# Critical Majorana Fermion at a Topological Quantum Hall Bilayer Transition

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Quantum Hall bilayers are a uniquely tunable platform that can realize continuous transitions between distinct topological phases of matter. One prominent example is the transition between the Halperin state and the Moore-Read Pfaffian, long predicted to host a critical theory of Majorana fermions but so far not verified in unbiased microscopic simulations. Using the fuzzy-sphere regularization, we identify the low-energy spectrum at this transition with the 3D gauged Majorana conformal field theory. We show that the transition is driven by the closing of the neutral fermion gap, and we directly extract the operator content in both integer and half-integer spin sectors. Our results resolve the long-standing question of the nature of a topological phase transition in a setting relevant to quantum Hall experiments, while also providing a realization of emergent fermionic fields on the fuzzy sphere, previously limited to bosonic fields.

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*Introduction*—The fractional quantum Hall (FQH) effect has long served as a fertile ground for discovering new phases of matter that defy the Landau paradigm [1,2]. It brought into the spotlight many-body phases with exotic quasiparticles called anyons [3,4], some of which could prove useful for fault-tolerant quantum computing [5,6]. A prominent example is the even-denominator state at filling factor $\nu = 5/2$ [7], believed to be described by the Moore-Read wave function [8]. The elementary excitations of the Moore-Read state are anyons with charge $e/4$ and non-Abelian braiding statistics [9,10]. Because of the existence of anyons, it is generally challenging to study topological phase transitions between different FQH states. For example, despite theoretical evidence [11,12], experiments have yet to unambiguously determine the nature of the $\nu = 5/2$ state [13–23].

Multicomponent quantum Hall systems, such as bilayers and wide quantum wells, offer a complementary platform for exploring a broader class of FQH states and transitions between them [24,25]. These systems are highly tunable, as the effective interactions and tunneling can be controlled via electrostatic gates and by varying the magnetic length; see Fig. 1(a). Such tuning can induce quantum phase transitions between FQH phases, including the one at filling $\nu = 1/2$, observed in numerous experiments [26–31]. The transition is believed to occur between a two-component Halperin state [32] and a single-component Moore-Read state [33–43] [Fig. 1(b)]. Both of these gapped phases represent weakly paired states of composite fermions [44]. The tunneling-driven critical point between

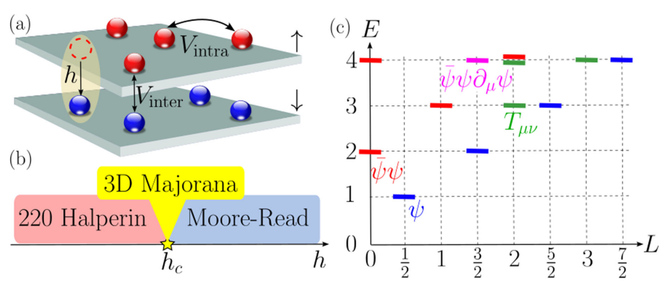


FIG. 1. (a) Quantum Hall bilayer system with the layers ↑, ↓. The particles interact via intralayer ($V_{\rm intra}$) and interlayer ($V_{\rm inter}$) interactions, and they can tunnel between the layers with an amplitude $h$. (b) At a critical tunneling $h_c$, the bosonic particles at filling $\nu = 1$ undergo a transition between the 220 Halperin state and the Moore-Read state. (c) At criticality, the energy spectrum on the fuzzy sphere, resolved as a function of angular momentum, matches that of the free 3D Majorana fermion conformal field theory, with the characteristic towers of primary fields (labeled) and their descendants.

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them is predicted to be governed by a massless Majorana fermion [10,45–47]—the long-sought particle that is its own antiparticle [48,49]. While the $\nu = 1/2$ transition was analyzed at the level of mean-field theory in Ref. [10] more than two decades ago, its identification in a microscopic model has remained elusive.

