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The Inaugural Address

Rosanna Keefe

Vague Beliefs and Reasoning

This paper considers the vagueness of belief itself as a category of mental state (rather than beliefs with vague *contents*), arguing that such vague beliefs are pervasive and strikingly diverse, as well as important both philosophically and to our mental lives. It begins by considering some fuzzy doxastic boundaries, and the borderline cases that they generate, as well as some of the reasons this vagueness is significant. It then explores an approach to vague beliefs which shows, I argue, that recognizing vague beliefs can play a key role in understanding how we ought to reason.

I

Introduction. Vague beliefs are pervasive, interestingly diverse and important both philosophically and to our mental lives, or so I will argue. Beliefs with vague *contents* are inevitably thoroughly ubiquitous, but are not the focus here. Instead, we will be exploring beliefs as a vague category of mental state. Subjects can be in states that are borderline beliefs and there are fuzzy boundaries to those states that are beliefs. Correspondingly, the term 'believes' is vague: our belief ascription is a vague matter, with sentences of the form '*S* believes that *p*' sometimes having borderline status.

I will sketch some of the many potential borderline cases and fuzzy boundaries to beliefs in §II. As I acknowledge, whether particular situations or phenomena are taken to involve vague beliefs can turn on the right theory of belief or of other doxastic states with which belief may share a boundary. But I will illustrate the multiplicity of different ways in which belief ascription is vague and some of the reasons this is significant for thinking about, among other things, our mental states, our descriptions of others, our cognitive behaviour and belief acquisition. The prevalence of vague beliefs may seem to have modest, if any, bearing on logic and reasoning, which are typically

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realms in which vagueness is of particular concern. In §III, however, we focus on an application where, I argue, recognizing vague beliefs *is* important to rational inference. While the vagueness of the *contents* of beliefs would typically be expected to bear on matters of logic and inference, I argue that the vagueness of the states themselves also plays a key role in understanding how we ought to reason.

II

Borderline Beliefs and Fuzzy Boundaries. Paradigm vague predicates (like 'bald', 'orange', 'tall') have borderline cases and lack sharp boundaries; 'believes' shares these features. There are clear cases of '*S* believes that *p*' (for example, 'Fran believes London is in England' is clearly true, even, note, when she isn't considering the matter), clear non-cases and borderline cases. Such borderline cases will not typically fall on a fuzzy boundary between believing that *p* and believing that not-*p*, as there can be cases where the subject believes neither *p* nor its negation (perhaps suspending judgement on it): borderline cases of believing *p* can fall between these cases and clear cases of belief.

Though we may have now hoped to demonstrate the existence of borderline beliefs by establishing some examples, the best we can do is describe *putative* examples. It is very common with evidently vague predicates that there are few, if any, cases that are uncontroversially agreed to be borderline cases: we may agree that there are borderline red patches between red and orange ones but disagree on which cases are in that category, with you regarding as non-red those candidate borderline cases I offer. We can, though, offer recipes for generating putative borderline cases to illustrate the potential breadth of the vagueness, seeking, up to a point, to remain neutral about the correct theory of beliefs and theory of vagueness. We start with the most natural source of vague beliefs.

Our beliefs vary in strength. Fran believes both that cats are mammals and that armadillos are mammals, but she believes the former (even) more strongly than the latter. Strengths of belief are often identified with degrees of confidence or credences, where some degrees of confidence correspond to borderline beliefs for which it is not clear whether or not they qualify as beliefs. Fran may be fairly sure, but not certain, that Kate has a sister, generally agreeing that she believes

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this and being typically disposed to act as if it is true, but occasionally being cautious, especially if the stakes are high.[1] Does she believe that Kate has a sister? This could be a borderline case: even with a much fuller description of the case, we might be unclear whether to say that Fran believes *p* or not, and indeed Fran herself might think it is unclear whether or not she counts as believing it.

Some terminology used in capturing the contrast between beliefs and credences can misleadingly encourage us to regard beliefs as non-vague, in particular talk of 'all-or-nothing belief'.[2] For talk of something as 'all-or-nothing' is often employed explicitly to exclude borderline cases. And it does not help to point out the connection between belief and assertion and then claim that assertion is all-or-nothing. For even if this is true of assertion (or at least that any borderline assertions, perhaps mumbling, will not suitably line up with putative borderline beliefs), the *disposition* to assert or to affirm if asked can come in degrees, and that, rather than actual assertion, is what matters for belief.

