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Updated constraints on omnipotent dark energy: A comprehensive analysis with CMB and BAO data

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In this work, we present updated observational constraints on the parameter space of the DMS20 dark-energy model, a member of the *omnipotent dark-energy* (ODE) class. Our analysis combines multiple cosmic microwave background datasets—including measurements from the Planck satellite (PL18), the South Pole Telescope (SPT), and the Wilkinson Microwave Anisotropy Probe (WMAP)—with Type Ia supernova data from the Pantheon + catalog (PP), and baryon acoustic oscillation measurements from the Dark Energy Spectroscopic Instrument and Sloan Digital Sky Survey. We find that certain data combinations, such as SPT + WMAP + BAO and PL18 + BAO, can reduce the significance of the H_0 tension below 1σ , but with considerably large uncertainties. However, the inclusion of PP data restores the tension in H_0 . To provide a comprehensive view of the ODE phenomenology, we also investigate the evolution of its energy density, emphasizing its dynamical behavior at low redshifts. Our results generically exhibit multiple phantom divide line crossings in a single expansion history; if confirmed, this points beyond the simplest minimally coupled canonical single-field quintessence/phantom descriptions and motivates more general dark-sector realizations.

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I. INTRODUCTION

The Λ cold dark matter (Λ CDM) model has emerged as the dominant and most widely accepted cosmological framework. Its success is largely attributed to its remarkable accuracy in explaining a broad range of astrophysical and cosmological observations without excessive model complexity. Despite these successes, however, the Λ CDM

model faces some unresolved challenges, particularly in capturing a consistent expansion history when various combinations of mainstream cosmological observables, such as the cosmic microwave background (CMB), baryon acoustic oscillations (BAO), large-scale structure, and local distance ladder measurements of the Hubble constant are simultaneously considered [1–5].

With the increasing precision of modern observations [6], it is anticipated that deviations from the standard Λ CDM model will become more apparent. Indeed, various discrepancies in the estimation of key cosmological parameters have already emerged, some of which show statistically significant departures from the predictions of the model [1,2,5,7–10]. Among these issues, one of the most prominent and statistically significant is the ongoing discrepancy related to the Hubble constant, H_0 (for a detailed review, see Refs. [1–3,5,9,11–17] and references therein). A notable conflict exists between the values of H_0 inferred from the CMB measurements [18–20] assuming the Λ CDM model and those derived from direct,

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independent measurements obtained through local astrophysical observations [21–47]. This tension, commonly referred to as the H_0 tension, has reached a statistical significance greater than 5σ [36,37,48] and is now considered one of the most pressing issues within the context of the Λ CDM model. To solve this tension, cosmologists have been exploring both possible systematic errors [29,36,38,39,48–57] and alternative cosmological models [1,2,5,12,14,16,58–63]. In addition to the H_0 tension, another point of discussion in cosmology is the S_8 tension [2,7,64,65] (see also Refs. [66,67]), which concerns discrepancies between the amplitude of matter fluctuations inferred from early-Universe measurements, such as the Planck CMB data, and the S_8 values derived from late-time observations, including weak gravitational lensing and galaxy clustering, though it seems to have lost significance after the new KiDS-Legacy release [68–84]. Furthermore, recent results from the DESI Collaboration analyzing the BAO feature have found evidence for dynamical dark energy (DDE) when their measurements are combined with other major observations [85–89]. This evidence for DDE is relevant only in the postrecombination era, with a statistical significance of 3.1σ when combined with the CMB. The significance ranges from 2.7σ to 4.2σ when different supernovae samples are included in the dataset combination. The indication persists even without the CMB; for example, only the DES supernovae sample [90] when combined with DESI BAO yields a 3.3σ significance. It is important to note that the evidence for DDE does not hinge on DESI BAO either; e.g., see Refs. [90,91] that report preference of a DDE without this dataset. A plethora of models have been proposed in the literature to address the existing cosmological tensions [5], a substantial portion of which incorporate a DDE component that is non-negligible in the late and/or early Universe [85,92–131].

The simplest phenomenological DDE model replaces the cosmological constant, whose effective equation of state (EOS) parameter is $w = -1$, with a dark energy (DE) fluid that has a constant EOS parameter able to satisfy $w_{\text{DE}} \neq -1$; this is often referred to as the w CDM model. Many models extend this approach by allowing a varying EOS parameter, with the most well-known example being the Chevallier-Polarski-Linder (CPL) parametrization [132,133], in which w_{DE} is assumed to be a linear function of the scale factor in the Robertson-Walker (RW) metric. Notably, unless restricted to specific regions of parameter space, such extensions generically allow w_{DE} to cross the phantom divide line (PDL), $w_{\text{DE}} = -1$. However, parametrizing DDE solely via its EOS parameter fails to capture the phenomenology of models in which DDE attains negative effective energy densities in the past, as such components would exhibit a singular EOS parameter [134]. From both theoretical and phenomenological perspectives, models predicting negative energy densities in the dark sector have garnered increasing attention in recent years, particularly for their potential to resolve the H_0 tension [5,86,135–155,155–159]. DDE models

capable of simultaneously incorporating all of these features were dubbed *omnipotent dark-energy* (ODE) models in Ref. [160], where it was argued that such a flexible DDE component may be necessary for a satisfactory resolution of the prevailing cosmological tensions.

In Ref. [161], the DMS20 model was proposed as a promising solution to the cosmological tensions and was found to prefer a PDL crossing at $z \sim 0.1$. The recent analysis in Ref. [160] identified DMS20 as an ODE model and showed that its ability to reach negative energy densities for $z \gtrsim 2$ —mimicking a negative cosmological constant at high redshifts—plays a crucial role in alleviating the tensions. This behavior is consistent with the predictions of the Λ_s CDM model [141,142,146], which suggests a transition from an anti-de Sitter to a de Sitter phase. This transition can be interpreted either as an emergent effect from modified gravity or as the result of an actual field within the framework of general relativity (GR).

In this paper, we revisit and update the observational constraints on the DMS20 model using Type Ia supernova data from the Pantheon+ catalog, BAO measurements from the DESI and SDSS surveys, and CMB temperature and polarization data from Planck, the South Pole Telescope, and the Wilkinson Microwave Anisotropy Probe. Motivated by previous studies of the DMS20 model that reported posterior distributions leaning against the edges of the prior ranges for certain dataset combinations, we extend these prior ranges. This not only relaxes the constraints on the model parameters but also enables the model to exhibit richer phenomenology that is qualitatively different—for example, a crossing of the PDL, occurring either from the quintessence regime to the phantom regime or vice versa. Nevertheless, we also present our results after excluding the Markov-chain Monte Carlo (MCMC) samples corresponding to the extended priors, to facilitate direct comparison with previous studies.

The structure of this paper is organized as follows. In Sec. II, we review the key physical features of the omnipotent dark-energy model, focusing on the aspects most relevant for cosmological tests. In Sec. III, we describe the observational datasets used in our analysis and outline the methodology employed to constrain the model’s free parameters. In Sec. IV, we present and discuss our main results, highlighting their implications for cosmological tensions and the broader landscape of dark-energy models. Finally, in Sec. V, we summarize our conclusions and offer some perspectives on future research directions.

II. OMNIPOTENT DARK ENERGY: REVIEW

In this section, we review the class of ODE models and the DMS20 model as a concrete member of this class. The term ODE describes a family of phenomenological proposals that allow energy densities to transition between positive and negative values and to exhibit oscillatory or nonmonotonic evolution histories. ODE models permit

arbitrary EOS, including singularities and PDL crossings, unconstrained by standard energy conditions. As an effective source in the Friedmann equations, ODE provides a flexible framework to address the limitations of the cosmological constant. More precisely, a DE model can be considered as an ODE model if, within the same expansion history, it exhibits all six possible combinations of the following conditions on the energy density and the equation of state of DE: $\rho_{\text{DE}} > 0$, $\rho_{\text{DE}} < 0$, and $w_{\text{DE}} > -1$, $w_{\text{DE}} = -1$, $w_{\text{DE}} < -1$. Here, ρ_{DE} denotes the energy density of the DE component [160].

Two distinctive features of ODE are its capacity to realize a nonmonotonic energy density evolution and to reach negative energy densities, as suggested by various observational reconstructions [156,162–167]. Such behavior has been shown to alleviate key cosmological tensions, including the H_0 and S_8 discrepancies [135,136,138,141,142,145,161,163,168–172]. Hereafter, we consider an ODE component, whether as an effective source from modified gravity or a fluid within GR, that satisfies the usual continuity equation that follows from the local conservation of the energy-momentum tensor for the RW metric. Given this weak assumption, unlike usual phantom DE [173], which maintains $w_{\text{DE}} < -1$, the nonmonotonicity feature of ODE models requires that its EOS parameter crosses the PDL. Moreover, since its energy density vanishes during the transition between negative and positive density regions at a scale factor a_p , its EOS parameter exhibits a pole of the form $\lim_{a \rightarrow a_p^\pm} w_{\text{DE}}(a) = \mp\infty$; this also corresponds to a PDL crossing, but a discontinuous one [134].