In this Letter, we employ the recently developed "fuzzy-sphere" regularization [50] to provide the first microscopic verification of the 3D gauged Majorana conformal field theory (CFT) emerging in the quantum Hall bilayer in the presence of interlayer tunneling. To minimize finite-size effects, we assume bosonic particles at filling $\nu = 1$ and study the transition between the two-component 220 Halperin (i.e., akin to two decoupled Laughlin states at $\nu = 1/2$) and a single-component Moore-Read state. We numerically demonstrate the closing of the gap and extract the operator content of the CFT, including both integer and half-integer spin sectors, as summarized in Fig. 1(c). These results establish fingerprints of an emergent Majorana fermion in a setting applicable to quantum Hall experiments. Beyond quantum Hall, they also open a new direction for the fuzzy-sphere studies of 3D CFTs, which so far have been restricted to bosonic fields.

*Gauged Majorana field theory*—The continuous Halperin-Pfaffian transition is driven by charge-neutral pairs of quasiparticles and quasiholes shared between the two layers [10,45]. The critical theory is defined by the Lagrangian

$$\mathcal{L} = \bar{\psi}(i\gamma_\mu \partial^\mu - m)\psi, \tag{1}$$

in which the Majorana fermion $\psi$ appears as a two-component spinor that is indistinguishable from its antiparticle. The massless $m = 0$ case enjoys an enlarged conformal symmetry. This symmetry imposes strong constraints on the operator content, organizing operators in conformal towers corresponding to different primaries of the theory [50,51], as summarized in Fig. 1(c). The noninteracting Majorana field brings additional constraints. On the one hand, the equation of motion, $\gamma_\mu \partial^\mu \psi = 0$, fixes the descendants of $\psi$ to only $\partial_{\mu_1} \ldots \partial_{\mu_l} \psi$, with scaling dimension $\Delta = l + 1$ and SO(3) angular momentum $L = l + 1/2$. On the other hand, the reality condition also reduces the number of operators. For example, this theory does not have a conserved vector current, as $J^\mu = \bar{\psi}\gamma^\mu \psi$ vanishes identically. Through Fierz identities, this also implies that any simple multiple-fermion terms beyond $\psi$ and $\bar{\psi}\psi$ vanish.

The emergence of conformal symmetry can be observed in microscopic models defined on spherical manifolds $S^{d-1} \times \mathbb{R}$ through the state-operator correspondence [52,53]. While historically this has presented difficulties for lattice models in $d > 2$, the standard setup of FQH spherical geometry with a magnetic monopole placed at the center of the sphere [54] elegantly removes these problems. This new fuzzy-sphere method of regularization [50] has already proven successful at extracting spectral data of many bosonic 3D CFTs [50,55–70]. These studies realized CFT critical points by tuning an integer quantum Hall state of electrons through a spin transition. However, as shown in Ref. [71], the fuzzy-sphere regularization remains powerful at fractional fillings, which also allows to use bosons as underlying degrees of freedom. We will follow the latter approach to directly extract the universal description of the Halperin-to-Pfaffian transition from the fuzzy-sphere numerics.

We note that the Halperin-Pfaffian critical point differs in a few subtle respects from the conventional 3D Majorana CFT. The gauging of the global $\mathbb{Z}_2$ layer-exchange symmetry [47,72] constrains the excitations of half-integer spin, such as $\psi$, to live in a different Hilbert space compared to the integer-spin ones, such as $\bar{\psi}\psi$ and $T^{\mu\nu}$. This is equivalent to the insertion of a gauge charge at the center of the sphere; specifically, increasing both the magnetic flux and the number of particles by 1, the insertion of a $2\pi$-flux of U(1) electric charge is bounded with a $\mathbb{Z}_2$ gauge charge. For this reason, the elementary field $\psi$ is a nonlocal fermion that carries gauge charge, and our critical theory is the "gauged" 3D Majorana fermion. In our numerics below, we will identify the Hilbert space sectors and focus on the bare fermion field $\psi$, the fermion mass term $\bar{\psi}\psi$, the stress energy tensor $T^{\mu\nu} = \bar{\psi}\gamma^\mu \partial^\nu \psi$, and also the next half-integer spin primary $\bar{\psi}\psi\partial^\mu\psi$; see Supplemental Material (SM) [73] for further details.

