
Published paper

Gunn, H.F., Mackie, P.J., and Ortuzar, J. de D. (1980) Assessing the Value of Time Travel Savings – A Feasibility Study on Humberside. Institute of Transport Studies, University of Leeds, Working Paper 137

Working Paper 137

1980

Assessing the Value of Time Travel Savings – A Feasibility *Study on Humberside*

Gunn, H.F, P.J. Mackie and J.D. Ortuzar

The work reported here was carried out with the support of the E.H. Division of D.Tp.

ABSTRACT

GUNN, H.F., P.J. MACKIE and J.D. ORTUZAR (1980)
Assessing the value of travel time savings - a
feasibility study on Humberside. Leeds: University
of Leeds, Inst. Transp. Stud., WP 137 (unpublished)

It is expected that the opening of the Humber Bridge will cause major changes to travel patterns around Humberside; given the level of tolls as currently stated, many travellers will face decisions involving a trade-off between travel time, money outlay on tolls or fares and money outlay on private vehicle running costs; this either in the context of destination choice, mode choice or route choice.

This report sets out the conclusions of a preliminary study of the feasibility of inferring values of travel time savings from observations made on the outcomes of these decisions. Methods based on aggregate data of destination choice are found to be inefficient; a disaggregate mode choice study is recommended, subject to caveats on sample size.

The work reported here was carried out with the support of the E.H. Division of D.Tp.

W.P. 137 - "Assessing the Value of Travel Time Savings - A Feasibility Study on Humberside".

H. F. Gunn, P. J. Mackie, J. D. Ortuzar.

p. 63 (foot)

" $\rho^2(\alpha_0', \alpha_i') = -0.20$ " should be " $\rho(\alpha_0', \alpha_i') = -0.20$ "

" $\frac{\rho^2(\alpha_0', \alpha_i')}{t_0 t_i}$ " should be " $\frac{2\rho(\alpha_0', \alpha_i')}{t_0 t_i}$ "

p. 64 (top)

" $\rho^2 = 0.20$ " should be " $\rho = -0.20$ "

N.B. The arithmetic stands; these were transcription errors.

H. F. G.
16/3/81.

TABLE OF CONTENTS

1. INTRODUCTION

1.1 The Humber Bridge is a major transport infrastructure investment which is expected to have a significant impact upon the pattern of travel in the Humberside region. Previous studies (Halcrow Fox and Associates, 1977; 1979; Martin and Voorhees Associates, 1979) have been concerned with predicting the levels of traffic using the facility, assessing the economic benefits of the Bridge, and determining which tolls policy should be pursued. With the opening of the Bridge now quite close, the emphasis of any future study must be on understanding the actual consequences rather than on prediction. There are many aspects which could be examined - the levels of traffic and revenue in relation to the predictions, the origins and destinations and journey purposes of traffic in the region, and the impact of the Bridge upon economic development, for example. However, the particular intention of this study is to determine whether the opening of the Bridge provides an opportunity to improve our understanding of travel behaviour. A particular question is whether to use the sub-region as a test-bed from which to draw inferences about the value which transport users place on savings in time spent travelling. The terms of reference for the study are set out in Appendix 1.1.

1.2 The intention of this Introduction is to give a brief overview of the issues involved when attempting to understand travel behaviour, the reasons for doing so, and the possible approaches in our study area. Two of these approaches are given detailed consideration in Chapters 2 and 3. We do not attempt a full scale review of the theory and practice of the valuation of travel time savings. Several such reviews have appeared in the past, the most recent, excellent, example only last year (see, for example, Harrison and Quarmby, 1969; Dalvi and Daly, 1977; Hensher, 1978; and Bruzelius, 1979). We do, however, show in Chapter 3 how the methods of analysis of travel behaviour which we propose to use are consistent with the general theory of consumer behaviour.

1.3 The basic proposition is that travel choices can be explained in terms of various parameters. Some of these are the characteristics of the choices themselves - the relative costs, journey times, comfort and convenience of the alternatives available to the traveller. Others will be the characteristics of the traveller - his income, car ownership, the size of the household from which he comes, his tastes and preferences, and so on. Individuals are characterised as utility maximisers. Each individual, faced with a choice, is held to make his decision according to his perception of the differences between the options in terms of their relevant attributes and the weight which he attaches to each attribute. The aim, then, is to define for the choice-making population as a whole, the relevant attributes of the travel options, and the set of weights placed on the attributes which best reflects the pattern of choices made. This process is the most common form of 'behavioural' modelling.

1.4 Understanding travel behaviour in this way is important for both the connected processes of travel forecasting and economic evaluation. If all the attributes of the transport system which influence travel choices are known and the relative weights placed on the various attributes by travellers are well understood, then it is theoretically possible to forecast the impact upon travel choices of changes in the transport system. Obviously, the extent to which results are transferable from one study area to another depends upon whether the relevant attributes of the system and characteristics of travellers are correctly represented in the model.

1.5 The link between behavioural modelling and economic evaluation is less direct. At the simplest level, once the set of coefficients of the travel attributes which best explains behaviour has been found, the <u>relative</u> weights on the attributes follows directly. If one of the attributes is the price, or cost of travel, then the unit values of the other attributes may be expressed in monetary terms. In a perfect world with adequate consumer perception, no problems of direct or indirect taxes infringing marginal conditions, and an ideal distribution of income, the values found by observing human behaviour could be used directly in economic evaluation. For, if it was found that for example, the mean trade-off rate between time and money which best explained consumer choices in a particular situation was one penny per minute, then that would be the best estimate of the unit social value of creating such a time saving in that situation. Since the ideal conditions mentioned above are not, in practice, satisfied, adjustments are required in order to take account of sub-optimalities (McIntosh and Quarmby, 1970). Furthermore, the mean trade-off rate itself may be expected to vary at any given point in time, depending on the circumstances of the traveller (the time constraints he is under, etc.) and the conditions of travel. Nevertheless, knowledge of the set of values which individuals appear to place on time, operating costs, comfort, and other relevant travel attributes, remains an essential starting point for the process of evaluation.

1.6 The valuation of travel time savings is an extremely significant practical issue for the allocation of resources to and within the transport sector. Some 80% of the measured benefits of trunk road investments come in the form of time savings (Department of the Environment, 1976; Department of Transport, 1978a). Much of the betterment element in railway investment is devoted to time savings, though in this case, improvements in the quality of service are converted into fare revenue. In urban transport studies, time savings remain a significant indicator of improvement to the system. It follows that values of travel time savings are not just a matter of theoretical interest; major questions of resource allocation should be dependent on the values which transport users attach to savings in travel time.

1.7 If the postulated weights or values attached to the various attributes of travel, especially travel time, are important for the reasons stated, we must now address the question - how are such values to be inferred from travellers' behaviour? In principle, values may be inferred from any decisions which involve choices between alternatives with different compositions of time, cost, comfort and other relevant attributes. As long as the traveller is involved in making a choice which requires him to sacrifice something in terms of at least one of the attributes - so-called 'trading' behaviour - useful information may be gleaned from the choice he makes. In Beesley's original study (Beesley, 1965), only two travel attributes, time and cost, were considered, and less than a third of the sample was found to be in a position of trading between time and cost.

1.8 One of the most hotly debated issues in the literature is whether only 'traders' are relevant. It is clear that, in the context of explaining travel behaviour, non-traders are as relevant as traders, since they form part of the sample of travellers whose behaviour is to be explained. The controversy arises in the context of estimating the value of time. On one side of the debate stand those who argue that only those actually involved in trading behaviour provide relevant information (see, for example, Rogers, 1976; Dalvi and Beesley, 1978). On the other, it is argued powerfully that it does not follow from this that those travellers whose chosen option is preferable in all the measured attributes to the alternatives should be deleted:

> "The common observation that individuals exist whose chosen option is worse than the alternative in all measured attributes is irrefutable evidence of the importance of unmeasured attributes. Thus the deletion of non-traders will not only reduce the accuracy of estimates of attribute values by discarding data but also bias those estimates by specifically deleting individuals for whom the unmeasured attributes are particularly important." (Daly, 1978)

1.9 Methods of analysis have become steadily more sophisticated over the last twenty years but the propositions about the nature of travel behaviour as an example of utility maximisation have remained central. Let us take the simplest case in which an individual faces a choice between a fast, expensive method of travel, or a slow, cheap method, which are exactly equal in every other respect. Then, for any one individual,

$$\begin{aligned} t_1 &< t_2 \\ c_1 &> c_2 \end{aligned} \qquad (1.1)$$

If the individual chooses mode 1, then the inference is drawn that the value of the time difference is greater than the value of the cost difference and so a minimum value of travel time consistent with the choice made can be defined. Conversely, for a choice of mode 2, a maximum value of travel time may be inferred. The aim is to find the set of weights placed on time and cost which is most consistent with the observed travel choices (*revealed preferences*). Beesley (1965) used a graphical technique to find the slope of the straight line passing through the origin which minimised the number of misclassified observations (i.e. which minimised the number of cases in which the value of time savings for the sample as a whole was less than an observed minimum value of time for an individual, or greater than an observed maximum). Though beautifully simple, this method has its limitations, notably the difficulty of handling more than two attributes(*), the restrictive nature of the 'straight line through the origin' assumption,

(*) *Although as Daly (1978) has noted, the method is simply a special* case of Manski's score maximisation method (Manski, 1975).

the lack of a statistical criterion of goodness of fit, and more importantly, evidence on the standard errors of the coefficients estimated.

1.10 The choice between travel options can clearly be formulated as a multivariate problem. Thus, for example, the choice of route an individual makes for travel between a given origin and destination could be expressed as a function of a number of independent variables - the time difference between the routes, the cost difference, variations in environmental characteristics, and so on. Why cannot this problem be handled using multiple regression techniques? The difficulty of course is that, where individual decisions are being made the dependent variable is not continuous, but may (for binary choices) take only two values. Either the traveller is observed to travel by route 1 or by route 2. This creates two problems. The first is that of heteroscedasticity; the variance around the Y values is a function of the level of Y (Tobin, 1955) and hence the resulting estimates are inefficient. Secondly, since the regression equation is a linear function of the independent variables, it is possible for certain combinations of values of these variables to produce values outside the range 0-1 for the dependent variable. It is then hard to give a 'probability of choosing mode X' interpretation to the results.

1.11 A solution to this problem is to transform the function from a linear into a sigmoid, S-shaped, curve, for example using either a cumulative normal (probit analysis) or a logistic (logit analysis) transformation(*). In logit analysis the probability of choosing a particular travel option, P_i is expressed as

$$P_i = \frac{e^{f(x_i)}}{1 + e^{f(x_i)}} \qquad (1.2)$$

where $f(x_i)$ is typically of the form

$$f(x_i) = a_o + a_1x_1 + a_2x_2 + \ldots\ldots a_nx_n, \qquad (1.3)$$

And x_i ...... x_n represent the relative attributes of the available choices.

(*) Some studies have used the technique of discriminant analysis, notably (Quarmby, 1967), but this is now felt to be an unsuitable method in this context (Daly, 1978; Stopher and Meyburg, 1976).

Now, for grouped data - say for the purpose of explaining mode choice at the zonal level - the task may be interpreted as explaining the variations between zones in the proportions of travellers choosing each mode. From the above equation it follows that the odds on choosing i

$$\frac{P_i}{1 - P_i} = e^{f(x_i)} \tag{1.4}$$

and the natural log of the odds (the logit)

$$ln\left\{\frac{P_i}{1 - P_i}\right\} = f(x_i) \tag{1.5}$$

Given observations of the proportions of travellers from different zones choosing each mode and the relevant modal attributes, the best fit coefficients for the attributes may then be readily found. (This is known as the Berkson-Theil method, see, McFadden, 1976.)

1.12 However, if individual choices are to be modelled, it is no longer possible to use this method, since it is a (0,1) decision to travel or not by mode i which is observed, rather than the probability of choosing that mode. In these conditions, the method of maximum likelihood is utilised to find the estimates of the coefficients of the attributes $x_1 \ldots x_n$ which maximise the likelihood of the observed choices being what they are. The estimation procedure and allied statistical measures are described in detail in Appendix 3.2.

1.13 Suppose that the attributes of the transport system which are determinants of travel choice can be defined and best fit coefficients estimated. Then, as mentioned above, the ratio of the time and cost coefficients can be interpreted as a mean value of travel time for the sample under consideration (Daly, 1978; Daly and Zachary, 1975). Simple logit models, however, will produce only a single-valued estimate of the value of time (or any other attribute) for each sample or sub-sample of travellers. Intuitively, one would expect that for any set of travellers of given income and other household characteristics, a distribution of time values around a mean would exist. Methods of handling variation in tastes and of estimating the distribution of the value of travel time as well as its mean value are dealt with in

Chapter 3. So too are some properties of logit and probit models not mentioned here.

1.14 Valid inferences can only be drawn from observation of travel behaviour if certain basic conditions are satisfied. Harrison (1974) has set out a number of such conditions:

1. "The choices analysed must be real ones".
   This is crucial, and implies that responses should not be forced into (say) a mode choice framework. If the real choice faced is between shopping by car now, or consolidating the journey with another shopping trip later in the week, drawing inferences from the comparison between car and bus journey times and costs will obviously be invalid. One of the advantages of studying the journey to work is that the range of possible responses is more limited than for other journey purposes.

2. "Where choices exist, they must be fully perceived, and there must be grounds for believing that individuals are aware of the alternatives".

   An area of controversy has been the use of engineering or reported data to model individuals' choices, and we comment on this further in Chapter 3. We hope that the Humber Bridge, with the new public transport arrangements associated with it, will cause people to consider their options, and to be reasonably aware of them.

3. "The effects of all variables thought likely to affect choices must be explicitly considered".

   Thus, such matters as whether the car is to be used during the day, or whether a car is available for the journey, are key elements in determining people's effective choices. Furthermore, socio-economic determinants of choice such as income levels must be handled either by scaling cost elements or by stratifying the data into sub-samples.

4. "There must be perceptible differences between alternatives".

   In a corridor where the average journey length of those sampled will be high, relative to urban commuting studies, this condition may be quite well satisfied.

5. "The variables considered relevant must not be too closely correlated".

This is a critical requirement, and one which frequently hinders study of road user behaviour. However, in the Humberside area, the road network is such that some origins and destinations are linked by high speed roads, while others have a lower grade network. So the relative components of time and cost are different. Furthermore, when the Bridge is open, some origin-destination pairs will be linked at a cost including a toll element, while others will not. Thus, an important precondition for successful value of travel time estimation is better satisfied here than in many other locations, and this gives Humberside considerable prima facie attractiveness as a potential study area.

6. "The variables affecting choice must show a fair amount of variation in the sample. For example, it might seem obvious that a value of time could be estimated from a tolled crossing situation because it presents a simple time/money trade-off. In practice it is rarely possible because in nearly all cases, the crossing offers a single price to all categories of user".

This is a serious problem with route choice studies which might, however, be overcome if individuals with different origins face different time/money trade offs in deciding whether to use the crossing. It was with this in mind that the Department asked us to consider whether a route choice study would be possible, and we discuss this in Appendix 1.2.

7. "The sample under consideration must be assumed similar with respect to factors not included specifically in the analysis".

The only way of validating this assumption would be by means of home interviews to establish whether people's attitudes to the non-measured attributes of the modes or routes under consideration were homogeneous across the population.

8. "The sample analysed must show a reasonable proportion choosing each of the relevant options, otherwise random elements are likely to dominate the analysis".

9. "As a check on validity the number of choices explained by the analysis must be high".

It should be noted that the status of these conditions differs. It is possible to verify whether some of them are satisfied (for example 5,6 above) given knowledge of the network. Others (1,2,8) might be validated by pilot testing. But (9) could only be established after the data was gathered and alternative models calibrated. These conditions do, therefore, have implications for the shape and sequencing of any studies which are carried out.

1.15 Values of time have been inferred from a number of different choice-making situations. These include:

choice of route
choice of mode
choice of destination
choice of home location

Recent surveys of past studies include (Hensher, 1978; Bruzelius, 1979).

1.16 We have not considered the possibility of using this area to study home location choices. We expect the numbers responding to the Bridge by changing the location of residence to be small, in view of the magnitude of the tolls, and the delays which are foreseen in industrial and commercial development of sites on the South Bank of the estuary. Furthermore, there is the serious difficulty of incorporating the environmental and other attributes of a house before any value of time could be inferred.

Modelling individual choices of destination has also been ruled out for similar reasons; determining the relevant choice set for individuals, and attaching values to the attractiveness of alternative locations for shopping or recreational trips poses very difficult problems which we are not confident of resolving.

1.17 However, one of the main points of initial interest in the Humberside Study Area was whether it might be possible to infer values of time from models of trip distribution. That is to say, could one find the relative weights on time, cost and other parameters in the deterrence function which gave the best explanation of the pattern of trip-making between origins and destinations in the study area? This approach avoids the need to describe the attractiveness of individual destinations, since this could be regarded as fixed independently of the travel origins. Some work in this line has been undertaken by the Department of Transport within the context of the RHTM project. Clearly, its philosophy is rather different from the other types of study mentioned, since it considers choice-making behaviour at an aggregated rather than a disaggregate level. This requires certain assumptions to be made about the homogeneity of the trip-making population (see Chapter 2). The attraction of this approach is that in principle, it could be used to shed light on road users' values of time and therefore bears directly on the question of the benefits of road investments which reduce journey time. One critical requirement is that Harrison's condition 5 be met. In the road network in general, this condition is not satisfied; times and costs of attaining alternative destinations are too closely correlated for the relative value of one to the other to be distinguished. As mentioned above, however, in the Humberside area, there is a good network of roads linking certain zones, but a poorer one linking others. In addition there will be a substantial toll element for certain traffics. The possibility of using the study area to infer values of time from the distribution of trips was therefore felt to be a good one. Our investigations into this possibility are reported in Chapter 2.

1.18 Studies of travel mode used for the journey to work have been the most popular single approach to travel choice explanation and value of time derivation. In the Grimsby-Hull corridor, we have found that a wide range of travel modes will be available when the Bridge is opened, and a study of this kind was judged to be worth considering in detail (see Chapter 3).

1.19 The final possibility is for a study of route choice. Route choice is in principle attractive because, again, it could shed light directly on the values of time for road users, and because the non-measured elements in road users costs (relative route attractiveness, etc.) might not be too important in determining choices. In general, though, route choice in the U.K. has not been a profitable area of work, because few or no explicit trade-offs between time and money exist. However, on the face of it, the Humberside area does offer an opportunity for a study of this kind, travellers between South Humberside and the Hull area facing a choice between going over the toll bridge or making the time-consuming journey around the estuary. Therefore, during the course of our work, the Department asked us to examine this opportunity, and our review is shown at Appendix 1.2. The conclusion is that, unfortunately, the number of travellers facing a trade-off between time and cost is likely to be quite restricted and what trading there is will take place at a similar rate. Harrison's condition 6 (see 1.14 above) is therefore unlikely to be satisfied in a route choice study.

1.20 We are therefore left with two remaining possibilities - a study of the distribution of road trips between origins and destinations within the study area, conducted at an aggregated (zonal) level, and a study of individuals' mode choice for the journey to work. These approaches are described and assessed fully in Chapters 2 and 3.

2. ESTIMATING A VALUE OF TIME FROM CONVENTIONAL AGGREGATE MODELS OF TRAVEL DEMAND

By definition, the concept of a value of time (associated with some specific activity) is connected with individual actions and individual decisions. Moreover, it is a concept which has meaning only within the notional framework of utility maximising theories of behaviour in which the individual weighs time against money costs when choosing between travel options.

That the structure of the conventional aggregate gravity model is consistent with 'utility maximising' individual behaviour was demonstrated by several authors in the mid-70's (see, for example, Cochrane, 1975; Domencich and McFadden, 1975; Williams, 1977) as a consequence, it has been noted in the literature that it is possible, in principle, to use such a model to estimate 'values of time'. (Bruzelius, 1979). However, to our knowledge, the only reported analysis of this sort was performed by Economic Highways Division of the U.K. Department of Transport, in the course of the RHTM project. (See Department of Transport, 1978b).

In this chapter, we discuss briefly the several theoretical assumptions that must be made to interpret the gravity model as a behavioural model, and the practical requirements in terms of levels of flows and nature of the network that must be met before reliable estimates of the model co-efficients may be achieved. Appendix 2.1 presents the results of an exercise designed to establish the suitability of the pattern of trip distribution on Humberside after the Bridge has been completed as a basis for drawing inferences about average 'values of time'.

Firstly, to simplify the argument, we shall restrict discussion to circumstances in which it is reasonable to hypothesise that there is a single 'value of time' which applies to all members of our population. In practice, many researchers have demonstrated that it is necessary to allow for a range of such values over the population, where the exact value of time to a given individual may be a function of characteristics of that individual (income level in particular) but may also vary randomly as between individuals of the same characteristics. We shall return to this point at a later stage in the discussion.

The derivation of models of individual choice which are consistent with 'utility maximising' behaviour is discussed in detail in the following chapter. To illustrate the equivalence of the conventional gravity model form with one such individual choice model, we shall use only the result that if options 1 to N are characterised by 'utilities' U_1 to U_N where each U_i is assumed by the analyst to be a random variable drawn from a Weibull distribution with mean $\bar{U}_i$ and variance s^2 (i.e. independent of i) then the probability that U_i will be larger than all other U's is given by

$$\exp(\bar{U}_i) \;/\; \sum_j \exp(\bar{U}_j)) \tag{2.1}$$

Hence if we assume that a rational individual, confronted with options affording him 'utilities' U_1, U_2 ....., U_N will choose that option which has maximum utility and if we can observe only the $\bar{U}_j$ j=1 .. N, (and so do not know which U_j is largest with certainty), and if the nature of our uncertainty about the 'true' values U_j is adequately described by the Weibull distribution, then we can estimate the probability with which option i is chosen as in equation (2.1) above. A fuller description of the process, and an explanation of the suitability of the Weibull distribution in these circumstances, is given by Cochrane (1975) and is generalised by Williams (1977). The resulting model (2.1) is, of course, the well-known logit model. If we write

$$U_{ij} = \bar{U}_j - \lambda s_{ij} + \varepsilon_{ij} = \bar{U}_{ij} + \varepsilon_{ij} \tag{2.2}$$

where U_{ij} represents the net 'utility' of a trip made to zone J, for a traveller starting from zone i; $\bar{U}_j$ (zone attractiveness) and λ are constants, with s_{ij} being a measure of the separation of zones i and j, and ε_{ij} is assumed to be an independent Weibull variable, we have specified a model form to account for the probability that a trip starting in zone i will be made to zone j. The form of the model may be written as

$$p_{ij} = pr(i \rightarrow j) = \exp(\bar{U}_{ij}) \;/ \sum_k \exp(\bar{U}_{ik}) \tag{2.3}$$

or, in more familiar gravity model notation, as

$$p_{ij} = K\, e^{\bar{U}_j}\, e^{-\lambda s_{ij}} = Kq_j\, e^{-\lambda s_{ij}} \tag{2.4}$$

where $K = (\sum_k \exp(\bar{U}_{ik}))^{-1}$, and q_j, are constants.

