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Proceedings Paper:

Zhao, Y., Song, Y., Wang, A. et al. (2024) Scalable Reliable Control for Cyber- Physical Microgrids Using Chordal Graph. In: 2024 IEEE 13th Data Driven Control and Learning Systems Conference (DDCLS). 2024 IEEE 13th Data Driven Control and Learning Systems Conference (DDCLS), 17-19 May 2024, Kaifeng, China. IEEE, pp. 1131-1136. ISBN: 979-8-3503-6168-1. ISSN: 2767-9853. EISSN: 2767-9861.

<https://doi.org/10.1109/ddcls61622.2024.10606585>

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Scalable Reliable Control for Cyber-Physical Microgrids Using Chordal Graph

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Abstract: We consider a microgrid system with sparse structure, where the security is susceptible to the impact of potential faults. To enhance the reliability and scalability of the microgrid system, this paper investigates the structured controller design problem for the microgrid system under fault conditions using chordal decomposition. By establishing a microgrid fault model with a sparse structure, the controller design problem for large-scale microgrid systems is decomposed into a set of smaller-scale problems using chordal decomposition, and then solved clique-by-clique in a sequential approach. The results demonstrate that this approach reduces the sensitivity of the microgrid system to faults and improves solution efficiency and scalability.

Key Words: Chord Decomposition, Scalable Reliable Control and Cyber-Physical Microgrid

1 Introduction

The increasing prevalence of emerging distributed energy sources, renewable energy, and direct current loads has garnered growing attention for DC microgrids [4]. Microgrids (MGs), as an evolutionary direction of cyber-physical systems (CPSs) are consists of electrically coupled physical systems and communication-coupled cyber systems [5-6]. In the development of MGs, both physical and cyber levels of evolution are involved. For example, at the physical level, system flexibility is heightened through the stabilization of renewable energy output and the provision of grid support services, achieved by efficiently employing energy storage systems [3]. At the cyber level, the extensive application of networking and distributed control theories empowers microgrid systems to adeptly adapt to fluctuations in power demand, facilitate collaborative control within distributed environments, and bolster overall system adaptability [7]. Although MGs have many advantages as a form of CPSs, it also faces challenges in scalability and reliability.

To enhance the performance of microgrid systems, the reference [18] implements the plug-and-play method through a global decentralized strategy, without extending it in terms of scalability. Furthermore, factors such as limited bandwidth of communication channels, network uncertainties, external interference, and noise environment are taken into consideration in reference [8]. Reference [10] investigates a distributed coordinated control strategy with strong adaptability. However, its scalability remains to be further analyzed. Despite the robust scalability of these research methods, they do not take into account the sparsity of the system model. Moreover, for large-scale microgrid systems, these methods exhibit significant limitations, hindering the efficiency of comprehensive system analysis. As microgrid systems become increasingly large, traditional modeling, analysis, and control techniques face challenges in providing reasonable computational workloads. Therefore, it is imperative to employ decomposition techniques to address stabil-

ity analysis issues in microgrid systems. The introduction of chordal graph theory proves effective in tackling sparsity concerns and significantly enhancing the efficiency of solving large-scale systems. The application of this theory provides an effective and innovative solution for the analysis of large-scale microgrid systems.

In practice, communication networks encounter numerous challenging factors and constraints that directly impact system reliability. Past research has extensively explored issues such as communication delays, noisy environments, and potential failures in actuators and sensors. In reference [15], a distributed fault-tolerant control (FTC) scheme for AC-MGs has been proposed with the aim of mitigating the impact of actuator failures on the system. Reference [14] effectively tackles communication delay and robust finite-time control in microgrids (MGs). Additionally, in reference [13], a fault-tolerant control scheme for a segregated microgrid is proposed, focusing on potential sensor and actuator failures. Although these research approaches have made significant progress in enhancing system reliability, there is still a relative lack of research on scalable reliability control. Therefore, there is a need for more research on the scalable reliability of microgrid systems.

The primary contributions of this paper are outlined as follows:

1) The MGs have many decentralized components and nodes, which leads to the sparsity of MGs. We consider that chordal graph theory is used to solve the sparsity of the system structure.

2) We build a microgrid fault model and then use chordal decomposition for structured design, which is scalable. For large microgrid systems, the efficiency of reliability control design increases significantly.

