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Anisotropic stress-induced permeability change in cracked rock during the crack closure stage: A tensorial approach incorporating tortuosity and bottlenecks

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1. Introduction

1.1 Background

Cracks in rocks are formed as a consequence of tectonic activity, including faulting and jointing, excess pore pressure and thermal stress. Therefore, the cracks observed in rocks from outcrops and borehole cores record their damage history. Even today, tectonically-induced brittle cracks and grain boundary cracks continue to form, contributing to the evolving geological history of rocks at depth. Cracks act as pathways, supplying fluid to faults and reducing effective stress due to excess pore pressure, and are a key factor in geophysics when considering fault mechanics (e.g., Chen and Nur, 1992; Evans and Wong, 1992; Gratier et al., 2003; Healy, 2009; Rice, 1992; Zimmerman and Paluszny, 2023). In CCS (Carbon dioxide Capture and Storage) and geothermal development, it is also important to evaluate fluid migration in cracked rocks subjected to confining pressure and tectonic stress in the subsurface, and many studies have been conducted on the coupling between crack geometry, permeability and stress (Feitz et al., 2022; Ma et al., 2023; Rizzo et al., 2024; Sarout et al., 2017; Suzuki et al., 2024a; Suzuki et al., 2024b).

The permeability of cracked rock can be mathematically described as a function of the crack aperture and orientation using the parallel plate model (Snow, 1969). Oda (1985) proposed a permeability tensor concept (Oda's permeability tensor) that incorporates observable three-dimensional (3D) geometric information of cracks, i.e., the orientation, aperture and crack density, into an equivalent permeability model based on the parallel-plate model, and this model has been applied in many studies (e.g. Akamatsu et al., 2023; Brown, 1987; Pakdel and Pietruszczak, 2024) （see Section 2.1）. From the perspective of statistical methods and percolation theory, Guéguen and Dienes (1989) proposed simplified microstructural models using pipes or cracks to describe the microstructure of porosity and used these to calculate permeability and conductivity of a cracked rock in the isotropic case. These permeability models are applied to evaluate the permeability of cracked rocks under stress as an equivalent permeability (e.g., Brown and Bruhn, 1998; Delle Piane et al., 2015; Oda et al., 2002b; Shao et al., 2005; Simpson et al., 2001). Here, representing the permeability of cracked rock by an equivalent permeability is essential for describing fluid flow behavior within the framework of continuum mechanics. The general form of a stress-dependent permeability tensor has been proposed by Oda et al. (2023) considering the closing or opening of crack apertures as a result of the change in the effective normal stress acting on crack surfaces （see Section 2.2）.

Under polyaxial stress states, which must be the general condition within the crust, the intermediate principal stress significantly influences the hydro-mechanical behavior of rocks (e.g., Haimson and Rudnicki, 2010; Healy et al., 2015; Mogi, 2006; Panaghi et al., 2021; Sato et al., 2018). The relationship between cracks and permeability in such a general 3D stress state can be evaluated using a permeability model under anisotropic stress conditions. Consequently, the development and validation of a 3D stress-dependent permeability tensor is of key importance in establishing the reliability of modeled permeability under polyaxial stresses condition.

1.2 Rationale

In the permeability tensor concept in a 3D Cartesian coordinate system, fluid flow, for example, along the z- (=x_3-) axis, is dominated by cracks oriented normal to the x- (=x_1-) and y- (=x_2-) axes, which are the dominant fluid pathways; i.e., cracks that have components normal to the flow direction do not contribute to permeability. Therefore, considering the stress distribution of a laboratory conventional triaxial test on a cylindrical rock sample, with axial σ_z > radial P_c (= $\sigma_x = \sigma_y$), the permeability in the σ_z-direction should increase since cracks parallel to the σ_z-axis increase in number and dilate with increasing differential stress. Here, the stress distributions used in this study are summarized in Fig. 1. In addition, σ_x, σ_y, and σ_z denote the stresses acting along the x-, y- and z-axes, respectively, while $\sigma_{\max}$, σ_{int} and $\sigma_{\min}$ represent the maximum, intermediate, and minimum principal stress values. When stress is applied normal to a crack, the crack closes and significantly impacts the permeability coefficient. Fig. 2 shows the variation in permeability along the σ_{max}-direction for conventional triaxial tests in cracked granitic rocks and massive basalt as reported in previous studies (Chen, 2018; Fortin et al., 2011; Jia et al., 2021; Kiyama et al., 1996; Mitchell and Faulkner, 2008; Shao et al., 2005; Wang et al., 2014; Zoback and Byerlee, 1975). At present, no studies reporting the results of permeability tests on cracked rocks under polyaxial stress conditions have been found. The permeability decreases up to the dimensionless failure level, FL, of approximately 0.4 in most samples, after which it transitions to a rapid or gentle increase. This is because crack growth begins at around $FL \approx 0.4$ in brittle rocks such as granite, increasing crack surface density through interactions and eventually leading to brittle failure (e.g., Martin and Chandler, 1994; Paterson and Wong, 2005). For FL<0.4, pre-existing cracks normal to the σ_{max}-direction are closed, and pre-existing cracks parallel to this axis are opening and preparing for crack growth. This regime is referred to as the crack-closure stage in this study and corresponds to the range indicated by the solid line in Fig. 2. Therefore, the decrease in permeability along the σ_{max}-direction, which is influenced by the cracks parallel to the σ_{max}-direction, cannot be explained by the conventional permeability tensor concept introduced at the beginning of this section. This suggests that the cracks normal to the σ_{max}-direction contribute to the decrease in permeability. Thus, the cracks parallel to the σ_{max}-direction act as the main fluid pathways, while the cracks normal to the σ_{max}-direction act as bottlenecks that prevent or restrict fluid flow within the overall crack network. In this study, the term bottleneck is used to describe a local aperture constriction, representing the narrowest segment along a flow path, and is quantitatively expressed in terms of constrictions. In addition, cracks normal to the σ_{max}-direction are expected to act as a factor that alters the effective length of favorable flow paths when aperture variations are taken into account. This effect can be evaluated in terms of changes in aperture-weighted tortuosity. The effect of cracks normal to the σ_{max}-direction on permeability needs to be given significant attention, particularly in cracked rocks subjected to anisotropic stress and below the percolation or failure threshold. In this study, we attempt to develop a more comprehensive permeability tensor concept by treating tortuosity and bottleneck effects as a new crack network parameter. (see Section 2.3).

In this study, 3D crack models were developed and numerical simulations were performed to investigate the effects of connectivity, tortuosity, and bottlenecks on the equivalent permeability of cracked rock in which fluid flow occurs through the crack network, based on the permeability tensor concept. Here, the equivalent permeability is required to link permeability behavior with stress and elastic wave velocities within the framework of continuum mechanics. On the other hand, the permeability of cracked rock can also be evaluated by explicitly representing the crack network in a discrete manner and calculating permeability using a discrete fracture network (DFN) approach. By applying the same crack model, we compare the equivalent permeability with the permeability obtained from the DFN method, in order to assess the effectiveness of the modified equivalent permeability proposed in this study (see Section 3). In these numerical simulations, the changes in crack aperture under anisotropic stress applied to various crack models were calculated using the method proposed by Oda et al. (2023). Subsequently, a crack network parameter accounting for tortuosity and bottleneck effects was calculated, and its directional dependence was evaluated and discussed to ensure its validity (see Section 4.1). Furthermore, the effects of variations in crack density and anisotropy in cracked rock on the crack network parameter, incorporating tortuosity and bottleneck effects, were discussed (see Section 4.2). Using the equivalent permeability calculated from a proposed modified correction factor that accounts for tortuosity, bottleneck effects, and crack density dependent connectivity, we discuss how anisotropic stress conditions and anisotropy in crack fabric influence permeability. In this study, the term connectivity refers to the fraction of geometrically connected cracks. We further propose an approach for evaluating permeability in the loading direction in triaxial compression experiments (Section 4.3). The proposed modified correction factor for permeability tensor cannot be directly measured from natural rock samples. How, then, can it be determined? If the permeabilities in multiple directions and the geometric information of cracks are provided, it can be back-calculated using the permeability tensor concept (see Section 4.4). The limitations and key considerations when applying the proposed method are summarised (see Section 4.5).

The change in connectivity and the crack network parameter within rocks is important not only from the perspective of permeability evolution but also as an indicator for evaluating crack interactions during fault development. The aim of this study is to propose the crack network parameter incorporated into the modified correction factor, with a particular focus on understanding crack interactions under anisotropic stress conditions through their effect on permeability. The key findings are summarized in Section 5, and the symbols used in this study are summarized in Table S1.