Given a particular set of trip interchanges, we can fit such a model and then assess the support that the data provides for the hypothesis about the model form. (Note that we cannot make any corresponding inference about the process by which the model form has arisen. As Williams and Ortuzar, 1980, have pointed out, the very same model form is consistent with several entirely different interpretations about the process which has given rise to the observed pattern of flows. Our interpretation of the process as 'behavioural', and 'utility maximising' in particular, must remain an act of faith.) If O_i trips in total start in zone i, then the model predicts

$$t_{ij} = O_i\, K\, q_j \exp(-\lambda s_{ij}) = p_i q_j \exp(-\lambda s_{ij}) \qquad (2.5)$$

trips between zone i and zone j. In the conventional process of fitting gravity models of this form, the end product is usually taken to be the set of best-fitting p, q and λ parameters. U.K. practice for some years has been to form the separation matrix $S = [s_{ij}]$ by taking s_{ij} as a weighted sum of time and out-of-pocket costs. The resulting 'generalised cost' function can be written as

$$s_{ij} = \sum_{k=1}^{K} (\tau_{ij}^{k}\, \theta_k + C_{ij}^{k}) \qquad (2.6)$$

where τ_{ij}^{k} is taken to represent the time component of the i-j trip spent in activity K, and C_{ij}^{k} to represent the out-of-pocket cost of that activity. The weights θ_1m .., θ_K have conventionally been derived from disaggregate studies of travel choice, and represent 'values of time' spent in the appropriate activites, 1, .., k.

Where the trip can be characterised by a single activity, we would have the expression for the separation given by

$$s_{ij} = \tau_{ij}\theta + C_{ij} \qquad (2.7)$$

If we write the gravity model as

$$t_{ij} = p_i q_j \exp(-\lambda_1 \tau_{ij} - \lambda_2 C_{ij}) \qquad (2.8)$$

and choose values for the extended parameter set $\lambda\{p_i\}$, $\{q_j\}$, λ_1, λ_2, which 'best fit' the observed interaction data, the ratio λ_1/λ_2 can be taken as an estimate of the corresponding value of time to the individual trip makers, on our interpretation of the process.

The gravity model seeks to represent long-run, *average* flow patterns by a simple model of the form of equation (2.8); in practice, we shall always expect such a model to be no more than approximate, but to simplify the calculations here we shall ignore this complication, and proceed as if the simple model were indeed 'absolutely correct' - i.e. as if there were some values of the p, q, λ_1 and λ_2 parameters which would reproduce the long-run flows exactly.

If the underlying model is perfectly specified, then with sufficient data (i.e. complete knowledge of long run flows) both λ_1 and λ_2 and thus λ_1/λ_2 could be known without error. In practice, even assuming that the model is absolutely correct, the parameters are estimated from a data base which is subject to sampling errors, and as a consequence the fitted parameter values will also be subject to estimation errors. Thus a third issue arises; not only must we consider whether or not we believe the utility maximising hypothesis underlying the proposed model structure, and then satisfy ourselves that such a structure does fit the aggregate data, but as with any other approach, we must also ensure that the error with which the unknown parameters are estimated is such that the resulting estimate of the value of time is sufficiently accurate for our purposes. The issue of the accuracy of the fitted parameters concerns both the absolute numbers of observed trips in the interaction matrix and the nature of the variation in the explanatory variables in the model.

The *credibility* of the *'utility maximising'* family of models amongst the set of rival explanations of choice behaviour will be discussed in Chapter 3; for the purposes of a preliminary assessment of the practical feasibility of *estimating* a 'value of *time*' *from trip* distribution patterns on Humberside, we shall assume that these models are realistic. We shall further assume that they will adequately explain observed aggregate flows. The attempt to ensure the *validity* of this latter requirement has important implications for design of the survey, and for the level at which aggregation is performed.

Firstly, we are implicitly assuming that all travellers have the same value of time, when there is clear evidence from other studies that income and journey purpose have some influence. By restricting the data to car driver trips, we can hope to reduce the error that income differences will introduce, as compared to, say, a mode-split study. Further, we should certainly aim to treat work trips, and trips on employers' business, separately from all other trips. The fact that each destination zone is to be characterised by a single 'attraction factor' also emphasises the need to treat different purposes in separate models. Another issue for survey design concerns the adequacy of a single time/cost pair to describe zone to zone separation for every traveller in the origin zone. This is an issue that has received a great deal of attention in the literature of disaggregate models; it is well established that, for most models, zonal averaging does not provide sufficiently sensitive estimates of individual level-of-service variables, and, further, that this is a common and serious source of model misspecification (Horowitz, 1980a). However, we can minimise this effect by choosing zones which are relatively remote one from another at the survey design stage. (In passing, it can also be hoped that choosing zones in this way will remove the need to consider re-assigning flows for each different value-of-time that is considered.)

The last issue for design of the survey concerns the need to ensure that observations take place over a fair range of the explanatory variables, and further that these explanatory variables are not too highly correlated to allow the separate effects of each to be distinguished. This last consideration points to the suitability of the Humberside area for a study of this sort; as has already been remarked, the opening of the Humber Bridge and the completion of the motorway network around Humberside will provide a basic structure of fast, high quality road links. Some trips, such as those to and from York and Lincoln, will continue to use at least part of an existing, slower, road network, providing the range of speeds necessary to distinguish between time and cost effects.

Yet another major advantage of conducting an analysis of this sort in the Humberside region is the existence of the toll on the Bridge; as is demonstrated in Appendix 2.1, the fact that a proportion of the town to town movements will use a tolled bridge would allow us to separately identify the component of cost associated with distance travelled. A comparison of the 'behavioural' value of a mile of travel with the estimated true running costs would in itself provide valuable information about the way in which running costs are perceived.

Finally, we turn to the question of the accuracy of the fitted model parameters, and the resulting accuracy of the implied value of time. Since we are making the assumption that our models will indeed fit the data, the errors in the fitted parameters will arise solely as a result of sampling error. By 'fitting the data', here we mean that the true, long-run average flows will be given in a form compatible with the fitted model. Given an estimate of the resulting trip distribution matrix, and any particular sampling strategy, we could form estimates of the accuracy to be expected from a fitted model in at least two different ways;

(a) by Monte Carlo methods, repeatedly simulating the proposed sampling strategy on the anticipated trip distribution matrix, and fitting models to each simulation, or

(b) by calculating the theoretical expressions for the accuracy of a single out-turn of the sampling strategy.

Given that we were restricting attention to a small number of zones, the second approach was the most appealing; for a simulated sample from an expected trip distribution pattern, gravity models were fitted by the GLIM package, yielding estimates of model co-efficients and of the accuracy of these co-efficients. The expected trip distribution pattern was that produced by consultants (Martin and Voorhees Associates, 1979) for the 1981 (post bridge opening) flows. Appendix 2.1 sets out the results of this exercise in some detail.

The conclusions of the analysis are somewhat discouraging; even on the (strong) assumption that the models will fit and that the sampling scheme would consist of single-direction interviewing of some 50% of vehicles crossing each of three cordons (round Hull, Grimsby and Scunthorpe) and crossing the Bridge in either direction, on one day, we would only establish values of time to around ± 100% with 95% confidence, by our estimate.

There is also some indication of particular linkages between towns which might invalidate the form of model being fitted.

The single most important reason for the low accuracy that we anticipate from even such a considerable survey effort is the fact that, in general, traffic flows around the area are expected to be relatively minor. Thus, despite the favourable existence of speed variations in the road network and the 'convenient' (for our purposes) existence of the toll, it will remain difficult to achieve a satisfactory estimate, even of a single value of time for major purpose groups, from anticipated flow patterns in the Humberside area. Repeated surveying on different days might improve the accuracy, at extra cost.

3. THE USE OF INDIVIDUAL CHOICE MODELS

As we mentioned in the Introduction any review of the literature in the field of travel time valuations reveals numerous controversies which relate to such problems as: the theoretical premises underpinning the concept; the formulation of models (of behaviour?) which can be used to measure the value of time; problems with the applications of these models, type of data needed, etc.; and last but not least, problems of implementation, i.e. how to incorporate the values obtained into the planning techniques used in practice (Bruzelius, 1979). The fact that in spite of the many reviews and studies which have been performed the situation, in all the aforementioned respects, is still very much open, can only serve to remind us that as in most cases within the social or behavioural sciences, the context and the data available will usually emerge as the strongest determinants of how to proceed and which methods to apply in any given circumstance.

In this section we will not concern ourselves with these issues directly, perhaps except in the discussion of some particular model forms (e.g. random coefficient models) which have implications in terms of degrees of generality within the sketchy accepted time valuation theory (Daly and Zachary, 1975; Bruzelius, 1979). What we will do is, firstly, briefly describe the usual micro-economic theory within which the concept of "value of time" finds its most natural niche; we will show how different econometric models, ranging from the simple but restricted multinomial logit model, to the powerful and general but computationally embarrassing multinomial probit model, can be generated from assumptions consistent with the theory. We will then make obvious why we prefer to undertake a modal choice study in the context of journey to work trips than, for example, a destination choice study for attempting to compute values of time. Next, we will discuss briefly the implications of using rather general model forms, within the theory, and of using alternative theories altogether.

Before moving to the practical aspects of this study, we also wish to discuss briefly the problems of using 'engineering' or 'reported' data in the estimation of models; the possible effects on modal parameters of including attitudinal and/or not usually measured explanatory variables in the data; and the possible implications of estimating models from panel-data rather than at a single cross-section. We will finally conclude the section with the consideration of practical issues in the proposed study, such as amount and type of data needed, and hence data gathering costs.

3.1 A theoretical view of models

In recent years a considerable advance has been made in the construction of travel demand models from choice-theoretic principles. One particular and convenient framework is that provided by random utility theory (see, for example, Domencich and McFadden, 1975; Williams, 1977). In Appendix 3.1 we present a formal description of the theory and show how to generate within it alternative model structures. Although only its most basic form, the theory has tended to be associated with the concept of 'homo economicus', that is a perfectly rational man, endowed with perfect information who considers all alternatives before taking a decision. As such not only the concept, but the theory itself, has been subject of enormous criticism(*). A brief general statement of the theory is in order:

(i) individuals in a given market segment (same choices and same constraints) are considered to associate with each option a net utility Ui, $i=1,\ldots,N$; and to select that option with the highest value of U(**);

(ii) to account for unobserved factors and interpersonal variation, the modeller considers the variables Ui to be randomly distributed over the population in the market segment;

(iii) therefore, the probability that a particular individual selects a particular option i is simply:

$$Pi = \text{Prob}\left\{ Ui > Uj, \forall j \in N \right\} \qquad (3.1)$$

and a formal choice model may be derived when the density function $f(U_1, \ldots U_N)$ of the utility components is specified.

A convenient way to incorporate the difference between what can be measured (and is therefore observable by the modeller) and the unobservable elements in any choice situation, has been to postulate that each utility component U_i is made up of a 'representative' or 'mean', or 'measurable' part, $\bar{U}_i$, and a stochastic residual, ϵ_i, such that:

$$U_i = \bar{U}_i + \epsilon_i \qquad (3.2)$$

---

* Recently Williams and Ortuzar (1980) have shown that the theory is far less restrictive than most critics consider it, and that some of the main criticisms are testable in a simulation framework.

** Note that in a modal choice for the journey-to-work situation, we can assume fixed destinations for each individual, and therefore the attractiveness or utility of the destinations can be ignored. For this reason, in this case, we actually deal with disutilities or costs of travel which are simply treated as negative values without changing the argument or the methodology.

In the Appendix 3.1 we discuss in some detail the effect that several assumptions concerning the distribution and patterns of association of the residuals, ϵ_i, have in the formation of: logit or probit models, and fixed or random coefficient models; and the characteristics and special features that each of these classes has.

Let us examine closely now the 'measurable' component, $\bar{U}_i$. A typical convenient assumption has been to consider it as 'linear-in-the parameters', that is:

$$\bar{U}_i(\underline{\Theta}, \underline{Z}^i) = \sum_k \Theta_k \cdot Z_k^i \tag{3.3}$$

where: $\underline{\Theta}$ = parameters of the model, to be estimated from observed choices.

Z_k^i = attribute K of alternative i for the individual (e.g. in-vehicle-time). Notice that attributes of the individual can also enter here (e.g. number of cars owned by his household).

This form implies that a linear trade-off mechanism operates between different attributes when making a decision, and has been challenged by many authors as an unreasonable form (Louviere, 1980b; Foerster, 1979). However, there is no doubt that it is the most convenient form and also the most widely used to date. (We will come back to this issue on 3.3). An example of mean (dis)utility of travel of this form is the well known <u>generalised cost</u> formulation, where typically:

$$\bar{U}_i = \Theta_1 t_i + \Theta_2 wt_i + \Theta_3 C_i \tag{3.4}$$

or alternatively,

$$\bar{U}_i = \Theta_3 \left\{ \frac{\Theta_1}{\Theta_3} t_i + \frac{\Theta_2}{\Theta_3} wt_i + C_i \right\} \tag{3.5}$$

where: t_i = in-vehicle-time on alternative i

wt_i = walking-and-waiting time on alternative i

C_i = monetary cost of travel using alternative i

Θ_k, k=1,2,3 = parameters to be estimated

$\frac{\Theta_1}{\Theta_3}$ = value of in-vehicle time

$\frac{\Theta_2}{\Theta_3}$ = value of walking and waiting time

Of course a utility expression can have many more explanatory variables than that in (3.4), although there are limits imposed by current software to this number (20 to 30 is the maximum). In section 3.4 we will comment on recent findings about the effect on the values of θ_1, say, of incorporating to $\bar{U}_i$ attitudinal variables like comfort θ_3 and reliability.

The most widely used individual choice model is the multinomial logit (MNL) model, where

$$P_i = \frac{\exp(\bar{U}_i)}{\sum_j \exp(\bar{U}_j)} \tag{3.6}$$

In Appendix 3.2 we discuss the estimation of this model in some detail. The model (3.6) is endowed with a well-known property of cross-substitution, the 'independence from irrelevant alternatives' (IIA) property, where the ratio

$$\frac{P_i}{P_j} = \frac{\exp(\bar{U}_i)}{\exp(\bar{U}_j)}, \ \forall i, \forall j \in N \tag{3.7}$$

is independent of the utility values associated with other options. This IIA property was once seen as an advantage to be exploited in 'new mode' situations, but now is recognised as a potential hazard when certain alternatives are more 'similar' than others(*). There are some generalizations of the model, within the logit family, and these are discussed in Appendix 3.1. The MNL also assumes that all parameters θ_k in (3.3) do not vary across individuals, i.e. there is no 'taste variation'. The practical implication is that the MNL is only capable of yielding the mean value of the parameters and does not say anything about its distribution (in fact, it <u>assumes</u> explicitly no distribution!). We will look at this issue further below.

The most powerful random utility model is the multinomial probit (MNP) model, which lacks a closed-form expression and which is very difficult to estimate for more than 3 travel options and almost impossible for more than 8 options, even in cases when the full generality of the model is not needed or postulated. We discuss the estimation and solution of MNP models in Appendix 3.3, and the software available at Leeds for handling MNL and MNP models in Appendix 3.4.

(*) An extreme illustrative example is the blue bus/red bus conundrum.

There is a strong belief in the literature (which is intuitively very sound) that, in particular for the value-of-time, it is not adequate to assume that the model parameters will be constant for all individuals, i.e. it is felt that there exists taste variations among individuals. This is consistent with the notion of a distribution rather than a single value of time, and makes compulsory the use of a 'random coefficients' model, instead of a 'fixed coefficients' model like the MNL(*). The simplest such a model is the CRA Hedonics model (Cardell and Reddy, 1977) which can be written as follows:

$$P_i(\underline{\theta}) = \frac{\exp\{\bar{U}_i(\underline{\theta}, \underline{Z}^i)\}}{\sum_j \exp\{\bar{U}_j(\underline{\theta}, \underline{Z}^j)\}} \qquad (3.8)$$

$$P_i = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} P_i(\underline{\theta}) \cdot f(\underline{\theta})\, d\underline{\theta} \qquad (3.9)$$

where: $P_i(\underline{\theta})$ = logit choice probability given the parameter vector $\underline{\theta}$
$f(\underline{\theta})$ = probability density function of the parameters of the individual utility functions.

Expression (3.9) is evaluated through Monte-Carlo methods by simply specifying a distribution function for the parameter vector $\underline{\theta}$. The approach is computationally and conceptually simple, although ... "it is somewhat time consuming", (Cardell and Reddy, 1977). The MNP also permits variations in tastes across individuals, but it is considerably more general than the CRA Hedonics, because it does not constrain the stochastic residuals in (3.2) to be as in the MNL specification (see Appendix 3.1), but permits them to be completely general, albeit with a multivariate normal joint distribution. Other

(*) It is important to mention that in his analysis of mis-specification errors, Horowitz (1980a) found that errors due to the existence of taste variation were only second in severity to errors in measurement and were far more important than structural errors, or not inclusion of important explanatory variables.

differences, advantages and disadvantages of the models are discussed at length by Cardell and Reddy (1977).

3.2 Mode choice and destination choice modelling using disaggregate techniques

Two disaggregate modelling approaches have been suggested for the Humber Bridge Value of Time study:

- modal choice only, for home-based journey to work trips
- choice of mode and destination for other home-based journey purposes (e.g. shopping).

Although the theory sketched in the previous section is rather well established, it is by no means the only one that has been under discussion. In fact, the concept of a perfectly informed, utility maximizing, rational man ('homo economicus') is not very easy to swallow. Recent work aimed at discovering if it is possible to discriminate, at the cross-section, between say an MNL and models derived from alternative theories of behaviour, have reported negatively and suggested that random utility models can be considerably robust for short-term applications (Williams and Ortuzar, 1980). It has however been found that although it is not, again, possible to discriminate amongst alternative specifications, the generation of choice sets and in general the problem of incomplete information (which is consistent with relaxing the assumption that every individual faces <u>exactly</u> the same choices, has the same constraints and knows all about each alternative), can produce significant bias in the estimates of model coefficients (Williams and Ortuzar, 1980). Hensher (1979) has noted that the models do not have the facility to determine the alternative decision structures and the options in each individual's choice set, but rather they can only take into account and test the alternative assumptions imposed by the modeller.

> ...."Choice set determination and the degree of independence of various decisions is the most difficult of all the issues to resolve. It reflects the complexity of behaviour and the dilemma which a modeller has to tackle in arriving at a suitable trade-off between modelling relevance and modelling complexity. Usually, however, data availability acts as the yardstick" (Hensher, 1979).

It is extremely difficult to decide on an individual's choice set unless one is prepared to ask him; therefore this problem has something to do with the well-known dilemma of using reported or measured data, on the attributes of the alternatives, for building the models. We will discuss this issue at some length in 3.4.

In the case of mode choice modelling, fortunately, the number of alternatives is generally small and therefore this problem should not be too critical. Also there is a broad agreement among experts that in this case the linear-in-parameters form of the 'representative' utility (3.3) should present few difficulties. It remains only to sort out two final obstacles: what model structure will be used (i.e. probit or logit), and given the structure, what variables will enter the utility functions and in what form. This is especially relevant for the case of variables describing the individual (e.g. socioeconomic variables). Until the mid 1970's, the most common approach was, to add these variables as additional linear terms. This is consistent with the hypothesis that the trade-off mechanism involving say, time and cost, is the same for all individuals. Two alternative approaches allow different trade-off functions for groups of people with different characteristics. The first, which is fully consistent with the requirement of observing groups of individuals with the same choices and constraints, is to stratify the sample on the basis of the individual characteristics and calibrate a model for each market segment. In this way, the coefficients of time and cost are allowed to vary for the different market segments thus resulting in possibly different trade-off mechanisms(*). The problem is, as usual, one of data; the larger the number of market segments the smaller the number of observations on each for a given sample size(**). The second one, which can be used in conjunction with the first, is to express the coefficient of the time or cost variable(s) as a function of an individual descriptor, usually income (see the discussion by Train and McFadden, 1978). In value-of-time terms, this would for example result in time valued as a percentage of the wage rate (McFadden, 1976).

The question of model structure can only be resolved by examining the particular situation under study. The variables entering the model and their form, the form of the utility functions themselves, etc., are all matters for testing (see, Leamer, 1978); again, it is quite often a question of data availability. Linear-in-the-parameters (logit and

(*) This is not to be confused with the issue of random vs. fixed coefficients models.

(**) A tremendous effort has been spent, in the field of individual choice models, in devising more efficient sampling and data collection methods. Choice-based samples are considered much more efficient than the traditional uniform sampling strategies. The state-of-the-art in this context has been summarized brilliantly by Lerman and Manski (1979).

simple probit) models can easily be estimated using available software (Ben-Akiva, 1973; Howe and Liou, 1975; Daganzo and Schoenfeld, 1978), whilst other more general forms present enormous difficulties (see Appendix 3.3. Good and well documented examples of the former are provided by Ben-Akiva and Atherton (1977); Hensher (1979); and Talvitie and Kirschner (1978).

In the case of destination choice modelling, the problems become much more complicated. Firstly, the identification of alternatives in the choice set is a much more crucial matter, and this is not simply because the total number of possibilities is usually very high(*). For example, consider the case of modelling the behaviour of a group comprised of individuals who vary a great deal in terms of their knowledge of potential destinations (owing to varying lengths of residency in the area), or when there exist some alternatives which completely dominate others in terms of their qualities. Because of this model coefficients, which attempt to describe the relationship between predicted utilities and observed choices, may be influenced as much by variation in choice sets among individuals (which are not fully accounted for in traditional models) as by variations in preferences (which are accounted for). Because changes in the nature of destinations may affect both choice set and preferences to different degrees, this confusion is likely to play havoc with the possibility of using the models in forecasting or in the transference of results over space (see, for example, Ben-Akiva, 1980; Louviere, 1980a).

Fortunately, McFadden (1978) has shown that for an MNL, the model parameters can be estimated without bias by sampling alternatives at random from the full set of alternatives, with appropriate adjustment in the estimation mechanism. This is not possible however for the MNP model, precisely because of its improved nature that allows for interaction between all alternatives. Another important drawback, in the context of destination choice modelling, is that almost all experts agree that the assumptions of linear-in-the-parameters utility functions is not a valid one in this case (Daly, 1980; Louviere and Meyer, 1979). The problem here is that there is no available software (anywhere to the best of our knowledge, and certainly not in Leeds) for estimating MNL or MNP models with non-linear utility functions (the problem is specifically that for non-linear utility expressions there is no guarantee that the likelihood function has a unique optimum). Finally, even if we were to

(*) See the discussion on 3.3, with respect to elimination-by-aspects models, in this context.

use a linear-in-the parameters model, another big challenge remains in this case and that is how to measure or represent the attractiveness of alternative destinations. If individuals are trading-off increased time and/or cost against the higher attractiveness of a more distant destination, there is a need to measure the relative attractiveness of destinations in order to determine the rate of trade-off taking place. As far as we are aware there are no satisfactory answers to this problem. In a mode choice to work context, in contrast, this important issue is not a problem since, as we mentioned before, it can plausibly be assumed that each <u>fixed</u> destinationexerts the same attraction to all competing modes and does not, therefore, influence choice.