The DMS20 model proposed in Ref. [161] serves as a concrete example of an ODE model. In this model, the DE density, ρ_{DE} , is parametrized to ensure an extremum at a scale factor a_m , satisfying the condition $\frac{d\rho_{\text{DE}}}{da} \big|_{a=a_m} = 0$. The DE density is then expressed by expanding ρ_{DE} around a_m ,

$$\rho_{\text{DE}}(a) = \rho_{\text{DE}0} \frac{1 + \alpha(a - a_m)^2 + \beta(a - a_m)^3}{1 + \alpha(1 - a_m)^2 + \beta(1 - a_m)^3}, \quad (1)$$

where α and β are constants defining the polynomial terms (for further details, see Refs. [160,161]). The absence of a linear term in the expansion follows from the vanishing of the first derivative at $a = a_m$. The parameter a_m has significant physical implications: from the continuity equation, it follows that $w_{\text{DE}}(a_m) = -1$, provided that ρ_{DE} is nonzero at the extremum. For $\alpha > 0$, this corresponds to a transition from $w_{\text{DE}} > -1$ to $w_{\text{DE}} < -1$ as the Universe expands, while for $\alpha < 0$, the crossing occurs in the opposite direction. The EOS for the ODE model is given by

$$w_{\text{DE}}(a) = -1 - \frac{a[2\alpha(a - a_m) + 3\beta(a - a_m)^2]}{3[1 + \alpha(a - a_m)^2 + \beta(a - a_m)^3]}, \quad (2)$$

which yields $w_{\text{DE}}(a = 0) = -1$ and $w_{\text{DE}}(a \rightarrow \infty) = -2$.

This model introduces three extra parameters: $\{a_m, \alpha, \beta\}$. Depending on the values of these parameters, certain features of ODE may remain dormant. For instance, when $\alpha = \beta = 0$, the DE density reduces to the cosmological constant Λ , recovering the standard Λ CDM model. For a detailed discussion of the dynamical behavior of the model across different values of $\{a_m, \alpha, \beta\}$, see Ref. [160]. Here, we simply note that the EOS of DMS20 cannot, in general, be represented *exactly* by the CPL parametrization, $w_{\text{DE}} = w_0 + w_a z / (1 + z)$, where $z = (1 - a)/a$ is the redshift. Here, $w_0 = w_{\text{DE}}(z = 0)$, and w_a controls the leading time variation of w_{DE} within the CPL ansatz [equivalently, $w(a) = w_0 + w_a(1 - a)$, so $dw/da = -w_a$ is constant by construction]. More generally, (w_0, w_a) are best regarded as a convenient two-parameter *data compression* of the model predictions for a given set of observables and redshift sensitivities (see, e.g., Refs. [174–176]). These two parameters are essential for characterizing the behavior of dark energy across a wide range of models, as discussed in Ref. [177]. However, as shown in Fig. 1, for any choice of $\{a_m, \alpha, \beta\}$, we observe an oscillatory pattern in w_{DE} , which cannot be described by the linear CPL form. In fact, w_a would need to acquire a redshift dependence to account for the behavior of the EOS in Eq. (2). It is seen from Fig. 1 that variations in α significantly influence the present-day values of both w and w_a , while having a negligible effect at higher redshifts. Specifically, as α increases, w shifts further into the phantom regime, and w_a also increases. In the middle panel, we observe that β induces more pronounced oscillations in w , although its effect on w_a is relatively minor. Lastly, the parameter a_m shows a similar trend to α , highlighting its role in shaping the present-day dynamics of dark energy. It is evident that the linear CPL parametrization fails to capture the nonlinear features of the ODE model, whose oscillatory behavior cannot be reproduced by such a simplified form. With only one additional parameter, our ODE parametrization is thus able to recover a complementary class of DE models with a richer phenomenology.

To understand the background evolution of this model, we begin with the expansion rate of the Universe, governed by the modified Friedmann equation,

$$\frac{H^2(a)}{H_0^2} = \Omega_{m0}a^{-3} + \Omega_{r0}a^{-4} + \Omega_{\text{DE}0}f(a), \quad (3)$$

where

$$f(a) = \frac{1 + \alpha(a - a_m)^2 + \beta(a - a_m)^3}{1 + \alpha(1 - a_m)^2 + \beta(1 - a_m)^3}. \quad (4)$$

In Eq. (3), the subscript m refers to all forms of matter (including both baryonic and cold dark matter), while r denotes radiation (photons and other relativistic relics).

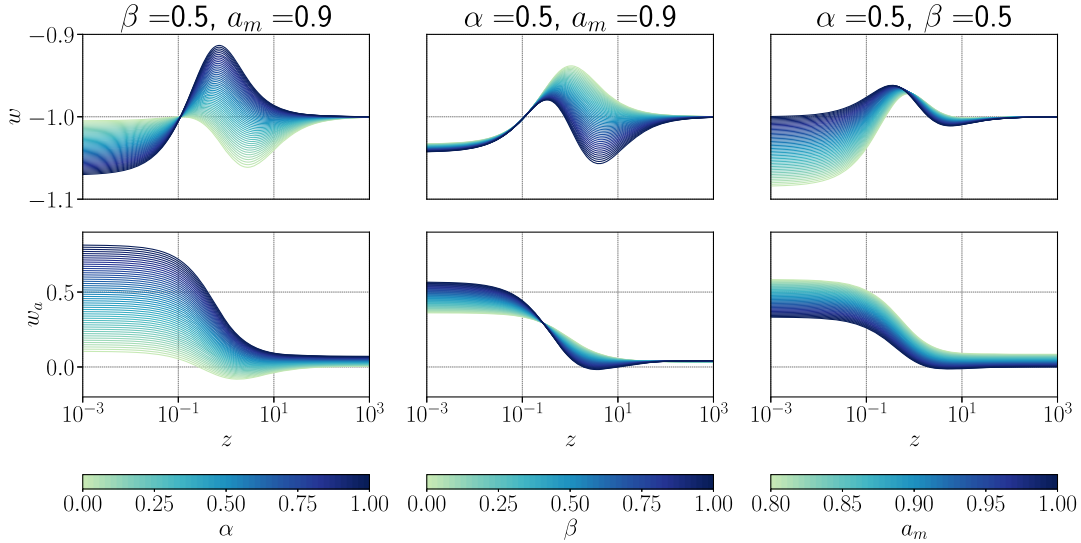


FIG. 1. The redshift evolution of w_{DE} and the CPL parameter w_a are shown for different values of $\{a_m, \alpha, \beta\}$. Clearly, w_a cannot remain constant in this model, so the linear CPL parametrization of w_{DE} fails to capture the oscillatory behavior characteristic of ODE.

The density parameters $\Omega_{i0} \equiv \rho_{i0}/(3H_0^2)$ represent the present-day values of the respective energy densities.

The DMS20 model does not introduce modifications to other sectors of the Universe or its constituent species. Consequently, the background evolution remains unchanged for all components other than DE itself. The linear evolution of DE perturbations follows the standard prescription, where we impose synchronous gauge conditions for metric perturbations. The continuity and Euler equations governing the DE fluid are given by

$$\begin{aligned} \dot{\delta}_x = & -(1+w) \left(\theta + \frac{\dot{h}}{2} \right) - 3(\hat{c}_s^2 - w) \mathcal{H} \delta_x \\ & - 9(1+w)(\hat{c}_s^2 - c_a^2) \mathcal{H}^2 \frac{\theta_x}{k^2}, \end{aligned} \quad (5)$$

$$\dot{\theta}_x = -(1 - 3\hat{c}_s^2) \mathcal{H} \theta_x + \frac{\hat{c}_s^2 k^2}{1+w} \delta_x - k^2 \sigma_x. \quad (6)$$

These equations are quite general, as they assume only a noninteracting fluid and allow for the presence of shear stress σ_x , a nonadiabatic sound speed, and a time-dependent equation of state parameter w . Henceforth, we assume the fluid is shear free and that w is given by Eq. (2).

III. DATA AND METHODOLOGY

We generate theoretical predictions for the ODE model using a modified version of the Boltzmann solver CAMB [178,179], while parameter estimation is performed with the publicly available sampler Cobaya [180]. The sampling of the posterior distributions is carried out using the MCMC method, originally developed for CosmoMC [181]. This implementation incorporates the “fast dragging” technique [182,183], which improves efficiency in

exploring parameter spaces with varying computational complexity.

In varying combinations, the likelihoods employed in this analysis use data from the following cosmological surveys:

- (i) *Planck 2018 (PL18)*. We include the full Planck 2018 temperature and polarization likelihoods [temperature auto-correlation spectrum (TT), temperature and polarization cross-correlation spectrum (TE), polarization auto-correlation spectrum (EE)], along with the Planck 2018 lensing likelihood [18], reconstructed from the four-point correlation function of temperature fluctuations.
- (ii) *South Pole Telescope (SPT)*. We incorporate CMB temperature and polarization anisotropy measurements (TT, TE, EE) from the SPT Collaboration [19,184].
- (iii) *Wilkinson Microwave Anisotropy Probe (WMAP)*. We utilize the CMB temperature and polarization data from the WMAP 9-year release [185]. To mitigate dust contamination, we exclude WMAP low- ℓ polarization data by setting the minimum multipole for TE and EE to $\ell = 24$. For this reason, whenever we use the WMAP spectra we consistently apply a Gaussian prior on the optical depth to reionization, $\tau = 0.0544 \pm 0.0073$, which is the baseline choice used by the SPT-3G Collaboration when they include the WMAP data in their analysis [184]. We also assume that there is no correlation between the SPT and WMAP datasets, as they probe largely different regions of the sky [184].
- (iv) *Dark Energy Spectroscopic Instrument (DESI)*. We include BAO measurements from the DESI Collaboration, based on galaxy and quasar observations [186], as well as Lyman- α tracers [187], compiled in Table I of Ref. [85]. These measurements span the redshift range $0.1 < z < 4.2$, divided into seven

bins, and include both isotropic and anisotropic BAO constraints. The isotropic BAO measurements are expressed as $D_V(z)/r_d$, where D_V is the volume-averaged distance normalized to the comoving sound horizon at the drag epoch, r_d . The anisotropic constraints include $D_M(z)/r_d$ and $D_H(z)/r_d$, where D_M is the comoving angular diameter distance and D_H is the Hubble horizon. Correlations between D_M/r_d and D_V/r_d are accounted for.

