lag Adaptive LASSO (WLAdaLASSO), which incorporates an exponential lag-decay structure within high-dimensional autoregressive distributed lag (ADL) models. Their results demonstrate that incorporating lag-structured shrinkage can substantially improve forecast accuracy in macroeconomic and financial time series. Despite these advances, existing lag-weighting schemes—most commonly polynomial or exponential—may not adequately capture the steep decline in predictive relevance often observed in economic time series, where short-run dynamics tend to dominate and the influence of distant lags diminishes rapidly. Moreover, the empirical understanding of how the shape of lag decay affects sparsity patterns and forecasting performance remains limited.

This study addresses these limitations by proposing a factorial-decay weighting scheme within the adaptive LASSO framework for high-dimensional ADL models. The proposed weighting structure generalizes conventional exponential decay by imposing a sharper penalty on distant lags while preserving the flexibility of adaptive penalization. Through extensive Monte Carlo simulations and an empirical application to inflation data from Thailand, we demonstrate that the proposed method improves lag selection, enhances model parsimony, and delivers stable estimation with competitive forecasting performance, particularly in settings characterized by strong short-run dependence.

The remainder of this paper is organized as follows. Section 2 introduces the ADL framework and reviews the adaptive LASSO methodology. Section 3 presents the proposed factorial-decay lag adaptive LASSO estimator. Section 4 reports the simulation design and results. Section 5 provides the empirical analysis, and Section 6 concludes with a discussion of the findings and implications.

METHODOLOGY

Time series model

For general time series models with no seasonality, it is important to account for the influence of lagged observations, as their effect typically decreases with increasing lag order. The autoregressive (AR) model is a fundamental method in time series analysis, in which the current observation is expressed as a linear combination of its own previous values (*2*). The coefficients assigned to lagged terms quantify the influence of past observations on the present value, thereby capturing the temporal dependence structure within the series. The autoregressive model of order q, denoted as AR(q), consists of q lagged response variables and a stochastic error term. It is formally expressed as:

$$\begin{aligned} Y_t &= \alpha + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \cdots + \beta_q Y_{t-q} + e_t \\ &= \alpha + \sum_{l=1}^{q} \beta_l Y_{t-l} + e_t \end{aligned} \tag{1}$$

where Y_t represents the dependent variable at time t, α is a constant term, β_l are the autoregressive coefficients, and e_t is a white noise process having a zero mean and a constant variance σ^2, $cov(e_t, Y_{t-1}) = 0$. To account for the influence of external factors, the model can be extended by including one explanatory variable X_t and its lagged terms. This results in an Autoregressive Distributed Lag (ADL) model, formulated as:

$$Y_t = \alpha + \sum_{l=1}^{q_0} \beta_{0,l} Y_{t-l} + \sum_{l=0}^{q_1} \beta_{1,l} X_{1,t-l} + e_t \tag{2}$$

where $\beta_{1,l}$ denote the distributed lag coefficients associated with the explanatory variable X_t. This specification captures both the autoregressive behavior of Y_t and the dynamic (lagged) impact of X_t. Furthermore, the ADL model can be generalized to include multiple explanatory variables $(X_{1,t}, X_{2,t}, ..., X_{k,t})$, each with its own lag structure as follows:

$$\begin{aligned} Y_t &= \alpha + \sum_{l=1}^{q_0} \beta_{0,l} Y_{t-l} + \sum_{l=0}^{q_1} \beta_{1,l} X_{1,t-l} + \sum_{l=0}^{q_2} \beta_{2,l} X_{2,t-l} + \cdots + \sum_{l=0}^{q_k} \beta_{k,l} X_{k,t-l} + e_t \\ &= \alpha + \sum_{j=0}^{k} \sum_{l=0}^{q_j} \beta_{j,l} X_{j,t-l} + e_t, \end{aligned} \tag{3}$$

where we assume that $\beta_{0,0} = 0$, $X_{0,t} = Y_t$, $X_{0,t-l} = Y_{t-l}$, Y_t denotes the dependent variable at time t, $X_{j,t-l}$ represents the j-th explanatory variable lagged by l periods, $\beta_{0,l}$ are the lag coefficients corresponding to Y_t, $\beta_{j,l}$ are the lag coefficients corresponding to the j-th explanatory variable, q_0 indicates the number of lag of Y_t, q_j denotes the number of lags for X_j, e_t is the error term, and the following three assumptions hold:

1. $E(e_t | Y_{t-1}, Y_{t-2}, ..., X_{1,t}, X_{1,t-1}, ..., X_{k,t-1}, X_{k,t-2}, ...) = 0$
2. $(Y_t, X_{1,t}, ..., X_{k,t})$ are stationary.
3. The correlation coefficients between $(Y_t, X_{1,t}, ..., X_{k,t})$ and $(Y_{t-l}, X_{1,t-l}, ..., X_{k,t-l})$ decline as l increase.

In this generalized form in equation 3, the model allows for the inclusion of several predictors, each potentially exerting both immediate and delayed effects on the dependent variable. This flexibility makes the ADL framework particularly suitable for modeling dynamic and time-dependent relationships. Based on the assumptions outlined above, we introduce a factorial-decay lag weight for adaptive LASSO that incorporates the lag effect by treating it as the lag length in a non-seasonal time series model.

Regularization techniques

Regularization techniques have become essential tools in modern regression analysis, particularly in high-dimensional settings where the number of predictors is large relative to the sample size. By incorporating a penalty term into the objective function, these methods effectively control model complexity, mitigate overfitting, and enhance predictive performance. In the context of time series models such as ARDL, regularization offers a practical solution for selecting relevant lagged variables and covariates. Before incorporating the lag effect into the model, it is essential to review existing regularization techniques. In the following sections, we briefly describe several prominent regularization approaches, including Ridge regression, LASSO, Adaptive LASSO, and Weighted Lag Adaptive LASSO.

Ridge

Ridge regression is a regularization method developed to address the instability of ordinary least squares estimates, particularly when predictors exhibit high multicollinearity or when the number of predictors approaches or exceeds the sample size. By adding a penalty proportional to the squared magnitude of the coefficients, ridge regression reduces variance and produces more reliable estimates, though at the cost of introducing some bias (*4*). The ridge estimator is defined as

$$\hat{\theta}^{\text{Ridge}} = \underset{\alpha,\ \{\beta_{j,l}\}}{\arg\min} \left\{ \sum_{t=1}^{T} \left(y_t - \alpha + \sum_{j=0}^{k} \sum_{l=0}^{q_j} \beta_{j,l} x_{j,t-l} \right)^2 + \lambda \left(\sum_{j=0}^{k} \sum_{l=0}^{q_j} \beta_{j,l}^2 \right) \right\}, \tag{4}$$

where the second term is the $L2$-penalty and $\lambda \geq 0$ represents the tuning parameter that controls the strength of the penalty. A higher λ leads to more shrinkage of coefficients towards zero.

Although ridge regression effectively reduces the variance of parameter estimates by shrinking coefficients toward zero, it does not eliminate any predictors. Consequently, ridge regression does not perform variable selection, as all predictors remain in the model regardless of their relevance (*3*). In practice, reducing the number of predictors can improve model interpretability and emphasize the most important variables underlying the data relationships. This limitation has motivated the development of alternative regularization methods, such as the LASSO and its extensions, which incorporate both shrinkage and variable selection.

Least Absolute Shrinkage and Selection Operator (LASSO)

In 1996, LASSO is a regularization method proposed by Tibshirani to achieve both coefficient shrinkage and variable selection simultaneously. In contrast to ridge regression, which continuously reduces coefficient magnitudes toward zero, LASSO applies an $L1$-penalty that can force some coefficients to become exactly zero. This results in a sparse model that includes only the most relevant predictors, improving interpretability and often predictive performance in high-dimensional settings where many variables may be unnecessary. The LASSO estimator is formulated as follows:

$$\hat{\theta}^{\text{LASSO}} = \underset{\alpha,\ \{\beta_{j,l}\}}{\arg\min} \left\{ \sum_{t=1}^{T} \left(y_t - \alpha + \sum_{j=0}^{k} \sum_{l=0}^{q_j} \beta_{j,l} x_{j,t-l} \right)^2 + \lambda \left(\sum_{j=0}^{k} \sum_{l=0}^{q_j} w_{j,l}^{L} |\beta_{j,l}| \right) \right\}, \tag{5}$$

where $\lambda \geq 0$ represents the tuning parameter and $w_{j,l}^{L} = 1$.

The tuning parameter λ in LASSO is crucial for determining the extent of shrinkage applied to the model coefficients. A larger λ imposes stronger penalization, driving more coefficients to exactly zero and producing a simpler model. In contrast, smaller values of λ produce estimates closer to the ordinary least squares solution with less shrinkage. Selecting an appropriate λ is vital to balance model sparsity with predictive accuracy. Typically, this parameter is chosen using data-driven methods such as cross-validation or information criteria (*1*).

Adaptive LASSO (AdaLASSO)

Zou (*13*) pointed out that the standard LASSO estimator does not have the oracle property and proposed the AdaLASSO as an effective alternative. Unlike LASSO, which applies the same penalty to all coefficients through the $L1$-penalty term, AdaLASSO assigns distinct weights to each coefficient. Zou showed that when these weights are appropriately selected based on the data, AdaLASSO can achieve the oracle property, leading to consistent variable selection and improved estimation performance. The AdaLASSO estimator is defined as

$$\hat{\theta}^{\text{AdaLASSO}} = \underset{\alpha,\ \{\beta_{j,l}\}}{\arg\min} \left\{ \sum_{t=1}^{T} \left(y_t - \alpha + \sum_{j=0}^{k} \sum_{l=0}^{q_j} \beta_{j,l} x_{j,t-l} \right)^2 + \lambda \left(\sum_{j=0}^{k} \sum_{l=0}^{q_j} w_{j,l}^{A} |\beta_{j,l}| \right) \right\}, \tag{6}$$

where $w_{j,l}^{A} = |\hat{\theta}^{\text{Ridge}}|^{-\tau}$, $\tau > 0$ and $\lambda \geq 0$ represents the tuning parameter. Individual weights $w_{j,l}^{A}$ play a crucial role in identifying relevant predictors. For variables x_j that are important, the ridge estimate $\hat{\theta}^{\text{Ridge}}$ tends to be large, which results in a smaller weight $w_{j,l}^{A}$ and thus a weaker penalty applied to that coefficient. Conversely, for variables that are likely irrelevant, $\hat{\theta}^{\text{Ridge}}$ is typically small, producing a larger $w_{j,l}^{A}$ and imposing a stronger penalty. This weighting mechanism allows the AdaLASSO to effectively discriminate between relevant and irrelevant variables, improving variable selection and estimation accuracy.