3.3 <u>Extensions to the theory and alternative behavioural frameworks</u>

In the conventional models discussed so far, each individual confronted by a choice is considered to have the same <u>deterministic</u> choice set available. As we commented in 3.2 it is increasingly recognised that in location choice contexts individuals do in fact act under a restricted knowledge of the alternatives and their attributes (see, for example, Williams and Thrift, 1980; Richardson, 1978; Kirby, 1979; and the references cited therein). Models which explicitly recognise these aspects of choice have tended to emphasize the search process (Richardson, 1980), in conjunction with aspiration levels and satisficing behaviour. We will not discuss this problem further but refer the reader to the papers by Weibull (1978) and Williams and Ortuzar (1980).

Another issue relates to the assumption of linear-in-parameters functional forms ('mean utility') in the models. We also mentioned in 3.2 that it has been strongly argued (Louviere and Meyer, 1979) that other forms (e.g. multiplicative) maybe more adequate. Three general approaches have been proposed to deal with this problem: the use of functional measurement conjoint analysis techniques with experimental design data (Lerman and Louviere, 1978; Hensher and Louviere, 1979; Louviere and Meyer, 1979); the use of 'form searches' by means of statistical transformations (e.g. the Box-Cox method) as in the work of Gaudry and Wills (1977) and more recently Dagenais, Gaudry and Liem (1980); and finally through the constructive use of the economic theory itself for the derivation of form (Train and McFadden, 1978; Hensher and Johnson, 1980). We are not going to explore the issue further except to mention that non-linear utility forms, not only imply trade-off mechanisms different from those usually associated with a concept like 'value-of-time', but also that model elasticities and forecasting power have been shown to vary dramatically with functional form, and hence the issue has important implications for model design and hypothesis testing.

The limitations of 'simple scaleable choice models', typified by the MNL function have been one of the prime motivations behind the construction of alternative models of the decision process. The development of more general structures, as outlined before, which exhibit more realistic cross-substitution properties than the MNL has removed some of the original justifications for building alternative decision models. Of course, this does not mean that current models are therefore and necessarily appropriate. As we will argue below, it is also desirable always to examine competing frameworks in order to get insights which would not have been obtained had any single framework been used (Koppelman and Hauser, 1978).

The general problem of a decision model can best be seen by reference to the solution of a multicriterion problem (Williams and Ortuzar, 1980). An individual contemplating a decision is considered to have a set of goals or objectives and a set of constraints. How does he resolve this problem? For example, he might be interested in finding an option, out of N, which simultaneously minimises travel time, minimises cost, maximises comfort, safety, etc. These attributes might, in addition, be required to satisfy 'absolute constraints', such as

"the trip cannot cost more than 3 pence/km"

in general this sort of constraint can be formally represented as:

$$\underline{g}(\underline{Z}) \leq \underline{b} \tag{3.10}$$

If a single alternative is found which simultaneously optimises all the functions (e.g. time, cost, comfort, etc) and whose attributes are feasible in terms of (3.10) then an unambiguous optimum is obtained. The norm, however, is to have conflicting objectives, that is options which are better in some respects and worse in others, and this of course ...."gives the multicriterion problem its flavour" (Williams and Ortuzar, 1980). Several researchers have discussed these issues (Eilon, 1972; Foerster, 1979) and the debate is an old one in cost-benefit analysis. In the linear-in-parameters 'compensatory models', by definition, high levels of satisfaction with one attribute can compensate for low levels of satisfaction with others, as in the case of the generalised cost formulation. Alternative approaches involve the conversion of some or all of the objectives to constraints or thresholds. A satisficing model will be generated by considering these thresholds and by establishing a structured search for the desired alternative in conjunction with an elimination strategy. The best known such model is the elimination-by-aspects (EBA) decision model proposed by Tversky (1972), which has been recently implemented by several researchers (Makowsky et al. 1977; Gensch and Svestka, 1978; Recker and Golob, 1979; Young and Richardson, 1980). The interest in these models would be purely academic except for the fact that their consideration may result in policy directions not suggested by traditional model forms. As Golob and Richardson (1980) have remarked

> ..."If a non-compensatory choice process is assumed to exist, then .... in order to have the most effective use of resources ... these should be used to improve the attributes of the system which are presently not satisfactory ... because the improvement of attributes which are already satisfactory will ... have no effect on the overall choice. This is at variance with ... compensatory choice models which would suggest that resources should be directed at the most important attribute".

They have gone further to point out that,

> ..."If a satisficing search process is assumed to exist, then ... in order to force a decision-maker to consider new alternatives, it is necessary to make the existing choice worse... This necessitates the use of disincentives ... as well as incentives ... (i.e. a stick as well as a carrot). This is in contradiction to existing models which suggest that the determinant of change ... is simply a variation in the difference in utility (no matter how it is achieved)" (Golob and Richardson, 1980).

It is clear then that these notions have important implications for value-of-time studies. On the other hand it is probably safe to assume that EBA-like decision mechanisms are more likely to assert themselves in destination choice, rather than mode choice contexts, due to the increased number of alternatives in the former. In this sense perhaps it is also important to mention that Young and Richardson (1980) concluded, in a destination choice study, that

> ...."the EBA model parameters appear to be slightly more stable than those obtained from a comparable logit model..... . Importantly, the measures of elasticity derived from each model appear to be different, with the logit elasticities being consistently higher than the EBA model elasticities."

3.4 Representation and measurement of travel choice attributes

The discussion so far, although cast in rather general terms, has implicitly assumed that models are estimated on the basis of revealed preferences observed at a single cross-section; this is overwhelmingly the most popular approach encountered in the literature. Firstly, let us mention that this assumption is not necessary, the discussion being general enough to cover other methods of obtaining data. We wish, however, to discuss briefly the implications for parameter estimates (and hence value-of-time) of several 'unconventional' measurement techniques and philosophies. We refer the reader once more to excellent discussions by Daly (1978) and Bruzelius (1979).

The problems involved in obtaining measures of the explanatory variables (e.g. cost and time requirements by alternative modes) are shown schematically in Figure 3.1. Ideally we would like to have the information on these variables as perceived by the consumer when taking his decision, this being especially true if we are not interested in forecasting (i.e. how do you get 'perceived' information about a future situation?). Our current understanding of the mechanisms by which 'perceived', 'reported' and 'measured' values are related is very limited (in fact the figure may well be the state-of-the-art). We are therefore made to choose between reported and measured (or 'engineering' or 'synthesized') data, and while models estimated on each type of data may prove reasonable in themselves,

> ... "it is very difficult to postulate relationships that will allow models calibrated on reported data to be applied to synthesised data or vice versa" (Daly, 1978).

Probably the safest way out is to try and collect information on both reported and engineering values, and make comparisons in order to gain insight from the two approaches. However, this is, of course, more costly and time consuming.

An old issue in the use of choice models to estimate values of time is the trader/non-trader question. As Daly (1978) has clearly pointed out, there is not, in fact, a problem!; all observations should be used(*). The main difficulty has actually been based on a misunderstanding, in the sense that only the observable, and hence measured (or measurable)

(*) Notice that this has nothing to do with the issue of captive travellers who should indeed be trimmed out of the sample (if identified!).

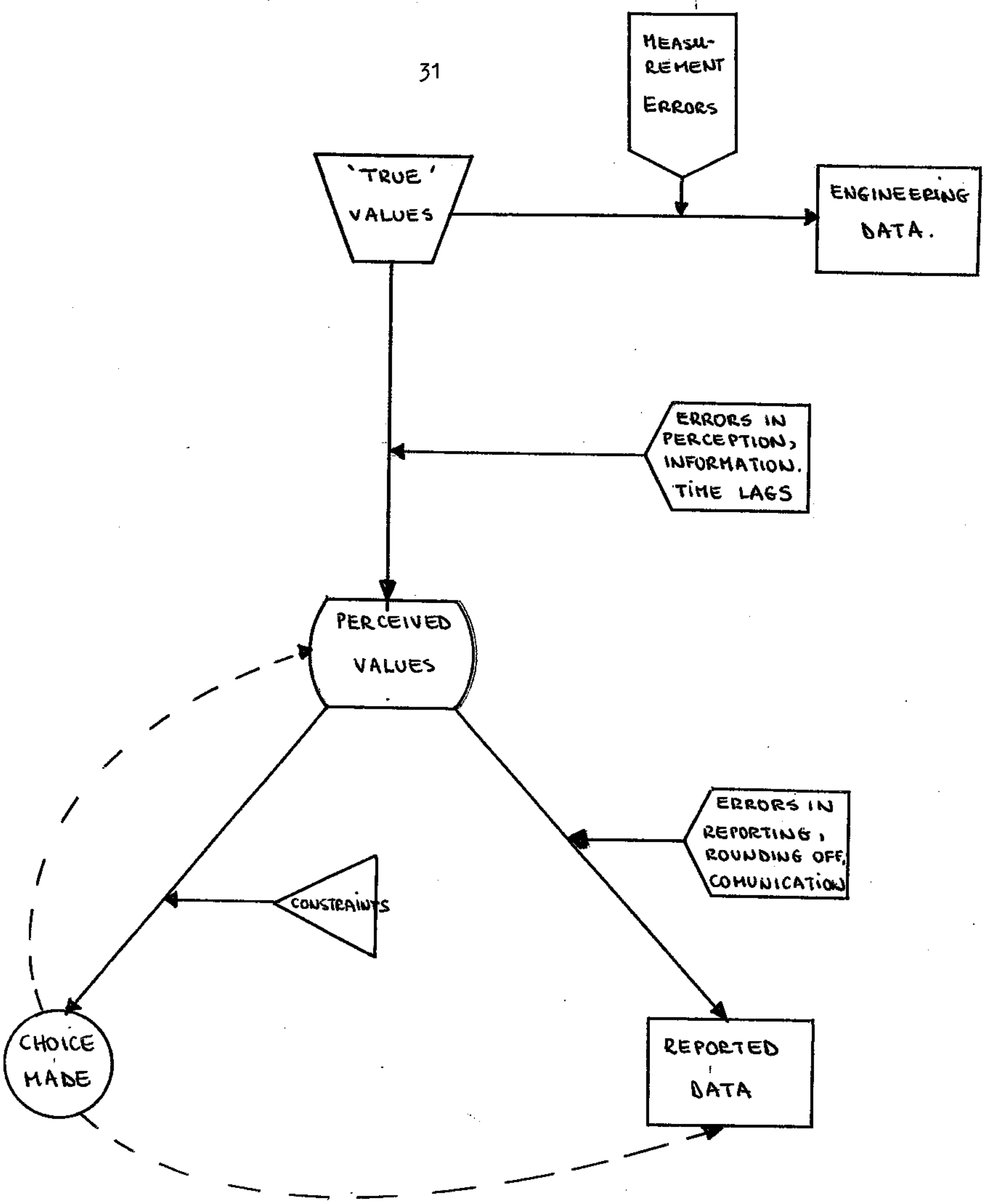


FIGURE 3.1: NOTIONAL RELATIONSHIP BETWEEN CHOICE AND DIFFERENT ATTRIBUTE MEASUREMENTS.

(Source: marriage of diagrams in Daly, 1978; and Bruzelius, 1980) attributes had been considered when defining whether a person is or is not a trader, leaving out the crucial unobservables and/or unmeasured characteristics. The larger the number of measured attributes incorporated in the model, the smaller will be the number of apparent non-traders and, better still, the lesser the influence of the unobserved factors (simply because more of those are incorporated). This brings us naturally into the question of the use of attitudinal variables to improve our models. Again, this is an area which has received much more attention than we could possibly attempt to do here. We refer the reader to papers by Dix (1980) and Hartgen (1979) which adequately discuss the state-of-the-art. In relation to the influence of attitudinal measures in the value of other parameters, there is conflicting evidence in the literature. McFadden et al (1979) concluded that choice was explained, to a great extent, by the typical level-of-service variables used in traditional studies, and that attitudinal measures did add very little explanation. (It is fair to say though that the models discussed by McFadden et al have been heavily criticised by, for example, Talvitie and Kirschner, 1978; in that the mode-specific constants tended to account for over 60% of the explanatory power of the models!). More recently, Prashker (1979) has found that including measures of reliability (e.g. reliability of finding a parking space), substantially increased the explanatory power of the model (i.e. produced insignificant mode-specific constants) and changed significantly the values of some parameters, in particular the value of in-vehicle time. Again here, probably the safest recommendation is to examine the possibility of measuring some unconventional factors, as exemplified in the literature and test for their effects in the parameter estimates, model explanatory power, etc. The trade-off is once more naturally against higher data collection and analysis costs.

We wish finally to mention briefly very recent evidence (Johnson and Hensher, 1980) that parameters estimated from 'panel-data' (i.e. information on choices for a given population at two or more points in time) may well be very (up to ten times in preliminary results) different from parameters estimated from a single cross-section. Among other reasons quoted, it appears that the time series data would enable the existence of habit in the population confronted by choice to be taken into account (Goodwin, 1977; Blase, 1979). Whether we accept that habit should influence (and indeed lower) the parameter estimates and hence probably value-of-time estimates, or whether we want to find out values 'in the absolute', is a matter of policy. The question, however, is unfortunately a serious one.

It is interesting to note that the Humber Bridge context offers a unique opportunity to collect data on those that cross to date, as a panel, and follow them through their new choices when the Bridge opens. Interestingly enough in this case the existence of habit should not be an important factor (after all that is precisely the main raison-d'être for choosing this particular circumstance for a value-of-time study, i.e. individuals would actually do a reappraisal of their choice patterns due to a dramatic change in their choice sets!): however, the 'before-and-after' data would be extremely useful in learning about response. Models estimated on data ex-ante could be tested with predictions post-hoc. Model parameters and value-of-time estimates could be checked for consistency over time, etc. Finally, and as we will argue in the next section, if anything we can treat the pre-Bridge survey/exercise as a pilot study which would be extremely useful in improving our chances of conducting a more successful study after the Bridge opens.

3.5 Practical considerations

Fairly early on, in this preliminary appraisal, it was considered that the corridor between Grimsby/Cleethorpes and Hull appeared as a natural candidate for conducting the study. This view, which has been confirmed by the preceding discussions, is based on the following reasons:

(i) The corridor is extremely appropriate in that even now there are a substantial number of journey-to-work trips made which cross the river in both directions.

(ii) The characteristics of the area, encourage a strong competition between alternative modes/combinations of modes nowadays. As we will comment below this trend can only be reinforced when the bridge opens.

(iii) A corridor, by definition, is a study area where the rather crucial assumption of the need for 'a group of individuals with similar choices and constraints', in the generation of our models is reasonably satisfied (or at least, it has a better chance of being satisfied than in an area-wide context).(*)

Before considering the problems of data collection method, questionnaire design, etc., which will constitute the core of this section, we believe

(*) As we will discuss later, we may need to go to an area-wide study after all due to lack of enough data in the corridor.

it is important to stress some of the major transport related facts observable to date in the area and what are reasonable expectations for the post-bridge situation.

An informal fact-finding expedition by a team of researchers of the Institute for Transport Studies, which comprised a one-day visit to the area, observing several ferry trips, the physical characteristics of the public transport and road networks, etc, suggested the following:

(i) The present number of morning peak-period ferry crossings is approximately 350 persons in each direction(*) Of these some 90% are journey-to-work commuters.

(ii) The majority of the morning peak-period travellers (some 95%) seem to be lower income people. This, in fact, constituted a surprise, we were expecting to find a large proportion of executive/managers, high income travellers. Some of the return travellers (e.g. from Hull to Barton) are night shift workers at Hull returning to residences in the South shore.

(iii) The present ferry charges, for a trip that lasts only 15-20 minutes, are fairly substantial (e.g. 60p/person; 2.50£/car; 4.0£/van). In the case of cars crossing, where it was suggested to us that drivers needed to be on the pier at least 15 minutes in advance of the trip to ensure a place, it would appear that the opening of the bridge will be a real blessing. It seems certain that these car drivers are, and will continue to be, captive to their cars (for whatever reason) and therefore will not give any information on trade-offs.

(iv) A fair amount of park-and-sail goes on (we observed some 60 cars in the car park next to the pier), which would suggest the possibility of park-and-ride in the future. The parking charge was quite high (60p/day), although it seems it is charged on a rather informal basis.

(v) The present range of modes/combinations of modes used is the following:

* car-sail with car - car (<u>most</u> probably captives)
* car - park - ferry - other (park-and-sail)
* train (park-and-ride?) - ferry - other
  kiss-and-ride?

(*) This figure is somewhat higher than the value suggested by the Humberside Ferry Service Passenger Survey.

* bus-ferry-other (?)

As it can be seen, several of these modes are almost certainly correlated. As we will discuss further below this would introduce the need to use more general model structures than the simple multinomial logit model (e.g. a nested logit structure), even if we do not allow for taste variation; this has consequences in terms of more difficult estimation and more data requirements.

(vi) It has been estimated that the number of commuter trips by car between Grimsby and Hull will be of the order of 60 in each direction after the Bridge opens (i.e. in 1981). This figure does not take into account trips from other parts of the corridor and thus would appear a reasonable if perhaps optimistic estimate, on the light of the present number of crossings. We have no estimates of non-car trips.

(vii) Although the ferry service is going to be discontinued when the Bridge opens, it appears quite possible that the rail link between Grimsby/Cleethorpes and the river side will continue operating slightly streamlined abandoning the station at New Holland pier and providing a better link to Barton where a car park/bus link to Hull will be provided. For this reason it appears reasonable to assume that the range of modes after the Bridge opens will be the following:

* car all-the-way (driver and/or passenger and/or car pool?).
* car-park-bus (P & R).
* bus-bus (?).
* train-bus (feeder train!) (P & R?, K & R?).

Again, it is easy to see that several of these modes will be, most probably, strongly correlated, raising once more the question of appropriate model structure.

We will now briefly analyze the general implications of this information, before considering the important and difficult questions of data collection methodologies and needs.

The characteristics of the present crossing behaviour would lead us to believe that, if anything, the opening of the bridge should encourage more trips. Also, because of the expected magnitude of the toll, it would appear quite clear that a range of modes/combinations, will be in operation with the consequent possibility of detecting trade-offs. However, the fact that the modes are and will be almost certainly correlated (thus violating the crucial assumptions needed in the generation of the most simple model forms), should rule out the possibility of using the simple multinomial logit model (MNL). This is because the correlation among alternatives would imply that the model would yield biased parameters (and hence biased estimates of the value-of-time). One, not so complex, alternative is to use the hierarchical or nested logit model (Williams, 1977 ; Daly and Zachary, 1978) discussed in Appendix 3.1. However, as discussed in Appendix 3.2, there are some problems associated with the model in that current estimation methods calibrate the model in a heuristic fashion (e.g. first the lower nests, then calculation of composite utilities, and then higher nests, etc.) and this is known, in some cases, to produce also biased estimates.(*) Further, usually more data is needed, and there are problems interpreting the exact meaning of parameters which have different values in different nests. Again, we can always consider the possibility of using a probit model, which has the advantage of allowing us to test for the existence of taste variations (i.e. distributed values of time). However, as we mention in Appendix 3.3, the estimation problems in this case are much more serious than for the logit models. We will now move on to the equally difficult question of data collection methodology.

We mentioned in 3.4 that McFadden <u>et al</u> (1979), in probably the most comprehensive study of individual choice models to date, concluded that mode choice was mostly explained (if we do not consider the mode-specific constants) by the typical level-of-service variables of conventional models. More to the point, the variables that they found important, which has been confirmed by several other studies,(**) were:

(*) However, we may obtain in the near future a recently developed estimation method which solves this problem (Berkman, Brownstone, <u>et.al</u>., (1979).

(**) Although Talvitie and Kirshner(1978) claim that there is a built-in 'wisdom' inside the profession in reporting only 'successful' modelling, in the sense of being not inconsistent with previous efforts.

- in-vehicle-time (1)
- walking time (2)
- waiting time (3)
- cost/wage rate or cost/income (4)
  (cost being separated sometimes into 'out-of-pocket' costs, e.g. fares, parking charges; and 'running costs', e.g. car operating costs).
- car competition = No. of cars/No. of licensed drivers (5)

of less importance they found sex, age, the number of residents in the house, and the characteristics of the destination of the trip (CBD or non-CBD) among others. Another crucial matter was to try and discover captivity and/or availability of alternative modes.

The <u>return</u> journey from work to home (with possible diversions) being probably as important as the home to work journey in the determination of mode choice, we recommend that the choice context for the model be that of mode (or modes) of travel for the home work <u>tour</u>, with corresponding implications for the explanatory variables.

Basically, we have then, between 5 and 10 explanatory variables (without counting mode-specific constants) we would consider a priori. Of course, how many will actually appear in any model will be a matter of search and trial-and-error. On the other hand it has been mentioned that a good rule-of-thumb is that one needs approximately 30 observations per parameter. If we were to consider a simple MNL model then, we would need at least between 150 and 300 observations for the simple case of generic variables (*). For mode-specific values of the parameters the number of observations required increase linearly. If we were to consider more complex models (as it appears we should) the problem is a lot more serious. The point is that quite rapidly we may find ourselves in a situation where the number of available data points (the whole of them, not just a sample!) is in our case not enough to estimate what we want. If this is so we should have to consider:

(i) Taking into account travellers from other parts of the general study area, thus increasing the data measurement costs.

(ii) Travellers in the corridor not necessarily crossing the bridge with the problem that there is no guarantee that they have <u>recently</u> revised their preferences, as is the case with the bridge users.

(*) As would be the case if we do not distinguish between in-vehicle time in bus and car.

It is a safe assumption that we shall require a relatively high response rate, given the likely number of travellers. Bearing this in mind, we suggest the following approach:

(i) To ensure data on trips crossing the bridge, it is clear we have to identify those who actually cross, that is we are restricting ourselves necessarily to a mail-back questionnaire distributed at the toll-booth and/or the bus.

(ii) It is well-known that response to this type of questionnaire is low, so we are proposing that the introductory letter mentions that answers will participate in a lottery with a substantial prize (e.g. £50).

(iii) A mail-back questionnaire must be short and easily understandable.

(iv) The questionnaire will include a question which asks respondents whether they would be willing to take part in a further (home) interview.

Examples of the types of questionnaires previously used for studies of this kind are shown in Appendix 3.5.

Given the doubts expressed above, we think it is a sensible strategy to obtain further information before the final decision to proceed with the study is made. One relevant piece of information will be the volume of traffic on the Bridge in the early months of operation. Secondly, it would be useful to know more than we do about the characteristics of commuters in the corridor.

One way of achieving this which, we believe has merit, is to carry out a survey of ferry users. Such a survey would fulfil a number of purposes:

(i) It would give us an up-to-date idea of the size of the existing (pre-Bridge) market for commuter travel.

(ii) It would provide an indication of the likely response rate from a reply-paid questionnaire of the general type which would be used for the survey proper. The sensitivity of the response rate to the inclusion of certain questions (eg. income) could be tested.

(iii) Since ferry users currently face a choice of mode for the access journey to/from the ferry, the data would provide an opportunity for the study team to use and become familiar with the software.