2 Preliminaries and Problem Formulas

2.1 Chordal Graphs

$\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{A})$ is a graph with $\mathcal{V} = \{1, 2, \dots, n\}$ denoting the vertices, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ denoting the edges and denoting adjacency matrix of the graph $\mathcal{G}$ and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n \times n}$, a_{ij} is the weight of an edge. If each vertices satisfies $(j, i) \in \mathcal{E}$ and $(i, j) \in \mathcal{E}$, then the graph is undirected. $|\mathcal{V}|$ and $|\mathcal{E}|$ denote both the count of vertices and the count of edges, re-

This work was partially supported by National Natural Science Funds of China(62173217), Natural Science Foundation of Shanghai Municipality(21ZR1423400), NSFC/Royal Society Cooperation and Exchange project(62111530154, IEC\NSFC\201107). (*Corresponding author: Yang Song).

spectively. The neighbors of vertex $i \in \mathcal{V}$ are defined as $\mathbb{N}_i = \{j \in \mathcal{V} \mid (j, i) \in \mathcal{E}\}$. A chord refers to an edge connecting two vertices that are not adjacent within a cycle. A clique of graph $\mathcal{G}$ is a subset of vertices $C \subseteq \mathcal{V}$ where any two vertices are connected by an edge. A clique is deemed maximal if it cannot be included as a subset within another clique. Given a chordal graph, there are very efficient algorithms to find maximal cliques [16]. The length of a cycle refers to the number of edges in the cycle. For example, the length of a cycle with vertices $\mathcal{V} = \{v_1, v_2, v_3\}$ is three.

Definition 1: An undirected graph is considered chordal if every cycle of length at least 4 contains a chord [22].

Chordal extension is the process of adding edges to a non-chordal graph $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{A})$ in a way that transforms it into a chordal graph $\tilde{\mathcal{G}}(\mathcal{V}, \tilde{\mathcal{E}}, \mathcal{A})$, i.e., $\mathcal{E} \subset \tilde{\mathcal{E}}$. For example, the graph in Fig. 1 (a) is a nonchordal and can be extended to a chordal graph by adding undirected edges (1,3) (see Fig. 1 (b)). Fig. 1 (c) shows a clique tree composed of two maximal cliques: $C_1 = \{1, 2, 3\}$ and $C_2 = \{1, 3, 4\}$.

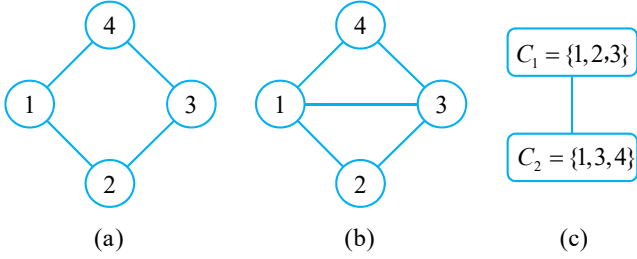


Fig. 1: Example of chordal graph and clique tree: (a) non-chordal graph, (b) chordal graph, and (c) clique tree.

Definition 2: Consider C be an arbitrary subset of $\mathcal{V}$ and define a set $\mathcal{M}(C) = \{(i, j) \in C \times C \mid i \leq j\}$, we use $\mathcal{H} = \{(i, j, p, q) \mid (i, j) \in J(C_p \cap C_q)\}$ to denote the minimal set of overlapping elements.

Definition 3: Consider a chordal graph $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{A})$ with a set of maximal cliques $\mathcal{F} = \{C_1, C_2, \dots, C_m\}$. If clique C_k is adjacent to clique C_i and also connected to clique C_j in the clique tree $\mathcal{T} = (\mathcal{F}, \Xi)$, these maximal cliques can be reorganized by running intersection property $C_i \cap C_j \subseteq C_k$ [11].