- (v) *Sloan Digital Sky Survey (eBOSS)*. We incorporate the galaxy, quasar, and Lyman- α measurements of D_M/r_d and D_H/r_d from the eBOSS experiment, as presented in Table 3 of Ref. [188].
- (vi) *Pantheon+ (PP)*. We include distance modulus measurements of Type Ia supernovae from the Pantheon+ sample [189], consisting of 1550 supernovae over the redshift range $0.001 \leq z \leq 2.26$.

For the DMS20 model parameters, we adopt the agnostic, flat priors outlined in Table I, except, as noted above, for an informative Gaussian prior on τ whenever the SPT+WMAP dataset combination is considered. Specifically, the parameter space of the DMS20 model extends the standard Λ CDM framework by introducing the three parameters defined in Eq. (1), namely, α , β , and a_m , in addition to the six baseline Λ CDM parameters: the physical baryon and cold dark matter densities ($\Omega_b h^2$, $\Omega_c h^2$), the optical depth to reionization (τ), the amplitude and spectral index of the primordial scalar fluctuations ($\log(10^{10} A_s)$, n_s), and the angular size of the sound horizon at last scattering (θ_{MC}). Compared to Refs. [160,161], the priors on α and β are extended to include negative values, and a_m is allowed to exceed 1, thereby enabling the possibility of PDL crossings occurring in the future (i.e., for $a > 1$). These choices are discussed in more detail in the next section.

All of the 1D and 2D posteriors were calculated and visualized using the `Getdist` code [190], while the functional posteriors for $w_{DE}(z)$ and $\rho_{DE}(z)$ shown in the next section were produced using the `FGIVENX` plotting package [191].

TABLE I. Flat priors are adopted to test the ODE model against all of the likelihood combinations presented in Sec. IV, with the exception of those involving the SPT+WMAP dataset combination, where a Gaussian prior is imposed on the optical depth: $\mathcal{P}(\tau) = \mathcal{N}(0.0544, 0.0073^2)$.

Parameter	Prior
$\Omega_b h^2$	[0.017, 0.027]
$\Omega_c h^2$	[0.09, 0.15]
τ	[0.01, 0.8]
n_s	[0.9, 1.1]
$\log(10^{10} A_s)$	[2.6, 3.5]
$100\theta_{MC}$	[1.03, 1.05]
α	$[-8, 8]$
β	$[-8, 8]$
a_m	[0, 1.4]

IV. RESULTS AND DISCUSSION

We begin our discussion by justifying our choice of priors for the extra parameters of DMS20, namely, $\{a_m, \alpha, \beta\}$. In contrast to Refs. [160,161], the prior ranges for $\{\alpha, \beta\}$ are chosen to include negative values, and the prior range for a_m is extended to include values greater than 1. In Refs. [160,161], the choice of priors enforces $\alpha > 0$, $\beta > 0$, and $a_m \in [0, 1]$. This restricts ρ_{DE} to an evolution history with certain properties, the most important being the guaranteed existence of a PDL crossing at $a = a_m$, where the EOS parameter transitions from quintessence ($w_{DE} > -1$) to phantom ($w_{DE} < -1$) as the Universe expands. However, in both of the previous studies, the reported posteriors for these parameters do not exclude the prior bounds for many of the dataset combinations, particularly those that do not include the SH0ES H_0 measurement. Moreover, recent indications of a dynamical dark-energy component suggest evidence for a preference of a PDL crossing in the opposite direction,¹ i.e., where the EOS parameter transitions from phantom ($w_{DE} < -1$) to quintessence ($w_{DE} > -1$) as the Universe expands [85,88]. In fact, the PDL crossing at $a = a_m$ occurs in this opposite direction when $\alpha < 0$. More generally, various qualitatively different DE density evolution histories become available in the DMS20 model when the parameters are allowed to explore this extended parameter space; see Table II and Fig. 1 of Ref. [160] and the discussions therein.

In this work, we chose the extended priors in Table I to take a more conservative approach that does not guarantee the existence of a_m within the expansion history and allowed the MCMC analysis to explore the parameter space of the model more freely. Indeed, the results we report suggest that these extended regions of parameter space are always consistent with our dataset combinations and are sometimes even preferred. To better compare our results with previous works, we also report results obtained after filtering the MCMC chains by dropping any samples with $\alpha < 0$, $\beta < 0$, or $a_m > 1$. We ensured that all of the filtered chains satisfied the Gelman-Rubin convergence criterion with $R - 1 < 0.07$; this approach is equivalent to performing the analysis with the corresponding restricted priors. We refer to the results from the full chain as the “full-prior results” and to those from the postprocessed chains as the “postfiltered results.”

In this section, we present the main results of our analysis using different combinations of datasets. We analyze constraints on our dynamical dark-energy model

¹It is important to note that the prior ranges chosen in Refs. [160,161] allow (but do not guarantee) for a second PDL crossing in this opposite direction in addition to the one at $a = a_m$, and even a third crossing may occur in a discontinuous way if the DE density attains negative values in the past. In fact, when the model is constrained with data, all three of these crossings were found to play an important role in the evolution of the DE density.

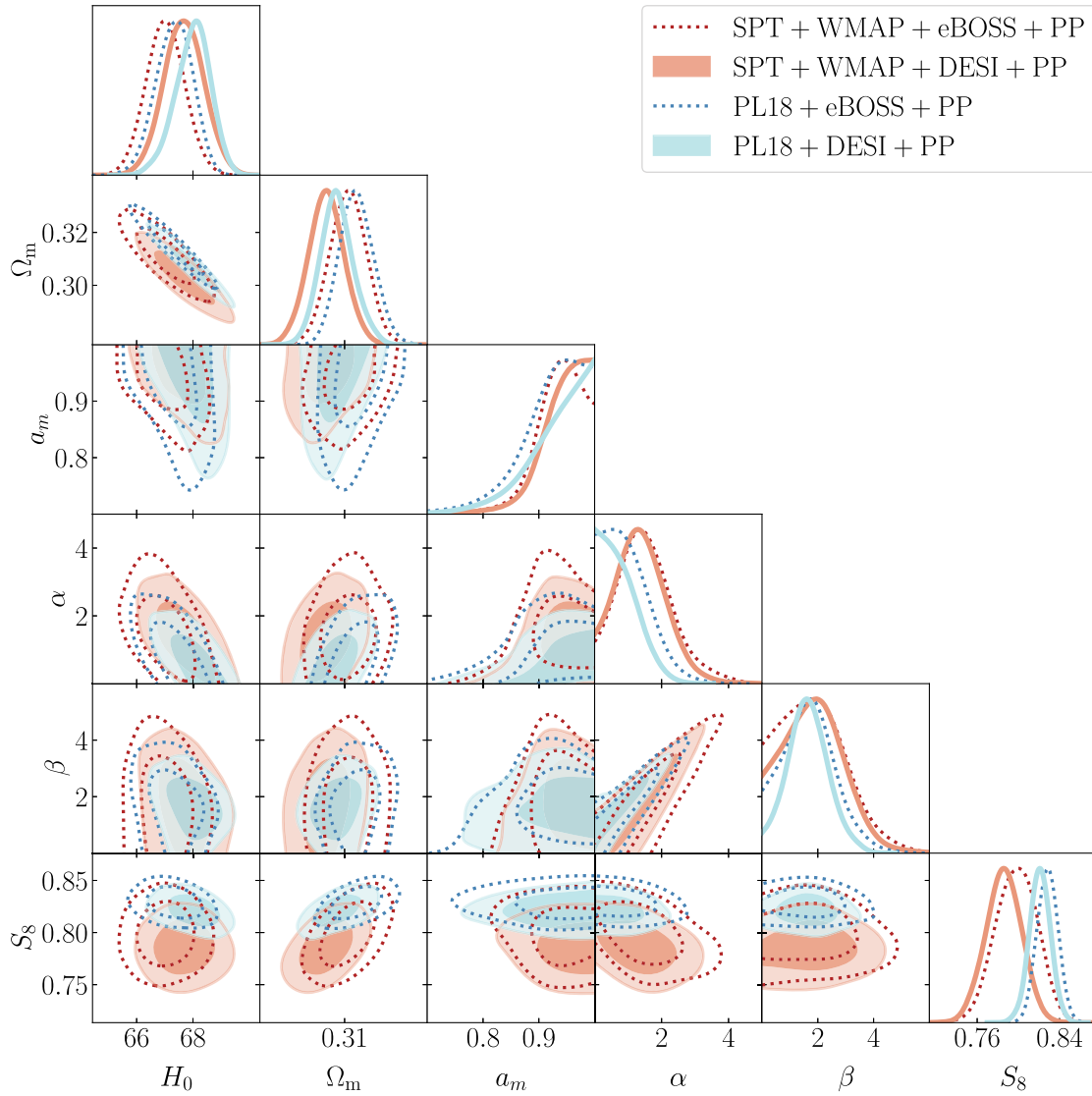


FIG. 2. 1D and 2D marginalized constraints on the ODE parameter space, derived from the postfiltered chains that include only samples with $\alpha > 0$, $\beta > 0$, and $a_m \in [0, 1]$.

ODE, characterized by the parameters α , β , and a_m , using various combinations of the CMB and BAO measurements described in the previous section, with and without supernovae distance moduli.

