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An analytical heterogeneous agent macro model of concentration, markups, and falling labour shares*

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Abstract

This paper presents an analytical heterogeneous agent model (HAM) to explore micro-level concentration dynamics and their effects on macro-level distribution and growth. It contributes to recent work on macroeconomic agent-based models (ABMs) by comparing a small-scale ABM with bilateral firm-customer matching to a simplified HAM without direct agent interactions. The HAM replicates key ABM results while allowing for analytical solutions. In the model, larger firms benefit from lower unit costs due to economies of scale, gain larger market shares, and set higher markups. More profitable firms temporarily grow faster, leading to endogenous changes in the firm size distribution. The resulting right-skewed distributions create a divergence between the average profit share across firms and the aggregate profit share, which is driven by the largest firms. The decline in the labour share reduces aggregate consumption and slows growth. Despite its simplicity, the model captures key empirical patterns related to superstar firms and declining labour shares.

Keywords: labour share, profit share, income distribution, markups, concentration, macroeconomic agent-based model, heterogeneous agents

JEL codes: C60, D33, E12, E17, E25

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1 Introduction

Over the last two decades, macroeconomic agent-based modelling has developed into a well established research programme. There are now several benchmark macroeconomic agent-based models (MABMs) that have been applied to diverse topics such as technological change, financial instability, climate change, inequality, and more (see Dawid & Delli Gatti 2018, Delli Gatti et al. 2018, Di Guilmi 2017, for overviews). While the field appears to be flourishing, it is not free of challenges. Benchmark models are increasingly complex and are often regarded as 'black boxes' (Dawid et al. 2018): it is difficult for readers who are not directly familiar with a specific model to understand its results. Some papers do not provide source code and, even if they do, their complexity can make it impracticable to replicate and scrutinise the reported results.[1]

One way of dealing with these issues is to build smaller scale 'hybrid' MABMs, in which only a few sectors are disaggregated into heterogeneous agents, while the remaining sectors are modelled at the aggregate level (e.g., Assenza & Delli Gatti 2013, Botta et al. 2021, Di Guilmi & Carvalho 2017, Michell 2014, Fierro et al. 2023, Reissl 2020, Suresh & Setterfield 2015). While such hybrid MABMs are substantially less complex than models where all sectors are disaggregated, limited efforts have been undertaken to study these models analytically. This is not surprising given that the presence of nonlinearities and stochasticity renders the derivation of analytical solutions challenging even for small-scale hybrid MABMs.

While the recent turn towards hybrid MABMs is helpful for understanding the mechanisms that generate aggregate outcomes, there is scope for studying the workings of these models analytically. More specifically, constructing reduced-form versions of fully agent-based models can yield a more precise understanding of the key assumptions driving the results. Rigorous analysis of such 'analytical' models facilitates an understanding of how specific causal mechanisms generate aggregate outcomes. This analytical approach could complement (rather than replace) existing numerical techniques for isolating key mechanisms.

The present paper contributes to recent attempts at building more tractable hybrid MABMs by developing a small-scale model of concentration dynamics among firms and their effects on distribution and growth at the aggregate level. We compare two versions of the model:

[1] In the words of Dawid et al. (2018, p. 469), 'articles based on large agent-based models typically present the main intuitions, and the logic and assumptions of the model, without truly providing all the details that would be necessary to fully understand all aspects of the emerging dynamics, or to actually facilitate a full re-implementation of the model from scratch. Even if the source code of the model were available, it remains costly to run the simulation model properly and to reproduce the exact same setup as the analyses presented in the paper.'

an agent-based version with direct interaction between firms and customers, and a reduced-form version without direct interaction. We show that the reduced-form version, which we label 'heterogeneous agent model' (HAM), produces qualitatively similar results to the ABM, while admitting analytical solutions.

Our model characterises the interaction between pricing, investment, and concentration dynamics at the micro level, and distribution and growth at the macro level. Despite its simplicity, it reproduces key stylised facts documented in the empirical literature on superstar firms and rising markups – in particular the fact that part of the fall in the aggregate labour share can be attributed to a change in the cross-sectional distribution of profit and market shares (Autor et al. 2020, Barkai 2020, De Loecker et al. 2020). In the model, firms are heterogeneous in capital stocks and market shares. We assume that unit labour costs at the firm level decline as a firm's capital stock increases – a phenomenon we refer to as 'economies of scale'. This enables larger firms to offer lower prices, thereby capturing greater market shares. Firms with greater market shares, in turn, use their market power to set higher markups. However, customers are price sensitive, so that firms that price above the market average tend to lose market shares.

These mechanisms give rise to endogenous changes in the distributions of firm-level variables, which become right-skewed due to the emergence of highly productive big firms with high markups and low labour shares. The (unweighted) *average* profit share across firms diverges from the *aggregate* profit share, which is driven up by the biggest firms. The resulting fall in the aggregate labour share depresses consumption demand and growth.

In order to demonstrate that these dynamics can be captured in an analytically tractable model, we proceed in three steps. We first develop a core model that is common to both the agent-based and heterogeneous-agent versions and consists of behavioural specifications for aggregate consumption, firm-level pricing and investment, a set of aggregation rules, and laws of motion of relative firm sizes and market shares. Second, we develop an agent-based closure of the core model in which market shares are determined through bilateral matching on the goods market, where customers choose firms based on their price competitiveness. We estimate the parameters of the ABM to match several empirical moments from Compustat data for US listed firms. In the third step, we study an alternative heterogeneous-agent closure of the core model, in which market shares are allocated by a reduced-form stochastic equation. For the HAM, we derive a number of results analytically.

We find that the HAM produces qualitatively similar results to the ABM: both models can generate concentration dynamics under similar conditions. The intensity of economies of scale is a key determinant of the degree of concentration. Once concentration dynamics are in motion, both models generate a rise in the markup and profit share of the dominant firm(s), leading to macroeconomic stagnation. However, the ABM provides a richer representation of different degrees of concentration and of the influence of factors beyond economies of scale, such as the price sensitivity of consumers on the distribution of firm sizes and market shares. We conclude that there are benefits to deriving reduced-form versions of ABMs as these allow for a more precise understanding of the key causal mechanisms of a model. At the same time, ABMs provide more flexibility, allowing for a richer modelling of the mechanisms of interest.

The paper's main contribution is to build a bridge between fully disaggregated MABMs and traditional macroeconomic models with either representative agents or aggregate behavioural equations. We provide a small-scale macroeconomic model that allows us to trace the macro effects of the micro behaviour of heterogeneous agents. This is accomplished through a mix of analytical and numerical techniques. In this way, our analysis contributes to the recent literature on hybrid MABMs by providing a rigorous understanding of how some of the results observed in structurally similar models may arise (e.g. Assenza & Delli Gatti 2013, Botta et al. 2021, Di Guilmi & Carvalho 2017, Michell 2014, Fierro et al. 2023, Reissl 2020, Suresh & Setterfield 2015). A secondary contribution is to provide a theoretical framework that explains the link between rising concentration, profit shares, and macroeconomic stagnation from a behavioural Keynesian perspective. Our models reproduce, in a stylised way, an empirical pattern whereby the aggregate markup and profit share increase due to a reallocation of market shares towards large firms with high markups and high profit shares, as documented in Autor et al. (2020), Barkai (2020) and De Loecker et al. (2020). Recent supply-driven Schumpeterian endogenous growth models have linked these concentration dynamics to lower physical investment and productivity growth due to factors such as a decline in knowledge diffusion (Akcigit & Ates 2021, 2023), increased use of intangible inputs (De Ridder 2024), and barriers to entry (Covarrubias et al. 2020). By contrast, our demand-driven model highlights endogenous labour-saving technical change as a driver of market concentration. Economies of scale reallocate market shares to large and profitable firms, which take advantage of their market position to further raise their markups. The concentration-driven rise in the aggregate profit share then depresses consumption, leading to slower investment and productivity growth.[2]

Related literature. Our paper is related to several strands of literature. First, it engages with the theoretical literature using hybrid MABMs (Assenza & Delli Gatti 2013, Botta et al.

[2]Steindl (1952) provides an early discussion of these trends in the US.

2021, Di Guilmi & Carvalho 2017, Dosi et al. 2010, Fierro et al. 2023, Michell 2014, Pedrosa & Lang 2021, Reissl 2020, Seppecher et al. 2018, Suresh & Setterfield 2015, Terranova & Turco 2022). Within that literature, Terranova & Turco (2022) is closest to our paper in terms of narrative. Their MABM generates a rise in market concentration, leading to a rise in the aggregate profit share and a slowdown in growth. As in our model, the rise in concentration is driven by productivity differentials across firms; however, changes in productivity stem from R&D investment and knowledge gaps rather than economies of scale. The model is also significantly more complicated – it features heterogeneous households as well as a division between firms producing capital and consumption goods – so that its properties can only be studied via simulations.

Botta et al. (2021), Di Guilmi & Carvalho (2017), Michell (2014), Fierro et al. (2023), Reissl (2020) and Suresh & Setterfield (2015) present models in which only one sector is disaggregated and there is no direct interaction between agents. While these models are structurally similar to ours, most are larger and all are studied entirely through simulations. Michell (2014), develops a model with a heterogeneous firm sector to study the effects of concentration on markups, distribution and aggregate economic activity. Suresh & Setterfield (2015) develop a simple model with heterogeneous firms and assess how the amplitude of aggregate fluctuations depends on the sensitivity of firm-level investment to the aggregate rate of capacity utilisation. Reissl (2020) conducts a sensitivity analysis of a hybrid ABM, where the disaggregated firm sector is replaced by a representative firm, and finds that the aggregate model produces endogenous cycles even in the absence of heterogeneous agents. While these exercises do cast some light on the models' workings, they are not conducted analytically, thereby limiting their ability to pin down the exact conditions under which certain aggregate outcomes occur.

The ABMs closest to ours in terms of method are Assenza & Delli Gatti (2013) and Di Guilmi & Carvalho (2017). Assenza & Delli Gatti (2013) develop a small-scale model with an aggregate household sector and heterogeneous firms differing in net worth, which affects funding costs for investment. They derive closed-form aggregate equations via second-order Taylor approximations around aggregate variables, allowing cross-sectional variances to influence macroeconomic outcomes. Unlike our paper, however, they do not derive analytical solutions for the model's micro-dynamics. Di Guilmi & Carvalho (2017) propose a stochastic aggregation approach in which heterogeneous agents are grouped into a small number of discrete states. Applying a master-equation framework to a model with firms in either a financially robust or fragile state, they derive a partial differential equation describing the distribution of firms across states. Combined with analytical solutions for aggregate variables, this yields an approximate analytical solution for the model. While this approach provides a rigorous framework for small-scale MABMs, it relies on reducing heterogeneity through a grouping of agents.

A second related strand of literature empirically investigates aggregate trends in markups, profit shares, and investment and their micro-level sources.[3] Autor et al. (2020), Barkai (2020) and Grullon et al. (2019) document a rise in market concentration correlated with an increase in profit shares. Autor et al. (2020) show that the fall in the aggregate US labour share is largely driven by the reallocation of market shares between firms, rather than a fall in labour shares for the average firm. De Loecker et al. (2020) use data for listed firms and document a rise in the aggregate markup between 1980 and 2016. They show that this increase is largely driven by a reallocation of economic activity towards large firms with high and rising markups, whereas median markups have stayed relatively constant. Taken together, these papers identify changes in the distribution of market shares towards high-profit share firms to be a key driver of aggregate patterns in markups and income distribution. While the present paper does not add to this empirical literature, it offers a simple model that reproduces some of the stylised facts – in particular a rise in aggregate markups and profit shares driven by a change in the distribution of market shares. While Autor et al. (2020)'s model exogenously imposes a heavy-tailed productivity distribution, we provide a mechanism whereby skewed distributions emerge endogenously due to economies of scale.

Our paper is also linked to macroeconomic models that provide explanations for rising concentration and profit shares along with falling (productivity) growth. With a stylized model, Covarrubias et al. (2020) identify increased barriers to entry as a source of 'bad concentration' that results in lower investment. Recent Schumpeterian endogenous growth models theorise such dynamics in general equilibrium frameworks with optimising microfoundations. Akcigit & Ates (2021, 2023) highlight a decline in knowledge diffusion from lead to laggard firms, whereas De Ridder (2024) considers increased use of intangible inputs that reduce marginal cost. While our model exhibits a more stylized supply side with a generic form of economies of scale associated with the use of physical capital, it offers a Keynesian perspective on the concentration-cum-stagnation phenomenon using behavioural microfoundations. Output in our model is demand determined. Firms make heuristic investment decisions based on the rate of capacity utilisation and the profit rate, independently of saving decisions. We thereby link concentration dynamics to the Keynesian literature on macroeconomic growth regimes (Bhaduri & Marglin 1990, Blecker 2016, Hein 2014, Napoletano et al. 2012). We show that

[3] Grossman & Oberfield (2022) provide a review of the literature on declining labour shares.

concentration-cum-stagnation most likely occurs in wage-led growth regimes with sufficiently strong economies of scale, combined with a moderate responsiveness of investment to profitability. This configuration temporarily allows profitable firms to strengthen their position via faster growth, leading to increased concentration. In a wage-led macro-economy, rising profit shares will eventually lead to reduced productivity growth via depressed aggregate demand, adding a demand-side channel not yet explored in Schumpeterian general-equilibrium growth models.