2. Permeability from viewpoint of crack geometry

2.1 Permeability tensor proposed by Oda (1985)

If a cracked rock is idealised as an anisotropic porous medium whose rock matrix is impermeable, the following assumptions are adopted: 1) Laminar flow of fluid occurs through a crack bounded by impermeable rock matrices according to the cubic law. 2) The gradient of the total hydraulic head Φ, expressed as $-\partial\Phi/\partial x_i$, is homogeneously distributed within a REV. 3) The crack aperture *t* represents the hydraulically equivalent aperture and varies with the normal stress acting on each crack. Under these assumptions, a permeability tensor k_{ij} [m^2] in Darcy's law can be expressed as follows:

$$v_i = -k_{ij}\frac{\partial\Phi}{\partial x_j} \qquad (1)$$

$$k_{ij} = \frac{\lambda}{12}\left(P_{kk}\delta_{ij} - P_{ij}\right) \qquad (2)$$

where, v_i [m s^{-1}] is the apparent seepage velocity, δ_{ij} is the Kronecker's delta, and Einstein's summation convention is implied. Eq. (2) was proposed by Oda (1985) and is referred to as Oda's permeability tensor. Here, P_{ij} [m^2] is a symmetric second-rank tensor depending only on the geometrical aspects of the cracks and is given by

$$P_{ij} = \pi\rho\int_0^\infty\int_0^\infty\int_{\Omega/2} r^2 t^3 n_i n_j 2E(\boldsymbol{n}, r, t)\,d\Omega\,dr\,dt \qquad (3)$$

Here, this approach makes the assumption of flat, penny-shaped cracks with a uniform aperture. In Eq. (2), λ is a dimensionless parameter $(0 < \lambda \leq 1)$ that represents the connectivity, which is the fraction of geometrically connected cracks. Within this framework, a density function $E(\boldsymbol{n}, r, t)$ is introduced, and Ω is the entire solid angle equal to 4π. Instead of $E(\boldsymbol{n}, r, t)$, $2E(\boldsymbol{n}, r, t)$ is sometimes used since $E(\boldsymbol{n}, r, t) = E(-\boldsymbol{n}, r, t)$. Here, $\boldsymbol{n}$ is a unit vector normal to the crack surface, r is the diameter of a circular crack [m], t is the crack aperture, and ρ is the crack density [m^{-3}], defined as the number of cracks per unit volume. If the three variables $\boldsymbol{n}$, r, and t are statistically independent, $E(\boldsymbol{n}, r, t)$ can be written as $E(\boldsymbol{n})\,f(r)\,g(t)$, where $E(\boldsymbol{n}), f(r)$ and $g(t)$ are density functions of $\boldsymbol{n}$, r, and t, respectively. Then, Eq. (3) can be rewritten as

$$\begin{aligned} P_{ij} &= \pi\rho\int_0^\infty r^2 f(r)dr\int_0^\infty t^3 g(t)dt\int_{\Omega/2} n_i n_j 2E(\boldsymbol{n})d\Omega \\ &= \pi\rho < r^2 >< t^3 > N_{ij} \qquad (4) \end{aligned}$$

with using the following notations:

$$< r^n > \equiv \int_0^\infty r^n f(r)dr \qquad (5)$$

$$N_{ij} \equiv \int_{\Omega/2} n_i n_j 2E(\boldsymbol{n})d\Omega = \int_\Omega n_i n_j E(\boldsymbol{n})d\Omega \qquad (6)$$

In Eqs.(4) – (6), ρ and r are still retained as 3D geometric information. Oda (1985) replaced ρ and r with physically measurable quantities using stereology (Kanatani, 1984) and rewrote Eq. (4) as follows.

$$P_{ij} = \frac{m(\boldsymbol{q})}{< |\boldsymbol{n}\cdot\boldsymbol{q}| >} < t^3 > N_{ij}$$

$$= S_0 < t^3 > N_{ij} \quad (7)$$

Here, $m(\boldsymbol{q})$ represents the number of cracks intersecting a scan line in direction $\boldsymbol{q}$, $< |\boldsymbol{n} \cdot \boldsymbol{q}| >$ is the average of the inner product between $\boldsymbol{n}$ and $\boldsymbol{q}$, referred to as the correction term (also see Terzaghi (1965)), and S_0 [m^{-1}] is the crack surface area of all cracks per unit volume. Since N_{ij} can be determined *in-situ* from measurements, for example of strike and dip using a clinometer, it demonstrates that P_{ij} can be determined from the observable crack geometry. Here, N_{ij} can also be determined using stereological methods when three sectional trace maps can be prepared from a rectangular specimen (Oda, 1982). To use Eq. (7) in theory, two requirements must be satisfied: 1) Cracks must be distributed in a statistically homogeneous manner within a representative volume, and 2) variables $\boldsymbol{n}$, r and t must be statistically independent.

2.2 Stress-dependent permeability tensor

The depth dependency of crack aperture was proposed by Bianchi and Snow (1968), and they formulated an equation for crack aperture, t, as a function of depth z based on empirical data. Oda et al. (2023) then rewrote this equation to the more general condition of the stress dependency of crack aperture on effective normal stress, as follows.

$$t = t_0 \left(1 - \frac{\sigma'_n}{\gamma' H + \sigma'_n} \right) \quad (8)$$

where t_0 is the aperture at the ground surface; H [m] denotes a mechanical parameter that depends on crack stiffness, roughness, and filling material; γ' [N m^{-3}] is the submerged unit weight of the rock; and σ'_n [N m^{-2}] is the effective normal stress acting on the crack surface. Using Eq. (8) in Eq. (7), the P_{ij} was given by

$$P_{ij} = S_0 \left\{ t_0 \left(1 - \frac{\sigma'_n}{\gamma' H + \sigma'_n} \right) \right\}^3 N_{ij} \quad (9)$$

Here, for simplicity, we assume an idealised model of three mutually-orthogonal sets of cracks, whose normal vectors are oriented along the principal axes. Here, $\gamma' H$ in Eq. (9) is the pseudo-elastic modulus related to crack closing (discussed further in Suzuki et al. (2024b)). Suzuki et al. (2024b) reported that stress-dependent mean hydraulic conductivity $K_{ij} (= (g/\nu) k_{ij})$, where g [m s^{-2}] is the gravitational acceleration and ν [m^2 s^{-1}] is the kinematic viscosity of the fluid, corresponds well with field measurements in two sites in Japan and Sweden using the same idealisation of a cracked rock mass with a three mutually orthogonal crack-set model. In addition, their study assumes λ=1, indicating that the cracks are fully connected to each other. Furthermore, Oda (2025) reported validations of the stress dependence of permeability at multiple field sites.

2.3 A new crack network parameter T_{ij} incorporating tortuosity and bottleneck in tensor form

Cracks can be considered to form a network, and the connectivity between cracks significantly affects the permeability. In the Oda's permeability tensor concept, the degree of connectivity is represented by λ. When $\lambda = 1$, all cracks contribute to fluid flow, whereas $\lambda = 0$ if none contribute, rendering the cracked medium impermeable. Specifically, when $\lambda = 1$, the relationship between the hydraulic gradient and local velocity follows the cubic law flow (Long et al., 1982; Oda, 1985; Snow, 1969). The parameter λ is a function of crack density, as considered in percolation theory. The functional form of λ as a function of crack density, $\lambda(F_0)$, derived theoretically by Robinson (1984), is given as follows.

$$\lambda(F_0) = 1 - \frac{c_1}{F_0} \tag{10}$$

Here, c_1 denotes the crack density at the percolation threshold, and a value of $c_1 = 1.39$ was derived by Robinson (1984). In addition, its validity has been verified by Oda et al. (1987) and Suzuki et al. (1998) based on empirical data, however, they noted that $\lambda(F_0)$ cannot be expressed as a simple function of crack density. Here, F_0 is the dimensionless crack density weighted by the radius of the cracks and is expressed as $F_0 = 2\pi\rho\langle r^3\rangle$, called a crack tensor (Oda, 1983) (The definition of the crack tensor is described in detail in the following section). Similarly, many authors have discussed the percolation threshold using $\rho\langle r^3\rangle$ in numerical studies (e.g. Guéguen and Palciauskas, 1994). Robinson (1984) reported a percolation threshold of $\rho\langle r^3\rangle$ =0.154 (F_0 =0.967) for a system composed of square cracks with a side length, and Charlaix (1986) reported a threshold of $\rho\langle r^3\rangle$ =0.180 (F_0 =1.130) for a system of randomly oriented discs.

Guéguen and Dienes (1989) introduced the probability of intersection between pipes or cracks, which has nearly the same meaning as λ. Assuming that the centers of pipes or cracks are distributed on a random network, the percolation threshold is described as depending on the mean coordination number. Furthermore, the connectivity can also be defined in terms of tortuosity τ for porous network, as $\lambda = \tau^{-1}$ (Caravella et al., 2023). In their study, both λ and tortuosity exhibit anisotropy and are expressed as tensors. Tortuosity characterises the complexity of network pathways, expressed as the ratio of path length to straight-line Euclidean distance. In other words, when λ is low, the network becomes more complex, and τ increases, and when λ = 1, the crack network should behave like parallel plate flow. Walsh and Brace (1984) and Zhu and Wong (1996) have suggested that the permeability of low-porosity rocks is related to a geometric factor and tortuosity. Walsh and Brace (1984) demonstrated that increasing external pressure leads to crack closure, resulting in a decrease in permeability, and discussed the relationship between permeability, crack aperture, and tortuosity in granitic rocks. Moreover, Zhu and Wong (1996) indicated that the increase in stress-induced microcracks leads to a rise in tortuosity due to the geometric complexity of the evolving crack network, and that the permeability could decrease in a dilating rock with tubular pores and microcracks. In addition, permeability and tortuosity are often related through τ^{-2}, which serves as a connectivity coefficient in porous or cracked rocks (e.g., Keller et al., 2011a; Scholes et al., 2007; Walsh and Brace, 1984; Zhu and Wong, 1996), therefore, we also accept this relationship. Moreover, tortuosity exhibits anisotropy, Keller et al. (2011a) and Keller et al. (2011b) demonstrated that tortuosity has anisotropy in the Opalinus clay, and it is lower in the direction parallel to bedding planes and higher in the normal direction. Furthermore, Keller et al. (2011a) define a tortuosity tensor as $L_{ij}/\|a_{ij}\|$, where a_{ij} is the corresponding straight-line distance tensor [m] in each direction, and L_{ij} is the transport path length tensor [m], representing the effective path length. However, the physical definition of the concept remains unclear, and thus the following relationship is introduced in the present study to define the anisotropy of tortuosity more explicitly.