Weighted lag adaptive LASSO (WLAdaLASSO)

WLAdaLASSO, a variant of the LASSO estimator, was proposed by Konzen and Ziegelmann(*6*), drawing inspiration from the work of Park and Sakaori(*9*). This method is specifically designed for time-series modeling that incorporates a lag structure. Its concept aligns with that of AdaLASSO and is constructed within the time-series ARDL framework. The key idea is that lags that are further away have less impact on predicting the dependent variable, and thus, larger penalties are applied to them. This approach helps in enhancing the model's predictive accuracy. The WLAdaLASSO estimator is defined as

$$\hat{\theta}^{\text{WLAdaLASSO}} = \underset{\alpha,\ \{\beta_{j,l}\}}{\arg\min} \left\{ \sum_{t=1}^{T} \left(y_t - \alpha + \sum_{j=0}^{k} \sum_{l=0}^{q_j} \beta_{j,l} x_{j,t-l} \right)^2 + \lambda \left(\sum_{j=0}^{k} \sum_{l=0}^{q_j} w_{j,l}^{w} |\beta_{j,l}| \right) \right\}, \tag{7}$$

where $w_{j,l}^{w} = |\hat{\theta}^{\text{Ridge}} e^{-\delta l}|^{-\tau}$, $\tau > 0, \delta, \lambda \geq 0$ and l represents the lag order.

To select δ, Konzen and Ziegelmann (*6*) recommend fitting the model over a range of candidate values $(0, 0.5, 1, \cdots, 10)$ for a given λ and choosing the option that achieves the lowest BIC. Similarly, the λ parameter is selected based on the criterion of minimizing BIC. Typically, τ is set to 1, following the approach used in the AdaLASSO.

Proposed method

Factorial-decay weighted lag adaptive LASSO (FLAdaLASSO)

To incorporate a sharper structure on lag relevance, we modify the weighting scheme used in the Weighted Lag Adaptive LASSO of Konzen and Ziegelmann (6). The original method employs exponential lag decay, defining the adaptive weight for coefficient $\beta_{j,l}$ as $\left|\hat{\theta}^{Ridge} e^{-\delta l}\right|^{-\tau}$, where l denotes the lag order, and $\delta \geq 0$ and $\tau > 0$ control the steepness of the decay and adaptive penalization. Although exponential decay downweights higher-order lags, its rate of decline may not be sufficiently steep to capture the rapidly diminishing contribution of distant lags in short-memory time-series processes.

To address this limitation, we introduce a factorial-decay lag-weighting function that penalizes higher-order lags more aggressively. The proposed weighting scheme is defined as $\left|\hat{\theta}^{Ridge}(l!)^{-\delta}\right|^{-\tau}$, with $\delta \geq 0$ and $\tau > 0$. Using Stirling's approximation $l! \approx \sqrt{2\pi l}(l/e)^l$, the factorial-decay term behaves approximately as $\exp\{-\delta l \log l\}$, which decreases substantially faster than the exponential term $\exp(-\delta l)$. As a result, the proposed penalty places significantly greater emphasis on low-order lags while almost entirely eliminating the influence of distant lags.

Substituting these weights into the Adaptive LASSO framework yields the Factorial-decay weighted lag adaptive LASSO estimator:

$$\hat{\theta}^{\text{FLAdalasso}} = \underset{\alpha,\ \{\beta_{j,l}\}}{\arg\min}\left\{ \sum_{t=1}^{T}\left(y_t - \alpha + \sum_{j=0}^{k}\sum_{l=0}^{q_j}\beta_{j,l}x_{j,t-l}\right)^2 + \lambda\left(\sum_{j=0}^{k}\sum_{l=0}^{q_j} w^F_{j,l}|\beta_{j,l}|\right)\right\}, \tag{8}$$

where $w^F_{j,l} = |\hat{\theta}^{\text{Ridge}}(l!)^{-\delta}|^{-\tau}, \tau > 0, \delta, \lambda \geq 0$ and l represents the lag order.

When $\delta = 0$ or when the lag order satisfies $l = 1$, the factorial-decay weighting term reduces to unity, and the proposed FLAdaLASSO estimator coincides with the standard Adaptive LASSO. Consequently, the Adaptive LASSO arises as a special case of the proposed framework.

This modification imposes substantially stronger penalties on more distant lags, reflecting the empirical regularity that remote lag coefficients contribute little to predicting current outcomes in time series data. By concentrating the effective penalization mass on lower-order lags, the proposed approach enhances lag-sparsity, improves covariate selection, and yields forecasting gains in settings with high lag dimensionality. We assess the performance of the method through extensive simulation experiments and an empirical application, benchmarking it against the LASSO, AdaLASSO, and WLAdaLASSO.

Beyond differences in magnitude, the two weighting schemes differ fundamentally in their structural design. The WLAdaLASSO weights, shown by the black line and defined as $w^W_{j,l} = \left|\hat{\theta}^{\text{Ridge}} e^{-\delta l}\right|^{-\tau}$, employ an exponential adjustment based solely on the lag order, resulting in a smooth and gradually increasing penalization across lags. In contrast, the FLAdaLASSO weights, shown by the red line and defined as $w^F_{j,l} = \left|\hat{\theta}^{\text{Ridge}}(l!)^{-\delta}\right|^{-\tau}$, incorporate a factorial term that grows at a factorial rate. Figure 1 displays the weight trajectories with $\hat{\theta}^{\text{Ridge}} = 1$ and $\tau = 1$ for two representative values of the decay parameter, $\delta = 0.5$ and $\delta = 1.0$. This setting allows the figure to focus on how the weights vary with the lag order. As illustrated, the FLAdaLASSO weights remain lower than those of the WLAdaLASSO for lower-order lags up to approximately $l = 5$, indicating milder penalization in this range and thus greater tolerance for nearby lags. Beyond this threshold, however, the factorial structure induces a markedly sharper increase in the penalty weights, resulting in substantially stronger penalization for higher-order lags relative to the WLAdaLASSO.

This behavior reflects the design of the proposed FLAdaLASSO, which assigns relatively low penalties to proximate lags while aggressively penalizing more distant and potentially irrelevant lags. These differences arise directly from the mathematical forms of the weighting functions: exponential decay yields gradual lag-based penalization, whereas factorial decay produces a more abrupt separation between lower- and higher-order lags. Each formulation therefore represents a distinct modeling choice for introducing lag-dependent penalization, reflecting different structural assumptions about how penalties should vary across lag orders.

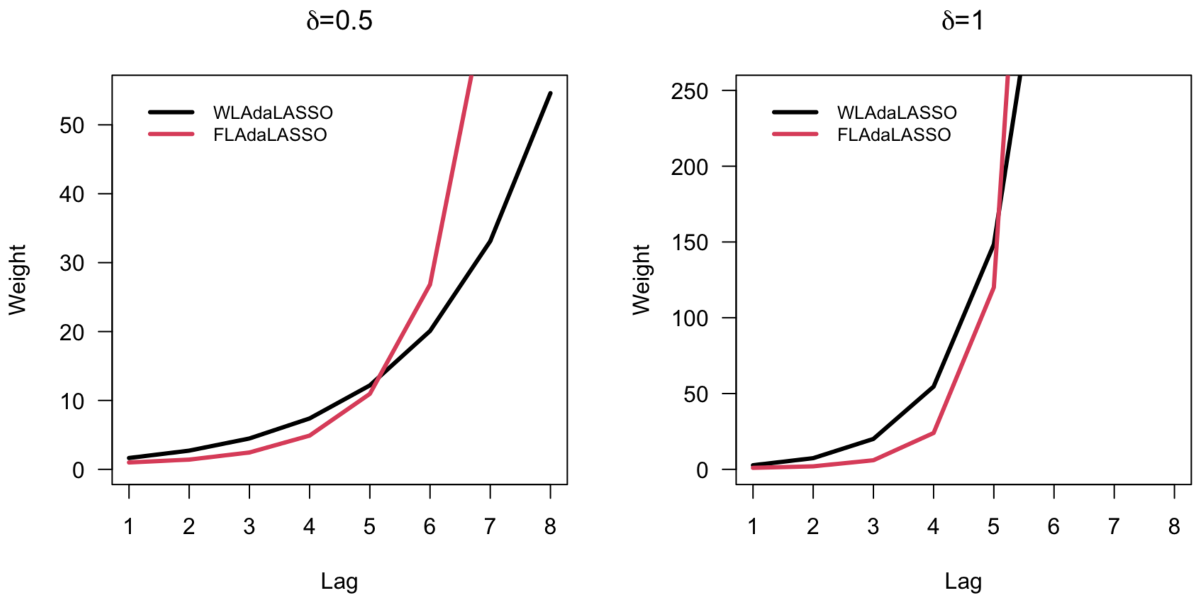

FIGURE 1 Penalty weight patterns in WLAdaLASSO (black) and FLAdaLASSO (red) when $\hat{\theta}^{\text{Ridge}} = 1$, $\tau = 1$, and $\delta = 0.5, 1$.

It is worth noting that the proposed FLAdaLASSO may face limitations in settings where the true data-generating process involves relatively long lag structures. In particular, when the number of relevant lags exceeds five, its forecasting performance tends to be less favorable relative to that of WLAdaLASSO. Evidence consistent with this observation can be found in the Monte Carlo forecasting simulations with true lag orders ranging from one to six, as reported in the Supplementary Material.

These modeling frameworks collectively provide a basis for assessing how alternative penalization schemes address lag proliferation and covariate redundancy in high-dimensional ADL settings. In the next section, we evaluate their performance using Monte Carlo simulation experiments, complemented by an illustrative empirical example.

Parameter Estimation and Optimization Procedure

Following the penalization structures defined in the preceding section, the estimation of the parameter vector $\hat{\theta}$ is performed through a unified computational framework that combines preliminary ridge estimation, adaptive weight construction, and penalized optimization. For all adaptive methods considered in this study, including AdaLASSO, WLAdaLASSO, and the proposed FLAdaLASSO, the procedure begins with a ridge regression fitted for each candidate lag order l, although only WLAdaLASSO and FLAdaLASSO incorporate the lag structure directly into their weighting schemes. The resulting preliminary ridge coefficients serve as the basis for constructing adaptive penalty weights, which determine the degree of shrinkage applied to each coefficient and allow penalization to vary across predictors and lag orders.

Once the adaptive weights are generated, the final parameter estimates are obtained by minimizing the corresponding penalized least-squares objective using a coordinate descent algorithm. In this study, all penalized estimations are implemented through the `glmnet` package, which provides an efficient and numerically stable coordinate descent routine designed for high-dimensional penalized regression. The algorithm updates parameters one at a time while holding the others fixed, applies soft-thresholding to penalized coefficients, and updates the intercept without penalty. Iterations continue until numerical convergence is achieved according to $\|\hat{\theta}^{(k+1)} - \hat{\theta}^{(k)}\|_\infty < 10^{-6}$, consistent with the convergence criteria adopted within the `glmnet` framework.

Model selection is performed by evaluating a sequence of candidate λ values (and δ values for adaptive methods) using the Bayesian Information Criterion (BIC). For WLAdaLASSO and FLAdaLASSO, this selection is conducted across all candidate lag orders, and the combination (l, λ, δ) yielding the minimum BIC is chosen as the final model (*6*). The resulting estimator simultaneously determines the coefficient magnitudes, the set of relevant predictors, and the optimal lag structure.