Caveats. Transparency is an important aspect of our approach: we build a simple model that makes all major assumptions explicit and allows for a precise derivation of key properties. This approach imposes limitations on the features that can be incorporated. First, following the hybrid approach, only the firm sector is meaningfully disaggregated. The only source of heterogeneity is micro-level shocks to the allocation of demand to firms. Second, our baseline model abstracts from entry and exit dynamics, but we relax this restriction in a robustness check. Third, for the sake of tractability, we only incorporate one type of good, abstract from intermediate inputs and overhead costs, and treat nominal wages as homogeneous across firms. Finally, to permit aggregation, some of the micro-level equations are cast in linear form, precluding nonlinear effects. We intend to investigate extensions that address some of these limitations in future work.

Outline. Section 2 presents the core model comprising the equations common to both versions of the model. Two alternative closures are then considered: in section 3, an agent-based version with bilateral matching between firms and customers on the goods market, and in section 4, an analytical heterogeneous agent version with a reduced-form market share allocation equation. Section 5 discusses the relationship between concentration dynamics and the macroeconomic growth regime. Sections 6 and 7 analyse the models further and present some robustness tests. Section 8 concludes. Proofs of propositions and auxiliary analytical results are collected in Online Appendix A. Further details on the main simulation results can be found in Online Appendix B, and on extensions and robustness tests in Online Appendices C and D.

2 The core model

We group the core model into a macro bloc, a micro bloc, aggregation, and dynamics.[4]

[4]See the Appendix for a concise overview of the notation and all variable definitions.

Macro bloc. A homogeneous good can be used for both investment and consumption. Aggregate demand consists of consumption and investment and there is instantaneous Keynesian quantity adjustment on the supply side. Aggregate goods market equilibrium is given by:

$$Y_t = C_t + I_t,$$

where Y_t, C_t, and I_t are aggregate output, aggregate consumption, and aggregate investment, respectively; all are measured in physical units.

We use a Keynesian aggregate consumption function with a constant marginal propensity to consume out of income. For simplicity, we assume that all profits are retained in the corporate sector, so that there is only consumption out of wage income:[5]

$$C_t = c_y Y_t(1 - \pi_t),$$

where c_y is the marginal propensity to consume out of income and $\pi_t = \frac{\Pi_t}{Y_t}$ is the aggregate profit share, such that $(1 - \pi_t)$ is the aggregate wage share.

In the following, we normalise all flows by the lagged capital stock K_{t-1}, giving:

$$u_t = \nu^{-1}(c_t + g_t), \tag{1}$$

and

$$c_t = c_y \nu u_t(1 - \pi_t), \tag{2}$$

where $u_t = \frac{Y_t}{Y_t^P}$ is the aggregate rate of capacity utilisation and $\nu = \frac{Y_t^P}{K_{t-1}}$ is the potential output-to-capital ratio we assume to be constant and homogeneous.

Micro bloc. N firms, indexed by $i = 1, .., N$, are heterogeneous with respect to the size of their capital stocks K_{it}. The law of motion of the capital stock is given by $K_{it} = (1+g_{it})K_{it-1}$, where g_{it} is the investment rate. Here we abstract from depreciation for simplicity, but relax this restriction in section 7. Two key variables capture the relative positions of firms: relative firm size $s_{it} = \frac{K_{it}}{K_t} \in (0, 1)$ and market share $q_{it} = \frac{Y_{it}}{Y_t} \in [0, 1]$. These variables evolve

[5] We initially abstract from consumption out of wealth and financial income streams; we relax this assumption in section 7.

endogenously, playing a key role in the transmission mechanism between the micro- and the macro dynamics of the model discussed below. The firm-level rate of capacity utilisation is defined as $u_{it} = \frac{Y_{it}}{Y_{it}^P}$ and can be written as $u_{it} = \frac{q_{it}u_t}{s_{it-1}}$, illustrating the role of market shares in allocating aggregate demand (represented by u_t) over the firms. Two further important firm-level variables are the profit share, $\pi_{it} = \frac{\Pi_{it}}{Y_{it}}$, and the profit rate $r_{it} = \frac{\Pi_{it}}{K_{it-1}} = \pi_{it}\nu u_{it}$.

With respect to the investment rate g_{it}, we adopt a common specification in the Keynesian MABM literature (Caiani et al. 2016, Di Guilmi & Carvalho 2017, Michell 2014, Suresh & Setterfield 2015),[6] where the investment rate is linearly related to an autonomous component γ_0, the firm-level profit rate r_{it}, and the rate of capacity utilisation u_{it}:

$$g_{it} = \gamma_0 + \gamma_r r_{it} + \gamma_u u_{it}. \tag{3}$$

Firms are assumed to operate in imperfectly competitive markets and charge a markup τ_{it} on nominal unit labour costs:

$$P_{it} = (1 + \tau_{it})\frac{w_{it}}{\delta_{it}}, \tag{4}$$

where w_{it} is the nominal wage rate and δ_{it} is firm-level labour productivity.

With this pricing equation, the firm-level profit share is given by:

$$\pi_{it} = \frac{\tau_{it}}{1 + \tau_{it}}. \tag{5}$$

The determination of firm-level labour productivity is a key element of the model. We assume that labour productivity is increasing in the level of the capital stock:

$$\delta_{it} = K_{it-1}^{\beta}. \tag{6}$$

Equation (6) could be regarded as a simplified reduced-form specification for conventional learning-by-doing effects as theorised in Arrow (1962) and Metcalfe (1994), but also 'scale advantages related to the growth of intangible capital and advances in information technol-

[6] Botta et al. (2021) use a similar version with the profit share instead of the profit rate. See Hein (2014) for an extensive discussion of this class of investment functions in aggregate models. The main advantage of using the profit rate in our context is that it facilitates aggregation as all variables in the investment function are normalised by the capital stock. Similarly, the use of a simple linear specification allows for exact aggregation.

ogy' (Autor et al. 2020, p. 649).[7] As a shorthand, we will refer to the mechanism represented by (6) as 'economies of scale' and to β as the 'economies of scale' parameter (representing the intensity of economies of scale).

Following a common approach in the literature (Autor et al. 2020, Dosi et al. 2010, Pedrosa & Lang 2021, Terranova & Turco 2022), we assume that wages are homogeneous across firms: $w_{it} = w_t$. Dynamically, (6) implies that the growth rate of productivity is positively related to the investment rate, $1 + \hat{\delta}_{it} = (1 + g_{it-1})^{\beta}$. To ensure that the wage share is stationary in long-run equilibrium, we assume that the nominal wage grows in line with the aggregate growth rate, $1 + \hat{w}_t = (1 + g_{t-1})^{\beta}$. Behaviourally, this could stem from a collective wage-setting rule that matches nominal wage growth with aggregate productivity growth so as to achieve non-inflationary real wage growth. These simplifying assumptions capture, in a reduced-form way, incomplete productivity-wage pass-through at the firm-level.[8] In a situation of disequilibrium, individual firm-level productivity growth rates deviate from the aggregate rate: $1 + \hat{\delta}_{it} = (1 + g_{it-1})^{\beta} \neq (1 + g_{t-1})^{\beta}$. This gives fast-growing firms a competitive advantage in price setting because their productivity temporarily grows faster than their nominal wage cost, i.e. $1 + \hat{\delta}_{it} > 1 + \hat{w}_t = (1 + g_{t-1})^{\beta}$. This mechanism is a key driver of concentration dynamics in the model.

The last key component of the micro bloc is the determination of markups τ_{it}. We follow a long-standing theoretical and empirical literature which argues that firms with higher market shares exhibit market power, allowing them to charge higher markups (see the discussion in Autor et al. (2020) and De Loecker et al. (2020), as well as Dosi et al. (2010), Pedrosa & Lang (2021), Reissl (2020) and Terranova & Turco (2022) for MABMs in which markups are positively related to market shares).[9] We formalise this assumption as follows:

$$\tau_{it} = \tau_0 + \tau_q N q_{it-1}. \tag{7}$$

As a firm's market share increases, its markup rises, with the sensitivity to market shares

[7] Lashkari et al. (2024) present evidence for French firms that reductions in the cost of ICT have disproportionately benefited large firms. More generally, Bartelsman et al. (2013) and Pagano & Schivardi (2003) review and present empirical evidence for a positive relationship between firm size and productivity. Chirinko & Fazzari (1994) find a positive relationship between market power and returns to scale with concentrated industries characterised by increasing returns.

[8] See Jarosch et al. (2024), Gouin-Bonenfant (2022) for theoretical models in which bigger firms have greater labour market power over workers. Chan et al. (2023) provide empirical evidence that bigger and more productive firms have lower productivity-wage pass-through.

[9] Seppecher et al. (2018) study an ABM that follows an alternative approach in which markups are adjusted entirely randomly.

specified by τ_q. With τ_0 and τ_q homogeneous across firms, (7) implies a constant (unweighted) average markup of $\bar{\tau} = \tau_0 + \tau_q$.[10] Equation (7) represents an indirect form of interaction between firms who take their *relative* market position into account when setting markups.

Aggregation. The aggregate capital stock and aggregate output are obtained by summing over the corresponding firm-level variables: $\sum_{i=1}^{N} K_{it} = K_t$ and $\sum_{i=1}^{N} Y_{it} = Y_t$. Aggregate ratios are defined as the appropriate weighted average over the firm-level counterparts, giving $\pi_t = \sum_{i=1}^{N} q_{it}\pi_{it}$, $u_t = \sum_{i=1}^{N} s_{it-1}u_{it}$, and $r_t = \nu \sum_{i=1}^{N} s_{it-1}\pi_{it}u_{it} = \pi_t \nu u_t$ for the profit share, the rate of capacity utilisation, and the profit rate, respectively.

A first important result concerns the aggregate investment rate, which is given by:

$$g_t = \sum_{i=1}^{N} s_{it-1} g_{it} = \gamma_0 + \gamma_r r_t + \gamma_u u_t. \tag{8}$$

The use of a linear investment function permits exact aggregation, so that the macro-level investment function is the exact counterpart to the micro-level function. This means that the analytical equilibrium solutions of the aggregate model given by (1), (2), and (8) that have been extensively studied in the literature on aggregate Keynesian models (see Hein 2014) carry over to our disaggregated model (see Online Appendix A.2). As established in this literature, the aggregate model permits categorisation of macroeconomic regimes based on the effect of a change in the aggregate profit share on aggregate demand and growth. In the following, we will focus on the 'wage-led' case where an increase in the profit share reduces both aggregate demand and growth due a strong negative effect on consumption demand, i.e. $\text{sgn}\left(\frac{\partial u_t^*}{\partial \pi_t}\right) = \text{sgn}\left(\frac{\partial g_t^*}{\partial \pi_t}\right) = -1$. We discuss these macroeconomic regimes in more detail in section 5.

A second main result concerns the aggregate markup, which we define as the weighted average over firm-level markups, where the weights are given by the lagged market shares: $\tau_t = \sum_{i=1}^{N} q_{it-1}\tau_{it}$.[11] Using this definition along with (7) we can state the following proposition:[12]

[10]Keeping τ_0 constant means that we abstract from secular trends in markups that could have contributed to the rise in aggregate profit shares along with changes in market shares. We acknowledge the rich literature, part of which is reviewed in Grossman & Oberfield (2022), proposing various mechanisms such as financialisation (Kohler et al. 2019), changes in labour market institutions and falling union density (Guschanski & Onaran 2021, Stockhammer 2015), and technical change. Our aim is to theoretically analyse the market-share channel, without diminishing the empirical relevance of other channels.

[11]De Loecker et al. (2020) use a similar definition of the aggregate markup with the contemporaneous market shares as weights.

[12]Note that the variance of market shares, $\text{Var}(q_t) = \frac{1}{N}\left(\sum_{i=1}^{N} q_{it}^2 - \frac{1}{N}\right)$, is directly related to the (nor-

Proposition 1 (Aggregate markup.) *The aggregate markup is increasing in market concentration as measured by the variance of market shares,* $\mathrm{Var}(q_{t-1})$*:*

$$\tau_t = \sum_{i=1}^{N} q_{it-1}\tau_{it} = \bar{\tau} + \tau_q N^2 \mathrm{Var}(q_{t-1}). \tag{9}$$

While it is not possible to obtain an exact expression for the aggregate profit share as a function of the aggregate markup due to the nonlinearity in (5), a first-order approximation around the average markup $\bar{\tau}$ gives:

$$\pi_t \approx \pi^{*2} + \frac{\tau_t}{(1+\bar{\tau})^2}, \tag{10}$$

where $\pi^* = \frac{\bar{\tau}}{1+\bar{\tau}}$ (see Online Appendix A.4). Thus, the aggregate profit share will generally be increasing in the aggregate markup and will thus also increase when market concentration rises.