2.2 Sparsity Structures

We first consider the sparse structure of block matrices. The set of $m \times n$ blocks matrices of a graph $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{A})$ is given by [9].

$$\mathbb{R}_{m,n}^N(\mathcal{E}, 0) = \{Z \in \mathbb{R}^{mN \times nN} \mid Z_{ij} = 0 \text{ with } (i, j) \notin \mathcal{E}\}$$

For the case where $m = n$ (the most common scenario), we define the sparse structure of a symmetric matrix of graph $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{A})$ as follows.

$$\mathbb{S}_n^N(\mathcal{E}, 0) = \{Z \in \mathbb{S}^{nN} \mid Z_{ij} = 0 \text{ with } (j, i) \notin \mathcal{E}\}$$

$$\mathbb{S}_{n,+}^N(\mathcal{E}, 0) = \{Z \in \mathbb{S}_n^N(\mathcal{E}, 0) \mid Z \succeq 0\}$$

Further, the block matrix with vertex set C which is the subsets of $\mathcal{V}$ is defined as follows.

$$\mathbb{S}_n^C = \{Z \in \mathbb{S}^{nN} \mid Z_{ij} = 0 \text{ with } (i, j) \notin C \times C\}$$

$$\mathbb{S}_{n,+}^C = \{Z \in \mathbb{S}_n^C \mid Z \succeq 0\}$$

Next, we define the basis matrix(also called the incidence matrix) $\mathcal{I}_{C_k} \in \mathbb{R}^{|C_k| \times n}$ of clique a clique C_k as follows.

$$(\mathcal{I}_{C_k})_{ij} = \begin{cases} 1, & C_k(i) = j \\ 0, & \text{otherwise} \end{cases}$$

where $|C_k|$ is the number of vertices in clique C_k , and $C_k(i)$ represents the i -th vertex of clique C_k . The statement of the chordal decomposition is as follows.

Theorem 1: Consider a chordal graph $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{A})$ with a set of maximal cliques $\{C_1, \dots, C_m\}$. If and only if there exists a set of matrices Z_k such that $Z = \sum_{k=1}^m \mathcal{I}_{C_k}^T Z_k \mathcal{I}_{C_k}$, where $Z_k \in \mathbb{S}_{n,+}^{|C_k|}(\mathcal{E}_k, 0)$, $k = 1, \dots, m$, then matrix $Z = [Z_{ij}]_{N \times N} \in \mathbb{S}_n^N(\mathcal{E})$ is positive semidefinite (see [22]).

According to Theorem 1, positive definite (or semidefinite) matrices are intricately connected with the maximal cliques in chordal graphs, and we can leverage the sparsity of the problem to decompose the original large-scale problem into structurally simpler and more manageable sub-problems. The application of chordal decomposition significantly reduces the computational complexity of semidefinite programming (SDP), especially when dealing with problems characterized by sparse matrix inequality constraints.

3 Fault Models for MGs with Sparsity Structures

We initially present the microgrid model with actuator faults in this section, then give the closed-loop state space model, and finally state the problem addressed in this paper.

3.1 Fault Models for MGs

This section explores the dynamic models of MGs with N distribution generation units (DGUs) interconnected via power lines. Fig. 2 illustrates the electrical configuration of MGs. Each DGU comprises a converter, a resistive load R_{Li} , an RLC filter and a direct current voltage source. Additionally, unknown current disturbances I_{Li} on the load side are taken into account [18].

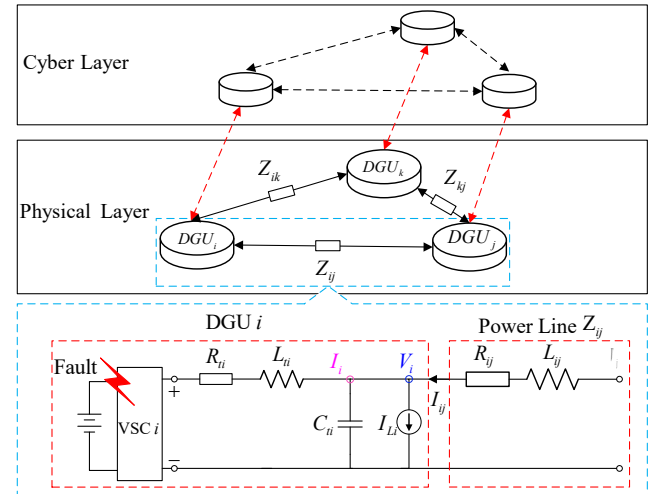


Fig. 2: An illustration of three interconnected DGUs.