(iv) More speculatively, the survey could be used to generate a cohort or panel of commuters for whom the impact of the opening of the Bridge or their travel and activity patterns could be monitored. Opportunities to understand the impact of major changes in the transport system as individuals occur only infrequently, and we think that although the results will inevitably be qualitative in character, they could throw light on such issues as the nature and significance of the constraints in people's time budgets.

The final merit of the Ferry survey is that it would be a relatively low cost way of proceeding. As well as providing useful information, it would enable the risks of proceeding with the main survey with an unacceptably low total market, or a market which is reluctant to respond, to be cut down. As such, we think it is a sensible way forward.

CHAPTER 4. CONCLUSIONS AND RECOMMENDATIONS

Our general conclusions are as follows:-

(i) We are unable to recommend further study of drivers' values of time on the basis of their choice of route. The numbers trading off time against cost are small and most trading will take place at a single rate. Thus one of the essential preconditions for a successful value of time study is not satisfied.

(ii) We are also unable to recommend a study of drivers' values of time based on the distribution of trips between origins and destinations (the aggregate approach) at least as a free-standing exercise.

(iii) We do think that the conditions for a successful study of individual travel behaviour in the context of mode choice for the home-work tour may be met, and we recommend that further work, including pilot studies, be carried out in order to verify this.

The Aggregate Approach

At the outset of the study it was expected that a key issue would be how well the various modelling exercises which have been carried out in connection with the opening of the Bridge performed. That is, how well did the distribution models reproduce the observed data on flows between origins and destinations. To this end, the Study Team familiarised themselves with the various Consultants' Reports. With the agreement of the Department of Transport no additional review of the transport models used to predict traffic volumes in the Study area is presented here, the topic having been thoroughly covered in the Report 'Review of Traffic and Tolls on the Humber Bridge', (Martin and Voorhees Associates,1979).

It was also discovered during the course of the project that the initial intention of using 1976 raw trip data to test the feasibility of the approach could not be carried through. The major difficulties were the low traffic volumes between the relevant origins and destinations and the problem of collinearity between times and costs, given the state of the network in 1976. In view of this, and also as a consequence of the practical difficulties in obtaining clean data, no effort was devoted to consideration of the base year flows.

The main thrust of the investigation into the feasibility of using aggregate data to estimate time values centred around the most recent modelled output by the Consultants of the post Bridge traffic flow pattern. The conclusion was (see Appendix 2.1) that even sampling 50% of the traffic travelling between the major towns in the area using the recommended sampling scheme, the mean value of time could only be predicted with 95% statistical confidence to within $\pm$ 100%.

Accordingly, unless this study were to take place as a by-product of an area-wide transportation study with other purposes, such as establishing the pattern of origins and destinations on Humberside, or monitoring the impact of the Bridge, we do not think there is a case for proceeding with the 'aggregate' study.

The Disaggregate Approach

A number of paradigms of individual choice behaviour have been developed in the literature. Of these 'utility maximisation' is the most commonly postulated and leads to the most practically tractable models. Within this framework, choice of route, choice of mode and choice of destination may all be studied given suitable conditions. Of these, we judge that a study of modal choice for the home-work tour is most likely to be fruitful in our study area.

When the Bridge is opened, a good range of alternative modes will be available for commuters and it is reasonable to suppose that regular travellers will have a good knowledge of the characteristics of the alternatives, and will have reviewed their travel choices. The issues which are in doubt are how large the commuter travel market across the Bridge will be, how satisfactory a response rate can be achieved from different types of questionnaires or interviewing techniques, and how adequately choices can be represented in a behavioural model.

We recommend that the question of sample size requirements receive detailed attention prior to any Study, and that an approach, similar to that outlined above in the context of aggregate data, be taken to establish the relationship between sample design and size, and the resulting accuracy of estimated parameters, for disaggregate data.

Pages 42 and 43 contain detail of staffing arrangements and have been removed from this version.

REFERENCES

ALBRIGHT, R.L., S.R. LERMAN and C.F. MANSKI (1977) Report on the development of an estimation program for the multinomial probit model. Prepared for the Federal Highway Administration, U.S. Department of Transportation, by Cambridge Systematics Inc.

BAKER, K. (1978) Calibration and modification of an urban transport model. B.Sc. Thesis. Department of Theoretical Mechanics, University of Nottingham. (unpublished)

BEESLEY, M.E. (1965) The value of time spent in travelling: Some new evidence, Economica, 32 (12b).

BEN-AKIVA, M.E. (1973) Program for maximum likelihood estimation of the multinomial logit model. Working paper, Department of Civil Engineering, M.I.T.

BEN-AKIVA, M.E. (1974) Structure of passenger travel demand models. Transportation Research Record, 526, 26-42.

BEN-AKIVA, M. (1980) Issues in transferring and updating travel behaviour models. In Stopher, P.R., A.H. Meyburg and W. Brog (Eds.) New Horizons in Travel Behaviour Research. Lexington Books, Lexington, Massachusetts.

BEN-AKIVA, M. and T.J. ATHERTON (1977) Methodology for short-range travel demand predictions: analysis of carpooling incentives, Journal of Transport Economics and Policy, 11(3), 224-61.

BERKMAN,J., D. BROWNSTONE and Associates (1979) QUAIL 4.0 User's Manual, Department of Economics, University of California at Berkeley.

BLASE, J.H. (1979) Hysteresis and catastrophe theory: a demonstration of habit and threshold effects in travel behaviour. PTRC Proceedings, Summer Annual Meeting, University of Warwick.

BOUTHELIER, F. (1978) An efficient methodology to estimate and predict with multinomial probit models: applications to transportation problems. Ph.D. Thesis. Department of Ocean Engineering, M.I.T.

BRUZELIUS, N. (1980) The Value of Travel Time: Theory and Measurement, Croom Helm, London.

CARDELL, N.S. and B.J. REDDY (1977) A multinomial logit model which permits variation in tastes across individuals. Charles River Associates Inc. (unpublished)

COCHRANE, B.A. (1975) A possible economic basis for the gravity model. Journal of Transport Economics and Policy, 9, 34-49.

DAGANZO, C.F. (1979) Multinomial Probit - The Theory and its Applications to Demand Forecasting, Academic Press Inc., New York.

DAGANZO, C.F., F. BOUTHELIER and Y. SHEFFI (1977) Multinomial probit and qualitative choice: a computationally efficient algorithm. Transportation Science, 11, 338-58.

DAGANZO, C.F. and L. SCHOENFELD (1978) CHOMP's user manual, Research Report UCB-ITS-RR-78-7, I.T.S., University of California at Berkeley.

DAGENAIS, M.G., M.J.I. GAUDRY and T.C. LIEM (1980) Multiple regression analysis with Box-Cox transformations and non-spherical residual errors: a transportation application. Publication No. 166, Centre de Recherche sur les Transport, Universite de Montreal.

DALVI, M.Q. and BEESLEY, M.E. (1978) Concluding comments. In Hensher, D.A. and M.Q. Dalvi (Eds.) Determinants of Travel Choice, Saxon House, Sussex.

DALVI, M.Q. and DALY, A.J. (1977) Valuation of Travelling Time: Theory and Estimation, LGORU Report T72, Reading.

DALY, A.J. (1978) Issues in the estimation of journey attribute values. In Hensher, D.A. and M.Q. Dalvi (Eds.) Determinants of Travel Choice, Saxon House, Sussex.

DALY, A.J. (1980) Some issues in the implementation of advanced travel demand models. In Stopher, P.R., A.H. Meyburg and W. Brog (Eds.) New Horizons in Travel Behaviour Research. Lexington Books, Lexington, Massachusetts.

DALY, A.J. and ZACHARY, S. (1975) Commuters Values of Time, LGORU Report T55, Reading.

DALY, A.J. and ZACHARY, S. (1978) Improved multiple choice models, In D.A. Hensher and M.Q. Dalvi (Eds.) Determinants of Travel Choice, Saxon House, Sussex.

DEPARTMENT OF THE ENVIRONMENT (1976) Transport Policy: A Consultation Document, Vol. 2, Para. 5.13, H.M.S.O., London

DEPARTMENT OF TRANSPORT (1978a) Report on the Advisory Committee on Trunk Road Assessment, Para. 4.17, H.M.S.O., London.

DEPARTMENT OF TRANSPORT (1978b) Regional Highway Traffic Model: Project Review, Roads and Local Transport Directorate.

DIX, M.C. (1980) Structuring our understanding of travel choices - the use of psychometric and social research techniques, In Stopher, P.R., A.H. Meyburg and W. Brog (Eds.) New Horizons in Travel Behaviour Research. Lexington Books, Lexington, Massachusetts.

DOMENCICH, T.A. and McFADDEN, D. (1975) Urban Travel Demand: a Behavioural Analysis, North Holland, Amsterdam.

EILON, S. (1972) Goals and constraints in decision-making. Operational Research Quarterly, 23(1). 3-15.

FOERSTER, J.F. (1979) Mode choice decision process models: a comparison of compensatory and non-compensatory structures. Transportation Research, 13A, 1, 17-28.

GAUDRY, M.J.I. and WILLS, M. (1977) Estimating the functional form of travel demand models. Publication 63, Centre de Recherche sur les transports, Universite de Montreal.

GENSCH, D.H. and SVESTKA, J.A. (1978) An exact hierarchical algorithm for determining aggregate statistics from individual choice date. Working paper, University of Wisconsin at Milwaukee.

GOLOB, T.F. and RICHARDSON, A.J. (1979) Non-compensatory and discontinuous constructs in travel behaviour models. In Stopher, P.R., A.H. Meyburg and W. Brog (Eds.) New Horizons in Travel Behaviour Research, Lexington Books, Lexington, Massachusetts.

GOODWIN, P.B. (1977) Habit and hysteresis in mode choice. Urban Studies, 14, 95-8.

GREEN, P.E. and CARROLL, J.D. (1976) Mathematical Tools for Applied Multivariate Analysis, Academic Press Inc., London.

GUNN, H.F. and BATES, J.J. (1980) Some statistical considerations in fitting travel demand models, International Conference on Research and Applications of Disaggregate Travel Demand Models, University of Leeds, 14-16 July, 1980.

HALCROW FOX AND ASSOCIATES (1977) Humber Bridge Toll Study, Report to the Humber Bridge Board, London.

HALCROW FOX AND ASSOCIATES (1979) Humber Bridge Economic Evaluation, Report to the Humber Bridge Board, London.

HARRISON, A.J. (1974) Economics and Transport Appraisal, Croom Helm, London.

HARRISON, A.J. and QUARMBY, D.A. (1969) The value of time in transport planning: A review, 6th Round table on Transport Economics, E.C.M.T., Paris, November, 1969.

HARRISON, I.B. (1977) A model of choice with distributed costs, Ph.D. Thesis, Department of Civil Engineering, University of Nottingham, (unpublished).

HARRISON, I.B. and CULLINGFORD, G. (1978) Modal choice with distributed costs, PTRC Summer Annual Meeting, University of Warwick.

HARTGEN, D.T. (1979) Behavioural models in transportation: perspective problems and prospects, prepared for presentation at the Transport Studies Unit, Oxford University, July 1979.

HAUSER, J.R. (1978) Testing the accuracy, usefulness, and significance of probabilistic choice models: an information theoretic approach, Operations Research 26(3), pp 406-421.

HAUSMAN, J.A. and WISE, D.A. (1978) A conditional probit model for qualitative choice: discrete decisions recognising inter-dependence and heterogeneous preferences. Econometrica, 46, 403-26.

HENSHER, D.A. (1978) Valuation in journey attributes: Some existing empirical evidence. In Hensher, D.A. and M.Q. Dalvi (Eds.) Determinants of Travel Choice, Saxon House, Sussex.

HENSHER, D.A. (1979) Individual choice modelling with discrete commodities: theory and application to the Tasman Bridge reopening, The Economic Record, Sept. 1979, pp 243-260.

HENSHER, D.A. and JOHNSON, L.W. (1980) Behavioural response and functional form, North Ryde: MacQuarie University, School of Economics and Financial Studies.

HENSHER, D.A. and LOUVIERE, J.J. (1979) Behavioural intentions as predictors of very specific behaviour, Transportation, 8(2), 167-82.

HOROWITZ, J. (1980a) Sources of error and uncertainty in behavioural travel demand models, In Stopher, P.R., A.H. Meyburg and W. Brog (Eds.) New Horizons in Travel Behaviour Research, Lexington Books, Lexington, Massachusetts.

HOROWITZ, J. (1980b) Specification tests for probabilistic choice models, International Conference on Research and Applications of Disaggregate Travel Demand Models, University of Leeds, 14-16 July 1980.

HOWE, S.M. and P.S. LIOU (1975) Documentation of PROLO and MLOGIT, two new calibration programs for building disaggregate choice models, Preliminary Research Report 98, Planning Research Unit, New York State Department of Transportation.

JANSEN, G.R.M., P.H.L. BOVY, J.P.J. van EST and F. le CLERQ (Eds.)(1979) New Developments in Modelling Travel Demand and Urban Systems, Saxon House, Sussex.

JOHNSON, L.W. and HENSHER, D.A. (1980) Application of multinomial probit to a two-period panel data set, International Conference on Research and Applications of Disaggregate Travel Demand Models, University of Leeds, 14-16 July 1980.

KENDALL, M.G. and STUART, A. (1969) The Advanced Theory of Statistics, Charles Griffin and Co. Ltd., London.

KIRBY, H.R. (1979) Is distance no object? Location behaviour and the journey to work. Presented to the 11th Annual Conference, University Transport Study Group, University of Southampton. (Unpublished).

KIRBY, H.R. and LEESE, M.N. (1978) Trip distribution calculations and sampling error: Some theoretical aspects, UTSG 10th Annual Conference, City University, London, January 1978.

KOPPELMAN, F.S. and HAUSER, J.R. (1978) Destination choice behaviour for non-grocery shopping trips, Transportation Research Record (forthcoming)

LANGDON, M. (1976) Modal split models for more than two modes, PTRC Summer Annual Meeting, University of Warwick.

LANGDON, M.G. (1978) Some developments in modelling multiple choice situations, PTRC Summer Annual Meeting, University of Warwick.

LEAMER, E. (1978) Specification Searches: Ad Hoc Inference with Nonexperimental Data, Wiley, New York.

LERMAN, S.R. and LOUVIERE, J. (1978) The use of functional measurement to identify the form of utility functions in travel demand models Transportation Research Record, 673, 78-85.

LERMAN, S.R. and C.F. MANSKI (1979) Sample design for discrete choice analysis of travel behaviour: The state of the art. Transportation Research 13A(1), pp 29-44.

LOUVIERE, J.J. (1980a) Some comments on premature expectations regarding spatial, temporal, and cultural transferability of travel choice models. In Stopher, P.R., A.H. Meyburg and W. Brog (Eds.) New Horizons in Travel Behaviour Research. Lexington Books, Lexington, Massachusetts.

LOUVIERE, J.J. (1980b) On the identification of the functional form of the utility expression and its relationship to discrete choice, Appendix B of D.A. Hensher and L.W. Johnson Applied Discrete Choice Modelling, Croom Helm, London.

LOUVIERE, J.J. and R.J. MEYER (1979) Behavioural analysis of destination choice: theory and empirical evidence, Technical Report 112, Institute of Urban and Regional Research, University of Iowa.

MAKOWSKY, G.G., BAKER, M.M. and SINHA, K.C. (1977) Estimating transportation mode utilities. Transportation Engineering Journal of ASCE, 103 (TE6), 807-23.

MANSKI, C.F. (1975) Maximum score estimation of the stochastic utility model of choice, Journal of Econometrics, 3(3), pp 205-228.

MANSKI, C.F. (1977) The structure of random utility models. Theory and Decision, 8, 229-54.

MANSKI, C.F. and S.R. LERMAN (1977) The estimation of choice probabilities from choice-based samples, Econometrica 45, pp 1977-88.

MARTIN AND VOORHEES ASSOCIATES (1979) Review of Traffic Tools on the Humber Bridge, Report to the Humber Bridge Board, London.

McFADDEN, D. (1976) The theory and practice of disaggregate demand forecasting for various modes of urban transportation, Working Paper 7623, Institute of Transportation Studies, University of California at Berkeley.

McFADDEN, D. (1978) Modelling the choice of residential location in A. Karlqvist, L. Lundqvist, F. Snickars and J.W. Weibull (Eds.) Spatial Interaction Theory and Planning Models, North Holland, Amsterdam.

McFADDEN, D. (1979) Quantitative methods for analysing travel behaviour of individuals: some recent developments. In Hensher, D.A. and Stopher, P.R. (Eds.) Behavioural Travel Modelling, Croom Helm, London.

McFADDEN, D., REID, F.A., TALVITIE, A.P., JOHNSON, M.A. and ASSOCIATES (1979) Overview and Summary: Urban Travel Demand Forecasting Project, Research Report UCB-ITS-RR-79-6, Institute of Transportation Studies, University of California at Berkeley.

McINTOSH, P.T. and QUARMBY, D.A. (1970) Generalised costs and the estimation of movement choice and benefits in transport planning, Department of the Environment, MAU Note 179, London.

ORTUZAR, J.D. (1979) Testing the theoretical accuracy of travel choice models using Monte Carlo simulation. Working Paper 125, Institute for Transport Studies, University of Leeds.

ORTUZAR, J.D. (1980) Modelling park 'n ride and kiss 'ride as submodel choices, Transportation, 9(3) (in press).

PRASHKER, J.N. (1970) Mode choice models with perceived reliability measures. Transport Engineering Journal of ASCE,105(TE3).

QUARMBY, D.A. (1967) Choice of travel mode for the journey to work: Some findings, Journal of Transport Economics and Policy, 1(3), pp 273-314.

RECKER, W.W. and GOLOB, T.F. (1979) A non-compensatory model of transportation behaviour based on sequential consideration of attributes, Transportation Research, 13B, 269-80.

RICHARDSON, A.J. (1978) A comparative analysis of transport choice models, Working Paper 78/11, Department of Civil Engineering, Monash University, Clayton.

RICHARDSON, A.J. (1980) Search models and choice set generation, International Conference on Research and Applications of Disaggregate Travel Demand Models, University of Leeds, 14-16 July 1980.

ROGERS, K.C. (1976) The use of behavioural models for deriving values of travel time: The state of the art. In Stopher, P.R. and A.H. Meyburg (Eds.) Behavioural Travel Demand Models, Lexington Books, Lexington, Massachusetts.

SHEFFI, Y., HALL, R. and DAGANZO, C.F. (1980) On the stimation of the multinomial probit model, International Conference on Research and Applications of Disaggregate Travel Demand Models, University of Leeds, 14-16 July 1980.

SOBELL, K.L. (1980) Travel demand forecasting with the nested multinomial logit model. Fiftieth Annual Meeting of the Transportation Research Board, Washington, DC. January.

SPEAR, B.D. (1977) Applications of new travel demand forecasting techniques to transportation planning: a study of individual choice models, Federal Highway Administration, U.S. Department of Transportation, Washington DC.

STOPHER, P.R. and MEYBURG, A.H. (Eds.) (1976) Behavioural Travel Demand Models, Chapter 1, Lexington Books, Lexington, Massachusetts.

STOPHER, P.R., WILMOT, C.G. and ASSOCIATES (1978) Final Report, Randburg/Sandton Bus Study, National Institute for Transport and Road Research, South Africa.

TALVITIE, A. and KIRSHNER, D. (1978) Specification, transferability and the effect of data outliers in modelling the choice of mode in urban travel, Transportation, 7(3), 311-32.

TARDIFF, T.J. (1976) A note on goodness-of-fit statistics for probit and logit models, Transportation 5(4), pp 377-388.

TOBIN, J. (1955) The application of multivariate probit analysis to economic survey data, Cowles Foundation Discussion Paper No.1, (unpublished).

TRAIN, K. and McFADDEN, D. (1978) The goods/leisure tradeoff and disaggregate work trip mode choice models, Transportation Research 12(5), 349-53.

TRUONG, T.P. and HENSHER, D.A. (1979) Valuation of commodity time savings and budget time, School of Economic and Financial Studies, Macquarie University, Australia.

TVERSKY, A. (1972) Elimination by aspects: a theory of choice, Psychological Review, 79(4), 281-99.

WEIBULL, J.W. (1978) A search model for microeconomic analysis with spatial application. In Karlqvist, A., Lundqvist, L., Snickars, F. and Weibull, J.W. (Eds.) Spatial Interaction Theory and Planning Models, North Holland, Amsterdam.

WILLIAMS, H.C.W.L. (1977) On the formation of travel demand models and economic evaluation measures of user benefit, Environment and Planning A, 9, 285-344.

WILLIAMS, H.C.W.L., and ORTUZAR, J.D. (1980) Behavioural theories of dispersion and the misspecification of travel demand models, Working Paper 277, School of Geography, University of Leeds.

WILLIAMS, H.C.W.L. and SENIOR, M.L. (1977) Model based transport policy assessment, 2: removing fundamental inconsistencies from the models, Traffic Engineering and Control, 18(10), 464-9.

WILLIAMS, H.C.W.L. and THRIFT, N.J. (1980) On the development of behavioural location models, Economic Geography (forthcoming).

WILSON, A.G. (1974) Urban and Regional Models in Geography and Planning, John Wiley and Sons, London.

YOUNG, W. and RICHARDSON, A.J. (1980) Residential location preference models: compensatory and non-compensatory approaches, PTRC Summer Annaul Meeting, University of Warwick, 7-10 July 1980.

APPENDIX 1.1 <u>Contract terms of reference</u>

DEPARTMENT OF TRANSPORT

SCHEDULE 1 - PROGRAMME OF RESEARCH

OBJECTIVES

To investigate the methods to be employed in assessing driver's valuation of time savings in relation to their perceptions of tolls and other motoring costs.

PROGRAMME TO BE CARRIED OUT BY THE CONTRACTOR

1. A thorough analysis of the models used in the Humber Bridge Tolls study and the subsequent review will be made. The extent to which these models and the data already collected can serve the objectives of the main study will be assessed and the need for further modelling work and data collection will be established. It must be recognised that because of cost constraints and the substantial amount of data already collected on traffic patterns on Humberside, the scope for data collection before the Bridge opens is very limited. The amount of data required after the opening of the Bridge will be described, together with methods of collection and a broad estimate of the costs of collection.

2. An examination of existing techniques and a clear statement of the methods to be used to estimate the trade off between tolls and time savings will form an important part of the work, having regard to the misperception of vehicle operating costs and the extent of redistributed traffic. The contractor will use as a starting point for this examination a description of the models used in the Humber Bridge Tolls study and the subsequent review, together with the planning data which formed the basis of this work.

3. The feasibility of determining a method to isolate the effects of changes in the location of trip origins and destinations, both in respect of planning changes and in respect of changes in the road network other than those directly related to the Bridge will be considered.

4. The contractor will provide a full specification of the proposed research detailing the stages of the project and the costs expected at each stage.

APPENDIX 1.2: Time/Cost trading for Humber Bridge

It has been suggested that a value of time study based on route choice could be considered, should it be that a reasonable number of trips would take place between Hull and areas for which routes which used the Bridge and routes which did not, had broadly similar "generalised costs". Given the road network, such areas would, of course, lie to the South of the Humber.