*Model and phase diagram*—We consider bosons at Landau level filling $\nu = 1$ on a fuzzy sphere. The monopole strength $Q$ is related to the number of particles $N$ as $2Q = \nu^{-1}N - \mathcal{S}$, where $\mathcal{S} = 2$ is the Wen-Zee shift [90]. As illustrated in Fig. 1(a), the Hamiltonian $H = H_{\text{intra}} + H_{\text{inter}} + H_{\text{t}}$ contains intra- and interlayer interactions, as well as the tunneling term. Using the bosonic operator $b_a^\dagger(\mathbf{\Omega})$—which creates a particle in layer $a = \uparrow, \downarrow$ at a spherical angle $\mathbf{\Omega}$—and the corresponding density operator, $n_a(\mathbf{\Omega}) \equiv b_a^\dagger(\mathbf{\Omega}) b_a(\mathbf{\Omega})$, the three terms in the Hamiltonian are given by

$$\begin{aligned} H_{\text{intra}} &= \sum_{a=\uparrow,\downarrow} \int V^{\text{intra}}(\mathbf{\Omega}_{12}) : n_a(\mathbf{\Omega}_1) n_a(\mathbf{\Omega}_2) : \mathrm{d}^2\mathbf{\Omega}_1 \mathrm{d}^2\mathbf{\Omega}_2, \\ H_{\text{inter}} &= 2 \int V^{\text{inter}}(\mathbf{\Omega}_{12}) n_\uparrow(\mathbf{\Omega}_1) n_\downarrow(\mathbf{\Omega}_2) \mathrm{d}^2\mathbf{\Omega}_1 \mathrm{d}^2\mathbf{\Omega}_2, \\ H_{\text{t}} &= -h \int \left( b_\uparrow^\dagger(\mathbf{\Omega}) b_\downarrow(\mathbf{\Omega}) + \text{H.c.} \right) \mathrm{d}^2\mathbf{\Omega}. \end{aligned} \tag{2}$$

Here, $h$ is the tunneling amplitude, and we take the interactions to be short-ranged, parametrized exclusively by $V_0^{\text{intra}}$ (set to 1) and $V_0^{\text{inter}}$ Haldane pseudopotentials [54]. The model above is invariant under SO(3) rotations, while finite $h$ leaves an additional $\mathbb{Z}_2$ symmetry that exchanges the two layers.

At weak tunneling $h \ll V^{\text{intra}}, V^{\text{inter}}$, the ground state is the 220 Halperin state [32],

$$\Psi_{220}(\{u_i^a, v_i^a\}) = \prod_{i<j}^{N/2}\left(u_i^{\uparrow} v_j^{\uparrow} - v_i^{\uparrow} u_j^{\uparrow}\right)^2 \prod_{i<j}^{N/2}\left(u_i^{\downarrow} v_j^{\downarrow} - v_i^{\downarrow} u_j^{\downarrow}\right)^2, \quad (3)$$

written in terms of standard spinor coordinates $u_j = \cos(\theta_j/2)\exp(i\phi_j/2)$, $v_j = \sin(\theta_j/2)\exp(-i\phi_j/2)$ on the fuzzy sphere. This state is a direct generalization of the Laughlin state [2] to two species of particles and, for our model, it is the *exact* ground state in the limit $h = 0$ and $V_0^{\text{inter}} = 0$ [54], while there is strong numerical evidence for its stability over a wider range of interactions [38].