SIMULATION STUDY AND RESULTS

Simulation study

In this section, we evaluate the performance of the modified method by comparing it with other regularization techniques using a Monte Carlo simulation study based on 1000 replications. The simulation is conducted in the free statistical software R. We employ the `glmnet` package to estimate equations (5), (6), (7) and (8) optimizing the parameters $\beta_{j,l}$ and λ using the Bayesian Information Criterion (BIC), with the parameter τ fixed at 1. For each candidate value of δ in the set $[0, 0.5, 1, \ldots, 10]$, the λ minimizing the BIC is chosen, and the δ yielding the lowest BIC overall is selected.

The simulation design follows the data-generating process (DGP) introduced by Konzen and Ziegelmann (6) to provide a consistent basis for statistical comparison. We generate 10 independent time-series covariates $x_{j,t}$, each following an AR(1) process ($x_{j,t} = \phi x_{j,t-1} + u_{j,t}, u_{j,t} \sim N(0,1), j = 1, 2, \ldots, 10$) with autoregressive coefficients set to 0.3, 0.6, and 0.9. Sample sizes T are varied across 50, 100, 200, 500, and 2000. The true model includes several lagged covariates and an autoregressive component:

$$\begin{aligned} y_t = {} & 0.8\, y_{t-1} + 0.6\, x_{1,t-1} + 0.3\, x_{1,t-2} - 0.5\, x_{2,t-1} - 0.2\, x_{2,t-2} \\ & + 0.4\, x_{3,t-1} + 0.3\, x_{3,t-2} + 0.4\, x_{4,t-1} - 0.3\, x_{5,t-1} + 0.2\, x_{6,t-1} + e_t. \end{aligned} \tag{9}$$

where $e_t \sim N(0,1)$, t = 1,2,. . . ,T.

In addition, we employ LASSO, AdaLASSO, WLAdaLASSO and FLAdaLASSO to estimate the equation (9) using L lags of y_t and L lags of each covariate $x_{j,t}$, for $j = 1, 2, \ldots, 10$. This leads to a total of 11L candidate predictors considered in the estimation process. We consider scenarios with L= 5,10 and 20 to examine the performance of each method across different lag lengths. We adopt variable selection metrics proposed by Medeiros and Mendes (7) and Medeiros and Mendes (8), consistent with the approach used by Konzen and Ziegelmann (6), to compare variable selection performance. The criteria used for this assessment are presented in Table 1.

TABLE 1 Variable selection metrics

Metric	Abbreviation	Description
Fraction of variables correctly identified	FVCI	It is the sum of the number of relevant variables included and the number of irrelevant variables excluded, divided by the total number of variables, in every replication, averaged across all replications.
Fraction of relevant variables included	FRVI	For each replication, the fraction of relevant variables that were correctly included, averaged across all replications.
Fraction of irrelevant variables excluded	FIVE	For each replication, the fraction of irrelevant variables that were correctly excluded, averaged across all replications.
Average number of included variables	NIV	The mean count of predictors selected across all replications.
True model included	TMI	The fraction of replications in that all relevant variables were correctly included.

In addition to the variable selection metrics, we assess the accuracy of coefficient estimation using the mean squared error (MSE) and mean absolute error (MAE). The MSE represents the average squared difference between the estimated and true coefficients across all replications, calculated as the true parameter values.

$$\mathrm{MSE} = \frac{1}{1000p} \sum_{i=1}^{1000} \sum_{j=1}^{p} \left(\hat{\beta}_j^{(i)} - \beta_j \right)^2 . \tag{10}$$

The MAE measures the average absolute difference, given by

$$\mathrm{MAE} = \frac{1}{1000p} \sum_{i=1}^{1000} \sum_{j=1}^{p} \left| \hat{\beta}_j^{(i)} - \beta_j \right| . \tag{11}$$

In the expressions for the MSE and MAE, the variables have the following meanings:

- p: the number of coefficients to be estimated.

- β_j: the true value of the j-th coefficient in the data-generating process.
- $\hat{\beta}_j^{(i)}$: the estimate of β_j obtained from the i-th Monte Carlo replication.

These criteria clearly indicate how accurately each method estimates. Furthermore, to evaluate forecasting performance, we remove the last ten observations from each simulated series and perform one-step-ahead out-of-sample forecasts.

Results

This section presents the simulation results evaluating the finite-sample performance of the proposed FLAdaLASSO in comparison with LASSO, Adaptive LASSO, and WLAdaLASSO. The results are summarized in tables 2 to 10 and are discussed with respect to variable selection accuracy, model parsimony, and predictive performance across different sample sizes, lag dimensions, and degrees of covariate dependence.

Tables 2 to 4 compare the model selection performance of LASSO, Adaptive LASSO, WLAdaLASSO, and the proposed FLAdaLASSO under a wide range of simulation settings. The designs vary in sample size ($T = 50, 100, 200, 500$, and 2000), the number of candidate lags ($L = 5, 10$, and 20), and the degree of covariate dependence ($\phi = 0.3, 0.6$, and 0.9). Overall, FLAdaLASSO shows the most consistent or near-best performance in terms of variable selection accuracy, particularly in small to moderate samples ($T = 50$–200) and when the lag dimension is large ($L = 10, 20$). These gains become more pronounced as covariate dependence increases ($\phi = 0.6, 0.9$), whereas performance differences across methods largely disappear in large samples ($T = 2000$), indicating convergence. From the perspective of relevance and parsimony, both lag-weighted methods clearly outperform LASSO and Adaptive LASSO in removing irrelevant variables, as reflected by higher FIVE values and lower NIV. When comparing WLAdaLASSO and FLAdaLASSO directly, their NIV values are generally very close. Accordingly, the main strength of FLAdaLASSO lies in improving selection accuracy while maintaining a level of parsimony comparable to existing lag-weighted approaches. These patterns are especially evident in high-dimensional settings where the total number of covariates ($11 \times L$) exceeds the sample size T.

Tables 5 to 7 report the estimation error measures of the estimated parameter vectors based on MSE and MAE. Overall, the results indicate that FLAdaLASSO most frequently attains the smallest estimation errors across the configurations considered. In several cases, WLAdaLASSO yields slightly lower error values than FLAdaLASSO, although the differences between the two lag-weighted methods are generally negligible. A notable exception occurs in the configuration with $\phi = 0.9$, $L = 5$, and $T = 50$, where LASSO achieves the smallest estimation error.

The results of the one-step-ahead forecasts are reported in tables 8 to 10. Overall, FLAdaLASSO and WLAdaLASSO exhibit very similar forecasting performance across most configurations. In moderate and large samples ($T = 100, 200, 500$, and 2000), FLAdaLASSO most frequently attains the lowest forecasting errors, although the differences relative to WLAdaLASSO are generally small. For the smallest sample size ($T = 50$), forecasting accuracy becomes more sensitive to the lag dimension, and WLAdaLASSO occasionally yields slightly better results. A notable exception occurs for $\phi = 0.9$, $L = 5$, and $T = 50$, where LASSO achieves the lowest forecasting error. Aside from this case, both lag-weighted approaches tend to outperform LASSO and Adaptive LASSO, particularly as the number of candidate lags increases, indicating that the proposed FLAdaLASSO provides stable and competitive forecasting performance across different lag dimensions and dependence levels.

Taken together, the evidence across all tables and figures suggests that the proposed

FLAdaLASSO estimator offers a favorable balance between accurate variable selection, precise coefficient estimation, and reliable forecasting performance across a wide range of data-generating scenarios.

TABLE 2 Model selection summary metrics comparing four methods (LASSO, AdaLASSO, WLAdaLASSO, and FLAdaLASSO) across sample sizes $T = 50, 100, 200, 500$, and 2000, using variable selection metrics when the level of covariate dependence is 0.3.

	5 candidate lags					10 candidate lags					20 candidate lags				
T	50	100	200	500	2000	50	100	200	500	2000	50	100	200	500	2000
FVCI															
LASSO	0.8517	0.8964	0.9251	0.9472	0.9622	0.9030	0.9330	0.9545	0.9699	0.9797	0.9423	0.9575	0.9730	0.9839	0.9892
AdaLASSO	0.8355	0.9236	0.9637	0.9855	0.9948	0.8905	0.9257	0.9738	0.9907	0.9972	0.9307	0.9524	0.9713	0.9941	0.9986
WLAdaLASSO	0.9057	0.9507	0.9817	0.9945	0.9991	0.9527	0.9701	0.9905	0.9972	0.9995	0.9763	0.9849	0.9927	0.9985	0.9997
FLAdaLASSO	0.9066	0.9512	0.9824	0.9947	0.9991	0.9527	0.9709	0.9909	0.9972	0.9995	0.9772	0.9868	0.9938	0.9985	0.9997
FRVI															
LASSO	0.6423	0.8608	0.9699	0.9981	1.0000	0.5617	0.8070	0.9499	0.9969	1.0000	0.4825	0.7476	0.9270	0.9956	1.0000
AdaLASSO	0.5806	0.8403	0.9517	0.9951	1.0000	0.5099	0.7460	0.9350	0.9936	1.0000	0.4332	0.6902	0.8757	0.9891	1.0000
WLAdaLASSO	0.6092	0.8159	0.9560	0.9965	1.0000	0.6161	0.7647	0.9535	0.9969	1.0000	0.6271	0.7822	0.9189	0.9965	1.0000
FLAdaLASSO	0.6129	0.8189	0.9568	0.9965	1.0000	0.6228	0.7902	0.9556	0.9967	1.0000	0.6514	0.8281	0.9467	0.9967	1.0000
FIVE															
LASSO	0.8982	0.9043	0.9152	0.9359	0.9538	0.9371	0.9456	0.9550	0.9673	0.9776	0.9642	0.9675	0.9751	0.9833	0.9886
AdaLASSO	0.8922	0.9421	0.9664	0.9833	0.9936	0.9286	0.9437	0.9777	0.9904	0.9969	0.9544	0.9649	0.9759	0.9943	0.9985
WLAdaLASSO	0.9716	0.9806	0.9874	0.9940	0.9989	0.9863	0.9907	0.9942	0.9972	0.9994	0.9929	0.9946	0.9962	0.9985	0.9997
FLAdaLASSO	0.9718	0.9806	0.9881	0.9943	0.9989	0.9857	0.9890	0.9945	0.9972	0.9995	0.9927	0.9944	0.9961	0.9986	0.9997
TMI															
LASSO	0.0000	0.0060	0.0250	0.1070	0.1880	0.0000	0.0000	0.0110	0.0650	0.1500	0.0000	0.0000	0.0090	0.0840	0.1550
AdaLASSO	0.0000	0.0090	0.1410	0.4820	0.7600	0.0000	0.0000	0.0580	0.4090	0.7440	0.0000	0.0000	0.0020	0.3400	0.7480
WLAdaLASSO	0.0030	0.0500	0.3560	0.7430	0.9540	0.0020	0.0300	0.3330	0.7410	0.9520	0.0000	0.0340	0.2280	0.7260	0.9440
FLAdaLASSO	0.0020	0.0490	0.3710	0.7530	0.9560	0.0020	0.0330	0.3510	0.7380	0.9540	0.0020	0.0490	0.2550	0.7340	0.9400
NIV															
LASSO	11.0030	12.9150	13.5150	12.8650	12.0790	11.9050	13.5130	14.0020	13.2440	12.2380	12.3430	14.3070	14.4890	13.4650	12.3860
AdaLASSO	10.6570	11.0090	11.0310	10.7020	10.2880	12.2430	13.0950	11.5780	10.9000	10.3110	13.9050	14.2830	13.8200	11.0790	10.3090
WLAdaLASSO	7.3680	9.0310	10.1250	10.2340	10.0500	7.5280	8.5800	10.1160	10.2500	10.0560	7.7570	8.9630	9.9850	10.2710	10.0600
FLAdaLASSO	7.3960	9.0640	10.1050	10.2230	10.0500	7.6570	9.0050	10.1080	10.2460	10.0550	8.0400	9.4670	10.2890	10.2580	10.0670

TABLE 3 Model selection summary metrics comparing four methods (LASSO, AdaLASSO, WLAdaLASSO, and FLAdaLASSO) across sample sizes $T = 50, 100, 200, 500$, and 2000, using variable selection metrics when the level of covariate dependence is 0.6.