$$\tau^{(k)} = \left(\frac{L^{(k)}}{a^{(k)}}\right) \qquad (11)$$

where k are the directional cosines of a unit vector $\boldsymbol{k}$, and $\tau^{(k)}$ [m/m, dimensionless] is the tortuosity in the k-direction, and $a^{(k)}$ is the straight-line distance between two planes normal to the k-direction, and $L^{(k)}$ is the flow path length in the k-direction, representing the effective path length. Tortuosity on its own cannot adequately describe flow behavior under conditions in which the apertures of cracks normal to the loading axis are significantly reduced, such as under anisotropic stress conditions. This limitation is due to the fact that conventional tortuosity is evaluated solely based on topological connectivity and does not account for changes in hydraulic efficiency or flow capacity associated with aperture variations. In such cases, although these cracks may not be completely closed, a reduction in aperture can substantially decrease their hydraulic contribution. This suggests that aperture-controlled effects need to be considered in the evaluation of tortuosity. Accordingly, in order to evaluate the ease of fluid flow, it is necessary to consider flow paths weighted by the inverse cube of the aperture, i.e. flow paths are selected based on aperture-weighted tortuosity. Figs 3(a)–(c) illustrate the flow paths under hydrostatic and anisotropic stress conditions, while Fig. 3(d) summarises the parameters related to tortuosity and bottleneck. In this figure, the lengths of three flow paths under each of the two stress conditions are compared. $L^{(k)}$ denotes the unweighted path length, whereas aperture-weighted path length, $L^{(k)}/t$, represents the weighted length, incorporating aperture via the reciprocal weighting. In Model A, all cracks have the same aperture, resulting in identical rankings for $\tau^{(k)}$ and $L^{(k)}$ with A-3 being the most favourable flow path, where $\tau^{(k)}|_{k=0,0,1}$=1.54. However, in Model B, the influence of crack closure normal to the loading axis is reflected in the weighted path lengths, resulting in B-1 being the most favourable flow path, in which $\tau^{(k)}|_{k=0,0,1}$=1.56 should be adopted. This result suggests that anisotropic crack closure impedes fluid flow and forces it along more tortuous routes, thereby reflecting the presence of bottlenecks. Notably, the aperture does not affect tortuosity itself, but it does influence the selection of flow paths. The bottleneck, which is a distinct consequence of aperture constriction, should be considered separately from tortuosity. Here, the bottleneck refers to the restriction of transport properties caused by narrow pore or crack throats or reduced cross-sectional areas within the network (e.g., Holzer et al., 2023; Meredith et al., 2012). To account for the resistive effects of bottlenecks in fluid flow, Petersen (1958) introduced constrictivity, β, as a correction parameter in transport equations (Holzer et al., 2023). Moreover, β can be defined for each flow path in the k-direction and is expressed as $\beta^{(k)}$ [m/m, dimensionless] for each direction, as follows.

$$\beta^{(k)} = \frac{A_{min}^{(k)}}{< A >^{(k)}} = \frac{t_{min}^{(k)}}{< t >^{(k)}} \quad (12)$$

where, $< A >^{(k)}$ [m] and A_{min} [m] represent the average and minimum cross-sectional areas of the flow path in the k-direction, respectively. In this study, assuming a constant width of the flow paths in the cracks network, then $A_{min}^{(k)}$ corresponds to the minimum aperture $t_{min}^{(k)}$, while $< A >^{(k)}$ corresponds to the average aperture $< t >^{(k)}$, and $< t >^{(k)}$ represents the weighted average along the flow path within the cracks, therefore, $\beta^{(k)}$ is a geometrical index that represents the influence of the constricted portion $t_{min}^{(k)}$ relative to a reference aperture in the *k*-direction. In this formulation, the reference value may be taken as $t_{\max}$, or it has also been proposed to be defined as an average weighted by the aperture cubed (e.g., Holzer et al., 2023; Petersen, 1958; Talon et al., 2010). Moreover, because the lengths of intersecting cracks are largely controlled by random factors, the width is assumed to be constant here. In a more rigorous treatment, however, the bottleneck effect should be expressed in terms of cross-sectional area, and thus the resulting value of $\beta^{(k)}$ is likely to be somewhat smaller than that obtained under the present assumption.

In the case of cracked rock subjected to anisotropic stress, when permeability is evaluated within the framework of the permeability tensor concept (Eq. (2)), only those cracks that are oriented normal to both the *x*- and *y*-axes (i.e., aligned parallel to the *z*-axis, the loading direction) contribute to fluid flow. However, cracks oriented normal to the loading axis (*z*-axis), whose apertures decrease under compression stress, do not contribute to permeability. Accordingly, the parameter $\beta^{(k)}$ reflects the influence of cracks that are oriented normal to the loading direction on the overall permeability. Here, $\beta^{(k)}$ is measurable using mercury injection porosimetry as the minimum value of $\beta^{(k)}$ without directional information, that is, as a scalar value, however, it is recommended to combine this method with three-dimensional imaging or numerical simulations for more detailed analysis (e.g., Holzer et al., 2023).

The parameters $\tau^{(k)}$ and $\beta^{(k)}$ are used as coefficients in diffusion coefficients, electrical conductivity, and permeability to relate experimental results to ideal models. For cracked rocks, Walsh and Brace (1984) and Zhu and Wong (1996) introduced a geometric factor to relate the model to the experimental values, and Oda et al. (2002b) pointed out that some kind of coefficient, such as a scaling factor, is also needed in addition to connectivity. In this study, these geometric or scaling factors are considered to be influenced by tortuosity and bottlenecks, and the following dimensionless second-rank tensor T_{ij} as a coefficient is introduced.

$$\frac{\beta^{(k)}}{(\tau^{(k)})^2} = T_{ij} k_i k_j \quad (13).$$

Here, T_{ij} is a new crack network parameter, representing a directional modification of permeability under anisotropic stress conditions. As the geometric or scaling factor, we adopted the form β/τ^2 proposed by van Brakel and Heertjes (1974). It characterizes the reduction in transport efficiency due to both tortuosity and bottlenecks within the crack network.

Fig. 3 is here examined from the perspective of bottlenecks, with the values of $\beta^{(k)}$ summarised in Fig. 3(d). In Model A under hydrostatic conditions, all cracks close uniformly in every direction, and therefore there is no bottleneck. As a result, $\beta^{(k)}$ is constant at 1. In contrast, in Model B under triaxial stress conditions, cracks normal to the loading axis are closed, which act as bottlenecks and lead to a reduction in the value of $\beta^{(k)}$. Thus, in this study, $\beta^{(k)}$ is used as a correction factor reflecting the influence of cracks normal to the loading axis, which the conventional permeability tensor concept does not account for.

Furthermore, by introducing T_{ij} as a coefficient of λ or $\lambda(F_0)$, a modified correction factor for Oda's permeability tensor Λ_{ij}, is defined as follows, incorporating the connectivity as well as tortuosity and bottleneck effects.

$$\Lambda_{ij} \equiv \lambda(F_0)T_{ij} \qquad (14)$$

In this study, since $\lambda(F_0)$ is defined using an expression that depends on crack density (Eq. 10), the above equation is rewritten as follows.

$$\Lambda_{ij} \equiv \left(1 - \frac{1.39}{F_0}\right)T_{ij} \qquad (14')$$

Here, if the crack aperture is constant, then $\beta^{(k)} = 1$,and if the crack density is sufficiently high, the flow paths become nearly straight, so that $\tau^{(k)} \cong 1$. Consequently, $T_{ij} \cong 1$, indicating that Λ_{ij} becomes nearly equivalent to $\lambda = 1$.

In this study, we consider an idealised model consisting of three mutually orthogonal crack sets(e.g., Oda et al., 2023; Suzuki et al., 2024a) as a model for calculating the permeability tensor k_{ij}, and assume that the principal stress axes are coaxial with the principal axes of the crack fabric. As a result, the off-diagonal elements of k_{ij} are zero, i.e., $k_{12} = k_{13} = k_{23} = 0$. In addition, because the aperture t depends on direction, it is necessary to define P_{ij} separately for each direction. Finally, the stress-dependent modified permeability tensor incorporating tortuosity and bottleneck is expressed as follows.

$$
\begin{aligned}
k_{11} &= \frac{1}{12}\Lambda_{11}S_0\left[\left\{t_0\left(1-\frac{\sigma_{22}'}{\gamma' H+\sigma_{22}'}\right)\right\}^3 N_{22}+\left\{t_0\left(1-\frac{\sigma_{33}'}{\gamma' H+\sigma_{33}'}\right)\right\}^3 N_{33}\right]\\
k_{22} &= \frac{1}{12}\Lambda_{22}S_0\left[\left\{t_0\left(1-\frac{\sigma_{11}'}{\gamma' H+\sigma_{11}'}\right)\right\}^3 N_{11}+\left\{t_0\left(1-\frac{\sigma_{33}'}{\gamma' H+\sigma_{33}'}\right)\right\}^3 N_{33}\right] \qquad (15)\\
k_{33} &= \frac{1}{12}\Lambda_{33}S_0\left[\left\{t_0\left(1-\frac{\sigma_{11}'}{\gamma' H+\sigma_{11}'}\right)\right\}^3 N_{11}+\left\{t_0\left(1-\frac{\sigma_{22}'}{\gamma' H+\sigma_{22}'}\right)\right\}^3 N_{22}\right]
\end{aligned}
$$