Tunneling can be viewed as a symmetrizer over the layer indices. Symmetrization of Eq. (3), via a Cauchy identity [91], results in the Moore-Read Pfaffian state [8]

$$\Psi_{\text{Pf}}(\{u_i, v_i\}) = \text{Pf}\left(\frac{1}{u_i v_j - v_i u_j}\right) \prod_{i<j}^{N}(u_i v_j - v_i u_j). \quad (4)$$

One expects this wave function to describe the ground state in the limit $h \gg V^{\text{intra}}, V^{\text{inter}}$ since layer symmetrization leads to an effective single-component system described by the interaction $V_0 = (V_0^{\text{inter}} + V_0^{\text{intra}})/2$ [35], whose ground state is in the Moore-Read phase [92,93].

The topological transition between the states in Eqs. (3) and (4), driven by $h$, lacks a local order parameter. We use the overlap with the model wave functions to map out the phase diagram in Fig. 2(a). The phase is identified by $\max(|\langle 0|\Psi_{220}\rangle|^2, |\langle 0|\Psi_{\text{Pf}}\rangle|^2)$, where $|0\rangle$ is the exact ground state obtained by exact diagonalization, and we draw an approximate phase boundary where the two overlaps are equal.

As additional evidence, we identify each of the topological phases from their entanglement spectra [94]. Under a real-space bipartition [95,96], the counting of the energy levels of the entanglement Hamiltonian should reflect the edge theory of the bulk phase. Accordingly, the 220 Halperin phase exhibits two chiral bosons on the edge, while the Pfaffian phase has a chiral boson and Majorana fermion [97,98]; the replacement of the neutral bosonic edge mode by a single Majorana field at the transition is consistent with the 3D gauged Majorana description of the critical point. We checked the sector with half the total number of bosons in a hemisphere, where we obtain the expected state countings, e.g., $(2, 4, 10, 20, \ldots)$ for the Halperin state and $(1, 2, 4, 7, \ldots)$ for the Pfaffian state, when each subsystem contains an odd number of particles; see Fig. 2(b). We note that the exact ground state is generally expected to reproduce these countings only in a finite number of sectors, below the so-called "entanglement gap" [94], as seen in Fig. 2(b).

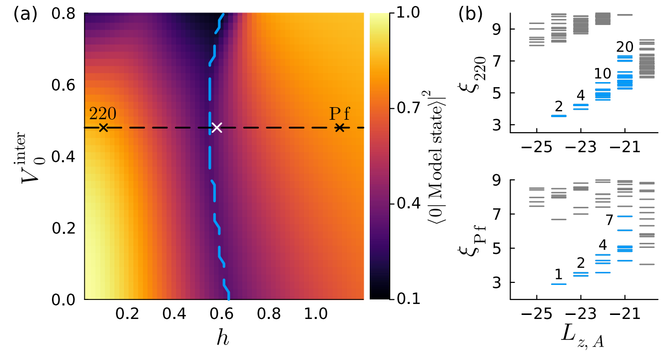


FIG. 2. (a) Phase diagram of the model in Eq. (2), calculated using overlaps with the states in Eqs. (3) and (4). The color intensity represents the maximum overlap, $\max(|\langle 0|\Psi_{220}\rangle|^2, |\langle 0|\Psi_{\text{Pf}}\rangle|^2)$, while the blue dashed line is the approximate phase boundary where the two overlaps are equal. The white cross at $(V_0^{\text{inter}}, h) = (0.48, 0.58)$ marks the gap closing point in Fig. 3. (b) The real-space entanglement spectra for one point inside each phase, denoted by black crosses in (a). The multiplicities of the low-lying entanglement levels $\xi$, resolved by the $z$ component of angular momentum in the subsystem $A$, are indicated by numbers and match those of the respective model states (see text). Data in (a) are obtained by exact diagonalization for $N = 12$ bosons, while (b) is for $N = 14$, the largest accessible system size with Hilbert space dimension 87 150 620.