In Figs. 2 and 3, we present the parameter constraints derived from both the full-prior and postfiltered analyses. We find that using full priors results in multimodal posterior distributions for certain parameters, while the postfiltered analysis yields unimodal distributions. The multimodality in the full results arises from the phenomenological degeneracies in the parameter space; i.e., different regions of the parameter space correspond to similar expansion histories (see Table II in Ref. [160]). The Λ CDM model is nested in DMS20 and corresponds to the $\alpha = \beta = 0$ section of the parameter space. It is seen from Fig. 3 that there is always consistency, in all of the dataset combinations shown in the triangular plot, with the part

of the parameter space corresponding to Λ CDM, and there is no evidence for dynamics in the DE density evolution. Overall, despite the different prior choices (to be discussed in detail later), the results remain statistically consistent across all analyses. A comparative analysis of the SPT+WMAP+DESI+PP and PL18+DESI+PP combinations shows that the latter yields tighter parameter constraints, as illustrated by the red and blue contour plots. Moreover, the inferred value of S_8 is systematically lower when using the SPT+WMAP+DESI+PP combination compared to PL18+DESI+PP. Substituting DESI with eBOSS leads to additional shifts in the best-fit values of several parameters (see the following discussions for the respective σ deviations), highlighting the sensitivity of the results to the choice of BAO dataset.

We present the complete set of parameter estimates from our analysis in Table II–V. Postfiltered results show

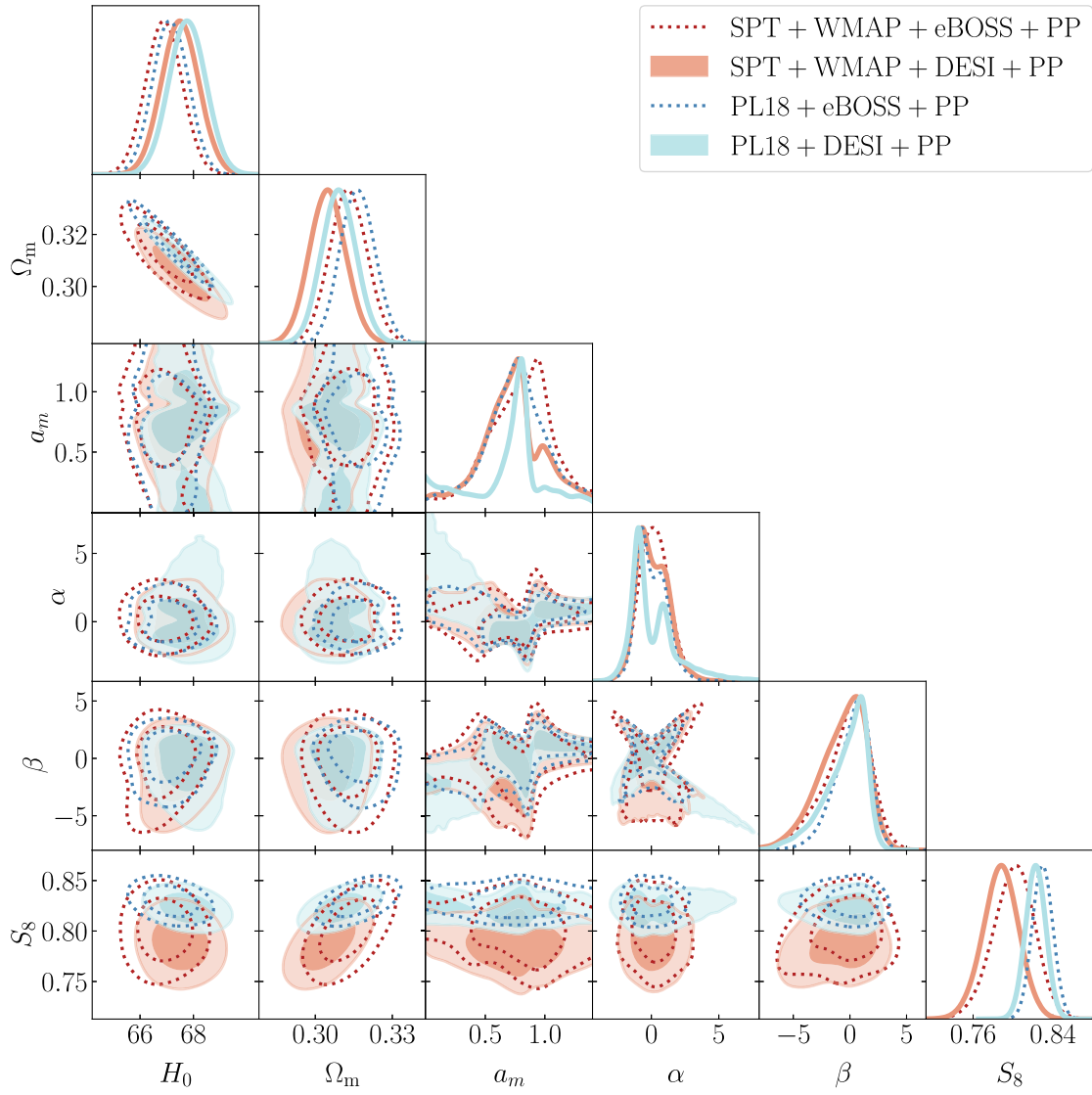


FIG. 3. 1D and 2D marginalized constraints on the ODE parameter space, obtained from the chains sampled with the full-prior ranges listed in Table I, without any postprocessing cuts (full-prior results).

significantly tighter constraints on cosmological parameters compared to the full-prior setup. In both the SPT and PL18 cases, the parameters α , β , and a_m are more precisely determined when the prior ranges on model parameters are restricted. This is expected as the postfiltering removes a significant portion that is within 68% C.L. of the mean values of the full-prior marginalized posteriors. The inclusion of PP data further enhances the precision of these constraints. For instance, the SPT+WMAP+DESI+PP combination with postfiltered priors (see Table II) yields, at 68% C.L., $\alpha = 1.40^{+0.65}_{-0.84}$, $\beta = 1.88^{+0.95}_{-1.2}$, and $a_m > 0.930$. In contrast, the same dataset under the full-prior setup gives $\alpha = 0.1^{+1.0}_{-1.3}$, $\beta = -0.7^{+2.5}_{-1.5}$, and $a_m = 0.73 \pm 0.28$ at 68% C.L. (see Table III). This corresponds to deviations of 1.06σ , 0.93σ , and 0.71σ for α , β , and a_m , respectively, when

comparing the prior cutoff case with the full-prior analysis for the same dataset combinations.

While the full-prior results are better for interpreting the true phenomena that our datasets prefer, the postfiltered results are easier to interpret and match the previous studies [160,161]. The postfiltered results enforce the existence of a DE EOS parameter that crosses from a quintessence regime to phantom regime at $a = a_m$. For instance, we see in Table IV that PL18+eBOSS yields a clear peak for the scale of the PDL crossing at $a_m = 0.76^{+0.17}_{-0.11}$ and the addition of PP yields $a_m > 0.904$. This is accompanied by a reduction in the uncertainty of H_0 , which changes from $H_0 = 69.4^{+1.4}_{-2.4}$ km/s/Mpc to $H_0 = 67.33^{+0.69}_{-0.62}$ km/s/Mpc with 0.83σ deviations, closer to Planck's Λ CDM value.

TABLE II. 68% C.L. constraints on the free parameters (above the line) and derived parameters (below the line), obtained from the postfiltered results using SPT as the baseline CMB dataset. *BE* is the Bayesian evidence.

	SPT+WMAP+eBOSS	SPT+WMAP+eBOSS+PP	SPT+WMAP+DESI	SPT+WMAP+DESI+PP
$\Omega_b h^2$	0.02242 ± 0.00021	0.02241 ± 0.00020	0.02244 ± 0.00020	0.02244 ± 0.00019
$\Omega_c h^2$	$0.1163^{+0.0021}_{-0.0019}$	0.1167 ± 0.0019	$0.1157^{+0.0016}_{-0.0014}$	0.1157 ± 0.0016
$100\theta_{MC}$	1.04018 ± 0.00065	1.04015 ± 0.00063	1.04028 ± 0.00062	1.04029 ± 0.00064
τ	0.0532 ± 0.0072	0.0532 ± 0.0072	0.0538 ± 0.0070	0.0537 ± 0.0070
$\ln(10^{10} A_s)$	3.031 ± 0.015	3.032 ± 0.015	3.030 ± 0.015	3.030 ± 0.015
n_s	0.9679 ± 0.0061	0.9673 ± 0.0057	0.9690 ± 0.0055	0.9691 ± 0.0055
α	< 3.12	$1.54^{+0.66}_{-0.94}$	< 1.80	$1.40^{+0.65}_{-0.84}$
β	< 3.94	$1.88^{+0.81}_{-1.5}$	< 2.98	$1.88^{+0.95}_{-1.2}$
a_m	$0.826^{+0.13}_{-0.069}$	$0.934^{+0.056}_{-0.023}$	$0.74^{+0.16}_{-0.12}$	> 0.930
Ω_m	$0.296^{+0.022}_{-0.016}$	0.3113 ± 0.0070	$0.276^{+0.025}_{-0.016}$	0.3030 ± 0.0069
H_0 (km/s/Mpc)	$68.8^{+1.7}_{-2.7}$	67.02 ± 0.68	$71.0^{+1.8}_{-3.2}$	67.68 ± 0.71
S_8	$0.787^{+0.024}_{-0.021}$	0.798 ± 0.020	$0.775^{+0.020}_{-0.017}$	0.786 ± 0.018
r_d (Mpc)	148.03 ± 0.53	147.93 ± 0.50	148.16 ± 0.44	148.18 ± 0.45
<i>BE</i>	-0.31	1.04	-0.95	0.84

Interestingly, replacing the eBOSS data with the newer DESI dataset further tightens the constraints on the ODE parameters (α , β) and H_0 . The Hubble constant shifts slightly toward higher values, reflecting the preference of DESI for a lower matter density relative to eBOSS, and the well-known anticorrelation between H_0 and Ω_m . We note that a PDL crossing from $w_{DE} < -1$ to $w_{DE} > -1$ always seems to occur in our postfiltered results, and it is then followed by a second PDL crossing at $a = a_m$ (see Figs. 5 and 7), which is well constrained in most cases.