A relatively natural position on the relation between beliefs and credences is to hold that believing that *p* is having a sufficiently high level of confidence (or strength of belief) in *p*, often presented as a credence in *p* at or above a threshold. What counts as 'sufficiently high' is a vague matter, and the threshold correspondingly vague. This vagueness is crucial for belief ascription practices (explicit and implicit), including self-ascription. A 'credence-first' position regards all-or-nothing beliefs as reducible to beliefs in this way, but the recognition of borderline beliefs in some cases of high but non-maximal confidence levels does not depend on that specific theoretical view on the relation between beliefs and credence.[3] We will return to consider structural features of such thresholds in relation to reasoning in §III.

[1] It is generally agreed that beliefs do not require certainty, though some theorists have defended accounts of belief as certainty. See Keefe (2024) for discussion and rejection of that position.

[2] Other terms include 'qualitative belief' (in contrast with quantitative degrees of belief), 'outright belief', 'flat-out belief', 'plain belief', 'binary belief', 'belief *simpliciter*' and 'categorical state'.

[3] See, for example, Jackson (2020) for a summary of positions on the relation between beliefs and credences, including the alternatives of belief-first and pluralist positions. Each, I would argue, can and must accommodate vague beliefs.

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A sorites-like series can be constructed by imagining a subject who starts off nearly completely confident that *p* and then very gradually loses a tiny degree of confidence each minute and ends up not believing *p*. In the manner of a sorites paradox, it may be compelling to say that if she believes *p* at one moment, then she still believes it the next minute—there is no non-arbitrary point at which a tiny drop in confidence marks a sudden shift from believing that *p* to not believing it. And this need not assume that other factors besides degree of confidence are irrelevant to belief ascriptions, just as a sorites paradox on 'bald' involving the removal of hairs one by one does not need to deny that the arrangement of hair matters to baldness as well as the number of hairs. Now, the chosen response to such a sorites will depend on one's theory of vagueness. But the availability of such a compelling instance of the paradox—and the corresponding apparent lack of sharp boundaries—illustrates the thesis that there are vague beliefs.

The sorites-like structure can indicate a significant role for vague beliefs, either in relation to a resilience of beliefs that a subject is losing confidence in or—running a series in the opposite direction—in the acquisition of beliefs. Take an affirmation case where someone repeatedly affirms 'I can do this' and inculcates in themselves that self-belief through a process in which they gradually gain confidence until it passes into a state of belief. Intermediate, borderline stages can have a vital role in establishing the belief by being, or seeming, close enough to belief to anchor that doxastic state.

Other fuzzy boundaries to beliefs may not be well captured in terms of credences.[4] These could include cases that involve belief acquisition through a gradual process in which a child gets to grips with new concepts and truths involving them. A child may be told that everything is made of atoms, parrot it but not understand this at first,

[4]Consider a case where, over many years, Eric slowly forgets that his neighbour at college was called Guericke, increasingly struggling to recall the fact until eventually he has lost the belief and can no longer recall the name at all. There may be no sharp boundary to the times when he can be truly ascribed that belief without it being the case that the intermediate stages correspond to intermediate levels of confidence. This is one of Schwitzgebel's examples of what he calls 'in-between beliefs'—cases 'in which it is neither quite right to describe a person as having a particular belief nor quite right to describe her as lacking it' (2010, p. 531; see also Schwitzgebel 2001, 2002 and elsewhere). My examples of vague beliefs in this section overlap with Schwitzgebel's illustrations of in-between beliefs, though I disagree with his position on their significance in relation to theories of belief.

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passing through stages where it is a vague matter whether she can be truly ascribed the belief. Such a phenomenon is not a process of gradually increasing confidence which starts from a position of low credence. The vagueness of beliefs is multi-dimensional and cannot be accurately captured using credences alone.