Applying Kirchoff's voltage law and current law, we obtain the following model of DGU i :

$$\begin{cases} \frac{dV_i}{dt} = \frac{I_{ti}}{C_{ti}} + \sum_{j \in \mathbb{N}_i} \frac{V_j - V_i}{C_{ti} R_{ij}} - \frac{I_{Li}}{C_{ti}} \\ \frac{dI_{ti}}{dt} = -\frac{V_i}{L_{ti}} - \frac{R_{ti}}{L_{ti}} I_{ti} + \frac{V_{ti}}{L_{ti}} \end{cases} \quad (1)$$

where V_i , I_{ti} , and V_{ti} are the capacitor voltage, output current and the command to the converter, respectively, and $\mathbb{N}_i$ is the set of neighbors. R_{ti} , L_{ti} and C_{ti} are the electrical parameters of the RLC filter.

Since the power line L_{ij} of the DC system is very small, the dynamics of the power line can be ignored, which is feasible [2]. Consequently, the state space equation of DGU i is as follows.

$$\begin{cases} \dot{x}(t) = A_{ii}x_i(t) + B_i u_i(t) + M_i d_i(t) + \xi_i(t) \\ y_i(t) = C_i x_i(t) \end{cases} \quad (2)$$

with

$$A_{ii} = \begin{bmatrix} \sum_{j \in \mathbb{N}_i} -\frac{1}{R_{ij}C_{ti}} & \frac{1}{C_{ti}} \\ -\frac{1}{L_{ti}} & -\frac{R_{ti}}{L_{ti}} \end{bmatrix} M_i = \begin{bmatrix} -\frac{1}{C_{ti}} \\ 0 \end{bmatrix}$$

$$A_{ij} = \begin{bmatrix} \frac{1}{R_{ij}C_{ti}} & 0 \\ 0 & 0 \end{bmatrix} B_i = \begin{bmatrix} 0 \\ \frac{1}{L_{ti}} \end{bmatrix} C_i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

where $x_i = [V_i, I_{ti}]^T$ is the state, $u_i = V_{ti}$ is the control input, $d_i = I_{Li}$ is the exogenous input. Moreover, $y_i(t)$ is the measurable output and $y_i = x_i$, $\xi_i(t) = \sum_{j \in \mathbb{N}_i} A_{ij}x_j(t)$.

However, in real situations, microgrid have potential fault. The actuator output voltage V_{ti} is easily affected. The control outcomes from the controller influence the actuator, and a malfunction in the actuator results in partial information loss within the channel from the controller to the actuator, leading to partial degradation in control effectiveness. Define V_{ti}^F as a fault output, the fault model of the microgrid controller is as follows.

$$u_{[i]}^F = \delta_i u_{[i]} \quad (3)$$

The fault model of the DGU i is

$$\dot{x}_i(t) = A_{ii}x_i(t) + B_i \delta_i u_i^F(t) + M_i d_i(t) + A_{ij}x_j \quad (4)$$

where δ_i is indicative of the effective control coefficient arising from actuator faults. When $\delta_i = 1$, $u_i^F(t) = u_i(t)$ signifies the proper functioning of the actuator. When $0 < \delta_i < 1$, $u_i^F(t) = \delta_i u_i(t)$ suggests that the actuator is experiencing a partial fault. $\delta_i = 0$ signifies complete failure, akin to an open-loop system when the controller is non-operational.

3.2 The Closed-loop MGs Model with Sparsity Structures

The electrical interactions among multiple MGs can be illustrated by a plant graph $\mathcal{G}_p(\mathcal{V}, \mathcal{E}_p, \mathcal{A})$ where each MG is represented as a node set $\mathcal{V} = [1, 2, \dots, N]$ and the edges $\mathcal{E}_p$ depict electrical connections. Further, the communication graph $\mathcal{G}_c(\mathcal{V}, \mathcal{E}_c)$ is modeled as the communication network of the MGs, where $\mathcal{V}$ represents the controller for each node in the MGs and $\mathcal{E}_c$ represents the communication links between MGs. The mathematical features of graphs $\mathcal{G}_p$ and $\mathcal{G}_c$ are represented by the corresponding adjacency matrices $\mathcal{A}$, respectively.

To ensure the attainment of the desired performance, we assume that the topology of $\mathcal{G}_c$ corresponds to the electrical network $\mathcal{G}_p$ of MGs. This assumption has garnered widespread acceptance in relevant literature [17].

