Article:
Glass, A. and Kenjegalieva, K. (2026) The Great British productivity puzzle and regional industries: a dynamic multilevel spatial frontier analysis. Journal of Regional Science.
ISSN: 0022-4146

https://doi.org/10.1111/jors.70077

Journal of Regional Science

WILEY

JOURNAL OF REGIONAL SCIENCE

RESEARCH ARTICLE OPEN ACCESS

The Great British Productivity Puzzle and Regional Industries: A Dynamic Multilevel Spatial Frontier Analysis

Anthony J. Glass[1] | Karligash Kenjegalieva[2]

[1]Sheffield University Management School, Sheffield, UK | [2]Loughborough Business School, Loughborough University, Leicestershire, UK

Correspondence: Karligash Kenjegalieva (k.a.kenjegalieva@lboro.ac.uk)

Received: 22 June 2025 | **Revised:** 1 June 2026 | **Accepted:** 9 June 2026

Keywords: regional economic performance | regional industries | spillovers | stochastic frontier analysis | UK productivity puzzle

ABSTRACT

Using two approaches, we contribute to sub-national economic performance measurement by netting out the influence of a region's performance on its industries. The first approach measures performance using technical efficiency and involves introducing the first multilevel spatial stochastic frontier model. The second involves using the results from the first approach to introduce a dynamic multilevel spatial TFP growth decomposition. Using panel data, we apply the approaches to a production frontier for the eight industries nested in Great Britain's NUTS3 regions (2004 − 20). The motivation is the well-known UK productivity puzzle, which concerns a lack of definitive reasons why its post-2008 productivity slowdown has been noticeably bigger than comparator countries. One suggested contributing factor are the regional economic performance differences within the country, where both our approaches provide clearer evidence on this by reporting estimates of net regional industry economic performance. For three of the eight industries (Construction; Distribution, hotels and restaurants; Other services), we observe negative mean own and spillover TFP growth, which is consistent with the country's productivity slowdown and its economy being service oriented.

JEL Classification: C23, C43, D24, R12

1 | Introduction

It is well-known that non-negligible disparities in regional economic performance can lead to social unrest and reduce national economic growth and social mobility. Easily accessible, off-the-shelf measures of regional economic performance (e.g., real gross value added [GVA] per head) are therefore routinely used to track such disparities and steer policy interventions to manage them. Other measures of regional economic performance that are not as easy to obtain, as they are unobserved and must be estimated, are also used to provide further insights into these disparities. The first of two such estimated measures from a regional stochastic production frontier (e.g., Cardoso and Ravishankar 2015; Tsionas and Michaelides 2016; Ramajo and Hewings 2018; Gude et al. 2018) is technical efficiency. The second involves using a fitted regional stochastic production frontier model to conduct a total factor productivity (TFP) growth decomposition (Zhang and Ye 2015; Njuki et al. 2019; Glass and Kenjegalieva 2024). That is, from the fitted model, regional TFP growth is obtained by calculating its constituent components (which include, among others, the growth rate of returns to scale) and, depending on the theoretical form of the decomposition, multiplying or summing them.

An appropriately specified stochastic production frontier model for a sample of regional industries would, among other things, (i) account for the impact of the geographical links between the industries; and (ii) model the multilevel structure of the data to account for the regional level impact on the industries. Accounting for (i) and (ii) is important because if they were overlooked, various estimates, such as the industries' efficiencies, may be biased. To illustrate, if (ii) is not modeled, the estimate of an industry's efficiency may be upwardly (downwardly) biased if it also picks up the omitted upward (downward) impact of the regional efficiency where the industry is located. Such biased industry efficiency estimates can then lead to inaccurate policy recommendations to manage disparities in industries' economic performance.

As we expand on in the discussion of related stochastic frontier studies in the next section, in more recent times there has been a notable expansion of the spatial stochastic frontier literature (see Ayouba 2023, for a comprehensive literature review). This literature, however, consists of only single level models. Additionally, and as we also discuss further in the next section, Naderi (2022) is the only study to employ a non-spatial multilevel stochastic frontier model. The empirical application therein is also not a regional analysis as it focuses on how the technical efficiencies of departments in a university in Iran are influenced by higher administrative units (i.e., departments are nested within faculties and faculties are within colleges). This motivates our first contribution, where we highlight how the three contributions of the paper are related and cross-cut different literatures. The latter broadens the paper, while also giving a reader the opportunity to focus on a particular part of our analysis. The first two contributions are on empirical methods, where the first involves introducing the first multilevel spatial stochastic frontier model.

As growth of technical efficiency is one of the components of TFP growth, the latter is a broader measure of regional economic performance. Not too dissimilar to the aforementioned spatial stochastic frontier literature, the extant non-spatial and spatial TFP growth decompositions are for a single level analysis. As noted above, depending on the underlying economic theory, TFP growth is obtained by multiplying or summing its constituent components. Single level regional industry decompositions would yield biased values of regional industry TFP growth due to the decomposition only comprising regional industry components and thus omitting higher level regional components. To address this bias by including the impacts of the corresponding regional level measures among the constituent components, our second contribution uses our stochastic frontier to set out the first multilevel spatial TFP growth decomposition.

The third contribution is empirical, as we apply the methods from our first two contributions using data on the eight industries in each of the 134 NUTS3 regions in Great Britain (2004 − 20) (see footnotes 1 and 2).[1] The regional industries in Northern Ireland are not part of our sample due to missing data for a number of variables.[2] The three reasons for the choice of the above regional industry case for the empirical analysis are as follows. First, compared to countries whose pre-2008 productivity growth tracked the UK's growth (Germany, France and the U.S.), the UK's 2008-crisis induced productivity slowdown has been deep and persistent. This is very concerning as productivity is an important determinant of country competitiveness and social issues (i.e., the availability of housing and funding of healthcare, social services, education and pensions). This comparative UK slowdown, which is commonly presented using a country labor productivity index (e.g., Haldane 2018), is evident from Figure 1 (1998 − 2022).[3,4] From 2008 to 2022, the indices for the U.S. and Germany increased from 100 to 118.1 and 111.3, respectively. Over this period, the index for the UK increased from 100 to 105.9, which highlights its relatively weak post-crisis rise in labor productivity.

Second, from our analysis of Britain's NUTS3 industries, we provide a new way to steer the development of disaggregated policies to enhance productivity. This is important because, as we highlight in our coverage of the related literature in the next section, the post-2008 UK productivity slowdown is known as a 'puzzle', as there is a lack of consensus on why this slowdown has been more acute than for countries with comparable pre-2008 labor productivity growth. We conclude from this coverage that there is a need to address some imbalance in this literature: namely, a relatively smaller number of studies that use the results from rigorous micro empirical analysis to suggest a clear steer of

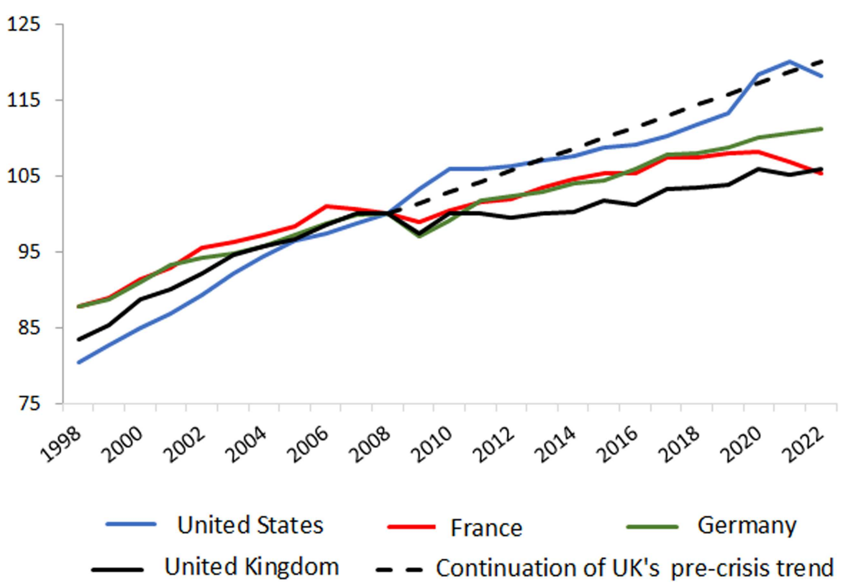


FIGURE 1 | Output per hour worked index (2008 = 100). [Color figure can be viewed at wileyonlinelibrary.com]

the process to develop specific actionable micro level policies to promote productivity improvement. To this end, the motivation for our empirical analysis is the post-2008 increases in the UK's regional productivity differences and their contributions to the national slowdown, which, among others, McCann (2020) highlights. On average, this points to the relatively marked slowdown for lower productivity regions driving the national picture. As Haldane (2020) notes that the UK's left-behind places are in a vicious circle, the left-behind NUTS3 regions and industries may find it challenging to improve their productivity by themselves. Hence our focus on productivity spill-ins, but as we expand on below, only some regional industries have sufficiently large relative productivity growth spill-ins.

As we discuss further in the paper, we obtain the TFP growth spill-in (spill-out) for each regional industry from (to) all the other industries. Using these results, we uncover the industries whose output ranking is below that of its TFP growth induced output spill-in (spill-out). As this indicates where these industries perform relatively better, we suggest that steering disaggregated policy development towards increasing these spill-ins and spill-outs would be a promising route to productivity improvement. Third, in support of our choice of Britain over other countries for our regional industry application of the TFP growth decomposition, Bryson and Forth (2016) note that the UK's post-2008 productivity performance is a low TFP growth problem.

The remainder of this paper is organized as follows. In Section 2 we (a) discuss the three groups of studies that relate to the above contributions; and (b) set out our hypotheses for the empirical analysis. Section 3 presents the spatial empirical methodology. This methodology comprises three subsections which present (i) the dynamic multilevel spatial autoregressive (SAR) frontier model; (ii) the dynamic multilevel elasticities and inefficiency spill-ins and spill-outs; and (iii) the dynamic multilevel spatial TFP growth decompositions. In Section 4, we present and discuss the empirical results from our granular geographical industry analysis, and in Section 5 we summarize and conclude.

2 | Related Literature and Hypotheses

2.1 | Stochastic Frontier Models With Spatial or Multilevel Structures

At the outset, consider a standard single level regional industry stochastic frontier, with the usual time-varying composed error comprising regional industry inefficiency and noise. By augmenting this model with the spatial lag of the dependent variable, more aggregate higher level regional regressors, and regional level time-varying inefficiency and noise components, we move to a two-level SAR frontier. This guards against inaccurate policy recommendations from this two-level model to manage disparities in the industries' economic performance by (i) providing useful estimates of spillovers and higher level impacts; and (ii) addressing any bias in the estimates of the industry level coefficients and inefficiencies from the standard model due to it overlooking the aforementioned spatial and hierarchical data structures. Given the absence of a multilevel spatial stochastic frontier from the literature, our first contribution is to introduce this type model. While there have been various advances in the spatial stochastic frontier analysis (SSFA) literature on, among other things, the SAR model (e.g., Glass et al. 2016; Kutlu 2018; Gude et al. 2018; Ramajo and Hewings 2018; Kutlu and Nair-Reichert 2019), non-spatial and/or spatial determinants of inefficiency (e.g., Tsukamoto 2019; Galli 2024; 2023a; 2023b), Bayesian estimation (e.g., Tsionas and Michaelides 2016; Carvalho 2018) and dynamic approaches (e.g., Skevas 2020; Glass and Kenjegalieva 2024; Emili and Galli 2025), this literature exclusively comprises single level models.

By including higher level explanatory variables, we follow a number of multilevel studies in the regional literature (e.g., Chung and Hewings 2015; Tselios et al. 2015; Giannakis and Bruggeman 2020; Lavoratori and Castellani 2021; Mádr 2025).[5] As nearly all multilevel studies are not concerned with estimating technical efficiency, the composed error structure in these models comprises an idiosyncratic error for each level in the hierarchy. In the regional literature, two empirical studies of hospitals (Martini et al. 2014) and banks (Aiello and Bonanno 2018) successively estimate two models to sequentially analyze efficiency and the impact of the hierarchical data structure. A standard single level stochastic frontier model is estimated to obtain the efficiency estimates, which are then used as the dependent variable (or a regressor) in a multilevel model. It is evident, therefore, that the regional literature lacks a stochastic frontier model for hierarchical data. Besides following multilevel models in the regional literature by including higher level explanatory variables, our stochastic frontier framework extends these models by including time-varying composed errors for the two levels. The components of the time-varying composed level 1 error are a NUTS3 industry's disturbance and inefficiency; while the time-varying composed level 2 error comprises the common impacts of the corresponding NUTS3 regional disturbance and inefficiency on the region's industries.

Our first contribution also extends Naderi's (2022) non-spatial multilevel SFA study of an Iranian university. To the best of our knowledge, Naderi is the only study to use a single model to jointly analyze efficiency and the impact of the hierarchical data structure. Naderi's approach involves constructing multilevel frontiers via post-estimation shifting of the fitted multilevel model. Such post-estimation shifting is not common in SFA, so we avoid this by estimating efficiency using multilevel normal-half-normal distributional assumptions for the lower and higher level noise and inefficiency terms. Consequently, there is a direct correspondence between our multilevel distributional assumptions and the widely used single level normal-half-normal error structure in regional SFA applications (e.g., Tveteras and Battese 2006; Duvivier 2013; Buch-Gómez and Cabaleiro-Casal 2020; Bashford-Fernández and Rodríguez-Álvarez 2024).

To account for contemporaneous and dynamic spatial dependence, we further extend Naderi's approach by including the spatial lag of the dependent variable and its time lag. As there are a large number of other multilevel regional cases where there is SAR dependence, there is scope to apply our spatial frontier model more widely. With or without including the spatial variables, a further area which our production frontier approach could be applied to, and which has received a lot of attention in the multilevel literature, is pupil attainment (e.g., Steele et al. 2007; Taylor 2007; Huang and Zhu 2020), as pupils are nested within classes which are nested within schools. Our approach could also be used to analyze the performance of subsidiary companies nested within holding corporations.

2.2 | Related Total Factor Productivity Growth Decompositions

Using our frontier model, our second contribution extends O'Donnell's (2016, 2018, 2022) static single level non-spatial TFP growth decomposition to our dynamic multilevel spatial setting. The initial spatial TFP growth decompositions (Glass et al. 2013; Glass and Kenjegalieva 2019; Glass et al. 2020) involved extending a static single level non-spatial Malmquist TFP growth decomposition (Orea 2002) to include spillover components. Using, for instance, a standard non-spatial stochastic production frontier, a simple Malmquist approach decomposes TFP growth into components that include the growth rates of technical change, returns to scale and technical efficiency. Extending the Malmquist approach to the static spatial setting involves augmenting the non-spatial TFP growth decomposition with the corresponding contemporaneous spill-in (or spill-out) components (Glass et al. 2013; Glass and Kenjegalieva 2019; Glass et al. 2020). Reflecting that it may take some time for spillovers to occur, Glass and Kenjegalieva (2024) extend a static single level spatial Malmquist TFP growth decomposition by augmenting it with the corresponding dynamic spill-in (or spill-out) components. The dynamic spill-in components represent the contributions to a unit's TFP growth in the current time period that spill-in from other units components in a previous period. Conversely, the dynamic spill-out components are the contributions from a unit's spill-out components in a previous period to other units TFP growth in the current period.

The Malmquist approach to decompose spatial/non-spatial TFP growth does not, therefore, include components that relate to the growth of environmental variables and the difference in the magnitudes of unexplained economic shocks from one period to the next.[6] The latter component indicates the sensitivity of a unit's TFP growth to economic shocks, which is useful to compare between regional industries. To ascertain whether the aforementioned components have non-negligible impacts on TFP growth, it is prudent to include them in the decomposition. One motivation for this is to account for the impact of the marked variations in the regional productivity environments in Britain on the productivity growth of regional industries. As O'Donnell's (2016, 2018, 2022) TFP growth decomposition includes components that measure the growth of environmental variables and differences in economic shocks from one period to the next, we extend this decomposition to our dynamic multilevel spatial setting.[7]

2.3 | UK Productivity Puzzle

Two broad groups of studies of the UK productivity puzzle have emerged. One group consists of reviews of the UK's productivity situation (e.g., Haldane 2018; McCann 2020; Van Ark and Venables 2020; Van Ark 2021; Williams et al. 2024) and the other comprises macro through to micro level empirical studies (e.g., Blundell et al. 2014; Coyle et al. 2025; Coyle and Mei 2023; Douch et al. 2023; Glass and Kenjegalieva 2024; Goldin et al. 2024; Goodridge et al. 2018; Goodridge and Haskel 2023; Harris and Moffat 2017; Patterson et al. 2016; Pessoa and Van Reenen 2014; Riley et al. 2018; Turrell et al. 2021; Van Reenen and Yang 2024). Turning to how both groups motivate the policy focus of our empirical analysis. The reviews in the first group cover a wide range of reasons for the UK's productivity situation, thereby highlighting the lack of consensus on the contributing factors to the UK's lagging productivity. This lack of consensus, together with these papers being reviews, are the reasons why they do not report and use new empirical evidence to make specific actionable micro level policy recommendations.

Concentrating on the micro level studies within the second group (e.g., Coyle et al. 2025; Douch et al. 2023; Harris and Moffat 2017) as this is where our paper sits. As a likely response to the aforementioned lack of consensus, the extant micro level studies in the second group tend to focus on reporting empirical evidence on the contributions of particular factors to the UK's post-2008 productivity performance. For example, Douch et al. (2023) find that cohorts of newer established firms in the services sector are a driver of the UK's post-crisis productivity slowdown. It is, of course, imperative to first gain a clear understanding of such contributions, but by concentrating on these, we suggest that micro level studies in the second group have not focused enough on the challenging task of using their results to make specific actionable micro level policy recommendations to promote productivity improvement (i.e., at the plant, firm and NUTS3 industry and regional levels). Along these lines, a recent article in the Financial Times (2025) concludes that there is a puzzle relating to the most appropriate policies to improve UK productivity.

There are cases though where micro level studies in the second group use their results to inform specific actionable micro level policy recommendations. Two examples include, focusing on improving firm level management quality in the UK (e.g., Bloom et al. 2016) and utilizing the NUTS3 regions in Britain that generate the largest TFP growth spill-ins to the country's left-behind regions (Glass and Kenjegalieva 2024). Rather than use regional data like the latter, which represents the closest productivity study to our paper, we use more disaggregated NUTS3 industry data. As a result, we provide a clearer steer of the process to develop specific actionable micro level policies to harness TFP growth spill-ins to support regions who may find it challenging to increase their productivity by themselves.

Turning to further compare our study with Glass and Kenjegalieva (2024). In line with technical inefficiency impacting productivity and Pessoa and Van Reenen (2014) noting that underutilization of resources is a factor driving the UK productivity slowdown, Glass and Kenjegalieva (2024) is the only other analysis of Britain's productivity puzzle to explicitly estimate technical inefficiency.[8] Whilst they also estimate a spatial stochastic frontier, their frontier analysis differs from ours in two further respects. First, unlike our multilevel spatial stochastic frontier, their spatial efficiency study is the usual single level analysis. Second, whereas we employ the widely used approach of distributional assumptions to jointly estimate the frontier and efficiencies in a single step, their less common and simpler second step approach to estimate efficiency involves shifting the fitted frontier using the unit specific effects.

2.4 | Hypotheses

Below we present and provide the rationales for the two empirical hypotheses (H1 and H2).

H 1. *The impacts of the contemporaneous and dynamic spatial lags of the industry output dependent variable in the frontier model are positive and significant.*

H1 is based on the frequently observed positive SAR dependence, whereas a negative estimate of this spatial relationship is regarded as "curious" (Kao and Bera 2013). Moreover, the positive contemporaneous and dynamic SAR dependence in the stochastic production frontier model points to positive output spill-ins. In and of themselves, these will lead to positive TFP spill-ins, which we suggest can be harnessed to support the left-behind regional industries that may find it challenging to increase their productivity by themselves.

H 2. *Within an industry classification (agriculture & fishing combined with energy and water; manufacturing; etc.), there is evidence of regional industries with high ranking output levels having high efficiency rankings.*

Some likely reasons why regional industries with high ranking output levels have high efficiency rankings include the following. (i) If a region has relatively high infrastructure investment and more developed logistics systems, this will aid its industries' output levels and operational performance. (ii) Regional industries with high ranking output levels are more likely to have better access to resources and technology (e.g., they will be more attractive to skilled labor and better capitalized, aiding investment in modern technologies), which will promote higher efficiency. (iii) Regional industries where agglomeration economies are high will benefit from knowledge spillovers, proximity to suppliers and customers, and shared infrastructure, raising both industry output and efficiency.

3 | Empirical Methodology

3.1 | Dynamic Multilevel Spatial Stochastic Production Frontier

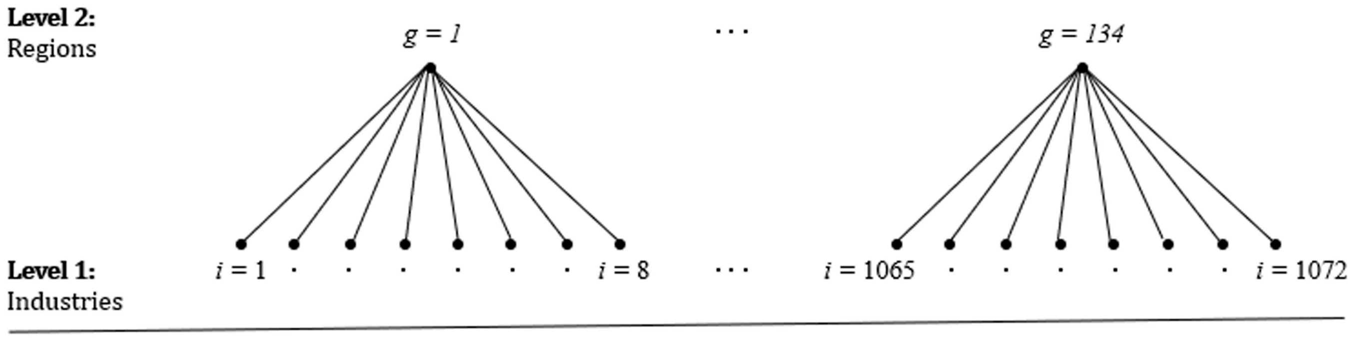
The general form of the dynamic multilevel SAR stochastic production frontier we estimate is given in Eq. (1), where throughout the presentation of the methodology data points denoted using lower case letters are in logs. In summary, and as we elaborate on below, our model has two levels as this gives a five component inefficiency-noise error structure. Our model does not have more levels as it would be computationally challenging to split an error structure with more components using distributional assumptions. The most aggregate level 2 comprises NUTS3 regions, which nest the level 1 industries, that is, each NUTS3 region has the eight industries in footnote 1. In Eq. (1) we use subscripts 1 and 2 to denote the industry and regional level terms. These regional level terms include, among others, the regional variables that account for further observed heterogeneity and hence shift the level 1 industry frontier.

$$\begin{aligned} y_{1,igt} = \alpha &+ \beta x'_{1,igt} + \gamma x'_{2,gt} + \eta \sum_{j=1}^{N} w_{1,ij}\, y_{1,jgt} \\ &+ \theta \sum_{j=1}^{N} w_{1,ij}\, y_{1,jgt-1} + \delta \sum_{j=1}^{N} w_{2,ij}\, y_{1,jgt} \\ &+ \rho \sum_{j=1}^{N} w_{2,ij}\, y_{1,jgt-1} + b_{1,t} + d_{1,ig} + v_{2,gt} - u_{2,gt} \\ &+ v_{1,igt} - u_{1,igt}. \end{aligned} \tag{1}$$

We use balanced panel data covering the 17 −year period 2004 − 20, where the time periods are indexed $t \in 1, \ldots, T = 17$. There are 1072 level 1 industries in our sample, which corresponds to Britain's 134 granular NUTS3 regions in level 2 multiplied by the eight industries in each region. The 1072 industries for each of the 17 years yields a large sample size of 18, 224 observations. Reflecting the two levels, the two indexations of the industries and regions are $i, j \in 1, \ldots, N = 1072$ for $i \neq j$ and $g, h \in 1, \ldots, G = 134$ for $g \neq h$, respectively. Note that in order to later denote the asymmetric level 1 spill-in and spill-out between any two industries and the asymmetric level 2 spill-in and spill-out between all the industries in one region and all those in another, we distinguish between any two industries (i, j) and regions (g, h). For a diagram of the above-described two level structure, see Figure 2.