	5 candidate lags					**10 candidate lags**					**20 candidate lags**				
T	50	100	200	500	2000	50	100	200	500	2000	50	100	200	500	2000
							FVCI								
LASSO	0.8490	0.8976	0.9235	0.9428	0.9606	0.8988	0.9323	0.9526	0.9696	0.9793	0.9388	0.9549	0.9715	0.9830	0.9890
AdaLASSO	0.7989	0.9136	0.9562	0.9792	0.9906	0.8522	0.9011	0.9695	0.9882	0.9951	0.9040	0.9333	0.9619	0.9925	0.9974
WLAdaLASSO	0.9046	0.9552	0.9831	0.9950	0.9991	0.9525	0.9681	0.9913	0.9975	0.9995	0.9760	0.9837	0.9904	0.9986	0.9998
FLAdaLASSO	0.9066	0.9558	0.9839	0.9952	0.9991	0.9533	0.9693	0.9915	0.9976	0.9995	0.9770	0.9860	0.9929	0.9987	0.9997
							FRVI								
LASSO	0.7049	0.8924	0.9823	0.9995	1.0000	0.6216	0.8448	0.9708	0.9993	1.0000	0.5432	0.7999	0.9542	0.9987	1.0000
AdaLASSO	0.6380	0.8698	0.9633	0.9971	1.0000	0.5529	0.7924	0.9510	0.9959	1.0000	0.4581	0.7424	0.8918	0.9934	1.0000
WLAdaLASSO	0.6236	0.8335	0.9588	0.9984	1.0000	0.6387	0.7603	0.9578	0.9984	1.0000	0.6509	0.7854	0.9207	0.9980	1.0000
FLAdaLASSO	0.6283	0.8364	0.9590	0.9985	1.0000	0.6591	0.8047	0.9568	0.9985	1.0000	0.6816	0.8397	0.9546	0.9982	1.0000
							FIVE								
LASSO	0.8810	0.8988	0.9104	0.9302	0.9519	0.9266	0.9410	0.9508	0.9666	0.9772	0.9576	0.9623	0.9723	0.9822	0.9885
AdaLASSO	0.8346	0.9234	0.9546	0.9753	0.9885	0.8822	0.9120	0.9713	0.9875	0.9946	0.9252	0.9424	0.9653	0.9925	0.9973
WLAdaLASSO	0.9671	0.9822	0.9885	0.9942	0.9989	0.9838	0.9889	0.9946	0.9974	0.9995	0.9915	0.9932	0.9937	0.9986	0.9998
FLAdaLASSO	0.9685	0.9823	0.9894	0.9944	0.9989	0.9828	0.9858	0.9950	0.9975	0.9995	0.9911	0.9930	0.9947	0.9987	0.9997
							TMI								
LASSO	0.0010	0.0010	0.0240	0.0970	0.1560	0.0000	0.0010	0.0130	0.0680	0.1650	0.0000	0.0010	0.0080	0.0510	0.1420
AdaLASSO	0.0000	0.0060	0.0960	0.3500	0.6190	0.0000	0.0000	0.0420	0.3180	0.6020	0.0000	0.0000	0.0010	0.2490	0.5870
WLAdaLASSO	0.0030	0.0870	0.4130	0.7750	0.9570	0.0030	0.0320	0.4050	0.7720	0.9520	0.0020	0.0280	0.1860	0.7570	0.9570
FLAdaLASSO	0.0040	0.0880	0.4340	0.7780	0.9560	0.0040	0.0370	0.4160	0.7810	0.9550	0.0050	0.0440	0.2420	0.7720	0.9520
							NIV								
LASSO	12.4030	13.4780	13.8530	13.1370	12.1660	13.5590	14.3440	14.6330	13.3330	12.2770	14.3270	15.9260	15.3510	13.7240	12.4210
AdaLASSO	13.8210	12.1470	11.6740	11.0840	10.5160	17.3130	16.7240	12.3790	11.2140	10.5430	20.2880	19.5160	16.2080	11.5080	10.5730
WLAdaLASSO	7.7180	9.1350	10.1070	10.2450	10.0480	8.0040	8.7140	10.1180	10.2440	10.0540	8.2990	9.2860	10.5210	10.2720	10.0510
FLAdaLASSO	7.7010	9.1590	10.0650	10.2350	10.0500	8.3140	9.4660	10.0710	10.2350	10.0500	8.6820	9.8730	10.6640	10.2610	10.0560

TABLE 4 Model selection summary metrics comparing four methods (LASSO, AdaLASSO, WLAdaLASSO, and FLAdaLASSO) across sample sizes $T = 50, 100, 200, 500$, and 2000, using variable selection metrics when the level of covariate dependence is 0.9.

	5 candidate lags					**10 candidate lags**					**20 candidate lags**				
T	50	100	200	500	2000	50	100	200	500	2000	50	100	200	500	2000
								FVCI							
LASSO	0.8437	0.8821	0.9145	0.9338	0.9544	0.8933	0.9261	0.9443	0.9627	0.9752	0.9348	0.9468	0.9689	0.9771	0.9859
AdaLASSO	0.7061	0.8341	0.9019	0.9431	0.9725	0.7967	0.7962	0.9099	0.9591	0.9802	0.8804	0.8598	0.8945	0.9742	0.9893
WLAdaLASSO	0.8533	0.9558	0.9856	0.9970	0.9997	0.9473	0.9562	0.9916	0.9983	0.9999	0.9736	0.9774	0.9794	0.9991	0.9998
FLAdaLASSO	0.8551	0.9573	0.9860	0.9973	0.9997	0.9472	0.9545	0.9920	0.9985	0.9998	0.9748	0.9802	0.9855	0.9990	0.9984
								FRVI							
LASSO	0.6890	0.9208	0.9907	0.9998	1.0000	0.6069	0.8557	0.9867	0.9996	1.0000	0.5416	0.8097	0.9652	0.9995	1.0000
AdaLASSO	0.5114	0.9075	0.9811	0.9972	1.0000	0.4208	0.6855	0.9765	0.9966	1.0000	0.3385	0.6149	0.8446	0.9966	1.0000
WLAdaLASSO	0.5297	0.8558	0.9666	0.9980	1.0000	0.6361	0.6839	0.9636	0.9979	1.0000	0.6724	0.7572	0.8780	0.9979	1.0000
FLAdaLASSO	0.5343	0.8591	0.9668	0.9982	1.0000	0.6956	0.7744	0.9655	0.9978	1.0000	0.7334	0.8394	0.9478	0.9984	1.0000
								FIVE							
LASSO	0.8781	0.8736	0.8976	0.9192	0.9442	0.9219	0.9331	0.9401	0.9591	0.9727	0.9535	0.9533	0.9691	0.9760	0.9852
AdaLASSO	0.7494	0.8178	0.8843	0.9310	0.9664	0.8343	0.8073	0.9032	0.9553	0.9782	0.9062	0.8715	0.8969	0.9731	0.9888
WLAdaLASSO	0.9252	0.9780	0.9898	0.9968	0.9997	0.9784	0.9835	0.9945	0.9983	0.9999	0.9880	0.9879	0.9842	0.9992	0.9998
FLAdaLASSO	0.9264	0.9791	0.9902	0.9971	0.9997	0.9723	0.9725	0.9946	0.9986	0.9997	0.9863	0.9869	0.9872	0.9991	0.9983
								TMI							
LASSO	0.0000	0.0020	0.0120	0.0470	0.0980	0.0000	0.0040	0.0070	0.0410	0.1000	0.0000	0.0010	0.0080	0.0280	0.0820
AdaLASSO	0.0000	0.0030	0.0190	0.0780	0.2070	0.0000	0.0000	0.0050	0.0480	0.1220	0.0000	0.0000	0.0000	0.0320	0.1110
WLAdaLASSO	0.0010	0.1020	0.4960	0.8590	0.9870	0.0010	0.0060	0.4470	0.8450	0.9900	0.0010	0.0050	0.0110	0.8430	0.9740
FLAdaLASSO	0.0010	0.1120	0.5120	0.8670	0.9870	0.0010	0.0080	0.4690	0.8620	0.9780	0.0020	0.0100	0.0340	0.8230	0.6900
								NIV							
LASSO	12.3750	14.8980	14.5150	13.6350	12.5090	13.8780	15.2470	15.8570	14.0910	12.7320	15.1870	17.8950	16.1400	15.0360	13.1010
AdaLASSO	16.3920	17.2720	15.0180	13.0750	11.5140	20.7770	26.1290	19.4450	14.4350	12.1790	23.0830	33.1360	30.1070	15.6100	12.3490
WLAdaLASSO	8.6640	9.5460	10.1240	10.1230	10.0140	8.5170	8.4940	10.1910	10.1500	10.0120	9.2530	10.1220	12.0900	10.1500	10.0520
FLAdaLASSO	8.6560	9.5330	10.1070	10.1140	10.0140	9.7230	10.4930	10.1920	10.1210	10.0270	10.2200	11.1490	12.1570	10.1790	10.3580

TABLE 5 Summary statistics of parameter estimates comparing four methods (LASSO, AdaLASSO, WLAdaLASSO, and FLAdaLASSO) across sample sizes $T = 50, 100, 200, 500,$ and 2000, using MSE and MAE when the level of covariate dependence is 0.3.