Combining the expressions for the aggregate markup (9) and the aggregate profit share (10) with the equilibrium solution for the aggregate rate of capacity utilisation (A1) implies that aggregate economic activity as measured by the rate of capacity utilisation is a function of the variance of market shares:

Proposition 2 (Aggregate rate of capacity utilisation.) *The aggregate rate of capacity utilisation is a function of market concentration as measured by the variance of market shares,* $\mathrm{Var}(q_{t-1})$*:*

$$u_t^* = \Phi(\mathrm{Var}(q_{t-1})) \approx \frac{\gamma_0}{\nu\{1 - c_y + \left[\pi^* + \frac{\tau_q N^2 \mathrm{Var}(q_{t-1})}{(1+\bar{\tau})^2}\right](c_y - \gamma_r)\} - \gamma_u}.$$

Proposition 2 establishes the dependence of a macro aggregate, the rate of capacity utilisation, on the cross-sectional distribution of a micro variable, the market share. This general relationship holds across both versions of the model introduced below, independently of the specific mechanism by which market shares are determined.

Dynamics. From the law of motion of the capital stock follows a first-order difference

malised) Herfindahl-Hirschman Index, $\mathrm{HHI} = \frac{\sum_{i=1}^{N} q_i^2 - \frac{1}{N}}{1-\frac{1}{N}}$, which is a common measure of market concentration.

equation for the dynamics of relative firm size:

$$s_{it} = s_{it-1} \left(\frac{1 + g_{it}}{1 + g_t} \right). \tag{11}$$

The final component of the model is a closure containing assumptions about the determination of market shares. Before examining an agent-based closure and a heterogeneous agent closure, respectively, we first state a generic version that encompasses both specifications. Following a common assumption in the MABM literature (Dosi et al. 2010, Pedrosa & Lang 2021, Reissl 2020), we assume that market shares are a function of price competitiveness:

$$q_{it} = f\left(\frac{P_{it}}{\bar{P}}, \epsilon_{it} \right), \quad f_{\frac{P_{it}}{\bar{P}}} < 0, \tag{12}$$

where $\frac{P_{it}}{\bar{P}}$ is the firm's price relative to the average price $\bar{P}$ and ϵ_{it} is a random variable. In Online Appendix A.5, we derive the following linear approximation for relative prices:

$$\begin{aligned} \frac{P_{it}}{\bar{P}_t} &\approx \beta + \frac{1 + \tau_0 + \tau_q N q_{it-1}}{1 + \bar{\tau}} - \beta N s_{it-1} \\ &= \Psi(q_{it-1}, s_{it-1}), \quad \Psi_{q_{it-1}} > 0, \Psi_{s_{it-1}} < 0, \end{aligned} \tag{13}$$

which are thus positively related to q_{it-1}, because higher market shares induce higher markups, and negatively related to s_{it-1} due to economies of scale.

Combining (12) and (13) gives $q_{it} = \Psi(q_{it-1}, s_{it-1}, \epsilon_{it})$, yielding a two-dimensional dynamic system in s_{it} and q_{it}, whose behaviour will depend on the exact specification for market shares in (12).

3 The agent-based version

3.1 Determination of market shares through bilateral matching

Our agent-based specification for the dynamics of market shares takes the form of a bilateral matching algorithm that links buyers and firms on the market for goods. We keep this agent-based component as simple as possible to facilitate comparison with the reduced-form HAM and to isolate the mechanism of interest: the emergence of concentration among firms. We model household-firm matching (purchases of consumer goods) and firm-firm matching (purchases of capital goods) identically. In line with the assumption that wages are uniform, we distribute aggregate consumption (2) evenly over $j = 1, ..., N^H$ households, so that $c_{jt} = \frac{c_t}{N^H}$. Given the investment demand for each firm determined in (3), we have a total of $N^H + N$ customers, each with a specific level of demand for the single good.

In the initial period, each firm, as producer, is randomly assigned an equal number of customers. An entirely elastic supply side ensures that every customer – whether firm or household – has all of their demand met by the firm they are matched with. Re-matching then occurs in each subsequent period as follows. First, firms set prices according to (4). Based on those prices, an exogenously fixed proportion of buyers α^p searches for cheaper sellers. Each of these buyers compares the price offered by the firm they are currently matched with to the prices available among ρ percent of the remaining $N - 1$ firms, and switches supplier if a lower price is available. Another exogenous share α^ϵ of buyers switches randomly to a different firm, and the remaining population share $1 - \alpha^p - \alpha^\epsilon$ sticks with their current supplier.

After re-matching has concluded, the market share for each firm i can be computed as follows:

$$q_{it} = \frac{\sum_{j=1}^{N^H} c_{ijt} + \sum_{k=1}^{N} s_{kt-1} g_{ikt}}{\nu u_t}, \tag{12 ABM}$$

where $k = 1, ..., N$ are the firms as buyers of investment goods from firm i.[13]

3.2 Simulation of the agent-based model

We estimate 7 of the core model's parameters with the Method of Simulated Moments (MSM) using data for US listed firms from Compustat as well as the aggregate rate of capacity utilisation (see Table 1). The moments are reported in Table A1 in the Appendix and the estimation procedure is described in Online Appendix B. Our estimation is meant to provide a stylised empirical anchor for the model. It does not aim to accurately match a full set of micro and macro moments, nor provide a quantitative forecast.

[13]Total demand for each firm i is given by $Y_{it} = K_{t-1} \sum_{j=1}^{N^H} c_{ijt} + \sum_{k=1}^{N} K_{kt-1} g_{ikt}$, where we permit firms to buy investment goods from themselves. Using the definition of market shares $q_{it} = \frac{Y_{it}}{Y_t}$ and the identity $Y_t = K_{t-1} \nu u_t$ yields (12 ABM).

Table 1: Parameterisations

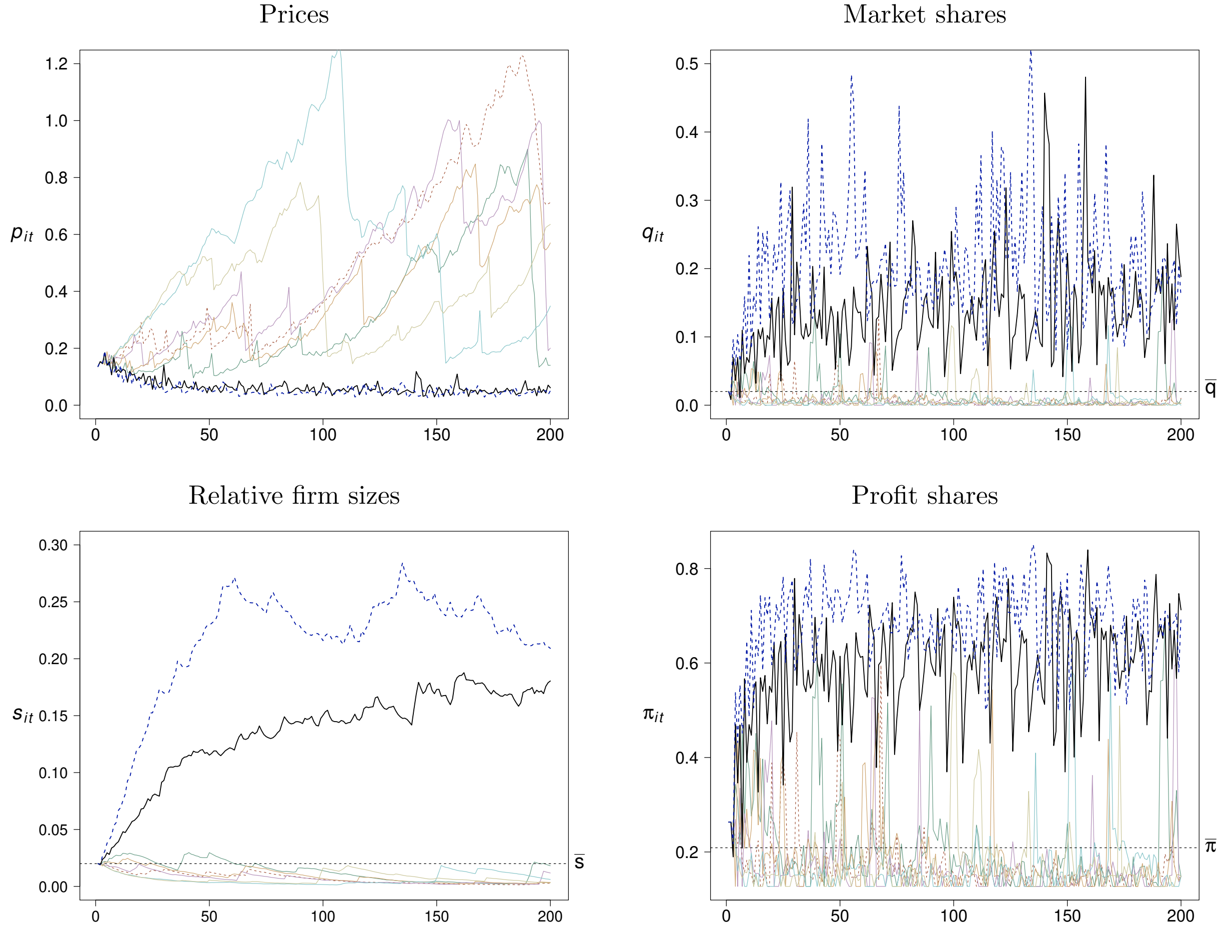
Parameter	Description	ABM	HAM	Source
N	Number of firms	50	50	Fixed
N^H	Number of households	150	–	Fixed
q_{i0}	Initial market shares	$1/N$	$1/N$	Fixed
s_{i0}	Initial relative firm size	$1/N$	$1/N$	Fixed
w_0	Initial nominal wage	0.1	0.1	Fixed
c_y	Marginal propensity to consume	0.85	0.85	Fixed
γ_0	Autonomous investment	0.046	0.046	Estimated
γ_r	Sensitivity of investment to profit rate	0.179	0.179	Estimated
γ_u	Sensitivity of investment to utilisation rate	0.050	0.050	Estimated
τ_0	Autonomous component of markup	0.146	0.146	Estimated
τ_q	Sensitivity of markup to market share	0.212	0.212	Estimated
$\beta(=\zeta_2)$	Increasing returns parameter	0.848	1.1	Estimated/Fixed
ν	Potential output to capital ratio	0.235	0.235	Estimated
α^p	Share of buyers comparing prices	0.4	–	Fixed
α^ϵ	Share of buyers switching randomly	0.15	–	Fixed
ρ	Share of firms sampled for price comparison	0.2	–	Fixed
$\zeta_1 = \frac{\tau_q}{1+\bar{\tau}}$	Sensitivity of market share to its lag	–	0.156	Estimated

Figure 1 displays the simulated dynamics of prices, market shares, relative firm sizes, and profit shares of the top eight firms (by size in the final period) over the first 200 periods of a single run of the model.[14] Starting from an initialisation where all firms are homogeneous, firms quickly become heterogeneous due to the random switching of customers. There is a rapid tendency towards concentration; two firms (represented by the black and blue lines) quickly undercut the prices of other firms, allowing them to gain market share, become more profitable, and grow faster. This feedback loop allows the two top firms to accumulate larger stocks of capital than the other firms, raising their productivity and thus enabling them to reduce their prices despite rising markups, thereby further increasing their market shares. All other firms follow the opposite trajectory, losing out in terms of price competitiveness, market shares, size, and profitability. After about 90 periods, concentration dynamics stabilise with the two top firms together taking about 35 percent of the market, leaving the remainder to the other firms, whose average individual market shares are well below the average market share.[15]

[14] Further firm-level variables are reported in Online Appendix B.2. Note that simulated firm-level rates of utilisation may exceed unity as no empirical moments were used to achieve more realistic values. Instead, we match the mean of the aggregate rate of capacity utilisation. Achieving realistic equilibrium values at both the micro and macro levels is difficult with our stylised model, where the macro variables are dominated by a small number of firms.

[15] Simulations over a longer horizon of 3,000 periods generate more complex episodes where up to four

Figure 1: Dynamics of top 8 firms, ABM



Notes: Data from a single run of the ABM with parameterisation reported in Table 1. Top 8 firms by size of capital stock in final period ($T = 3000$). The dashed horizontal lines represent the average market share, relative firm size, and average profit share, respectively.