Vague beliefs may similarly be appropriately ascribed to an animal. Barney the cat may count as believing that I have filled his bowl (for example, as an explanation for why he has run straight to the bowl); he will not count as believing propositions with other more complex and less immediate contents; and between these there will be borderline cases of Barney believing that *p*. Some species may have a borderline region involving contents that are either more or less complex than a cat's borderline region. With other, less sophisticated creatures, there may be a question whether beliefs are ascribable at all, and vagueness over whether or not they count as having a belief system.[5]

Other fuzzy boundaries to belief fall in the vicinity of imaginings and other kinds of states that are representational without being beliefs. Although paradigm imaginings and paradigm beliefs play very different roles in our cognitive lives and in producing and explaining our behaviour, the difference between other cases can also be less stark, suggesting a fuzzy boundary between belief and states of imagining. A child's 'belief' in Father Christmas can often hover round the boundary between imaginative play and something she believes. It may also be on a boundary between belief and desire—an instance of wishful thinking, perhaps, and a charming version of a generally epistemically problematic tendency to believe what is desirable with insufficient dispassionate regard for its likely truth. A fuzzy boundary between representing *p* for imaginative purposes and a representation amounting to a belief—taking *p* to be true—could be important for the way that children learn from imaginative play.[6]

[5] Dretske (1988, p. 107) talks of 'a terminological boundary dispute of negligible philosophical interest' in relation to the question of which animals have beliefs. There are clearly empirical questions relevant to these cases, some of which will bear on the question whether the subject can be said to *represent* the content, which, as discussed below, is a central question in relation to representationalist theories of belief. The relation between empirical facts of which we are ignorant and whether certain states count as *beliefs* is beyond the scope of this paper, however.

[6] Schellenberg (2013), for example, argues that there is 'a continuum between imaginings and beliefs', and that this is important for explaining the way we can engage in 'imaginative

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People's attitudes to superstitions could provide further examples. Fran may reject as rationally indefensible the claim that walking under a ladder will bring bad luck and yet sometimes acts in ways the belief would dictate (in the absence of another explanation of that action), such as avoiding walking under the ladder even if it involves a rather inconvenient detour. This mix of stated commitments and behaviour may again indicate a borderline belief.

Similarly, with certain delusions there can be reasons to count the subject as having a certain belief, reasons to deny they are beliefs and an option to treat them as vague, or in-between, beliefs. Take a sufferer of Cotard's syndrome who professes the belief that she is dead. Her declaration of that belief, and some of her related attitudes—that she claims to smell rotting flesh and wants to be in a morgue—seems to support the rather natural characterization of her as having the belief that she is dead. On the other hand, other theorists emphasize the way that she fails to do or infer the things you would expect if she genuinely had that belief, taking this as evidence that a state other than belief is at play.[7] One response to this tension is to categorize them as vague beliefs, perhaps on the boundary between belief and imagination or in that conceptual vicinity.

One's verdict on these and other cases will interact with the theory of belief that one adopts. For a dispositionalist about beliefs, *S* is truly said to believe that *p* if *S* possesses the dispositions characteristic of that belief (see, for example, Schwitzgebel 2002). The characteristic dispositions will typically be a body of interrelated dispositions to behaviour, assertion, other internal states and reasoning, and a subject can believe that *p* without exhibiting all those dispositions. This leaves plenty of space for vague belief by satisfying a proportion of those dispositions or satisfying them to a borderline degree. Indeed,

immersion' and the learning we can do from this activity. See Liao and Doggett (2014) for objections to this position.

[7] See, for example, Currie (2000) for a treatment of delusions in terms of imagination and Borlotti (2010) for a defence of delusions as beliefs, which includes arguing that other non-delusional beliefs can commonly share features of irrationality, lack of responsiveness to evidence, and encapsulation. Egan (2008) accommodates the delusions in question via a category of propositional attitudes—bimagination—with some features of beliefs and some features of imagination. Although these have been presented as a variety of borderline belief, Egan's position that bimaginings are *not* beliefs suggests otherwise. With a vague belief, it is a vague matter *whether* it is a belief, and vague beliefs should not be seen as a new category of state on top of belief, disbelief and suspended judgement.

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Schwitzgebel (2001) draws on 'in-between' beliefs to argue for dispositionalism over representationalism. Representationalism can and must also accommodate vague beliefs, however.

According to the representationalist, believing that *p* essentially involves having a representation with the content *p* stored within the mind. To be a belief, rather than a different representational state, *p* must be in the subject's 'belief box', which is a matter of bearing the appropriate belief-like relation to that representation, understood in terms of the functional role of belief. The image of representations in a belief box is not amenable to admitting borderline cases, since it is hard to make sense of something hovering between being inside and outside a box as a borderline case would seem to need to do.[8] But of course the 'belief box' is just a metaphor; 'belief clouds' may work better for the metaphor. Clouds in the sky are paradigm candidates for items with vague spatial boundaries, and there will be water droplets that aren't clearly part of the cloud or clearly not. What it takes to count as being in *S*'s belief box is a matter of being treated by *S* in the ways characteristic of beliefs. There is, of course, no physical location in the brain (neither box-like nor cloud-like), so whether there is scope for borderline cases will depend on quite what the characterization of the function is. There is no definitive account of the role, but any plausible attempt to specify it itself will involve vague criteria or concepts that admit of borderline cases.[9]