	5 candidate lags					**10 candidate lags**					**20 candidate lags**				
T	50	100	200	500	2000	50	100	200	500	2000	50	100	200	500	2000
								MSE							
LASSO	0.01285	0.00611	0.00283	0.00110	0.00028	0.00785	0.00388	0.00184	0.00070	0.00017	0.00462	0.00239	0.00113	0.00043	0.00010
AdaLASSO	0.01670	0.00558	0.00207	0.00061	0.00012	0.01089	0.00458	0.00126	0.00034	0.00006	0.00719	0.00285	0.00111	0.00020	0.00003
WLAdaLASSO	0.01044	0.00432	0.00159	0.00047	0.00010	0.00514	0.00258	0.00080	0.00024	0.00005	0.00258	0.00125	0.00056	0.00012	0.00002
FLAdaLASSO	0.01046	0.00430	0.00156	0.00047	0.00010	0.00523	0.00243	0.00079	0.00024	0.00005	0.00256	0.00111	0.00046	0.00012	0.00002
								MAE							
LASSO	0.04814	0.03178	0.02100	0.01271	0.00626	0.02806	0.01850	0.01219	0.00723	0.00350	0.01574	0.01063	0.00689	0.00405	0.00194
AdaLASSO	0.05625	0.02866	0.01641	0.00846	0.00363	0.03473	0.02029	0.00933	0.00450	0.00184	0.02128	0.01195	0.00665	0.00244	0.00093
WLAdaLASSO	0.03844	0.02305	0.01347	0.00722	0.00327	0.01917	0.01268	0.00676	0.00362	0.00164	0.00961	0.00628	0.00397	0.00182	0.00082
FLAdaLASSO	0.03840	0.02296	0.01334	0.00719	0.00327	0.01933	0.01234	0.00669	0.00362	0.00164	0.00952	0.00591	0.00363	0.00181	0.00082

TABLE 6 Summary statistics of parameter estimates comparing four methods (LASSO, AdaLASSO, WLAdaLASSO, and FLAdaLASSO) across sample sizes $T = 50, 100, 200, 500$, and 2000, using MSE and MAE when the level of covariate dependence is 0.6.

	5 candidate lags					**10 candidate lags**					**20 candidate lags**				
T	50	100	200	500	2000	50	100	200	500	2000	50	100	200	500	2000
								MSE							
LASSO	0.01180	0.00506	0.00220	0.00083	0.00021	0.00748	0.00319	0.00138	0.00051	0.00012	0.00450	0.00198	0.00084	0.00031	0.00007
AdaLASSO	0.02099	0.00552	0.00193	0.00057	0.00011	0.01767	0.00481	0.00115	0.00030	0.00006	0.01290	0.00318	0.00104	0.00017	0.00003
WLAdaLASSO	0.01036	0.00402	0.00150	0.00042	0.00010	0.00488	0.00258	0.00075	0.00021	0.00005	0.00247	0.00121	0.00054	0.00011	0.00003
FLAdaLASSO	0.01029	0.00400	0.00147	0.00041	0.00010	0.00480	0.00229	0.00075	0.00021	0.00005	0.00244	0.00104	0.00042	0.00011	0.00002
								MAE							
LASSO	0.04689	0.02875	0.01850	0.01103	0.00535	0.02798	0.01673	0.01045	0.00610	0.00293	0.01601	0.00976	0.00591	0.00335	0.00159
AdaLASSO	0.06726	0.02933	0.01623	0.00837	0.00367	0.04862	0.02264	0.00911	0.00434	0.00184	0.03170	0.01412	0.00678	0.00231	0.00093
WLAdaLASSO	0.03842	0.02196	0.01289	0.00682	0.00334	0.01884	0.01278	0.00649	0.00342	0.00167	0.00948	0.00624	0.00394	0.00172	0.00084
FLAdaLASSO	0.03820	0.02185	0.01275	0.00676	0.00333	0.01864	0.01209	0.00645	0.00339	0.00166	0.00937	0.00573	0.00354	0.00171	0.00082

TABLE 7 Summary statistics of parameter estimates comparing four methods (LASSO, AdaLASSO, WLAdaLASSO, and FLAdaLASSO) across sample sizes $T = 50, 100, 200, 500,$ and 2000, using MSE and MAE when the level of covariate dependence is 0.9.

	5 candidate lags					10 candidate lags					20 candidate lags				
T	50	100	200	500	2000	50	100	200	500	2000	50	100	200	500	2000
								MSE							
LASSO	0.01247	0.00424	0.00166	0.00060	0.00015	0.00801	0.00298	0.00097	0.00035	0.00009	0.00486	0.00191	0.00071	0.00020	0.00005
AdaLASSO	0.08059	0.01023	0.00234	0.00062	0.00012	0.04530	0.02426	0.00197	0.00036	0.00007	0.02717	0.01487	0.00571	0.00020	0.00003
WLAdaLASSO	0.03406	0.00388	0.00140	0.00047	0.00022	0.00502	0.00332	0.00073	0.00023	0.00010	0.00245	0.00140	0.00076	0.00012	0.00005
FLAdaLASSO	0.03413	0.00384	0.00138	0.00044	0.00019	0.00472	0.00267	0.00071	0.00023	0.00009	0.00230	0.00108	0.00046	0.00011	0.00003
								MAE							
LASSO	0.04821	0.02679	0.01608	0.00937	0.00459	0.02929	0.01615	0.00889	0.00503	0.00245	0.01701	0.00984	0.00532	0.00274	0.00132
AdaLASSO	0.13818	0.04558	0.01994	0.00931	0.00386	0.08425	0.06234	0.01471	0.00531	0.00207	0.04863	0.03907	0.02087	0.00295	0.00105
WLAdaLASSO	0.07076	0.02144	0.01192	0.00684	0.00485	0.01948	0.01513	0.00609	0.00341	0.00232	0.00971	0.00696	0.00468	0.00172	0.00114
FLAdaLASSO	0.07070	0.02125	0.01181	0.00669	0.00439	0.01916	0.01361	0.00599	0.00336	0.00211	0.00943	0.00604	0.00369	0.00164	0.00081

TABLE 8 **Forecast performance statistics summary comparing four methods (LASSO, AdaLASSO, WLAdaLASSO, and FLAdaLASSO) across sample sizes $T = 50, 100, 200, 500$, and 2000, using Mean/Median of MSEs and Mean/Median of MAEs when the level of covariate dependence is 0.3.**

	5 candidate lags					10 candidate lags					20 candidate lags				
T	50	100	200	500	2000	50	100	200	500	2000	50	100	200	500	2000
								Mean of MSEs							
LASSO	2.0123	1.4519	1.2001	1.0966	1.0454	2.2404	1.5991	1.2646	1.1188	1.0502	2.4508	1.7489	1.3365	1.1435	1.0564
AdaLASSO	2.3062	1.3695	1.1325	1.0545	1.0289	2.8761	1.6763	1.1617	1.0584	1.0287	3.6898	1.8771	1.2999	1.0663	1.0293
WLAdaLASSO	1.7494	1.2689	1.1040	1.0436	1.0274	1.7626	1.3290	1.1041	1.0434	1.0275	1.7669	1.3299	1.1424	1.0437	1.0276
FLAdaLASSO	1.7592	1.2693	1.1019	1.0436	1.0274	1.7618	1.3247	1.1028	1.0431	1.0275	1.7643	1.3262	1.1407	1.0431	1.0276
								Median of MSEs							
LASSO	1.8660	1.3824	1.1139	1.0461	1.0073	2.0544	1.4919	1.1718	1.0633	1.0077	2.2045	1.6076	1.2567	1.0759	1.0150
AdaLASSO	2.1074	1.3057	1.0679	0.9893	0.9898	2.5695	1.5564	1.0805	0.9945	0.9902	3.0471	1.7276	1.2186	1.0027	0.9901
WLAdaLASSO	1.6153	1.1935	1.0400	0.9844	0.9929	1.6228	1.2467	1.0302	0.9884	0.9914	1.6185	1.2737	1.0742	0.9885	0.9893
FLAdaLASSO	1.6265	1.2016	1.0371	0.9894	0.9886	1.6215	1.2484	1.0307	0.9863	0.9898	1.6093	1.2554	1.0814	0.9889	0.9903
								Mean of MAEs							
LASSO	1.1283	0.9602	0.8757	0.8378	0.8169	1.1916	1.0046	0.8980	0.8464	0.8189	1.2449	1.0502	0.9230	0.8566	0.8215
AdaLASSO	1.2053	0.9338	0.8509	0.8216	0.8099	1.3443	1.0283	0.8613	0.8233	0.8098	1.5096	1.0860	0.9092	0.8264	0.8101
WLAdaLASSO	1.0558	0.8995	0.8415	0.8178	0.8094	1.0593	0.9195	0.8408	0.8176	0.8093	1.0623	0.9198	0.8551	0.8180	0.8094
FLAdaLASSO	1.0594	0.8999	0.8402	0.8177	0.8094	1.0584	0.9184	0.8404	0.8175	0.8093	1.0608	0.9182	0.8541	0.8176	0.8094
								Median of MAEs							
LASSO	1.1106	0.9572	0.8668	0.8315	0.8037	1.1714	0.9947	0.8860	0.8402	0.8070	1.2119	1.0295	0.9108	0.8475	0.8111
AdaLASSO	1.1789	0.9291	0.8442	0.8114	0.7984	1.3039	1.0279	0.8500	0.8156	0.7986	1.4253	1.0689	0.8975	0.8228	0.8025
WLAdaLASSO	1.0437	0.8881	0.8297	0.8064	0.7977	1.0441	0.9185	0.8325	0.8079	0.7976	1.0458	0.9187	0.8436	0.8083	0.8001
FLAdaLASSO	1.0449	0.8876	0.8335	0.8075	0.7965	1.0389	0.9139	0.8308	0.8089	0.7974	1.0423	0.9144	0.8439	0.8076	0.7991

TABLE 9 Forecast performance statistics summary comparing four methods (LASSO, AdaLASSO, WLAdaLASSO, and FLAdaLASSO) across sample sizes $T = 50, 100, 200, 500$, and 2000, using Mean/Median of MSEs and Mean/Median of MAEs when the level of covariate dependence is 0.6.