Since the load is unknown, the associated interference is not considered. According to the above microgrid model, consider a microgrid system with N DGUs. The subsystem state space model is as follows.

$$\begin{cases} \dot{x}_i(t) = A_{ii}x_i(t) + \sum_{j \in \mathbb{N}_i^p} A_{ij}x_j(t) + B_i u_i^F(t) \\ u_i^F = \delta_i u_i(t) \\ u_i(t) = k_{ii}x_i(t) + \sum_{j \in \mathbb{N}_i^c} k_{ij}x_j(t) \end{cases} \quad (5)$$

The state space model of the closed-loop system is given by

$$\dot{x}(t) = (A + B\delta K)x(t) \quad (6)$$

where $\mathbb{N}_i^p$ signifies the vertex set influencing the dynamics of vertex i in $\mathcal{G}^p$. Vertex i in graph $\mathcal{G}^c$ receives state information from the vertices denoted by $\mathbb{N}_i^c$. Note that $A \in \mathbb{R}_n^N(\mathcal{E}^p, 0)$, $K \in \mathbb{R}_n^N(\mathcal{E}^c, 0)$, $B = \text{diag}\{B_1, \dots, B_n\}$, and $\delta = \text{diag}\{\delta_1, \dots, \delta_n\}$.

Note that sparsity constraints are naturally present in microgrid systems, and feedback gains can be computed locally using the sparsity of graphs $\mathcal{G}^p$, $\mathcal{G}^c$, which helps ensure model privacy. Concisely, the objective of this paper is to identify $K \in \mathbb{R}_{m,n}^N(\mathcal{E}^c, 0)$, ensuring the asymptotic stability of $A + B\delta K$.

3.3 The MGs Model Analysis via Relaxation

In this section, problem (6) is reformulated into linear matrix inequalities (LMIs) through a relaxation strategy, while maintaining the sparsity inherent in problem (6). Using the Lyapunov function $V(x) = x^T P x$, the expression of stability can be equivalently stated as

$$P^{-1}A^T + AP^{-1} + (\delta K P^{-1})^T B^T + B(\delta K P^{-1}) \prec 0 \quad (7)$$

For convenience, let $Q = \delta K P^{-1}$

$$\begin{cases} P^{-1}A^T + AP^{-1} + Q^T B^T + BQ \prec 0 \\ QP \in \mathbb{R}_{n,m}^N(\mathcal{E}^c, 0) \\ P^{-1} > 0 \end{cases} \quad (8)$$

Assuming matrix P or P^{-1} is a diagonal matrix with a size corresponding to the scale of the microgrid system, the non-linear component of the structural constraint can be further relaxed. The result is as follows.

$$QP \Leftrightarrow Q \in \mathbb{R}_{n,m}^N(\mathcal{E}^c, 0) \quad (9)$$

Furthermore, inequality (8) can be transformed into

$$\begin{cases} P^{-1}A^T + AP^{-1} + Q^T B^T + BQ \prec 0 \\ Q \in \mathbb{R}_{n,m}^N(\mathcal{E}^c, 0) \\ P > 0, P \text{ is block diagonal} \end{cases} \quad (10)$$

Remark 1: For a positive system, the presence of a diagonal Lyapunov function is synonymous with stability. When computing the norm of a positive system, assuming the presence of a diagonal Lyapunov function does not introduce conservatism[12]. Therefore, it is feasible to assume the presence of a diagonal Lyapunov function for the microgrid closed-loop system. Furthermore, problem (6) maintains its sparse structure throughout the solving process, laying the foundation for the application of chordal decomposition. When the system size involved in problem (8) is relatively

small, a centralized approach can be directly employed for resolution. However, as the system size increases, computational efficiency may be impacted. Hence, for large-scale sparse systems, the next section will develop a scalable algorithm to address problem (8).

4 Scalable Analysis of MGs via Chordal Graph

In this section, we apply chord decomposition and a sequential approach [9] to the structured design of large-scale microgrid systems with faults. We first get a chordal characterization of the microgrid system data in (8). Then, semidefinite constraint decomposition is performed using Theorem 1. Finally, the sequential approach is utilized to solve each decomposed subsystem.