*Critical point*—At finite $h$, the charge-neutral pairs of quasiholes and quasiparticles lead to two species of decoupled fermions, which are even and odd, respectively, under $\mathbb{Z}_2$ layer exchange. At the critical point, one species becomes massless, with its parity selected by the sign of $h$. For $h > 0$ in Eq. (2), the gapless fermion lies in the $\mathbb{Z}_2$-odd sector. Consequently, to identify the operator spectrum of the 3D Majorana CFT we focus on two different sectors: the odd-particle, $\mathbb{Z}_2$-odd sector, corresponding to fermionic operators, and the even-particle, $\mathbb{Z}_2$-even sector, containing bosonic operators.

Before analyzing the spectrum, we need to identify the optimal point with minimal finite-size effects along the phase boundary in Fig. 2(a). We use two complementary approaches for this. First, to narrow down the space of parameters, we perform an optimization over the spectrum's conformal structure. We consider states in the (fixed) $N$-even sector with $\Delta \leq 4$ and $L \leq 2$, with the conformal tower cost function $\delta_{\text{tow}} = |\mathbf{\Delta}|^2 - |\mathbf{\Delta} \cdot \mathbf{E}|^2/|\mathbf{E}|^2$, where $\mathbf{\Delta}$ is the vector containing the operators' CFT scaling dimensions and $\mathbf{E}$ is the vector of the corresponding microscopic energies. We plot $\delta_{\text{tow}}$ across the phase diagram in Fig. 3(a), revealing a strong peak in the vicinity of $V_0^{\text{inter}} \approx 0.48$. We fix this value of $V_0^{\text{inter}}$ as optimal, and study the finite-size scaling of the excitation gap to determine the optimal $h$.

Based on the CFT prediction, $\Delta E \sim 1/R \sim 1/\sqrt{2Q+1}$, we construct the cost function $\delta_{\text{gap}} = \sum_i \Delta E_i(N \to \infty)$ that minimizes the sum of linear $1/R$ extrapolations for a set of gaps corresponding to $\{\bar{\psi}\psi, \partial^\mu(\bar{\psi}\psi), T^{\mu\nu}\}$.

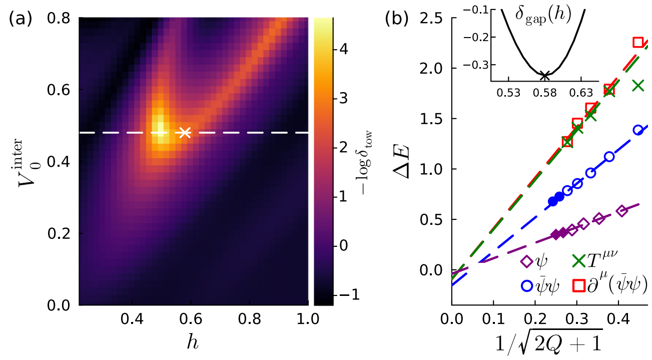


FIG. 3. (a) The cost function $\delta_{\rm tow}$ quantifying the conformal structure in the low-lying spectrum across the phase diagram at a fixed system size $N = 12$. A sharp peak occurs along the dashed line at $V_0^{\rm inter} = 0.48$. The optimal point (white cross) is further determined by the gap vanishing in (b). (b) Extrapolated gaps of $\{\bar\psi\psi, \partial(\bar\psi\psi), T\}$ at the optimal point $(V_0^{\rm inter}, h) = (0.48, 0.58)$, identified from the minimum of the gaplessness cost function $\delta_{\rm gap}$ shown in the inset. Although not explicitly enforced, the gap of the neutral fermion $\psi$ also converges to zero at this point, a signature of a quantum critical point in both even- and odd-particle sectors. Empty markers correspond to exact diagonalization data, while solid markers ($N = 15$–$18$) are obtained from the density matrix renormalization group algorithm and are not used in the extrapolations.