3. Numerical calculation

3.1 Crack Models and Numerical Framework

Numerical calculations were performed to evaluate Λ_{ij} under anisotropic stress, taking into account tortuosity and bottlenecks. A cubic domain with dimensions of 100 mm × 100 mm × 100 mm was generated numerically, and within this domain, a crack model was created by randomly placing $m^{(V)}$ cracks, each with a radius of 10 mm. Here, the numerical calculations for the model were implemented in Python, with random positions generated using a seed argument, resulting in the creation of 100 different instances. In this study, we employ a model consisting of three mutually orthogonal crack sets, in which crack orientations are aligned with the principal axes. That is, when represented on a stereonet, the crack normals are distributed with three dominant poles. Suzuki et al. (2024b) showed that crack distributions observed in granite and sedimentary rock outcrops can be reasonably approximated by three mutually orthogonal crack sets. Similarly, Takemura et al. (2003a), Nadan and Engelder (2009) and Freire-Lista and Fort (2017) reported that the major crack systems in granite can be represented by three mutually orthogonal crack sets. The normal vectors of the cracks were aligned with either the x ($=x_1$), y ($=x_2$), or z ($=x_3$) axes (three mutually orthogonal crack-set model), and in the crack models, their preferred orientations were configured as one isotropic case (Iso) and two anisotropic cases (Ani-1 and Ani-2). Under the Iso condition, the crack normals have an orientation ratio of 1:1:1, meaning that equal numbers of cracks are oriented along the x-, y-, and z-axes; consequently, the crack distribution is isotropic with $AI = 0$. Under the Ani-1 condition, the orientation ratio becomes 2:2:1, resulting in moderate anisotropy with $AI = 0.28$. Under the Ani-2 condition, the orientation ratio is 4:1:1, and the anisotropy is more pronounced, with $AI = 0.71$. Additionally, the stress distribution was modeled in three cases: one case each for hydrostatic (Hydro), conventional triaxial stress (Triaxial), and polyaxial stress (Polyaxial). For the Hydro case, the applied stresses were $\sigma_x = \sigma_y = \sigma_z = 10$ MPa. For the Triaxial case, $\sigma_x = \sigma_y = 10$ MPa and $\sigma_z = 50$ MPa. For the Polyaxial case, $\sigma_x = 10$ MPa, $\sigma_y = 20$ MPa, and $\sigma_z = 50$ MPa. A total of 9 crack models combining different orientations and stress distributions were generated, and the number of cracks $m^{(V)}$ was varied stepwise across 8 different levels.

In this study, to quantify crack density and the degree of crack orientation anisotropy, the dimensionless crack density F_0 and the anisotropy index AI were defined using the crack tensor F_{ij}. Since all of the data concerning the crack geometry are known a priori, F_{ij} for these models can easily be calculated as follows (Oda, 1986):

$$F_{ij} = \frac{1}{V}\sum_{k=1}^{m^{(V)}} 2s^{(k)}r^{(k)}n_i^{(k)}n_j^{(k)} \qquad (16)$$

$$F_0 = F_{ii} = F_1 + F_2 + F_3 = \frac{2\pi}{V}\sum_{k=1}^{m^{(V)}}\left(r^{(k)}\right)^3 = 2\pi\rho\langle r^3\rangle \qquad (16')$$

$$AI = \frac{\sqrt{6J_2^{(\hat{F})}}}{F_0} = \frac{\sqrt{3\hat{F}_{ij}\hat{F}_{ij}}}{F_0} = \frac{1}{F_0}\sqrt{3(F_{ij} - \frac{1}{3}F_{kk}\delta_{ij})(F_{ij} - \frac{1}{3}F_{ll}\delta_{ij})}$$

$$= \frac{\sqrt{[(F_1 - F_2)^2 + (F_2 - F_3)^2 + (F_3 - F_1)^2]}}{F_1 + F_2 + F_3} \quad (17)$$

where the superscript (k) stands for a k^{th} crack among $m^{(V)}$ in V. $n_i^{(k)}$ and $n_j^{(k)}$ are the direction cosines of $\boldsymbol{n}^{(k)}$ with respect to orthogonal reference axes x_i (i=1, 2, 3), and can be rewritten using their strikes and dips. Here, we use the relationship $n_i^{(k)}n_i^{(k)} = n_1^{(k)}n_1^{(k)} + n_2^{(k)}n_2^{(k)} + n_3^{(k)}n_3^{(k)} = 1$, and F_0 is the dimensionless crack density weighted by the radius of the cracks. In addition, $J_2^{(\hat{F})}$ in Eq. (17) is the second invariant of the deviatoric tensor of F_{ij}, denoted by $\hat{F}_{ij}$. Note that *AI* ranges from 0 to 1.41. If *AI* equals 0, for instance, the crack orientation is isotropic (i.e., $F_1 = F_2 = F_3$). On the other hand, if *AI* equals 1.41 (=$\sqrt{2}$), all cracks are oriented in the same direction, representing the highest degree of anisotropy (i.e., parallel, with $F_1 = 1$, $F_2 = F_3 = 0$). Incidentally, the numerator term in Eq. 17 represents the equivalent shear stress when employed with the stress tensor and the equivalent shear strain when employed with the strain tensor.

The conditions of these crack models are summarized in Table 1. Fig. 4 shows the crack traces observed on each cross-section (x=0, y=0, z=0 planes) for crack models generated under the conditions of Iso, Ani-1, and Ani-2 with varying degrees of anisotropy and different crack densities. Here, the black, red, and green crack traces represent cracks with normal vectors in the x-, y- and z-directions, respectively. Hereafter, the colors for the x, y and z directions will follow this scheme. Under the Iso condition, the crack distribution appears nearly uniform across all cross-sections (Fig. 4(a)). Under the Ani-1 condition, the number of cracks with normals oriented in the z-direction is half that of those oriented in the x- and y-directions. As a result, fewer crack traces with z-oriented normals (green crack traces in Fig. 4(b)) are observed on the x- and y-cross-sections. In contrast, under the Ani-2 condition, the number of cracks with normals oriented in the x-direction is four times that of those oriented in the y- and z-directions. Consequently, crack traces with x-oriented normals (black crack traces in Fig. 4(c)) are frequently observed on the y- and z-cross-sections.

3.2 Influence of Connectivity, Tortuosity and Bottleneck on the stress-dependent modified Permeability Tensor under Anisotropic Stress

In this study, numerical simulations were performed using the crack models generated in Section 3.1 to derive equivalent permeability that incorporates the effects of tortuosity and bottlenecks under hydrostatic, conventional triaxial, and polyaxial stress conditions. In the generated crack models, the cracks intersecting the boundary planes of the domain (e.g., z=0 and z=100) are represented as line segments (crack traces). The midpoint of each crack trace is defined as a node on the boundary surface. Here, an aperture is assigned to each crack (Fig. 5). The fluid pathway traverses the intersection lines between cracks within the domain. The midpoint of each intersection line is defined as a node on the crack intersection. A line segment connecting two nodes is defined as an edge, and fluid flows along these edges. In the numerical calculations for each model, $\tau^{(k)}$ is determined by considering the coefficients $a^{(k)}$ exclusively in the x, y and z-direction. Furthermore, for the sake of simplification, the distance is defined not based on the directional vector between the nodes on the opposing boundary surfaces, as originally formulated by Eq.11, but instead as the distance between boundary surfaces (Fig.5).

This $\tau^{(k)}$ was determined by performing a shortest-path search (using Dijkstra's algorithm (Dijkstra, 1959) implemented in Python's NetworkX) from nodes appearing at x=y=z=0 as the starting point to nodes appearing at x=y=z=100 as the endpoint. For example, if there are n nodes at x=0 and m nodes at x=100, shortest-path searches were performed for all $m \times n$ combinations, calculating paths up to the 10th shortest in order (a total of $m \times n \times 10$). A percolating flow path is formed by multiple edges connecting nodes, with each edge containing information about its aperture. The initial aperture of the cracks is arbitrarily set to 100 μm and is changed according to the normal stress applied to the crack surfaces, based on Eq. 8. In this study, stress values in each direction were substituted into σ'_n in Eq. 8, based on the stress conditions, hydrostatic (Hydro), triaxial (Tri), and polyaxial (Poly), for the models listed in Table 1. Here, the crack stiffness H was set to 20 m, a value used for hard rock. (Oda et al., 2025; Suzuki et al., 2024b). Specifically, the aperture is 4.79 μm at 10 MPa, 2.43 μm at 20 MPa, and 0.98 μm at 50 MPa. In the flow path search, the apertures are uniform in the hydrostatic model (Hydro), whereas they vary with direction in the anisotropic stress models (Triaxial, Polyaxial). Therefore, in the path search calculation, the reciprocal of the aperture was given as the weight for each edge, and the algorithm was designed to determine the path with the smallest total weight as the shortest path. The constrictivity $\beta^{(k)}$ was calculated for each path in the x, y, and z directions as the ratio of the weighted mean of the aperture to the minimum aperture, using Eq. 12. Here, in the hydrostatic (Hydro) model, there is no bottleneck effect, resulting in $\beta^{(k)}$=1 in all directions. The flowchart of the numerical procedure is shown in Fig. 6. Finally, Λ_{ij} and the crack geometry information, S_0, N_{ij} and stress dependent t, obtained from the crack model were substituted into Eq. 15 to calculate the permeability in the x, y and z-direction for each model.