For the i-th industry in the g-th region in period t, $y_{1,igt}$ in Eq. (1) is the output observation and we collect the observations for all the industry regressors (indexed $p \in 1, \ldots, P$) in the $(P \times 1)$ vector $x'_{1,igt}$. For the g-th region in period t, we also collect the observations for all the regional regressors (indexed $q \in 1, \ldots, Q$) in the $(Q \times 1)$ vector $x'_{2,gt}$, where the observations for each of these regressors are common across the eight industries in a region. We present the general form of the model in Eq. (1) by collecting the observations for the industry and regional regressors in $x'_{1,igt}$ and $x'_{2,gt}$ as this simplifies the notation for the procedure we present later for the model estimation. In the presentation of the specific form of the empirical model in 3.3, we distinguish between particular industry and regional regressors. These regressors for the industries include, for example, their inputs and $x'_{2,gt}$ comprises the regional observations that correspond to the industry regressors and also data for variables that are only observed at the regional level, such as a measure of how urbanized the NUTS3 region is.[9]

Time period effects $(b_{1,t})$ are included to account for common shocks across the industries. By including these effects these common shocks will not be conflated with the impacts of the contemporaneous SAR variables (see below for discussion of these spatial lags). Next, we note that the composed error in Eq. (1) is $\mathring{\varepsilon}_{M,igt} = d_{1,ig} + \varepsilon_{2,gt} + \varepsilon_{1,igt} = d_{1,ig} + v_{2,gt} - u_{2,gt} + v_{1,igt} - u_{1,igt}$, where in the parlance of a subscript denoting the corresponding level, the subscript M denotes that a measure has multilevel components. To simplify the estimation of the components of $\mathring{\varepsilon}_{M,igt}$ using distributional



$i = \{1, 9, \ldots, 1065\}$ is Agriculture & fishing combined with energy & water
$i = \{2, 10, \ldots, 1066\}$ is Manufacturing
$i = \{3, 11, \ldots, 1067\}$ is Construction
$i = \{4, 12, \ldots, 1068\}$ is Distribution, hotels & restaurants
$i = \{5, 13, \ldots, 1069\}$ is Transport & communications
$i = \{6, 14, \ldots, 1070\}$ is Banking, finance & insurance
$i = \{7, 15, \ldots, 1071\}$ is Public administration, education & health
$i = \{8, 16, \ldots, 1072\}$ is Other services

FIGURE 2 | Hierarchical structure.

assumptions, we limit the number of components in this term. We do so in two ways: first, we restrict the number of levels in our model to two. Second, we include only five error components (and not six) as we account for unobserved heterogeneity using industry random effects ($d_{1,ig}$) and omit regional random effects. Without too much difficulty, and using the same type of approach as we adopt to model $d_{1,ig}$ in the estimation procedure, our model could be extended to include regional random effects. Alternatively, given we omit regional random effects, one could account for unobserved industry heterogeneity in our model using fixed effects, but we do not pursue this for the following two reasons.

First, this would prevent further work from extending our two level framework to account for unobserved regional heterogeneity. This is because the impacts of the regional level regressors would be confounded with the regional fixed effects (Centre for Multilevel Modeling 2024), that is, it would not be possible for these impacts and effects to be identified as they cannot be separated. There is no such issue though with industry and regional random effects. Second, using industry level random effects rather than fixed effects avoids further complicating the model estimation. To appreciate the complications involved with industry fixed effects, note that, in contrast to the estimation of linear fixed effects models, the usual demeaning (i.e., within transformation) to eliminate these effects does not yield a tractable log-likelihood function for nonlinear models, for example, a stochastic frontier model (Wang and Ho 2010). To address this, the following three approaches have been used to model fixed effects in a stochastic frontier model.

The first approach retains the fixed effects dummies by not demeaning (Greene 2005), but this may yield inconsistent parameter estimates due to the well-known incidental parameters problem (Neyman and Scott 1948). Both the second and third approaches are more complex and involve using demeaned data to address the aforementioned potential inconsistency. The second approach involves modeling time-varying inefficiency as the product of the truncated normally distributed time-invariant inefficiency and a particular function that depends on time-varying exogenous variables (see Wang and Ho 2010, for details). For a spatial stochastic frontier using this approach, see Galli (2024). While the second approach requires determinants of inefficiency, the third recognizes that a number of stochastic frontier models do not include such determinants. In the third approach, as well as the widely employed normal-half-normal distributional assumptions for noise and time-varying inefficiency, these errors are collectively assumed to have a closed skew normal distribution (see Chen et al. 2014, for details). For a SSFA using the third approach, see Adetutu et al. (2025).[10]

Returning to Eq. (1), the i-th industry's inefficiency and disturbance are $u_{1,igt} \sim N^{+}\left(0, \sigma^2_{u_1}\right)$ and $v_{1,igt} \sim N\left(0, \sigma^2_{v_1}\right)$, respectively. $u_{2,gt} \sim N^{+}\left(0, \sigma^2_{u_2}\right)$ and $v_{2,gt} \sim N\left(0, \sigma^2_{v_2}\right)$ are the impacts on $y_{1,igt}$ of the unobserved inefficiency and disturbance from region g (i.e., the common impacts across the eight industries located in region g). Hence, the industry efficiency, benchmarked impact of efficiency from region g, and hierarchical adjusted industry efficiency are $IE_{1,igt} = \exp(-u_{1,igt})$,$RE_{2,gt} = \exp(-u_{2,gt})$, and $HE_{M,igt} = IE_{1,igt} \times RE_{2,gt}$, respectively.

The four spatial environmental variables we include comprise two contemporaneous SAR variables and their one period time lags, where the latter are included as it may take some time for spillovers to occur. The contemporaneous SAR variables are endogenous, which is accounted for in the estimation procedure. As will be evident in 3.2 and 3.3, in addition to contemporaneous asymmetric spill-in and spill-out measures (elasticities, inefficiencies and economic shocks as measured by noise), all of which are the result of including the contemporaneous SAR variables, by also including their time lags we obtain the corresponding dynamic measures for future in-sample periods. Using these measures we obtain the contemporaneous and dynamic asymmetric spill-in and spill-out components of the multilevel spatial TFP growth decompositions.

The observations for the contemporaneous SAR variables and their time lags are weighted averages of the outputs of the industries that are linked to the i-th industry.[11] These linkages are represented by the a priori specifications of the $(N \times N)$

row-normalized spatial weights matrices W_1 and W_2. The ij-th elements of these matrices are the weights $w_{1,ij}$ and $w_{2,ij}$, where the elements on the main diagonals are set to zero as an industry cannot be linked to itself. The off-diagonal elements in these matrices represent two different specifications of the linkages between the industries, where in each spatial weights matrix if two industries are linked the pre-normalised weight is 1 (and 0 otherwise). Using ~ to denote pre-normalised weights, $\widetilde{w}_{1,ij} = 1$ if the i-th and j-th industries are in the same region and are thus different industry types. See Figure 2 or footnote 1 for the eight industry types in each region. $\widetilde{w}_{2,ij} = 1$ if the two industries are of the same type in contiguous regions. Note here that a subscript 1 denotes that $\widetilde{w}_{1,ij}$ represents industry links *within* the same region, and a subscript 2 denotes that $\widetilde{w}_{2,ij}$ represents particular industry links *between* (contiguous) regions. The formal specification of these pre-normalised weights is given in Eq. (2).

$$\widetilde{w}_{1,ij} = \begin{cases} 1 & \begin{array}{l}\text{for two industries } (i \text{ and } j) \\ \text{in the same region } (g)\end{array} \\ 0 & \text{otherwise} \end{cases};$$
$$\widetilde{w}_{2,ij} = \begin{cases} 1 & \begin{array}{l}\text{for two industries } (i \text{ and } j) \\ \text{of the same type in any two} \\ \text{contiguous regions } (g \text{ and } h)\end{array} \\ 0 & \text{otherwise} \end{cases}. \tag{2}$$

We then normalize the weights in Eq. (2) by the corresponding row sum in $\widetilde{W}_1$ and $\widetilde{W}_2$. For all $\widetilde{w}_{1,ij} = 1$ this gives $w_{1,ij} = \frac{1}{8}$ as each row sum is the number of industries in a region. The mean row sum of $\widetilde{W}_2$ (or, in other words, the mean number of contiguous neighboring regions in the sample) is 4.30.

Corrado and Fingleton (2012) stress that to align with the nature of spatial econometrics applications to economic phenomena, the spatial weights should be based on the economic linkages between the units. While this can give rise to endogenous weights and the need for a more complex estimator to account for this than the one we use, there is no such issue with the weights we employ. We also use these weights for the NUTS3 industries, which is the most disaggregated available industry-regional NUTS data, as there is a lack of data on the economic linkages between the industries (e.g., an input-output matrix). While the weights we use, in and of themselves, do not reflect the economic linkages between the industries, by attaching equal weights to the linked industries across a row in W_1 and W_2, we allow the outputs of the linked industries to determine the magnitudes of their contemporaneous and dynamic interdependence. In doing so, we implicitly allow these interdependence estimates to be determined by the combined influence of the interindustry and spatial economics (e.g., value chains) that underpin the interconnnectivity of the outputs of the linked industries in W_1 and W_2. Given we use our stochastic frontier to obtain multilevel decompositions of the contemporaneous and dynamic TFP growth spill-ins and spill-outs, we implicitly allow the economics that underpins the interconnectivity of the outputs of linked industries to determine these spill-ins and spill-outs.

Whilst a lack of data on disaggregated economic linkages leads us to implicitly model the economics that drives the interconnectivity of the outputs of linked industries, using other data that is unsuitable for our purposes, Carrascal-Incera and Hewings (2024) and Bolea et al. (2022) explicitly model the underpinning economics. Whereas we use data for 2004 − 20, Carrascal-Incera and Hewings (2024) employ a multi-regional input-output model, and hence data for a single year as the model is calibrated for 2016, to analyze asymmetric income, goods and service flows between UK regions. Also using data for a single year (i.e., 2010 as this represented the most recent year of the EUREGIO input-output database for the EU NUTS2 regions), Bolea et al. (2022) analyze the impacts of the shares and positions of regions in global values chains.

As per the spatial econometrics literature, we impose the parameter limits $\{\eta, \theta, \eta + \theta\} \in \left(\frac{1}{\xi_{\min}}, \frac{1}{\xi_{\max}}\right)$ and $\{\delta, \rho, \delta + \rho\} \in \left(\frac{1}{\varpi_{\min}}, \frac{1}{\varpi_{\max}}\right)$, where $\xi_{\min}$ and $\varpi_{\min}$ ($\xi_{\max}$ and $\varpi_{\max}$) are the most negative (positive) real characteristic roots of W_1 and W_2, respectively. As each spatial weights matrix is row-normalised $\xi_{\max}$ and $\varpi_{\max}$ are both 1. The limits on each contemporaneous and dynamic SAR coefficient ensure that each individual spatial process is stationary (i.e., non-explosive across space), while the limits on $\eta + \theta$ and $\delta + \rho$ are so the collective contemporaneous and dynamic spatial processes are stationary. Among the other parameters to be estimated are the intercept (α) and vectors of coefficients on the industry and regional regressors (β and γ). Whilst β and γ represent the own impacts of changes in the level 1 and 2 x observations, and η, δ, θ and ρ measure the local spillover impacts of changes in the outputs of the geographically linked industries, these parameters overlook some form of spillover captured by the spatial multiplier matrix. These overlooked spillovers are the impacts of changes in the level 1 and 2 x observations for a single industry and single region on the outputs of all the other industries, and in the other direction, the impacts of changes in the level 1 and 2 x observations for all the other industries and regions on this single industry. To account for all these impacts, we compute two total elasticities for each of the level 1 and 2 x variables. As we elaborate on in 3.2, we also provide a breakdown of these elasticities into direct and asymmetric indirect (spill-in and spill-out) impacts of these variables, where we obtain a total elasticity by summing the direct and spill-in (spill-out) impacts.

For details of the simulated maximum likelihood procedure to estimate Eq. (1), see Appendix A.

3.2 | Total Elasticities and the Industry and Regional Level Inefficiency Spillovers

We obtain the contemporaneous and dynamic direct, indirect and total level 1 and 2 elasticities, inefficiencies and disturbances by building on the approach in Debarsy et al. (2012) to compute the corresponding elasticities for the single level setting. To do so, we use the data generating process (DGP) of the dependent variable in Eq. (3), which is the reduced form of Eq. (1). To obtain Eq. (3), first, for each time period we stack successive cross-sectional observations for the dependent variable. We then stack this data by time periods and write the dependent variable for the whole sample as the $(NT \times 1)$ vector $\mathbf{y}_1 = (\mathbf{y}_{1,1}, \ldots, \mathbf{y}_{1,T})'$. In the same way, for levels 1 and 2 we write the x variables for the whole sample as the $(NT \times P)$ and $(NT \times Q)$ matrices $\mathbf{X}_1 = (\mathbf{X}_{1,1}, \ldots, \mathbf{X}_{1,T})'$ and $\mathbf{X}_2 = (\mathbf{X}_{2,1}, \ldots, \mathbf{X}_{2,T})'$, respectively, where, to illustrate, $\mathbf{X}_{1,T}$ is the $(N \times P)$ matrix of observations for the level 1 x variables in period T. The inefficiencies and disturbances are also written as the $(NT \times 1)$ vectors $\mathbf{u}_1 = (\mathbf{u}_{1,1}, \ldots, \mathbf{u}_{1,T})'$, $\mathbf{u}_2 = (\mathbf{u}_{2,1}, \ldots, \mathbf{u}_{2,T})'$, $\mathbf{v}_1 = (\mathbf{v}_{1,1}, \ldots, \mathbf{v}_{1,T})'$ and $\mathbf{v}_2 = (\mathbf{v}_{2,1}, \ldots, \mathbf{v}_{2,T})'$.

$$\mathbf{y}_1 = \sum_{p=1}^{P}\sum_{q=1}^{Q} \Lambda_{NT}^{-1}(\beta_p I_{NT}\mathbf{x}_{1,p} + \gamma_q I_{NT}\mathbf{x}_{2,q}) + \Lambda_{NT}^{-1} \begin{pmatrix} \alpha\iota_{NT} + (\iota_N \otimes I_T)\mathbf{b}_1 + (\iota_T \otimes I_N)\mathbf{d}_1 + \\ \mathbf{v}_1 - \mathbf{u}_1 + \mathbf{v}_2 - \mathbf{u}_2 \end{pmatrix}, \tag{3}$$

where

$$\Lambda_{NT} = \begin{pmatrix} \mathbf{A}_{t=1} & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \mathbf{B}_{t=2} & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0} & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{B}_{t=T-1} & \mathbf{A}_{t=T} \end{pmatrix} \text{ and } \Lambda_{NT}^{-1} = \begin{pmatrix} \mathbf{A}_{t=1,\psi=0}^{-1} & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \mathbf{C}_{t=2,\psi=1} & \ddots & \ddots & \ddots & \vdots \\ \mathbf{C}_{t=3,\psi=2} & \mathbf{C}_{t=3,\psi=1} & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \mathbf{0} \\ \mathbf{C}_{t=T,\psi=T-1} & \mathbf{C}_{t=T,\psi=T-2} & \cdots & \mathbf{C}_{t=T,\psi=1} & \mathbf{A}_{t=T,\psi=0}^{-1} \end{pmatrix}.$$

Λ_{NT} is the time and space filtered $(NT \times NT)$ block matrix. The block nature of this matrix means that it comprises a number of matrices: namely, the same $\mathbf{A}_t = I_N - \eta W_1 - \delta W_2$ matrix for each time period $t \in 1, \ldots, T$, where η and δ are the contemporaneous SAR coefficients from the fitted model and I_N is the $(N \times N)$ identity matrix; the same $\mathbf{B}_t = -\theta W_1 - \rho W_2$ matrix for each time period $t \in 2, \ldots, T-1$, where θ and ρ are the dynamic SAR coefficients from the fitted model; and the $(N \times N)$ matrices of zeros are denoted $\mathbf{0}$. In Eq. (3), I_{NT} is the $(NT \times NT)$ identity matrix, $\mathbf{x}_{1,p}$ is the $(NT \times 1)p$-th column of $\mathbf{X}_1$, $\mathbf{x}_{2,q}$ is the $(NT \times 1)q$-th column of $\mathbf{X}_2$ and β_p and γ_q are the associated coefficients. ι_{NT} is the $(NT \times 1)$ vector of ones and ι_N and ι_T are the N and T dimensional vectors of ones. $\otimes$ denotes the Kronecker product operator, which is used to obtain the $(NT \times T)$ matrix $\iota_N \otimes I_T$. This matrix assigns each of the time period effects from the $(T \times 1)$ vector $\mathbf{b}_1 = (b_{1,1}, \ldots, b_{1,T})'$ to all the industries. Similarly, the $(NT \times N)$ matrix $\iota_T \otimes I_N$ assigns the relevant random effect from the $(N \times 1)$ vector $\mathbf{d}_1 = (d_{1,1}, \ldots, d_{1,N})'$ to the corresponding industry in each period.

In the $(NT \times NT)$ lower triangular block matrix Λ_{NT}^{-1}, the current and remaining time periods over the sample are indexed $\psi \in 0, 1, \ldots, T-2, T-1$. $\psi = 0$ is the contemporaneous period, where for any period the number of future in-sample time horizons lies in the range $[1, T-1]$. The block nature of the matrix Λ_{NT}^{-1} means that it comprises a number of matrices: namely, the same $\mathbf{A}_{t,\psi=0}^{-1} = (I_N - \eta W_1 - \delta W_2)^{-1}$ matrix for each time period $t \in 1, \ldots, T$; and the spatial multiplier matrix $\mathbf{C}_{t+\psi,\psi} = (-1)^{\psi}(\mathbf{A}_{t,\psi=0}^{-1}\mathbf{B}_t)^{\psi}\mathbf{A}_{t,\psi=0}^{-1}$ that varies with the time horizon ψ. Next, let Ψ denote the set of time horizons from the current period 0 to ψ periods ahead. Differentiating Eq. (3) with respect to a permanent industry level marginal change in $x_{1,pt}$ yields the matrices of the contemporaneous, cumulative one period ahead and cumulative ψ periods ahead industry level impacts in Eqs. (4a–c), respectively. For purposes of comparison with Eqs. (4a–c), in footnote 12 we use the same type of approach by differentiating Eq. (3) to obtain the matrices of the contemporaneous, cumulative one period and cumulative ψ periods ahead industry level impacts of a permanent regional level marginal change in $x_{2,qt}$.[12] It is clear that the form of each equation in this footnote is closely related to its counterpart in Eqs. (4a–c). Importantly though, and in contrast to a change in $x_{1,pt}$, when there is a change in $x_{2,qt}$, this change will be the same for each industry in a particular region.

Turning to the interpretation of Eqs. (4a–c) and the three equations in footnote 12.

$$\frac{\partial y_{1,t,\psi=0}}{\partial x_{1,pt}} = \mathbf{A}_{t,\psi=0}^{-1} I_N \beta_p, \tag{4a}$$

$$\frac{\partial y_{1,t+1,\psi=1}}{\partial x_{1,pt}} = \left(\mathbf{A}_{t,\psi=0}^{-1} + \mathbf{C}_{t+1,\psi=1}\right) I_N \beta_p, \tag{4b}$$

$$\frac{\partial y_{1,\Psi}}{\partial x_{1,pt}} = \sum_{\psi \in \Psi} \mathbf{C}_{t+\psi,\psi} I_N \beta_p. \tag{4c}$$

The spatial multiplier matrix is $\mathbf{A}_{t,\psi=0}^{-1}$ in the contemporaneous period, $\mathbf{A}_{t,\psi=0}^{-1} + \mathbf{C}_{t+1,\psi=1}$ is the cumulation of the spatial multiplier matrices in time horizons 0 and 1, and, more generally, $\sum_{\psi \in \Psi}\mathbf{C}_{t+\psi,\psi}$ is the cumulation of the spatial multiplier matrices across the time horizons in Ψ. The main diagonals of the three matrices Eqs. (4a–c) yield represent contemporaneous and dynamic (cumulative one period ahead and cumulative ψ periods ahead) direct elasticities for each industry. These direct elasticities comprise the own elasticity (i.e., the fixed β_p coefficient from the structural form of the model in Eq. (1)) plus any heterogeneous feedback (i.e., the effect of $\partial x_{1,pt}$ that reverberates back to the i-th industry from other industries). Each row (column) sum of the off-diagonal elements of these matrices are the heterogeneous contemporaneous and dynamic asymmetric indirect spill-in (spill-out) $x_{1,pt}$ elasticities. A spill-in (spill-out) elasticity permeates to (from) the i-th industry from (to) all the other industries. Summing the contemporaneous (dynamic) direct and indirect spill-in/spill-out elasticities yields asymmetric total elasticities for each industry. From the matrices which the equations in footnote 12 yield for a change in $x_{2,qt}$, the contemporaneous and dynamic (direct, asymmetric indirect and two total) elasticities for each industry are obtained and interpreted in the same type of way as the above corresponding elasticities from Eqs. (4a–c).

The spatial TFP growth decompositions are based on, among other things, contemporaneous and dynamic direct and asymmetric indirect elasticities for each industry. As this asymmetry between the indirect spill-in and spill-out elasticities relates to subsets of the industries, we evidence this asymmetry for selected variables due to space constraints. As a result of these space constraints, to present results for a wider range of variables we report mean contemporaneous and dynamic direct, indirect and total elasticities across all the industries in the sample. As the mean sums of the row and column (off-diagonal) elements in any matrix are equal, the two corresponding mean indirect (and total) elasticities across all the industries will be symmetric.

To calculate the t-statistics for the mean contemporaneous and dynamic direct, indirect and total elasticities across all the industries, we divide each one by the standard deviation of 1000 Monte Carlo simulated values of the mean elasticity (LeSage and Pace 2009). To obtain this standard deviation, we first draw 1000 combinations of values of the coefficients and variance of the composed error ($\mathring{\varepsilon}_{M,igt}$) for Eq. (1) from its variance-covariance matrix. For technical details of this matrix, see Eq. (A3) and the accompanying discussion in Appendix A. As noted in Eq. 2.16 in Elhorst (2014, p. 25) and the related discussion therein, a vector of drawn values is the transpose of the vector of fitted parameters plus the product of: (i) the upper triangular Cholesky decomposition of the variance-covariance matrix; and (ii) a vector whose length equals the number of parameters and comprises random values drawn from a normal distribution with zero mean and standard deviation of one. For each vector of drawn values, we then calculate the values of the mean contemporaneous and dynamic direct, indirect and total elasticities.

The approach to obtain the contemporaneous and dynamic level 1 (direct, asymmetric indirect and two total) inefficiencies for the industries is the same as Glass and Kenjegalieva (2024) set out for their single level analysis. While the approach to obtain the contemporaneous and dynamic impacts of the level 2 (direct, asymmetric indirect and two total) regional inefficiencies on the industries is broadly similar, there are key differences between the interpretation of these impacts and the corresponding level 1 inefficiencies. Due to these differences and our innovation on how the inefficiency results for levels 1 and 2 are used to compute hierarchical adjusted inefficiencies, we set out the approach to compute the contemporaneous and dynamic impacts of the level 2 regional inefficiencies on the industries.