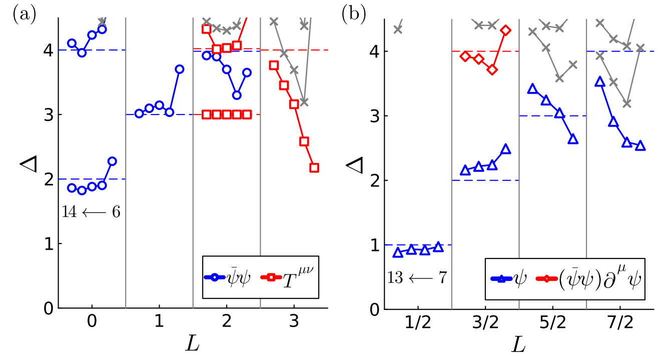


FIG. 4. Comparison between CFT data and the Hamiltonian spectrum at the optimal point $(V_0^{\rm inter}, h) = (0.48, 0.58)$. (a) Even-particle sector for $N = 6, 8, 10, 12, 14$, containing integer angular momentum states. The lowest energy state in $L = 2$ sector is taken to be the stress-energy tensor, whose energy was fixed to $\Delta = 3$. (b) Odd-particle sector for $N = 7, 9, 11, 13$, containing half-integer angular momentum states. Energy levels are normalized with respect to the mean energies of the vacuum and stress-energy tensors in the adjacent, $N \pm 1$, even-particle sectors. Both panels show the complete energy spectra for levels with $\Delta \leq 4.5$, $L \leq 7/2$. The expected CFT operators are labeled, with their scaling dimensions shown by dashed lines. Other signatures of the free Majorana CFT in these spectra also include the *absence* of certain states, such as the conserved vector current, $J^\mu = \bar\psi\gamma^\mu\psi$, at $(L = 1, \Delta = 2)$, and the state at $(L = 1/2, \Delta = 2)$, which is absent due to the equation of motion $\gamma_\mu\partial^\mu\psi = 0$.

The minimum of $\delta_{\rm gap}$ is achieved at the optimal $h = 0.58$, for which the gaps of previously mentioned states vanish [Fig. 3(b)]. The (slightly) negative values of these extrapolated gaps in Fig. 3(b) are attributed to the remnant finite-size effects that, as expected, are more pronounced for higher-energy states. As a consistency check, we also computed the lowest gap in the $N$-odd sector, corresponding to the neutral fermion $\psi$. Since this sector contains no conformal vacuum, the gap is calculated with respect to the averaged vacuum energy of the adjacent even system sizes ($N \pm 1$), and the convergence to zero is remarkably accurate.

*Conformal spectrum*—State-operator correspondence at the optimal critical point is shown in Fig. 4. The ($N$-even, $\mathbb{Z}_2$-even) sector can be seen in Fig. 4(a), where all energies have been rescaled such that the stress-energy tensor—the lowest-lying state at $L = 2$—has $\Delta = 3$ [50]. This sector is identified with the integer angular momentum sector of the 3D Majorana CFT, and a good agreement is obtained for the primaries $\bar\psi\psi$, $T^{\mu\nu}$, and their descendants. The absence of a conserved vector current, $J^\mu = \bar\psi\gamma^\mu\psi$ at $(L = 1, \Delta = 2)$, is also notable, further supporting the Majorana CFT nature of our critical point.

Similarly, Fig. 4(b) shows the ($N$-odd, $\mathbb{Z}_2$-odd) sector, which we identify as the half-integer angular momentum sector of the CFT. This sector contains no conformal vacuum, hence we rescale each system size using the average energies of the vacuum and stress-energy tensor of the adjacent *even* sizes ($N \pm 1$). State-operator correspondence is clearly manifested for the primaries $\psi$ and $\bar\psi\psi\partial^\mu\psi$ and a few of their descendants. The noninteracting nature of the critical point is also highlighted by the absence of the $\gamma_\mu\partial^\mu\psi$ descendant at $(L = 1/2, \Delta = 2)$.