3.3 Comparing a Discrete Fracture Network (DFN) Model with an Equivalent permeability Model

To clarify the extent to which equivalent permeability derived from a permeability tensor differs from that obtained by averaging a discrete crack model, seepage flow analyses were performed using a discrete fracture network (DFN) model based on the crack models employed in Section 3.2. The DFN-based flow analyses were conducted using DFN.lab (Le Goc et al., 2019; Le Goc and R., 2023). Because DFN.lab is a Python-based framework that enables both DFN model construction and seepage flow analysis, the coordinate data of the crack models used in Section 3.2 were exported as CSV files and imported into DFN.lab to generate the DFN models. Here, the analysis was conducted for the range $F_0 \geq 2.83$, because numerical solutions could not be obtained in regions where the fraction of instances with a percolating flow path is low. In addition, crack apertures were assigned to each crack according to direction based on each stress condition (Hydro, Triaxial, and Polyaxial). For each model, the flow rates in the *x*-, *y*-, and *z*-directions were evaluated by imposing a hydraulic gradient corresponding to a pressure difference of 1 MPa in each direction, with inflow, outflow, and impermeable boundary conditions specified accordingly. For example, in the x-direction, inflow and outflow boundary conditions were applied to the two faces perpendicular to the *x*-axis, while the remaining faces were treated as impermeable boundaries.

4. Results and discussion

4.1 3D Crack Connectivity and percolating flow path

Fig. 7 shows the fraction of geometrically connected cracks as a function of crack density for models with different degrees of crack orientation anisotropy, *AI*. When $F_0 = 5.65$, all cracks are connected even with anisotropy of approximately *AI*=0.28 (Fig. 7(a,b)). However, when the anisotropy increases to *AI*=0.71, not all cracks are connected, even at high crack density (Fig. 7(c)). Thus, the fraction of isolated cracks tends to increase under conditions of low crack density or strong anisotropy. In addition, from the perspective of the permeability tensor, focusing solely on connectivity, it can be considered that nearly all cracks (more than 90%) are connected when $F_0 \geq 5.65$.

Fig. 8 shows the fraction of instances in which a percolating flow path connecting the boundary surfaces appeared in the *x*, *y*, and *z* directions, plotted as a function of crack density, for each crack model. Here, the stress conditions are set to hydrostatic, since the formation of percolating flow paths is independent of crack aperture changes caused by the applied stress. The vertical axis represents the ratio of instances in which percolating flow paths were formed to the total of 100 random crack generation instances for each model. percolating flow paths are presented for the *x*, *y*, and *z* directions. For example, the percolating flow path in the x-direction refers to a path formed by edges connecting the planes at *x*=0 and *x*=100. The percolating flow paths are formed in all directions at $F_0 \geq 3.77$, regardless of the degree of anisotropy. At $F_0 < 3.77$, variability is observed across directions, and the fraction of percolating flow paths decreases as the degree of anisotropy increases. Here, focusing on $F_0 < 3.77$ in the Ani-2 series, the fraction of percolating flow paths in the x-direction, which is normal to the most abundantly oriented cracks, is observed to be low. This is likely due to stronger anisotropy reducing the number of cracks functioning as bridges between other cracks.

The percolation threshold, defined as the crack density at which the percolating flow path starts to form, corresponds to $F_0 = 0.94$ in the present study. Guéguen et al. (1997) reported a percolation threshold corresponding to a crack density of 0.1. In their study, the crack density is expressed as ρr^3, and when recalculated in terms of the crack density F_0 $(= 2\pi\rho\langle r^3\rangle)$ used in this study, it corresponds to $F_0 = 0.63$. Preliminary calculations for F_0 lower than 0.94 indicated that percolating flow paths formed with a very low probability. Considering that percolating flow paths were observed in 3 out of 100 models with a crack density of an isotropically distributed crack model, it can be inferred that the percolation threshold lies slightly lower. Therefore, the threshold is considered to be close to the value reported by Guéguen et al. (1997). When the percolation threshold is exceeded, the probability of percolating flow path formation increases sharply. However, when the crack orientation exhibits anisotropy, differences in the convergence curves emerge depending on the direction. Focusing on this transition region, which corresponds to the convergence curve, (Lang et al., 2014) conducted numerical simulations of discrete fracture networks (DFNs) to calculate permeability. They reported that percolation significantly increases permeability within the range of $2.2<\rho_{3-D,c}<4.2$. Here, they employed a dimensionless crack density based on excluded volume $\rho_{3-D,c}$. When recalculated in terms of the F_0 defined by Eq.(16'), the range of $\rho_{3-D,c}$ corresponds to $1.36 < F_0 < 2.59$. Although both the lower and upper bounds differ when compared to the range of $0.94 < F_0 < 3.77$ in this study, considering only isotropic models, the range is $0.94 < F_0 < 2.83$, which is relatively close to the range reported by Lang et al. (2014).

4.2 Effect of anisotropic stress on tortuosity and bottleneck

The connectivity of cracks and the formation of percolating flow paths is primarily governed by the orientation, size, and arrangement of cracks, irrespective of their aperture. Therefore, considering the aperture of cracks, the shortest percolating flow path tends to follow the route of least resistance. Fig. 9 illustrates percolating flow paths in the x-, y-, and z-directions for four crack models with low ($F_0 < 2.83$) and high ($F_0 > 4.17$) crack density under hydrostatic and triaxial stress conditions applied to an isotropically distributed crack model (Iso). In each quadrant of Fig. 9, the left-hand side presents the top 10 shortest paths in each direction, depicted with solid lines, while dashed lines represent their projections onto specific planes: x-direction paths are projected onto the z=0 plane, y-direction paths onto the x=0 plane, and z-direction paths onto the y=100 plane. On the right-hand side, the figures depict the shortest path in the z-direction along with the cracks traversed by this path. Comparing the cases of low crack density (Fig. 9(a)) and high crack density (Fig. 9(b)) under hydrostatic conditions, the length of the percolating flow path becomes shorter and more linear as the crack density increases. This point will be discussed in the following paragraph. Here, we focus on the changes in the percolating flow path induced by variations in the stress state. In the numerical calculations conducted here, the triaxial stress conditions result in the closure of cracks oriented in the z-direction. Since cracks with normals in the z-direction are closed, the percolating flow path preferentially selects wide-aperture cracks whose normals are in the x- or y-direction as flow paths. Therefore, in a low crack density model, the limited availability of nearby cracks causes the path to detour in search of cracks that offer easier fluid flow (Fig. 9(c)). Conversely, at a high crack density model, cracks with normals in the x- and y-directions are more abundant nearby, resulting in minimal changes to the trajectory of the path (Fig. 9(d)).

Fig. 10 shows the variations in $\tau^{(k)}$, $\beta^{(k)}$, and T_{ij} with crack density under hydrostatic and triaxial stress conditions for the isotropic crack distribution. This corresponds to the elastic behaviour observed in triaxial and polyaxial compression tests and indicates changes in the crack network at the crack closure stage, where a decrease in permeability is observed in Fig. 2. Here, we focus on the decrease in permeability in the loading direction at the crack closure stage, i.e., below the failure level of approximately 0.4 observed in the triaxial test (Fig. 2) and examine the characteristics of the crack network at this stage. This range should be within the elastic regime, where crack development is limited; therefore, the crack density could remain nearly unchanged from the initial state (e.g. Zoback and Byerlee, 1975). A comparison between Figs. 10(a) and (b) shows that, under triaxial stress conditions, the value of Λ_{ij} in the loading direction (z-direction) decreases sharply compared with the hydrostatic condition (initial state) when the initial crack density is $F_0 < 3.77$. This behavior is interpreted to be more strongly influenced by bottleneck effects than by an increase in tortuosity. In other words, this indicates that many of the selected permeability pathways preferentially include cracks oriented perpendicular to the z-axis. This result suggests that, during the crack closure stage of triaxial loading, cracked rocks with lower initial crack densities exhibit a larger reduction in permeability. In contrast, although changes in permeability in the lateral directions have not been reported in previous studies, the numerical results indicate that lateral permeability decreases markedly regardless of the initial crack density, due to the combined effects of increased tortuosity and bottleneck.

The samples shown in Fig. 2 are initially intact, with a low crack density—for example, $F_0 < 2.83$. Within the elastic regime, where the crack density does not change, it becomes inevitable to select closed cracks as flow paths. As a result, the flow paths become longer, and the permeability is likely to decrease due to the combined effects of increased tortuosity and bottlenecks.

Fig. S1 summarises the changes in $\tau^{(k)}$, $\beta^{(k)}$ and T_{ij} for each crack density in each model. Here, Λ_{ij} is a parameter directly used in determining the permeability, as outlined in Eq. 14, based on the permeability tensor. When considering T_{ij} for $i = j$, the permeability direction is assigned to the x-, y- and z-axes corresponding to $i = 1,2$ and 3, respectively. The characteristics of crack connectivity under each stress condition are summarised as follows.

· Hydrostatic stress condition (Fig. S1 (a-1)-(a-3)): In the isotropic case, the graphs for the three directions exhibit the same trend, with T_{ij} converging towards 1 as the crack density increases. This is because the $\tau^{(k)}$ converges towards 1, resulting in a more linear percolating flow path. On the other hand, in cases where the crack orientation exhibits anisotropy (Ani-1, Ani-2), the value of $\tau^{(k)}$ becomes higher in the direction corresponding to the higher crack density, indicating a more complex percolating flow path in that direction. As a result, the T_{ij} value is higher in the direction with lower crack density, allowing easier fluid flow.