The left-hand side of Eq. (5) is the part of the DGP in Eq. (3) that relates to the impacts of the regional inefficiencies on the industries in the contemporaneous period.

$$\psi = 0: \quad \underbrace{(I_N - \eta W_1 - \delta W_2)^{-1}}_{\mathbf{A}^{-1}_{t,\psi=0}} \times \underbrace{\begin{pmatrix} \mathbf{u}_{g=1} \\ \vdots \\ \mathbf{u}_{g=134} \end{pmatrix}_t}_{\mathbf{u}_{2,t}}$$
$$= \begin{pmatrix} \mathbf{u}^{Dir}_{gg=1,1}+ & \mathbf{u}^{Ind}_{gh=1,2}+ & \cdots & +\mathbf{u}^{Ind}_{1,134} \\ \mathbf{u}^{Ind}_{gh=2,1}+ & u^{Dir}_{2,2}+ & \cdots & +\mathbf{u}^{Ind}_{2,134} \\ \vdots & \ddots & & \vdots \\ \mathbf{u}^{Ind}_{134,1}+ & \cdots & & +\mathbf{u}^{Dir}_{134,134} \end{pmatrix}_t, \tag{5}$$

where $\mathbf{u}_g$ is the (8×1) vector of common contemporaneous impacts of the own inefficiency of the g-th region on the eight industries it nests. Using the decomposition on the right-hand side of Eq. (5) we obtain the following. (a) The (8×1) vector of common contemporaneous impacts of the direct inefficiency of the g-th region on its eight industries ($\mathbf{u}^{Dir}_{gg}$). (b) The two asymmetric (8×1) vectors of the common contemporaneous spill-in and spill-out impacts of the inefficiencies of the h-th and g-th regions that permeate to each of the g-th and h-th region's eight industries (e.g., $\mathbf{u}^{Ind}_{1,2}$ and $\mathbf{u}^{Ind}_{2,1}$, respectively).

Rather than focus on the spill-in impact of a particular region's inefficiency that permeates to another region's industries, it is more manageable to consider the asymmetric sums of the contemporaneous spill-in and spill-out impacts that relate to a particular region's inefficiency. The (8×1) vectors of these asymmetric sums are $\mathbf{u}^{Ind}_{In,g} = \sum_{h \in G} \mathbf{u}^{Ind}_{gh}$ and $\mathbf{u}^{Ind}_{Out,g} = \sum_{g \in G} \mathbf{u}^{Ind}_{gh}$, respectively. The elements of the former are the sums of the contemporaneous inefficiency spill-in impacts that permeate to the g-th region's eight industries from all the other level 2 regions. The elements of the latter are the sums of the contemporaneous inefficiency spill-out impacts from the g-th region to the eight industries across the other regions. The two asymmetric (8×1) vectors of the total inefficiency impacts are $\mathbf{u}^{Dir}_{gg} + \mathbf{u}^{Ind}_{In,g}$ and $\mathbf{u}^{Dir}_{gg} + \mathbf{u}^{Ind}_{Out,g}$.

Whereas Eq. (5) is for the contemporaneous time horizon $\psi = 0$, Eq. (6) is the corresponding general expression for any ψ. Along the same lines as Eq. (5), $(\mathbf{u}_{g=1}, \ldots, \mathbf{u}_{g=134})'_t$ is pre-multiplied in Eq. (6) by the spatial multiplier matrix which changes with ψ. For $0 < \psi \leq T - 1$, and in the same way as we do in Eq. (5) for $\psi = 0$, we decompose the vector from Eq. (6) to obtain the dynamic impacts in a future in-sample time horizon of the level 2 (direct, two indirect and two total) inefficiencies for the current period t. One can also sum Eq. (6) across successive in-sample time horizons to obtain the cumulative impacts of the level 2 direct, indirect and total inefficiencies.

$$\psi \in 0, \ldots, T-1: \quad \underbrace{(-1)^{\psi}[(I_N - \eta W_1 - \delta W_2)^{-1}(-\theta W_1 - \rho W_2)]^{\psi}}_{\mathbf{C}_{t+\psi,\psi}} \times \underbrace{\begin{pmatrix} \mathbf{u}_{g=1} \\ \vdots \\ \mathbf{u}_{g=134} \end{pmatrix}_t}_{\mathbf{u}_{2,t}}. \tag{6}$$

We obtain the contemporaneous and dynamic level 1 (direct, asymmetric indirect and two total) inefficiencies in the same above way as we compute the impacts of the corresponding level 2 inefficiencies.

The hierarchical adjusted direct and asymmetric indirect spill-in and spill-out inefficiencies of a particular industry for horizon ψ are $u^{Dir}_{ii} + u^{Dir}_{gg}$, $\sum_{j \in N} u^{Ind}_{ij} + \sum_{h \in G} u^{Ind}_{gh}$, and $\sum_{i \in N} u^{Ind}_{ij} + \sum_{g \in G} u^{Ind}_{gh}$, respectively, where u^{Dir}_{gg} and u^{Ind}_{gh} are individual elements of the vectors $\mathbf{u}^{Dir}_{gg}$ and $\mathbf{u}^{Ind}_{gh}$. Summing these hierarchical adjusted direct and indirect spill-in/spill-out measures gives the two hierarchical adjusted total inefficiencies.

3.3 | Dynamic Multilevel Spatial TFP Growth Decompositions

The specific form of the empirical model is given in Eq. (7) where continuous variables are in logs.

$$y_{1,igt} = \alpha + TL\left(t, m'_{1,igt}\right) + \kappa z'_{1,igt} + \zeta m'_{2,gt} + \frac{1}{2} m_{2,gt} \Xi m'_{2,gt} + \varrho m'_{2,gt} t + \varphi r'_{2,gt} + \eta \sum_{j=1}^{N} w_{1,ij}\, y_{1,jgt} + \theta \sum_{j=1}^{N} w_{1,ij}\, y_{1,jgt-1} + \delta \sum_{j=1}^{N} w_{2,ij}\, y_{1,jgt} + \rho \sum_{j=1}^{N} w_{2,ij}\, y_{1,jgt-1} + b_{1,t} + d_{1,ig} + v_{2,gt} - u_{2,gt} + v_{1,igt} - u_{1,igt}. \quad (7)$$

For the i-th industry and g-th region in period t, $m'_{1,igt}$ and $m'_{2,gt}$ are the $(K \times 1)$ vectors of observations of the factor inputs (indexed $k, l \in 1, \ldots, K$ for $k \neq l$). t in $TL(t, m'_{1,igt})$ is the time counter, where $TL(t, m'_{1,igt})$ is the industry translog (TL) function, which indicates that Eq. (7) includes $t, m'_{1,igt}$ and their squares and interactions. $z'_{1,igt}$ is the $(A \times 1)$ vector of observations for the industry environmental variables (indexed $a \in 1, \ldots, A$). The regional environmental variables comprise observations for the regional TL function (i.e., $\zeta m'_{2,gt} + \frac{1}{2} m_{2,gt} \Xi m'_{2,gt} + \varrho m'_{2,gt} t$) and the $(C \times 1)$ vector $r'_{2,gt}$, where the latter relates to variables that are indexed $c \in 1, \ldots, C$. The regional TL function is the higher level counterpart of $TL(t, m'_{1,igt})$, but with the latter also including t and t^2 (see below for further discussion of this). In addition to the parameters that were previously defined for Eq. (1), further coefficients to be estimated in Eq. (7) are those in the industry TL function, the vectors κ and φ, and the vectors ζ and ϱ and matrix Ξ in the regional TL function.

The impact of the two level technical change on $y_{1,igt}$ is captured by a multi-component non-linear time trend. The industry components of this trend are the $m'_{1,igt} \times t$ interactions in $TL(t, m'_{1,igt})$ and the corresponding regional components are the $m'_{2,gt} \times t$ interactions. The remaining t and t^2 components of this trend in $TL(t, m'_{1,igt})$ capture impacts that correspond to the industry and regional levels collectively, which is because due to perfect collinearity these terms cannot also be included in the regional TL function. We also follow the spatial econometrics literature and do not conflate common shocks with the impacts of the contemporaneous SAR variables, as we account for these shocks using time period effects ($b_{1,t}$). Note that by including a non-linear time trend and time period effects we follow the single level spatial stochastic frontier literature, for example, Glass et al. (2013). The rationale for this is that, given data for the entire study period are used to estimate the multilevel non-linear time trend, for a particular period, $b_{1,t}$ measures any common departure from this trend across all the industries.

It follows from the approaches in 3.2 that we can construct direct, two asymmetric indirect and two total equations for each time horizon. In Eq. (8) we demonstrate the form of these equations for the indirect output spill-in to the i-th industry in horizon ψ ($y^{Ind}_{In,1,ig\psi}$). We do not present the forms of the direct, indirect spill-out and two total equations, as they only involve replacing the $^{Ind}_{In}$ notation in Eq. (8) with ^{Dir}, $^{Ind}_{Out}$, $^{Tot}_{In}$ and $^{Tot}_{Out}$ because the independent variables remain the same. Note that unlike the structural form of a dynamic SAR model (e.g., Eq. (1)), the direct, two indirect and two total equations do not include any SAR variables and their time lags. This is because the impacts of these spatial variables are incorporated within various direct, indirect and total measures (coefficients, random and time period effects, inefficiencies, and disturbances).

$$y^{Ind}_{In,1,ig\psi} = \alpha^{Ind}_{In,\psi} + TL^{Ind}_{In,\psi}\left(t, m'_{1,igt}\right) + \kappa^{Ind}_{In,\psi} z'_{1,igt} + \zeta^{Ind}_{In,\psi} m'_{2,gt} + \frac{1}{2} m_{2,gt} \Xi^{Ind}_{In,\psi} m'_{2,gt} + \varrho^{Ind}_{In,\psi} m'_{2,gt} t + \varphi^{Ind}_{In,\psi} r'_{2,gt} + b^{Ind}_{In,1,\psi} + d^{Ind}_{In,1,\psi} + v^{Ind}_{In,2,\psi} - u^{Ind}_{In,2,\psi} + v^{Ind}_{In,1,\psi} - u^{Ind}_{In,1,\psi}. \quad (8)$$

In contrast to Eq. (1) which is a regression equation, we construct Eq. (8) as $y^{Ind}_{In,1,ig\psi}$ is unobserved. In Eq. (8), we use a subscript ψ to indicate the contemporaneous or future in-sample horizon the indirect spill-in occurs in. Using the same type of approach as we set out above for the contemporaneous and dynamic (direct, asymmetric indirect and two total) inefficiencies, we obtain the corresponding level 1 constants and effects, and level 1 and 2 disturbances.

The corresponding static, single level, non-spatial O'Donnell (2016, 2018, 2022) regional industry TFP index decomposition is $IndTFP_{it} = IndScale_{it} \times IndEnvir_{it} \times IndEff_{it} \times IndShock_{it}$. In this decomposition, the four industry level components are returns to scale, environment, technical efficiency and economic shock indices (i.e., the latter is an index of unexpected changes in industry level output). This O'Donnell decomposition and our dynamic multilevel spatial extension are both multiplicative. This is because this avoids with an additive TFP index decomposition obtaining fixed and homogenous $IndEnvir_{it}$ indices for the regional industries over the study period. That is, between all consecutive periods there is no change in the first order derivatives of the fitted stochastic production frontier with respect to each regional industry environmental variable. With an additive form of our dynamic multilevel spatial TFP index decomposition, and for the same reason, we would obtain fixed and homogenous higher level regional environmental indices over the study period. With an additive TFP index decomposition, one could avoid fixed and homogenous regional industry (and higher level regional) environmental indices over the study period by including regressors that interact with the environmental variables. This would likely involve including a large number of interaction terms leading to a model that is not parsimonious. We instead follow O'Donnell (2016, 2018, 2022) and adopt a multiplicative framework for our TFP index decomposition as this yields regional industry (and higher level regional) environmental indices that vary over the sample.

Turning to our dynamic multilevel spatial TFP index decomposition. For each time period t that represents time horizon ψ, Eq. (9) is the decomposition of the total industry TFP index, which is based, in part, on the indirect spill-in to an industry ($IndTFP^{Tot}_{In,t}$). The corresponding decomposition of the total industry TFP index that is based, in part, on the spill-out from an industry ($IndTFP^{Tot}_{Out,t}$) has the same form and is obtained by simply replacing the $^{Tot}_{In}$ notation in Eq.

(9) with ${}^{Tot}_{Out}$. An industry would be interested in $IndTFP^{Tot}_{In,t}$ as its calculation involves using the index of the indirect TFP spill-in to the industry. $IndTFP^{Tot}_{In,t}$ and $IndTFP^{Tot}_{Out,t}$ would also be of interest to policymakers. For instance, policymakers could be interested in $IndTFP^{Tot}_{Out,t}$ as it is calculated using the index of the indirect TFP spill-out from an industry and thus indicates which industries are the biggest drivers of other industries' TFP indices.

$$\begin{aligned} IndTFP^{Tot}_{In,1,igt} = {} & IndScale^{Tot}_{In,1,igt} \times IndEnvir^{Tot}_{In,1,igt} \\ & \times IndEff^{Tot}_{In,1,igt} \times IndShock^{Tot}_{In,1,igt} \\ & \times RegScale^{Tot}_{In,2,gt} \times RegEnvir^{Tot}_{In,2,gt} \\ & \times RegEff^{Tot}_{In,2,gt} \times RegShock^{Tot}_{In,2,gt} \\ & \times MultilevTC^{Tot}_{In,M,igt}. \end{aligned} \tag{9}$$

The total indices on the right-hand side of Eq. (9) are all based, in part, on the indirect spill-in to the i-th industry, where we use *Ind* and *Reg* (or equivalently the subscripts 1 and 2) to indicate whether a component relates to the industry or regional level of the hierarchy. Where a component relates to both levels, we use the subscript M to denote its multilevel nature.

Following the above notation for the corresponding industry level components in Eq. (9) are $IndScale^{Tot}_{In,1,igt}$, $IndEnvir^{Tot}_{In,1,igt}$, $IndEff^{Tot}_{In,1,igt}$ and $IndShock^{Tot}_{In,1,igt}$. These industry level components are returns to scale, environment, technical efficiency and economic shock indices (i.e., the latter is an index of unexpected changes in industry level output). The corresponding regional level components are $RegScale^{Tot}_{In,2,gt}$, $RegEnvir^{Tot}_{In,2,gt}$,$RegEff^{Tot}_{In,2,gt}$ and $RegShock^{Tot}_{In,2,gt}$. As the dependent variable in Eq. (1) is at the industry level, these components are not indices of the regional level returns to scale, environment, etc. Rather these components are indices of the *impacts* of the regional level returns to scale, environment, technical efficiency and economic shocks (i.e., unexpected changes in regional level output) on $IndTFP^{Tot}_{In,1,igt}$. The remaining component in Eq. (9) is $MultilevTC^{Tot}_{In,M,igt}$, which is the multilevel technical change index. We refer to this index as a multilevel measure because, as will be evident below when we show how to calculate each of the component total indices in Eq. (9), $MultilevTC^{Tot}_{In,M,igt}$ relates to both levels in the hierarchy. This is because some of the values we use to calculate $MultilevTC^{Tot}_{In,M,igt}$ are not level specific and instead correspond to levels 1 and 2 collectively.

One could calculate the component total indices on the right-hand side of Eq. (9) using the total impacts, efficiencies and economic shocks (i.e., disturbances). Alternatively, one could take the right-hand side industry and regional level total indices to be the product of the direct and indirect industry and regional components.[13] These direct and indirect components are calculated in the same way using the direct and indirect impacts, efficiencies and economic shocks. However, whilst the sum of the direct and indirect impacts of a variable is its total impact, the value of a right-hand side total component in Eq. (9) will differ depending on whether it is calculated using, for instance, a total impact, or is taken to be the product of the corresponding direct and indirect indices. As will be evident from what follows, this is because the direct, indirect and total component indices are ratios with different denominators. We therefore favor the richer decomposition using direct and indirect component indices.

In Eqs. (10a, 11a, 12a, 13a, and 14a), we present the right-hand side total indices in Eq. (9) as the product of the corresponding direct and indirect indices.

$$\begin{aligned} IndScale^{Tot}_{In,1,igt} \times RegScale^{Tot}_{In,2,gt} = {} & IndScale^{Dir}_{1,igt} \times IndScale^{Ind}_{In,1,igt} \\ & \times RegScale^{Dir}_{2,gt} \times RegScale^{Ind}_{In,2,gt}, \end{aligned} \tag{10a}$$

where the same type of approach is used to obtain each of the components on the right-hand side of Eq. (10a). This is also the case for the components on the right-hand sides of Eqs. (11a, 12a, 13a, and 14a). For brevity therefore, we only demonstrate in this subsection how we compute one of the components on the right-hand side of each of these equations. The formulas for these components are provided in Eqs. (10b, 11b, 12b, 13b, and 14b). For completeness, we provide the formulas for the other right-hand side components of Eqs. (10a, 11a, 12a, 13a, and 14a) in Appendix B.

$$IndScale^{Ind}_{In,1,igt} = \left[\prod_{k,l \in K} \left(\frac{\left(\frac{M_{1,kigt}}{M_{1,kigt-1}}\right)^{*^{Ind}_{In,k\psi}} \left(\frac{M^2_{1,kigt}}{M^2_{1,kigt-1}}\right)^{*^{Ind}_{In,k\psi}} \times}{\left(\frac{M_{1,kigt} M_{1,ligt}}{M_{1,kigt-1} M_{1,ligt-1}}\right)^{*^{Ind}_{In,k\psi}}} \right) \right] \left(\frac{Y_{1,igt}}{Y_{1,igt-1}} \right), \tag{10b}$$

where throughout the presentation of the formulas for the indirect spill-in component indices of $IndTFP^{Tot}_{In,1,igt}$, we use upper case letters to denote the move from logs in Eq. (8) to levels, for example, $M_{1,kigt}$ in Eq. (10b) and its log in Eq. (8) $m_{1,kigt}$.

In Eq. (10b), $Y_{1,igt}/Y_{1,igt-1}$ is the output scale factor and, to conserve notation, we use $*^{Ind}_{In,k\psi}$ to denote the indirect spill-in coefficient on the relevant variable in $TL^{Ind}_{In,\psi}(t, m'_{1,igt})$ in Eq. (8). We can also see from the numerators and denominators in the ratios in Eq. (10b) (and also Eqs. 11b, 12b, 13b, and 14b) that we are setting out a TFP growth decomposition.

$$\begin{aligned} IndEnvir^{Tot}_{In,1,igt} \times RegEnvir^{Tot}_{In,2,gt} = {} & IndEnvir^{Dir}_{1,igt} \times IndEnvir^{Ind}_{In,1,igt} \\ & \times RegEnvir^{Dir}_{2,gt} \times RegEnvir^{Ind}_{In,2,gt}, \end{aligned} \tag{11a}$$

where

$$IndEnvir^{Ind}_{In,1,igt} = \prod_{a=1}^{A} \left(\frac{Z_{1,aigt}}{Z_{1,aigt-1}} \right)^{\kappa^{Ind}_{In,a\psi}}. \tag{11b}$$

From Eq. (11b), we can see that $IndEnvir^{Ind}_{In,1,igt}$ relates to continuous industry level environmental variables and not dummies, which is also the case for $IndEnvir^{Dir}_{1,igt}$. In similar vein, $RegEnvir^{Dir}_{2,gt}$ and $RegEnvir^{Ind}_{In,2,gt}$ relate to continuous regional level environmental variables.

$$IndEff^{Tot}_{In,1,igt} \times RegEff^{Tot}_{In,2,gt} = IndEff^{Dir}_{1,igt} \times IndEff^{Ind}_{In,1,igt} \times RegEff^{Dir}_{2,gt} \times RegEff^{Ind}_{In,2,gt}, \tag{12a}$$

where

$$IndEff^{Ind}_{In,1,igt} = \frac{\exp\left(-\sum_{j \in N} u^{Ind}_{1,ijgt}\right)}{\exp\left(-\sum_{j \in N} u^{Ind}_{1,ijgt-1}\right)}. \tag{12b}$$

$$IndShock^{Tot}_{In,1,igt} \times RegShock^{Tot}_{In,2,gt} = IndShock^{Dir}_{1,igt} \times IndShock^{Ind}_{In,1,igt} \times RegShock^{Dir}_{2,gt} \times RegShock^{Ind}_{In,2,gt}, \tag{13a}$$

where

$$IndShock^{Ind}_{In,1,igt} = \frac{\exp\left(\sum_{j \in N} v^{Ind}_{1,ijgt}\right)}{\exp\left(\sum_{j \in N} v^{Ind}_{1,ijgt-1}\right)}. \tag{13b}$$

$$MultilevTC^{Tot}_{In,M,igt} = MultilevTC^{Dir}_{M,igt} \times MultilevTC^{Ind}_{In,M,igt}, \tag{14a}$$

where

$$MultilevTC^{Ind}_{In,M,igt} = \left(\frac{\exp\left(*^{Ind}_{In,\psi} t\right)}{\exp\left(*^{Ind}_{In,\psi}(t-1)\right)}\right)\left(\frac{\exp\left(*^{Ind}_{In,\psi} t^2\right)}{\exp\left(*^{Ind}_{In,\psi}(t-1)^2\right)}\right) \times \prod_{k=1}^{K}\left(\left(\frac{M_{1,kigt}\, t}{M_{1,kigt-1}\, t-1}\right)^{*^{Ind}_{In,k\psi}}\left(\frac{M_{2,kgt}\, t}{M_{2,kgt-1}\, t-1}\right)^{\varsigma^{Ind}_{In,k\psi}}\right). \tag{14b}$$

As we touched on previously, the values of the first two indices on the right of Eq. (14b) correspond to levels 1 and 2 collectively. Conversely, in the calculation of $\prod_{k=1}^{K}\left((\,\cdot\,)^{*^{Ind}_{In,k\psi}}(\,\cdot\,)^{\varsigma^{Ind}_{In,k\psi}}\right)$ in this equation, $(\,\cdot\,)^{*^{Ind}_{In,k\psi}}$ relates to level 1 and $(\,\cdot\,)^{\varsigma^{Ind}_{In,k\psi}}$ to level 2.

4 | Empirical Case and Findings

4.1 | Data

We use rich annual balanced panel data for the period $2004-20$, where the level 1 data comprise $18,224$ granular industry observations. These observations are for the previously listed eight industries nested in each of Britain's 134 NUTS3 regions in level 2. The most recent available data is for 2020, which we include to observe the impact of the COVID pandemic. The 2008 crisis was a common shock across the industries that led to a productivity slowdown in the UK, so including 2020 is interesting as the COVID pandemic was a further common shock (De Vries et al. 2021). To control for the common impacts of such shocks, we include time period effects in our stochastic frontier model.

In Table 1 we provide descriptions of the non-spatial variables, our notation, the data sources, and summary statistics. In this table the variables are split up by level and according to which type of component in the TFP growth decompositions they relate to. For instance, the components that measure the impacts of the direct and indirect regional level growth in returns to scale relate to the level 2 factor inputs.

Data on the capital stocks for levels 1 and 2 are not available, so we estimate this level 1 variable. From this variable, we obtain the estimate of each capital stock in level 2, which is the sum of the level 1 industry capital stocks in the relevant NUTS3 region. See Berlemann and Wesselhöft (2014) for a comprehensive presentation of the unified approach to the standard perpetual inventory method we use to estimate the level 1 capital stocks. In this estimation we use a depreciation rate of 3.0%, which is within the range they suggest. To estimate the level 1 capital stocks we use gross fixed capital formation data that begins in 2000. The data sample then begins in 2004 as this is when the labor factor input data is available from.

Using the GVA deflator the monetary variables are deflated to 2019 prices and the continuous variables are then logged. We then mean adjust all the continuous variables. The mean adjustment is so the (own and contemporaneous and dynamic direct, indirect and total) coefficients on t, l_1, k_1, l_2, and k_2 can be interpreted as elasticities at the sample mean.