	5 candidate lags					10 candidate lags					20 candidate lags				
T	50	100	200	500	2000	50	100	200	500	2000	50	100	200	500	2000
							Mean of MSEs								
LASSO	2.1746	1.4574	1.1907	1.0894	1.0432	2.4986	1.6020	1.2525	1.1114	1.0493	2.7585	1.7708	1.3194	1.1320	1.0542
AdaLASSO	3.2813	1.4304	1.1431	1.0565	1.0300	5.0387	1.9284	1.1813	1.0606	1.0302	7.0335	2.3648	1.3610	1.0643	1.0303
WLAdaLASSO	1.9588	1.2757	1.0997	1.0412	1.0281	1.9543	1.4099	1.1024	1.0416	1.0280	1.9511	1.4099	1.1768	1.0421	1.0282
FLAdaLASSO	1.9540	1.2777	1.0999	1.0415	1.0280	1.9485	1.4043	1.1026	1.0404	1.0279	1.9536	1.4050	1.1720	1.0421	1.0279
							Median of MSEs								
LASSO	2.0065	1.3693	1.1062	1.0276	1.0012	2.2053	1.4835	1.1712	1.0484	1.0077	2.4310	1.6364	1.2024	1.0660	1.0133
AdaLASSO	2.7780	1.3286	1.0712	0.9997	0.9916	3.9165	1.7086	1.1049	1.0036	0.9910	5.1466	1.9993	1.268224	0.9997	0.9920
WLAdaLASSO	1.7783	1.2188	1.0286	0.9872	0.9951	1.7790	1.3185	1.0287	0.9857	0.9915	1.7499	1.3147	1.0856	0.9819	0.9932
FLAdaLASSO	1.7933	1.2078	1.0262	0.9869	0.9921	1.7903	1.3123	1.0285	0.9848	0.9920	1.7789	1.3172	1.0885	0.9801	0.9920
							Mean of MAEs								
LASSO	1.1692	0.9634	0.8712	0.8354	0.8159	1.2556	1.0103	0.8935	0.8435	0.8184	1.3172	1.0569	0.9154	0.8514	0.8201
AdaLASSO	1.4236	0.9554	0.8546	0.8230	0.8104	1.7475	1.1038	0.8670	0.8239	0.8105	2.0565	1.2097	0.9284	0.8259	0.8105
WLAdaLASSO	1.1137	0.9025	0.8393	0.8162	0.8096	1.1090	0.9463	0.8394	0.8165	0.8095	1.1077	0.9449	0.8654	0.8166	0.8095
FLAdaLASSO	1.1124	0.9034	0.8394	0.8164	0.8094	1.1082	0.9441	0.8397	0.8161	0.8095	1.1086	0.9435	0.8640	0.8168	0.8095
							Median of MAEs								
LASSO	1.1504	0.9649	0.8620	0.8264	0.8056	1.2264	1.0003	0.8830	0.8346	0.8091	1.2800	1.0368	0.8969	0.8446	0.8074
AdaLASSO	1.3685	0.9373	0.8446	0.8123	0.7994	1.6235	1.0719	0.8503	0.8116	0.8005	1.8645	1.1558	0.9068	0.8157	0.7988
WLAdaLASSO	1.0914	0.9024	0.8319	0.8052	0.8006	1.0876	0.9381	0.8286	0.8059	0.8022	1.0879	0.9394	0.8506	0.8060	0.8011
FLAdaLASSO	1.0964	0.8969	0.8278	0.8052	0.8007	1.0945	0.9379	0.8313	0.8036	0.8021	1.0865	0.9383	0.8491	0.8054	0.8015

TABLE 10 Forecast performance statistics summary comparing four methods (LASSO, AdaLASSO, WLAdaLASSO, and FLAdaLASSO) across sample sizes $T = 50, 100, 200, 500$, and 2000, using Mean/Median of MSEs and Mean/Median of MAEs when the level of covariate dependence is 0.9.

	5 candidate lags					10 candidate lags					20 candidate lags				
T	50	100	200	500	2000	50	100	200	500	2000	50	100	200	500	2000
Mean of MSEs															
LASSO	2.7673	1.5214	1.1879	1.0771	1.0420	3.2511	1.8587	1.2423	1.0964	1.0452	3.5262	2.0936	1.4021	1.1133	1.0507
AdaLASSO	14.5029	1.8010	1.1930	1.0638	1.0366	15.7707	9.4568	1.3088	1.0731	1.0372	16.7418	11.8613	4.5276	1.0818	1.0370
WLAdaLASSO	7.6768	1.3186	1.1054	1.0440	1.0362	8.3659	3.5626	1.1104	1.0437	1.0358	9.2505	3.8438	1.9815	1.0446	1.0356
FLAdaLASSO	7.7038	1.3199	1.1046	1.0431	1.0356	8.3938	3.5679	1.1094	1.0439	1.0355	9.2422	3.8524	1.9713	1.0433	1.0362
Median of MSEs															
LASSO	2.4793	1.3822	1.1046	1.0241	0.9923	2.8824	1.6647	1.1603	1.0306	0.9976	3.0927	1.8786	1.2839	1.0539	1.0086
AdaLASSO	9.3514	1.5809	1.0975	1.0138	0.9962	10.8601	4.7553	1.2035	1.0176	0.9967	12.0866	6.7508	2.3132	1.0190	0.9954
WLAdaLASSO	4.0460	1.2186	1.0232	0.9880	0.9932	4.3061	2.2251	1.0287	0.9913	0.9924	4.7093	2.2900	1.6120	0.9911	0.9876
FLAdaLASSO	4.0269	1.2293	1.0290	0.9923	0.9875	4.3139	2.2131	1.0269	0.9942	0.9885	4.6472	2.2710	1.5845	0.9921	0.9881
Mean of MAEs															
LASSO	1.3186	0.9832	0.8696	0.8298	0.8151	1.4314	1.0838	0.8883	0.8374	0.8165	1.4914	1.1458	0.9429	0.8442	0.8183
AdaLASSO	2.9119	1.0632	0.8712	0.8247	0.8134	3.0951	2.2890	0.9094	0.8289	0.8136	3.1968	2.6069	1.5832	0.8329	0.8136
WLAdaLASSO	2.0210	0.9169	0.8409	0.8171	0.8132	2.0984	1.4203	0.8425	0.8172	0.8129	2.1930	1.4548	1.1069	0.8172	0.8127
FLAdaLASSO	2.0222	0.9181	0.8405	0.8169	0.8129	2.1029	1.4206	0.8422	0.8171	0.8131	2.1914	1.4548	1.1029	0.8168	0.8136
Median of MAEs															
LASSO	1.2838	0.9722	0.8554	0.8187	0.8098	1.4033	1.0558	0.8725	0.8250	0.8091	1.4402	1.1192	0.9138	0.8297	0.8102
AdaLASSO	2.5623	1.0369	0.8533	0.8121	0.8006	2.7616	1.8422	0.8852	0.8217	0.8020	2.8669	2.1735	1.2642	0.8202	0.8013
WLAdaLASSO	1.6668	0.9096	0.8283	0.8091	0.8052	1.7096	1.4203	0.8290	0.8123	0.8039	1.7718	1.2527	1.0487	0.8103	0.8013
FLAdaLASSO	1.6710	0.9100	0.8277	0.8086	0.8027	1.7222	1.2219	0.8301	0.8120	0.8009	1.7575	1.2423	1.0423	0.8092	0.8015

ILLUSTRATIVE EMPIRICAL EXAMPLE

For the real data analysis, we investigate the determinants of inflation in Thailand using the proposed modified method together with several regularization techniques. The empirical analysis is based on annual data from 1975 to 2024 obtained from the World Development Indicators. Previous studies on Thailand's macroeconomic forecasting often rely on a relatively narrow set of variables, for example GDP, inflation, and external debt (*11, 12*). In contrast, this study adopts a broader specification through a Generalized Unidentified Model (GUM) framework that incorporates ten potential determinants of inflation. The variables included in the analysis, together with the data transformations applied to each of them, are presented in Table 11.

TABLE 11 Variables and data transformations used in the analysis across countries.

Symbol	Variable	Thailand	China	USA
INF	Inflation	Level(INF)	Level(INF)	Level(INF)
IMP	Imports of goods and services	ΔIMP	ΔIMP	ΔIMP
EXP	Exports of goods and services	Δ log(EXP)	Δ log(EXP)	Δ^2 log(EXP)
EXR	Official exchange rate	Δ^2 log(EXR)	Δ^2 log(EXR)	–
CREDIT	Domestic credit to private sector	Δ^2 log(CREDIT)	Δ log(CREDIT)	Δ^2 log(CREDIT)
HCONS	Household and NPISH consumption	Δ^2 log(HCONS)	Δ^2 log(HCONS)	ΔHCONS
GCONS	Government final consumption expenditure	Δ^2 log(GCONS)	Level(GCONS)	ΔGCONS
RIR	Real interest rate	Level(RIR)	Level(RIR)	Level(RIR)
BM	Broad money	ΔBM	ΔBM	ΔBM
CAB	Current account balance	ΔCAB	–	–
GFCF	Gross fixed capital formation	Δ^2 log(GFCF)	ΔGFCF	ΔGFCF
TRD	Trade	–	Δ log(TRD)	ΔTRD
TAX	Tax revenue	–	–	ΔTAX

Note: Annual data from 1975–2024 obtained from the World Development Indicators (World Bank).

In the model specification, all transformed variables are incorporated together with their lagged values. The design includes ten regressors, each augmented with ten lagged terms, and the dependent variable is also included with ten lags. This results in a GUM comprising 120 covariates. The tuning parameters are selected according to the Bayesian Information Criterion. The sample from 1975 to 2014 is used for model training, whereas the observations from 2015 to 2024 are reserved for evaluating out-of-sample forecasting performance. Forecast accuracy is measured using the root mean squared error.

Prior to evaluating forecasting performance, the adequacy of the fitted models is examined through residual diagnostics. The Q–Q plots in Figure 2 show that the residuals produced by LASSO, WLAdaLASSO, and FLAdaLASSO adhere reasonably well to the theoretical quantiles, while those from AdaLASSO deviate markedly and display heavy-tailed behavior. The formal Shapiro–Wilk and Jarque–Bera statistics reported in Table 12 corroborate these observations: normality is not rejected for the LASSO-based estimators but is strongly rejected for AdaLASSO. These findings indicate that incorporating lag-structured adaptive penalties enhances residual stability in the presence of persistent temporal dependence.

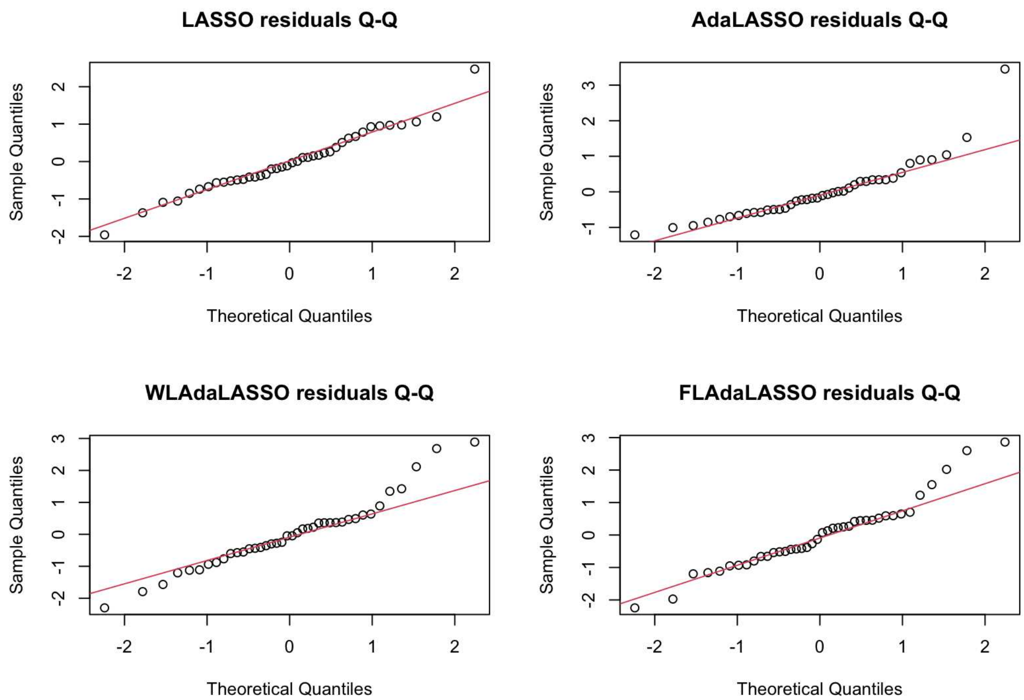

FIGURE 2 Q-Q plot of residuals for the Thailand Inflation data.