We observe that for the SPT+WMAP+eBOSS combination, the mean value of H_0 is noticeably lower compared

to the corresponding result with PL18. However, replacing eBOSS with DESI leads to a significant shift of the Hubble constant toward higher values, with $H_0 = 71.0^{+1.8}_{-3.2}$ km/s/Mpc, in agreement with local measurements with 1.56σ deviations as compared to $\sim 5\sigma$ for the Λ CDM case, along with a correspondingly lower matter density. Moreover, as in the results with PL18, the addition of PP improves the constraints on α , β , slightly deviating from α , $\beta = 0$ (Λ CDM). It also increases the preference for the ODE model over Λ CDM, in both the filtered and the full-prior cases, as one can see from the Bayesian evidence values in Tables II, III, IV, and V. However, such a

TABLE III. 68% C.L. constraints on the free parameters (above the line) and derived parameters (below the line), obtained from the full-prior results using SPT as the baseline CMB dataset. *BE* is the Bayesian evidence.

	SPT+WMAP+eBOSS	SPT+WMAP+eBOSS+PP	SPT+WMAP+DESI	SPT+WMAP+DESI+PP
$\Omega_b h^2$	0.02242 ± 0.00021	0.02241 ± 0.00020	0.02243 ± 0.00020	0.02244 ± 0.00020
$\Omega_c h^2$	0.1167 ± 0.0020	$0.1169^{+0.0020}_{-0.0017}$	0.1159 ± 0.0017	0.1158 ± 0.0016
$100\theta_{MC}$	1.04014 ± 0.00065	1.04012 ± 0.00065	1.04026 ± 0.00064	1.04029 ± 0.00064
τ	0.0531 ± 0.0071	0.0531 ± 0.0072	0.0535 ± 0.0072	0.0536 ± 0.0072
$\ln(10^{10} A_s)$	3.032 ± 0.015	3.032 ± 0.015	3.030 ± 0.015	3.030 ± 0.015
n_s	0.9674 ± 0.0059	0.9670 ± 0.0058	0.9686 ± 0.0056	0.9689 ± 0.0055
α	$0.6^{+1.9}_{-2.6}$	$0.2^{+1.0}_{-1.2}$	$0.6^{+2.3}_{-3.1}$	$0.1^{+1.0}_{-1.3}$
β	$-0.7^{+3.9}_{-3.5}$	$-0.3^{+2.5}_{-1.5}$	$-2.6^{+2.3}_{-4.6}$	$-0.7^{+2.5}_{-1.5}$
a_m	$0.66^{+0.26}_{-0.32}$	0.79 ± 0.27	0.58 ± 0.28	0.73 ± 0.28
Ω_m	$0.330^{+0.029}_{-0.038}$	0.3129 ± 0.0073	$0.323^{+0.036}_{-0.041}$	0.3048 ± 0.0071
H_0 (km/s/Mpc)	65.4 ± 3.3	66.90 ± 0.68	$65.9^{+3.6}_{-4.2}$	67.50 ± 0.70
S_8	0.804 ± 0.025	$0.801^{+0.022}_{-0.019}$	0.796 ± 0.024	0.788 ± 0.018
r_d (Mpc)	147.94 ± 0.53	147.88 ± 0.51	148.12 ± 0.47	148.16 ± 0.46
<i>BE</i>	0.43	0.67	0.25	1.62

TABLE IV. 68% C.L. constraints on the free parameters (above the line) and derived parameters (below the line), obtained from the postfiltered results using PL18 as the baseline CMB dataset. *BE* is the Bayesian evidence.

	PL18+eBOSS	PL18+eBOSS+PP	PL18+DESI	PL18+DESI+PP
$\Omega_b h^2$	0.02242 ± 0.00013	0.02241 ± 0.00014	0.02248 ± 0.00014	0.02247 ± 0.00014
$\Omega_c h^2$	0.1195 ± 0.0010	0.1195 ± 0.0010	0.11860 ± 0.00093	0.11883 ± 0.00098
$100\theta_{MC}$	1.04099 ± 0.00030	1.04097 ± 0.00031	1.04113 ± 0.00028	$1.04107^{+0.00030}_{-0.00026}$
τ	0.0553 ± 0.0072	0.0554 ± 0.0072	0.0574 ± 0.0076	0.0571 ± 0.0077
$\ln(10^{10} A_s)$	3.046 ± 0.014	3.046 ± 0.014	3.048 ± 0.015	3.048 ± 0.015
n_s	0.9664 ± 0.0038	0.9665 ± 0.0038	0.9688 ± 0.0038	0.9683 ± 0.0038
α	< 1.50	< 1.25	< 1.02	< 1.01
β	$2.44^{+0.91}_{-2.1}$	$1.71^{+0.84}_{-1.0}$	< 2.84	1.66 ± 0.72
a_m	$0.76^{+0.17}_{-0.11}$	> 0.904	$0.66^{+0.26}_{-0.13}$	> 0.913
Ω_m	$0.297^{+0.020}_{-0.013}$	0.3146 ± 0.0066	$0.277^{+0.030}_{-0.019}$	$0.3071^{+0.0058}_{-0.0067}$
H_0 (km/s/Mpc)	$69.4^{+1.4}_{-2.4}$	$67.33^{+0.69}_{-0.62}$	$71.7^{+2.3}_{-3.9}$	$67.99^{+0.68}_{-0.56}$
S_8	0.821 ± 0.012	0.828 ± 0.010	$0.806^{+0.015}_{-0.013}$	0.821 ± 0.010
r_d (Mpc)	147.17 ± 0.24	147.19 ± 0.24	147.35 ± 0.23	147.31 ± 0.24
<i>BE</i>	7.02	7.59	7.55	9.28

preference is not substantial when SPT is used instead of Planck as the CMB benchmark dataset, with the former case showing only slightly positive Bayesian evidence values when SPT CMB data are used alongside the Pantheon + supernovae measurements.

Different behaviors are observed when analyzing the model with the full priors. As shown in Tables III and V, β consistently prefers a mean value in the negative region, with the 68% C.L. for the SPT+WMAP+DESI case being negative, and the PL18+DESI case having an upper bound of -2.12 . Hence, we see that the postfiltering forces the β values to be larger. One could expect, in parallel, that postfiltering would shift the a_m constraints to lower values by dropping the $a_m > 1$ samples in the chain. However, in contrast, the favored values of a_m are smaller for the

full-prior, resulting in an upper limit when DESI data are used with Planck. This is due to the correlations between $\{\alpha, \beta, a_m\}$. Finally, although α appears to have significantly larger uncertainties, Fig. 3 shows that its posterior is bimodal around zero for the any dataset combination including PP (except for the SPT+WMAP+eBOSS case), with the peak for the negative values corresponding to a larger probability.

In our analysis, the inferred values of the Hubble constant H_0 remain consistently lower than the SH0ES measurement ($H_0 = 73.04 \pm 1.04$ km/s/Mpc), though the extent of the discrepancy varies across datasets. For example, the SPT+WMAP+eBOSS combination yields $H_0 = 68.8^{+1.7}_{-2.7}$ km/s/Mpc with postfiltered priors (Table II) and $H_0 = 65.4 \pm 3.3$ km/s/Mpc with full priors

 TABLE V. 68% C.L. constraints on the free parameters (above the line) and derived parameters (below the line), obtained from the full-prior results using PL18 as the baseline CMB dataset. *BE* is the Bayesian evidence.

	PL18+eBOSS	PL18+eBOSS+PP	PL18+DESI	PL18+DESI+PP
$\Omega_b h^2$	0.02241 ± 0.00014	0.02241 ± 0.00014	0.02245 ± 0.00014	0.02246 ± 0.00014
$\Omega_c h^2$	0.1196 ± 0.0010	0.11959 ± 0.00099	0.11904 ± 0.00098	0.11889 ± 0.00097
$100\theta_{MC}$	1.04096 ± 0.00030	1.04097 ± 0.00030	1.04105 ± 0.00029	1.04107 ± 0.00029
τ	0.0549 ± 0.0073	0.0551 ± 0.0073	0.0560 ± 0.0075	0.0566 ± 0.0075
$\ln(10^{10} A_s)$	3.045 ± 0.014	3.045 ± 0.014	3.046 ± 0.015	3.047 ± 0.015
n_s	0.9662 ± 0.0038	0.9663 ± 0.0038	0.9677 ± 0.0038	0.9681 ± 0.0038
α	$0.6^{+1.9}_{-3.3}$	$0.02^{+0.91}_{-1.2}$	1.1 ± 3.9	$0.2^{+1.2}_{-1.9}$
β	$-1.5^{+3.9}_{-3.2}$	$0.1^{+1.8}_{-1.1}$	< -2.12	$-0.5^{+2.3}_{-1.0}$
a_m	$0.58^{+0.28}_{-0.54}$	$0.75^{+0.28}_{-0.24}$	< 0.743	< 0.824
Ω_m	$0.334^{+0.030}_{-0.039}$	0.3165 ± 0.0068	0.329 ± 0.036	0.3093 ± 0.0067
H_0 (km/s/Mpc)	65.6 ± 3.3	67.14 ± 0.66	$66.1^{+3.1}_{-4.2}$	67.77 ± 0.69
S_8	0.836 ± 0.016	0.830 ± 0.010	$0.831^{+0.018}_{-0.016}$	0.823 ± 0.010
r_d (Mpc)	147.15 ± 0.24	147.17 ± 0.24	147.27 ± 0.23	147.29 ± 0.23
<i>BE</i>	7.65	7.90	9.84	9.59