4.2 | Frontier Model and (In)Efficiencies

Table 2 presents the estimated dynamic multilevel SAR production frontier. We find that the coefficients on the dynamic SAR variables (0.13 and 0.16) are larger than the contemporaneous SAR parameters (0.08 and 0.02). This indicates that it takes some time for the larger spillovers to occur. As the contemporaneous and dynamic SAR coefficients are positive and significant at the 1% level, we do not reject our first empirical hypothesis (H1). These contemporaneous and dynamic SAR coefficients point to positive output spill-ins. In and of themselves, these will lead to positive TFP growth spill-ins, which we suggest can be harnessed to support the left-behind regional industries that may find it challenging to increase their productivity by themselves.

We begin the discussion of the other results in Table 2 by noting that the non-spatial coefficients are contemporaneous parameters. We can see that the coefficients on the first order level 1 labor and capital inputs of the industries are, as expected, positive. These coefficients are also significant at the 1% level

TABLE 1 | Variables and summary statistics.

Variable	Notation	Data source	Mean	St. Dev.
Dependent variable: Level 1 industry output				
Real gross value added (million pounds)	y	ONS	1627	4269
Level 1 industry factor inputs				
Number of persons employed	l_1	Nomis	27,422	34,644
Real capital stock (million pounds)	k_1	ONS & authors' calculation	5389	9336
Level 1 industry environmental variables				
Percentage of persons employed in an industry who are female	$Female_1$	Nomis	38.3	20.0
Percentage of persons employed in an industry who are non-white	$Non\text{-}white_1$	Nomis	6.4	9.0
Number of persons employed in an industry aged 50 and over	$50plus_1$	Nomis	7982	9940
Annual real gross fixed capital formation for intangible assets (million pounds) to measure annual real investment in these assets	$Intang_1$	ONS	61.5	185.4
Annual real gross fixed capital formation for ICT equipment (million pounds) to measure annual real investment in this equipment	$ICTeq_1$	ONS	11.4	24.3
Level 2 regional environmental variables				
Level 2 factor inputs				
Number of persons employed in a NUTS3 region	l_2	ONS	219,374	191,536
Real capital stock of a NUTS3 region (million pounds)	k_2	ONS & authors' calculation	43,115	49,977
Other level 2 environmental variables				
Predominantly urban dummy: value of 1 if more than 80% of the region's population live in urban clusters (0 otherwise)	$80\%Urb_2$	Eurostat	0.60	0.49
Intermediate urban dummy: value of 1 if 50%-80% of the region's population live in urban clusters (0 otherwise)	$50\text{-}80\%Urb_2$	Eurostat	0.31	0.46
Percentage of persons employed in a NUTS3 region who are female	$Female_2$	Nomis	46.9	1.6
Percentage of persons employed in a NUTS3 region who are non-white	$Non\text{-}white_2$	Nomis	7.3	8.7
Number of persons employed in a NUTS3 region aged 50 and over	$50plus_2$	Nomis	63,856	51,986
Annual real gross fixed capital formation for intangible assets (million pounds)	$Intang_2$	ONS	491.6	872.2
Annual real gross fixed capital formation for ICT equipment (million pounds)	$ICTeq_2$	ONS	91.0	134.1

Note: Office of National Statistics (ONS).

and point to a reasonable order of returns to scale at the sample mean (1.01). As expected the coefficient on t is positive and significant, which points to annual technical progress for the sample average industry.

Turning to the results for the environmental variables for the level 1 industries. As expected, we find that $Intang_1$ and $ICTeq_1$ have significant positive impacts. Donald (2025) and Rubery et al. (2023) point to females having a key role in improving the UK's productivity, which is in line with the positive and significant coefficient we observe for $Female_1$. We also find that the coefficient on the level 2 environmental variable l_2 is positive and significant. This is reasonable as an increase in l_2 for a region will increase the size of the local markets for the region's industries.

TABLE 2 | Estimated dynamic multilevel spatial profuction frontier.

	Coeff.		**Coeff.**		**Coeff.**
Constant	−0.541***	$Non\text{-}white_1$	0.001×10^{-1}	$80\%Urb_2$	0.413***
l_1	0.343***	$50plus_1$	−0.008***	$50\text{-}80\%Urb_2$	0.246***
k_1	0.671***	$Intang_1$	0.037***	$Female_2$	0.004***
$l_1 \times l_1$	0.132***	$ICTeq_1$	0.284***	$Non\ \text{-}white_2$	0.009***
$k_1 \times k_1$	0.029***	l_2	0.036***	$50plus_2$	−0.039***
$l_1 \times k_1$	−0.138***	k_2	−0.065***	$Intang_2$	−0.041***
t	0.021***	$l_2 \times l_2$	0.276***	$ICTeq_2$	−0.176***
t^2	−0.002***	$k_2 \times k_2$	−0.251***	$W_1 y_t$	0.022***
$l_1 t$	−0.021***	$l_2 \times k_2$	−0.055***	$W_1 y_{t-1}$	0.163***
$k_1 t$	−0.064***	$l_2 t$	0.014***	$W_2 y_t$	0.082***
$Female_1$	0.003***	$k_2 t$	0.042***	$W_2 y_{t-1}$	0.133***
$\sigma_{v_2} = 0.064$	$\sigma_{u_2} = 0.110$	$\mathring{\sigma}_{\varepsilon_M} = 0.141$	$\lambda_1 = 2.274$	$LogL = 2591.65$	

Note: *** denote significance at 1% level.

As Britain's productivity has been lagging behind for some time, we would expect some of the results in Table 2 to be consistent with this. However, we would not immediately associate the results we have discussed thus far with Britain's lagging productivity. As we noted in the opening section, there is uncertainty surrounding the reasons why Britain's productivity is lagging behind, so we use our multilevel analysis to provide some further insights. In contrast with the aforementioned positive l_2 parameter, the significant coefficients on k_2, $Intang_2$ and $ICTeq_2$ are negative. These negative coefficients provide further insights because if any of these three variables increase for a region — while holding the corresponding variable for one of the region's industry's constant — the widening of the gap for this variable between this industry and its region leads to a decline in the industry's output. This impact for these three variables of the wider gap between an industry and its region may, therefore, be a contributing factor to Britain's lagging productivity.

Whilst the coefficient on a non-spatial variable in Table 2 is an own impact, the presence of at least one contemporaneous SAR regressor in our model means that this impact represents only part of the contemporaneous impact of the variable. This is because, via the spatial multiplier matrix, a change in the variable for one industry in the current period will also have a contemporaneous impact on the dependent variable of other industries. To collectively measure these contemporaneous own and spillover impacts, we compute a contemporaneous total coefficient across the periods that represent the contemporaneous time horizon $\psi = 0$. We decompose this coefficient into contemporaneous direct and indirect spillover coefficients, where a contemporaneous direct impact is the aforementioned own impact plus feedback which to an industry from all the other industries. A feature of our model is how including the time lag of at least one contemporaneous SAR regressor enables us to compute the dynamic direct, indirect and total impacts in future in-sample time horizons of a change in an x variable in the current horizon $\psi = 0$.

Due to space constraints, Table 3 reports the mean contemporaneous direct, indirect and total coefficients for selected x variables. These variables comprise those which the results in Table 2 suggest may be most relevant to Britain's lagging productivity, along with further variables for context. For marginal changes in these variables in $\psi = 0$, Table 3 also presents the cumulative dynamic direct, indirect and total parameters, where these coefficients measure the sum of the impacts across current and future in-sample time horizons. Note that as the results in Table 3 are not for subsamples, but averages across all the regional industries, the contemporaneous and dynamic indirect and total parameters are symmetric. We make three observations about the results in this table, where the long run (cumulative) parameters that persist through to the final future in-sample horizon $\psi = 16$ are in bold. First, the direct parameters for $\psi = 0$ are equal to (or not too different from) the corresponding non-spatial coefficient in Table 2. This suggests that the contemporaneous feedback is at best small. Second, the signs of the contemporaneous and dynamic direct, indirect and total coefficients are the same as the corresponding non-spatial coefficient in Table 2. Third, over current and future in-sample time horizons, the adjustment of the cumulative indirect and total parameters to the long run value in bold is larger and slower than we observe for the corresponding direct coefficient. This is due to the inclusion of the time lags of the SAR variables and the calculation of the total coefficients being based in part on the corresponding indirect estimate.

For our TFP growth decompositions, we use asymmetric contemporaneous and cumulative dynamic indirect spill-in and spill-out coefficients for individual industries and regions. In Figure 3, we demonstrate such asymmetry for subsamples. For marginal changes in the first order variables l_1 and l_2 in $\psi = 0$, Figure 3 reports the results for quintiles 1 and 5 of the distributions of the contemporaneous and cumulative dynamic indirect spill-in and spill-out coefficients. From this figure, we can see that for l_1 and l_2, the quintile 1 (quintile 5) short and long run spill-in coefficients (i.e., for individual time horizons and when there is no tendency to change, respectively) are larger (smaller) than the corresponding spill-out parameter. If we think of these results in terms of quintile 5's comparative advantage being in generating spill-outs and quintile 1's being

TABLE 3 | Cumulative direct, indirect and total parameters for selected variables.

	Horizon, ψ	l_1	k_1	$l_1 \times l_1$	$k_1 \times k_1$	$l_1 \times k_1$	$Intang_1$	$ICTeq_1$	l_2	k_2	$l_2 \times l_2$	$k_2 \times k_2$	$l_2 \times k_2$	$Intang_2$	$ICTeq_2$
	0	0.343***	0.673***	0.132***	0.029***	−0.138***	**0.038*****	0.285***	**0.036*****	−0.065***	0.276***	−0.252***	−0.055***	**−0.041*****	−0.176***
	1	0.345***	0.677***	0.133***	0.029***	−0.139***		0.287***		**−0.066*****	0.278***	−0.253***	−0.055***		−0.177***
Direct	2	**0.348*****	0.682***	**0.134*****	**0.030*****	**−0.140*****		**0.289*****			0.280***	**−0.255*****	**−0.056*****		**−0.179*****
	3		**0.683*****								0.280***				
	4										**0.281*****				
	0	0.038***	0.074***	0.015***	0.003***	−0.015***	0.004***	0.031***	0.004***	−0.007***	0.031***	−0.028***	−0.006***	−0.005***	−0.019***
	1	0.160***	0.313***	0.061***	0.014***	−0.064***	0.017***	0.133***	0.017***	−0.030***	0.129***	−0.117***	−0.026***	−0.019***	−0.082***
	2	0.198***	0.387***	0.076***	0.017***	−0.079***	0.022***	0.164***	0.021***	−0.038***	0.159***	−0.145***	−0.032***	−0.023***	−0.101***
Indirect	3	0.210***	0.413***	0.081***	**0.018*****	−0.085***	0.023***	0.175***	0.022***	−0.040***	0.169***	−0.154***	−0.034***	−0.025***	−0.108***
	4	0.215***	0.421***	**0.083*****		−0.086***	0.023***	0.178***	0.022***	**−0.041*****	0.173***	−0.157***	−0.034***	−0.025***	−0.110***
	5	0.216***	0.424***			**−0.087*****	**0.024*****	0.179***	0.022***		0.174***	−0.158***	**−0.035*****	**−0.026*****	**−0.111*****
	6	**0.217*****	**0.425*****					**0.180*****	0.022***		0.174***	**−0.159*****			
	7								**0.023*****		**0.175*****				
	0	0.381***	0.747***	0.147***	0.033***	−0.153***	0.042***	0.316***	0.040***	−0.073***	0.307***	−0.279***	−0.061***	−0.045***	−0.196***
	1	0.505***	0.990***	0.194***	0.043***	−0.203***	0.055***	0.419***	0.052***	−0.096***	0.407***	−0.370***	−0.081***	−0.060***	−0.259***
	2	0.545***	1.069***	0.210***	0.047***	−0.219***	0.060***	0.453***	0.057***	−0.104***	0.439***	−0.400***	−0.087***	−0.065***	−0.280***
Total	3	0.559***	1.095***	0.215***	**0.048*****	−0.225***	0.061***	0.464***	0.058***	−0.107***	0.450***	−0.409***	−0.090***	−0.066***	−0.287***
	4	0.563***	1.103***	0.217***		−0.226***	**0.062*****	0.467***	0.058***	−0.107***	0.453***	−0.413***	−0.090***	**−0.067*****	−0.289***
	5	0.564***	1.106***	0.217***		**−0.227*****		0.468***	**0.059*****	**−0.108*****	0.454***	**−0.414*****	**−0.091*****		**−0.290*****
	6	**0.565*****	1.107***	**0.218*****				**0.469*****			**0.455*****				
	7		**1.108*****												

Note: *** denotes significance at the 1% level. Bold is used to denote the value (to 3 d.p) of the significant long run cumulative parameter that persists through to horizon 16.

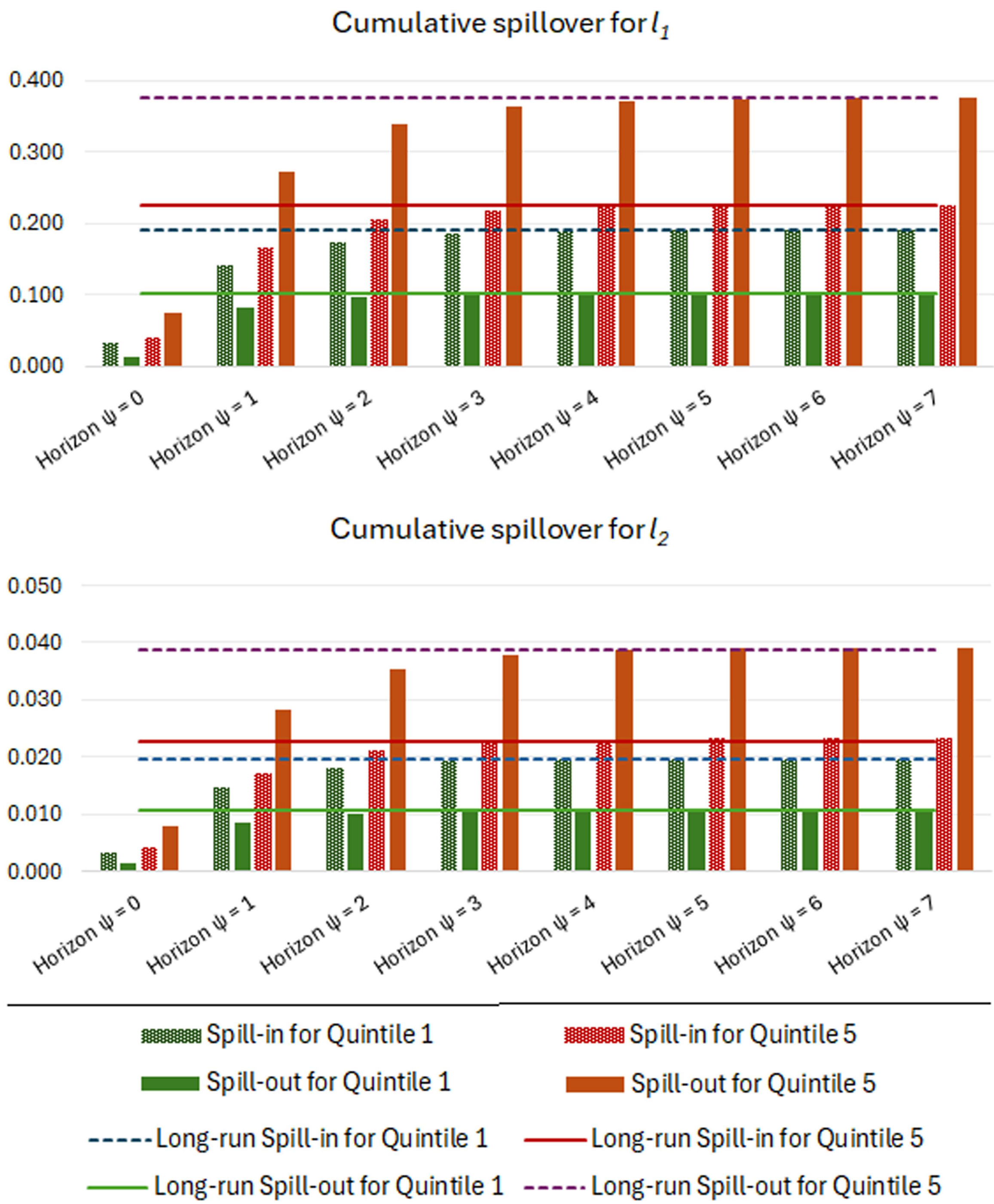


FIGURE 3 | Asymmetric cumulative spill-in and spill-out coefficients for the industry and regional labor variables. [Color figure can be viewed at wileyonlinelibrary.com]

in receiving spill-ins, this supports our rationale that spill-ins represent a vehicle to improve laggard regional industries.

To provide an insight into the estimated hierarchical adjusted industry efficiencies ($HE_{M,igt}$), for the eight industry classifications Table 4 reports the sample average, standard deviation and top and bottom five average scores by industry, where $HE_{M,igt} = IE_{1,igt} \times RE_{2,gt}$ (see 3.1).[14] In terms of the implications of the multilevel structure, as the average $RE_{2,gt}$ is less than 1 (see Figure 4), this indicates that the impact of the regions' technical inefficiencies is to push the mean $HE_{M,igt}$ below the corresponding $IE_{1,igt}$.

NUTS1 is the most aggregate regional level in Europe, so to provide clarity on where the NUTS3 regional industries in Table 4 are located in Britain, the NUTS1 regions that nest these industries are in parentheses in this table. In bold in Table 4, we also present the within industry classification ranking of the average real GVA of a regional industry over the study period. To test our second empirical hypothesis (H2), for the eight industry classifications we calculate the Spearman rank correlations between a regional industry's real GVA ranking in each period and its $HE_{M,igt}$ ranking. For six of the eight industries (i.e., with the exception of Distribution‾restaurants and Public admin‾health), we do not reject H2 as the six correlations, which range from 0.41 to 0.64, are significant at the 1% level.

TABLE 4 | Top and bottom 5 hierarchical adjusted regional industry efficiencies.

Agriculture–water		**Manufacturing**	
Top 5		*Top 5*	
Aberdeen City & Aberdeenshire (Scotland), **2**	0.685	Flintshire & Wrexham (Wales), **18**	0.704
Berkshire (SE, England), **1**	0.682	Cheshire East (NW, England), **4**	0.689
Perth & Kinross and Stirling (Scotland), **17**	0.677	Aberdeen City & Aberdeenshire (Scotland), **32**	0.687
Dumfries & Galloway (Scotland), **49**	0.676	South West Wales (Wales), **71**	0.666
Gwynedd (Wales), **74**	0.663	Clackmannanshire & Fife (Scotland), **43**	0.663
Bottom 5		*Bottom 5*	
Milton Keynes (SE, England), **120**	0.156	Torbay (SW, England), **131**	0.403
Luton (East of England), **130**	0.117	Brighton & Hove (SE, England), **129**	0.319
Blackburn with Darwen (NW, England), **131**	0.108	Shetland Islands (Scotland), **132**	0.311
Blackpool (NW, England), **134**	0.101	Eilean Siar, Western Isles (Scotland), **133**	0.272
Southend-on-Sea (East, England), **133**	0.082	Orkney Islands (Scotland), **134**	0.255
Average	*0.482*	*Average*	*0.577*
St. Dev.	*0.130*	*St. Dev.*	*0.076*
Construction		**Distribution– restaurants**	
Top 5		*Top 5*	
Inner London-West (London), **2**	0.745	Inner London-West (London), **1**	0.766
Surrey (SE, England), **6**	0.673	Milton Keynes (SE, England), **14**	0.675
Hampshire CC (SE, England), **9**	0.672	Berkshire (SE, England), **5**	0.670
Hertfordshire (East, England), **7**	0.672	Gwynedd (Wales), **124**	0.656
Berkshire (SE, England), **12**	0.670	Dumfries & Galloway (Scotland), **116**	0.645
Bottom 5		*Bottom 5*	
Orkney Islands (Scotland), **133**	0.383	Darlington (NE, England), **127**	0.440
Darlington (NE, England), **126**	0.383	Blackburn with Darwen (NW, England), **117**	0.431
Blackburn with Darwen (NW, England), **124**	0.349	Eilean Siar, Western Isles (Scotland), **133**	0.414
Eilean Siar, Western Isles (Scotland), **134**	0.348	Orkney Islands (Scotland), **134**	0.397
Shetland Islands (Scotland), **132**	0.305	Shetland Islands (Scotland), **132**	0.354
Average	*0.562*	*Average*	*0.573*
St. Dev.	*0.073*	*St. Dev.*	*0.051*
Transport & communications		**Banking–insurance**	
Top 5		*Top 5*	
Inner London-West (London), **1**	0.748	Inner London-West (London), **1**	0.832
Berkshire (SE, England), **4**	0.661	City of Edinburgh (Scotland), **12**	0.731
Outer London-West & North West (London), **2**	0.616	Aberdeen City & Aberdeenshire (Scotland), **26**	0.698
Suffolk (East, England), **16**	0.610	Glasgow City (Scotland), **16**	0.689
Inner London-East (London), **3**	0.601	Berkshire (SE, England), **8**	0.686
Bottom 5		*Bottom 5*	
Caithness & Sutherland[a] (Scotland), **129**	0.365	Darlington (NE, England), **121**	0.463
Dorset CC (SW, England), **103**	0.365	Blackburn with Darwen (NW, England), **122**	0.431
Torbay (SW, England), **131**	0.364	Eilean Siar, Western Isles (Scotland), **132**	0.429
Eilean Siar, Western Isles (Scotland), **134**	0.362	Orkney Islands (Scotland), **134**	0.419
Blackpool (NW, England), **128**	0.354	Shetland Islands (Scotland), **133**	0.336
Average	*0.498*	*Average*	*0.602*
St. Dev.	*0.069*	*St. Dev.*	*0.060*
Public admin–health		**Other services**	
Top 5		*Top 5*	
Inner London-West (London), **1**	0.702	Inner London-West (London), **1**	0.764

(Continues)

TABLE 4 | (Continued)

Public admin‒health		Other services	
Gwynedd (Wales), **122**	0.689	Greater Manchester South (NW, England), **4**	0.608
Conwy & Denbighshire (Wales), **95**	0.672	City of Edinburgh (Scotland), **19**	0.601
Dumfries & Galloway (Scotland), **116**	0.672	Berkshire(SE, England), **10**	0.595
South West Wales (Wales), **56**	0.658	Inner London-East (London), **2**	0.594
Bottom 5		*Bottom 5*	
Eilean Siar, Western Isles (Scotland), **132**	0.469	Thurrock (East of England), **125**	0.282
Outer London-East & North East (London), **5**	0.458	Isle of Anglesey (Wales), **131**	0.250
Blackburn with Darwen (NW, England), **117**	0.450	Orkney Islands (Scotland), **133**	0.160
Thurrock (East, England), **130**	0.425	Shetland Islands (Scotland), **132**	0.109
Shetland Islands (Scotland), **133**	0.410	Eilean Siar, Western Isles (Scotland), **134**	0.108
Average	*0.575*	*Average*	*0.465*
St. Dev.	*0.047*	*St. Dev.*	*0.092*

Note: 1. To provide clarity on where the NUTS3 regional industries in this table are located in Great Britain, the NUTS1 region that nests a regional industry is provided in parentheses. 2. The full name of the NUTS3 region with the a superscript is Caithness & Sutherland and Ross & Cromarty. 3. The acronyms are SE - South East; NW - North West; SW - South West; NE - North East. 4. In bold is the within industry classification ranking of the average real GVA of the regional industry over the study period.