We thus need to acknowledge vague beliefs on any theory of belief, even if the instances of vague beliefs—like the instances of beliefs—will sometimes vary across those theories. The controversial cases

[8] Representationalists and their opponents sometimes implicitly or explicitly do, however, seem committed to a non-vague picture of this kind, for example, in the discussion in Quilty-Dunn and Mandelbaum (2018), where they grant an assumption that 'for representationalism there is always a determinate fact of the matter about whether one stands in the belief relation to a given mental representation' (p. 2356); they question the metaphor on the separate grounds that it implies a single belief box rather than multiple boxes, corresponding to fragmented belief, as touched on below.

[9] For example, in Schwitzgebel's overview (2024, §1.4), the common candidate features are unavoidably peppered with occurrences of 'typically'. They also involve other vague notions that specify the circumstances in which a belief will yield the characteristic result, such as 'Believing that *P*, in conditions favoring sincere expression of that belief, will typically lead to an assertion of *P*' (it can be vague what counts as 'sincere' or what conditions favour such sincerity). Similar qualifications are need in spelling out the way that beliefs are 'caused by perception, serve as the premise of inferences, and interact with desire to cause behavior' (Mandelbaum 2014, p. 58).

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include some important phenomena such as implicit bias. Schwitzgebel offers the example of Juliet, who explicitly endorses the claim that all races are equal but behaves in discriminatory ways suggesting the opposite belief, claiming that she has an in-between or borderline belief in the equality claim (Schwitzgebel 2010). There is dissonance evident in her behaviour, pointing towards having the belief, on the one hand, and not having it, on the other, which could be put down to the state being a *vague* belief. On one opposing approach, Juliet believes both that claim and its negation, albeit in different 'fragments' of her mind which take hold of her behaviour in different contexts.[10] There is still, though, scope for vague beliefs within a fragment (perhaps, for example, a representation in which the subject has, in that fragment, a fairly high credence), and there will be other contexts in which the complexities and slipperiness within cases of implicit biases line up with borderline ascriptions of beliefs.

The vagueness of belief is multi-faceted and widespread, even if the cases appropriately classified as vague beliefs is itself uncertain and vague. In the next section, we look in some more detail at the significance of vague beliefs in relation to a fundamental role for beliefs, namely in reasoning.

III

Vague Thresholds and Reasoning. Inferential reasoning paradigmatically starts from beliefs as premisses, leading to new beliefs or the revision of others. The usual way that vagueness is taken to be significant in relation to practices of reasoning is through the vagueness of the *contents* of the beliefs that constitute our premisses, conclusions, and the steps in between. Different theories of vagueness tackle the logical challenges of vagueness in different ways, yielding

[10] Mandelbaum advocates the latter position in Mandelbaum (2016) and elsewhere. His defence focuses largely on the *propositional* structure of implicit bias—in opposition to associative accounts—a structure which is also amenable to vague beliefs. And though the associative alternative regards the fundamental, explanatory structure as non-propositional (as, rather, an association between *F* and *G*), it can still also matter how we describe the beliefs of a biased person which reflect their problematic associations (for example a putative belief that '*F*s are *G*'). Borderline cases will arise again there, especially as the exact nature and strength of the association between the concepts (or quite what concept is involved) is often hazy.

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different logics of vagueness and corresponding guides to our reasoning. We have put aside the vagueness of the contents of our beliefs for our discussion in this paper, though, and it might be expected that the vagueness of the state of belief—vague beliefs of the kind at issue here—will not be of logical interest. In this section, I argue that vague beliefs are, in fact, relevant to rational inference. While acknowledging the full variety of potential vague beliefs considered in the previous section, it is by focusing on the vagueness arising in relation to degrees of belief or credence that we can use a familiar Bayesian approach to the modelling of relevant doxastic states to explore one way that the vagueness of belief can be significant for issues concerning reasoning.