4.1 Chordal Characterization of MGs

The graph structure of a microgrid system may not be a chordal graph, it is necessary to perform a chordal extension on the graph structure of the microgrid to transform it into a chordal graph. A better chordal extension can be obtained through symbolic Cholesky factorization [19]. For instance, as illustrated in Fig. 3, a chordal extension is shown.

In addition, to illustrate the symmetry of the Lyapunov function, we need to consider the following contents.

Consider symmetric graphs $\mathcal{G}_\eta(\mathcal{V}, \mathcal{E}_\eta)$ of graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$, which shares an identical set of vertices and edges. Then, we have

$$\begin{aligned} P^{-1}A^T &\in \mathbb{R}_n^N(\mathcal{E}^p, 0), \quad Q^TB^T \in \mathbb{R}_n^N(\mathcal{E}^c, 0) \\ AP^{-1} &\in \mathbb{R}_n^N(\mathcal{E}_\eta^p, 0), \quad BQ \in \mathbb{R}_n^N(\mathcal{E}_\eta^c, 0) \end{aligned}$$

Further, we define a undirected supergraph $\mathcal{G}_s(\mathcal{V}, \mathcal{E}_s)$. Next, we construct a chordal graph $\mathcal{G}_{ms}(\mathcal{V}, \mathcal{E}_{ms})$ through chordal extension. For notational simplicity, we define

$$\mathcal{X}_{P,Q} = -(P^{-1}A^T + AP^{-1} + Q^TB^T + BQ + \varepsilon I)$$

Then, (10) can be rewritten into

$$\begin{cases} \mathcal{X}_{P,Q} \succcurlyeq 0 \\ Q \in \mathbb{R}_{m,n}^N(\mathcal{E}^c, 0) \\ P - \varepsilon I \succcurlyeq 0 \end{cases} \quad (11)$$

where I and $\varepsilon > 0$ represent the identity matrix of the suitable dimension and a constant number, respectively.

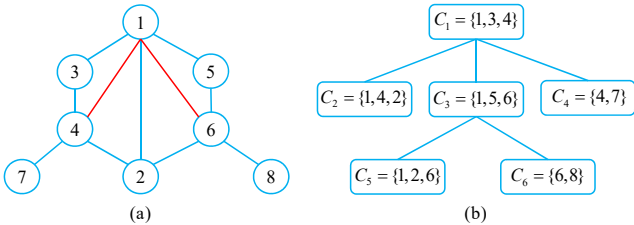


Fig. 3: Chordal extension: (a) chordal graph with 8 nodes, where two red edges are added. (b) clique tree with 6 cliques.

4.2 Structured Feedback Gain Design via Chordal Decomposition

After determining the chordal characteristics and performing chordal extension, we are prepared to decompose

the positive semidefinite constraints in (11) by Theorem 1. Then, according to Theorem 1, we can equivalently reduce (11) into (12).

$$\begin{cases} \sum_{k=1}^p \mathcal{I}_{C_k}^T Z_k \mathcal{I}_{C_k} = \mathcal{X}_{P,Q} \\ Z_k \succcurlyeq 0, \quad k = 1, \dots, p \\ Q \in \mathbb{R}_{m,n}^N(\mathcal{E}^c, 0) \\ P - \varepsilon I \succcurlyeq 0 \end{cases} \quad (12)$$

Compared to (11), (12) involves a smaller scale of system parameters. However, the cost is the addition of extra equality constraints in (12). Furthermore, relaxing these constraints can lead to the sequential approach discussed in the next section.

4.3 Sequential Approach on the Clique Tree

The sequential approach computes the corresponding feedback gains in the order of the maximal cliques, where each iteration involves the calculation of the respective feedback gain. The order of computation and the path of information transfer depend on the structure of the clique tree satisfying the running intersection property [9]. When overlapping elements exist, it indicates coupling between subsystems. Our approach is to partition the overlapping elements in the manner of maximal cliques. Relevant studies have indicated the feasibility and optimality of this partitioning of overlapping elements [20]. The specific steps are as follows.