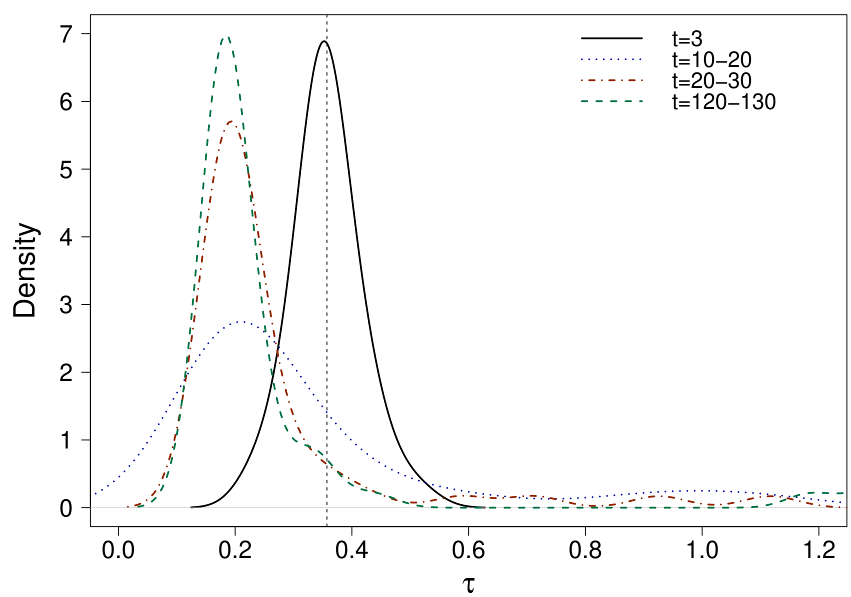
What prevents the biggest firm from becoming a full monopolist? While the top firms temporarily grow faster than the rest because higher profitability stimulates investment spending, all firms eventually grow at the same average rate. To understand why this happens, recall that investment is driven by both profitability and the rate of capacity utilisation. As the dominant firms increase their capital stocks, their rates of capacity utilisation decline (see Figure B1 in Online Appendix B.2). For the firms that lose out in terms of competitiveness, the opposite occurs: their capital stocks initially grow more slowly, but they still get randomly matched with customers with a positive probability, driving up their average rates

firms dominate the market as well as episodes where a dominant firm's position is replaced by a rising competitor; see Online Appendix B.2.

of capacity utilisation. The rise in their utilisation rates induces them to catch up in terms of investment growth rates, while the catch up in terms of size and productivity is only partial. As a result, firm-level growth rates tend to equalise in the stochastic equilibrium of the model, with dominant firms exhibiting high average profit shares and low utilisation rates and the opposite for non-dominant firms.

Figure 2 illustrates how the cross-sectional distribution of markups changes during the adjustment towards the stochastic equilibrium. At $t = 3$, right after stochastic dynamics have started to alter the initial uniform distributions, markups are distributed approximately symmetrically around the average $\bar{\tau}$. This changes quickly: between periods 10 and 20 the (averaged) distribution is already strongly right skewed. This process of concentration continues, with the centre of the distribution shifting to the left of the average markup and the right tail becoming thicker, representing the much higher markups of the dominant firms.

Figure 2: Distribution of firm-level markups (τ_{it}), ABM

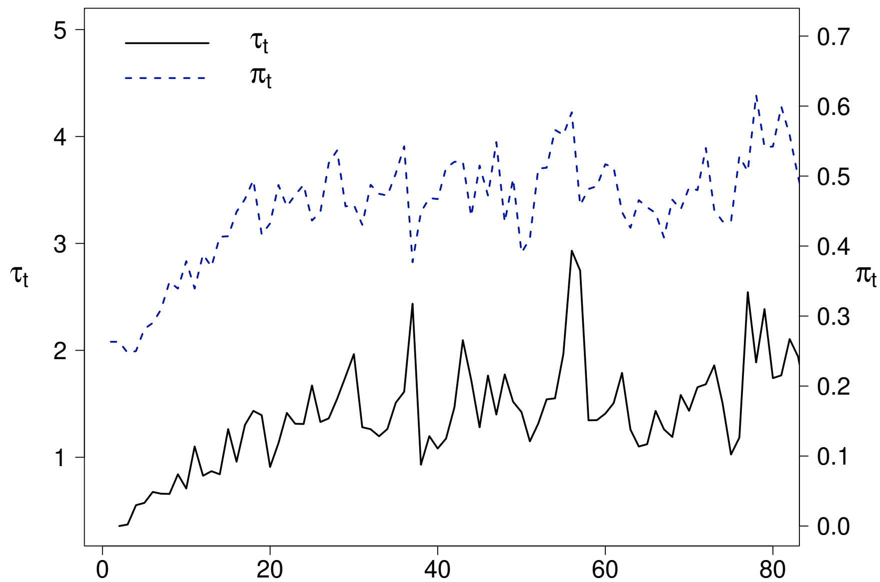


Notes: Kernel densities of markups from a single run of the ABM with parameterisation reported in Table 1. Gaussian kernel with bandwidth adjustment factor of 2. The labels $t = 10 - 20$ and so forth refer to the period over which a firm-level average has been taken. The vertical dashed line represents the (unweighted) average markup $\bar{\tau} = \tau_0 + \tau_q$.

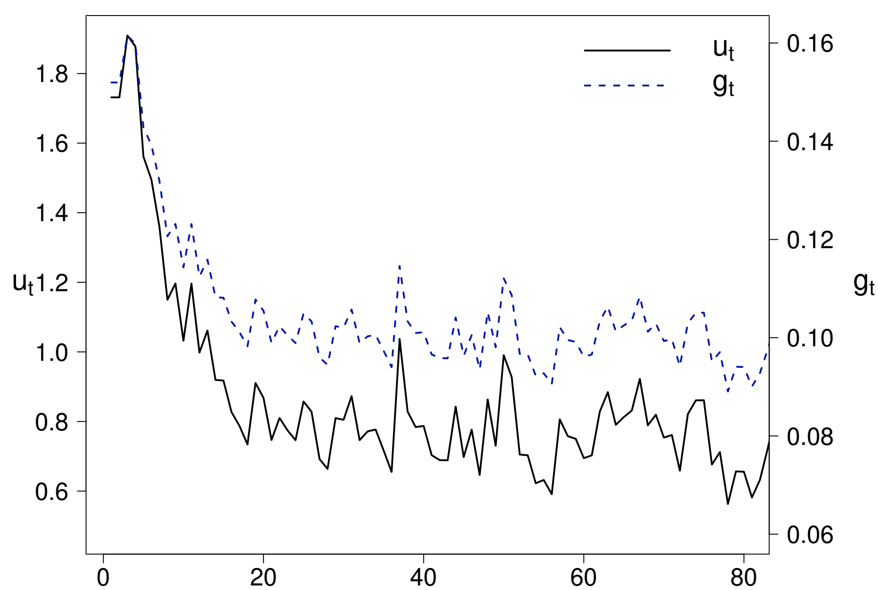
The macro effects of these micro-level dynamics are shown in Figure 3. The tendency towards concentration, allowing a handful of dominant firms to significantly raise their market shares, translates into a rise in the aggregate markup as stated in proposition 1. This is accompanied by a rise in the aggregate profit share (panel (a)). For the parameterisation under consideration, the macroeconomic regime is ‘wage-led’ so that the rise in the aggregate profit share must lead to a fall in aggregate demand and growth (see Online Appendix A.2). This is indeed the case (panel (b)).

Figure 3: Macroeconomic variables, ABM

(a) Aggregate markup (τ_t) and profit share (π_t)



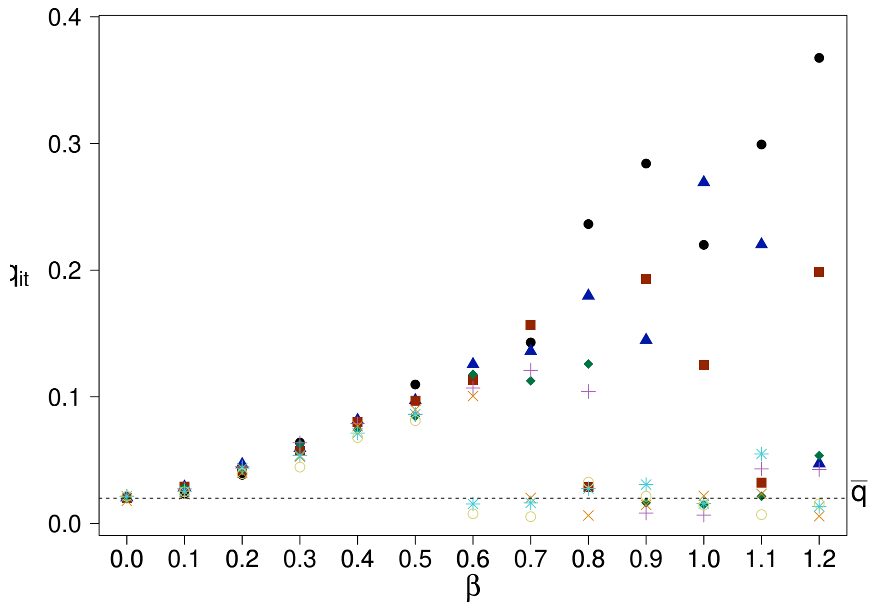
(b) Aggregate rate of capacity utilisation (u_t) and rate of growth (g_t)



Notes: Output from a single run of the ABM with parameterisation reported in Table 1.

Finally, we explore the impact of changes in β, which captures the intensity of economies of scale in the determination of unit cost in (6). Figure 4 displays the long-run market shares of the top eight firms for different values of β. Market concentration only arises for $\beta > 0$, and the degree of concentration tends to increase in β. The higher β is, the smaller the number of dominant firms and the larger the variation in market shares. This result is confirmed by a Monte Carlo analysis of the equilibrium Herfindahl-Hirschman index (HHI), which is indeed increasing in β (see Figure B2 in Online Appendix B.2).

Figure 4: Average long-run market shares of top 8 firms for different intensities of economies of scale (β), ABM



Notes: Average market shares over final 200 periods (with $T = 2000$) from a single run of the ABM with parameterisation reported in Table 1 for different values of β representing the intensity of economies of scale. Top 8 firms by size of capital stock in final period. The dashed horizontal line represents the average market share ($1/N = 0.02$).

In sum, our simple ABM predicts a tendency towards market concentration that is stronger when economies of scale are greater, allowing larger firms to undercut competitors while raising markups and thus profitability. At the macro level, an increase in market concentration translates into a rise in the aggregate profit share that depresses consumption demand, thereby reducing aggregate demand and thus growth. These aggregate distributional effects are stronger when concentration is greater.

4 The analytical heterogeneous agent version

In this section, we study what we call the 'analytical heterogeneous agent' version (HAM) of the model developed in the previous section. The HAM replaces direct interaction between agents with a reduced-form equation for the determination of market shares. Aside from this change in closure, all equations from section 2 are retained. The advantage of the HAM is that it admits precise analytical solutions. Despite the more stylised representation of goods market matching, the HAM produces qualitatively similar results to the ABM.

4.1 Determination of market shares through a reduced-form equation

In the HAM, the bilateral goods-market matching algorithm is replaced by a reduced-form stochastic market share allocation equation. Consistent with our assumption that market shares are determined by relative prices (see (12)), which in turn are a function of lagged market shares and relative size (see (13)), we use the following specification:[16]

$$q_{it} = -\zeta_1 q_{it-1} + \zeta_2 s_{it-1} + (1 + \zeta_1 - \zeta_2)\epsilon_{it}, \quad (1 + \zeta_1 - \zeta_2) \geq 0, \tag{12 HAM}$$

where the stochastic variable ϵ_{it} is based on a uniformly distributed random variable $\omega_{it} \sim U(0,1)$ that is normalised by $\sum_{i=1}^{N} \omega_{it}$, such that $\sum_{i=1}^{N} \epsilon_{it} = 1$ holds in every period, and $\mathbb{E}[\epsilon_{it}] = \frac{1}{N}$, $\text{Var}(\epsilon_{it}) = \frac{1}{3N^2}$, and $\text{Cov}(s_{it-1}, \epsilon_{it}) = \text{Cov}(q_{it-1}, \epsilon_{it}) = 0$ hold asymptotically.[17] This ensures that market shares sum to one in every period and are always bounded between zero and one.

What determines the parameters ζ_1 and ζ_2? The negative term $-\zeta_1 q_{it-1}$ reflects the loss of price competitiveness from increases in markups in response to past gains in market shares implied by (7). The positive term $\zeta_2 s_{it-1}$ captures the gains in price competitiveness from economies of scale as determined by (6). The coefficient on the random term, $(1 + \zeta_1 - \zeta_2)$, then governs the strength of factors other than price competitiveness in the determination of market shares. In line with the approximation for relative prices in (13), we set $\zeta_1 = \frac{\tau_q}{1+\bar{\tau}}$

[16]See Di Guilmi & Carvalho (2017), Dosi et al. (2010), Pedrosa & Lang (2021) and Reissl (2020) for similar reduced-form market share specifications in the MABM literature. Of those, Di Guilmi & Carvalho (2017) and Reissl (2020) also use functions with relative firm size as an argument, but without clarifying the underlying economic mechanism.