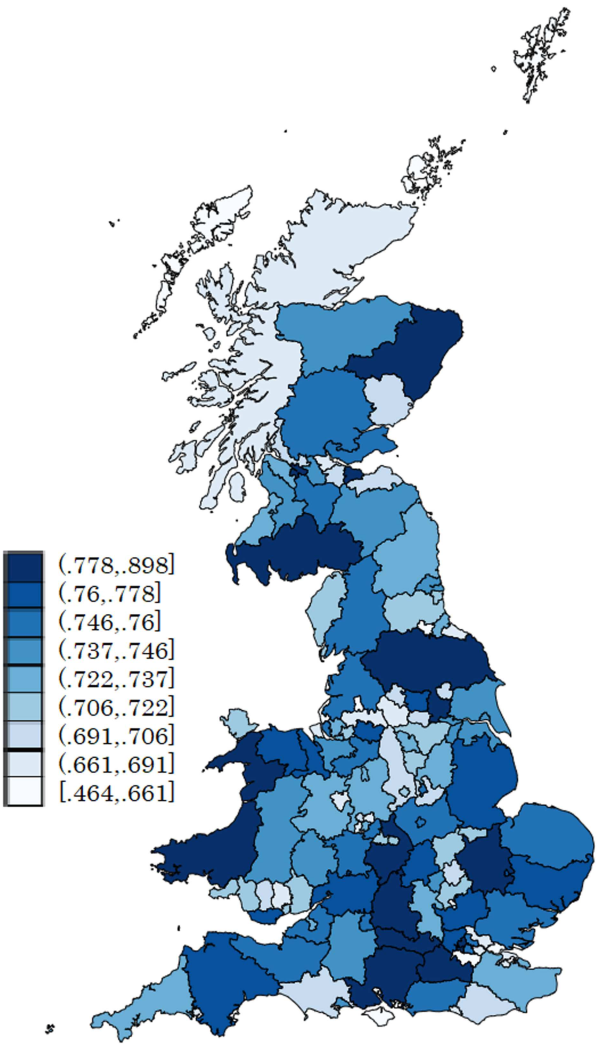
We make five remarks about the efficiency rankings in Table 4 to provide context and support for these findings. First, the top five Agriculture‒ water regional industries are not surprising, as relatively large proportions of the land in these regions is rural with well-established agricultural economic activities. Finding that Aberdeen City & Aberdeenshire is the top ranked region in the Agriculture‒water industry is also reasonable as this industry classification includes energy activities, which are key to this region due to its close proximity to the North Sea oil and gas fields.

Second, for the Banking‒insurance, the top ranked regional industry is in Inner London-West, which is what we would expect as London's financial centre, Canary Wharf, is a district in this region. Third, for five other industry classifications, Inner London-West has the top ranked regional industry (Construction; Distribution‒restaurants; Transport & communications; Public admin‒health; Other services), which aligns with London's economy consistently outperforming other regional economies in the country. Consistent with this, for five of the six industries where Inner London-West is the top ranked region (Construction; Distribution‒restaurants; Transport & communications; Banking‒insurance; Other services), we can see that its average *HE* is well above that of the second ranked region (7.2% – 15.6% higher).

Fourth, it is well-known that a contributing factor to the marked differences in regional productivity in England is the relatively low productivity in a number of regions in the Midlands and North of England. This fits with our findings because whilst 22 of the top five ranked regional industries in Table 4 are in England, only two of these are not in the South of England (Manufacturing in Cheshire East and Other services in Greater Manchester South). At the opposite end of the performance spectrum, and as expected, there are some regional industries in the North of England in the bottom five (e.g., Transport & communications in Blackpool, and Construction and Distribution‒restaurants in Darlington and Blackburn with Darwen).

Fifth, a number of regions in Table 4 have features that are consistent with them having industries in the top or bottom five. As we have discussed top five regional industries in previous remarks about Table 4, we briefly suggest that Distribution‒restaurants in Milton Keynes is in the top five as the region is a major hub for logistics and distribution due to its central location and close proximity to the M1 motorway. Additionally, the following three features are evident among those in the bottom five. (i) Industries on islands (Eilean Siar, Western Isles, i.e., Outer Herbrides; Orkney; Shetland; Anglesey), which makes them far less accessible. A number of these island industries are also reliant on tourism, which varies with the weather and faces strong competition from alternative destinations. (ii) Industries in mainland coastal regions (Blackpool; Brighton and Hove; Torbay; Dorset CC), which are also reliant on tourism and, to a lesser extent, have some accessibility issues. (iii) Thurrock is a region with two industries in the bottom five (Public admin‒health; Other services) and whose council has formally declared effective bankruptcy (The Guardian 2022).

Two of the components we split the estimate of the composed error in Eq. (1) ($\hat{\hat{\varepsilon}}_{M,igt}$) into are the estimate of the inefficiency of a regional industry ($\hat{u}_{1,igt}$) and the benchmarked impact of the inefficiency of the relevant higher level region on the output of the industry ($\hat{u}_{2,gt}$). From this split and using the estimates of these u's, we obtain the estimate of the hierarchical adjusted regional industry efficiency ($\widehat{HE}_{M,igt}$) (see Table 4). Given we focus on the hierarchical data structure, we provide insights into the benchmarked impacts of the higher level efficiencies on the outputs of the regional industries ($\widehat{RE}_{2,gt}$), as this higher level performance impact would be overlooked by a single level stochastic frontier model for these industries. To this end, Figure 4 presents the spatial distribution of the mean



NUTS 3 area	Mean impacts of the area efficiencies
Top 15	
Inner London - West (London)	0.898
Berkshire (South East, England)	0.827
Aberdeen City and Aberdeenshire (Scotland)	0.823
City of Edinburgh (Scotland)	0.802
Oxfordshire (South East, England)	0.799
Glasgow City (Scotland)	0.799
South West Wales (Wales)	0.797
Cambridgeshire (East of England)	0.797
Warwickshire (West Midlands, England)	0.790
Dumfries & Galloway (Scotland)	0.789
Gwynedd (Wales)	0.788
Hampshire (South East, England)	0.782
North Yorkshire (Yorkshire and The Humber, England)	0.781
Surrey (South East, England)	0.779
Suffolk (East of England)	0.778
Bottom 15	
Sefton(North West, England)	0.661
Southampton (South East, England)	0.659
Isle of Wight (South East, England)	0.653
Outer London - South (London)	0.653
Portsmouth (South East, England)	0.644
Luton (East of England)	0.631
Thurrock (East of England)	0.616
Blackpool (North West, England)	0.615
Torbay (South West, England)	0.613
Southend-on-Sea (East of England)	0.590
Darlington (North East, England)	0.544
Orkney Islands (Scotland)	0.519
Blackburn with Darwen (North West, England)	0.515
Eilean Siar, Western Isles (Scotland)	0.500
Shetland Islands (Scotland)	0.464
Average	**0.719**
St. Dev.	**0.062**

FIGURE 4 | Spatial distribution of the mean benchmarked impacts of the regional efficiencies. [Color figure can be viewed at wileyonlinelibrary.com]

benchmarked impacts of the NUTS3 regional efficiencies on the outputs of the industries they nest; the sample mean and standard deviation of these impacts; and the top and bottom 15 average impact scores.[15] We again in this figure provide in parentheses the NUTS1 regions where the top and bottom 15 NUTS3 regions are located.

When a region has more top 5 industries in Table 4, the impact of its regional efficiency on its industries tends to be among the highest in Figure 4 (darkest areas in this figure, where these benchmarked impacts range from 0.778 – 0.898). There are some interesting differences though as Oxfordshire is the fifth ranked region in Figure 4, but has no top five industries in Table 4. Conversely, Dumfries & Galloway, South West Wales, Gwynedd, and Inner London-East are outside the top five in Figure 4, but have multiple top five industries in Table 4. In terms of ranking, these industries are outperforming their higher level regional efficiency environment, while the industries in Oxfordshire are underperforming theirs. Moreover, in addition to the above coastal areas with industries in the bottom five in Table 4, there are further coastal areas in the bottom 15 in Figure 4 (Southend-on-Sea; Portsmouth; Southampton). Inner London-West and East also have top five industries in Table 4 and Outer London-South is among the bottom 15 in Figure 4, which is consistent with the big inequalities between some areas in major cities.

We next provide some insights into the lower level contemporaneous and dynamic asymmetric industry inefficiency spill-in and spill-out measures that we use to obtain the corresponding efficiency growth components of the industries' TFP growth. To this end, for the regional industries with a top five hierarchical adjusted own efficiency in Table 4, in Table 5 we present the contemporaneous and dynamic asymmetric hierarchical adjusted inefficiency spill-ins (spill-outs) from (to) all the other industries. These contemporaneous measures are means across the contemporaneous time horizons over the sample ($\psi = 0$) and a dynamic measure is the mean of the cumulative inefficiency spill-ins (spill-outs) that take effect in the next three future in-sample horizons ($\psi = 1 - 3$), but which relate to production in the contemporaneous horizon. To put the reported contemporaneous and dynamic inefficiency spill-ins and

TABLE 5 | Indirect spill-in and spill-out inefficiencies for selected regional industries.

Industries with a top 5 hierarchical adjusted own efficiency (Table 4)	Contemporaneous ($\psi = 0$)			Cumulative dynamic ($\psi = 1 - 3$, inclusive)	
	Direct ineff (rank)	Ind spill-in ineff (rank)	Ind spill-out ineff (rank)	Ind spill-in ineff (rank)	Ind spill-out ineff (rank)
Agriculture–water					
Aberdeen City & Aberdeenshire	0.379 (1)	0.067 (28)	0.026 (11)	0.296 (25)	0.130 (3)
Berkshire	0.383 (2)	0.073 (59)	0.039 (27)	0.289 (16)	0.193 (15)
Perth & Kinross and Stirling	0.391 (3)	0.085 (105)	0.115 (102)	0.343 (101)	0.402 (87)
Dumfries & Galloway	0.393 (4)	0.076 (74)	0.053 (50)	0.320 (64)	0.243 (35)
Gwynedd	0.413 (5)	0.063 (23)	0.035 (23)	0.288 (15)	0.172 (8)
Average	*0.800*	*0.075*	*0.079*	*0.321*	*0.373*
St. Dev.	*0.415*	*0.017*	*0.045*	*0.038*	*0.175*
Manufacturing					
Flintshire & Wrexham	0.352 (1)	0.064 (93)	0.032 (29)	0.298 (62)	0.158 (16)
Cheshire East	0.373 (2)	0.063 (83)	0.041 (47)	0.301 (67)	0.196 (36)
Aberdeen City & Aberdeenshire	0.376 (3)	0.065 (99)	0.025 (16)	0.290 (38)	0.129 (6)
South West Wales	0.408 (4)	0.063 (89)	0.051 (67)	0.283 (24)	0.221 (47)
Clackmannanshire & Fife	0.412 (5)	0.069 (120)	0.014 (2)	0.308 (84)	0.104 (1)
Average	*0.564*	*0.061*	*0.059*	*0.300*	*0.272*
St. Dev.	*0.162*	*0.010*	*0.033*	*0.033*	*0.111*
Construction					
Inner London-West	0.296 (1)	0.054 (10)	0.021 (7)	0.252 (7)	0.110 (1)
Surrey	0.397 (2)	0.057 (20)	0.058 (74)	0.277 (17)	0.262 (66)
Hampshire CC	0.399 (3)	0.061 (41)	0.059 (77)	0.292 (38)	0.258 (63)
Hertfordshire	0.399 (4)	0.063 (58)	0.111 (121)	0.283 (24)	0.399 (118)
Berkshire	0.402 (5)	0.051 (8)	0.041 (43)	0.250 (6)	0.203 (32)
Average	*0.588*	*0.062*	*0.061*	*0.302*	*0.282*
St. Dev.	*0.142*	*0.010*	*0.034*	*0.033*	*0.110*
Distribution–restaurants					
Inner London-West	0.267 (1)	0.064 (70)	0.019 (5)	0.270 (9)	0.099 (1)
Milton Keynes	0.393 (2)	0.065 (79)	0.040 (46)	0.324 (113)	0.187 (37)
Berkshire	0.401 (3)	0.056 (11)	0.041 (47)	0.257 (6)	0.203 (44)
Gwynedd	0.422 (4)	0.055 (10)	0.036 (38)	0.273 (12)	0.176 (22)
Dumfries & Galloway	0.439 (5)	0.068 (112)	0.059 (82)	0.302 (65)	0.272 (77)
Average	*0.562*	*0.062*	*0.060*	*0.303*	*0.277*
St. Dev.	*0.095*	*0.009*	*0.035*	*0.034*	*0.116*
Transport & communications					
Inner London-West	0.292 (1)	0.064 (8)	0.021 (6)	0.270 (6)	0.108 (2)
Berkshire	0.415 (2)	0.064 (9)	0.042 (39)	0.273 (7)	0.210 (26)
Outer London-West & North West	0.486 (3)	0.066 (15)	0.078 (80)	0.296 (16)	0.330 (72)
Suffolk	0.496 (4)	0.071 (31)	0.039 (33)	0.296 (17)	0.200 (20)
Inner London-East	0.511 (5)	0.067 (17)	0.047 (43)	0.299 (23)	0.226 (31)
Average	*0.709*	*0.075*	*0.077*	*0.322*	*0.353*
St. Dev.	*0.144*	*0.013*	*0.049*	*0.037*	*0.165*
Banking–insurance					
Inner London-West	0.184 (1)	0.055 (21)	0.013 (3)	0.256 (8)	0.068 (1)
City of Edinburgh	0.314 (2)	0.059 (57)	0.031 (36)	0.283 (35)	0.141 (14)

(Continues)

TABLE 5 | (Continued)

Industries with a top 5 hierarchical adjusted own efficiency (Table 4)	Contemporaneous ($\psi = 0$)			Cumulative dynamic ($\psi = 1 - 3$, inclusive)	
	Direct ineff (rank)	Ind spill-in ineff (rank)	Ind spill-out ineff (rank)	Ind spill-in ineff (rank)	Ind spill-out ineff (rank)
Aberdeen City & Aberdeenshire	0.360 (3)	0.059 (64)	0.024 (23)	0.280 (28)	0.123 (7)
Glasgow City	0.378 (4)	0.052 (11)	0.039 (50)	0.249 (6)	0.191 (48)
Berkshire	0.381 (5)	0.048 (8)	0.023 (19)	0.267 (12)	0.134 (10)
Average	*0.514*	*0.059*	*0.055*	*0.297*	*0.253*
St. Dev.	*0.107*	*0.009*	*0.033*	*0.033*	*0.110*
Public admin–health					
Inner London-West	0.356 (1)	0.069 (106)	0.026 (25)	0.279 (15)	0.132 (10)
Gwynedd	0.374 (2)	0.051 (9)	0.032 (33)	0.266 (8)	0.156 (20)
Conwy & Denbighshire	0.398 (3)	0.053 (11)	0.034 (38)	0.276 (14)	0.163 (29)
Dumfries & Galloway	0.398 (4)	0.064 (56)	0.054 (72)	0.297 (48)	0.247 (66)
South West Wales	0.419 (5)	0.053 (10)	0.052 (69)	0.266 (7)	0.226 (54)
Average	*0.558*	*0.064*	*0.062*	*0.306*	*0.282*
St. Dev.	*0.086*	*0.010*	*0.040*	*0.035*	*0.134*
Other services					
Inner London-West	0.271 (1)	0.064 (7)	0.019 (2)	0.271 (7)	0.100 (1)
Greater Manchester South	0.501 (2)	0.084 (89)	0.039 (25)	0.330 (62)	0.202 (12)
City of Edinburgh	0.512 (3)	0.089 (120)	0.050 (39)	0.334 (77)	0.230 (23)
Berkshire	0.525 (4)	0.064 (8)	0.053 (44)	0.270 (6)	0.265 (36)
Inner London-East	0.525 (5)	0.065 (11)	0.049 (37)	0.296 (13)	0.233 (25)
Average	*0.800*	*0.079*	*0.084*	*0.328*	*0.387*
St. Dev.	*0.267*	*0.016*	*0.051*	*0.042*	*0.178*

spill-outs into context, Table 5 also reports the mean direct inefficiency. We consider the same regional industries in Tables 4 and 5 to draw parallels between these two sets of results, that is, using a direct inefficiency from Table 5, we obtain a close approximation of the corresponding own efficiency in Table 4, $\exp(-u^{Dir})$, which indicates that the direct inefficiencies contain very little inefficiency feedback.

The within industry classification ranks of the direct and indirect inefficiencies are reported in parentheses in Table 5. From these ranks, we can see that if a regional industry has a high ranking direct inefficiency, this does not preclude the industry from having a notably lower ranked indirect inefficiency. Moreover, whereas a number of regional industries with a top 5 hierarchical adjusted own efficiency are well-known and well-developed (e.g., the Agriculture‾water industries in Berkshire and Aberdeen City & Aberdeenshire), it is less clear which regional industries would be expected to have the lowest inefficiency spillovers. Hence, a key motivation of our spatial performance analysis is to uncover such regional industries. Finally on Table 5, as expected and relative to the direct inefficiency, the contemporaneous indirect spill-in and spill-out inefficiencies have smaller downward effects on the performance of a regional industry, although in a number of cases these spillovers are non-negligible. Interestingly, we can see that a contemporaneous indirect spill-in or spill-out inefficiency is substantially smaller than the corresponding cumulative dynamic indirect inefficiency.

4.3 | Contemporaneous and Dynamic Multilevel Spatial TFP Growth Decompositions

Before turning to the mean annual series for the regional industries from the TFP growth decompositions, Table 6 summarizes the components used to calculate these series. Specifically, Table 6 presents means of the contemporaneous and dynamic direct and asymmetric indirect TFP indices and their components. When there is a change in the contemporaneous period, each dynamic measure in this table represents the cumulative effect over the first three future in-sample time horizons.

As expected, some of the measures of TFP growth in Table 6 are negative. These often include dynamic TFP_{In}^{Ind} and TFP_{Out}^{Ind}, which highlights the merits of our dynamic spatial analysis. Notable examples of negative TFP growth from this table include contemporaneous TFP^{Dir} and dynamic TFP_{In}^{Ind} and TFP_{Out}^{Ind} for Construction and Distribution‾restaurants, and also all six of the reported TFP growth measures for Other services. Such findings for Construction and Distribution‾restaurants may reflect Britain's productivity challenge as the performance of these industries may be

TABLE 6 | Mean contemporaneous and dynamic multilevel spatial TFP growth decompositions.

	IndScale	*IndEnvir*	*IndEff*	*IndShock*	*RegScale*	*RegEnvir*	*RegEff*	*RegShock*	*MultilevTC*	*TFP*
Agriculture–water										
$Dir\psi=0$	102.77	99.15	99.86	100.79	99.14	100.02	99.74	98.33	97.84	101.50
$Ind_{In}\psi=0$	100.58	99.84	99.94	99.84	100.20	99.98	99.97	99.78	99.76	100.02
$Ind_{Out}\psi=0$	100.58	99.84	99.92	99.79	100.20	99.98	99.97	99.78	99.76	100.25
$Dir\psi=1\text{-}3$	100.45	99.98	99.99	99.97	100.32	100.00	100.00	99.97	99.97	100.81
$Ind_{In}\psi=1\text{-}3$	101.34	99.40	99.82	99.76	99.73	99.94	99.86	99.02	98.61	97.51
$Ind_{Out}\psi=1\text{-}3$	101.30	99.42	99.70	99.44	99.72	99.95	99.86	99.05	98.61	99.15
Manufacturing										
$Dir\psi=0$	103.42	99.90	100.03	101.56	99.14	100.03	99.74	98.33	97.84	100.86
$Ind_{In}\psi=0$	101.02	99.95	99.99	100.07	100.20	99.98	99.97	99.78	99.76	100.91
$Ind_{Out}\psi=0$	101.02	99.95	100.00	100.10	100.20	99.98	99.97	99.78	99.76	101.03
$Dir\psi=1\text{-}3$	100.82	99.99	100.00	100.01	100.32	100.00	100.00	99.97	99.97	101.27
$Ind_{In}\psi=1\text{-}3$	101.94	99.86	99.90	100.12	99.73	99.94	99.86	99.02	98.61	99.15
$Ind_{Out}\psi=1\text{-}3$	101.93	99.87	100.01	100.59	99.72	99.95	99.86	99.05	98.61	100.07
Construction										
$Dir\psi=0$	101.39	100.11	99.55	98.47	99.14	100.03	99.74	98.33	97.84	95.81
$Ind_{In}\psi=0$	99.35	99.98	99.96	99.82	100.20	99.98	99.97	99.78	99.76	99.05
$Ind_{Out}\psi=0$	99.34	99.98	99.95	99.76	100.20	99.98	99.97	99.78	99.76	99.04
$Dir\psi=1\text{-}3$	99.17	100.00	99.99	99.97	100.32	100.00	100.00	99.97	99.97	99.58
$Ind_{In}\psi=1\text{-}3$	100.17	99.97	99.85	99.72	99.73	99.94	99.86	99.02	98.61	97.23
$Ind_{Out}\psi=1\text{-}3$	100.18	99.97	99.78	99.06	99.72	99.95	99.86	99.05	98.61	96.91
Distribution–restaurants										
$Dir\psi=0$	100.79	99.55	100.23	102.24	99.14	100.03	99.74	98.33	97.84	97.90
$Ind_{In}\psi=0$	99.96	99.91	100.01	100.14	100.20	99.98	99.97	99.78	99.76	99.94
$Ind_{Out}\psi=0$	99.97	99.91	100.02	100.20	100.20	99.98	99.97	99.78	99.76	100.04
$Dir\psi=1\text{-}3$	99.89	99.99	100.00	100.03	100.32	100.00	100.00	99.97	99.97	100.37
$Ind_{In}\psi=1\text{-}3$	100.29	99.65	99.93	100.22	99.73	99.94	99.86	99.02	98.61	97.51
$Ind_{Out}\psi=1\text{-}3$	100.30	99.67	100.11	101.01	99.72	99.95	99.86	99.05	98.61	98.57
Transport & communications										
$Dir\psi=0$	105.13	101.90	99.72	100.11	99.14	100.03	99.74	98.33	97.84	102.61
$Ind_{In}\psi=0$	102.39	100.16	99.96	99.92	100.20	99.98	99.97	99.78	99.76	102.31
$Ind_{Out}\psi=0$	102.39	100.16	99.96	99.91	100.20	99.98	99.97	99.78	99.76	102.38
$Dir\psi=1\text{-}3$	102.13	100.02	100.00	99.99	100.32	100.00	100.00	99.97	99.97	102.60
$Ind_{In}\psi=1\text{-}3$	103.53	100.83	99.85	99.87	99.73	99.94	99.86	99.02	98.61	101.31
$Ind_{Out}\psi=1\text{-}3$	103.54	100.84	99.83	99.76	99.72	99.95	99.86	99.05	98.61	101.61
Banking–insurance										
$Dir\psi=0$	103.98	100.61	100.52	105.18	99.14	100.02	99.74	98.33	97.84	104.61
$Ind_{In}\psi=0$	101.21	100.01	100.04	100.40	100.20	99.98	99.97	99.78	99.76	101.45
$Ind_{Out}\psi=0$	101.21	100.01	100.06	100.55	100.20	99.98	99.97	99.78	99.76	101.64
$Dir\psi=1\text{-}3$	100.95	100.00	100.01	100.07	100.32	100.00	100.00	99.97	99.97	101.39
$Ind_{In}\psi=1\text{-}3$	102.38	100.15	99.98	100.64	99.73	99.94	99.86	99.02	98.61	100.22
$Ind_{Out}\psi=1\text{-}3$	102.39	100.16	100.28	102.59	99.72	99.95	99.86	99.05	98.61	102.47
Public admin–health										
$Dir\psi=0$	101.64	100.80	100.47	105.97	99.14	100.02	99.74	98.33	97.84	100.65
$Ind_{In}\psi=0$	100.43	100.03	100.03	100.28	100.20	99.98	99.97	99.78	99.76	100.53

(Continues)

TABLE 6 | (Continued)