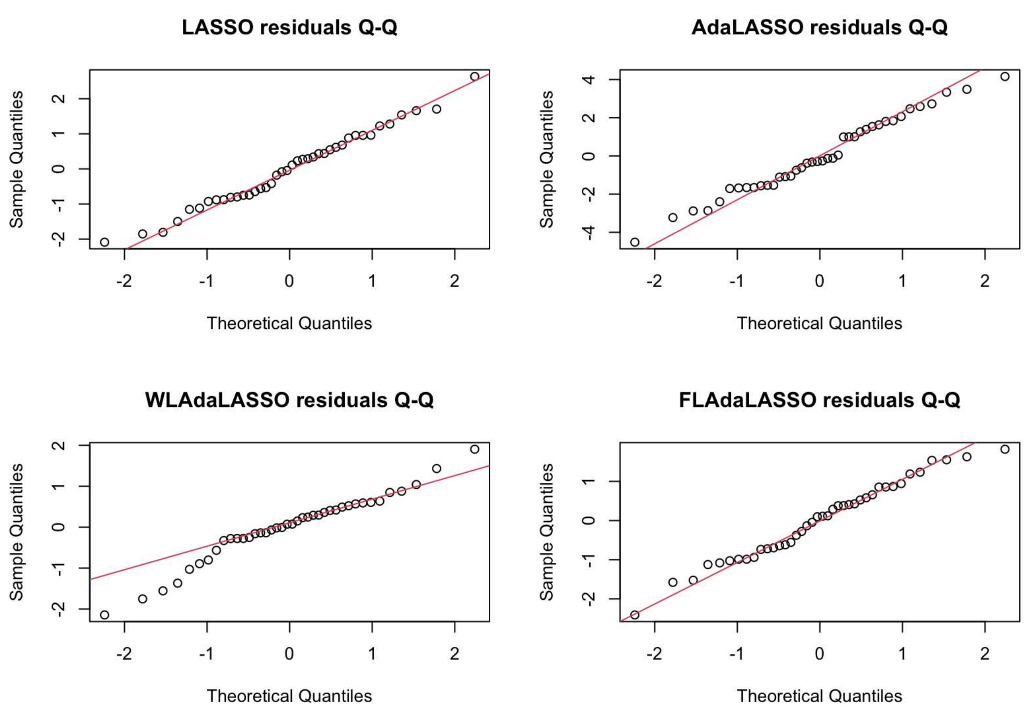

FIGURE 3 Q-Q plot of residuals for the China Inflation data.

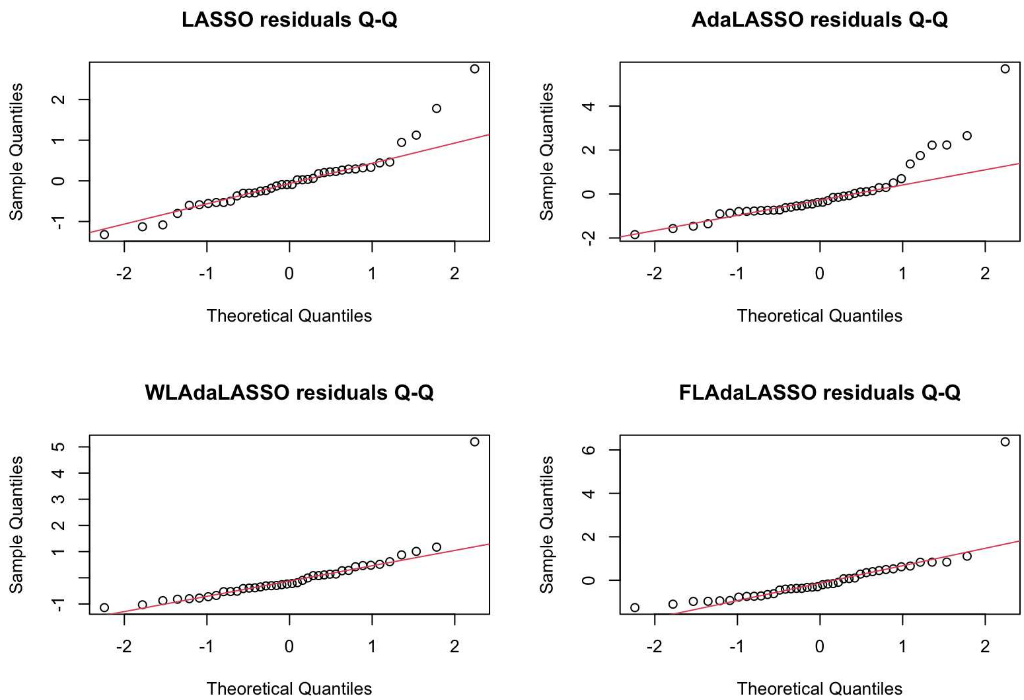

FIGURE 4 Q-Q plot of residuals for the USA Inflation data.

TABLE 12 Normality tests of residuals for the Thailand inflation based on Shapiro-Wilk and Jarque-Bera statistics.

Model	**Test**	**Statistic**	**p-value**	**Decision** (α = 0.05)
LASSO	Shapiro–Wilk	$W = 0.9754$	0.5250	Fail to reject H_0
	Jarque–Bera	$\chi^2 = 2.3385$	0.3106	Fail to reject H_0
AdaLASSO	Shapiro–Wilk	$W = 0.8542$	< 0.001	Reject H_0
	Jarque–Bera	$\chi^2 = 81.2960$	< 0.001	Reject H_0
WL AdaLASSO	Shapiro–Wilk	$W = 0.9565$	0.1270	Fail to reject H_0
	Jarque–Bera	$\chi^2 = 3.8337$	0.1471	Fail to reject H_0
FLAdaLASSO	Shapiro–Wilk	$W = 0.9544$	0.1075	Fail to reject H_0
	Jarque–Bera	$\chi^2 = 3.4930$	0.1744	Fail to reject H_0

TABLE 13 Normality tests of residuals for the China inflation dataset based on Shapiro-Wilk and Jarque-Bera statistics.

Model	Test	Statistic	p-value	Decision ($\alpha = 0.05$)
LASSO	Shapiro–Wilk	$W = 0.9850$	0.8654	Fail to reject H_0
	Jarque–Bera	$\chi^2 = 0.4771$	0.7878	Fail to reject H_0
AdaLASSO	Shapiro–Wilk	$W = 0.9817$	0.7508	Fail to reject H_0
	Jarque–Bera	$\chi^2 = 0.6607$	0.7187	Fail to reject H_0
WL AdaLASSO	Shapiro–Wilk	$W = 0.9624$	0.2020	Fail to reject H_0
	Jarque–Bera	$\chi^2 = 2.3252$	0.3127	Fail to reject H_0
FLAdaLASSO	Shapiro–Wilk	$W = 0.9802$	0.6978	Fail to reject H_0
	Jarque–Bera	$\chi^2 = 0.6390$	0.7265	Fail to reject H_0

TABLE 14 Normality tests of residuals for the US inflation dataset based on Shapiro-Wilk and Jarque-Bera statistics.

Model	Test	Statistic	p-value	Decision ($\alpha = 0.05$)
LASSO	Shapiro–Wilk	$W = 0.8839$	0.0007	Reject H_0
	Jarque–Bera	$\chi^2 = 38.9170$	< 0.001	Reject H_0
AdaLASSO	Shapiro–Wilk	$W = 0.7915$	< 0.001	Reject H_0
	Jarque–Bera	$\chi^2 = 90.6170$	< 0.001	Reject H_0
WL AdaLASSO	Shapiro–Wilk	$W = 0.6686$	< 0.001	Reject H_0
	Jarque–Bera	$\chi^2 = 501.8700$	< 0.001	Reject H_0
FLAdaLASSO	Shapiro–Wilk	$W = 0.6307$	< 0.001	Reject H_0
	Jarque–Bera	$\chi^2 = 623.3300$	< 0.001	Reject H_0

The forecasting results are summarized in Table **??**. Among the four estimators, FLAdaLASSO achieves the lowest RMSE, followed closely by WLAdaLASSO, while LASSO and AdaLASSO exhibit noticeably larger forecast errors. In terms of model size, FLAdaLASSO and LASSO both select 19 variables, compared with 20 for WLAdaLASSO and 32 for AdaLASSO. Accordingly, the proposed approach combines low forecasting error with a parsimonious specification. The joint comparison of predictive accuracy and model complexity, illustrated in Figure 5, highlights a favorable trade-off between error reduction and variable selection. By retaining a compact set of informative lagged predictors while discarding redundant components, the resulting model remains interpretable and competitive in forecasting performance.

TABLE 15 Forecast evaluation based on RMSE and the number of selected variables (Vars) across countries.

	Thailand		China		USA	
Methodology	**RMSE**	**Vars**	**RMSE**	**Vars**	**RMSE**	**Vars**
LASSO	1.6181	19	0.5866	9	2.7067	19
AdaLASSO	4.1257	32	2.2929	26	3.1759	16
WLAdaLASSO	1.1559	20	0.8426	20	2.8554	8
FLAdaLASSO	1.0026	19	0.5211	11	2.6673	8

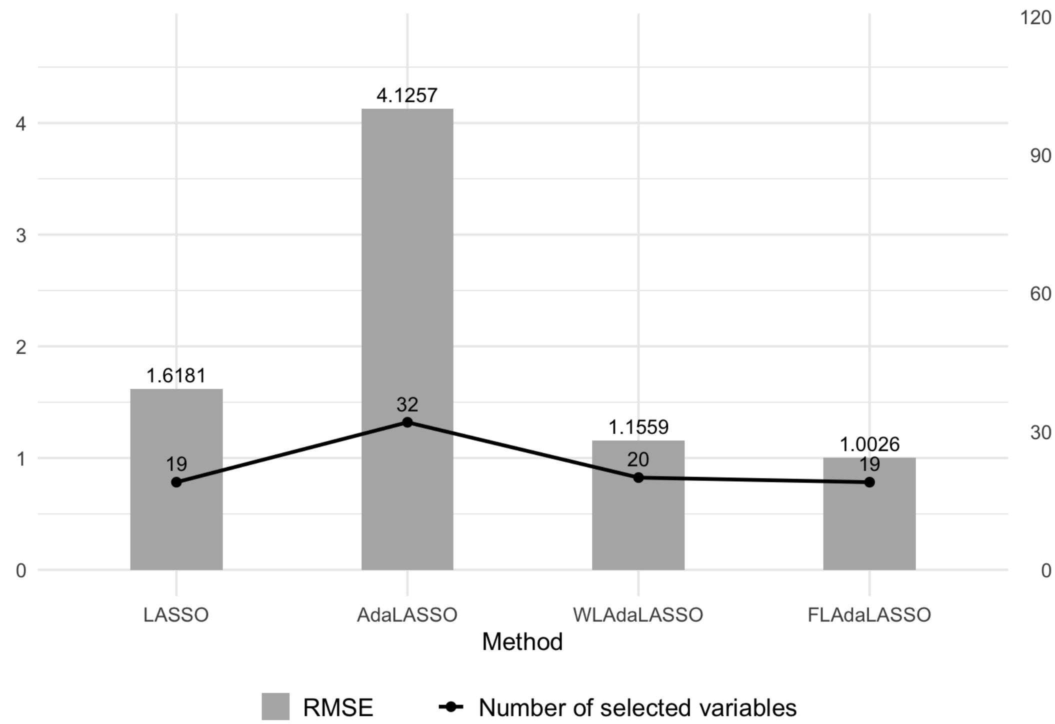

FIGURE 5 Plot of RMSE and number of selected variables for the Thailand inflation dataset.