(Table III) having the deviations of 2.13σ and 2.21σ , respectively, with the inferred SH0ES value. In contrast, the PL18+DESI+PP combination provides a more precise full-prior estimate of $H_0 = 67.77 \pm 0.69$ km/s/Mpc (Table V) with the deviation of 4.12σ with the SH0ES value and 0.43σ with the Planck value. Although certain configurations formally yield a significantly reduced tension with the SH0ES measurement (e.g., the analyses without PP), these tension reductions are mainly due to increased uncertainties of the H_0 constraints, a direct consequence of the three extra parameters of the DMS20 model. Nevertheless, it is worth noting that both the SPT+WMAP+DESI and PL18+DESI postfiltered results reduce the discrepancy with SH0ES to roughly 1σ , due

to both a genuine shift of the mean H_0 value toward higher values and the increased uncertainties.

Top panels of Fig. 4 illustrate the evolution of the Hubble parameter for the PL18+DESI, PL18+DESI+PP, SPT+WMAP+DESI, and SPT+WMAP+DESI+PP dataset combinations of the postfiltered results, while the bottom panels present the corresponding results for the full-prior case. While all these scenarios, along with the baseline Λ CDM model, remain discrepant with the DESI LRG measurement at redshift $z = 0.510$, we can see some improvement in the full-prior results. These figures show, as it was also discussed in the previous paragraph, that the postfiltered results for PL18+DESI and SPT+WMAP+DESI reduce the H_0 tension relative to Λ CDM albeit with large

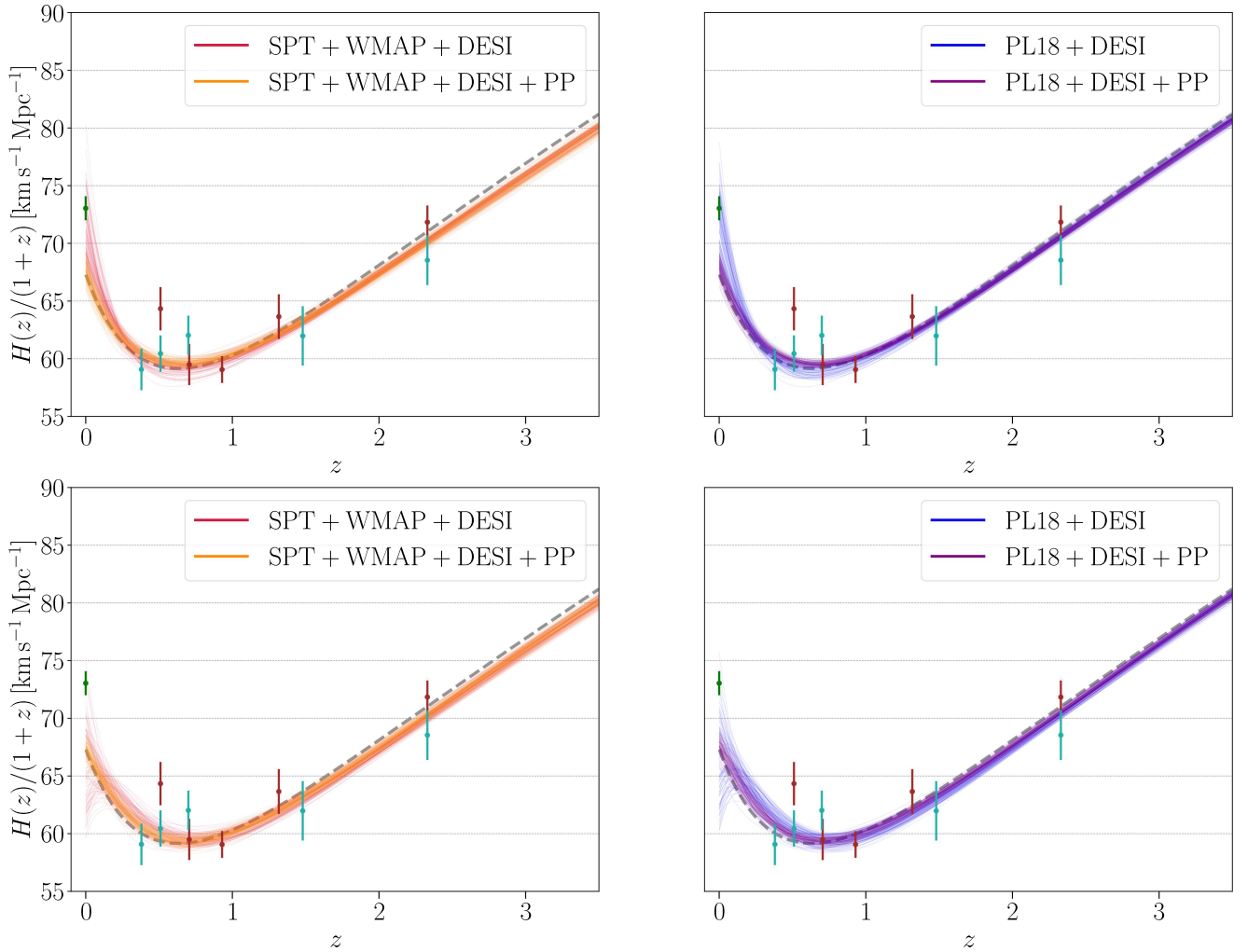


FIG. 4. ODE constraints (top panels for postfiltered results and bottom panels for full-prior results) on the redshift evolution of the $H(z)/(1+z)$ function, derived from the SPT+WMAP+DESI and PL18+DESI data combinations, with and without supernovae. Also shown are D_H/r_d measurements from eBOSS [192] (green) and DESI YR1 [85] (red), obtained using luminous red galaxies, emission line galaxies, and quasars as tracers; the point at $z = 0$ is the SH0ES constraint $H_0 = 73.04 \pm 1.04$ km s $^{-1}$ Mpc $^{-1}$ [48]. These measurements are converted into $H(z)$ using the Planck constraint $r_d = 147.09 \pm 0.26$ Mpc. The gray dashed line represents the Λ CDM best-fit curve from TTTEEE + lowE + lensing [18].

uncertainties. As shown in the figures, the inclusion of PP data reduces the uncertainty and lowers the mean value of H_0 , putting it in strong tension with the SH0ES measurement. Comparing the top panels with the bottom ones, it is evident that restricting the priors to satisfy $\alpha > 0$ and $\beta > 0$

significantly limits the flexibility of DMS20 to describe different expansion histories.

Next, we present the evolution of the dark-energy density, $\rho_{\text{DE}}(z)$, and the corresponding EOS parameter, $w_{\text{DE}}(z)$, derived from the posterior chains of our analysis,

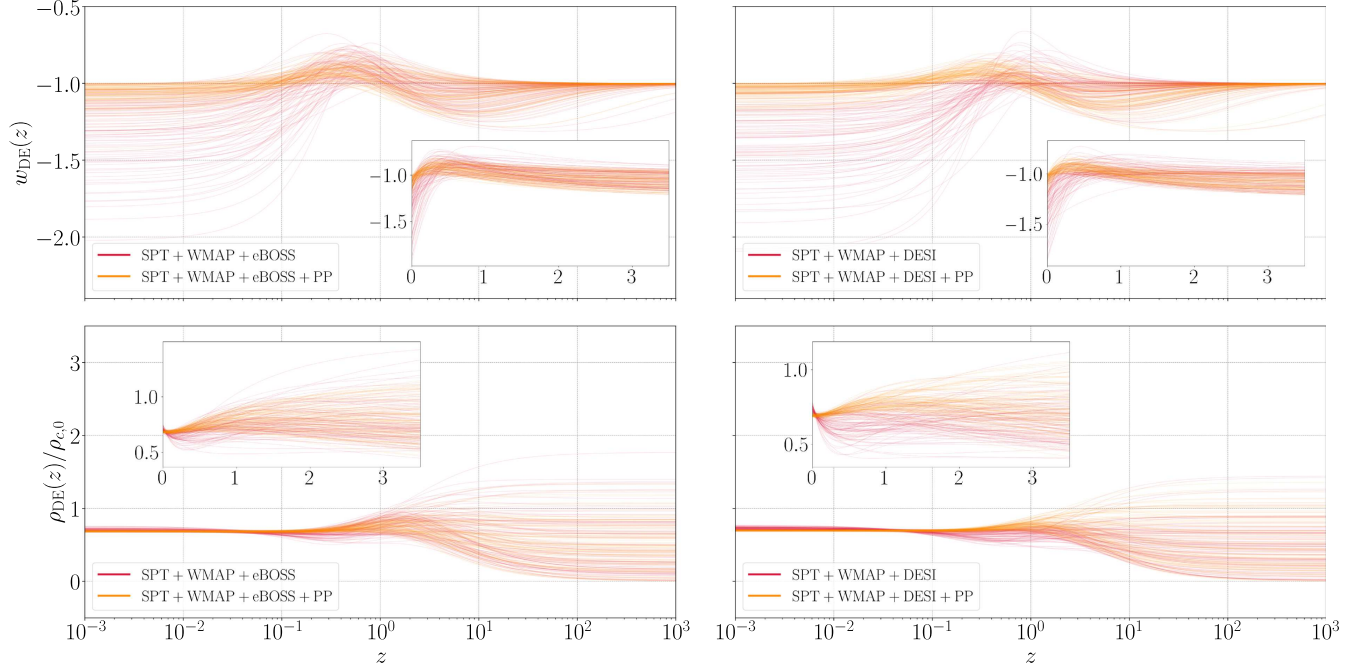


FIG. 5. Redshift evolution of $w_{\text{DE}}(z)$ (top panels) and $\rho_{\text{DE}}(z)/\rho_{c,0}$ (bottom panels) obtained from the postfiltered results using SPT as the baseline CMB dataset. In each panel, the reader can find the same plot contained in the panel itself, reexpressed through a linear but shorter redshift range.