	IndScale	*IndEnvir*	*IndEff*	*IndShock*	*RegScale*	*RegEnvir*	*RegEff*	*RegShock*	*MultilevTC*	*TFP*
$Ind_{Out}\,\psi$=0	100.42	100.04	100.06	100.39	100.20	99.98	99.97	99.78	99.76	100.66
$Dir\psi$=1-3	100.30	100.00	100.01	100.05	100.32	100.00	100.00	99.97	99.97	100.70
$Ind_{In}\,\psi$=1-3	100.96	100.26	99.97	100.44	99.73	99.94	99.86	99.02	98.61	98.70
$Ind_{Out}\,\psi$=1-3	100.93	100.28	100.25	101.89	99.72	99.95	99.86	99.05	98.61	100.24
Other services										
$Dir\psi$=0	101.15	99.95	98.55	95.94	99.14	100.02	99.74	98.33	97.84	93.93
$Ind_{In}\,\psi$=0	98.99	99.93	99.86	99.54	100.20	99.98	99.97	99.78	99.76	98.33
$Ind_{Out}\,\psi$=0	98.98	99.92	99.82	99.39	100.20	99.98	99.97	99.78	99.76	98.24
$Dir\psi$=1-3	98.81	99.99	99.98	99.92	100.32	100.00	100.00	99.97	99.97	99.22
$Ind_{In}\,\psi$=1-3	99.83	99.81	99.69	99.28	99.73	99.94	99.86	99.02	98.61	96.08
$Ind_{Out}\,\psi$=1-3	99.82	99.79	99.21	97.53	99.72	99.95	99.86	99.05	98.61	94.78

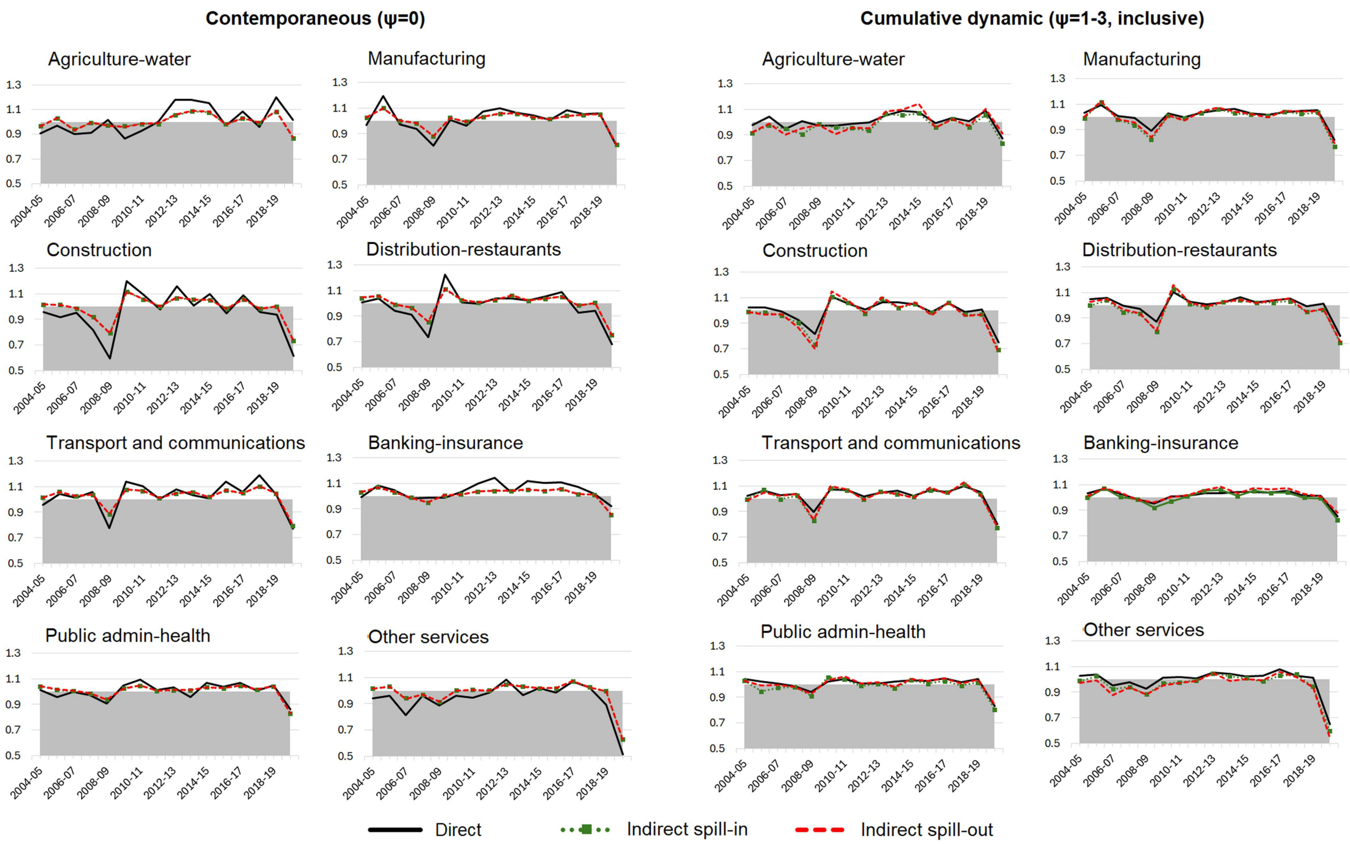


FIGURE 5 | Mean regional industry TFP indices. [Color figure can be viewed at wileyonlinelibrary.com]

indicative of the general economic climate. The negative TFP growth measures for Other services may also reflect this challenge as Britain's economy is service oriented. Focusing on contemporaneous TFP^{Dir} in Table 6, we can see that for Construction, Distribution‐restaurants and Other services, one of the larger drivers of this measure is $MultilevTC^{Dir}$. A bigger driver of contemporaneous TFP^{Dir} for Other services is $IndShock^{Dir}$, while $IndShock^{Dir}$ and $RegShock^{Dir}$ are among the largest drivers of contemporaneous TFP^{Dir} for Construction. This sensitivity of contemporaneous TFP^{Dir} to negative economic shocks may point to both these industries lacking resilience.

There are a few notable positive TFP growth measures in Table 6, which highlights the more promising parts of Britain's productivity situation. These include contemporaneous TFP^{Dir} for Banking‐insurance and Transport & communications (and also for the latter dynamic TFP^{Dir} and contemporaneous TFP^{Ind}_{In} and TFP^{Ind}_{Out}). These findings differ from the above-discussed negative TFP growth measures in Table 6, which indicates that going forward services in Transport & communications and Banking‐insurance may help drive improvement in Britain's productivity.

Figure 5 presents the annual means of the TFP index measures in the final column of Table 6. During notable portions of the

TABLE 7 | Regional industries with top five mean TFP growth spill-ins (spill-outs) from (to) all the other industries.

Contemporaneous, $\psi = 0$ (in 2019 million £s)				**Cumulative dynamic, $\psi = 1 - 3$, inclusive (in 2019 million £s)**			
	Indirect spill-in		**Indirect spill-out**		**Indirect spill-in**		**Indirect spill-out**
Agriculture–water							
Aberdeenshire[a] (**2**; *5.6%*)	114.63	Aberdeenshire[a] (**2**; *6.6%*)	136.64	Aberdeenshire[a] (**2**; *2.4%*)	50.10	Aberdeenshire[a] (**2**; *7.8%*)	160.88
Warwickshire (**18**; *7.9%*)	79.05	Warwickshire (**18**; *10.1%*)	100.73	Warwickshire (**18**; *4.5%*)	45.22	Warwickshire (**18**; *13.8%*)	138.32
Berkshire (**1**; *2.3%*)	49.02	Berkshire (**1**; *2.9%*)	63.23	Coventry (**33**; *4.5%*)	30.11	Coventry (**33**; *8.9%*)	60.33
Coventry (**33**; *6.7%*)	45.61	Inner London-West (**7**; *4.6%*)	57.84	North Lanarkshire (**55**; *5.7%*)	25.85	North Lanarkshire (**55**; *12.2%*)	55.82
Inner London-West (**7**; *3.5%*)	43.67	Coventry (**33**; *7.7%*)	51.75	West Northamptonshire (**50**; *4.3%*)	20.36	Inner London-West (**7**; *4.0%*)	50.94
Manufacturing							
Warwickshire (**15**; *5.9%*)	161.01	Warwickshire (**15**; *7.0%*)	193.03	Warwickshire (**15**; *5.6%*)	155.21	Warwickshire (**15**; *10.7%*)	295.43
Cheshire East (**4**; *2.9%*)	117.47	Cheshire East (**4**; *2.9%*)	118.38	West Cumbria (**47**; *8.1%*)	114.65	Flintshire& Wrexham (**18**; *4.5%*)	117.91
Flintshire& Wrexham (**18**; *3.6%*)	94.10	Flintshire& Wrexham (**18**; *4.1%*)	105.33	Cheshire East (**4**; *2.4%*)	98.89	Cheshire East (**4**; *2.8%*)	115.76
S&W Derbyshire[b] (**17**; *3.2%*)	83.49	S&W Derbyshire[b] (**17**; *3.7%*)	98.45	Flintshire& Wrexham (**18**; *1.9%*)	49.91	S&W Derbyshire[b] (**17**; *3.7%*)	98.83
CheshireW & Chester[c] (**38**; *4.0%*)	71.52	CheshireW & Chester[c] (**38**; *4.5%*)	79.93	CheshireW & Chester[c] (**38**; *2.4%*)	42.16	CheshireW & Chester[c] (**38**; *5.2%*)	92.69
Construction							
Inner London-West (**2**; *4.4%*)	194.82	Inner London-West (**2**; *4.7%*)	210.43	Inner London-West (**2**; *2.4%*)	105.12	Inner London-West (**2**; *3.1%*)	136.51
Inner London-East (**3**; *3.6%*)	144.55	Inner London-East (**3**; *3.7%*)	150.39	Inner London-East (**3**; *1.5%*)	60.73	Inner London-East (**3**; *1.9%*)	76.20
Outer London-W& NW[d] (**1**; *1.5%*)	74.62	Outer London-W& NW[d] (**1**; *1.2%*)	61.97	Milton Keynes (**81**; *1.8%*)	7.67	Medway (*65;2.1%*)	10.95
Hertfordshire (**7**; *1.0%*)	36.02	Hertfordshire (**7**; *0.6%*)	23.58	Darlington (**126**; *1.3%*)	1.72	City of Bristol (**46**; *1.3%*)	10.70
Kent CC (**8**; *0.7%*)	27.11	City of Bristol (**46**; *2.3%*)	18.65	Central Valleys (**84**; *0.3%*)	1.28	Central Valleys (**84**; *2.7%*)	10.50
Distribution–restaurants							
Inner London-West (**1**; *3.6%*)	640.29	Inner London-West (**1**; *4.3%*)	756.19	Inner London-West (**1**; *0.9%*)	159.89	Inner London-West (**1**; *3.9%*)	694.31
Inner London-East (**3**; *3.7%*)	310.98	Inner London-East (**3**; *3.8%*)	327.72	Inner London-East (**3**; *1.2%*)	104.51	Inner London-East (**3**; *2.6%*)	220.74

(Continues)

TABLE 7 | (Continued)

Contemporaneous, $\psi = 0$ (in 2019 million £s)				Cumulative dynamic, $\psi = 1 - 3$, inclusive (in 2019 million £s)			
	Indirect spill-in		**Indirect spill-out**		**Indirect spill-in**		**Indirect spill-out**
GM South[e] (**6**; *2.0%*)	130.81	GM South[e] (**6**; *2.1%*)	134.45	Milton Keynes (**14**; *0.4%*)	15.01	GM South[e] (**6**; *0.8%*)	49.35
Berkshire (**5**; *1.4%*)	94.17	Berkshire (**5**; *1.6%*)	108.28	Leicester (**74**; *1.1%*)	11.01	Nottingham (**54**; *2.9%*)	37.42
Hampshire CC (**8**; *1.4%*)	85.15	Milton Keynes (**14**; *2.3%*)	83.17	Nottingham (**54**; *0.7%*)	8.72	City of Bristol (**40**; *1.8%*)	33.03
Transport & communications							
Inner London-West (**1**; *6.8%*)	1509.05	Inner London-West (**1**; *7.6%*)	1670.49	Inner London-West (**1**; *5.6%*)	1231.79	Inner London-West (**1**; *9.4%*)	2074.24
Inner London-East (**3**; *7.0%*)	876.05	Inner London-East (**3**; *7.2%*)	902.62	Inner London-East (**3**; *5.7%*)	717.67	Inner London-East (**3**; *7.4%*)	935.52
Berkshire (**4**; *10.6%*)	683.65	Berkshire (**4**; *11.6%*)	745.02	Berkshire (**4**; *9.4%*)	602.45	Berkshire (**4**; *13.9%*)	892.27
Outer London-W& NW[d] (**2**; *3.7%*)	470.96	Outer London-W& NW[d] (**2**; *3.9%*)	493.78	Outer London-W& NW[d] (**2**; *2.2%*)	273.66	Outer London-W& NW[b] (**2**; *3.6%*)	454.63
GM South[e] (**6**;*6.3%*)	225.01	GM South[e] (**6**; *6.5%*)	232.10	GM South[e] (**6**; *5.7%*)	203.29	GM South[e] (**6**; *6.4%*)	230.64
Banking-insurance							
Inner London-West (**1**; *5.8%*)	6128.07	Inner London-West (**1**; *6.1%*)	6471.36	Inner London-West (**1**; *4.8%*)	5091.93	Inner London-West (**1**; *8.6%*)	9153.06
Inner London-East (**2**; *5.4%*)	2855.70	Inner London-East (**2**; *5.6%*)	2935.80	Inner London-East (**2**; *4.8%*)	2523.37	Inner London-East (**2**; *8.2%*)	4312.12
GM South[e] (**5**; *3.3%*)	560.56	Outer London-W& NW[d] (**3**; *2.8%*)	646.93	GM South[e] (**5**; *2.6%*)	443.33	Outer London-W& NW[d] (**3**; *5.5%*)	1279.56
Outer London-W& NW[b] (**3**; *2.3%*)	527.12	GM South[e] (**5**; *3.3%*)	562.13	Outer London-W& NW[d] (**3**; *1.3%*)	297.71	Hampshire CC (**7**; *7.9%*)	1070.27
Hertfordshire (**10**; *3.0%*)	360.17	Hampshire CC (**7**; *3.7%*)	494.41	Hertfordshire (**10**; *1.4%*)	174.69	Hertfordshire (**10**; *6.6%*)	798.22
Public admin–health							
Inner London-West (**1**; *4.5%*)	847.45	Inner London-West (**1**; *5.2%*)	977.77	Inner London-West (**1**; *2.8%*)	528.65	Inner London-West (**1**; *6.9%*)	1294.68
Inner London-East (**2**; *3.1%*)	458.27	Inner London-East (**2**; *2.9%*)	428.75	Inner London-East (**2**; *1.7%*)	245.66	Inner London-East (**2**; *2.0%*)	301.15
GM South[e] (**4**; *2.1%*)	189.33	GM South[e] (**4**; *2.4%*)	211.16	Milton Keynes (**65**; *2.9%*)	47.02	Berkshire (**17**; *4.1%*)	198.49
Berkshire (**17**; *2.6%*)	126.73	Berkshire (**17**; *3.0%*)	145.28	Berkshire (**17**; *0.7%*)	32.74	GM South[r] (**4**; *2.2%*)	198.05
Outer London-W& NW[d] (**3**; *1.2%*)	112.86	Outer London-W& NW[d] (**3**; *1.5%*)	132.83	North Lanarkshire (**60**; *1.9%*)	31.76	Hampshire CC (**10**; *3.3%*)	193.80
Other services							

(Continues)

TABLE 7 | (Continued)

Contemporaneous, $\psi = 0$ (in 2019 million £s)				Cumulative dynamic, $\psi = 1 - 3$, inclusive (in 2019 million £s)			
	Indirect spill-in		Indirect spill-out		Indirect spill-in		Indirect spill-out
Inner London-West (**1**; *2.0%*)	143.91	Inner London-West (**1**; *2.6%*)	184.55	Stoke-on-Trent (**38**; *3.5%*)	16.23	Stoke-on-Trent (**38**;*7.2%*)	33.47
Inner London-East (**2**; *1.2%*)	45.24	Inner London-East (**2**; *1.1%*)	44.10	Milton Keynes (**29**; *2.2%*)	12.67	Sandwell (**47**; *5.0%*)	18.41
Stoke-on-Trent (**38**; *5.2%*)	24.33	Stoke-on-Trent (**38**; *6.5%*)	30.11	Sandwell (**47**; *1.1%*)	4.05	Milton Keynes (**29**; *2.3%*)	13.21
Milton Keynes (**29**; *3.8%*)	22.15	Milton Keynes (**29**; *4.2%*)	24.36	Dudley (**88**; *1.5%*)	2.55	Dudley (**88**; *7.4%*)	12.83
GM South^e (**4**; *0.9%*)	16.97	GM South^e (**4**; *1.2%*)	22.64	Leicester (**64**; *0.7%*)	1.62	Southampton (**72**; *3.5%*)	7.78

Note: The full names of the NUTS3 regions with the $a - e$ superscripts are ^{a} Aberdeen City and Aberdeenshire; ^{b} South & West Derbyshire; ^{c} Cheshire West & Chester; ^{d} Outer London-West & North West; ^{e} Greater Manchester South. In parentheses are (i) the within industry classification rank of a regional industry's mean real output from the 134 such industries (bold); and (ii) the mean contemporaneous and dynamic TFP growth spill-ins (spill-outs) measured in terms of the real output spill-in (spill-out) as a percentage of the regional industry's mean real output (italics).

study period, the contemporaneous and dynamic industry indices in this figure follow the expected paths. These include declines due to the 2008 financial crisis and the COVID pandemic in the final year of the study period, as well as some evidence of flatlining (or variation around the flatline) in the intervening years. Interestingly, in Figure 5, an industry's contemporaneous and dynamic indirect spill-in and spill-out TFP indices tend to follow the same path as the corresponding direct index. This is a feature of these annual mean TFP indices, as in the next subsection we highlight the asymmetry between the top five contemporaneous (dynamic) indirect TFP growth spill-ins and spill-outs for individual regional industries.

4.4 | Largest Regional Industry TFP Growth Spill-Ins and Spill-Outs

For individual regional industries, we now consider the sums of the contemporaneous and dynamic TFP growth spill-ins (spill-outs) from (to) the other 1, 071 regional industries. By industry classification, Table 7 presents the regional industries with the top five means of these spill-ins and spill-outs, which are measured in terms of real output the regional industry receives and generates, respectively. In parentheses in this table and for context, we also report the within industry classification rank of a regional industry's mean real output $(1 - 134)$ (bold), and the aforementioned mean real output spill-ins and spill-outs as a percentage of a regional industry's real output (italics).

Table 7 reveals that, whilst for each of the eight industry classifications the same regional industries are often in the top five for the corresponding spill-in and spill-out, there is an asymmetry between the magnitudes of the spill-in and spill-out, which in a number of cases is non-negligible. As expected, a number of the top five regional industries have a high ranking mean real output in the eight industry classifications. For example, for five of the classifications, Inner London-West is the regional industry with the largest mean real output and the largest mean contemporaneous spill-in and spill-out.

Interesting other cases are where the ranking of a regional industry's TFP growth spill-in (spill-out) within its industry classification is notably higher than the ranking of its real output. These cases are interesting as the spill-in (spill-out) represents relative high performance for the regional industry that may not have been focused on and may represent unexploited potential to improve productivity. For example, we can see from Table 7 that Agriculture‾water in Coventry has both a top five contemporaneous TFP growth spill-in and spill-out and a notably lower mean real output ranking in this industry (33) (see below for further discussion of this example).

To better inform policymaking to improve productivity via spillovers, we suggest a further two-step process to obtain the more detailed results that drive those in Table 7. Specifically, this process can uncover individual firms that receive (generate) the largest TFP growth spill-ins (spill-outs). We undertake the analysis for the first step here and illustrate how to conduct the second step for a regional industry using the case of Agriculture‾water in Coventry.

TABLE 8 | Largest flows of TFP growth spillovers for regional agriculture-water industries.

Contemporaneous, $\psi = 0$ (in 2019 million £s)					
	Indirect spill-in			**Indirect spill-out**	
Aberdeen City and Aberdeenshire	*114.63*		*Aberdeen City and Aberdeenshire*	*136.64*	
Caithness & Sutherland and Ross	40.57	35.4%	Inverness & Nairn and Moray	66.41	48.6%
Angus and Dundee City	37.10	32.4%	Lochaber, Skye & Lochalsh	65.13	47.7%
Perth & Kinross and Stirling	31.51	27.5%	Caithness & Sutherland and Ross	2.84	2.1%
Warwickshire	*79.05*		*Warwickshire*	*100.73*	
Gloucestershire	9.11	11.5%	Coventry	28.88	28.7%
Oxfordshire	9.09	11.5%	Solihull	15.05	14.9%
West Northamptonshire	8.96	11.3%	West Northamptonshire	9.63	9.6%
Berkshire	*49.02*		*Berkshire*	*63.23*	
Buckinghamshire CC	8.77	17.9%	Hampshire CC	10.01	15.8%
Outer London - West and North West	1.57	16.7%	Oxfordshire	9.99	15.8%
Wiltshire CC	0.28	16.0%	Surrey	9.99	15.8%
Coventry	*45.61*		*Inner London - West*	*57.84*	
Warwickshire	22.38	49.1%	Inner London - East	25.68	44.4%
Solihull	20.63	45.2%	Outer London - South	17.24	29.8%
Birmingham	0.71	1.6%	Outer London - West and North West	12.98	22.4%
Inner London - West	*43.67*		*Coventry*	*51.75*	
Outer London - West and North West	15.42	35.3%	Solihull	34.27	66.2%
Outer London - South	13.49	30.9%	Warwickshire	15.41	29.8%
Inner London - East	13.00	29.8%	Birmingham	0.59	1.1%
Cumulative dynamic, $\psi = 1 - 3$, inclusive (in 2019 million £s)					
Aberdeen City and Aberdeenshire	*50.10*		*Aberdeen City and Aberdeenshire*	*160.88*	
Caithness & Sutherland and Ross	16.19	32.3%	Inverness & Nairn and Moray	71.38	44.4%
Angus and Dundee City	13.23	26.4%	Lochaber, Skye & Lochalsh	67.15	41.7%
Perth & Kinross and Stirling	12.34	24.6%	Caithness & Sutherland and Ross	11.78	7.3%
Warwickshire	*45.22*		*Warwickshire*	*138.32*	
Gloucestershire	4.61	10.2%	Coventry	34.03	24.6%
Oxfordshire	4.60	10.2%	Solihull	19.52	14.1%
Birmingham	4.60	10.2%	West Northamptonshire	11.36	8.2%
Coventry	*30.11*		*Coventry*	*60.33*	
Warwickshire	12.76	42.4%	Solihull	34.91	57.9%
Solihull	4.82	37.8%	Warwickshire	16.21	26.9%
Birmingham	0.23	4.8%	Birmingham	2.23	3.7%
North Lanarkshire	*25.85*		*North Lanarkshire*	*55.82*	
South Ayrshire	4.26	16.5%	East Ayrshire	24.23	43.4%
East Ayrshire	3.74	14.5%	Dumfries & Galloway	18.36	32.9%
West Lothian	3.48	13.5%	South Ayrshire	2.71	4.9%
West Northamptonshire	*20.36*		*Inner London - West*	*50.94*	
Milton Keynes	3.34	16.4%	Inner London - East	20.14	39.5%
Buckinghamshire CC	3.25	16.0%	Outer London - South	13.81	27.1%
Oxfordshire	2.82	13.8%	Outer London - West and North West	10.49	20.6%

To set the scene for the first step, recall that in Table 7 a TFP growth spill-out (spill-in) from (to) a regional industry is the sum of those to (from) all the other industries. The first step goes further by uncovering the origin-destination TFP growth spill-out (spill-in) flows that make the biggest contributions to the sums in Table 7. The approach to calculate these flows begins by obtaining own TFP growth for the regional industries ($IndTFP_{1,igt}$). Having made the relevant simple adjustments to the notation in Eq. (9) and the equations for its nine right-hand side components, we calculate the nine own components of a standard O'Donnell (2016; 2018; 2022) TFP growth decomposition from the model in Table 2 (i.e., $IndScale_{1,igt}$; $IndEnvir_{1,igt}$; $IndEff_{1,igt}$; $IndShock_{1,igt}$; $RegScale_{2,gt}$; $RegEnvir_{2,gt}$; $RegEff_{2,gt}$; $RegShock_{2,gt}$; $MultilevTC_{M,igt}$). Following Eq. (9) these components are then multiplied to obtain $IndTFP_{1,igt}$. Next, we obtain the contemporaneous and cumulated dynamic TFP growth spill-outs (spill-ins) from (to) a regional industry to (from) each of the other industries, i.e., for horizon 0 and the sum across horizons $1-3$, respectively. To obtain these spill-outs (spill-ins), for each horizon we decompose the product of the spatial multiplier matrix and the own TFP growth vector. The decomposition of this product is along the same lines as that in Eq. (5), where the form of the spatial multiplier matrix changes with the time horizon.