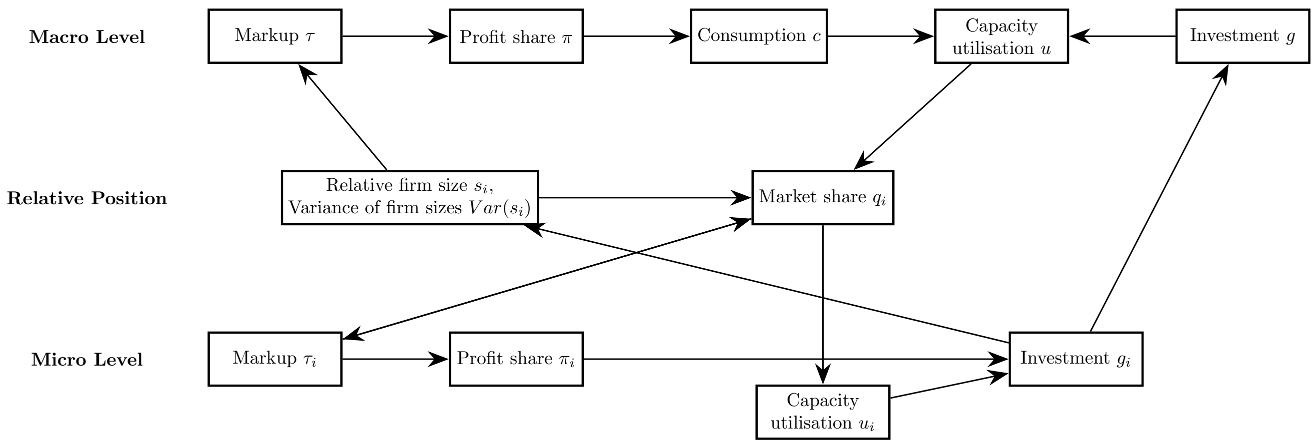
[17]To see this, note that since $\omega_{it} \sim U(0,1)$, we have $\mathbb{E}[\omega_{it}] = \frac{1}{2}$ and $\text{Var}(\omega_{it}) = \frac{1}{12}$. Thus, asymptotically $\mathbb{E}[\omega_{it}]N = \frac{N}{2}$. From this, it follows that $\mathbb{E}[\epsilon_{it}] = \frac{1}{N}$, $\text{Var}(\epsilon_{it}) = \frac{1}{3N^2}$.

and $\zeta_2 = \beta$ to account for the effects of markup adjustment and of economies of scale on price competitiveness, respectively.

Dynamics are now given by (12 HAM) for market shares along with (11) for the dynamics of relative firm size. These two equations constitute a non-linear system of first-order difference equations in q_{it} and s_{it} whose dynamics determine the micro-level concentration dynamics, which in turn generate macroeconomic outcomes that are symmetric to the ABM.

Figure 5 provides a visual representation of the causal structure of the HAM, highlighting the the micro-macro linkages. It shows the key roles of relative firm size and market shares in mediating between the micro and the macro levels.

Figure 5: Micro-macro linkages in HAM



Notes: Flowchart of the causal structure of the heterogeneous agent model. Relative size and market shares determine the relative position of firms, mediating between the micro and the macro levels. Firms' relative power in markets (their market shares) determines their markup pricing decisions, which in turn affects their profitability and investment. In addition there is a direct effect of market shares on rates of utilisation. Firm-level differences in investment then drive changes in relative sizes (giving rise to changes in the variance of the firm size distribution), which not only affects the allocation of market shares, but also the aggregate markup and profit share. The aggregate distributional effects then influence consumption demand and thus aggregate demand, which feed back into the micro-level via market shares.

4.2 Analytical results for the heterogeneous agent model

We proceed by formally stating some key analytical results of the HAM, which we then illustrate and compare to the ABM by means of numerical simulations. Proposition 3 specifies the mean steady states of the HAM.[18]

Proposition 3 (Mean steady states of HAM.) *The dynamic system* (11)–(12 HAM) *in s_{it} and q_{it} admits the following mean steady states:*

(i) a no-concentration steady state: $\mathbb{E}[s_i^*] = \mathbb{E}[q_i^*] = \frac{1}{N}$ *for all i firms,*

(ii) a concentration steady state: $1 \geq \tilde{s}_j^* > \frac{1}{N} > \tilde{s}_k^* \geq 0$ *with $j = 1, ..., N^j$ firms at $\tilde{s}_j^*$ and the other $k = 1, ..., N^k = N - N^j$ firms at $\tilde{s}_k^*$, and $\tilde{s}_j^* = \frac{1-\tilde{s}_k^* N^k}{N^j}$. The corresponding steady state market share is $\tilde{q}_i^* = \left(\frac{\zeta_2}{1+\zeta_1}\right)\tilde{s}_i^* + \left(\frac{1+\zeta_1-\zeta_2}{1+\zeta_1}\right)\frac{1}{N}$, where $i = j, k$.*

If $\tilde{s}_j^* > 1/2$, then there is only one firm j at $\tilde{s}_j^*$ (so that $N^j = 1$) and the $N^k = N - 1$ other k firms are at $\tilde{s}_k^*$, such that $\tilde{s}_j^* = 1 - \tilde{s}_k^*(N - 1)$.

In the no-concentration steady state, all firms are on average homogeneous in terms of size and market shares. Correspondingly, firms are also (on average) homogeneous in terms of markups, profit shares, and rates of utilisation. As a result, all firms grow on average at the same rate, such that $\mathbb{E}[s_i^*] = \frac{1}{N}$ is an (interior) equilibrium.

In the concentration steady state, firms reach one of two possible equilibria, which sit to the left and right of the average market share given by $\bar{q} = 1/N$. This steady state has two sub-cases. In the fully deterministic case where $1 + \zeta_1 - \zeta_2 = 0$, market shares are exclusively determined by price competitiveness. We can derive the following corollary:

Corollary 1 (Concentration steady state under deterministic allocation of market shares.) *With $1 + \zeta_1 - \zeta_2 = 0$, the allocation of market shares is fully deterministic. Then the concentration steady state is a corner solution such that one firm j converges to $s_j^* = q_j^* \to 1$, while the other $N - 1$ firms k converge to $s_k^* = q_k^* \to 0$.*

In the fully deterministic case, there is no mechanism other than price competitiveness by which aggregate demand is allocated. Any firm that gains a sufficient advantage vis-à-vis the other firms in terms of relative size and thus price will get a higher market share, allowing for higher markups, higher profit shares and higher growth. The smaller firms will experience not only lower profit shares but also declining rates of capacity utilisation, leading to a

[18] In the case of the concentration steady state, we approximate the stochastic steady state by solving the deterministic system obtained by replacing the shock ϵ_{it} with its mean value $\mathbb{E}[\epsilon_{it}] = 1/N$ (see Online Appendix A.6). The approximated mean steady-state values are denoted by a tilde.

permanent differential in growth rates where $g_j > g_k$, such that the dominant firm will converge asymptotically to a relative size of one, becoming the monopolist.

By contrast, in the stochastic case where $1 + \zeta_1 - \zeta_2 > 0$, the partially random allocation of market shares prevents any firm that gains a significant size advantage from permanently growing faster. Concentration comes to a halt before the limiting case is reached, because the random allocation of aggregate demand drives up the rates of capacity utilisation of the small firms. This process leads to an alignment of the growth rates of smaller less profitable firms with those of the more profitable dominant firms until growth rates are equalised on average. This leads to an interior equilibrium, where $1 > \tilde{s}_j^* > \frac{1}{N} > \tilde{s}_k^* > 0$.

What determines which of the two steady states are reached in the stochastic case? Proposition 4 states the conditions under which the no-concentration steady state is unstable, such that any displacement will lead to a transition towards the concentration steady state:[19]

Proposition 4 (Condition for concentration in HAM.) *Consider the no-concentration steady state where $\mathbb{E}[s_i^*] = \mathbb{E}[q_i^*] = \frac{1}{N}$. This steady state is unstable if*

$$\begin{aligned} &(1 - \zeta_2 + \zeta_1)(\gamma_r \nu \pi^* + \gamma_u) - \frac{\zeta_1 \zeta_2 \gamma_r \nu}{1 + \bar{\tau}} \\ &= \left(1 - \beta + \frac{\tau_q}{1 + \bar{\tau}}\right)(\gamma_r \nu \pi^* + \gamma_u) - \frac{\tau_q \beta \gamma_r \nu}{(1 + \bar{\tau})^2} \leq 0. \end{aligned} \tag{14}$$

Corollary 2 further states that in the deterministic case, concentration must occur as soon as β, γ_r, and τ_q are all positive, i.e. there are economies of scale, investment responds positively to profits, and markups are increasing in market shares:

Corollary 2 (Condition for concentration under deterministic allocation of market shares.) *With $1 + \zeta_1 - \zeta_2 = 0$, the allocation of market shares is fully deterministic. Then the no-concentration steady state $s^* = \frac{1}{N}$ is unstable if $\zeta_2 \gamma_r \tau_q = \beta \gamma_r \tau_q > 0$, which is always satisfied.*

In the stochastic case where $1 + \zeta_1 - \zeta_2 > 0$, the condition for concentration given by (14) is more stringent. Assuming β, γ_r, and τ_q are all positive, the condition requires sufficiently large values of β for concentration to emerge, as stated in corollary 3:

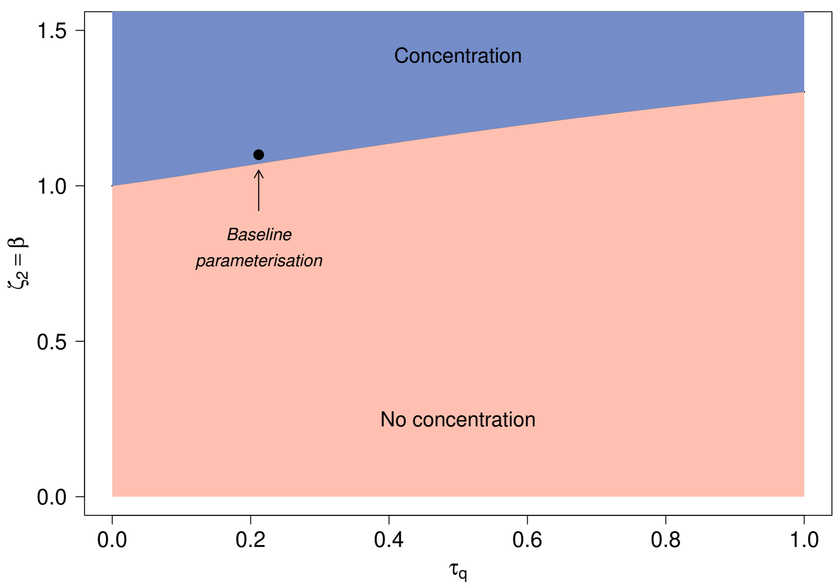
[19] Note that condition (14) only holds locally around the $\mathbb{E}[s_i^*] = \mathbb{E}[q_i^*] = \frac{1}{N}$ steady state. In numerical simulations, stochastic shocks can push the system away from the equilibrium so that (14) is no longer strictly valid. In this case, concentration can arise even if (14) is positive but very close to zero.

Corollary 3 (Critical value of β in HAM.) *Concentration will arise if the economies of scale parameter $\zeta_2 = \beta$ exceeds the following critical value:*

$$\zeta_2 \geq \frac{(1+\zeta_1)(\gamma_r \nu \pi^* + \gamma_u)}{\frac{\gamma_r \nu}{1+\bar{\tau}}(\zeta_1 + \bar{\tau}) + \gamma_u}$$
$$\Leftrightarrow \beta \geq \frac{(1+\frac{\tau_q}{1+\bar{\tau}})(\gamma_r \nu \pi^* + \gamma_u)}{\frac{\gamma_r \nu}{1+\bar{\tau}}\left(\frac{\tau_q}{1+\bar{\tau}} + \bar{\tau}\right) + \gamma_u}. \tag{15}$$

Figure 6 plots condition (15) in (τ_q, β)-space, illustrating the combinations of parameter values under which concentration occurs. The relationship is positive: increases in τ_q reduce the parameter space for which concentration will occur due to the negative effects of higher markups on price competitiveness.

Figure 6: Condition for concentration in HAM



Notes: Plot of (15) using the parameterisation reported in Table 1. The figure shows whether concentration occurs under combinations of parameter values for β, the intensity of economies of scale, and τ_q, the sensitivity of the markup to market share. The black dot represent the baseline parameterisation for β and τ_q used in the simulations.