· Triaxial stress condition (Fig. S1 (b-1)-(b-3)): The z-direction corresponds to the maximum principal compressive stress, and in models with any crack orientation, the tortuosity in the z-direction is low. Additionally, the bottleneck becomes more significant as the value of $\beta^{(k)}$ decreases, and under triaxial stress conditions, the bottleneck is more pronounced in the two directions of the two minimum principal stresses. Consequently, the T_{ij} in the two directions of the minimum principal stresses is lower compared

to the direction of the maximum principal stress, with all T_{ij} values beginning to converge towards a constant from approximately $F_0 = 5$. Notably, the shapes of the T_{ij} graphs are similar across all crack orientation models, suggesting that T_{ij} is more strongly influenced by the stress conditions than by the initial crack orientations.

<u>*· Polyaxial stress condition (Fig. S1 (c-1)-(c-3)):*</u> $\tau^{(k)}$ exhibits a trend similar to that observed under triaxial stress conditions. However, $\beta^{(k)}$ demonstrates more complex behaviour as the crack density increases, while remaining smaller than the values observed under triaxial conditions. Consequently, T_{ij} does not increase significantly even as crack density increases. This indicates that polyaxial stress conditions are more complex than triaxial conditions and lead to a reduction in T_{ij}. In other words, assuming that crack density remains unchanged, it suggests that a slight increase in the intermediate principal stress under triaxial stress conditions would result in a significant decrease in T_{ij}.

4.3 Equivalent Permeability Calculated Using Λ_{ij} Based on the Permeability Tensor

Permeability was calculated in four different ways: using the modified correction factor Λ_{ij}, which considers connectivity, tortuosity, and bottlenecks; using connectivity $\lambda(F_0)$, where λ depends on crack density; assuming λ =1, corresponding to a fully connected crack network; and directly from DFN-based seepage flow analyses. In this study, we examine the relationships between the equivalent permeability tensors $k_{ij}^{(\Lambda_{ij})}$, $k_{ij}^{(\lambda(F_0))}$, and $k_{ij}^{(\lambda=1)}$, which are calculated using the correction factors Λ_{ij}, $\lambda(F_0)$, and $\lambda = 1$, respectively, and the permeability tensor k_{ij}^{DFN} obtained from DFN-based seepage flow analysis. Figure 11 shows the values of $k_{33}^{(\Lambda_{33})}$, $k_{33}^{(\lambda(F_0))}$, and $k_{33}^{(\lambda=1)}$, normalized by k_{ij}^{DFN}, as a function of the dimensionless crack density F_0. As a representative example, the normalized permeabilities in the *z*-direction under hydrostatic and triaxial stress conditions are presented. In the range $F_0 \geq 3.77$, both $k_{33}^{(\Lambda_{33})}$and $k_{33}^{(\lambda(F_0))}$ differ from k_{33}^{DFN} by approximately a factor of 10–20. Even at $F_0 = 2.83$, $k_{33}^{(\Lambda_{33})}$ remains within roughly a factor of 20 of k_{33}^{DFN}, a level of discrepancy that can be regarded as small from the perspective of permeability estimation. At the lowest crack density considered ($F_0 = 2.83$), the differences increase to approximately a factor of 50 for $k_{33}^{(\lambda(F_0))}$ and a factor of 60 for $k_{33}^{(\lambda=1)}$. These results indicate that, when restricted to the range $F_0 > 3.77$, either $\lambda(F_0)$ or Λ_{ij} can be used to suppress discrepancies relative to the DFN results, implying that the medium behaves effectively as a porous continuum. However, when the range $F_0 \leq 3.77$ is also considered, the correction factor Λ_{ij}, which explicitly accounts for tortuosity and bottleneck effects, should be adopted. In particular, $k_{33}^{(\Lambda_{33})}$ rapidly converges toward the DFN-derived value, which is interpreted as a consequence of incorporating tortuosity and bottleneck effects into the correction factor. It should also be noted that $\lambda = 1$ exhibits large discrepancies over the entire range of crack densities. Moreover, even at high crack densities such as $F_0 = 7.54$, $k_{33}^{(\Lambda_{33})}$ differs from k_{33}^{DFN} by approximately one order of magnitude. This discrepancy is most likely attributable to differences in flow representation. In the permeability tensor formulation used to compute equivalent permeability, the entire crack disc is assumed to act as a fluid pathway, whereas in the DFN model fluid transport is localized, as shown in Fig. S2, with only a portion of the disc actually contributing to flow. As a result, the effective flow area is limited to approximately one-tenth of the total disc area, which is considered the primary cause of the observed discrepancy. In addition, Thomas et al. (2020) pointed out that equivalent permeability formulations do not explicitly represent the discrete, dominant flow channels that emerge in DFN simulations. Nevertheless, when pronounced bottleneck structures and high tortuosity are present, the influence of this restricted effective flow area is expected to be mitigated through the correction factor, thereby reducing its impact on the estimated permeability.

Fig. 12 shows the variation of $k_{ij}^{\Lambda_{ij}}$ with dimensionless crack density as solid lines, while $k_{ij}^{\lambda(F_0)}$ and the k_{ij}^{DFN} are indicated by broken and dotted lines, respectively. We compare the equivalent permeabilities obtained under each stress condition—namely, those derived from the formulations $k_{ij}^{\lambda(F_0)}$ and $k_{ij}^{\Lambda_{ij}}$—with the permeabilities obtained directly from DFN simulations, k_{ij}^{DFN}. Under hydrostatic stress conditions, introducing the coefficient Λ_{ij} significantly reduces the discrepancy with the DFN results, and the difference is limited to approximately one order of magnitude (about 10–20 times). Even in the presence of anisotropy, the directional trends of permeability are well preserved. Under triaxial compression stress conditions, the permeability in the *z*-direction tends to be slightly overestimated; however, the discrepancy remains on the order of 20 times, and the directional trends are still maintained even under anisotropic conditions. Furthermore, as observed in K_{33}^{DFN}, the rate of permeability reduction becomes pronounced for $F_0 \lesssim 4.71$. This tendency is also observed in both $k_{33}^{\lambda(F_0)}$ and $k_{33}^{\Lambda_{33}}$, suggesting that not only a reduction in connectivity but also the effects of tortuosity and bottlenecks contribute to the permeability decrease. Under polyaxial stress conditions, the trends in the *z*- and *y*-directions are consistent with those obtained from the DFN results, with differences of roughly one order of magnitude. In contrast, the equivalent permeability in the *x*-direction is underestimated by about an order of magnitude relative to the DFN results, regardless of crack anisotropy. The *x*-direction corresponds to the σ_{min} direction, and the cracks that constitute the primary flow paths in this direction are closed by the combined effects of the maximum and intermediate principal stresses. In the DFN results, the permeabilities in the x- and y-directions are nearly identical for the Iso and Ani-1 cases, whereas for the Ani-2 case, the permeabilities in the x- and z-directions are nearly the same. In other words, the DFN results indicate that permeability in the σ_{int}-direction is consistently low, while that in the σ_{max}-direction is high, and that permeability in the σ_{min}-direction varies depending on the degree of anisotropy. In contrast, the equivalent permeability consistently follows the order of σ_{max}-direction, σ_{int}-direction, and σ_{min}-direction, from highest to lowest. This discrepancy suggests that, under polyaxial stress conditions, the σ_{min}-direction exhibits particularly complex behavior because cracks with different degrees of closure act as flow paths. Therefore, a more detailed investigation of tortuosity and bottleneck effects in the σ_{min}-direction under polyaxial stress conditions will be required in future studies.

These results indicate that the equivalent permeability incorporating correction factors for tortuosity and bottlenecks can reproduce the same directional trends as the DFN-derived permeability, although quantitative differences of up to approximately one order of magnitude remain. This agreement holds at least for all three directions under hydrostatic and triaxial compression stress conditions, as well as for the σ_{max}- and σ_{int}-directions under polyaxial stress conditions. This finding further suggests that, when considering the decrease in permeability in the axial loading direction during the crack-closure stage under triaxial stress—such as that observed in Fig. 2—it is necessary to evaluate equivalent permeability formulations that explicitly account for the effects of tortuosity and bottlenecks.

Table 2 summarizes the ratios of permeability in the loading-axis direction under hydrostatic and triaxial stress conditions. These results indicate that variations in permeability along the loading axis cannot be adequately evaluated using $K_{ij}^{\lambda(F_0)}$ calculated solely from the pre-existing λ or $\lambda(F_0)$. In contrast, incorporation of Λ_{ij} enables a quantitative assessment of permeability reduction. Directional permeability ratios based on Λ_{ij} further show that, under triaxial compressive stress, permeability decreases not only in the loading axis direction but more strongly in directions perpendicular to the loading axis. To our knowledge, permeability test results obtained during the crack-closure stage under triaxial compression have not been reported, and future experimental verification of these behaviors is therefore anticipated.

4.4 Method to determine $\boldsymbol{\Lambda_{ij}}$ by means of experimental data

As demonstrated above, evaluating permeability under hydraulic, triaxial or polyaxial stress conditions from the viewpoint of the crack network requires careful consideration of closed cracks. In this study, the crack network parameterT_{ij} was proposed to be incorporated into $\lambda(F_0)$. While T_{ij} can be easily calculated through numerical modelling, its experimental estimation remains problematic. Fig. 13 shows a diagram that summarises the method for determining permeability from the perspective of crack geometry in rocks. The methods for obtaining the parameters necessary to determine the permeability tensor from crack information are summarised as follows.