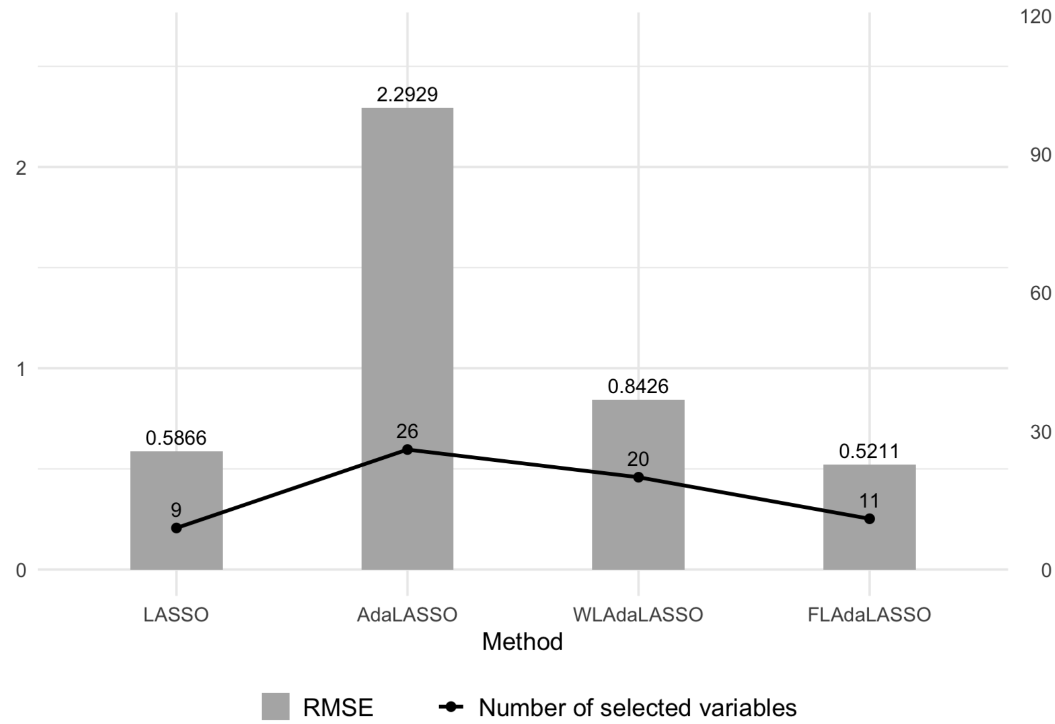

FIGURE 6 Plot of RMSE and number of selected variables for the China inflation dataset.

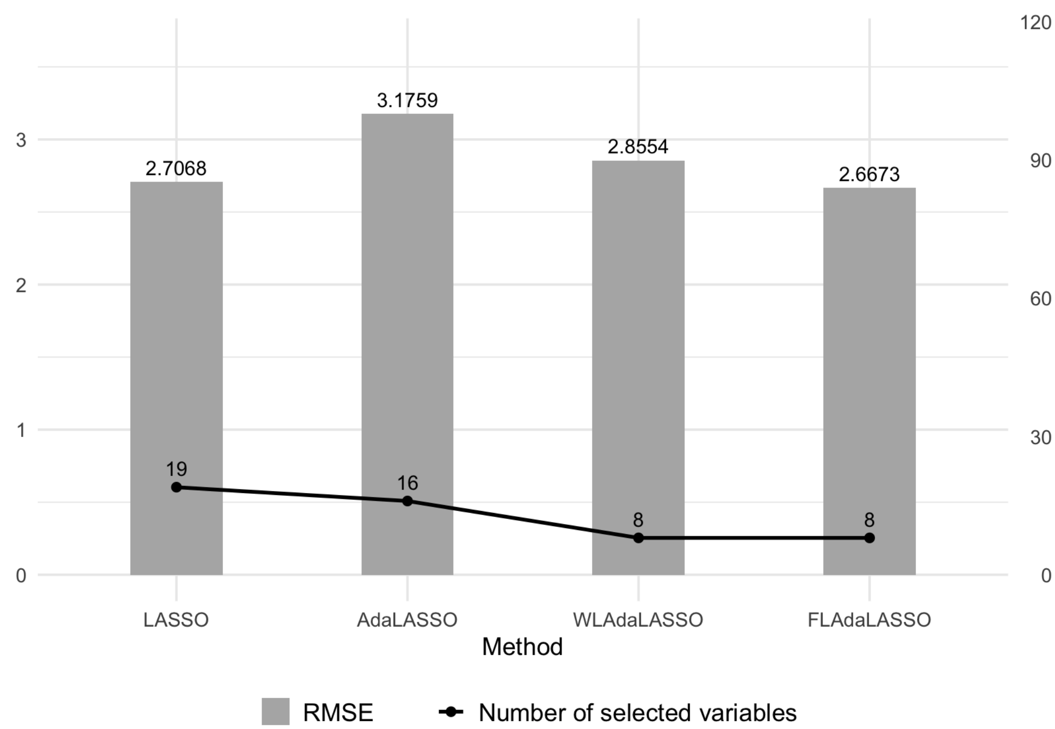

FIGURE 7 Plot of RMSE and number of selected variables for the USA inflation dataset.

Overall, the empirical evidence highlights meaningful differences among the competing estimators. In terms of forecasting accuracy, FLAdaLASSO attains the lowest RMSE, followed closely by WLAdaLASSO, while LASSO and AdaLASSO exhibit noticeably larger forecast errors. When predictive accuracy and model parsimony are considered jointly, FLAdaLASSO offers a particularly attractive performance, achieving near-minimal forecast error while selecting a compact set of predictors comparable in size to LASSO and smaller than that of WLAdaLASSO and AdaLASSO. With respect to residual behavior, LASSO, WLAdaLASSO, and FLAdaLASSO exhibit acceptable normality, whereas AdaLASSO performs poorly across the diagnostic measures considered. Taken together, these results suggest that FLAdaLASSO effectively balances predictive accuracy, model parsimony, and residual stability, making it an empirically attractive approach for high-dimensional macroeconomic forecasting settings where interpretability and robustness are important considerations.

DISCUSSION

The simulation experiments and the empirical application jointly provide insight into the role of lag-aware penalization in high-dimensional autoregressive settings. Across the simulation designs, the results demonstrate that incorporating structured lag-dependent weights improves the recovery of the true lag structure, particularly in small to moderate samples and in environments characterized by strong dependence or lag proliferation. In these settings, the proposed FLAdaLASSO consistently delivers selection accuracy that is comparable to, or slightly better than, existing lag-weighted approaches, while avoiding the instability observed for AdaLASSO when initial estimates are noisy. These findings align with the theoretical arguments of Konzen and Ziegelmann (2016), highlighting the importance of controlling overselection in persistent time-series models.

The simulation evidence further indicates that the benefits of factorial lag weighting extend beyond variable selection. In terms of parameter estimation, FLAdaLASSO most frequently attains the lowest estimation errors, with differences relative to WLAdaLASSO being small and occasionally reversed in specific configurations. Importantly, these differences diminish as the sample size increases, suggesting that the proposed approach does not sacrifice estimation accuracy relative to existing lag-weighted methods. In high-dimensional regimes where the number of candidate lagged covariates exceeds the available sample size, both lag-weighted procedures remain substantially more stable than LASSO and Adaptive LASSO, reinforcing the value of incorporating structural information about lag relevance.

The empirical application to Thai inflation data corroborates the simulation findings and illustrates their practical relevance. In the forecasting exercise, FLAdaLASSO achieves the lowest RMSE, followed closely by WLAdaLASSO, while LASSO and AdaLASSO produce noticeably larger forecast errors. When forecasting accuracy and model parsimony are considered jointly, the proposed approach exhibits a favorable trade-off: it delivers near-minimal forecast error while selecting a compact set of predictors comparable in size to LASSO and smaller than those selected by WLAdaLASSO and AdaLASSO. This result is consistent with the arguments of Park and Sakaori (2013), who emphasize that weighting schemes aligned with lag relevance can enhance both interpretability and predictive performance.

Residual diagnostics provide further support for the proposed method. The residuals obtained from LASSO, WLAdaLASSO, and FLAdaLASSO exhibit acceptable normality, whereas AdaLASSO performs poorly across the diagnostic measures considered. This pattern suggests that incorporating lag-structured adaptive penalties contributes to greater residual stability in the presence of temporal dependence, an aspect that is often overlooked when evaluation focuses exclusively on forecast accuracy.

Taken together, the evidence from both simulation and empirical analyses suggests that FLAdaLASSO offers an effective compromise between accurate variable selection, reliable parameter estimation, and competitive forecasting performance. Rather than uniformly dominating existing methods, the proposed estimator achieves a balanced performance across multiple evaluation criteria, making it particularly attractive for high-dimensional macroeconomic forecasting applications where interpretability, robustness, and stability are central considerations.

CONCLUSION

This study introduces the FLAdaLASSO estimator for autoregressive variable selection in high-dimensional time-series settings. Simulation results demonstrate that incorporating flexible lag-dependent weighting improves lag identification, reduces overselection, and stabilizes estimation relative to standard LASSO-type procedures. The empirical application to Thai inflation data further supports these findings, showing that FLAdaLASSO attains the lowest forecasting error, followed closely by WLAdaLASSO, while maintaining a compact model specification comparable in size to LASSO and smaller than those selected by WLAdaLASSO and Adaptive LASSO. These empirical results reinforce the simulation evidence, indicating that the benefits of lag-aware adaptive shrinkage extend across both controlled and real-world environments. Taken together, the findings suggest that FLAdaLASSO offers a balanced combination of parsimony, stability, and predictive performance, making it a computationally attractive and practically appealing alternative to existing penalization methods. Future research may explore extensions to multivariate or nonlinear dynamic models to further assess its applicability in broader forecasting contexts.

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DECLARATION

Conflict of interest We declare that there are no Conflict of interest in this article.

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