1) Obtaining coupling coefficients through mathematical analysis on overlapping elements. First, we consider $\mathcal{F} = \{C_1, C_2, \dots, C_m\}$ to be a set of maximal cliques in graph $\mathcal{G}_{ms}$, we then define $\theta \in \mathbb{S}_n^N$ to represent the number of repetitions of nodes and edges, i.e.,

$$\begin{cases} \theta_{ii} = \text{times node } i \text{ appears in } \mathcal{F} \\ \theta_{ij} = \text{times edge } (i, j) \text{ appears in } \mathcal{F} \end{cases}$$

Furthermore, we define the matrix $\mathcal{R}$, and each of its elements is represented as follows.

$$\mathcal{R}_{ij} = \begin{cases} \frac{1}{\theta_{ij}}, & \theta_{ij} \neq 0 \\ 0, & \text{otherwise} \end{cases}$$

2) Obtain the set of LMIs on the maximal cliques. Based on the above, Z_k can be represented as follows.

$$Z_k = \mathcal{X}_{P,Q}(C_k, C_k) \times \mathcal{R}_{C_k} \quad k \in \{1, 2, \dots, m\}$$

where $\times$ denotes the Hadamard product.

Through this approach, (12) is transformed into a set of smaller-dimensional LMIs Ψ_k , and Ψ_k is represented as follows.

$$\Psi_k : \begin{cases} \mathcal{X}_{P,Q}(C_k, C_k) \times \mathcal{R}_{C_k} \succcurlyeq 0 \\ Q - \varepsilon I \succcurlyeq 0, \quad j \in C_k \\ Q(C_k, C_k) \in \mathbb{S}_{n,+}^N(\mathcal{E}^c, 0) \end{cases} \quad (13)$$

where the dimension of Ψ_k corresponds to the size of the maximal clique, and there are shared design parameters among different Ψ_k .

3) Sequential approach on the clique tree. We can solve them successively, addressing one clique at a time following the structure of a clique tree $\mathcal{T}$. We initiate the traversal with the root clique, integrating overlapping parameters

from the upper layer throughout the tree structure. This systematic traversal thoroughly considers the parameter associations among cliques. Fig.3 illustrates this tree structure and strategy. We initially address the root clique $C_1 = \{1, 3, 4\}$, embedding it into the next layer of the tree and thereby acquiring solutions for cliques C_2, C_3 and C_4 . Subsequently, the results of the first layer are embedded into the cliques of the second layer, thereby determining the feedback gains for cliques C_5 and C_6 .

Remark 2: 1) The structure of the clique tree provides us with flexibility, allowing us to initiate exploration from any maximal clique and systematically delve into the characteristics of other cliques. 2) Maximal cliques play a crucial role in enhancing both efficiency and feasibility. Our objective is to improve solution efficiency by identifying a chordal extension with minimal added edges while maintaining a relatively small size of maximal cliques. 3) In a chordal graph, there is no unique clique tree, and the structure of the clique tree can influence the feasibility of sequential approach, especially with regard to the equal splitting strategy.

5 Simulation Examples

we will present two examples to illustrate the scalability of the sequential approach, aiming to showcase its effectiveness and reliability in addressing problems of various scales in this section. Simulations were executed on a single computer equipped with an Intel Core i5 central processing unit (CPU) running at a speed of 3.2 gigahertz (GHz) and has 32 gigabytes (GB) of random access memory (RAM).

approach

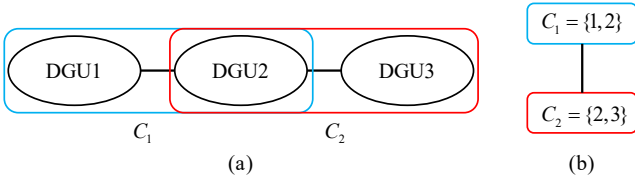


Fig. 4: Chordal decomposition of the microgrid system: (a) maximal cliques; (b) clique tree.

Example 1: A specific example is being considered. The microgrid system with fault consisting of three DGUs (see Fig. 4). Using the microgrid fault model of Section 2, the linearized dynamics can be described as

$$A_{ii} = \begin{bmatrix} -\frac{1}{R_{ij}C_{ti}} & \frac{1}{C_{ti}} & 0 \\ -\frac{1}{L_{ti}} & -\frac{R_{ti}}{L_{ti}} & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$A_{ij} = \begin{bmatrix} \frac{1}{R_{ij}C_{ti}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad B_i = \begin{bmatrix} 0 \\ \frac{1}{L_{ti}} \\ 0 \end{bmatrix}$$

where $(i, j) = (j, i) \in \mathcal{E} = \{(1, 2), (2, 3), (2, 1), (3, 2)\}$ and $\delta = [0.1 \ 0 \ 0; 0 \ 1 \ 0; 0 \ 0 \ 0.5]$ denotes the fault parameter. We assume an overlap between the communication graph and its plant graph. Fig. 4 shows the chordal decomposition.