A final analytical result concerns the aggregate markup. Using (12 HAM) and imposing equilibrium ($q^*_{it} = q_{it} = q_{it-1}$), we can derive a more precise version of proposition 1:

Proposition 5 (Equilibrium aggregate markup in HAM.) *In the HAM, the (equilibrium) aggregate markup is increasing in the variance of the firm size distribution:*

$$\tau_t^* = \sum_{i=1}^{N} q_{it}^* \tau_{it}^* = \bar{\tau} + \tau_q \left[\left(N \frac{\zeta_2}{1+\zeta_1} \right)^2 \mathrm{Var}(s_t^*) + \left(\frac{1+\zeta_1-\zeta_2}{1+\zeta_1} \right)^2 \frac{1}{3} \right].$$

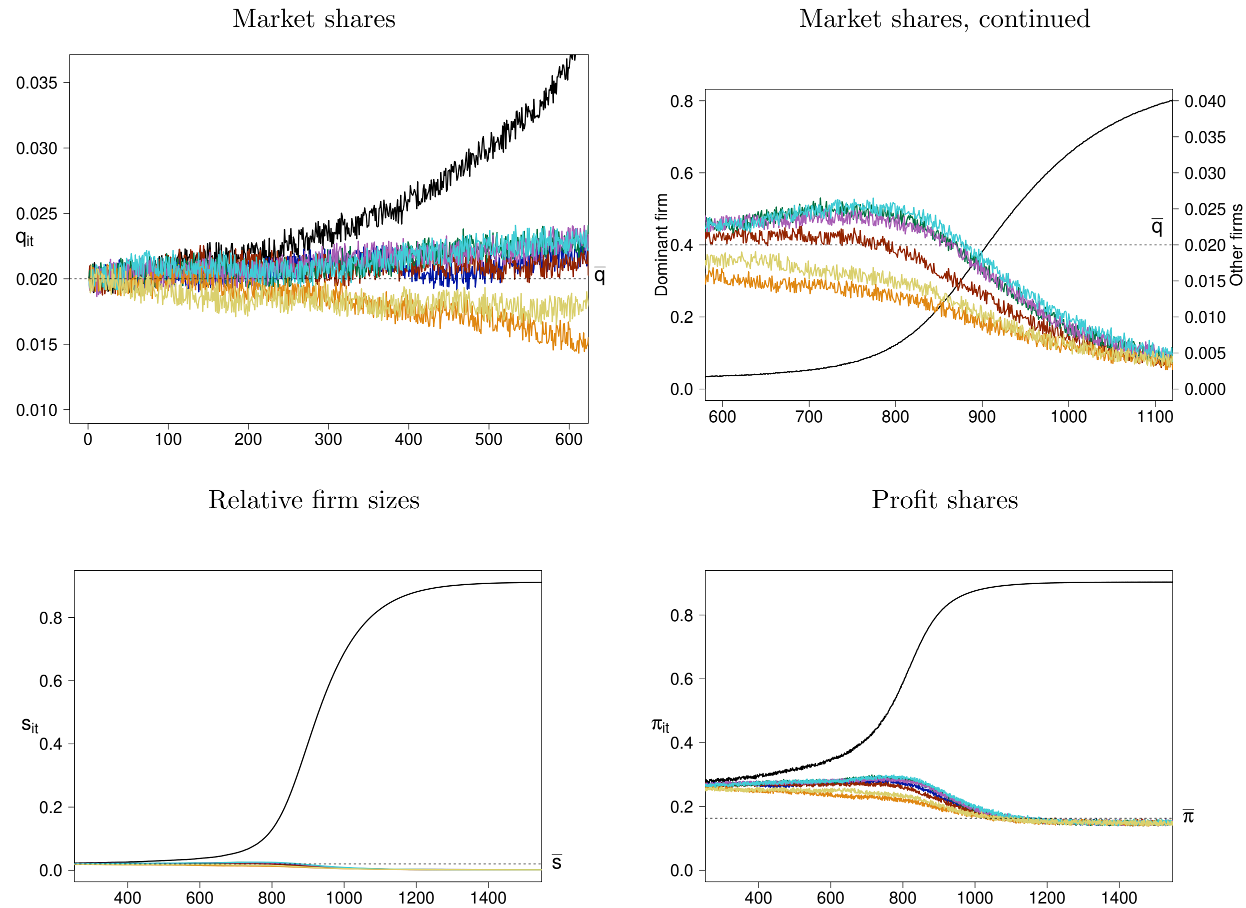
With a constant average markup, changes in the aggregate markup are entirely driven by changes in the variance of the size distribution of firms, reflecting the critical role of firm size for concentration dynamics in our model.

4.3 Simulation of the heterogeneous agent model

Figure 7 shows simulation results for the HAM focussing on the top eight firms (by size in the final period).[20] Starting from the no-concentration steady state where firms are homogeneous, a dominant firm gradually emerges, taking a higher market share and accumulating a much larger capital stock than the remaining firms. As a result, its profit share increases, driving up the aggregate profit share. This has the same implications for the aggregate system as in the ABM, as discussed in section 3.2. Figure B5 in Online Appendix B.3 further shows the joint increase in the variance of firm sizes, the aggregate markup and profit share, illustrating the link between increasing variance of firm sizes and the aggregate markup stated in proposition 5.

[20]The parameterisation is identical to that for the ABM, with the only difference that $\beta = \zeta_2$ has been set to a higher value of 1.1 so that the condition for concentration (14) is satisfied.

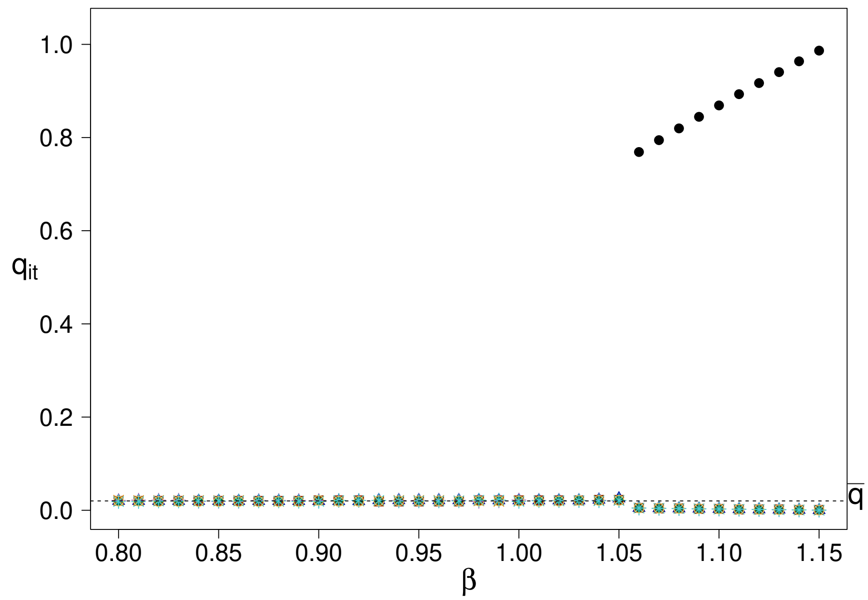
Figure 7: Dynamics of top 8 firms, HAM



Notes: Output from a single run of the HAM with parameterisation reported in Table 1. The top 8 firms by size of capital stock in the final period ($T = 2000$) are included. The top-right panel has separate axes for the dominant firm (left) and the other firms (right). The dashed horizontal lines represent the average market share, relative firm size, and average profit share, respectively.

In Figure 8, we show the impact of the economies of scale parameter β on the equilibrium degree of concentration, measured by the average long-run market shares of the top eight firms. Concentration is positively related to β, but in a non-linear way. For values of β below the critical value in (15), the condition for concentration (14) is not satisfied and the long-run equilibrium does not exhibit concentration. As β crosses the threshold, there is a phase transition where the system switches from the no-concentration to the concentration steady state. Further increases of β above the threshold raise the equilibrium degree of concentration.

Figure 8: Average long-run market shares of top 8 firms (by size) for different intensities of economies of scale (β), HAM



Notes: Average market shares for different values of β over the final 200 periods of a single 2000-period simulation run of the HAM with parameterisation reported in Table 1. The top 8 firms by size of capital stock in the final period are included. The dashed horizontal line represents the average market share ($1/N = 0.02$).

4.4 Comparison of the heterogeneous agent and agent-based models

The HAM produces qualitatively similar results to the ABM. Both models, under certain parameter combinations, generate concentration dynamics. In both, the economies of scale parameter β is a key factor in the emergence of concentration: $\beta > 0$ is a necessary condition for concentration to occur, and the degree of concentration is generally increasing in β. Once concentration dynamics are in motion, both models generate a rise in the markup and profit share of the dominant firm(s), driving up the aggregate markup and profit share and lowering aggregate growth.

A notable difference is that the concentration steady state of the HAM only allows for two possible average market shares $\tilde{q}_j^*$ and $\tilde{q}_k^*$. If $\tilde{s}_j^* > 1/2$, which is always the case in our simulations, there will be a single dominant firm. By contrast, the ABM allows for a continuum of possible market shares and thus a richer representation of different degrees of concentration. Similarly, while the HAM exhibits a phase transition where the equilibrium suddenly shifts from no concentration to strong concentration, the ABM always generates some concentration if $\beta > 0$, and the degree of concentration increases more gradually as β (or other model parameters) are increased.

In the HAM, the reduced-form market share equation (12 HAM) makes the economies-of-scale parameter β and the sensitivity of markups to market shares τ_q the two key parameters governing the existence and degree of concentration. By contrast, the ABM's bilateral matching algorithm contains three additional parameters that influence the degree of competition: the share of buyers comparing prices α^p, the share of buyers choosing firms randomly α^ϵ, and the percentage of firms ρ that buyers consider when comparing prices. This renders the ABM more flexible for explaining specific concentration episodes. Indeed, according to Autor et al. (2020, pp. 648-649), concentration could also occur 'if consumers have become more sensitive to quality-adjusted prices due to, for example, greater product market competition (e.g., through globalization) or improved search technologies (e.g., greater availability of price comparisons on the internet)'. Additional simulation results in Online Appendix B.2 show that the degree of concentration generated by the ABM is increasing also in α^p and ρ, and falling in α^ϵ, demonstrating that the ABM is capable of capturing the greater price competition/price sensitivity mechanism as well.

We conclude that the HAM, while not capable of completely matching the richness of the ABM, does succeed in capturing its key dynamics. The main advantage of the HAM is its tractability, enabling a rigorous analysis of the critical role of economies of scale for concentration dynamics and of the firm size distribution for aggregate markups.

5 Concentration dynamics and the macroeconomic growth regime

The Keynesian literature on macroeconomic growth regimes explores the implications of changes in functional income distribution on growth in demand-led economies (Bhaduri & Marglin 1990, Blecker 2016, Hein 2014, Napoletano et al. 2012). Growth regimes are classified as either 'profit-led' or 'wage-led', depending on whether aggregate growth increases or decreases in response to an increase in the profit share. The macroeconomic core of both the ABM and HAM allows for both regimes (see Online Appendix A.2). In several recent MABMs, a rise in income inequality reduces aggregate demand (Botta et al. 2021, Fierro et al. 2023, Terranova & Turco 2022). We have followed this literature by considering 'wage-led' parameterisations where an increase in the profit share has a negative effect on growth because of a strong negative effect on consumption that is not offset by a profitability-induced increase in investment. In this section, we analyse the relationship between the macroeconomic growth regime and the condition for microeconomic concentration in the HAM.

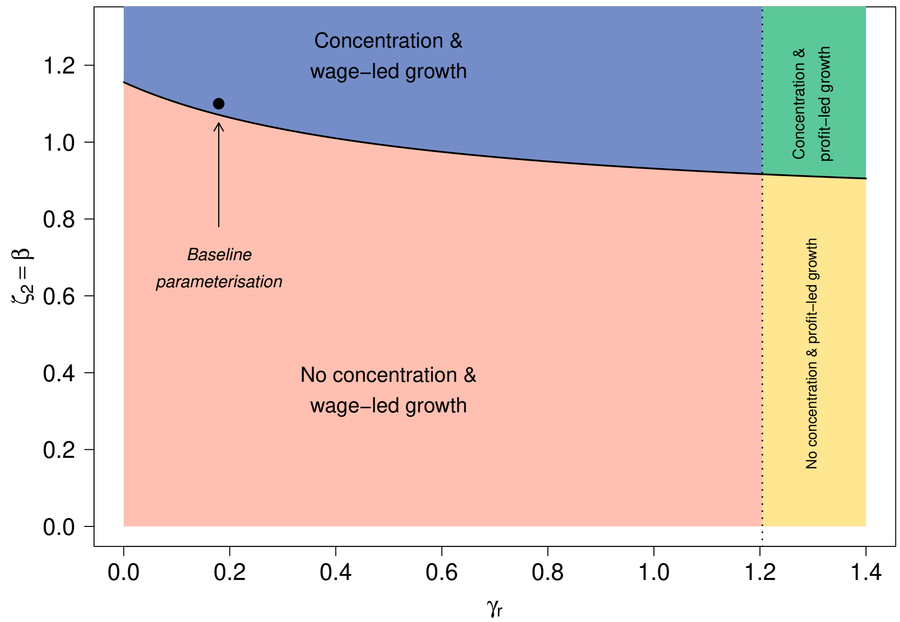
Figure 9 plots both the condition for concentration (14) and for a wage-led growth regime in the (γ_r, β)-space (panel a) and in the (γ_r, γ_u)-space (panel b). For our baseline parameterisation, the HAM produces concentration dynamics within a wage-led macroeconomic regime (blue-shaded area), leading to the concentration-cum-stagnation outcomes seen in the previous section. However, Figure 9 shows that for other parameterisations, the HAM may yield other combinations of the micro concentration regime and the macro growth regime. A higher economies-of-scale parameter β generally increases the probability of concentration, but it leaves the macroeconomic regime unaffected. This is because β influences the aggregate profit share, but not how consumption and investment respond to it. For example, for a β below the critical value (14) and γ_r as in the baseline, the macroeconomic regime would be wage-led, but no stagnation would occur because there is no endogenous tendency towards concentration.

By contrast, a higher sensitivity of investment to the profit rate, γ_r, increases the likelihood of both concentration and a profit-led growth regime. This is because of the dual role of this parameter: it enters into the positive concentration feedback loop in which price competitive firms become more profitable, invest more, and become even more competitive. It also enters into aggregate demand determination whereby increased profitability directly raises aggregate investment demand. Thus, for a sufficiently high γ_r, the macroeconomic regime is profit-led and also generates concentration. The increase in the profit share resulting from concentration then stimulates aggregate investment demand enough to offset the negative effects on consumption demand.