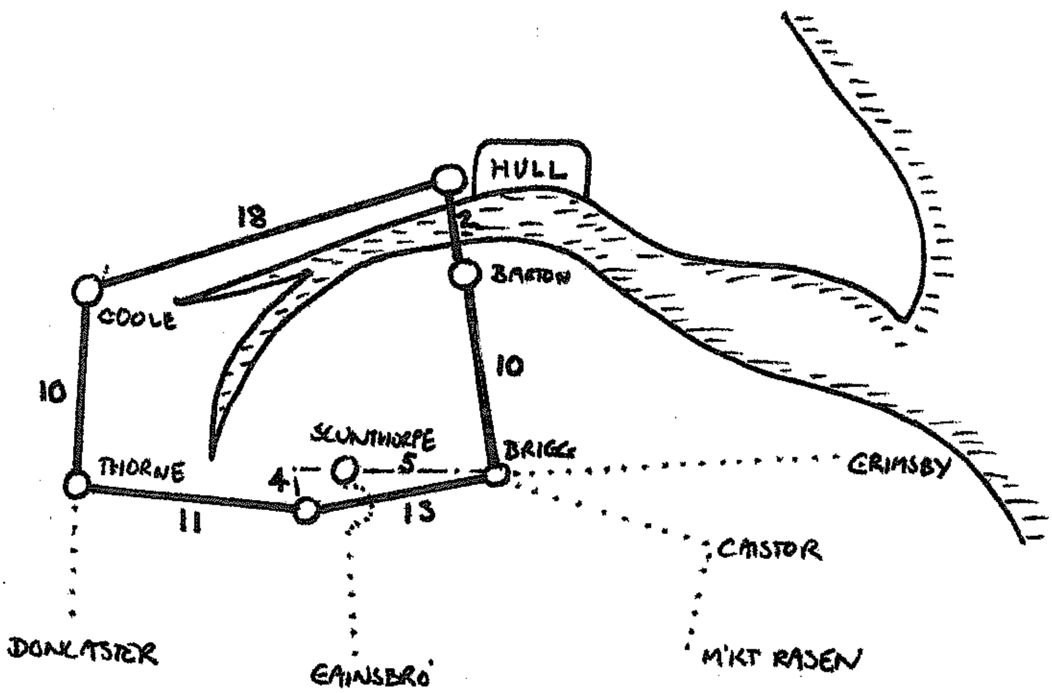


Figure A.1: Road Network to South of Humber for Route Choice (distance in miles)

The routes shown in Figure A.1 dotted are routes which would be used whatever the decision to cross the bridge or not; as such their lengths are immaterial. The main centres of population and access points to the network are shown; it is assumed that all movements between Hull and the Southwest/far South will face the same choice as a movement starting at Doncaster (and thus also as one starting at Thorne) whereas movements from the immediate South and the South East will correspond to route choice decisions from Brigg or Scunthorpe. Using this approximate network, Table A.1 gives the distance advantage of using the bridge crossing, converted into generalised cost at a high value of £1.50 per hour and a low value of 50p per hour, for the centres of Goole, Thorne, Scunthorpe and Brigg.

	Distance Advantage	Time (1)	Cost (2)	Gen.Cost High (3)	Advantage Low (4)
Goole	-26	-26 mins	-£2.80	-£3.45	-£3.02
Thorne	-6	-6 mins	-£1.80	-£1.95	-£1.85
Scunthorpe	26	+26 mins	-£0.20	+£0.48	+£0.02
Brigg	40	+40 mins	+£0.50	+£1.50	+£0.83

Notes:

(1) at 60 mph

(2) at 30 mpg and £1.50 per gallon, minus toll (£1.50)

(3) at £1.50 per hour, plus cost advantage

(4) at £0.50 per hour, plus cost advantage

TABLE A.1: Bridge advantage

The last two columns indicate that one or other option dominates at these four points, regardless of value of time (providing it lies between £0.50 per hour and £1.50 per hour).

The A1077 potentially offers a shorter route between Scunthorpe and the Bridge, but the distance saving (of about 1 mile) would almost certainly be outweighed by the increased time/lower speeds.

Clearly, Scunthorpe is the only access point/urban area where a choice might be perceived. Areas west of the Trent will find the route around the Humber more attractive/cheaper, and areas to the north and east of Scunthorpe will find the Bridge crossing even more attractive than from Scunthorpe. The rough calculations of Table A.1 indicate a small advantage for the Bridge crossing from Scunthorpe, as a result of a (roughly) 26 mile advantage and a net £0.20 extra "out-of-pocket" cost. However, given that no account has been taken of possible delays at the Bridge, or details of access points to the network, it is certainly not wise to dismiss the possibility of a route choice being perceived from Scunthorpe, and thus for movements from the south arriving at Scunthorpe. Only the Scunthorpe/Hull movements are likely to be of any significance, which would give rise to the problem that all choices were being made about a single time/distance trade-off; this in itself would make the situation unsuitable for a value-of-time determination from route choice.

APPENDIX 2.1: The Aggregate Approach

The preliminary evaluation of the feasibility of the aggregate approach required the following assumptions:

Assumption 1

The flows after bridge opening can be satisfactorily represented by a model of the form

$$t_{ij} = p_i q_j \, e^{-\lambda c_{ij}}$$

$p_i \; q_j$ constant, c_{ij} = generalised cost of 1-j trip

in which the generalised cost of an i-j trip is composed of a weighted sum of the time, instance and toll costs of that trip. Thus,

$$c_{ij} = \alpha_o \; \text{TIME}_{ij} + \alpha_1 \; \text{DISTANCE} + 1.0 \; \text{TOLL}_{ij}$$

Assumption 2

The forecast flows from the revised Humber Bridge Toll Study (HBTS) model (Martin and Voorhees Associates, 1979), latest version, for the 1981 position, will be a fair estimate of the actual outturn flows.

Assumption 3

Roadside interviews will produce estimates of the 'true' flows which will have a Poisson distribution about those 'true' flows. (Appendix 2.2 discusses this further.)

Based on these assumptions, and further expecting that the actual values of time, distance and toll will be in the region of those assumed in the HBTS work, we can calculate the relative accuracy with which we would be able to estimate each coefficient based on any given survey design. Hence we can directly relate survey costs to the accuracy with which the coefficients would be estimated. The assessment of accuracy will not depend on the HBTS model being exactly correct, but rather on the HBTS forecasts being of broadly the right magnitude. The actual fitted coefficients from this exercise are, of course, of no value. We expect to recover the values that the consultants used to create the forecasts. The standard errors of these values are the statistics we want to consider.

The analyses described below have used the total number of vehicle generations and attractions for each of the categories Home based Work (HBW) and Other Home Based (OHB) separately. The effect of taking an X% sample is to increase standard errors by a factor $\sqrt{\frac{100}{X}}$ (*), so we can make simple corrections to the derived figures to estimate the accuracy of various sampling strategies. (More complex strategies would involve different sampling fractions at different cordons, and would need a more detailed analysis along the same lines.) We must also <u>halve</u> the consultants' figures, which are for Generations and Attractions, to give the numbers of trips distributed between the various destinations, thus the appropriate correction factor becomes $\sqrt{\frac{200}{X}}$.

Five different sampling strategies have been explored, in the context of the HBW and the OHB trip matrices. These are:

1) cordons around Hull, Grimsby, Scunthorpe, Lincoln, York and Beverley
2) cordons around Hull, Grimsby, Scunthorpe and Lincoln
3) cordons around Hull, Grimsby and Scunthorpe
4) cordons around Hull, Grimsby plus Bridge interviewing.

It has been assumed that all cordons would be one way, and the 'out' directions interviewed. It may be seen from the forecast trip matrices that it makes little difference which direction is chosen for the interview; there are a similar number of trips forecast in each direction. Interviewing in both directions is not the most efficient use of resources in this study, it will be argued.

Table A.1 sets out the estimated standard errors of fitted model coefficients for six data sets, (as calculated from the full G/A model).

	Data Set 1	Data Set 2	Data Set 3	Data Set 4	Data Set 5	Data Set 6
Distance	.0077	.0095	.0101	.0070	.0059	.0220
Time	.0102	.0119	.0131	.0091	.0077	.0250
Toll	.1520	.5050	.5170	.2470	.1870	.5600

TABLE A.1: <u>Standard Errors - Other Home Based Case</u>

(*) We have, as usual, $\gamma = k/\ n$ where n is the sample size. If we take $\left(\frac{X}{100}\,n\right)$ instead, γ becomes $k/\sqrt{\frac{X}{100}\,n}$, or $\sqrt{\frac{100}{X}}\ \frac{k}{\sqrt{n}}$.

The first data set consists of all movements to and from the six towns indicated in the first sampling strategy. The second and third data sets represent all movements to these six towns from towns in strategies two and three. As might be expected, accuracy decreases as the amount of data input. (The size of these errors relative to the magnitude of the fitted coefficients is discussed below.) Data set four expanded the information from data set one to include trips from the three cordoned towns to Selby Goole, Barton and Immingham; including the extra four destinations has the effect of decreasing standard errors by about 40%. Data set five represents the same flows as in data set four, but with some movements re-duplicated by interviewing at the bridge (in both directions). This has had the effect of increasing the accuracy of the Toll coefficient by most, as might be expected, but of reducing standard errors of the time and distance coefficients by about 10% also. The last data set consisted of movements from Hull and Grimsby only, reduplicated where appropriate by two directional Bridge interviewing. The loss in accuracy is dramatic.

We can tentatively conclude that we should not go below three cordons, and further that the larger the number of destination zones (and hence the larger and more varied the set of generalised costs involved) the better. This is why the strategy of two-directional interviews is not the most effective here; the extra information that it supplies is about trips over costs (time, distance and toll) which have already been observed.

There are three restricting considerations which affect the number of destination zones that should be considered. Firstly, it is not worth including zones unless they attract a reasonable amount of traffic from at least two of the generating zones. Secondly, it must be possible to characterise each zone-to-zone cost by a single time and a single distance. We are thus directed towards looking for 'concentrated' attractors, at a fair amount of mutual separation - in other words, we should be looking at the major town-to-town flows. Rural zones around Hull, for example, do in some cases give rise to comparatively large flows, but fall down in respect of the requirement for single values of time and distance over which the flows are taking place. Towns like Louth and Bridlington could be given single time/distance separations from the ten other towns we presently consider (although they are not individually identified in the existing HBTS zone system) but the flows involved are vanishingly small. (The mean trip length for work trips in the 1976 RHTM data was around 20 minutes.

Only the Hull-Beverley flows are below this level, in our data sets. Most of the other flows have time separations of around one hour. Correspondingly, we are dealing with small numbers of vehicles.)

The third restricting consideration on the number of zones/towns involved is the need to collect a relatively large proportion of relevant flows, given the errors involved. It was hoped that restricting interest to only a few destinations could allow most interviews to be conducted in the space of a few seconds; traffic to other areas could be identified and allowed to leave. The larger the list of areas of interest, the longer it will take to eliminate non-interesting traffic, and the lower our sampling fraction.

Returning to the figures in Table A.1, data set 5 seems the most appropriate sampling strategy to continue our examination; increasing the number of destination zones, if possible, will reduce these errors still further, but for the present we can proceed with these. The same strategy can be implemented with the HBW trips; Table A.2 presents the corresponding standard errors, along with our expectation of the absolute size of the coefficients involved.(*) We can thus compare the two to assess relative accuracy. (Note that the situation for OHB is more favourable than that for HBW, in that the coefficients of time and distance are more nearly equal, and relatively more important in comparison with the toll. The relative accuracy of the fitted coefficients will thus be higher.)

	OHB (st. error)	HBW (st. error)	Expected HBW(≠) coefficients	't' values HBW
Distance	.0059	.0064	.056	8.8
Time	.0077	.0083	.036	4.3
Toll	.1870	.1420	3.547	25.0

TABLE A.2: <u>OHB/HBW Relative Accuracy (for data set 5)</u>

In the above we have been fitting models to vehicles movements, and explaining the distribution of these in terms of a gravity model. In fact, the modelling work in the HBTS was based on distributing person trips.

(*) we use 'expectation' rather than 'fitted' values here

(≠) standardised so that TOLL expected = TOLL fitted

There is a difference inasmuch as we have been assuming an average vehicle occupancy to assign a single 'toll per head' for each trip. This problem will be ignored here. We are thus considering average CAR DRIVER VALUES OF TIME, assuming each driver to be carrying the appropriate fraction of a passenger, and charging the corresponding fraction of the toll.

From Table A.2, we can calculate the approximate accuracy of the fitted coefficients on the basis of an X% sample: this is set out in Table A.3.

	X%			
Coefficient	60	50	40	30
Distance	4.8	4.4	3.9	3.4
Time	2.4	2.2	1.9	1.7
Toll	13.7	12.5	11.2	9.7

TABLE A.3: <u>'t' Values for X% Samples - Home Based Work trips</u>

The 't' values are the ratio of the expected coefficient to its estimated standard error; Table A.4 illustrates the size of 't' values required to measure any variable to the stated accuracies with 95% or 90% confidence.

	Accuracy (%)			
t-value	± 10	± 25	± 50	± 100
95% Confidence	20	8	4	2
90% Confidence	16.7	6.7	3.3	1.7

TABLE A.4: <u>% Accuracy at given Confidence Level and t Value</u>(*)

Comparing these values with our estimates from Table A.3 indicates that, on our current assumptions and with the flows anticipated between the towns selected for data set 5, we would have to be planning for a sampling fraction around 50% to be expecting 95% confidence limits even as wide as plus or minus 100% for the value of the time coefficient!

(*) For example, if we are to be 95% certain of measuring to within ± 10% we want to have 2 s.e's = 10% of the true value: thus $\frac{\text{true value}}{\text{s.e.}} = \frac{2}{.1} = 20$ = 't'. For 90% confidence, we would want 1.67 s.e's = 10%, giving 't' $= \frac{1.67}{.1} = 16.7$.

Actual 'values of time' (and of distance) will involve standardising the fitted coefficients so as to scale the 'toll' penalty to the appropriate money units (ignoring the non-money costs of bridge crossing for the moment). Thus we require the 't' values appropriate to the ratio of the time coefficient to the toll coefficient, and a similar ratio for the distance coefficient. The effect is to reduce the expected 't' values by about 5% (see Appendix 2.3) so broadly similar conclusions derive for value of time in money units; 3 cordons, plus two-way Bridge interviews, are required, with a sample fraction of around 50% to establish the 'value of time' to ± 100%.

If we are correct in attributing the source of the observed variation around the model to day-to-day variability in combination with sampling errors (presumably after correcting for any gross trends and seasonality) then we can further reduce the errors in our determination of the model parameters by surveying on more than one day. Table A.5 presents the 't' values corresponding to surveying on each of two days on this basis.

	X(%)			
Coefficient	60	50	40	30
Distance	6.8	6.2	5.5	4.8
Time	3.4	3.1	2.7	2.4
Toll	19.4	17.7	15.8	13.7

TABLE A.5: 't' Values for X% Samples on Each of Two Days

Tables A.6 and A.7 present the major town to town flows in the area for the HBW trip (these are in G/A form, so that an entry of N represents N/2 vehicles) for the 1976 RHTM data and for the forecast 1981 HBTS.

	Hull	York	Grimsby	Sc/pe	Linc.
Hull	X	84	32	48	0
York	89	X	4	13	0
Grimsby	9	0	X	1258	0
Sc/pe	14	2	440	X	0
Lincoln	0	0	9	3	0

TABLE A.6: RHTM 1976

	Hull	York	Grimsby	Sc/pe	Linc.
Hull	X	138	65	32	10
York	85	X	8	7	1
Grimsby	22	4	X	221	41
Sc/pe	24	16	244	X	62
Lincoln	13	13	126	76	X

TABLE A.7: HBTS 1981

The flows are generally similar, except that the RHTM data showed virtually no trips between Lincoln and the other towns, and that far higher flows were observed in 1976 between Grimsby and Scunthorpe than are being forecast for 1981, on the basis of the simple gravity model. This may be a warning of the existence of a 'special link' between these two towns, possibly as a result of available skills of a certain kind in one and corresponding opportunities in the other. If this is true, we would <u>not</u> want to model HBW flows between these two towns by a simple gravity model.

Finally, the equivalent of Table A.2 for Other Home Based trips leads to 't' values of 9.2, 6.4 and 18.2 for distance, time and toll respectively. (The HBW values from Table A.2 were 8.8, 4.3 and 2.5.) The improved 'precision' on the time coefficient arises principally as a result of the relative sizes of the expected coefficients. If the expectations are well founded, the fitted values of time for the OHB purpose should be relatively more accurate than for the HBW trips.

APPENDIX 2.2: Discussion of the Assumptions of the Aggregate Approach

The analysis described in Appendix 2.1 has been based on the three assumptions stated at the outset. It is hoped that the results will be reasonably robust to departures from assumption 2, i.e. that the standard errors are not sensitive to exact flows, but rather to the overall amounts of traffic and the broad patterns of movement. Assumption 3 was that the sampled flows would be related to the 'true' flows with a Poisson error structure. This is the conventional assumption, deriving from the expectation of an underlying Poisson variation in traffic on any link/interchange, and Binomial sampling from this. (See Kirby and Leese, 1978)

In practice, roadside interviews are almost always conducted over less than 24 hours, and then 'grossed-up' according to the indicated total from an automatic counter. This process introduces errors, especially into the estimate of O/D patterns and the breakdown into trip purposes. It is also conventional procedure to assume that trips observed crossing the cordon in one direction will make a corresponding trip back in the opposite direction. It would be highly desirable if more were known about the errors that these assumptions introduce into the data; we would certainly recommend that such an analysis be performed on the data gathered for any value of time study of this form. However, the trip reversal procedure could only be checked if at least one of the cordons were interviewed in both directions.

In this analysis, we cannot anticipate the effects of departures from assumption 1; if the model does not fit well, none of the results hold.

APPENDIX 2.3: <u>Accuracy of Ratios of Fitted Coefficients</u>

The fitted coefficients in the model are α_o^1(TIME), α_1^1(DISTANCE) and α_2^1(TOLL), say, and they are related to the general form

$$t_{ij} = p_i q_j \, e^{-\lambda c_{ij}}$$

where $c_{ij} = \alpha_o \text{TIME} + \alpha_1 \text{DIST} + \text{TOLL}$

in that

α_o^1 estimates $\alpha_o \lambda$, α_1^1 estimates $\alpha_1 \lambda$, and α_2^1 estimates λ.

Thus, because the 'toll' has been entered in the data as a zero-one variable, the units of measurement for both time and distance are 'units of toll'. However, all three, time, distance and toll components of generalised cost have been estimated <u>multiplied by the parameter</u> λ. Thus to re-derive 'values of time', for example, we have to divide α_o^1 by α_2^1 ; this then gives the 'value of time' in units of toll, (that is, if the toll were £1, the value would be £α_o^1/α_2^1 , if the toll were £5, the value would be £5 x α_o^1/α_2^1 .

Thus, we should not just consider the accuracy with which we can measure the coefficients α_o^1 , α_1^1 , α_2^1 , but also the accuracy of the ratios α_o^1/α_2^1 , α_1^1/α_2^1 since these are what we are really concerned about.

The GLIM package prints out statistics sufficient to give the variance-covariance matrix of the estimated coefficients; for the OHB model on the selected data set, we can derive (see Kendall and Stuart, (1969)

$$\rho^2(\alpha_o^1, \alpha_2^1) = -0.20$$

Now, denoting $\alpha_o^1/\sqrt{\text{var}\,\alpha_o^1}$ by t_o and $\alpha_2^1/\sqrt{\text{var}\,\alpha_2^1}$ by t_2, we have

$$t_r = \frac{\alpha_o^1/\alpha_1^1}{\sqrt{\text{var}\,\dfrac{\alpha_o^1}{\alpha_2^1}}} \doteq \left[\frac{1}{t_o^2} + \frac{1}{t_2^2} - \frac{\rho^2(\alpha_o^1, \alpha_2^1)}{t_o t_2}\right]^{-\frac{1}{2}}$$

With $t_o = 4.3$, $t_2 = 25.0$ and $\rho^2 = 0.20$, we derive

$t_r = 4.11$; i.e. we estimate the 'value of time' with 'relative accuracy' about 96% of that of the time coefficient, α_o^1 .

We should also note that the relative accuracies have been calculated on the basis of the expected values of time and distance, as supplied by the consultants. The 'fitted' model, which would ideally have simply recovered exactly the same values as were used to construct the forecasts, reached a value of the time coefficient some 25% lower than that input: the explanation for this seeming illogicality is, for the most part, due to the use of a negative exponential deterrence function in place of the original power deterrence function. There is some evidence that the power function may be more appropriate for inter-town flows (see, for example, Gaudry and Wills, 1977; Wilson, 1974). If we wish to estimate the α_o and α_1 coefficients within a power deterrence function, we shall have to supply a purpose written non-linear optimisation program. This need not be too difficult; however, it is to be hoped that such a refinement will not be necessary. The sample sizes derived here would no longer be strictly applicable if a power deterrence function were adopted. It is not known how different they would be; however, it is unlikely that they would be too large.

Appendix 3.1: Random utility models of choice

1. General statement

In recent years a considerable advance has been made in the construction of travel demand models from choice theoretic principles. Much interest has centred on the relationship between the structure of the model and the behavioural principles associated with its formation; one particular framework within which this relation has been sought is that provided by random utility theory (for a review see Domencich and McFadden, 1975).

In this quantal choice theory individuals in a given market segment, Q, are considered to associate with each member A_n; n=1, ..., N of a discrete set of options $\underline{A}$, a net utility U_n; n=1, ...,N, and to select that member with the highest value of U^n (*). To account for interpersonal variation in the value of attributes incorporated in the utility functions, and the influence of unobserved factors, the modeller considers the variables $(U_1, \ldots, U_n, \ldots, U_N)$ to be randomly distributed over the population confronted by a choice. The probability P_n that an individual with particular characteristics selects an alternative A_n is then simply expressed in terms of the probability that U_n be greater than those values associated with all other options. A formal choice model may be derived when the density function $f(\underline{U}) = f(U_1, \ldots, U_N)$ of the utility components is specified.

Formally, we can express the model generator equations of random utility theory as follows:

$$P_n = \text{Prob}\ (U_n > U_{n'}, \forall A_{n'} \ \varepsilon \underline{A}) \qquad (1)$$

$$= \int_{R_n} d\underline{U}\ f(\underline{U}) \qquad (2)$$

in which $f(\underline{U})$ is the joint distribution function of $(U_1, \ldots, U_N)$ and R_n is that region of utility space defined by

$$R_n : U_n \geq U_{n'}, \quad , \forall A_{n'}, \ \varepsilon \underline{A} \qquad (3)$$

$$U_n \geq 0 \qquad (4)$$

(*) Individuals are taken as rational decision makers, with perfect information who always maximise their utilities ('homo economicus')

If only those cases in which a trip is actually made are considered, the non-negativity restriction in (4) can be considered inoperative. For the distribution functions considered later this will involve a neglibible inconsistency, which does not affect the argument to be presented.