In §II we considered a constitutive threshold thesis that beliefs are credences at or above a vague threshold. The closely related 'Lockean thesis' states that it is rational to believe p if it is rational to have credence in p at or above a threshold.[11] A commitment to *some* threshold of belief does not typically involve commitment to a *specific* threshold. It is typically assumed to be at least 0.5 but below 1, and is sometimes taken to be context-dependent, so that when stakes are high, a higher threshold is required for rational belief. The threshold is also sometimes acknowledged to be vague, but this is not typically discussed in any detail, and the consequences this vagueness might have for reasoning have been overlooked.[12]

One prominent reason to reject the Lockean thesis has been its incompatibility with the logical closure of rational beliefs, the thesis that if some premisses are among your beliefs, then it is rational to believe any logical consequence of them. The preface paradox provides a striking illustration of an apparent failure of this thesis

[11] Foley (1992) coined the term for this *normative* thesis, though the label is also sometimes used for the distinct constitutive threshold thesis; these theses should be kept apart, though their overall status is beyond the scope of this paper.

[12] Weatherson (2005) argues in favour of a context-dependent threshold, but against its vagueness. See Sturgeon (2020) for some discussion and exploration of the vagueness of the threshold. A different source of vagueness and imprecision that has been more extensively modelled is the phenomenon of 'imprecise credences', which reflect the concerns with an assumption that my credence in p is accurately captured by a unique precise credence (0.87 rather than 0.86). We put these aside for now: subsequent work could consider the interaction between imprecise credences and vague thresholds.

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(Makinson 1965).[13] Take author Ali, who believes each of the statements in her book individually but, as she acknowledges in her preface, resists believing their conjunction—she realizes she will have got *something* wrong—a conclusion that she knows to follow from those premisses. Arguably, she is in a rational overall doxastic state with high credences in each assertion and low credence in their long conjunction. The fact that each of the premisses of the conjunction introduction exceeds the threshold for belief is compatible with the conclusion (their conjunction) falling a long way short of that threshold. Indeed, in this case, she believes the *negation* of that consequence, leaving her with an inconsistent set of beliefs.

Some theorists respond by accepting the failure of the logical closure of beliefs.[14] The preface scenario is rational, and plausibly an instance of a common distribution of doxastic states. I agree that logical closure fails, and here explore the impact of vague beliefs on the matter. This structural modelling is significant even if the Lockean thesis fails for other reasons, such as the existence of certain kinds of cases where it can be rational to refrain from believing something in which a suitably high credence is rational.[15]

The failure of logical closure can seem an unpalatable bullet to bite. Surely following a valid argument should be a fundamental way of expanding our beliefs by adding those propositions that follow from the beliefs we already have? Yet Ali's situation shows that it can be irrational to form beliefs based on inference from our beliefs: in considering the entailed conjunction, it is not the case that she should believe the conclusion, and yet nor should she thereby reject any of her individual beliefs in the premisses. For our purposes here, we are interested in how the *vagueness* of the threshold of rational belief fits into this picture. I argue that attending to vague beliefs is important in understanding and modelling the extent and types of cases in which closure fails. The vagueness of the threshold significantly mitigates the threat to the reasoning we engage in.

[13] See Keefe (2021) for some discussion, from a different angle, of the preface paradox in relation to vagueness.
[14] See, for example, Foley (1992) and Christensen (2004).
[15] See, for example, Buchak's cases involving belief and blame in contexts of statistical evidence (2014). The Lockean thesis may also be challenged via some §II cases of vague beliefs that aren't well modelled in terms of credences.

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With a *sharp* threshold, even simple two-premiss arguments would defy logical closure: cases are not limited to those like the preface paradox with large numbers of premisses. For example, suppose the threshold is 0.9 and rational subject *S* has degrees of belief of 0.94 in each of two probabilistically independent propositions, 'John has a sister' and 'Mark has a sister'. *S*'s rational degree of belief in 'John and Mark both have sisters' will drop to 0.88, which is below the postulated threshold for belief *simpliciter*. In this simple, two-premiss case, then, *S* believes the conjuncts but her rational degree of belief in the conjunction falls short of the level required for belief.

Whatever the (sharp) threshold, there will be clear counterexamples to the logical closure of belief *simpliciter* for a rational subject *S*, where some premisses validly entail a conclusion but *S* believes those premisses and does not believe the conclusion she knows to follow from them. For the conclusion of a valid argument can inherit uncertainty from each premiss, with those uncertainties adding up to yield a conclusion that drops below the threshold. With a threshold of n, if a subject *S* has credence of c ($\geq n$) in each of the two premisses of a rational argument, *S*'s rational credence in the conclusion may drop below n if the gap between 1 and c is greater than half the gap between 1 and n.