For the first step analysis, we calculate the contemporaneous and dynamic TFP growth spill-outs (spill-ins) between each pair of regional industries. The results for this complete set of flows are available from the corresponding author on request as due to space constraints we present a small snapshot of our findings. To this end, Table 8 presents the three flows that make the biggest contributions to the Agriculture‒water results in Table 7. All the spill-ins and spill-outs in Table 8 flow between relatively nearby regional Agriculture‒water industries, which is consistent with the notable productivity relationships within an industry classification reported by Coyle and Mei (2023) and Goodridge and Haskel (2023). The results in Table 8 therefore point to the intuitive finding that the biggest sources (recipients) of TFP growth spill-outs (spill-ins) are the nearest and most similar regional industries. The flows in Table 8 with the largest percentage contributions to the corresponding result in Table 7 also point to the regional industries which policies can target to harness these spillovers to foster productivity improvement (e.g., 48.6% of the contemporaneous TFP growth spill-out from Agriculture‒water in Aberdeen City and Aberdeenshire flows to the corresponding industry in Inverness & Nairn and Moray).

Within the regional industries that correspond to the largest flows of TFP growth spillovers from the first step, the second step focuses on uncovering individual firms that are the most likely recipients (sources) of the largest spill-ins (spill-outs). A good starting point for this is knowledge of the biggest firms in a regional industry, as bigger firms tend to be associated with larger spillovers. The second step is therefore more manageable for regional industries outside London with a smaller number of large firms, which is convenient as it is productivity outside London and the South East of England that is most concerning. We demonstrate the second step for the Agriculture‒water industry in Coventry, where from footnote 1 this industry classification covers agriculture, fishing, energy and water. As there is little agriculture and fishing activity in Coventry, and given the aforementioned firm size‒spillover nexus, the top five contemporaneous spill-in and spill-out of TFP growth in Table 7 for Coventry's Agriculture‒water industry fits with it being where three major energy and water public utilities have (head) offices (see footnote 16 for details).[16] We therefore suggest that complementary factors driving these spillovers are management activities at the companies' Coventry offices and the geographical network characteristics of their business operations.

Within other regional industries that correspond to some of the largest flows of TFP growth spillovers, the same type of second step approach can be used to uncover individual firms that are the most likely recipients (sources) of the largest spill-ins (spill-outs), for example, TFP growth spill-in (spill-out) flows for Manufacturing in Flintshire & Wrexham, as this is where JCB, Tata Steel, Toyota and Kellogg's have plants.

5 | Concluding Remarks

This paper adds to the literature on regional economic performance analysis in three respects. The first respect builds on the expanding literature on single level spatial efficiency analysis by introducing the first multilevel spatial stochastic frontier model. We account for the regional impact on the industries a region nests by including higher level NUTS3 regional regressors and region and industry inefficiency and noise components. This is important because if the impact of the hierarchical data structure is overlooked, the industry level coefficients and efficiencies may be biased, which can lead to inaccurate policy recommendations to manage disparities in industries' economic performance. A single level stochastic frontier model would also not yield the informative hierarchical adjusted industry efficiencies. This efficiency measure adjusts an industry efficiency for the benchmarked impact of the higher level regional efficiency on an industry's output. Given this benchmarked impact will typically lie below the higher level frontier (i.e., be less than 1), there will tend to be a downward hierarchical adjustment of an industry's efficiency, which is a key feature of our multilevel stochastic frontier.

Using our stochastic frontier, the second respect is the extension of O'Donnell's (2016, 2018, 2022) static, single level, non-spatial, TFP growth decomposition to our setting. The standard Malmquist TFP growth decomposition omits the impacts of changes in continuous environmental variables, which for our analysis include higher level regional determinants. As continuous environmental variables may therefore have non-negligible impacts on the decomposition of regional industries' TFP growth, one should not overlook them. O'Donnell's decomposition includes these impacts, so we extend his approach to our dynamic multilevel spatial setting.

Bryson and Forth (2016) conclude that the UK productivity puzzle relates to the country's lower TFP growth. Drawing on this, the third respect is our more granular frontier and TFP growth analysis of Britain's productivity puzzle. We conduct the first regional industry frontier analysis of this puzzle using data for the eight industries in the 134 NUTS3 regions in Britain ($2004-20$). The only other frontier study of this puzzle is a single level analysis using more aggregate regional data (Glass and Kenjegalieva 2024). Using the Agriculture‒water industry in one NUTS3 region as an example, we show how our more granular findings can uncover a more well-defined starting point to develop policies to foster improvement in TFP growth and its spillovers for particular regional industries.

Acknowledgments

The authors thank the handling co-editor and three anonymous reviewers for their valuable comments on an earlier version of this paper.

Conflicts of Interest

The authors declare no conflicts of interest.

Data Availability Statement

The data that support the findings of this study are available in ONS at https://www.ons.gov.uk/economy/regionalaccounts/grossdisposablehouseholdincome/datasets/experimentalregionalgrossfixedcapitalformationgfcfestimatesbyassettype. These data were derived from the following resources available in the public domain: - ONS, https://www.ons.gov.uk/economy/regionalaccounts/grossdisposablehouseholdincome/datasets/experimentalregionalgrossfixedcapitalformationgfcfestimatesbyassettype.

Endnotes

[1] For each of the NUTS3 regions, the eight broad industries we consider may be better thought of as sectors, but we remain consistent with one of the data sources (Nomis) and refer to them as industries. The eight industries are single or combined sections of the 2007 UK Standard Industrial Classification (SIC). Specifically, the eight industries are (i) Agriculture & fishing (section A) combined with energy & water (sections B, D & E combined); (ii) Manufacturing (section C); (iii) Construction (section F); (iv) Distribution, hotels & restaurants (sections G & I combined); (v) Transport & communications (sections H & J combined); (vi) Banking, finance & insurance (sections K-N combined); (vii) Public administration, education & health (sections O-Q combined); (viii) Other services (sections R-U combined).

[2] Data for a number of variables for Northern Ireland's regional industries are only available from 2009. The data for these variables does not therefore include the negative shock to productivity in 2008 and the different productivity situation in the preceding years. Moreover, the most recent available data are for 2020, which we include to observe the impact of the COVID pandemic. The 2008 crisis was a shock to Britain's regional industries that led to a productivity slowdown, so including 2020 is interesting as the COVID pandemic was a further shock (De Vries et al. 2021).

[3] The indices in Figure 1 are based on 2015 U.S. dollars and PPPs.

[4] See Figure 1 in Glass and Kenjegalieva (2024) for a regional view of the UK's post−2008 productivity slowdown (i.e., NUTS1 regional labor productivity indices, 2000 − 20).

[5] In contrast to our micro multi-regional industry analysis, Bhattacharjee et al. (2024) conduct an interesting macro multi-regional analysis of the UK. Whereas we adopt the common approach to multilevel modeling by accounting for the hierarchical data structure within a single model, they link a national macro model to a spatial econometric multi-regional model.

[6] Whilst accounting for changes in the magnitudes of economic shocks over time does not fit within the framework of a Malmquist TFP growth decomposition, to some extent, the effects of environmental variables can be incorporated into such a decomposition by interacting these variables with the time trend and inputs in the stochastic production frontier. This would not though account for the effects of the environmental variables in and of themselves, and would only take account of the effects of these variables through their impacts on the growth rates of technical change and returns to scale.

[7] The familiar other components we extend to our setting are the growth rates of technical change, returns to scale and technical efficiency.

[8] In line with the findings of Glass and Kenjegalieva (2024), a recent article in the Financial Times (2025) cites “inefficiencies” as a contributing factor to the UK's lagging productivity performance.

[9] Whilst non-spatial dynamic panel data models involve including the time lag of the dependent variable, we do not include $y_{1,igt-1}$ and also $y_{2,gt-1}$ (or $x'_{1,igt-1}$ and $x'_{2,gt-1}$). This is so the non-spatial part of the production frontier is static and thus consistent with its counterpart in production theory (Glass and Kenjegalieva 2024).

[10] For the non-spatial setting, hierarchical random coefficients models are common. Random coefficients for all the variables in our model was not an option as this model does not exist. Central to this is the identification and estimation problems associated with a random component in the coefficient on a (spatial) lagged dependent variable (e.g., Elhorst 2014, p. 74). Relatedly, Aquaro et al.'s (2021) spatial heterogeneous coefficients estimator could be extended to our spatial multilevel setting, which is an area for further work.

[11] In the empirical analysis we use a multilevel translog functional form. Compared to the corresponding single level translog model, our model specification has quite a number of additional regressors, i.e., level 2 regional variables and for a number of these variables squares and interactions. For parsimony, therefore, we omit from the set of regressors in Eq. (1), $\sum_{j=1}^{N} w_{1,ij}x'_{1,jgt}$, $\sum_{j=1}^{N} w_{2,ij}x'_{1,jgt}$, $\sum_{j=1}^{N} w_{1,ij}x'_{2,jgt}$ and $\sum_{j=1}^{N} w_{2,ij}x'_{2,jgt}$.

[12] $\frac{\partial y_{1,t,\psi=0}}{\partial x_{2,qt}} = \mathbf{A}_{t,\psi=0}^{-1} I_N \gamma_q$; $\frac{\partial y_{1,t+1,\psi=1}}{\partial x_{2,qt}} = (\mathbf{A}_{t,\psi=0}^{-1} + \mathbf{C}_{t+1,\psi=1}) I_N \gamma_q$; $\frac{\partial y_{1,\Psi}}{\partial x_{2,qt}}$.

$= \sum_{\psi \in \Psi} \mathbf{C}_{t+\psi,\psi} I_N \gamma_q$

[13] Whilst $MultilevTC_{In,M,igt}^{Tot}$ differs from the other right-hand side total indices in Eq. (9) as it is neither an industry or regional level component, but a multilevel one, $MultilevTC_{In,M,igt}^{Tot}$ can be calculated in the same type of way as the other components (i.e., as the product of the direct and indirect multilevel technical change indices).

[14] The full set of estimates of $HE_{M,igt}$ for the regional industries by time period are available from the corresponding author on request.

[15] The full set of estimates by time period of $RE_{2,gt}$ for the regions and $IE_{1,igt}$ for the regional industries are available from the corresponding author on request.

[16] Head offices of Cadent Gas and the FTSE 100 company Severn Trent Water, and a corporate office of the FTSE 100 company National Grid (owner and operator of Britain's electricity transmission and distribution).

References

Adetutu, M. O., A. J. Glass, K. Kenjegalieva, and K. A. Odusanya. 2025. “Product Level Market Power Spillovers Among U.S. Banks.” *Papers in Regional Science* 104: 100093.

Aiello, F., and G. Bonanno. 2018. “Multilevel Empirics for Small Banks in Local Markets.” *Papers in Regional Science* 97: 1017–1038.

Aquaro, M., N. Bailey, and M. H. Pesaran. 2021. “Estimation and Inference for Spatial Models With Heterogeneous Coefficients: An Application to US House Prices.” *Journal of Applied Econometrics* 36: 18–44.

Ayouba, K. 2023. “Spatial Dependence in Production Frontier Models.” *Journal of Productivity Analysis* 60: 21–36.

Bashford-Fernández, J. M., and A. Rodríguez-Álvarez. 2024. “Measuring Local Employment Multipliers and Informal Employment: A Stochastic Frontier Approach.” *Regional Studies* 58: 78–90.

Berlemann, M., and J. -E. Wesselhöft. 2014. “Estimating Aggregate Capital Stocks Using the Perpetual Inventory Method—A Survey of Previous Implementations and New Empirical Evidence for 103 Countries.” *Review of Economics* 65: 1–34.

Bhattacharjee, A., A. Pabst, T. Szendrei, and G. J. Hewings. 2024. “NiReMS: A Regional Model at Household Level Combining Spatial Econometrics With Dynamic Microsimulation.” *Spatial Economic Analysis* 19: 436–461.

Bloom, N., R. Sadun, and J. Van Reenen. 2016. "Management as a Technology?" National Bureau of Economic Research, Working Paper 22327.

Blundell, R., C. Crawford, and W. Jin. 2014. "What Can Wages and Employment Tell us About the UK's Productivity Puzzle?" *Economic Journal* 124: 377–407.

Bolea, L., R. Duarte, G. J. D. Hewings, S. Jiménez, and J. Sánchez-Chóliz. 2022. "The Regional Character of Global Value Chains: An Analysis for the European Union." *Papers in Regional Science* 101: 771–794.

Bryson, A., and J. Forth. 2016. "The UK's Productivity Puzzle." In Productivity Puzzles Across Europe, edited by P. Askenazy, L. Bellmann, A. Bryson, and E. Moreno Galbis. Oxford University Press.

Buch-Gómez, E. J., and R. Cabaleiro-Casal. 2020. "Turnout, Political Strength, and Cost Efficiency in Spanish Municipalities of the Autonomous Region of Galicia: Evidence From an Alternative Stochastic Frontier Approach." *Papers in Regional Science* 99: 533–553.

Cardoso, C. M. E. S., and G. Ravishankar. 2015. "Productivity Growth and Convergence: A Stochastic Frontier Analysis." *Journal of Economic Studies* 42: 224–236.

Carrascal-Incera, A., and G. J. D. Hewings. 2024. "Income Interdependence in the UK Multi-Regional Economy: A Meso-Level Analysis." *International Regional Science Review* 47: 591–621.

Carvalho, A. 2018. "Efficiency Spillovers in Bayesian Stochastic Frontier Models: Application to Electricity Distribution in New Zealand." *Spatial Economic Analysis* 13: 171–190.

Centre for Multilevel Modelling 2024 "What are Multilevel Models and Why Should I Use Them?" University of Bristol. Accessed September 3, 2025. https://www.bristol.ac.uk/cmm/learning/multilevel-models/what-why.html#:~:text=In%20a%20fixed%20effects%20model,of%20variable%20can%20be%20estimated.

Chen, Y. Y., P. Schmidt, and H. -J. Wang. 2014. "Consistent Estimation of the Fixed Effects Stochastic Frontier Model." *Journal of Econometrics* 181: 65–76.

Chung, S., and G. J. Hewings. 2015. "Competitive and Complementary Relationship Between Regional Economies: A Study of the Great Lake States." *Spatial Economic Analysis* 10: 205–229.

Corrado, L., and B. Fingleton. 2012. "Where is the Economics in Spatial Econometrics?" *Journal of Regional Science* 52: 210–239.

Coyle, D., J. McHale, I. Bournakis, and J. C. Mei. 2025. "Recent Trends in Firm-Level Total Factor Productivity in the UK: New Measures, New Puzzles." *Economica* 92: 348.

Coyle, D., and J. C. Mei. 2023. "Diagnosing the UK Productivity Slowdown: Which Sectors Matter and Why?" *Economica* 90: 813–850.

De Vries, K., A. Erumban, and B. Van Ark. 2021. "Productivity and the Pandemic: Short-Term Disruptions and Long-Term Implications." *International Economics and Economic Policy* 18: 541–570.

Debarsy, N., C. Ertur, and J. P. LeSage. 2012. "Interpreting Dynamic Space–Time Panel Data Models." *Statistical Methodology* 9: 158–171.

Donald, A. 2025. "Women, Innovation and Productivity." Bennett Institute for Public Policy, Cambridge. https://www.productivity.ac.uk/wp-content/uploads/2025/05.

Douch, M., H. Edwards, and S. Mallick. 2023. "Productivity Debacle in the UK: Do Post-Crisis Firm Cohorts Explain the Performance Puzzle?" *British Journal of Management* 34: 1459–1487.

Duvivier, C. 2013. "Does Urban Proximity Enhance Technical Efficiency? Evidence From Chinese Agriculture." *Journal of Regional Science* 53: 923–943.

Elhorst, J. P. 2014. *Spatial Econometrics—From Cross-Sectional Data to Spatial Panels*. Springer Heidelberg New York Dordrecht London.

Elhorst, J. P. 2009. "Spatial Panel Data Models." In The Handbook of Applied Spatial Analysis, edited by M. M. Fischer and A. Getis, Springer.

Emili, S., and F. Galli. 2025. "Modelling Spatio-Temporal Dynamics in Multi-Output Stochastic Frontiers for the European Agribusiness Industry." *Journal of Agricultural, Biological and Environmental Statistics* 30: 540–575.

Filippini, M., and W. Greene. 2016. "Persistent and Transient Productive Inefficiency: A Maximum Simulated Likelihood Approach." *Journal of Productivity Analysis* 45: 187–196.

Financial Times. 2025. The UK Doesn't Have a Productivity Puzzle. 1 June, 2025. Accessed September 6, 2025. https://www.ft.com/content/583a30e1-f411-40b2-bf60-8102634a6a3c.

Galli, F. 2024. "Accounting for Unobserved Individual Heterogeneity in Spatial Stochastic Frontier Models: The Case of Italian Innovative Start-Ups." *Spatial Economic Analysis* 19: 620–645.

Galli, F. 2023a. "A Spatial Stochastic Frontier Model Introducing Inefficiency Spillovers." *Journal of the Royal Statistical Society Series C: Applied Statistics* 72: 346–367.

Galli, F. 2023b. "A Spatial Stochastic Frontier Model Including Both Frontier and Error-Based Spatial Cross-Sectional Dependence." *Spatial Economic Analysis* 18: 239–258.

Giannakis, E., and A. Bruggeman. 2020. "Regional Disparities in Economic Resilience in the European Union Across the Urban–Rural Divide." *Regional Studies* 54: 1200–1213.

Glass, A. J., and K. Kenjegalieva. 2024. "The Role of Inefficiency in a Productivity Puzzle: Regional Evidence for Great Britain." *Journal of Regional Science* 64: 1287–1322.

Glass, A. J., and K. Kenjegalieva. 2019. "A Spatial Productivity Index in the Presence of Efficiency Spillovers: Evidence for US Banks, 1992–2015." *European Journal of Operational Research* 273: 1165–1179.

Glass, A. J., K. Kenjegalieva, and R. C. Sickles. 2016. "A Spatial Autoregressive Stochastic Frontier Model for Panel Data With Asymmetric Efficiency Spillovers." *Journal of Econometrics* 190: 289–300.

Glass, A. J., K. Kenjegalieva, and M. Douch. 2020. "Uncovering Spatial Productivity Centers Using Asymmetric Bidirectional Spillovers." *European Journal of Operational Research* 285: 767–788.

Glass, A. J., K. Kenjegalieva, and J. Paez-Farrell. 2013. "Productivity Growth Decomposition Using a Spatial Autoregressive Frontier Model." *Economics Letters* 119: 291–295.

Goldin, I., P. Koutroumpis, F. Lafond, and J. Winkler. 2024. "Why is Productivity Slowing Down?" *Journal of Economic Literature* 62: 196–268.

Goodridge, P., and J. Haskel. 2023. "Accounting for the Slowdown in UK Innovation and Productivity." *Economica* 90: 780–812.

Goodridge, P., J. Haskel, and G. Wallis. 2018. "Accounting for the UK Productivity Puzzle: A Decomposition and Predictions." *Economica* 85: 581–605.

Greene, W. H. 2005. "Reconsidering Heterogeneity in Panel Data Estimators of the Stochastic Frontier Model." *Journal of Econometrics* 126: 269–303.

Gude, A., I. Álvarez, and L. Orea. 2018. "Heterogeneous Spillovers Among Spanish Provinces: A Generalized Spatial Stochastic Frontier Model." *Journal of Productivity Analysis* 50: 155–173.

Haldane, A. G. 2020. "Foreword to UK Regional Productivity Differences: An Evidence Review." Industrial Strategy Council, Research Paper, February.

Haldane, A. G. 2018. "The UK's Productivity Problem: Hub no Spokes." Speech Given by Andrew G. Haldane, Chief Economist, Bank of England, Academy of Social Sciences Annual Lecture, London, June 28th.

Harris, R., and J. Moffat. 2017. "The UK Productivity Puzzle, 2008–2012: Evidence Using Plant-Level Estimates of Total Factor Productivity." *Oxford Economic Papers* 69: 529–549.

Huang, B., and R. Zhu. 2020. "Peer Effects of Low-Ability Students in the Classroom: Evidence From China's Middle Schools." *Journal of Population Economics* 33: 1343–1380.

Kao, Y. -H., and A. K. Bera. 2013. *Spatial Regression: The Curious Case of Negative Spatial Dependence*. University of Illinois, Urbana-Champaign, Mimeo.

Kutlu, L. 2018. "Estimating Efficiency in a Spatial Autoregressive Stochastic Frontier Model." *Economics Letters* 163: 155–157.

Kutlu, L., and U. Nair-Reichert. 2019. "Agglomeration Effects and Spatial Spillovers in Efficiency Analysis: A Distribution-Free Methodology." *Regional Studies* 53: 1565–1574.

Lavoratori, K., and D. Castellani. 2021. "Too Close for Comfort? Microgeography of Agglomeration Economies in the United Kingdom." *Journal of Regional Science* 61: 1002–1028.

LeSage, J., and R. K. Pace. 2009. *Introduction to Spatial Econometrics*. CRC Press, Taylor and Francis Group.

Mádr, M. 2025. "Electoral Consequences of Globalization for Social Democratic Parties Across European Regions." *Journal of Regional Science* 65: 43–74.

Martini, G., P. Berta, J. Mullahy, and G. Vittadini. 2014. "The Effectiveness–Efficiency Trade-Off in Health Care: The Case of Hospitals in Lombardy, Italy." *Regional Science and Urban Economics* 49: 217–231.

McCann, P. 2020. "Productivity Perspectives: Observations From the UK and the International Arena." In *Productivity Perspectives*, edited by P. McCann and T. Vorley. Edward Elgar.

Naderi, A. 2022. "Efficiency Measurement of Higher Education Units Using Multilevel Frontier Analysis." *Journal of Productivity Analysis* 57: 79–92.

Neyman, J., and E. L. Scott. 1948. "Consistent Estimates Based on Partially Consistent Observations." *Econometrica* 16: 1–32.

Njuki, E., B. E. Bravo-Ureta, and C. J. O'Donnell. 2019. "Decomposing Agricultural Productivity Growth Using a Random-Parameters Stochastic Production Frontier." *Empirical Economics* 57: 839–860.

O'Donnell, C. J. 2022. "Productivity Change in the Market Sectors of the Australian Economy." Accessed April 9, 2024. https://www.pc.gov.au/__data/assets/pdf_file/0007/338056/sub040-productivity.pdf.

O'Donnell, C. J. 2018. *Productivity and Efficiency Analysis: An Economic Approach to Measuring and Explaining Managerial Performance*. Springer Nature.