A few comments are in order. For higher levels, $\Delta \geq 4$, the state-operator correspondence becomes less clear, with strong finite-size effects and additional states mixing in with the conformal towers. In other theories such as the 3D Ising CFT, conformal perturbation theory was useful in quantifying such finite-size effects [99,100]. However, the sparse operator-product expansion structure of the free Majorana CFT implies that the fermion mass $\bar\psi\psi$—the only relevant scalar operator—does not produce corrections to the lowest excited states in either even- or odd-particle sectors. Hence, the conformal perturbation approach cannot be directly applied to improve the matching of microscopic spectra with CFT beyond Fig. 4. Furthermore, as in previous work [71], our model realizes a CFT that is space-time parity symmetric, but with no corresponding symmetry at the microscopic level. A potential operator that implements the parity symmetry may be related to the particle-hole symmetry of spinful bosons at $\nu = 1$ [101,102], which is only emergent in the thermodynamic limit. Moreover, the microscopic parity operator appears nontrivial to write down since our calculations are performed at a non-particle-hole-symmetric flux value, dictated by the Wen-Zee shift of the Pfaffian and Halperin states.

*Experimental implications*—Droplets of bosonic FQH states have recently been realized in ultracold atoms [103] and cavity-QED experiments [104]. While the ultra-short-range $V_0$ interactions in our model are native to such platforms, the two-component and Moore-Read states have yet to be realized. Moreover, such platforms are currently limited to a few bosons and far from the scaling regime required to observe the critical behavior.

In solid state materials, the fermionic version of the transition studied above occurs at filling $\nu = 1/2$ between the 331 Halperin and (fermionic) Moore-Read state. There is extensive experimental evidence for the 331 Halperin state and transition in both GaAs bilayers and wide quantum wells [26–29]. Although recent studies found evidence for the one-component state being the Moore-Read state based on the observation of Pfaffian daughter states surrounding $\nu = 1/2$ [30,31], the nature of the one-component state past the transition has remained unclear. However, these studies were limited to transport, which probes the charge gap of the system and the latter may remain open throughout the transition [50]. By contrast, light-scattering probes, as in the recent experiment [105], directly probe neutral excitations and would detect the transition. In a bilayer heterostructure with independent electric contacts to each layer [106], it is possible to directly measure the interlayer tunneling current. The temperature dependence of this current is predicted to be $\langle \varphi_\uparrow^\dagger \varphi_\downarrow(x) \rangle \sim T^2$, since the tunneling term effectively relates to the mass of Majorana fermions, $\bar{\psi}\psi(x)$. Verification of this scaling would not only serve as evidence for the Majorana nature of the critical point, but it would also lend strong support to the identification of the Moore-Read state in the gapped phase.

*Conclusions*—We have microscopically established the emergence of a gauged 3D Majorana fermion at the Halperin-Pfaffian transition. Using the fuzzy-sphere regularization, we have identified the hallmarks of the corresponding CFT: gap closing and operator content. These results not only offer a fresh perspective on the long-standing problem of non-Abelian states in quantum Hall bilayers, they also provide the first unbiased microscopic dissection of a continuous FQH transition, which could be directly applied to other universality classes [47,107,108]. It would also be important to study interfaces between Halperin and Pfaffian states [109–111], which are directly tied to the critical theory discussed here. Beyond the quantum Hall setting, our Letter realizes the emergent fermionic fields within the fuzzy-sphere framework, setting the stage for exploring interacting theories such as Yukawa CFTs [112,113].

*Note added*—We recently became aware of Ref. [114], which realized the free Majorana fermion CFT in a different microscopic model.

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*Data availability*—The data that support the findings of this article are openly available [117].

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