To explore the importance of the *vagueness* of the threshold, we need to move away from modelling it with a single point value. Instead, in the first instance, we move to a *range* or interval of points—a threshold range. Beliefs at or above the top of the range are clear or definite cases of belief, and those below the bottom of the range are clearly/definitely not beliefs, while those within the threshold range are borderline cases of beliefs where it is unclear whether or not the subject has sufficient confidence to count as believing.

Through the lens of the supervaluationist theory of vagueness, the threshold range will encompass all those ways of making 'belief' precise by fixing an admissible sharp threshold to the credences. The supervaluationist maintains that a sentence is true if it is true on all admissible precisifications, so '*S* believes that *p*' will be (definitely) true if true on all precisifications of 'belief' (at or above the top of the range), false if false on all precisifications (below the bottom of the range), and neither true nor false otherwise. Borderline beliefs will fall above some admissible thresholds and not above others, so belief ascription in such cases will be neither true nor false.

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With a range rather than a precise threshold point, we may have two premisses with credences above the top of the range (so definitely qualifying as beliefs *simpliciter*) and a conclusion within the borderline region. This will then *not* be a clear case in which logical closure fails. Recognizing this possibility at the least cuts down on clear counterexamples to the logical closure, where it is rational for a subject to definitely believe both premisses but definitely not believe a conclusion she knows to follow from those premisses. We will see that if the threshold range meets certain conditions, clear two-premiss counterexamples of this type will be eliminated entirely.

Now, the conclusion of a valid argument can inherit some uncertainty from each premiss but *no more than that*.[16] Suppose the threshold range was 0.75–0.9, so that credence between 0.75 and 0.9 corresponds to a vague belief. If *S* definitely believed two premisses of a valid argument (so has credence of at least 0.9 in each), then her rational credence in the conclusion may drop to 0.8 (inheriting uncertainty from each premiss), but not below. So her rational credence in something following from two definitely believed premisses cannot drop below the lower threshold, and will be at least a vague belief. If this were the threshold range, there would be no clear counterexamples to logical closure with two premisses.

For other candidate threshold ranges, whether all clear counterexamples are similarly ruled out will depend on the breadth and location of the range. Suppose the top of the range is $1-m$ and the bottom is $1-n$ ($m<n$). If two premisses at or above the upper boundary (that is, definitely believed) are to guarantee that their conclusion is above the lower boundary (so at least a vague belief), the breadth of the boundary range ($n-m$) must be at least m, that is, at least as large as the gap between the upper boundary and 1. For two premisses with credence of $1-m$ could result in a conclusion with credence of $1-2m$, so to avoid dropping below the lower boundary, that lower boundary must be at least m below the upper boundary. For example, if the top of range is 0.95, then the bottom must be at most 0.9 to rule out clear counterexamples. A range such as 0.9–0.95 will be of this type, as will 0.8–0.9, 0.6–0.8 and 0.5–0.75. I use the label 'c_2 ranges' for those that rule out clear counterexamples to logical closure for arguments

[16] Adams (1966) shows that the uncertainty (1 minus its subjective probability) of the conclusion of a valid argument cannot exceed the sum of the uncertainties of the premisses.

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with two premisses (with the subscript indicating the number of premisses involved).

Threshold ranges may vary with context, but it could be that those threshold ranges, corresponding to vague beliefs, are always c_2 ranges. The lower the upper boundary is, the more uncertainty in p there can be while still counting as believing that p, so, informally, the laxer the standards for belief are. And, as we have seen, the lower that upper boundary is, the wider the threshold range needs to be to rule out clear counterexamples. This pattern is to be expected, given the vagueness of belief and patterns of vagueness more generally. It is natural to interpret the loose notion of 'more vague', or 'vaguer', in terms of the width of the threshold range: a wider range amounts to more borderline cases and thus a vaguer notion (in the context). It is appropriate that the threshold range changes with context and the lower the standard, the vaguer the notion of 'belief' in that context. In high-stakes contexts where belief ascription requires near certainty, the idea of 'near certainty', while not precise, encompasses a relatively narrow range of credences. By contrast, a rougher requirement of a 'high' credence will go along with a larger part of the credence scale falling into the range of the borderline. This pattern is shared by other vague terms with upper limits, such as 'flat' and 'bald' (which intuitively have limits at completely flat or zero hairs comparable to the limit of credence 1 for belief): stricter standards will tolerate less vagueness. The further that something can drop below the limit and still classify as flat or bald, the wider will be the range of cases that are borderline. The pattern exemplified by the c_2 ranges—wider ranges for lower upper boundaries—is typical for vague expressions of this type.