O'Donnell, C. J. 2016. "Using Information about Technologies, Markets and Firm Behavior to Decompose a Proper Productivity Index." *Journal of Econometrics* 190: 328–340.

Orea, L. 2002. "Parametric Decomposition of a Generalized Malmquist Productivity Index." *Journal of Productivity Analysis* 18: 5–22.

Patterson, C., A. Sahim, G. Topa, and G. L. Violante. 2016. "Working Hard in the Wrong Place: A Mismatch-Based Explanation to the UK Productivity Puzzle." *European Economic Review* 84: 42–56.

Pessoa, J. P., and J. Van Reenen. 2014. "The UK Productivity and Jobs Puzzle: Does the Answer Lie in Wage Flexibility?" *Economic Journal* 124: 433–452.

Ramajo, J., and G. J. D. Hewings. 2018. "Modelling Regional Productivity Performance Across Western Europe." *Regional Studies* 52: 1372–1387.

Riley, R., A. Rincon-Aznar, and L. Samek. 2018. "Below the Aggregate: A Sectoral Account of the UK Productivity Puzzle." ESCoE Discussion Paper 2018-06.

Rubery, J., I. Bi, and A. Rafferty. 2023. "Gender and Productivity." Working Paper No. 032, The Productivity Institute. https://www.productivity.ac.uk/wp-content/uploads/2023/03/WP032-Gender-and-productivity-FINAL-070323.pdf.

Skevas, I. 2020. "Inference in the Spatial Autoregressive Efficiency Model With an Application to Dutch Dairy Farms." *European Journal of Operational Research* 283: 356–364.

Steele, F., A. Vignoles, and A. Jenkins. 2007. "The Effect of School Resources on Pupil Attainment: A Multilevel Simultaneous Equation Modelling Approach." *Journal of the Royal Statistical Society Series A: Statistics in Society* 170: 801–824.

Taylor, J. 2007. "Estimating the Impact of the Specialist Schools Programme on Secondary School Examination Results in England." *Oxford Bulletin of Economics and Statistics* 69: 445–471.

The Guardian 2022. "Thurrock Becomes the Latest Council to Declare Effective Bankruptcy." 19 December, 2022. Accessed April 9, 2024. https://www.theguardian.com/society/2022/dec/19/thurrock-latest-council-declare-effective-bankruptcy.

Tselios, V., I. Noback, J. van Dijk, and P. McCann. 2015. "Integration of Immigrants, Bridging Social Capital, Ethnicity, and Locality." *Journal of Regional Science* 55: 416–441.

Tsionas, E. G., and P. G. Michaelides. 2016. "A Spatial Stochastic Frontier Model With Spillovers: Evidence for Italian Regions." *Scottish Journal of Political Economy* 63: 243–257.

Tsukamoto, T. 2019. "A Spatial Autoregressive Stochastic Frontier Model for Panel Data Incorporating a Model of Technical Inefficiency." *Japan and the World Economy* 50: 66–77.

Turrell, A., B. Speigner, D. Copple, J. Djumalieva, and J. Thurgood. 2021. "Is the UK's Productivity Puzzle Mostly Driven by Occupational Mismatch? An Analysis Using Big Data on Job Vacancies." *Labour Economics* 71: 102013.

Tveteras, R., and G. E. Battese. 2006. "Agglomeration Externalities, Productivity, and Technical Inefficiency." *Journal of Regional Science* 46: 605–625.

Van Ark, B. 2021. "Putting Together the Pieces of the Productivity Puzzle: Review Article of Productivity Perspectives and Productivity and the Pandemic." *International Productivity Monitor* 40: 118–133.

Van Ark, B., and A. J. Venables. 2020. "A Concerted Effort to Tackle the UK Productivity Puzzle." *International Productivity Monitor* 39: 3–15.

Van Reenen, J., and X. Yang. 2024. "Cracking the Productivity Code: An International Comparison of UK Productivity." *International Productivity Monitor* 46: 60–82.

Wang, H. -J., and C. -W. Ho. 2010. "Estimating Fixed-Effect Panel Stochastic Frontier Models by Model Transformation." *Journal of Econometrics* 157: 286–296.

Williams, S., A. J. Glass, M. Matos, T. Elder, and D. Arnett. 2024. "The UK Productivity Puzzle: A Survey of the Literature and Expert Views." *International Journal of the Economics of Business* 32: 31–65.

Zhang, Z., and J. Ye. 2015. "Decomposition of Environmental Total Factor Productivity Growth Using Hyperbolic Distance Functions: A Panel Data Analysis for China." *Energy Economics* 47: 87–97.

Appendix A

Appendix 1

We estimate Eq. (1) using simulated maximum likelihood (SML). This involves applying the usual type of parameterizations to estimate a single level stochastic frontier using distributional assumptions to level 1 of our model: namely, $\lambda_1 = \sigma_{u_1}/\sigma_{v_1}$ and $\sigma_{u_1} + \sigma_{v_1} = \sigma_{\varepsilon_1}$. Adapting the notation used in Filippini and Greene (2016) for their non-spatial SML procedure to estimate a unit's time-invariant and time-varying inefficiencies, we also parameterize $\varepsilon_{2,gt} = v_{2,gt} - u_{2,gt}$ as follows:

$$\varepsilon_{2,gt} = \sigma_{v_2} V_{2,gt} - \sigma_{u_2}|U_{2,gt}|. \tag{A1}$$

$V_{2,gt}$ and $U_{2,gt}$ are both normally distributed with a zero mean and variance of one, where the parameterization in Eq. (A1) is useful for the simulation of standard normal populations. Similarly, we use the further parameterization $d_{1,ig} = \sigma_{d_1} D_{1,ig}$, where $D_{1,ig}$ is normally distributed with zero mean and variance of one and $\sigma^2_{d_1}$ is the variance of $d_{1,ig}$.

Eq. (A1) allows us to define the following simulated log-likelihood function for Eq. (1).

$$\begin{aligned}\log L = \sum \log \frac{1}{S}\sum_{s=1}^{S} & \left\{\prod_{t=1}^{T}\left[\frac{2}{\sigma_{\varepsilon_1}}\phi\left(\frac{\mathring{\varepsilon}_{M,igt} - \sigma_{d_1} D_{1,igs} - (\sigma_{v_2} V_{2,gts} - \sigma_{u_2}|U_{2,gts}|)}{\sigma_{\varepsilon_1}}\right)\times \Phi\left(\frac{-(\mathring{\varepsilon}_{M,igt} - \sigma_{d_1} D_{1,igs} - (\sigma_{v_2} V_{2,gts} - \sigma_{u_2}|U_{2,gts}|))\lambda_1}{\sigma_{\varepsilon_1}}\right)\right]\right\} \\ & + \log|I_N - \eta W_1 - \delta W_2|,\end{aligned} \tag{A2}$$

where recall from the discussion of Eq. (1), $\mathring{\varepsilon}_{M,igt} = d_{1,ig} + \varepsilon_{2,gt} + \varepsilon_{1,igt} = d_{1,ig} + v_{2,gt} - u_{2,gt} + v_{1,igt} - u_{1,igt}$. As Filippini and Greene (2016) note that the accuracy of their SML estimation procedure increases with T, we add some context to the $T = 17$ we use in our empirical analysis by noting that in Filippini and Greene's application $T = 13$. We account for the endogeneity of the two contemporaneous SAR variables in Eq. (1) and the fact that the composed error $\varepsilon_{1,igt}$ is unobserved, where the corresponding variance of this composed error over the entire sample (ε_1) is σ_{ε_1} (see Eq. (A2)). As is standard, in Eq. (A2) this involves including the logged determinant of the Jacobian of the transformation from ε_1 to the dependent variable (namely, $\log|I_N - \eta W_1 - \delta W_2|$, e.g., Elhorst 2009), where I_N is the $(N \times N)$ identity matrix. In Eq. (A2), $D_{1,igs}$, $V_{2,gts}$ and $U_{2,gts}$ are three simulated draws of pseudo-random numbers from each respective standard normal population, where s denotes a draw and the total number of draws (S) is 2, 000 in the empirical analysis. In the same equation, ϕ is the standard normal density function, Φ is the standard normal cumulative density function and

$$\begin{aligned}\mathring{\varepsilon}_{M,igt} = y_{1,igt} - \alpha - \beta x'_{1,igt} - \gamma x'_{2,gt} - \eta \sum_{j=1}^{N} w_{1,ij}\, y_{1,jgt} \\ - \theta \sum_{j=1}^{N} w_{1,ij}\, y_{1,jgt-1} \\ - \delta \sum_{j=1}^{N} w_{2,ij}\, y_{1,jgt} - \rho \sum_{j=1}^{N} w_{2,ij}\, y_{1,jgt-1} - b_{1,t}.\end{aligned}$$

The estimation of Eq. (A2) involves simulating the latent heterogeneity ($d_{1,ig}$) and unobserved inefficiency components ($u_{1,igt}$ and $u_{2,gt}$) using 2, 000 random draws. Following Filippini and Greene (2016), the asymptotic variance-covariance matrix (AVCM) of the estimated parameters is computed using the outer product of their simulated values (i.e., scores). This approach is robust to the use of simulation and the presence of non-Gaussian errors.

Let $\widehat{\mathbf{b}}$ denote the estimate of the full parameter vector, which comprises estimates of the structural coefficients, $(\alpha, \beta, \gamma, \eta, \theta, \delta, \rho)$, time and group effects, $(b_{1,t}, d_{1,ig})$, and variances of $u_{1,igt}$, $u_{2,gt}$, $v_{1,igt}$ and $v_{2,gt}$. The AVCM is estimated as

$$\widehat{\text{AVCM}}(\widehat{\mathbf{b}}) = \left[\sum_{i=1}^{N}\left(\frac{\partial \log \hat{L}(\widehat{\mathbf{b}})}{\partial \mathbf{b}}\right)\left(\frac{\partial \log \hat{L}(\widehat{\mathbf{b}})}{\partial \mathbf{b}'}\right)\right]^{-1}. \tag{A3}$$

$\log \hat{L}(\widehat{\mathbf{b}})$ is the simulated log-likelihood contribution for a regional industry, averaged over all the simulation draws, $\log \hat{L}(\widehat{\mathbf{b}}) = \frac{1}{S}\sum_{s=1}^{S} \log L_i^{(s)}(\widehat{\mathbf{b}})$. The score vectors for the parameters in Eq. (A3) are computed numerically for each regional industry and incorporate regional industry spillover dependence and the hierarchical inefficiency structure. The aforementioned dependence is due to within region industry spillover dependence (i.e., $W_1 -$ equal weights linking each industry to all the other industries in a region) and between region industry spillover dependence (i.e., $W_2 -$ equal weights linking an industry and the same type of industries in contiguous regions).

The block structure of the AVCM reflects contributions from the industry level error components ($u_{1,igt}$ and $v_{1,igt}$), common regional level error components across the eight industries in a region ($u_{2,gt}$ and $v_{2,gt}$), cross-sectional heterogeneity ($d_{1,ig}$), and the aforementioned regional industry spillover dependencies through W_1 and W_2. The corresponding simulated score function is

$$\frac{\partial \log \hat{L}(\widehat{\mathbf{b}})}{\partial \mathbf{b}} = \frac{1}{\log \hat{L}(\widehat{\mathbf{b}})} \cdot \frac{1}{S}\sum_{s=1}^{S}\frac{\partial \log L_i^{(s)}(\widehat{\mathbf{b}})}{\partial \mathbf{b}}. \tag{A4}$$

Eq. (A4) captures the gradient of the simulated log-likelihood and each simulation draw contributes to the overall derivative. These gradients are not computed analytically, but numerically using finite differences, as closed form expressions for the derivatives of the simulated log-likelihood function are not available. Specifically, each gradient component is approximated by

$$\frac{\partial \log \hat{L}(\widehat{\mathbf{b}})}{\partial \mathbf{b}_k} \approx \frac{\log \hat{L}_i(\mathbf{b} + \Delta_k \mathbf{1}_k) - \log \hat{L}_i(\mathbf{b})}{\Delta_k},$$

where $\mathbf{1}_k$ is the unit vector that corresponds to the k-th parameter and Δ_k is a small finite step size. These simulated and numerically approximated gradients are then used to construct the AVCM. As noted by Aquaro et al. (2021), even under non-Gaussianity, spatial heteroskedasticity, and parameter heterogeneity, and provided regularity conditions are satisfied, consistent standard errors for quasi-maximum likelihood and simulation-based estimators can be obtained from the above formulation.

We next turn to the method to estimate the inefficiency of each level 1 industry and the common impact of the inefficiency of each NUTS3 region in level 2 on the industries it nests. To obtain these estimates we use Eq. (A5), which is based on the moment generating function for the closed skewed normal distribution (e.g., Filippini and Greene 2016). For the multivariate normal integration to compute Eq. (A5), we follow Filippini and Greene (2016) and use the GHK simulator, where below Eqs. (A5) and (A6) we turn to their interpretation.

$$E\left(\exp(\tau_n' \mathbf{u}_{n_{gt}})|\varepsilon^*_{M,gt}\right) = \frac{\overline{\Phi}_n\left(\Gamma\varepsilon^*_{M,gt} + \Omega(\tau_n), \Omega\right)}{\overline{\Phi}_n\left(\Gamma\varepsilon^*_{M,gt}, \Omega\right)}\exp\left[(\tau_n)'\Gamma\varepsilon^*_{M,gt} + \frac{1}{2}(\tau_n)'\Omega(\tau_n)\right], \tag{A5}$$

where

$$\mathbf{u}_{n_{gt}} = \begin{bmatrix} u_2 \\ u_{1,1} \\ \vdots \\ u_{1,8}\end{bmatrix}_{gt} \text{ and } \tau_n = \begin{bmatrix}-1 \\ 0 \\ \vdots \\ 0\end{bmatrix}, \begin{bmatrix}0 \\ -1 \\ \vdots \\ 0\end{bmatrix}, \ldots, \begin{bmatrix}0 \\ 0 \\ \vdots \\ -1\end{bmatrix}. \tag{A6}$$

In Eq. (A5), the eight elements of the column vector of composed errors $\varepsilon^*_{M,gt}$ correspond to the eight industries in region g (agriculture & fishing combined with energy and water, manufacturing, etc., see

Figure 2 for the other six industries). For $\varepsilon^*_{M,gt}$, the subscript M indicates that each of its elements have multilevel components. We also use the superscript * to indicate that each of these composed errors do not have a random effect component. That is, $\overset{*'}{\varepsilon}_{M,gt} = (\varepsilon_{1,1} + \varepsilon_2, \ldots, \varepsilon_{1,8} + \varepsilon_2)_{gt}$, where 1, .., 8 denote the eight different types of industry in each region.

Using the estimated parameters, we obtain the estimate of each of the eight elements of $\varepsilon^*_{M,gt}$ (i.e., $\hat{\varepsilon}^*_{M,igt}$) from Eq. (A7).

$$\hat{\varepsilon}^*_{M,igt} = y_{igt} - \hat{\alpha} - \hat{\beta}\, x'_{1,igt} - \hat{\gamma}\, x'_{2,gt} - \hat{\eta} \sum_{j=1}^{N} w_{1,ij}\, y_{1,jgt} - \hat{\theta} \sum_{j=1}^{N} w_{1,ij}\, y_{1,jgt-1} - \hat{\delta} \sum_{j=1}^{N} w_{2,ij}\, y_{1,jgt} - \hat{\rho} \sum_{j=1}^{N} w_{2,ij}\, y_{1,jgt-1} - \hat{b}_{1,t} - \hat{d}_{1,ig}. \tag{A7}$$

Using a bar to denote means and $\mathbf{L}$ to denote the one period lag operator, we obtain $\hat{d}_{1,ig}$ in Eq. (A7) as follows:

$$\hat{d}_{ig} = \bar{y}_{ig} - \hat{\beta}\, \bar{x}'_{ig} - \hat{\gamma}\, \bar{x}'_{g} - \hat{\eta} \sum_{j=1}^{N} w_{1,ij}\, \bar{y}_{1,jg} - \hat{\theta} \sum_{j=1}^{N} w_{1,ij}\, \overline{\mathbf{L}y}_{1,jg} - \hat{\delta} \sum_{j=1}^{N} w_{2,ij}\, y_{1,jg} - \hat{\rho} \sum_{j=1}^{N} w_{2,ij}\, \overline{\mathbf{L}y}_{1,jg} - \hat{\bar{b}} + \sqrt{\frac{2}{\pi}}\, (\hat{\sigma}_{u_1} + \hat{\sigma}_{u_2}).$$

From the first of the two equations in A6, we can see that the elements of the vector $\mathbf{u}_{n_{gt}}$ in Eq. (A5) are (i) the common impact of the inefficiency of region g in level 2 on the region's industries ($u_{2_{gt}}$); and (ii) the inefficiencies of the region's eight industries in level 1. $\mathbf{u}_{n_{gt}}$ therefore has multilevel elements. Accordingly, the dimension of $\mathbf{u}_{n_{gt}}$ in Eqs. (A5) and (A6), which is denoted by the subscript n, is nine (i.e., the element from (i) above and the eight from (ii)).

Eq. (A5) also includes the matrices Ω and Γ which are computed as follows:

$$\Omega = \left[\begin{bmatrix} \sigma^2_{u_2} & \mathbf{0}'_n \\ \mathbf{0}_n & \sigma^2_{u_1} I_n \end{bmatrix}^{-1} + \Theta' \left[\sigma^2_{v_1} I_n + \sigma^2_{v_2} \mathbf{1}_n \mathbf{1}'_n \right]^{-1} \Theta \right]^{-1}, \tag{A8}$$

$$\Gamma = \Omega \Theta' \left[\sigma^2_{v_1} I_n + \sigma^2_{v_2} \mathbf{1}_n \mathbf{1}'_n \right]^{-1}. \tag{A9}$$

With regard to the notation in Eqs. (A8) and (A9), $\mathbf{0}_n$ and $\mathbf{1}_n$ are n dimensional vectors of zeros and ones, I_n is the $n \times n$ identity matrix, and $\Theta = -[\mathbf{1}_n \mid I_n]$. We can now define in Eq. (A5), $\overline{\Phi}_n(\Gamma \varepsilon^*_{M,gt} + \Omega(\tau_n), \Omega)$ and $\overline{\Phi}_n(\Gamma \varepsilon^*_{M,gt}, \Omega)$, which are the joint probabilities of a n variate normal vector with mean vectors $(\Gamma \varepsilon^*_{M,gt} + \Omega(\tau_n)$ and $\Gamma \varepsilon^*_{M,gt}$ and covariance matrix Ω that belongs to the non-negative orthant.

For the dynamic multilevel spatial TFP growth decompositions, we require noise estimates for the industries in level 1 ($\hat{v}_{1,igt}$) and regions in level 2 ($\hat{v}_{2,gt}$). Using the estimates of $\mathbf{u}_{n_{gt}}$, $\varepsilon^*_{M,igt}$ and $\varepsilon_{2,gt}$, we retrieve $\hat{v}_{1,igt}$ and $\hat{v}_{2,gt}$.

Appendix B

Appendix 2

Other right-hand side components of Eq. 10a

$$IndScale^{Dir}_{1,igt} = \left[\prod_{k,l \in K} \left(\left(\frac{M_{1,kigt}}{M_{1,kigt-1}} \right)^{*^{Dir}_{k\psi}} \left(\frac{M^2_{1,kigt}}{M^2_{1,kigt-1}} \right)^{*^{Dir}_{k\psi}} \left(\frac{M_{1,kigt} M_{1,ligt}}{M_{1,kigt-1} M_{1,ligt-1}} \right)^{*^{Dir}_{kig\psi}} \right) \right] \left(\frac{Y_{1,igt}}{Y_{1,igt-1}} \right),$$

$$RegScale^{Dir}_{2,gt} = \left[\prod_{k,l \in K} \left(\left(\frac{M_{2,kgt}}{M_{2,kgt-1}} \right)^{\zeta^{Dir}_{k\psi}} \left(\frac{M^2_{2,kgt}}{M^2_{2,kgt-1}} \right)^{\xi^{Dir}_{kk\psi}} \left(\frac{M_{2,kgt} M_{2,lgt}}{M_{2,kgt-1} M_{2,lgt-1}} \right)^{\xi^{Dir}_{kl\psi}} \right) \right] \left(\frac{Y_{2,gt}}{Y_{2,gt-1}} \right),$$

$$RegScale^{Ind}_{In,2,gt} = \left[\prod_{k,l \in K} \left(\left(\frac{M_{2,kgt}}{M_{2,kgt-1}} \right)^{\zeta^{Ind}_{In,k\psi}} \left(\frac{M^2_{2,kgt}}{M^2_{2,kgt-1}} \right)^{\xi^{Ind}_{In,kk\psi}} \left(\frac{M_{2,kgt} M_{2,lgt}}{M_{2,kgt-1} M_{2,lgt-1}} \right)^{\xi^{Ind}_{In,kl\psi}} \right) \right] \left(\frac{Y_{2,gt}}{Y_{2,gt-1}} \right).$$

Other right-hand side components of Eq. 11a

$$IndEnvir^{Dir}_{1,igt} = \prod_{a=1}^{A} \left(\frac{Z_{1,aigt}}{Z_{1,aigt-1}} \right)^{\kappa^{Dir}_{a\psi}},$$

$$RegEnvir^{Dir}_{2,gt} = \prod_{c=1}^{C} \left(\frac{R_{2,cgt}}{R_{2,cgt-1}} \right)^{\varphi^{Dir}_{c\psi}},$$

$$RegEnvir^{Ind}_{In,2,gt} = \prod_{c=1}^{C} \left(\frac{R_{2,cgt}}{R_{2,cgt-1}} \right)^{\varphi^{Ind}_{In,c\psi}}.$$

Other right-hand side components of Eq. 12a

$$IndEff^{Dir}_{1,igt} = \frac{\exp\left(-u^{Dir}_{1,igt}\right)}{\exp\left(-u^{Dir}_{1,igt-1}\right)},$$

$$RegEff^{Dir}_{2,gt} = \frac{\exp\left(-u^{Dir}_{2,gt}\right)}{\exp\left(-u^{Dir}_{2,gt-1}\right)},$$

$$RegEff^{Ind}_{In,2,gt} = \frac{\exp\left(-\sum_{h \in G} u^{Ind}_{2,ght}\right)}{\exp\left(-\sum_{h \in G} u^{Ind}_{2,ght-1}\right)}.$$

Other right-hand side components of Eq. 13a

$$IndShock^{Dir}_{1,igt} = \frac{\exp\left(v^{Dir}_{1,igt}\right)}{\exp\left(v^{Dir}_{1,igt-1}\right)},$$

$$RegShock^{Dir}_{2,gt} = \frac{\exp\left(v^{Dir}_{2,gt}\right)}{\exp\left(v^{Dir}_{2,gt-1}\right)},$$

$$RegShock^{Ind}_{In,2,gt} = \frac{\exp\left(\sum_{h \in G} v^{Ind}_{2,ght}\right)}{\exp\left(\sum_{h \in G} v^{Ind}_{2,ght-1}\right)}.$$

Other right-hand side component of Eq. 14a

$$MultilevTC^{Dir}_{M,igt} = \left(\frac{\exp\left(*^{Dir}_{\psi} t\right)}{\exp\left(*^{Dir}_{\psi} (t-1)\right)} \right) \left(\frac{\exp\left(*^{Dir}_{\psi} t^2\right)}{\exp\left(*^{Dir}_{\psi} (t-1)^2\right)} \right) \times \prod_{k=1}^{K} \left(\left(\frac{M_{1,kigt}\, t}{M_{1,kigt-1}\, t - 1} \right)^{*^{Dir}_{k\psi}} \left(\frac{M_{2,kgt}\, t}{M_{2,kgt-1}\, t - 1} \right)^{\varrho^{Dir}_{k\psi}} \right).$$