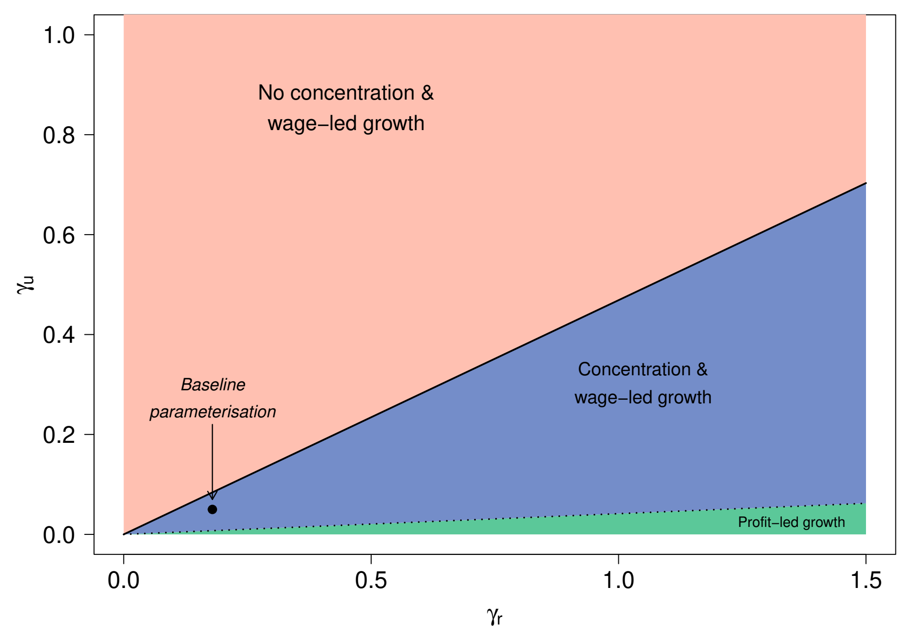
Panel b shows that the sensitivity of investment to utilisation, γ_u, plays the opposite role to γ_r: a higher γ_u reduces the probability of concentration and makes the growth regime more likely to be wage-led. This is because γ_u makes firms' investment sensitive to utilisation rates, which tend to be high for small firms, leading them to catch up with bigger firms, thereby curbing concentration. At the same time, negative effects of higher profit shares on consumption demand will have stronger negative effects on investment with a higher γ_u, making the macro-economy more likely to be wage-led.

Figure 9: Conditions for concentration and growth regimes

(a) Sensitivity of investment to profits and economies of scale



(b) Sensitivities of investment to profits and to utilisation



Notes: Panel (a) plots the conditions for concentration (14) (solid line) and for wage-led growth $\gamma_r < \frac{c_y \gamma_u}{\nu(1-c_y)}$ (dotted line; see Online Appendix A.2 for derivation) in (γ_r, β)-parameter space, where γ_r is the sensitivity of investment growth to the rate of profit and β is the intensity of economies of scale. The growth regime is wage-led to the left of the dotted line. Panel (b) plots the same conditions in (γ_r, γ_u)-parameter space, where γ_u is the sensitivity of investment growth to demand (capacity utilisation). Growth is wage-led above the dotted line. The remaining parameters are fixed at the values reported in Table 1.

Proposition 6 summarises the dual role of γ_r in connecting the microeconomic and macroeconomic regimes of the HAM.

Proposition 6 (Condition for concentration and wage-led growth in HAM.) *The HAM will generate microeconomic concentration dynamics and a macroeconomic wage-led growth regime for intermediate sensitivities of investment to the profit rate that satisfy:*

$$\frac{(1-\zeta_2+\zeta_1)\gamma_u}{\nu[\frac{\zeta_1\zeta_2}{1+\bar{\tau}}-(1-\zeta_2+\zeta_1)\,\pi^*]} \leq \gamma_r < \frac{c_y\,\gamma_u}{\nu(1-c_y)}$$
$$\Leftrightarrow \frac{(1-\beta+\frac{\tau_q}{1+\bar{\tau}})\gamma_u}{\nu[\frac{\tau_q\beta}{(1+\bar{\tau})^2}-(1-\beta+\frac{\tau_q}{1+\bar{\tau}})\,\pi^*]} \leq \gamma_r < \frac{c_y\,\gamma_u}{\nu(1-c_y)}. \tag{16}$$

For concentration to occur, γ_r needs to exceed the threshold on the left side of (16), so that profitable firms tend to grow in size, which further cements their profitability via economies of scale. At the same time, γ_r needs to be below the threshold defined on the right side for the macroeconomic regime to be wage-led, so that growth falls in response to a rise in the profit share.

In Online Appendix A.10, we further investigate analytically the conditions under which the aggregate growth regime coincides with the firm-level investment response to a shift in its profit share, i.e. $\text{sgn}\left(\frac{\partial g_t^*}{\partial \pi_t}\right) = \text{sgn}\left(\frac{\partial g_{it}}{\partial \pi_{it}}\right)$. Consider a situation in which the firm-level profit share has increased. How this affects firm-level versus aggregate growth depends on if and how the increase in the firm-level profit share raises the aggregate profit share. The latter, in turn, depends on whether all firms or just one increase their profit share, whether market shares are constant or not, and on the size of the market share of the firm in question.

The case most relevant for this paper is that where firms are heterogeneous in terms of market shares and one firm's profit share increases due to a rise in its market share. In this case, whether the firm-level growth response coincides with the macro growth regime will depend on the market share of that firm. If the firm has a market share close to the average, the aggregate profit share will not increase much (or could even fall) in response to an increase in the market share of the firm, making the aggregate demand effect small. This is what happens in the initial transition phase of the HAM, where, starting from a no-concentration equilibrium, a successful firm begins to increase its market and profit shares with little effect on aggregate dynamics. As a result, the firm grows faster than the others and captures higher market shares. Subsequently, further increases in the firm's profit share exert stronger effects on the aggregate profit share, eventually leading to contractionary aggregate demand effects that offset the positive effect of profitability on the firm's growth rate.

Our analysis of the relationship between concentration and the macroeconomic growth regime provides a Keynesian perspective on the concentration-cum-stagnation dynamics studied in Akcigit & Ates (2021, 2023), Covarrubias et al. (2020) and De Ridder (2024). In a demand-led economy with endogenous productivity via economies of scale, concentration dynamics accompanied by falls in physical investment and productivity growth are most likely to occur for intermediate sensitivities of investment demand to profitability. From a starting point of low concentration, this allows profitable firms to further cement their advantage via faster growth, initially with limited negative effects on the macroeconomy. However, as the economy becomes more concentrated, the investment response of the dominant firm becomes aligned with the macro growth regime. Rising profit shares will then reduce growth of both the dominant firm and the economy as a whole, eventually leading to stagnation.

6 Exploring the ABM further: Gibrat's law and the granular hypothesis

In this section, we deepen the analysis of the ABM and highlight connections to two other strands of literature: Gibrat's law and the origins of heavy-tailed firm size distributions, and the granular hypothesis on the microeconomic origins of macroeconomic fluctuations.

Gibrat's law. Heavy-tailed firm size distributions are a well-documented empirical fact (Axtell 2001, Fujiwara et al. 2004) that has received some attention in the theoretical MABM literature (e.g. Caiani et al. 2016, Delli Gatti et al. 2005, Dosi et al. 2018). Right-skewed distributions are also central to the superstar-firm argument, which claims that part of the rise in the aggregate profit share is due to a reallocation of market shares towards large firms (Autor et al. 2020, De Loecker et al. 2020). A canonical approach for explaining the emergence of such skewed distributions is Gibrat's law of proportional effect, according to which firm growth rates are stochastic and independent of size (Gabaix 2009). Suppose firm-level capital stocks evolve according to:

$$K_{it} = (1 + g_{it}^{\epsilon})K_{it-1},$$

where g_{it}^{ϵ} is an i.i.d. random sequence. If $g_{it}^{\epsilon} \sim N(\mu_g, \mathrm{Var}(g))$, then capital stocks asymptotically follow a log-normal distribution. The key feature of this process, which generates a right-skewed distribution, is the assumption that the rate of *change* of the capital stock is proportional to its size, while the rate of *growth* is independent of size – the so-called 'Law of Proportionate Effect' or simply 'Gibrat's law'. This postulated independence of growth rates from size has generally found empirical support (Fujiwara et al. 2004).

Simulation results reported in Online Appendix C.1 (Figure C1) suggest that during the transition towards the stochastic equilibrium of the ABM, Gibrat's law does not hold as bigger firms temporarily grow faster. However, Gibrat's law does hold in the steady state where firms tend to grow at the same average rate independently of size. A further aspect of Gibrat's law that has not found empirical support is the postulated independence of the *variance* of growth rates from size (Fujiwara et al. 2004). Our ABM reproduces this empirical finding: the log of the variance of firm-level growth rates is negatively related to firm size, i.e. larger firms tend to exhibit less variation in their growth rates (Figure C2).

To assess whether firm sizes in our ABM follow a (heavy-tailed) power law distribution, we construct a log-log plot of rank vs size (also called Zipf plot), which should yield a straight line if the distribution follows a power law (Figure C3). While the full distribution does not fit a straight line, the top tail does (with the exception of a handful of the largest firms). This is in line with empirical evidence that power law distributions provide a good fit for empirical distributions of large firm sizes (Fujiwara et al. 2004).

Overall, these results suggest that the ABM succeeds in the qualitative reproduction of key features identified in the literature on firm size distributions – in particular the existence of heavy-tails, the independence of growth rates from size, and the lower volatility of growth rates of large firms.

The granular hypothesis. In an influential paper that has also been debated in parts of the MABM literature (Dosi et al. 2018), Gabaix (2011, p. 737) proposed the 'granular hypothesis' that 'idiosyncratic shocks to large firms have the potential to generate nontrivial aggregate shocks'. While granularity has not been explicitly discussed in the literature on profit shares, a right-skewed firm size distribution is a key condition for both the superstar firm mechanism and the granularity hypothesis, reflecting the presence of a small number of very large firms which make up a nontrivial component of macroeconomic aggregates such as profit shares. Gabaix (2011) construct a 'granular' residual that serves as a proxy for the idiosyncratic shocks to the biggest firms. The aggregate variable of interest is then regressed on the granular residual to assess how much of its variation is explained by firm-level shocks to large firms.

To evaluate the applicability of the granular hypothesis to profit shares in the ABM, we introduce idiosyncratic shocks to firm-level markups by replacing the fixed parameter τ_0 in the markup equation (7) with a normally distributed random variable $\tau_0^{\text{rand}} \sim \mathcal{N}(\tau_0, \text{Var}(\tau_0))$. Using simulated data from this augmented ABM, we construct the granular residual χ_t for the profit share as follows:

$$\chi_t = \sum_{i=1}^{M} q_{it}(\pi_{it} - \bar{\pi}_t), \tag{17}$$

where M is the number of top firms (by size in the final period), which we set to $M = 5$. We then regress the aggregate profit share π_t on the granular residual. Regression results reported in Online Appendix C.2 show that the granular residual is statistically significant, with a high adjusted R-squared, across several specifications, confirming that the granular hypothesis applies to profit shares in our model.

7 Extensions and robustness tests

In this section, we present several extensions and robustness tests to the core model, ABM, and HAM.

Alternative specification for unit costs. An alternative specification for unit costs with heterogeneous pass-through from productivity to wages gives equivalent results to our baseline. In the common core of the two models, unit cost differentials across firms arise from heterogeneous labour productivities δ_{it} along with homogeneous nominal wages w_t (see (6)). An alternative specification is to assume constant homogeneous productivities, but heterogeneous pass-through of productivity to nominal wages: $w_{it} = \eta_{it}\delta$, where η_{it} is the pass-through coefficient. Based on the evidence reported in Jarosch et al. (2024), Gouin-Bonenfant (2022) and Chan et al. (2023), we assume that larger firms have lower pass-through relative to smaller firms: $\eta_{it} = (Ns_{it-1})^{-\beta}$. These alternative assumptions about unit cost determination yield the following pricing equation:

$$P_{it} = (1 + \tau_{it})(Ns_{it-1})^{-\beta}. \tag{4.Alt}$$

While this pricing equation is not equivalent to (4), both specification give identical *relative* prices (see Online Appendix D.1). Simulations of both the ABM and HAM confirm that both specifications indeed yield identical results for all variables except for the price level.

Exit, mergers and re-entry. Next we show that adding a simple exit, mergers and re-entry mechanism to the ABM amplifies the distributional dynamics of the baseline model. We first introduce a constant and homogeneous rate of depreciation ξ to the capital stock. We then assume that firms that make zero profits for m periods exit the model. Their customers are randomly re-matched to active firms. Whenever h or more firms have exited, the capital stocks held by these firms at the time of exit are merged to form a new active firm (with $h = 0$ representing the baseline model without exit and re-entry).