Now, we may note that even if clear counterexamples with two premisses are ruled out, the vague boundary still yields arguments that are not clear counterexamples to logical closure but are also not clearly not counter-examples, that is, ones where the rational subject believes two premisses but *merely* has a borderline belief in the conclusion they entail. The difference between clear and borderline counterexamples is pivotal, however. Vaguely believing is sort-of believing, rather than a form of *not* believing. Acknowledging vagueness can, here as in other cases of vagueness, indicate the need for caution: where there is vagueness, a sorites paradox is typically looming. And indeed, piling up enough premisses will still yield clear counterexamples to logical closure in the framework.

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Turning, then, to arguments with more than two premisses, we can ask: how many premisses would be enough to give a clear counterexample to logical closure? Again, that will depend on the breadth and location of the threshold range. In general, to ensure no clear counterexamples with three premisses—a c_3 range—a higher upper boundary or a broader range is needed than that needed for a c_2 range. The range will be at least twice the gap between the upper boundary and 1, for example, 0.7–0.9 or 0.85–0.95 or 0.55–0.85. For four premisses, even wider ranges or higher upper boundaries would be needed, and so on for arguments with more premisses. If we took the lower boundary to be 0.5, clear counterexamples with five premisses would be ruled out with an upper boundary at 0.9, that is, c_5 ranges include those with a lower boundary of 0.5 and an upper boundary greater than or equal to 0.9. Similarly, 0.5 to 0.95 will be a c_{10} range, ruling out clear counterexamples to arguments with ten believed premisses. With i premisses and a lower boundary of 0.5, the upper boundary would need to be at most $0.5/i$ below 1. For if the upper boundary is $1-m$, the conclusion will be at least $(1-i\times m)$ (each of the i premisses induces a maximum drop of m); for this to be >0.5, $i\times m$ must be <0.5, that is, $m<0.5/i$. So, for example, for 20 premisses, $m=0.5/20=0.025$, and the upper boundary would have to be at least 0.975.

As the number of premisses increases, these ranges soon get very wide—amounting to belief being very vague—unless the upper boundary is very close to 1. Even if the restriction to c_2 ranges seemed plausible, 'believes' is surely not as vague as, say, the c_{10} ranges would require.

For an argument with a large number of premisses, then, the logical closure of rational belief will fail regardless of the vagueness of belief. The inference to the conjunction of claims in Ali's book is such a case: this yields a conclusion that she doesn't believe (as she acknowledges in the preface). Such a pattern in which short arguments are fine and long ones are not, resembles a sorites-like situation characteristic of vagueness, where, intuitively or for practical purposes, a few steps down the series are fine but a lot of them are not. In our reasoning we need to be cautious if there are many premisses; but short inferences are cushioned by a guarantee that at the least a borderline belief is rational. If we gradually increase the number of premisses from two, we can expect no precise point at which we reach arguments where there are clear counterexamples. Here, again, there is

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indeterminacy, and we need to pay attention to vague beliefs to understand these patterns.

Now, the above model employed *precise* ranges for the threshold, which is to ignore higher-order vagueness. Surely the upper and lower boundaries to definite beliefs and non-beliefs are themselves vague? Putting that higher-order vagueness aside need not be problematic, however, if the modelling of patterns of beliefs above and within the threshold range does not rely on the artificial precision of the range. Recognizing second-order vagueness is naturally taken to expand the range of cases that are borderline at some order—there are borderline definite cases above the stipulated boundary to the definite cases. Just as recognizing vagueness suggested a move from a point to a range, so the shortcoming of modelling with a unique upper boundary could be tackled by replacing it with a second-order range within which the candidate upper boundaries fall. And so on for higher orders. We have seen that, in general, a wider threshold range rules out more possible clear counterexamples, so cases ruled out from being clear counterexamples because of the vagueness of the threshold will retain that status once second-order vagueness is acknowledged.

A different objection to the modelling here is that in attempting to preserve a neutrality in relation to theories of vagueness, I have masked the fact that claims I have made are not, in fact, true for the supervaluationist. For a supervaluationist, a sentence is true if and only if it is true on all precisifications. Now, take a claim such as 'a rational agent will have at least a *borderline* belief in a conclusion following from two premisses that they believe'. On any precisification, the objection will go, this claim is false. For, on any given precisification—a specific sharp threshold—there are no borderline cases, and the initial two-premiss failure of closure will hold. Since that is the case with any precisification, it is true on all precisifications, and the cushioning drop to mere borderline belief falls out of the picture.