① **Trace map (Direct), Clinometer and universal stage:** This is the most conventional method, involving direct two-dimensional (2D) crack mapping through field observation. In addition, it is possible to obtain 3D information on N_{ij} by measuring the strike and dip of individual cracks using a clinometer (Oda, 1985). Even under a microscope with thin sections, 3D N_{ij} data can be acquired using a universal stage (Takemura et al., 2003a). However, since observations are conducted on 2D sections, obtaining 3D N_{ij} requires the use of sections in different orientations or supplementary borehole data. Aperture width can also be measured in the field at outcrops (Snow, 1968), or in the laboratory using electron microscopy (Brace, 1977), even for hand-sized specimens. In such cases, care must be taken as the crack width observed on a 2D section represents the apparent aperture in 3D (Suzuki et al., 1998).

② **Trace map (Mixed)**: This method involves the indirect generation of t2D trace maps by means of computer-based image processing. For example, FracPaQ, developed by Healy et al. (2017), allows trace maps to be created from crack images, enabling easy extraction of 2D information such as orientation, connectivity, and tortuosity. It also facilitates the calculation of 2D permeability tensors. In recent years, crack extraction techniques incorporating machine learning have advanced, and it is expected that obtaining 2D crack trace maps will become increasingly accessible (e.g., Arena et al., 2014; Chen et al., 2017).

③ **Stereology**: If 2D trace maps from multiple sections are available, it is possible to determine the 3D fabric tensor N_{ij} and surface area density S_0 using stereological theory (e.g., Kanatani, 1985; Oda, 1984; Takemura and Oda, 2004). In the field, obtaining multiple sections is often difficult; in such cases, the number density of cracks intersecting a scanline on borehole cores should be used in combination. For rock specimens used in laboratory experiments, it is relatively easy to obtain three orthogonal sections, making it possible to estimate 3D crack geometry (Oda et al., 2002a).

④ **Wave velocity**: Elastic wave velocity is often used to obtain crack geometric information, as it is strongly related to crack density. For example, the 3D N_{ij} can be determined using the elastic stiffness tensor if wave velocities are measured in at least nine different directions. Crack density can also be theoretically derived using elastic constitutive equations, for instance in the form of $\langle r^3 \rangle$ in the effective medium theory (EMT)(e.g., O'Connell and Budiansky, 1974; Sayers and Kachanov, 1995) or F_0 in the crack tensor theory (Oda et al., 1986; Takemura and Oda, 2005). However, if water is present, Biot's theory of poroelasticity must be applied, and caution is required when using elastic wave measurements to estimate crack density (e.g., Sarout, 2012; Schubnel and Guéguen, 2003) .

⑤ **X-ray CT**: For hand-sized rock specimens, X-ray CT can be used to directly obtain 3D information on crack orientation and density (e.g., Takemura et al., 2007). For example, Takemura et al. (2003b) has observed crack closure under confining pressure using a carbon fibre pressure vessel, and McBeck et al. (2023) has conducted crack observations during triaxial testing using a high-intensity X-ray source. However, the image resolution of X-ray CT is typically on the order of several micrometres, making it insufficient for capturing closed cracks under load. Moreover, increasing the resolution reduces the observable volume, making it currently difficult to observe a sufficiently large region for evaluating permeability.

⑥ **Laboratory test**: Parameters obtainable through laboratory experiments include porosity and aperture width, which can be measured using porosimetry techniques such as mercury intrusion porosimeter. If porosity ϕ and aperture t are known, the surface area density S_0 can be calculated using the relationship $\phi = tS_0/V$ (Oda et al., 2002b). Furthermore, by separating the volumetric strain of a rock under stress into elastic and inelastic components, the change in porosity ϕ can be estimated (Oda et al., 2002b). When crack density remains unchanged, this change in porosity can be interpreted as a change in aperture.

To determine the 3D permeability tensor, crack information including the 3D orientation distribution, aperture, surface area, connectivity, and crack network parameter is required. Here, we summarise the procedure for estimating connectivity and crack network parameter in cases where the permeability has been obtained experimentally. In Fig. 13, the methods for determining orientation distribution, aperture, surface area and permeability are indicated by colour-coded arrows. The measurement techniques for each parameter are summarised below.

- *Orientation distribution (blue arrows in Fig. 13)*: The orientation of cracks can be obtained in the field using a clinometer on multiple outcrop surfaces with different strike and dip, or from trace maps (methods (1) and (2)). If cracks are observable via X-ray CT, the orientation distribution can also be derived from image data (method (5)). In addition, applying stereological techniques to trace maps from multiple section planes allows estimation of the 3D orientation distribution (Kanatani, 1984, 1985; Oda, 1984)(method (3)). It is also possible to derive this information from elastic wave velocities measured in multiple directions (Takemura and Oda, 2005) (method (4)). However, if the elastic wave measurements are taken only along three mutually orthogonal directions, only the diagonal components of the N_{ij} can be determined. Therefore, the principal axes of the crack network or permeability tensor should be made co-axial with the measurement directions of the elastic waves.
- *Aperture (yellow arrow in Fig. 13):* The initial aperture of cracks within the rock can be directly obtained from in-situ observations or under a scanning electron microscope (①), and, if feasible, via X-ray CT imaging (⑤). In addition, aperture values can also be obtained using experimental methods such as mercury intrusion porosimetry (e.g., Holzer et al., 2023) (⑥). Furthermore, an inverse estimation method has been proposed, whereby the aspect ratio is inferred by comparing theoretical equations with changes in rock stiffness estimated from reductions in P- and S-wave velocities (Sayers and Kachanov, 1995). Since the permeability tensor can also be expressed in terms of the aspect ratio, the permeability can be estimated if the aspect ratio is inferred from elastic wave measurements. However, when considering permeability, the cracks are assumed to be fluid-filled, consequently, an additional fourth-rank tensor must be introduced in the compliance formulation (Schubnel and Guéguen, 2003).

Surface area (green arrow in Fig. 13): The surface area of cracks within a volume can be estimated from multiple trace maps using stereological methods (Oda, 1984; Oda, 1985). Moreover, since this can be derived from the number of cracks intersecting scan lines, it can be estimated not only from trace maps but also from borehole cores and borehole TV images. In such analyses, it is necessary to use scan lines in multiple directions, and the total length of the scan lines must be selected with consideration of the representative elementary volume (REV) (Takemura and Oda, 2004). Although the surface area cannot be directly obtained from the dimensionless crack density, such as $\langle r^3 \rangle$ or F_0, determined from elastic wave velocities, it can be estimated by assuming the crack orientation distribution function and the crack diameter distribution function.

Permeability (purple arrow in Fig. 13): Permeability can be measured by laboratory experiments. In recent years, measurements in two or three directions have been conducted using prismatic samples, while cylindrical samples allow measurements only in a single direction (e.g. Nasseri et al., 2014; Sato et al., 2018). If the measurement directions correspond to the principal axes of permeability, the diagonal components of the permeability tensor can be determined from permeability measurements in three orthogonal directions. In this case, k_{ij} reduces to k_{11}, k_{22}, and k_{33}, and the values of P_{11}, P_{22}, and P_{33} can be back-calculated from Eq. 2, however, the Λ_{ij} remains an unknown parameter. Therefore, if S_0, t, and N_{ij} in Eq. 7 are known from the aforementioned methods, the Λ_{ij}can be determined. No study has been found that has measured crack geometry and permeability in three orthogonal directions under triaxial or polyaxial stress conditions using cracked rock such as granite. Suzuki et al. (1998) reported one-directional permeability under a confining pressure of 0.2 MPa, along with detailed 3D crack geometry in thermally damaged granitic rock with varying crack densities. According to their results, the permeability of the intact sample obtained from the experiment was 3.05×10^{-18} m^2, while the permeability calculated using the permeability tensor with measured rack geometry was 1.76×10^{-17} m^2. A comparison of the measured and calculated permeabilities suggests that a connectivity value of λ =0.173 is required. When the modified correction factor proposed in this study is applied to the results of Suzuki et al. (1998), the bottleneck factor is $\beta^{(k)}$=1.0 due to the hydrostatic condition, and the tortuosity is found to be $\tau^{(k)}$=2.40. The value is higher than the tortuosity obtained from the numerical calculations in this study, which is likely due to differences between the assumed and actual crack diameter distributions and the roughness of the crack surfaces.

In the future, it is expected that 3D crack geometry will be obtained in conjunction with experimental measurements of three-directional permeability. As shown in Fig. 13, if 3D crack geometry can be derived from either trace maps or P- and S-wave velocity measurements in three directions, the model proposed in this study can be applied to obtain the parameter Λ_{ij}. This, in turn, will advance the understanding of fluid transport mechanisms along cracks under anisotropic stress conditions.

4.5 Limitation

The following points should be carefully considered when estimating permeability from crack geometry based on the permeability tensor concept.

- If the matrix is permeable, a dual-porosity model should be considered to separate the permeability contributions of the matrix and the crack domains. Specifically, the permeability measured under a confining pressure at which cracks are fully closed can be regarded as the matrix permeability. The crack-domain permeability can then be experimentally estimated by subtracting the matrix contribution, weighted by crack density, from the permeability measured under low confining pressure. It should be noted that the permeability estimated using the permeability tensor corresponds to the crack domain.