To derive an explicit probabilistic choice model we need to know both the form of $f(\underline{U})$ and an expression for the utility function in terms of the attributes of alternatives in the set $\underline{A}$.

We shall take the components U_n to be of the following form:

$$U_n(\underline{\theta},\underline{Z}_n) = \bar{U}_n(\underline{\theta},\underline{Z}_n) + \varepsilon_n \qquad (5)$$

in which $\bar{U}_n$ is the so-called 'representative' utility of the population Q confronted by the choice, and ε_n is a stochastic residual. $\bar{U}_n$ is normally taken to be linear in terms of the attributes Z_n^μ characterising A_n. That is:

$$\bar{U}_n(\underline{\theta},\underline{Z}_n) = \sum_\mu \theta_\mu Z_n^\mu, \quad \forall A_n \in \underline{A} \qquad (6)$$

$$= \underline{\theta}.\underline{Z}_n \qquad (7)$$

The vector of parameters $\underline{\theta}$ is estimated from observed choices. It remains to specify the distribution function $f(\underline{U})$ or equivalently that of the stochastic residuals $\underline{\varepsilon}$.

An important class of random utility models includes those generated by identical and independent (IID) utility distributions for which we can decompose $f(\underline{U})$ as follows:

$$f(\underline{U}) = \prod_{n=1}^{N} g(U_n) \qquad (8)$$

Here $g(U_n)$ is the distribution of the utility component associated with A_n. The expression for P_n can now be written

$$P_n = \int_{-\infty}^{\infty} dU_n\, g(U_n) \prod_{n' \neq n} \int_{-\infty}^{U_n} dU_{n'}\, g(U_{n'}) \qquad (9)$$

Omission of the constraint (4) allows the lower limits of integration to be extended to minus infinity.

It is by now widely known that the much favoured multinomial logit model (MNL)

$$P_n = \frac{e^{\Delta \bar{U}_n}}{\sum_{n'} e^{\Delta \bar{U}_{n'}}} \tag{10}$$

is an IID model generated from Weibull (Gnedenko) probability distributions (Charles Rivers Associates, 1972) for which

$$g(U_n) = \Delta e^{-\Delta(U_n - \bar{U}_n)} e^{-e^{-\Delta(U_n - \bar{U}_n)}} \tag{11}$$

This is a skewed unimodal distribution, in which the dispersion parameter Δ is inversely related to the standard deviation, σ, as follows (Cochrane, 1975):

$$\Delta = \frac{\pi}{\sqrt{6}\sigma} \tag{12}$$

In general for the utility distributions U_n; n=1, ...,N we can define a variance-covariance matrix $\underline{\Sigma}$ with elements $\Sigma_{nn'}$, given by:

$$\begin{aligned} \Sigma_{nn'} &= E\,(U_n - \bar{U}_n;\ U_{n'} - \bar{U}_{n'}) \\ &= E\,(\varepsilon_n\ \ \varepsilon_{n'})\ \forall\ A_n, A_{n'}\ \varepsilon \underline{A} \end{aligned} \tag{13}$$

in which E(.) denotes an expectation value. In the case of IID utility components the matrix has, by construction, a simple diagonal form

$$\underline{\underline{\Sigma}} = \sigma^2\, \underline{\underline{I}} \tag{14}$$

where $\underline{\underline{I}}$ is the unit matrix of dimension N, and σ the common standard deviation of the distributions g(U), that is

$$\sigma^2 = E(\varepsilon_n\ \varepsilon_n) \quad \forall\ A_n\ \varepsilon \underline{A} \tag{15}$$

The MNL model (10) generated from IID Weibull distribution, which is therefore characterised by a matrix with a simple diagonal structure (14) has been widely applied in mode choice modelling (for a review, see Spear, 1977). It is well known, however, that the model suffers a restrictive property of cross-substitution, 'the independence from irrelevant alternative' (IIA) property, whereby the ratio

$$\frac{P_n}{P_{n'}} = e^{\Delta\,(\bar{U}_n - \bar{U}_{n'})} \qquad \forall\, A_n,\, A_{n'}\, \varepsilon\, \underline{A} \tag{16}$$

is independent of the utility values associated with other options. The ratio (16) will be unaffected by the expansion or contraction of the choice set $\underline{A}$. The IIA property, once seen as a positive advantage to be exploited in 'new option' situations, is now recognised to be a potential hazard when certain alternatives are more 'similar' than others in the set $\underline{A}$. In random utility theory this notion of 'similarity' is interpreted in terms of the presence of off-diagonal elements in the matrix $\underline{\underline{\Sigma}}$(*).

At the other end of the range an arbitrary covariance matrix, that is one with different standard deviations for each marginal density function $g_n(U_n)$, and allowing for correlation between the different utility members in $f(\underline{U})$ will, if $f(\underline{U})$ is multivariate Normal, generate the multinomial probit model (MNP). In this case the appropriate density function, for choice between N alternatives is given by:($\neq$)

$$f(\underline{U}) = (2\pi)^{-N/2}\, |\underline{\underline{\Sigma}}|^{-\frac{1}{2}} \exp\{-\tfrac{1}{2}(\underline{U} - \bar{\underline{U}})^{T}\, \underline{\underline{\Sigma}}^{-1}\, (\underline{U} - \bar{\underline{U}})\} \tag{17}$$

We shall immediately transform Equation (17) from $\underline{U}$- space into $\underline{\varepsilon}$- space using Equation (5), giving

$$f(\underline{\varepsilon}) = (2\pi)^{-N/2}\, |\underline{\underline{\Sigma}}|^{-\frac{1}{2}} \exp\{-\tfrac{1}{2}\, \underline{\varepsilon}^{T}\, \underline{\underline{\Sigma}}^{-1}\, \underline{\varepsilon}\,\} \tag{18}$$

If we define

$$\bar{U}_{nn'} = \bar{U}_{n'} - \bar{U}_n \tag{19}$$

then resorting to Equation (2) the model can be stated as

$$P_n = \int_{-\infty}^{\bar{U}_{1n} + \varepsilon_n} \int_{-\infty}^{\bar{U}_{2n} + \varepsilon_n} \cdots \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\bar{U}_{Nn} + \varepsilon_n} f(\underline{\varepsilon})\, d\underline{\varepsilon} \tag{20}$$

(*) We will examine the implications for model formation, of off-diagonal elements in section 2 below.

($\neq$) N.B. In equations (17) and following, the superscript T will stand for matrix transpose and the superscript -1 for matrix inverse.

Although the MNP (20) is completely general in its theoretical statement, it is considerably more cumbersome than the MNL (10) to implement. The difficulties of achieving a solution to the MNP by direct numerical integration for other than 'small' problems involving 3 or 4 options (Hausman and Wise, 1978) are well known, and have led to the formulation of approximate solution schemes. We will discuss these in Appendix 3.3.

2. Correlation and model structure

In the previous part of this Appendix, we outlined the derivation of two well-known models within the random utility framework: the simple and flexible (but theoretically restrictive) multinomial logit model (MNL) and the powerful and general (but rather intractable) multinomial probit model (MNP). We have been interested in random utility functions of the form

$$U_n = \bar{U}_n + \varepsilon_n \tag{5}$$

We note that in theory, the modeller could select any random structure, but because of its flexibility and analytical simplicity, additive disturbances have been assumed in all empirical applications.

In random utility theory, the observer (modeller) considers each individual to act rationally and consistently when repeatedly confronted by the same choice. In this sense he interprets the probability P_n, that an individual t selects alternative A_n, in terms of the proportion of a ficticious population T of individuals with observable attributes identical to t, selecting A_n. Dispersion is attributed to the observer's uncertainty of the true subjective utility values, which are taken to be probabilistically distributed over T(*).

The representative component $\bar{U}_n$ of the utility function (5), can be computed from the observable attributes $\underline{Z}_n$ given the taste parameters $\underline{\theta}$. The linear-in-the-parameters representation (6) is only one (extremely convenient) form for $\bar{U}_n$. Together with additive disturbances ε_n, which account for all deviation from the 'group average', (thus absorbing

---

* Note that this notion of probability is quite different to that of Tversky (1972) for example, which is interpreted in terms of the relative frequency of choice of A_n in repeated trials due to variability in the state of mind.

non-observed attributes including taste variations in the utility functions) this functional structure (linear-in-the-parameters with additive disturbances, or LPAD) is compatible with most observational situations as exemplified below (Manski, 1977).

Let $\underline{Z}_{tn}$ and $\underline{Y}_{tn}$ be vectors of real numbers describing the characteristics of alternative A_n for a decision maker of type t; and let $\underline{\phi}$ and $\underline{\tau}$ be vectors of real numbers that may be interpreted as vectors of taste parameters (i.e. vectors of parameters that change from type to type of decision maker). Now, consider the following observational situations:

(i) Omitted structure. Each decision-maker t is consistent with the maximization of a utility function $U\{\overline{U}(\underline{\phi}, \underline{Z}_{tn}), W(\underline{\tau}, \underline{Y}_{tn})\}$, where as usual $\overline{U}$ is known and $\underline{Z}$ is observed. In this example then, W is observationally a random variable as well as U. Now assume that

$$U_{tn} = \phi \cdot \underline{Z}_{tn} + W_{tn} \tag{21}$$

Then, to be consistent with the LPAD form (5), we must have

$$\varepsilon_{tn} = W_{tn} \tag{22}$$

and we note that ε_{tn} is distributed independently of $\underline{Z}_{tn}$ if and only if W_{tn} is independent of $\underline{Z}_{tn}$.

(ii) Cross-sectional preference variations. Here we take each decision; maker to be consistent with the maximization of a utility function $U(\underline{\phi}_t, \underline{Z}_{tn})$. Although $\underline{\phi}_t$ is fixed for each individual t, it varies across the population Q of the market segment to which individuals t belong. The distribution of $\underline{\phi}$ is unknown therefore, and both $\underline{\phi}_t$ and U have to be considered random.

Now, assume that

$$U_{tn} = \underline{\phi}_t \cdot \underline{Z}_{tn} \tag{23}$$

and let $E(\underline{\phi}_t)$ be the expected value of $\underline{\phi}_t$ taken over the population of decision-makers. Therefore, we can write

$$\underline{\phi}_t = E(\underline{\phi}_t) + \underline{\tau}_t \tag{24}$$

where $\underline{\tau}_t$ is an unobserved random vector with zero expected value. If we call

$$\varepsilon_{tn} = \underline{\tau}_t \cdot \underline{Z}_{tn} \tag{25}$$

we get the LPAD structure

$$U_{tn} = E(\underline{\phi}_t) \cdot \underline{Z}_{tn} + \varepsilon_{tn} \tag{26}$$

where ε_{tn} is clearly not independent of $\underline{Z}_{tn}$. Similarly, if we consider alternative n', we have

$$U_{tn'} = \underline{\phi}_t \cdot \underline{Z}_{tn'} \quad = \quad E(\underline{\phi}_t) \cdot \underline{Z}_{tn'} + \varepsilon_{tn'} \tag{27}$$

where

$$\varepsilon_{tn'} = \underline{\tau}_t \cdot \underline{Z}_{tn'} \tag{28}$$

So, the common appearance of $\underline{\tau}_t$ in ε_{tn} and $\varepsilon_{tn'}$ implies that these disturbances are not independently distributed.

The IID assumption of the MNL (10) is definitely not consistent with the cross-sectional preference variation. It will not be consistent with the omitted structure situation either, if the omitted function W has elements $\underline{Z}_{tn}$ as arguments, or if $\underline{Y}_{tn}$ and $\underline{Z}_{tn}$ are not independently distributed. Even assuming no taste variation, this would bring in off-diagonal elements to the variance-covariance matrix of the residuals $\underline{\varepsilon}$. As we will note below, when this happens we would expect the decision-maker to lump the more similar alternatives together, not treating them as independent; therefore, we would expect that strict application of the MNL could give unreasonable results (*). For the taste-variation case, the situation is, of course, even more serious.

The multinomial probit (MNP) model, with its completely general variance-covariance matrix allows both taste variations and dependence between alternatives, by assuming that the taste disturbances and the error terms are multivariate normally distributed across the population (Hausman and Wise, 1978). We mentioned, however, the problems of solving the model, and although much effort has recently been devoted to its development, it still remains unmanageable for more than a few alternatives (Daganzo et al, 1977).

There are many examples for which the generality of the MNP, even if it could be implemented, is an unnecessary luxury. In certain applications, specific forms for the utility functions tend to suggest themselves. Consider 'two dimensional' choice contexts involving, for example, combinations of destination (D) and mode (M). Alternatives in each dimension will be denoted by $(D_1, \ldots, D_n, \ldots, D_N)$ and $(M_1, \ldots, M_m, \ldots, M_M)$, respectively, and the combination of dimensions

(*) The most infamous example is that of the red bus/blue bus problem.

produces the NM discrete choice options ($D_1 M_1$, ..., $D_n M_m$, ..., $D_N M_M$), which comprise the set $\underline{A}$. The general element A_n is now $D_n M_m$ which might be a specific destination-mode combination for the purpose of performing an activity.

For such choice contexts, we shall be particularly interested in utility functions of the form

$$U(n,m) = U_n + U_m + U_{nm} \qquad \forall\ D_n M_m \varepsilon \underline{A} \tag{29}$$

here U_n and U_m may, for example, correspond to destination and mode specific utilities, respectively, while U_{nm} might be the travel disutility associated with $D_n M_m$ combination. This form was used in the shopping model developed by Ben Akiva (1974), and in a number of other applications in the United States since that time.

Writing U(n,m) in terms of a 'representative' term $\overline{U}(n,m)$ and a stochastic residual $\varepsilon(n,m)$ we have

$$U(n,m) = \overline{U}(n,m) + \varepsilon(n,m) \tag{30}$$

in which

$$\overline{U}(n,m) = \overline{U}_n + \overline{U}_m + \overline{U}_{nm} \tag{31}$$

and

$$\varepsilon(n,m) = \varepsilon_n + \varepsilon_m + \varepsilon_{nm} \tag{32}$$

We shall now assume that the residuals ε_n, ε_m and ε_{nm} are <u>separately</u> IID, with

$$\begin{aligned} E(\varepsilon_n \varepsilon_{n'}) &= \delta_{nn'}\, \sigma_D^2 \\ E(\varepsilon_m \varepsilon_{m'}) &= \delta_{mm'}\, \sigma_M^2 \\ E(\varepsilon_{nm} \varepsilon_{n'm'}) &= \delta_{nn'}\, \delta_{mm'}\, \sigma_{DM}^2 \\ E(\varepsilon_n \varepsilon_m) = E(\varepsilon_n \varepsilon_{nm}) &= E(\varepsilon_m \varepsilon_{nm}) = 0 \qquad \forall D_n M_m \varepsilon \underline{A} \end{aligned} \tag{33}$$

in which δ is the Kronecker delta. The elements of $\underline{\underline{\Sigma}}$ now become

$$\Sigma_{nm,n'm'} = \delta_{nn'} \sigma_D^2 + \delta_{mm'} \sigma_M^2 + \delta_{nn'} \delta_{mm'} \sigma_{DM}^2 \tag{34}$$

and the matrix is expressed in Figure A-2 together with those corresponding to the residual structures

$$\varepsilon(n,m) = \varepsilon_{nm} \tag{35}$$

$$\varepsilon(n,m) = \varepsilon_n + \varepsilon_{nm} \tag{36}$$

$$\varepsilon(n,m) = \varepsilon_m + \varepsilon_{nm} \tag{37}$$

which are clearly special cases of that defined in Equation (32). It is readily seen that the source of correlation in 'multiple dimension' cases is the existence of a common term or 'dimension specific' element (U_n or U_m) in the utility function. For the four cases (32), (35) - (37) we have developed in Figure A-2, a pictorial representation of the structure of the $\underline{\underline{\Sigma}}$ matrix with correlation between alternatives incorporated through common bonds as shown. This is the basis for a representation of the choice model itself (Williams, 1977). In the first case, both σ_D and σ_M are zero and a diagonal $\underline{\underline{\Sigma}}$ matrix results. This case which is consistent with Equation (35) will correspond to the MNL model (10) if the utility functions are drawn from IID Weibull distributions. It is clear that the use of the utility function (29) in a MNL model of the form (10) will be inconsistent because the appropriate $\underline{\underline{\Sigma}}$ matrix, corresponding to that utility function, is not of the diagonal form involved in the <u>generation</u> of the model.

Before treating the more general case (32), which is consistent with the utility function (29) and which corresponds to the fourth $\underline{\underline{\Sigma}}$ matrix of Figure A-2, we shall consider the derivation of a hierarchical or nested model from a function consistent with the residual structure (36),

$$
\begin{aligned}
U(n,m) &= U_n + U_{nm} \\
&= \overline{U}_n + \overline{U}_{nm} + \varepsilon_n + \varepsilon_{nm} \qquad (38)
\end{aligned}
$$

and which corresponds to the second representation in Figure A-2. In this case the component σ_M vanishes and the two parameters σ_D and σ_{DM} allow different degrees of cross-substitution between <u>intra</u> and <u>inter</u>-branch alternatives in the 'tree' form shown in Figure A-2(b); that is, between $D_n M_m$ and $D_n M_{m'}$, in the former case, and between $D_n M_m$ and $D_{n'} M_{m'}$ in the latter. It may be shown (Williams, 1977) that P(n,m), the probability of selecting $D_n M_m$, can be written

$$P(n,m) = P_n \cdot P_{nm} \qquad (39)$$

in which

$$P_{nm} = \text{Prob}\ (U_{nm} > U_{nm'},\ \forall M_{m'}\ \varepsilon \underline{M}) \qquad (40)$$

and

$$P_n = \text{Prob}\ (U_n + U_{n*} > U_{n'} + U_{n'*},\ \forall\ D_{n'}\ \ \varepsilon \underline{D}) \qquad (41)$$

with

$$U_{n*} = \max_m (U_{n1}, \ldots, U_{nm}, \ldots, U_{nM}) \qquad (42)$$

If the components U_{nm} are Weibull distributed variables $W(U,\overline{U}_{nm},\Delta)$ with mean $\overline{U}_{nm} + \gamma/\Delta$ (where γ is Euler's constant), and standard deviation $\pi/(\sqrt{6}\,\Delta)$, then it is readily shown (Cochrane, 1975) that U_{n*} is also Weibull distributed, with mean

$$\overline{U}_{n*} = \frac{1}{\Delta} \ln \left(\sum_m e^{\Delta \overline{U}_{nm}}\right) + \gamma/\Delta \tag{43}$$

and standard deviation given by

$$\sigma_{D*} = \frac{\pi}{\sqrt{6}\Delta} \tag{44}$$

The marginal distribution P_n is then derived from the sum of Weibull distributed variables U_{n*} and variables U_n, derived from some distribution $\Gamma(U,\overline{U}_n)$, n=1, ..., N to be specified.

Now the hierarchical logit (HL) model (Williams, 1977; Daly and Zachary, 1978; McFadden, 1979)

$$P(n, m) = \frac{e^{\beta(\overline{U}_n + \overline{U}_{n*})}}{\sum_{n'} e^{\beta(\overline{U}_{n'} + \overline{U}_{n'*})}} \cdot \frac{e^{\Delta \overline{U}_{nm}}}{\sum_{m'} e^{\Delta \overline{U}_{nm'}}} \tag{45}$$

can be generated by specifying that $\Gamma(U,\overline{U}_n)$ be that distribution of a variate which is formed from the difference between random variables drawn from Weibull functions $W(U, \overline{U}_n + \overline{U}_{n*}, \beta)$ and $W(U, \overline{U}_{n*}, \Delta)$.

Because U_n and U_{n*} are independent, the variance of their sum is given by

$$\frac{\pi^2}{6\beta^2} = \sigma_D^2 + \frac{\pi^2}{6\Delta^2}$$

or

$$\frac{\beta}{\Delta} = \left(1 + \frac{6\sigma_D^2\,\Delta^2}{\pi^2}\right)^{-\frac{1}{2}} \tag{46}$$

When $\sigma_D = 0$, the model collapses to the MNL, characterised by the single parameter Δ. It can also be seen that for a consistent model (and for $\Gamma(U, \overline{U}_n)$ to have a non-negative variance), we require (Williams, 1977)

$$\beta \leqslant \Delta \tag{47}$$

This condition is of particular importance, and its violation may imply cross-elasticities of the wrong sign. Violation has, in fact, been observed in conventional transport models (Williams and Senior, 1977).

We now turn to consider the choice model generated from the utility function (29). Because of the form of the random residuals, (32), we can say immediately that this model should contain as special cases the MNL and alternative HL functions. As far as the author is aware no explicit analytic function has been obtained for such a structure.

The cross-correlated logit function (CCL) was an ad-hoc model proposed by Williams (1977)(*) as a closed form approximation which corresponded to the utility function (29). It is defined by the equations.

$$P(n,m) = \frac{e^{\beta U^\dagger_n + \lambda U^\dagger_m + \Delta\bar{U}_{nm}}}{\sum_{n'm'} e^{\beta U^\dagger_{n'} + \lambda U^\dagger_{m'} + \Delta\bar{U}_{n'm'}}} \tag{48}$$

where

$$U^\dagger_n = \bar{U}_n + \frac{(\beta-\Delta)}{\beta}\bar{U}_{n*} \tag{49}$$

$$U^\dagger_m = \bar{U}_m + \frac{(\lambda-\Delta)}{\lambda}\bar{U}_{m*} \tag{50}$$

$$\bar{U}_{n*} = \frac{1}{\Delta}\ln\sum_{m'} e^{\Delta\bar{U}_{nm'} + \lambda\bar{U}_{m'}} \tag{51}$$

$$\bar{U}_{m*} = \frac{1}{\Delta}\ln\sum_{n'} e^{\Delta\bar{U}_{n'm'} + \beta\bar{U}_{n'}} \tag{52}$$

and

$$\frac{\beta}{\Delta} = \left(1 + \frac{6\sigma_D^2\,\Delta^2}{\pi^2}\right)^{-\frac{1}{2}} \tag{53}$$

$$\frac{\lambda}{\Delta} = \left(1 + \frac{6\sigma_M^2\Delta^2}{\pi^2}\right)^{-\frac{1}{2}} \tag{54}$$

(*) In that paper (section 5.3.2, pp. 321-323), the function was denoted General Choice Model. More recently, and in deference to the general probit model and to the class of General Extreme Value (GEV) models (McFadden, 1979), the function has been rechristened appropriately.

It may be checked that as σ_D^2 and σ_M^2, the variances of the residuals ε_n and ε_m, tend to zero the respective hierarchical logit models are formed. If both variances are zero, the CCL collapses to the multinomial logit form (10).

In summary, we note that within the framework of random utility theory in which behaviour if governed by rational choice between discrete alternatives, the structure of the model is determined uniquely by the underlying utility functions, and the structure of correlation or similarity between alternative choices is the essential feature which dictates the complexity of the model. Varying degrees of similarity may be accommodated within the logit family. To conclude this Appendix, we will now briefly introduce two other models which have been proposed to cope with some of the problems brought about by correlation and taste variations.