My response to this objection mirrors my response to a parallel objection to supervaluationism more generally. For that framework can appear to generate surprising results when applied to the very sentences naturally used in describing the framework itself. For example, '"tall" has sharp boundaries' may seem to count as true because 'tall' has sharp boundaries on all precisifications. '"Tall" has

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borderline cases' similarly threatens to come out false, because on any given precisification any person will be in either the positive or negative extension of 'tall' rather than classifying as a borderline case. In response to these concerns, the supervaluationist points out that once you talk explicitly of borderline and clear cases and counterexamples, you have stepped out of the regular object language in question and cannot simply assess the truth-value at a precisification purely by reference to the extensions of atomic predicates at that precisification. The truth-value of '"*a* is *F*" is borderline' at a precisification depends not just on whether *a* is *F* at *that* precisification, but whether it is *F* at some precisifications and not others. This can be captured with the 'D' operator, where D*p* is true at a precisification if and only if *p* is true at all precisifications and '*p* is borderline' is expressed as '¬D*p* & ¬D¬*p*', which *is* true at a precisification when different precisifications disagree over whether *p* (see, for example, Keefe 2000, pp. 186–8). Similarly, to formalize the point that two-premiss arguments will be, at worst, *borderline* counterexamples to the logical closure of beliefs, we would need to model the claim with the D operator or in the metalanguage, thereby reflecting the pattern of belief predications across the framework as a whole.

A vague belief thus can, and often will, be the rational response to scenarios involving reasoning from (definite) beliefs. Given certain assumptions, rational agents will always have at least a borderline belief in a conclusion that they know to follow from two premisses that they believe. In considering actual, sometimes irrational, belief and vague belief, patterns of borderline and definite failure of logical closure will diverge from the patterns sketched, whatever the threshold range. But objections and replies surrounding the failure of the logical closure of belief essentially centre on assumptions of rationality: the irrationality of a subject will let in any pattern of beliefs and vague beliefs in the premisses and conclusions of arguments over and above the challenges relating to thresholds on which we have focused. By exploring the role of vague beliefs on the assumption of rationality, we cast light on the kinds of caution we should—and often do—exercise in the face of arguments that employ more than a very small number of premisses we believe.

Similarly, other simplifying structural assumptions would be needed to *guarantee* the cushion of the borderline belief. In fact, though, a subject should not be assumed to have real degrees of confidence meeting the probabilistic Bayesian assumptions, and additionally, the Lockean thesis

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may not hold in full generality if rational beliefs can come apart from credences above a threshold. Indeed, as sketched in §II, there are all sorts of vague beliefs that may not be well captured in terms of credences and their vague thresholds. But the interest in vague beliefs does not rely on those assumptions, and the illustrated patterns of beliefs and vague beliefs may then be exhibited in slightly different ways. Fuzzy boundaries are typically amenable to modelling through a degree-like or quantitative structure, which opens the way for corresponding patterns, such as arguments with beliefs as premisses and mere vague beliefs as conclusions. Such modelling could be employed even when the relation to rationality is messier or when particularly controversial cases of vague beliefs, such as implicit bias, are at issue. Being taken as a premiss in reasoning is often taken as a defining feature of beliefs, so further investigation building on my discussion of the roles that *vague* beliefs have in reasoning is particularly needed for understanding our doxastic states.

IV

Conclusion. In categorizing and understanding our complex array of doxastic states, beliefs are a crucial category, with intimate connections to reasoning, action, assertion, knowledge, and so on. Attention needs to be paid to the *vagueness* of that category, and vague beliefs are, I argued, inescapable and strikingly varied.

Some philosophically and practically significant states fall in (or hover round) the borderline region of belief or turn on the fuzzy and context-dependent boundaries to beliefs. And vague beliefs should be taken into account in grappling with theories of belief and issues concerning belief ascription, belief acquisition, and transitions to and from other kinds of propositional states. In this paper I focused on showing how explicitly modelling *vague* belief can help illuminate the *reasoning* we do and should do. The vagueness of belief should not just be put aside.[17]

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School of History, Philosophy and Digital Humanities
University of Sheffield
9 Mappin Street
Sheffield S1 4DT
UK
r.keefe@sheffield.ac.uk

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