- When considering permeability obtained from triaxial or polyaxial compression tests, it is necessary to account for stress shadow effects due to the shape of the specimen. Oda et al. (2002b) pointed out that, in a cylindrical specimen with a diameter of 50 mm and a height of 120 mm, damage and crack generation are concentrated within a central 50 mm zone along the height. This implies that, when interpreting axial permeability based on a crack model in the failure zone, one must consider the presence of relatively intact rock at both ends of the specimen, effectively acting as caps.
- In laboratory experiments, it is realistic to measure permeability and elastic wave velocities in three directions. Accordingly, this study adopts a crack model consisting of three mutually orthogonal crack sets and considers only the diagonal components of N_{ij}. However, in reality, many cracks are likely to be obliquely oriented. In this study, such oblique cracks are implicitly accounted for within the tortuosity. To account for the influence of oblique cracks, it would be necessary to represent N_{ij} as a full tensor form and introduce directional dependence into the aperture expression in Eq. 15. While expressing the aperture as a tensor would require higher-order tensors and become increasingly complex, the use of the full N_{ij} tensor could provide a more detailed understanding of crack connectivity and network under anisotropic stress conditions.
- The permeability obtained from laboratory experiments reflects not only the effects of aperture, connectivity, tortuosity and bottlenecks, but also the reduction in permeability due to surface roughness on the crack surfaces. In the permeability tensor-based model, this effect is incorporated into the tortuosity, whereas it is not explicitly included in the open-form expression. Therefore, in addition to the tortuosity and bottlenecks associated with the crack network, tortuosity and bottlenecks arising from surface roughness within individual crack planes (e.g., Tsang, 1984) should also be considered. Additionally, crack intersections may lead to a change in permeability, and as demonstrated by Stanton-Yonge et al. (2023), incorporating the hydro-mechanical effects of crack intersections into permeability models is essential for improving predictions of fluid flow in cracked media (e.g., Takemura and Healy, 2026).

5. Concluding remarks

This study aimed to clarify, from the perspective of a crack model, the role of cracks oriented normal to the loading axis—which are not accounted for in conventional stress-dependent permeability tensor models—and the reduction in permeability observed within the elastic regime during triaxial compression tests. In addition, the equivalent permeability calculated based on Oda’s permeability tensor, using the correction factor that incorporates both tortuosity and bottleneck effects, reproduces the same overall trends as the DFN-derived permeability, despite differences of approximately one order of magnitude in absolute values. This holds, at a minimum, for hydrostatic conditions, for all three principal directions under triaxial compression, and for the maximum and intermediate principal stress directions under polyaxial stress conditions. Based on the numerical modelling results, we draw the following conclusions:

1) Under anisotropic stress conditions, such as triaxial loading, the closure of cracks normal to the loading axis forces flow paths to become more tortuous, resulting in increased tortuosity. Furthermore, if no open cracks are available nearby to serve as alternative flow paths, the flow must traverse closed cracks, creating bottlenecks. This effect is particularly pronounced when the crack density is low (e.g., $F_0 < 2.83$). In contrast, even if cracks normal to the loading axis are closed at higher crack densities, other open cracks aligned parallel to the loading axis are likely to be present nearby, resulting in lower tortuosity and more linear flow paths.
2) Under anisotropic stress conditions, the permeability in the direction of the loading axis is influenced by cracks oriented normal to the axis, especially in regions of low crack density. This effect is independent of the degree of crack orientation anisotropy. In polyaxial stress conditions, permeability in the direction of the maximum principal compressive stress decreases in low crack density regions and remains lower than in triaxial stress conditions even at higher densities. This is because bottlenecks appear along the flow paths in the direction of the maximum principal stress.
3) We proposed the modified correction factor Λ_{ij} by expressing the combined effects of tortuosity and bottlenecks as a tensorT_{ij}, which serves as a correction factor for the $\lambda(F_0)$. Incorporating Λ_{ij} into a stress-dependent permeability tensor allows for more accurate modelling of permeability under anisotropic stress conditions. Notably, the reduction in permeability observed in the loading direction within the elastic regime of triaxial compression tests, where no new cracks are developed, can be explained in terms of crack geometry using the proposed Λ_{ij}.

To represent permeability along the loading axis using an equivalent permeability framework and to couple it with elastic wave velocities or stress–strain behavior, it is necessary to introduce direction-dependent correction factors. A tensorial form, such as that proposed in this study, is therefore desirable. With the accumulation of experimental results together with detailed geometric information on crack networks, it is expected that more accurate and quantitatively constrained correction factors can be obtained in the future.

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Captions

Table 1 Simulation conditions

Table 2 Relative change in permeability along the loading axis associated with a change in stress state from hydrostatic to triaxial compression.

Fig. 1. Schematic illustration of the three types of stress conditions applied to the rock in this study.

Fig. 2. Change in permeability with increasing differential stress up to the failure stress under conventional triaxial testing of basalt (Fortin et al., 2011; Jia et al., 2021) and granitic rocks (Chen, 2018; Kiyama et al., 1996; Mitchell and Faulkner, 2008; Shao et al., 2005; Wang et al., 2014; Zoback and Byerlee, 1975). The abscissa represents the dimensionless failure level calculated from the axial strain, ε_a, or differential stress, $\sigma_1 - \sigma_3$ as reported in the respective studies. Here, the subscript 'max' denotes the value at failure, and the solid and broken lines correspond to the crack closure and dilatancy stages, respectively. In the legend, P'_C denotes the effective confining pressure.

Fig. 3 Schematic view of the conceptual model illustrating tortuosity and bottleneck in a crack model. The crack model comprises five cracks in total: two normal to the *z*-axis, two normal to the *y*-axis, and one normal to the *x*-axis. The crack model comprises five cracks in total: two normal to the *z*-axis, two normal to the *y*-axis, and one normal to the *x*-axis. (A) the model under hydrostatic conditions and (B) the model under anisotropic stress with σ_z (red cracks oriented normal to the *z*-axis are closed by σ_z, and their aperture decreases from 100 μm to 20 μm). The fluid is assumed to flow in through the cracks at the upper end and be discharged from the cracks at the lower end, with each model having three possible flow paths. (C) Diagram of individual flow paths with aperture, and (D) shows the calculated parameters for each flow path.

Fig. 4 Crack traces observed on the x-, y-, and z-cross-sections for each model. The crack generation conditions are (a) isotropic (Iso), (b) weakly anisotropic (Ani-1), and (c) strongly anisotropic (Ani-2), and stress condition set as hydrostatic stress. For comparison, three different crack densities (F_0=1.88, 5.65, 7.54) are presented for each model. In the figure, the black, red, and green crack traces represent cracks with normal vectors in the x-, y-, and z-directions, respectively. Here, the crack generation configuration uses the instance with seed=0.

Fig. 5 Conceptual schematic of the algorithm for determining tortuosity and bottlenecks numerically. The aperture of cracks not subjected to normal stress are denoted by t_x and t_y, and the red crack is subjected to normal stress with its aperture denoted by t_z. (a) Crack arrangement and percolating flow pathways in three dimensions, (b) Diagram showing only the percolating flow pathways.

Fig. 6 Flowchart illustrating the generation of the crack model and the output of parameters in this study.

Fig. 7 A box-and-whisker plot of the fraction of intersecting cracks for each crack density. Here, 100 instances are generated for each dimensionless crack density. Cracks that do not intersect become isolated cracks. Since the stress state does not influence crack intersections, results under hydrostatic conditions are shown.

Fig. 8 The fraction of instances in which a percolating flow path connecting the boundary surfaces in the *x*, *y*, and *z* directions appeared, for each crack model (Iso, Ani-1, Ani-2). Each model includes 100 randomly generated instances.

Fig. 9 3D visualisations of percolating flow paths corresponding to illustrative crack densities under hydrostatic and triaxial stress conditions in an isotropically distributed crack model (Iso). Specifically: (a) crack density $F_0 = 2.83$ under hydrostatic condition ($\sigma_x = \sigma_y = \sigma_z$ =10MPa), (b) crack density $F_0 = 5.65$ under hydrostatic condition ($\sigma_x = \sigma_y = \sigma_z$ =10MPa), (c) crack density $F_0 = 2.83$ under triaxial condition ($\sigma_x = \sigma_y$ =10MPa, σ_z =50MPa) and (d) crack density $F_0 = 5.65$ under triaxial condition ($\sigma_x = \sigma_y$ =10MPa, σ_z =50MPa). Here, the crack generation condition was set using seed = 12. The points in the graph represent nodes, and the dashed lines represent their projections onto specific planes: *x*-direction paths are projected onto the *z*=0 plane, *y*-direction paths onto the *x*=0 plane, and *z*-direction paths onto the *y*=100 plane.

Fig. 10 Examples of the variations in $\tau^{(k)}$, $\beta^{(k)}$, and T_{ij} with crack density under hydrostatic and triaxial conditions in the isotropic case.

Fig. 11 Variation of $k_{33}^{(\Lambda_{33})}$, $k_{33}^{(\lambda(F_0))}$, and $k_{33}^{(\lambda=1)}$, normalized by k_{33}^{DFN}, as a function of dimensionless crack density.

Fig. 12 Changes in $k_{ij}^{(\Lambda_{ij})}$(solid lines), $k_{ij}^{(\lambda(F_0))}$(broken lines), and k_{ij}^{DFN}(dotted lines) as a function of dimensionless crack density for different stress and crack fabric conditions.

Fig.13 Diagram illustrating the procedures for determining parameters associated with the permeability tensor.