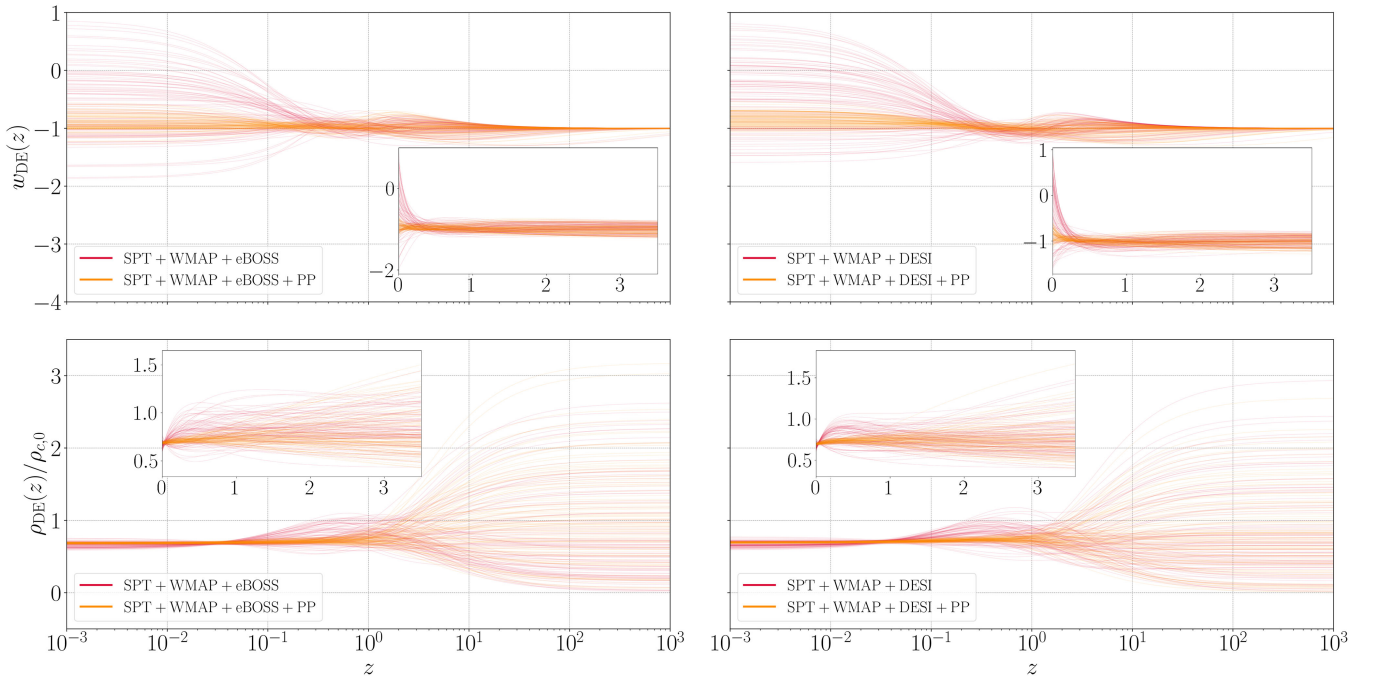


FIG. 6. Same as Fig. 5 but for the full-prior results.

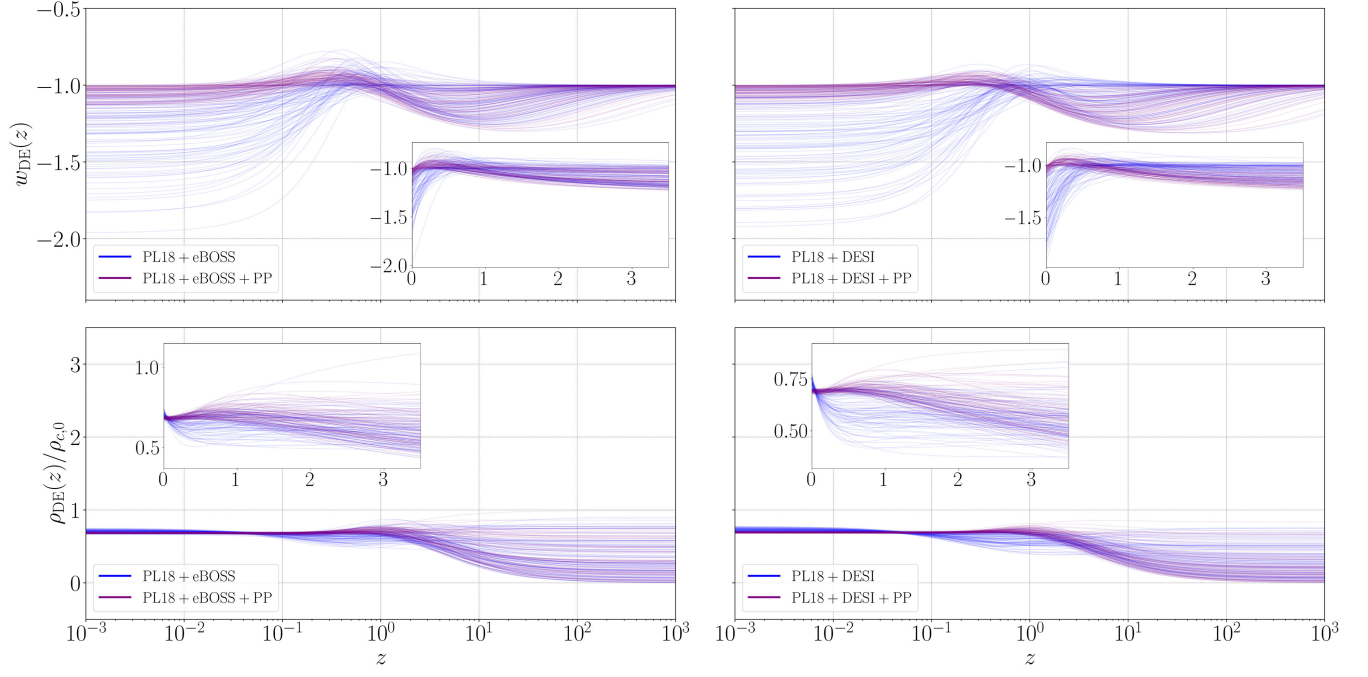


FIG. 7. Redshift evolution of $w_{\text{DE}}(z)$ (top panels) and $\rho_{\text{DE}}(z)/\rho_{c,0}$ (bottom panels) obtained from the postfiltered results using PL18 as the baseline CMB dataset. In each panel, the reader can find the same plot contained in the panel itself, reexpressed through a linear but shorter redshift range.

as shown in Figs. 5 and 7 for the postfiltered results and in Figs. 6 and 8 for the full-prior ones. While noticeable deviations from a cosmological constant ($w = -1$) are permitted across all results, such behavior is not necessarily favored, as discussed previously. A further striking feature

shared by all results is the generic occurrence of a double PDL crossing, characterized by opposite crossing directions in each instance. However, the full-prior and postfiltered results differ noticeably, as the order of the PDL crossings may be reversed between them. Specifically, the