The generalised extreme value (GEV) family of models, recently proposed by McFadden (1979), allows a fairly general pattern of dependence among alternatives while keeping the choice probabilistics in a closed form. The joint distribution function of the error terms for this model is:

$$F(\underline{\varepsilon}) = \exp \{-G (e^{-\varepsilon_1}, \ldots, e^{-\varepsilon_N}) \qquad (55)$$

where G is a non-negative, homogenous-of-degree-one function (McFadden, 1979). It can be shown that (55) yields probabilities of the form:

$$P_{tn} = \exp (\bar{U}_{tn}) \cdot G_n (e^{\bar{U}_{t1}}, \ldots, e^{\bar{U}_{tN}}) / G (e^{\bar{U}_{t1}}, \ldots, e^{\bar{U}_{tN}}) \quad (56)$$

where:

P_{tn} = probability that individual t selects alternative A_n out of the set $\underline{A}$ of available options.

G_n = derivative of G with respect to $\exp (\bar{U}_{tn})$.

Note that the special case

$$G = \sum_{n=1}^{N} \exp(\bar{U}_{tn}) \tag{57}$$

yields the MNL model. Similarly, although not simply in the latter case, expressions can be found for the nested logit and cross-correlated logit models (Williams and Ortuzar, 1980).

GEV models are well suited for tree-like decision structures (Sobel, 1980 has noted that the most useful GEV form is in fact the nested logit model), but do not allow for cross-sectional taste variations (Bouthelier, 1978).

The CRA hedonics model (Cardell and Reddy, 1977) has been developed as an extension of the MNL to cope with taste variations, by treating the model parameters to be estimated as random variables. If the utility of alternative n is written in its usuel LPAD form

$$U_n = \bar{U}_n + \varepsilon_n$$

$$= \sum_{\mu} Z_n^{\mu} \theta_{\mu} + \varepsilon_n$$

the CRA hedonics model assumes that the ε_n are IID Weibull random variables, just as the MNL does, but it further assumes that the parameters $\underline{\theta}$ are random variables with <u>any</u> specified well-behaved distribution.

The probability that individual t will choose alternative n takes, in this model, an hybrid form with a complexity that lies in between those for the MNL (10) and MNP (20) models. First define:

$$P_n(\underline{\theta}) = \frac{\exp\{\bar{U}_n(\underline{\theta}, \underline{Z}_n)\}}{\sum_{n=1}^{N} \exp\{\bar{U}_n(\underline{\theta}, \underline{Z}_n)\}} \tag{58}$$

The variable $P_n(\underline{\theta})$ is simply the logit choice probability given that the parameter vector is $\underline{\theta}$. The choice probability for the model is then given by

$$P_n = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} P_n(\underline{\theta}) \cdot f(\underline{\theta})\, d\underline{\theta} \tag{59}$$

where $f(\underline{\theta})$ is the probability density function of the parameters of the individual utility functions. If we may quote Cardell and Reddy (1977),

> "...This expression implies that the choice probability in the model is simply the expected value of the choice probability of the logit model, where the expectation is made over the parameters. As a result, the logit model is a special case of the CRA hedonics model"...

Expression (59) is evaluated through Monte-Carlo methods by simply specifying a distribution function for the parameter vector $\underline{\theta}$. The approach is computationally and conceptually straightforward, although ..."it is somewhat time consuming..." (Cardell and Reddy, 1977).

Although both the MNP and CRA hedonics model permit variations in tastes across individuals (they are thus 'random coefficient models'), the former is considerably more general because it does not constraint the ε_n to be IID Weibull variables, but permits them to be correlated and with unequal variances. Other differences, advantages and disadvantages are discussed at length by Cardell and Reddy (1977). We will look at the MNP in more detail in Appendix 3.3.

STRUCTURE	VARIANCE – COVARIANCE MATRIX (Σ)	ALTERNATIVE NOTATION
$\varepsilon(D,M) = \varepsilon_{DM}$	$\begin{bmatrix} \sigma_{DM}^2 & 0 & 0 & 0 \\ 0 & \sigma_{DM}^2 & 0 & 0 \\ 0 & 0 & \sigma_{DM}^2 & 0 \\ 0 & 0 & 0 & \sigma_{DM}^2 \end{bmatrix}$	$\underline{\underline{\Sigma}} = \sigma_{DM}^2 \underline{\underline{I}}$
$\varepsilon(D,M) = \varepsilon_D + \varepsilon_{DM}$	$\left[\begin{array}{cc:cc} (\sigma_D^2+\sigma_{DM}^2) & \sigma_D^2 & 0 & 0 \\ \sigma_D^2 & (\sigma_D^2+\sigma_{DM}^2) & 0 & 0 \\ \hdashline 0 & 0 & (\sigma_D^2+\sigma_{DM}^2) & \sigma_D^2 \\ 0 & 0 & \sigma_D^2 & (\sigma_D^2+\sigma_{DM}^2) \end{array}\right]$	$\underline{\underline{\Sigma}} = \left[\begin{array}{c:c} \underline{\underline{\Sigma}}_1 & 0 \\ \hdashline 0 & \underline{\underline{\Sigma}}_1 \end{array}\right]$ $\underline{\underline{\Sigma}}_1 = (\sigma_D^2+\sigma_{DM}^2)\begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$ $\rho = \sigma_D^2/(\sigma_D^2+\sigma_{DM}^2)$
$\varepsilon(D,M) = \varepsilon_{DM} + \varepsilon_M$	$\left[\begin{array}{cc:cc} (\sigma_{DM}^2+\sigma_M^2) & 0 & \sigma_M^2 & 0 \\ 0 & (\sigma_{DM}^2+\sigma_M^2) & 0 & \sigma_M^2 \\ \hdashline \sigma_M^2 & 0 & (\sigma_{DM}^2+\sigma_M^2) & 0 \\ 0 & \sigma_M^2 & 0 & \sigma_{DM}^2+\sigma_M^2 \end{array}\right]$	$\underline{\underline{\Sigma}} = \left[\begin{array}{c:c} \underline{\underline{\Sigma}}_1 & \underline{\underline{\Sigma}}_2 \\ \hdashline \underline{\underline{\Sigma}}_2 & \underline{\underline{\Sigma}}_1 \end{array}\right]$ $\underline{\underline{\Sigma}}_1 = (\sigma_{DM}^2+\sigma_M^2)\,\underline{\underline{I}}$ $\underline{\underline{\Sigma}}_2 = \sigma_M^2 \underline{\underline{I}}$
$\varepsilon(D,M) = \varepsilon_D + \varepsilon_{DM} + \varepsilon_M$	$\left[\begin{array}{cc:cc} (\sigma_D^2+\sigma_{DM}^2+\sigma_M^2) & \sigma_D^2 & \sigma_M^2 & 0 \\ \sigma_D^2 & (\sigma_D^2+\sigma_{DM}^2+\sigma_M^2) & 0 & \sigma_M^2 \\ \hdashline \sigma_M^2 & 0 & (\sigma_D^2+\sigma_{DM}^2+\sigma_M^2) & \sigma_D^2 \\ 0 & \sigma_M^2 & \sigma_D^2 & (\sigma_D^2+\sigma_{DM}^2+\sigma_M^2) \end{array}\right]$	$\underline{\underline{\Sigma}} \left[\begin{array}{c:c} \underline{\underline{\Sigma}}_1 & \underline{\underline{\Sigma}}_2 \\ \hdashline \underline{\underline{\Sigma}}_2 & \underline{\underline{\Sigma}}_1 \end{array}\right]$ $\underline{\underline{\Sigma}}_1 = (\sigma_D^2+\sigma_{DM}^2+\sigma_M^2)\begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$ $\rho = \sigma_D^2/(\sigma_D^2+\sigma_{DM}^2+\sigma_M^2)$ $\underline{\underline{\Sigma}}_2 = \sigma_M^2 \underline{\underline{I}}$

REPRESENTATION

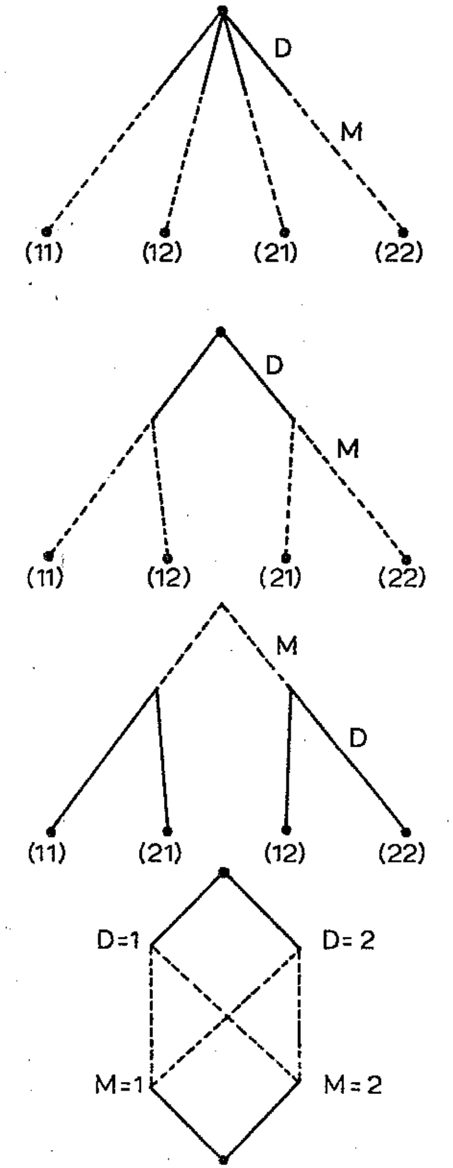


FIGURE A-2: The structure of choice models: a set of special cases.

Appendix 3.2: Statistical estimation in general

In this section we will assume that the modeller has gathered, following a certain sampling rule, information on the actual choices (e.g. alternative Ai, from the choice set $\underline{A}(q) \varepsilon \underline{A}$) of individuals q, and information on choice influencing variables Z_{iq}^{k} (these may be level-of-service attributes of the options and/or socioeconomic characteristics of the individual). In passing note that the issue of sampling method is a very important one, because although disaggregate models are certainly more efficient than traditional methods in the use of the data, to achieve their full capabilities they usually need better and more expensive information. This has attracted considerable attention recently and there are now firm grounds to believe that choice -based samples (given that the aggregate shares of each alternative are known) should be preferred to other methods (Lerman and Manski, 1979; Manski and Lerman, 1977).

The most widely used and more strongly recommended estimating procedure is maximum likelihood estimation (MLE) (Jansen et al., 1979; McFadden, 1976; 1979). This technique looks at the probability of obtaining the Q independent choices, Cq, q=1, ..., Q, given the model (along with its parameters $\underline{\theta}$): P(Cq, $\underline{\theta}$). Then the probability of obtaining the observations C_1, C_2 ..., C_Q is

$$L(C_1, \ldots, C_Q, \underline{\theta}) = \prod_{q=1}^{Q} P(C_q, \underline{\theta}) \qquad (1)$$

The usual way of looking at this function is to regard the vector of parameters $\underline{\theta}$ as known and L as a set of probabilities over possible observations. However, in the estimation context, the observations are known and $\underline{\theta}$ is unknown. When L is regarded as a function of $\underline{\theta}$ for given (observed) Cq, q=1, ...Q, it is called the likelihood function and is normally written as L($\underline{\theta}$), for short. Note that the observed dependent variable takes a value of 0 or 1. This brings in some problems for assessing goodness of fit, as it will be discussed below.

Assuming that L($\underline{\theta}$) is well behaved, it is possible to find a unique set of estimates of $\underline{\theta}$, $\hat{\underline{\theta}}$ which brings L($\underline{\theta}$) to a maximum. Such value depends on the observations. Now, if $\hat{\underline{\theta}}$ is that value of $\underline{\theta}$ that brings L($\underline{\theta}$) to a maximum and we define

$$\ell(\underline{\theta}) = \ln L(\underline{\theta}) \tag{2}$$

and $$\underline{\underline{V}} = \{\frac{\partial^2 \ell(\underline{\theta})}{\partial \underline{\theta}^2}\}^{-1} \tag{3}$$

then, on the assumption that the model correctly describes the data, $\hat{\underline{\theta}}$ is an asymptotically efficient estimator of $\underline{\theta}$ and is asymptotically distributed as Normal, $N(\underline{\theta},\underline{\underline{V}})$. Moreover $-2.\ell(\hat{\underline{\theta}})$ is asymptotically distributed χ^2 (Chi-squared) with Q degrees of freedom. This means that although $\hat{\underline{\theta}}$ may be biased for small samples, the bias is small for large enough Q (just how large is "large enough" is a function of the problem under examination, but generally data sets with 500 to 1000 observations have been found to be sufficient). The estimator $\hat{\underline{\theta}}$ is the best possible for large samples (McFadden, 1976) and there is a concrete expression $\underline{\underline{V}}$ for its variance-covariance matrix. Note, however, that for most model forms, including the easy to handle logit model, $\hat{\underline{\theta}}$ must be calculated by an iterative procedure. Fortunately $\underline{\underline{V}}$ is useful in this iterative calculation and is thus available when convergence occurs.

A word of caution is in order here, although it is well known that for a logit model with linear-in-parameters specification $\ell(\hat{\underline{\theta}})$ is well behaved, this has not been proven for probit models, except for the simplest independent binary case. Indeed it has been noted that the most widely used and efficient MNP estimation computer code available, may have problems in that the approximation to $\ell(\theta)$ used is not necessarily unimodal (Bouthelier 1978, Daganzo and Schoenfeld, 1978). We discuss this in more detail in Appendix 3.3.

The well understood properties of the maximum likelihood estimation method, for well behaved likelihood functions, allow a number of statistical significance tests which are of major importance:

i) <u>The t-test for significance of any component $\hat{\theta}_k$ of $\underline{\theta}$</u>

Equation (3) implies that $\hat{\theta}_k$ has an estimated variance $V_{kk'}$, where $\underline{\underline{V}} = (V_{kk'})$ which is calculated by the estimating program. Thus if $\theta_k = 0$,

$$t = \hat{\theta}_k V_{kk}^{-1} \tag{4}$$

is distributed Normal, N(0,1). For this reason, it is possible to test whether it is significantly different from zero (it is not exactly a t-test as this is a large sample approximation - t is tested with the Normal distribution). Large absolute values of t (e.g. bigger than 1.96 for 95% confidence levels) lead to the rejection of the null hypothesis and hence to accept that $\hat{\theta}_k$ is significantly different from zero.

ii) The likelihood ratio test of linear restriction of any general hypothesis

A number of important model properties can be expressed as linear restrictions on a more general linear-in-parameters model. Some important examples of such properties are:

Attribute genericity: There are two main types of explanatory variables, 'generic variables' and 'alternative-specific' variables. The former vary in value (or level) across choice alternatives, whereas the latter are those with an identifiable correspondence between choice alternatives, and because they may not vary across all alternatives, they can take on a zero value for certain elements of the choice set. Let us assume a model with three alternatives, car, bus and rail, and the following choice influencing variables:

TT = travel time OPC = out-of-pocket travel costs

Then, a general form of the model could be:

$$\bar{U}_{car} = \theta_1 \, OPC_{car} + \theta_2 \, TT_{car}$$
$$\bar{U}_{bus} = \theta_3 \, OPC_{bus} + \theta_4 \, TT_{bus}$$
$$\bar{U}_{rail} = \theta_5 \, OPC_{rail} + \theta_6 \, TT_{rail}$$

However, it might be hypothesised that costs should be generic. This can be expressed by writing the hypothesis as two linear equations in the parameters:

$$\theta_3 - \theta_1 = 0$$
$$\theta_5 - \theta_1 = 0$$

In general, it is possible to express attribute genericity by linear restrictions on a more general model. For extensive use of this type of test refer to Talvitie and Kirschner (1978).

Sample homogeneity: It is possible to test if the same model coefficients are appropriate for two subpopulations. For this, one formulates a general model using different coefficients for the two populations, and then equality of the coefficients is a linear restriction.

Because of the properties of the MLE, it is very easy to test any such hypothesis expressed by linear restrictions, by means of the well-known <u>likelihood ratio test</u>. To perform the test, the estimation program is first run in the more general case to give the estimates $\hat{\underline{\theta}}$ and log-likelihood at convergence $\ell^*(\hat{\underline{\theta}})$. Is is then run again to obtain estimates $\hat{\underline{\theta}}_r$ of $\underline{\theta}$, for the restricted case and the new log-likelihood at maximum $\ell^*(\hat{\underline{\theta}}_r)$. Now the likelihood ratio statistic is

$$-2\ \{\ell^*(\hat{\underline{\theta}}_r) - \ell^*(\hat{\underline{\theta}})\}$$

which is distributed as χ^2 with K - r degrees of freedom, where K is the number of elements in $\underline{\theta}$ and r is the number of linear restrictions.

iii) <u>The overall test of fit and the Rho square index</u>

A special case of likelihood ratio test is to find out whether all components of $\underline{\theta}$ are equal to zero (equally likely model), or better, if those components of $\underline{\theta}$ which do not correspond to model constants are equal to zero (best null model). Let us consider the first case, which is the most common and obvious one, to begin with:

If there are K parameters and $\ell^*(0)$ is the log-likelihood of the equally likely model, this means testing $-2.\{\ell^*(0) - \ell^*(\hat{\theta})\}$ which is distributed χ^2 with K degrees of freedom. Note that $\ell^*(0)$ does not require a special program run. It is usually calculated as the initial log-likelihood at the start of the program. This test is actually rather weak; if rejected it only says that the model with parameters $\underline{\theta}$ provides a better explanation of the data than a model which does not have any significant explanatory power (the equally likely model). It is obvious that when the model contains alternative-specific constants, the test in this simplest form is not appropriate. It is more relevant to test, as suggested above, whether the explanatory variables add anything to the explanation given by the constants alone (the best null model). It is rather discouraging to note that constants tend to account for 60% to 80% of the explanatory power of these models (Talvitie and Kirschner, 1978).

In general, an extra run is required to calculate $\ell^*(C)$, the log-likelihood of the model containing only alternative-specific constants, except for logit models when all individuals face the same alternatives where it has the following closed form equation:

$$\ell^*(C) = \sum_{j=1}^{J} Q_j \, \ell n \frac{Q_j}{Q} \tag{5}$$

where

Q_j = number of individuals choosing alternative A_j

It is felt by many that a coefficient of goodness of fit is useful. However, since we do not observe probabilities but (0,1) decisions, a goodness fit like R^2 in ordinary least squares, which is based on estimated residuals, does not exist. A goodness of fit coefficient should range from 0 to 1 (no fit, to perfect fit), be meaningful for comparing models calibrated with different samples, and hopefully be related to a statistic with a known probability distribution for purposes of statistical hypothesis testing. Such an index has been defined (McFadden, 1976) as

$$\rho^2 = 1 - \frac{\ell^*(\hat{\theta})}{\ell^*(0)} \tag{6}$$

However, it has been noted that although ρ^2 behaves nicely at the limits (e.g. 0 and 1), it does not have an intuitive interpretation between the limits (Hauser, 1978). A quotation by McFadden (1976) may be appropriate at this point:

> ..."Those unfamiliar with the ρ^2 index should be forewarned that its values tend to be considerably lower than those of the R^2 index (of regression analysis) and should not be judged by the standards for "good fit" in ordinary regression analysis. For example, values of 0.2 to 0.4 for ρ^2 represent an excellent fit"...

Because a ρ^2 - like index can in principle be computed relative to any null hypothesis, it is important to choose an appropriate one. For example, it is very easy to show that the minimum values of ρ^2 (with respect to the equally likely model), in models with alternative - specific constants, vary depending on the proportion of individuals choosing each alternative. Taking a simple binary case, Table A-8 (Tardiff 1976) shows the minimum values of ρ^2 for different proportions choosing option 1. It can be seen that ρ^2 is only appropriate for the 50/50 per cent case.

Sample proportion selecting the first alternative	Minimum value of ρ^2
0.5	0.00
0.6	0.03
0.7	0.12
0.8	0.28
0.9	0.53
0.95	0.71

Table A-8: Minimum value of ρ^2 for various relative frequencies (Source, Tardiff, 1976)

These values mean, for example, that a model calibrated with a 0.9/0.1 sample, yielding a ρ^2 of 0.55 would undoubtedly be much weaker than a model yielding a ρ^2 of 0.25 from a sample with a 0.5/0.5 split.

Fortunately, a rather simple adjustment exists (Tardiff, 1976) that overcomes these difficulties. This consists of defining a more appropriate index $\bar{\rho}^2$ as

$$\bar{\rho}^2 = 1 - \frac{\ell^*(\underline{\theta})}{\ell^*(C)} \tag{7}$$

This statistic lies between 0 and 1, is comparable across different samples and is also related to the χ^2 statistic; therefore it is recommended over ρ^2.

iv) <u>Other measures of goodness of fit</u>

McFadden (1976) mentions in his work a series of possible measures: Hauser (1978) has also given considerable thought to the problem. We will however only mention one other measure, this is the "percentage correctly predicted", or "percent right" for short. It is simply computed as follows: using the final model parameters, compute, for each individual, the predicted utilities and check if the largest corresponds to the chosen alternative. The "percent right" is the sum of all those cases where this happens, over the total number of cases.

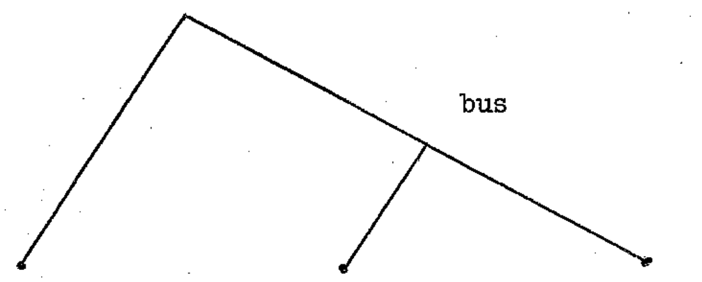
v) <u>Other issues</u>

This appendix gives only an introduction to the complex problem of model estimation and in general 'specification searches' (Leamer, 1978). Recently, two very good papers have treated in more detail aspects like the use of more powerful tests than the ones reported here: some may involve grouping the data ('saturated test'); others show how to get more information from the distribution of errors assumed in the model;

how to compare 'non-nested' models, i.e. those where the parameters of one model are not a subset of another as assumed in point iii); etc. The interested reader is referred to the papers by Gunn and Bates (1980); Horowitz (1980b), and Dagenais, Gaudry and Liem (1980).

vi) <u>Estimation of the nested logit model</u>

In Appendix 3.1, we studied a generalisation of the multinomial logit (MNL) model, the nested or hierarchical logit model (Williams, 1977; Daly and Zachary, 1978) which does not have the IIA restriction. If we take the well known red bus/blue bus case, as a simple example, a nested logit model would proceed in two stages. Firstly, a primary split between car (c) and a 'composite' bus mode (b) and secondly a sub split between the two bus options (rb and bb respectively) as shown in Figure A-3.