Simulation results reported in Online Appendix D.2 show that introducing exit, mergers, and re-entry reduces the number of active firms in equilibrium relative to the baseline. There is still a clear tendency towards concentration, but its degree as measured by the normalised HHI over the active firms tends to be lower relative to the baseline. Nevertheless, the aggregate markup and profit share are lower relative to the baseline, as fewer firms divide total sales between them, yielding stronger stagnation tendencies (see Online Appendix D.5).

Preferential attachment. Introducing preferential attachment of buyers to firms in the ABM increases the degree of concentration, reinforcing the macroeconomic effects of the baseline model. We incorporate this extension by assuming that the share of buyers α^{ϵ} that, in the baseline, switch randomly in each period now switch to a new firm with a probability that is increasing in the number of existing buyers of that firm. Formally, let d_{it} be the current number of buyers of firm i. The probability p_{it+1} that the firm i gets chosen by a random buyer is given by:

$$p_{it+1}\left(\text{chosen} \mid d_{it}\right)_{=} \frac{(1+d_{it})^{\psi}}{\sum_{i=1}^{N}(1+d_{it})^{\psi}}, \tag{18}$$

where ψ governs the strength of the preferential attachment. For $\psi = 0$ there is no preferential attachment as in the baseline. Simulation results reported in Online Appendix D.3 show that the equilibrium degree of concentration is generally increasing in the preferential attachment parameter. For high intensities of preferential attachment of $\psi \geq 2.25$, concentration becomes very strong, occasionally even leading to winner-takes-all results, where a single firm attains a market share of unity. The higher degree of concentration amplifies the distributional effects of the baseline (see Online Appendix D.5).

Nonlinear markup specification. Using a (more general) nonlinear specification for the markup preserves the main features of the HAM. Instead of the linear specification in (7), consider the following power law:

$$\tau_{it} = \tau_0 (N q_{it-1})^{\tau_q}. \tag{7.Alt}$$

Online Appendix D.4 analyses the HAM with this specification. The condition for concentration changes only marginally. The expression for the steady states admits more than two solutions for $\tau_q > 1$. Numerical simulations suggest qualitatively similar results to the linear specification, with the degree of concentration declining in τ_q.

Softmax market share equation. Next, we consider a softmax specification for the allocation of market share in the HAM:

$$q_{it} = \frac{e^{a_{it}}}{\sum_{i=1}^{N} e^{a_{it}}}, \tag{19}$$

where a_{it} is a latent variable. The softmax function guarantees that $q_{it} \in (0, 1)$ and $\sum_i q_{it} = 1$. Unlike the linear specification in (12 HAM), the softmax specification precludes analytical results, but allows for a richer set of outcomes.

In line with (12), we assume that a_{it} is a negative function of relative prices and a stochastic term:

$$a_{it} = -\zeta \ln \left(\frac{P_{it}}{\bar{P}_t} \right) + \epsilon_{it}, \tag{20}$$

where $\epsilon_{it} \sim \mathcal{N}(1/N, 0.01)$.

Simulation results reported in Online Appendix D.6 show that, similar to the HAM with the linear market share equation, there is a critical value for ζ above which concentration arises. Above the critical value, the degree of concentration is generally increasing in ζ, leading to a higher profit share and slower growth (see Online Appendix D.8). The two main differences with respect to the linear specification are, firstly, that polarisation is stronger with the market share of the non-dominant firms approaching zero, and secondly, that the softmax specification allows for multiple dominant firms that tend to split the market evenly between them. For example, for $\zeta = 1.4$, there are six dominant firms with equilibrium market shares of roughly 16.6% each.

Stock-flow consistency. In the core model in section 2, we allowed households to save out of wage income and firms to independently make investment decisions, without explicitly discussing the financial interactions between the two sectors. Many recent MABMs carefully pay attention to stock-flow consistency (e.g. Botta et al. 2021, Caiani et al. 2016, Di Guilmi & Carvalho 2017, Pedrosa & Lang 2021, Reissl 2020, Seppecher et al. 2018, Terranova & Turco 2022), following the framework developed by Godley & Lavoie (2006). Here, we briefly discuss the stock-flow consistency of our model and show that an extension of the HAM that includes a stock-flow feedback mechanism on aggregate consumption leaves the main results intact.

The simplest approach to making explicit the financial relationships in our core model is to let households hold deposits D_t and allow firms to take out loans L_t to finance investment in excess of retained profits, so that $D_t = L_t$. A no-profit banking sector provides loans and collects deposits, with exogenous and identical lending and deposit rates (see Online Appendix D.7 for the corresponding balance sheet and transactions flow matrices).

The core model abstracts from feedback effects of interest payments and wealth on expenditures. To gauge how introducing such effects would alter the model, consider the following augmented aggregate consumption function:[21]

$$c_t = c_y \left[\nu u_t(1-\pi_t) + d_{t-1} i^D\right] + c_d d_{t-1}, \tag{21}$$

where $d_t = \frac{D_t}{K_t}$ is the deposit-to-capital ratio, i^D is the interest rate on deposits, and c_d is the propensity to consume out of deposits.

The dynamics of the deposit-to-capital ratio can be derived from the budget constraint of the household sector $C_t + D_t - D_{t-1} = W_t + i^D D_{t-1}$, yielding:

$$d_t = \frac{d_{t-1}\left[1 + i^D(1-c_y) - c_d\right] + u_t \nu (1-c_y)(1-\pi_t)}{1+g_t^*}. \tag{22}$$

Online Appendix D.7 reports the corresponding equilibrium solutions and stability conditions. A higher deposit-to-capital ratio generally has positive effects on aggregate demand and growth due to the wealth effect (and the absence of a corresponding negative effect on investment). As the stock-flow effects only affect aggregate consumption, the concentration dynamics at the micro level are unchanged, so that all results from section 4.2 carry over.

Simulation results reported in Online Appendices D.7 and D.8 show that the stock-flow aug-

[21]We restrict stock-flow effects to aggregate consumption for tractability. Analogous effects in the firm-level investment function would preclude the derivation of analytical results.

mented HAM displays very similar results to the core version: concentration dynamics occur, driving up the dominant firm's markup and the aggregate profit share. These dynamics are accompanied by a fall in the deposit-to-capital ratio, depressing consumption demand in addition to the fall of the wage share. As a result, the aggregate rate of capacity utilisation and rate of growth fall.

8 Conclusion

Using a small tractable heterogeneous agent model we analyse the dynamic interactions between concentration among firms, aggregate income distribution and growth. Despite its simplicity, the estimated model generates results which broadly match contemporary stylised facts concerning corporate concentration, rising markups and falling labour shares. Importantly, the model allows for analytical solutions; we regard this as the major advantage of our approach and a step forward for heterogeneous agent modelling.

We compare the heterogeneous agent model to a more sophisticated agent-based model featuring direct interaction between agents in the goods market. Both models produce qualitatively similar results: starting from uniform distributions for market shares and the capital stock, both generate relaxation to a statistical equilibrium characterised by concentration among firms. We thus demonstrate that reduced-form heterogeneous agent models can be used to derive analytical results of more complex classes of agent-based models. While the matching mechanism in the agent-based model results in more complex emergent properties, it does not produce results that are significantly different to those of the more tractable heterogeneous agent version.

In both versions of the model, concentration dynamics and the structure of the resulting statistical equilibrium are driven by a small number of mechanisms and parameters. Concentration is the result of economies of scale coupled with an investment response that is sufficiently sensitive to profitability. The degree of concentration that emerges is increasing in the intensity of economies of scale.

The model lends itself to rigorous analysis of micro-macro linkages, providing important insights. Cross-sectional distributions affect macroeconomic outcomes: aggregate markups, and by extension profit shares, are positively related to the variance of the firm size distribution, which increases as the firm sector becomes more concentrated. Furthermore, there is a connection between micro-level dynamics and macroeconomic regimes. Concentration dynamics within a 'wage-led' macroeconomic regime are most likely to occur for intermediate sensitivities of investment demand to profitability. From a starting point of low concentration, this allows profitable firms to further cement their advantage via faster growth, initially with limited aggregate effects. As the economy becomes more concentrated, the aggregate profit share rises, which will eventually reduce the growth of both the dominant firm and the economy as a whole, leading to stagnation.

Methodologically, the paper contributes to the literature on smaller-scale hybrid macroeconomic agent-based models (e.g., Assenza & Delli Gatti 2013, Botta et al. 2021, Dosi et al. 2010, Di Guilmi & Carvalho 2017, Michell 2014, Fierro et al. 2023, Reissl 2020, Suresh & Setterfield 2015). Large-scale models that capture many features of the economy offer realism at the expense of tractability. We demonstrate that studying a reduced-form counterpart to an agent-based model is a worthwhile way to open up the 'black box' of complex models, allowing for the use of analytical techniques to identify and isolate key causal mechanisms. This provides a useful counterpart to the more experimental numerical simulation techniques that are commonly used. While not all of the results in this paper will carry over to other models, we demonstrate a useful and powerful method for analysing heterogeneous agent models with complex properties.

The paper also contributes to the literature on concentration, rising markups, and their macroeconomic effects (Akcigit & Ates 2021, 2023, Autor et al. 2020, Barkai 2020, Covarrubias et al. 2020, De Loecker et al. 2020, De Ridder 2024) and links it to the Keynesian literature on macroeconomic growth regimes (Bhaduri & Marglin 1990, Blecker 2016, Hein 2014, Napoletano et al. 2012). Our model offers a Keynesian perspective on the concentration-cum-stagnation phenomenon discussed in the Schumpeterian endogenous growth literature, highlighting cost advantages for large firms as a driver of concentration and the negative effects of rising profit shares on aggregate demand.

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Appendix

Notation. We use upper-case letters for variables in levels and lower-case letters for ratios/rates as well as parameters. All parameters are assumed to be non-negative. Firm-level variables are indexed by i. The corresponding macro aggregates lack the index i. We use $\bar{x}_t = N^{-1}\sum_{i=1}^{N} x_{it}$ for the unweighted cross-sectional average of a variable x_{it} (for finite N). We drop the subscript t if the cross-sectional average happens to be time-invariant. We further use $\mathbb{E}[x_{it}]$ for the (population) expected value, where the subscript indicates whether the expectation is taken over the cross-section ($\mathbb{E}[x_t]$), time ($\mathbb{E}[x_i]$), or both $\mathbb{E}[x_{it}]$. Analogously, $\mathrm{Var}(x_{it})$ and $\mathrm{Cov}(x_{it}, y_{it})$ represent the population variance and covariance, respectively. Asterisks x^* refer to the equilibrium or steady state value of a variable and hats $\hat{x}$ to the growth rate. We use the tilde $\tilde{x}^*$ for approximate mean steady state values that have been obtained from setting stochastic terms to their population mean values.

Variable definitions and identities.

Capacity utilisation	$u_{it} = \dfrac{Y_{it}}{Y_{it}^p}$	Aggregate utilisation	$u_t = \sum_{i=1}^{N} s_{it-1}u_{it}$
Pot. output–capital ratio	$\nu = \dfrac{Y_t^P}{K_{t-1}}$	Aggregate output	$Y_t = \sum_{i=1}^{N} Y_{it}$
Profit share	$\pi_{it} = \dfrac{\Pi_{it}}{Y_{it}}$	Agg. profit share	$\pi_t = \sum_{i=1}^{N} q_{it}\pi_{it}$
Profit rate	$r_{it} = \dfrac{\Pi_{it}}{K_{it-1}} = \pi_{it}\nu u_{it}$	Agg. profit rate	$r_t = \sum_{i=1}^{N} s_{it-1}\pi_{it}\nu u_{it}$
Capital accumulation	$K_{it} = (1+g_{it})K_{it-1}$	Agg. capital stock	$K_t = \sum_{i=1}^{N} K_{it}$
Relative firm size	$s_{it} = \dfrac{K_{it}}{K_t}$	Agg. growth rate	$g_t = \sum_{i=1}^{N} s_{it-1}g_{it}$
Market share	$q_{it} = \dfrac{Y_{it}}{Y_t}$	Agg. markup	$\tau_t = \sum_{i=1}^{N} q_{it-1}\tau_{it}$
Labour productivity	$\delta_{it} = \dfrac{Y_{it}}{L_{it}}$	Agg. consumption rate	$c_t = \dfrac{C_t}{K_{t-1}}$
Price	P_{it}		

Table A1: Empirical and Simulated Moments

Description	Empirical moment	Simulated moment
Mean firm-level profit share	0.1864	0.2067
Median firm-level profit share	0.1497	0.1513
Variance of firm-level profit share	0.0217	0.0211
Skewness of firm-level profit share	1.647	2.576
Normalised HHI of market shares	0.1456	0.1431
Skewness of market share	4.003	4.053
Mean firm-level capital growth rate	0.0914	0.1037
Mean aggregate capacity utilisation	0.7702	0.7049

Notes: Skewness is computed as the third standardised moment: $skew(x_i) = \frac{\sum_{i=1}^{N}(x_i-\bar{x})^3}{NSD(x)^3}$. The normalised HHI of market shares (q) is computed as: HHI $= \frac{\sum_{i=1}^{N} q_i^2 - \frac{1}{N}}{1-\frac{1}{N}}$.