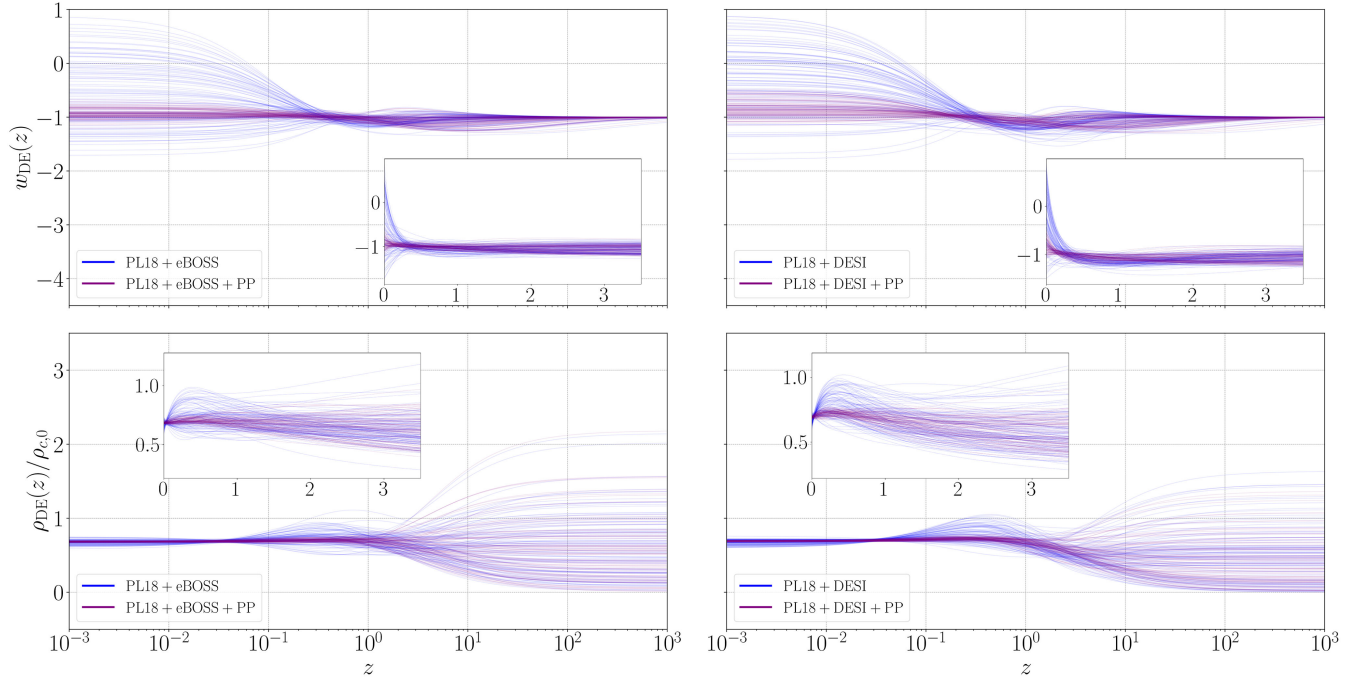


FIG. 8. Same as Fig. 7 but for the full-prior results.

postfiltered results show an initial crossing into the quintessence regime, followed by a second crossing into the phantom regime. This ordering is by construction, as the postfiltered results assume the existence of a PDL crossing from the quintessence to the phantom regime, which must be the latest crossing if multiple ones occur. In contrast, the full-prior results may instead exhibit an initial crossing into the phantom regime, followed by a second one into the quintessence regime—although they can also display the same crossing pattern as the postfiltered results. However, note that, while the double PDL crossing is generic, it is not present in all the MCMC samples for either full or postfiltered results. A double crossing for the postfiltered results would correspond to an initially increasing DE density that goes through a decreasing phase before falling back to a decreasing regime; while this behavior describes the majority of the samples in Figs. 5 and 7, DE density evolutions that start with an initially increasing phase are also ubiquitous, especially when DESI is not present in the dataset. Similar observations can be made for the full-prior results. As a corollary to these discussions, we see that the present day value of the EOS parameter is always in the phantom regime for the postfiltered results by construction, but it is mostly in the quintessence regime for the full-prior results especially when DESI is present in the dataset combination. The addition of PP exacerbates this behavior further. This late-time preference for a quintessence EOS parameter obtained from combining CMB, DESI, and PP data agrees with the findings of Refs. [85,88]. To quantify this preference (it also serves as a measure to quantify deviations from Λ CDM), we report constraints on the present-day dark-energy equation of state parameter w_0 in Table VI for the full-prior results. The inferred values range from $w_0 = -0.93^{+0.11}_{-0.13}$ for the SPT+WMAP+eBOSS+PP combination to $w_0 = -0.34^{+0.53}_{-0.73}$ for PL18+DESI, indicating, depending on the specific case, a mild preference for quintessencelike behavior

TABLE VI. Values of w_0 for different dataset combinations (full-prior results). Uncertainties correspond to 68% CL, with 95% CL shown in parentheses.

Dataset	w_0
PL18+eBOSS	$-0.55^{+0.50(+1.3)}_{-0.71(-0.99)}$
PL18+eBOSS+PP	$-0.92^{+0.087(+0.25)}_{-0.12(-0.20)}$
PL18+DESI	$-0.34^{+0.53(+1.2)}_{-0.73(-1.0)}$
PL18+DESI+PP	$-0.848^{+0.097(+0.27)}_{-0.16(-0.22)}$
SPT+WMAP+eBOSS	$-0.65^{+0.44(+1.3)}_{-0.71(-1.0)}$
SPT+WMAP+eBOSS+PP	$-0.93^{+0.11(+0.27)}_{-0.13(-0.23)}$
SPT+WMAP+DESI	$-0.44^{+0.58(+1.3)}_{-0.75(-1.0)}$
SPT+WMAP+DESI+PP	$-0.86^{+0.11(+0.27)}_{-0.15(-0.22)}$

within the 2σ range. Although $w_0 = -1$ remains consistent with all datasets at the 95% C.L., the central values point to slight deviations from a pure cosmological constant, in agreement with the evolving trends in $\rho_{\text{DE}}(z)$ and $w_{\text{DE}}(z)$ discussed earlier.

V. CONCLUSION

In this work, we have carried out a comprehensive analysis of a dynamical dark-energy model parametrized by the parameters $\{\alpha, \beta, a_m\}$, extending their prior ranges beyond those considered in previous studies, to enable a broader exploration of the parameter space of the model. This extended framework allows for qualitatively richer evolution histories of the dark-energy density $\rho_{\text{DE}}(z)$ and equation of state $w_{\text{DE}}(z)$, including possible phantom divide line crossings in both directions. By combining a variety of observational datasets—including CMB measurements from SPT, WMAP, and Planck; BAO data from eBOSS and DESI; and supernovae observations from Pantheon+ (PP)—we find mild deviations from a cosmological constant at intermediate redshifts, as our results are generally consistent with Λ CDM within 2σ or less. In particular, the EOS generically exhibits two PDL crossings in opposite directions within a single expansion history. These features are robust across different dataset combinations, suggesting that dynamical dark energy with physically nontrivial features might be present in a concordance model.

We also investigated the impact of prior choices by comparing the constraints obtained from full-prior and postfiltered MCMC chains. The postfiltered results assume the presence of a PDL crossing from quintessence to phantom regime and are better comparable to Refs. [160,161] that analyze the same DE model. The full priors reveal multimodal behavior highlighting the critical role of prior selection in capturing the full phenomenology of the model, as in this case the preferred present-day value of the EOS parameter is in the quintessence regime unlike the postfiltered results, which assume a present-day phantom value. Both cases are statistically consistent with -1 . For some dataset combinations, the H_0 tension is ameliorated formally due to significantly enlarged uncertainties in comparison to the Λ CDM model; however, the combinations of our CMB, BAO, and supernovae data give tight constraints on H_0 , in strong tension with the SH0ES measurements.

A robust qualitative feature that emerges across essentially all of our likelihood combinations is the frequent appearance of multiple (regular) crossings of the phantom divide line (PDL), $w_{\text{DE}} = -1$, within a single dark-energy evolution history. If this phenomenology persists with future data, it would carry nontrivial implications for fundamental model building, but it is crucial to state precisely what is excluded. In particular, a minimally coupled single scalar field with a standard kinetic structure,

$\mathcal{L}_\phi = \epsilon X - V(\phi)$ (with $\epsilon = +1$ for quintessence and $\epsilon = -1$ for a pure phantom), cannot realize a *stable* crossing of $w_{\text{DE}} = -1$ while maintaining $\rho_{\text{DE}} > 0$; such models satisfy $w_{\text{DE}} \geq -1$ (quintessence) or $w_{\text{DE}} \leq -1$ (phantom) as long as $\rho_{\text{DE}} > 0$ (see, e.g., Ref. [159]). Therefore, even a single smooth PDL crossing with strictly positive ρ_{DE} already points beyond the vanilla quintessence/phantom paradigm. Multiple crossings—especially when accompanied by oscillatory behavior and, in parts of parameter space, sign changes in ρ_{DE} —tighten these requirements further and naturally motivate multi-degree-of-freedom dark-sector realizations (e.g., *quintom* or effective multifluid descriptions) and/or an effective origin from modified gravity.² Importantly, this discussion should *not* be interpreted as excluding general single-field constructions: more elaborate frameworks—for instance, nonminimally coupled scalars or single-field EFTs with nontrivial kinetic structure/braiding and broken shift symmetry—can yield an *effective* equation of state that crosses -1 and may fit current data well (see, e.g., Refs. [194,195]). Our point is instead that the specific combination of features suggested by our constraints makes the simplest minimally coupled canonical realizations inadequate and provides concrete motivation to pursue fundamental ODE-like realizations, including noncanonical fields (e.g., Refs. [196,197]), coupled systems, or emergent dark-sector phenomena. Additionally, as a complementary diagnostic, one may map the ODE phenomenology onto an effective CPL description in the (w_0, w_a) plane by treating (w_0, w_a) as a data compression of the model predictions: for a range of ODE parameter choices, one can generate the corresponding observables and determine the best-fitting

(w_0, w_a) while verifying that the residuals remain small (e.g., Ref. [174]). For a meaningful comparison with data-driven constraints in the (w_0, w_a) plane, the mapping should also account for the redshift sensitivity and error budget of the probes employed (e.g., Refs. [175,176]). A dedicated implementation of this program is beyond the scope of the present analysis and is left to future work.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available because they are owned by a third party and the terms of use prevent public distribution. The data are available from the authors upon reasonable request.

²We note, however, that if ρ_{DE} is allowed to cross zero, a single-field phantom setup can accommodate transitions from $\rho_{\text{DE}} < 0$ with $w_{\text{DE}} > -1$ to $\rho_{\text{DE}} > 0$ with $w_{\text{DE}} < -1$ at late times; see Ref. [159]. In this case, the divergence of $w_{\text{DE}} = p_{\text{DE}}/\rho_{\text{DE}}$ at $\rho_{\text{DE}} = 0$ is a kinematic artifact of the ratio, rather than a singularity of the background expansion; see, e.g., Refs. [159,193].

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