<u>Figure A-3: A simple nested logit model.</u>

In this situation, individuals are, as in the case of the MNL, conceptually assumed to evaluate each alternative according to utility functions Uc, Urb and Ubb (with representative components $\bar{U}c$, $\bar{U}rb$ and $\bar{U}bb$) as we discussed at length in Appendix 3.1.(*) However, in this case we have also to consider a composite utility of the lower hierarchy or 'nest'. This composite utility includes the expected value of the maximum utility of the members of the nest, given by

$$I_b = \ln\{\exp(\bar{U}_{rb}) + \exp(\bar{U}_{bb}) \qquad (8)$$

The composite utility of bus is then

$$\bar{U}_b = \alpha I_b + \underline{\theta}\,\underline{Z}_b \qquad (9)$$

where α is an estimated coefficient, $\underline{\theta}$ is a vector of estimated coefficients and $\underline{Z}_b$ is a vector of attributes common to all the members

(*) Although note that in this case we are using a different notation for the nested logit model.

of the nest.

The nested logit model can be thus estimated with standard MNL software in two stages: firstly, a binary logit model between red bus and blue bus, the results of which allow us to calculate I_b from (8); then this value is entered as another independent variable along with the $\underline{Z}_b$ variables and the attributes of car in the primary split which in this simple case is another binary logit model. The secondary split will thus yield P(rb/b) and P(bb/b), the conditional probabilities of red bus or blue bus given that the choice is constrained to bus. The primary split yields P(c) and P(b), the marginal probabilities of car and bus respectively. It is clear that the probabilities of each mode are:

$$\left.\begin{aligned} P\ \text{car} &= P(c) \\ P\ \text{red bus} &= P(b).\ P(rb/b) \\ P\ \text{blue bus} &= P(b).\ P(bb/b) \end{aligned}\right\} \qquad (10)$$

An important feature of the model concerns acceptable values of α, the coefficient of the expected maximum utility of the nest. Williams (1977), (and see Williams and Ortuzar, 1980 for a full discussion) has shown that α must satisfy:(*)

$$0 < \alpha \leqslant 1 \qquad (11)$$

Furthermore, it has also been shown that if there are more than two levels of nesting, e.g. a case with more composite utilities and coefficients α, then

$$0 < \alpha_1 \leqslant \alpha_2 \leqslant \alpha_3 \leqslant \ldots\ldots \leqslant 1 \qquad (12)$$

where α_1 represents the coefficient of the expected maximum utility of the 'lowest' hierarchy. Note also that any hierarchical level, a value of $\alpha_i = 1$ implies that the limited nesting at level i is mathematically equivalent to a simple MNL at that level - for a good discussion of these issues see Sobel (1980), who has shown that for nested logit models there exist equivalent measures to the ρ^2 and $\bar{\rho}^2$ indices (eqs. (6) and (7)) given by:

---

* This is equivalent to condition (47) of Appendix 1.

$$\rho^2 = 1 - \frac{\ell_1^*(\hat{\theta}) + \ell_2^*(\hat{\theta}) + \ldots + \ell_j^*(\hat{\theta})}{\ell_1^*(0) + \ell_2^*(0) + \ldots + \ell_j^*(0)} \qquad (13)$$

and

$$\rho^{-2} = 1 - \frac{\ell_1^*(\hat{\theta}) + \ell_2^*(\hat{\theta}) + \ldots + \ell_j^*(\hat{\theta})}{\ell_1^*(C) + \ell_2^*(C) + \ldots + \ell_j^*(C)} \qquad (14)$$

where the subscripts 1 to j refer to the simple MNL models in the hierarchy of interest.

Notwithstanding the simplicity of the 'heuristic' or 'bottom up' (Williams,1977) calibration of the nested logit model it is known that the consequence of sequential estimation is a loss of statistical efficiency which may be severe (Daly and Zachary, 1978; Sobel, 1980). This results because the standard errors of lower level coefficient estimates permeate from lower hierarchies upwards embedded in the values of the expected maximum utilities I. When there are multiple hierarchies, successively 'higher' level I's will

> "...contain greater and greater proportions of random statistical 'noise'..." (Sobel, 1980)

For this reason it has been argued the necessity of a simultaneous estimation routine which would eliminate the compounding effects of these errors, thereby improving the statistical efficiency of the estimates of the α's.(*) Another powerful reason for such a software is the unpleasant possibility of obtaining different estimates for the same parameter at different hierarchical levels (which is quite possible due to different amounts of data used in each). An experimental simultaneous estimation software has been developed by Berkman, Brownstone et al. (1979), although so far it is only capable of dealing with a particular version of the nested logit model. However we understand at present it is being generalized.

(*) Remember also how crucial the α's are in the structural diagnosis of the model, i.e. conditions (11) and (12).

Appendix 3.3: Estimation and solution of the multinomial probit model

As we mentioned in Appendix 3.1, the multinomial probit model (MNP) can be stated as:

$$P_n = \int_{-\infty}^{\overline{U}_{1n}+\varepsilon_n} \int_{-\infty}^{\overline{U}_{2n}+\varepsilon_n} \cdots \int_{-\infty}^{\infty} \int_{-\infty}^{\overline{U}_{Nn}+\varepsilon_n} \cdots f(\underline{\varepsilon})\, d\underline{\varepsilon} \qquad (1)$$

where: $\overline{U}_{nn'} = \overline{U}_{n'} - \overline{U}_n$ (2)

and $f(\underline{\varepsilon}) = (2\pi)^{-\frac{N}{2}} |\underline{\underline{\Sigma}}|^{-\frac{1}{2}} \exp\{-\frac{1}{2}(\underline{U}-\underline{\overline{U}})^T \underline{\underline{\Sigma}}^{-1} (\underline{U}-\underline{\overline{U}})\}$ (3)

We also mentioned that, although completely general in terms of its theoretical statement, it is considerably cumbersome to implement. It has been known for some time that direct numerical integration, other than for 'small' problems involving 3 or 4 options is extremely difficult if at all possible (Hausman and Wise, 1978). This has led to the formulation of approximation schemes. One method involves Monte-Carlo simulation directly to evaluate the model (Albright et al, 1977).

The method is elegant, theoretically appealing and has the advantage of being completely general, in the sense that in principle any function can be integrated. However, it is not well suited for optimisation purposes near the neighbourhood of the optimum, it is biased, and very slow and expensive to use. (Bouthelier, 1978).

The second method, due to Daganzo et al (1977) invokes the Clark (1961) approximation, which essentially involves the replacement of the maximum of bivariate normal variables by one normally distributed variable. By repeated application of the Clark approximation, the multiple integral in Equation (1) may be reduced to a particular univariate integral.(*) When the correlation between variables is non-negative, this approximation which has been extensively examined by Manski and Lerman (1978), using Monte Carlo simulation, is apparently

---

* In this approach, only a somewhat restricted version of the MNP can be estimated though. It involves 'fixed parameters'.

accurate to a few per cent, for up to 10 alternatives. However, problems with the possible existence of multiple optima associated with the likelihood function of MNP models, for more than 2 alternatives, have recently been reported (Daganzo, 1979). These imply that in general, there is no guarantee that the model in its more general form can be calibrated.(*) The estimation of MNP models have been recently reviewed comprehensively by Sheffi, Hall and Daganzo (1980) to whom we refer the interested reader. Before leaving this Appendix we just wish to comment briefly on the use of transformations for solving MNP models.

When encountering normally distributed variables, it has often been the case that a transformation to a co-ordinate system in which the structure of variation in a data set is more appropriately described, has provided not only insight into the nature of factors giving rise to the variation, but has also formed the basis for approximation schemes. Principal component analysis is perhaps the best such example. (For a very didactic treatment of transformation theory in multivariate analysis, see Green and Carroll, 1976). Moreover, it is well known that the MNL and an uncorrelated, equal variance probit model (with suitably normalised standard deviation) are almost indistinguishable. That is, if we could transform general probit models into equivalent functions with diagonal variance-covariance matrices, it might be possible to establish conceptual links with the logit family, and in the process erase the burden of numerical integration.

In general, under the transformation

$$\underline{U} \longrightarrow \underline{T} \qquad (4)$$

the expression for P_n given by (5)

$$P_n = \int_{R_n} f(\underline{U})d\underline{U} \qquad (5)$$

becomes

$$P_n = \int_{R_{n*}} h(\underline{T})|J|d\underline{T} \qquad (6)$$

(*) Other methods have been proposed by British investigators (Langdon, 1976; 1978; Harrison, 1977; and Harrison and Cullingford, 1978; with a critique by Baker, 1978), but none has been implemented.

in which $h(\underline{T})$ is the transformed density function, J is the Jacobian and R_{n*} the new region of integration.

In the probit model (1), the algebraic manipulations and geometric interpretations of the required transformations are essentially those of principal component analysis. The surfaces of constant density in $\underline{\varepsilon}$-space are this time ellipsoids, given by the quadratic form.

$$QF = \underline{\varepsilon}^T \underline{\underline{\Sigma}}^{-1} \underline{\varepsilon} = \text{constant} \tag{7}$$

We wish to invoke an orthogonal transformation

$$\underline{T} = \underline{\underline{\Lambda}}\,\underline{\varepsilon} \tag{8}$$

such that the vectors $\underline{V}_1, \underline{V}_2, \ldots, \underline{V}_N$, which are the columns of $\underline{\underline{\Sigma}}$, are the principal axes of the ellipsoid. In the new coordinate system, the transformed matrix $\tilde{\underline{\underline{\Sigma}}}$, is written

$$\begin{aligned} \tilde{\underline{\underline{\Sigma}}} &= \underline{\underline{\Lambda}}\,\underline{\underline{\Sigma}}\,\underline{\underline{\Lambda}}^T \\ &= \begin{pmatrix} \lambda_1 & 0 & - & - & - & - & 0 \\ 0 & \lambda_2 & - & - & - & - & 0 \\ 0 & 0 & - & - & - & - & 0 \\ \vdots & & & & & & \\ \vdots & & & & & & \\ 0 & - & - & - & - & 0 & \lambda_N \end{pmatrix} \end{aligned} \tag{9}$$

in which $\lambda_1, \ldots, \lambda_N$ are the eigenvalues of $\underline{\underline{\Sigma}}$. The eigenvalues and corresponding eigenvectors are determined from the usual equation

$$\underline{\underline{\Sigma}}\,\underline{V}^r = \lambda_r\,\underline{V}^r \qquad r = 1, \ldots, N \tag{10}$$

The quadratic form (7) may now be written

$$QF = \sum_r \frac{T_r^2}{\lambda_r} \tag{11}$$

and the transformed probit model becomes

$$P_n = \int_{R_{n*}} \prod_r \exp\left(-\tfrac{1}{2}\,\frac{T_r^2}{\lambda_r}\right).d\underline{T} \tag{12}$$

the Jacobian of the orthogonal(*) transformation being unity.

The transformed region of integration becomes

$$R_{n*}:\bar{U}_n + (\underline{\underline{A}}^T\underline{T})_n > \bar{U}_{n'} + (\underline{\underline{A}}^T\underline{T})_{n'} \;,\; \forall A_n \varepsilon \underline{A} \qquad (13)$$

which is quite an unhospitable region involving all components of $\underline{T}$ on both sides of the inequality without possibilities of simplification, and therefore rendering useless the effort to decompose the multivariate density function (1) into the product of univariate functions (12).

An attempt to solve probit models with symmetric less-general covariance matrices (as had been discussed in Appendix 3.1 for the extended members of the logit family), by the transformation method, proved unsuccessful (Ortuzar, 1979) and will not be discussed here.

(*) Variance coveriance matrices are especially well-behaved. They are square symmetric and positive semidefinite. All their eigenvalues are real and non-negative, the transformations that diagonalise them are orthogonal, and further, their inverse is equal to their transpose. (Green and Carroll, 1976).

Appendix 3.4: Description of the software available at Leeds

The University of Leeds has acquired two disaggregate model calibration packages, MLOGIT, developed at MIT and CHOMP, released by the University of California at Berkeley.

i) MLOGIT

This is a MNL calibration program. The original computer code was written by C.F. Manski and later modified by M.E. Ben-Akiva (1973). The present program has been slightly streamlined and improved at Leeds. A more complete description of the program has been given by Howe and Liou (1975).

The program is written in FORTRAN and employs a Newton-Raphison iterative technique to determine parameter values which maximize the likelihood of a binary or multi-nomial logit function. Given the convexity of the likelihood function of linear-in-parameters logit models, the method always converge and to appropriate values.

At present the program can handle up to twenty parameters, seven alternatives and any number of observations. The independent variables can be continuous and/or discrete, and the number and characteristics of alternatives can vary from observation to observation. Core requirements are small. Time requirements increase fairly linearly with the number of observations processed, the average size of choice sets and the number of iterations performed. Time increases somewhat less than with the square of the number of parameters. If we may quote Ben-Akiva (1973)

> ..."As a rule of thumb, each iteration on a purely binary logit problem requires twice the time needed for a linear regression having the same number of observations and variables"...

The CPU times at Leeds after having it tried with problems containing 3 alternatives, 7 parameters and a 100 observations have always been less than 10 secs of an ICL 1906A.

The program produces as output, on each iteration, the current log-likelihood value; the coefficient estimates; their standard errors and t-ratios; the changes in the values of the coefficients relative to the previous iteration; values of the first derivatives; and an estimate of their variance-covariance matrix. At convergence it gives also the likelihood ratio relative to the equally likely model, the percent of choices correctly predicted, and a print out of the identification and values of the probabilities of the chosen option for those cases predicted incorrectly, in order to check for bias.

ii) CHOMP

This package is designed to estimate and predict with a MNP, and has also the capability of estimating a MNL. The Leeds version is a slightly streamlined and improved code of the program released by the University of California at Berkeley (Daganzo and Schoenfeld, 1978). It is still labelled 'experimental and research oriented' because, as mentioned in Appendix 3.3, the approximation used to the log-likelihood of the MNP (which is in itself a breakthrought) is unfortunately not guaranteed to be unimodal (Bouthelier, 1978). Daganzo and Schoenfeld (1978), claim that the program

> "... will admit any specification whatsoever for the measured utilities and the variance-covariance matrices..." (*)

This versatility causes it, however, to be considerably less efficient, for similar problems, than MLOGIT. In fact, our experience is that it takes at least 2.5 and may take several times longer. However, it is considerably more user-orientated than MLOGIT and this surely contributes to the loss in efficiency. Bouthelier (1978) has also pointed out that the computational effort of the MNP approximation grows with the square of the number of alternatives and not linearly as in the MNL. The program can deal with small to medium size problems (less than 8 options) quite satisfactorily. It incorporates a 'warming up' strategy which only uses the whole of the data for the last few iterations, thus making application to larger problems feasible.

The MNP model has, in general, several more parameters than the MNL. Table A-9(Bouthelier, 1978) shows the number of parameters to be estimated by each model for different values of N, the total number of alternatives, and K, the number of parameters in the utility functions.

---

* Unfortunately, there is no guarantee that such a model could be successfully estimated (Daganzo, 1979)

K	N	MNL	MNP
10	3 4 5	10 10 10	12 15 19
15	3 4 5	15 15 15	17 20 24
20	3 4 5	20 20 20	22 25 29

Table A-9: Number of parameters to be estimated for different values of K and N (Source: Bouthelier, 1978)

In order to minimise the number of times that the log-likelihood function of the MNP is computed (which is extremely time consuming), CHOMP incorporates a much more sophisticated search algorithm than MLOGIT. Basically it consists of a feasible direction steepest ascent algorithm which performs a non-dimensional Fibbonacci search at each iteration. In order to avoid hemstitching, a variable metric algorithm which uses Davidson-Fletcher-Powell's updating formula for the inverse hessian of the likelihood function was developed (Bouthelier, 1978). At present the program claims to handle 20 alternatives, 20 parameters and 1000 observations.

The output of CHOMP has been standardised at Leeds to be basically the same as that for MLOGIT; once again because the package is more user orientated it is easier to interpret the results, to check for bias and to try alternative specifications.

We have only tested, so far, the logit capabilities of CHOMP. Also we have managed to reproduce exactly the results of an extremely simple MNP example provided in the documentation (Daganzo and Schoenfeld, 1978). We have no experience with alternative ways of specifying the variance-covariance matrix of an MNP model. We only know that some forms may prove impossible to calibrate due to their leading to badly behaved likelihood functions. The only remedy to these problems is to experiment. The last point to mention here, is that this program cannot estimate MNP models which allow for taste variation. Therefore we lack at present software to estimate random coefficient models and EBA or satisficing models.

APPENDIX 3.5: Draft Questionnaires

(i) Both questionnaire drafts shown below, must contain a letter explaining why an answer is so important, and mentioning the existence of a prize to be won in a draw made from the questionnaire replies.

(ii) Both must be designed in such a way as to have a self-addressed-business reply-prepaid side, to make life very simple to the respondent.

(iii) The main body of Questionnaire A, follows:

Part 1: WE WOULD LIKE TO KNOW DETAILS ABOUT YOUR TRIP FROM HOME TO WORK

1) What time did you leave home today? ....h .... min
2) What time did you arrive at work today? ....h .... min
3) Please indicate the means of travel you used:

Car driver ☐ : Car passenger ☐ : Car pool ☐ :

Motorcycle ☐ : Car to park and bus ☐ :

Walk to station-train-bus ☐ : car to station-train-bus ☐ :

bus-bus ☐ : other ....(please specify)

4) How many times do you travel to work by this means?

Less than 1 day/week ☐ ; 1 day ☐ ; 2 days ☐ ; 3 days ☐

4 days ☐ ; 5 days ☐ ; more than 5 days/week ☐

Part 2: IF YOUR TRIP TO WORK WAS BY CAR, PLEASE ANSWER QUESTIONS 5 TO 9 IF ANY PART OF YOUR JOURNEY WAS BY BUS AND/OR TRAIN, PLEASE ALSO ANSWER QUESTIONS 10 TO 13.

Car Users:

5) Do you need the car at work as part of your activities? No ☐ Yes ☐
6) Did you come directly to work? No ☐ ; Yes ☐ ; If No, please indicate the reason for breaking the journey: took children to school ☐ ; Went shopping ☐ ; parked car and took train ☐ ; other .... (please specify)
7) Do you drive a company car to work today? No ☐ ; Yes ☐

8) Did you have to pay parking costs: No [ ] ; Yes [ ]

9) If yes, was it paid for by your company: No [ ] ; Yes [ ]

Bus Users:

10) How did you get from home to bus stop? Drove and parked? [ ] ; driven? [ ] ; walked? [ ] ; took train? [ ] ; Other ....(please specify)

11) If you drove and parked, were there enough parking spaces? No [ ] ; Yes [ ] ; and how much did it cost to park? ....pence

Train Users:

12) How did you get from home to station? Drove and parked? [ ] driven? [ ] ; walked? [ ] ; took bus? [ ] : Other.........(please specify)

13) If you drove and parked, were there enough parking spaces? No [ ] ; Yes [ ] ; and how much did it cost to park? .... pence

Part 3: IN THE FOLLOWING TABLE THERE ARE SEVERAL MEANS OF TRAVEL: IMAGINE YOU HAVE TO MAKE YOUR TRIP TO WORK BY EACH OF THESE. PLEASE COMPLETE THE TABLE FILLING APPROPRIATE BOXES

Mode description	Walking time (min)	Waiting time (min)	Travel time in vehicle (min)	Time spent looking for parking (min)	Parking cost (pence)	Travel cost excluding parking (pence)	No. of times used in the last 6 months
1) Car driver							
2) Car passenger							
3) Car pool							
4) Drove to park and took the bus							
5) as 4), but driven							
6) Walked to bus stop took first bus, got down at park, took another bus							
7) Drove to station, took train, took bus							
8) Walked to station, took train, took bus							
9) Took bus to station, took train, took bus							

Etc.

Part 4:

THE QUESTIONS BELOW ARE ONLY USED TO DETECT DIFFERENCES AMONG PEOPLE WITH RESPECT TO HOW THEY FEEL ABOUT DIFFERENT MODES ETC.

1) Are you male [ ] ; female [ ]

2) Is your age: 12-17 [ ] ; 18-24 [ ] ....

3) How many residents in your household? One [ ] 2 [ ] 3 [ ] 4 [ ]

5 or more [ ]

4) How many cars has your household? Ø [ ] ;1 [ ] ;2 [ ];3+ [ ]

5) Do you have a driving licence? No [ ] ; Yes [ ]

6) How many workers with driving licence excluding yourself? Ø [ ]; 1 [ ]; 2 [ ]; 3 [ ]; or more [ ]

7) What is <u>your</u> monthly take-home pay (strictest confidence)

Not employed [ ] ; less than [ ] [ ] [ ] [ ] [ ]

more than [ ]

8) Your address please ....

....

....

....

9) Place of work ....

....

....

....

Part 5:

WE WOULD GREATLY APPRECIATE THE CHANCE OF CLARIFYING SOME OF THE QUESTIONS AND MAYBE ASKING SOME MORE. FOR THIS REASON WE WOULD LASTLY LIKE TO KNOW IF YOU WOULD AGREE TO A FURTHER INTERVIEW IN YOUR HOUSE. <u>NO</u> [ ] <u>YES</u> [ ]

Thank you very much for your co-operation ......

(iv) It is clear that Questionnaire A is rather complicated and we are not sure how well, if at all, it would be answered (especially the rather odd question-table of Part 3). However a similar one was very successfully employed recently in South Africa (Stopher, Wilmot et al., 1978). This questionnaire should tell us both reported <u>and</u> synthesised (from data on origins and destinations) values for the important level-of-service attributes discussed in Chapter 3, <u>plus</u> information on captivity, choice set, perceptions of attributes of rejected modes, <u>and</u> socio-economic information including income.

(v) An alternative form is Questionnaire B, below

<u>Household Questions</u>

1. How many people live in your household: One ☐ 2 ☐ 3 ☐ 4 ☐ 5+ ☐

2. How many of them travelled to work today: None ☐ 1 ☐ 2 ☐

3. How many of those that travelled to work had a driving licence:

   None ☐ 1 ☐ 2 ☐ 3 ☐

4. How many cars are owned by your household: None ☐ 1 ☐ 2 ☐ 3+ ☐

5. Please indicate income bracket of your household:

   less than ☐ ☐ ☐ ☐ more than ☐

<u>Work Journey Questions</u>

6. Do you have a driving licence No ☐ Yes ☐

7. Where is your place of work. Please give No. and street, or name of factory, town etc. ....

   ....

   ....

8. At what time did you leave home to work today?...........h .... min

   At what time did you arrive at work?........... ...h .... min

9. Here is a list of different ways of making the journey. Please tick in the first column the one you used today, and in the other columns possible modes you might have used, had your preferred one not been available:

	Preferred mode	First alt.	Second alt.	Third alt.
1.				
2.				
3.				
4.				
5.				
6.				

10. For those who drove to work

- how much did it cost to park .... pence
- was the car used during the day? No ☐ Yes ☐

11. Would you be amenable to a further interview at your home No ☐ Yes ☐ . Thanks.

All cards completed and returned will be entered in a grand prize draw. Please write your name and address below:

Name ....

Address ....

....

....

Thank you for answering. Please return this card through the post. NO STAMP IS NECESSARY.

vi) Although it is considerably simpler, it only offers the opportunity of measuring the values of the attributes, has very little information on captivity, etc. and little information on rejected modes.