The simulated microgrid system parameters are shown in Table 1, and the inductance of the coupling between sub-systems is $L_{12} = 0.07\Omega$ and $L_{23} = 0.05\Omega$, the resistances are $R_{12} = 1.8\text{mH}$, $R_{23} = 2.1\text{mH}$. Then, Converting the problem into the form of equation (13), structured feedback gains

Table 1: Microgrid system parameters

DGU i	Resistance R_{ti}	Capacitance C_{ti}	Inductance L_{ti}
DGU 1	0.1Ω	1.7mF	2.2mH
DGU 2	0.2Ω	2.2mF	1.8mH
DGU 3	0.3Ω	1.9mF	2.0mH

are calculated in a centralized way, but this way requires all model information of the microgrid system. Moreover, we can employ a sequential method for solving. As shown in Fig. 4 (b), we initially solve clique C_1 using the information from nodes 1 and 2. Subsequently, we embed the information from node 2 into clique C_2 , and then solve node 3. Both of these approaches are feasible. The performance of the computed structured controllers based on the initial condition $x_0[1] = [0.2, 0.1, 0]$, $x_0[2] = [0.2, 0.2, 0]$ and $x_0[3] = 0$ is depicted in Fig. 5.

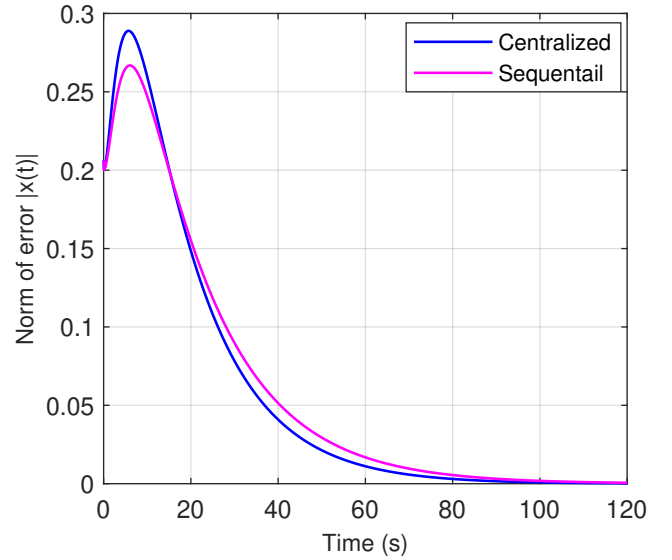


Fig. 5: Norm of error $|x(t)|$ using centralized and sequential ways.

As shown in Fig. 5, the time consumed by the two methods to make the microgrid system stabilise is relatively close, but the sequential approach based on chordal decomposition performs better than the centralized approach. Furthermore, the time difference between the two methods widens as the system size increases, which we will explore in the next example.

Example 2: In the simulation, we initially generate a random chordal graph with constraints imposed on the size of its maximal cliques. This process is aimed at capturing the graph structure characteristics of the microgrid system. Subsequently, we randomly remove some edges to make the graph structure more realistic, forming the final graph structure corresponding to the microgrid system. Such a design is intended to more accurately reflect the complexity and uncertainty that may exist in actual microgrid systems. Then, we consider a general microgrid system with 100 nodes, where the maximum size of any clique is restricted to be no more than 6. Inspired by reference [21], we assume that the dynamic model of the coupling between each node and

its neighboring nodes is as follows.

$$\dot{x}_i = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} x_i + \sum_{j \in \mathbb{N}_i^p} e^{-\frac{1}{10}(i,j)} x_j + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_i^F \quad (14)$$

As shown in Fig. 6, the runtime of centralized way and sequential way increases with the scale of the microgrid system. The sequential way is more efficient than the centralized way, aligning with expectations. Furthermore, a decrease in efficiency may occur for the sequential approach when dealing with large maximal cliques, as it corresponds to a centralized solving approach for each maximal clique.

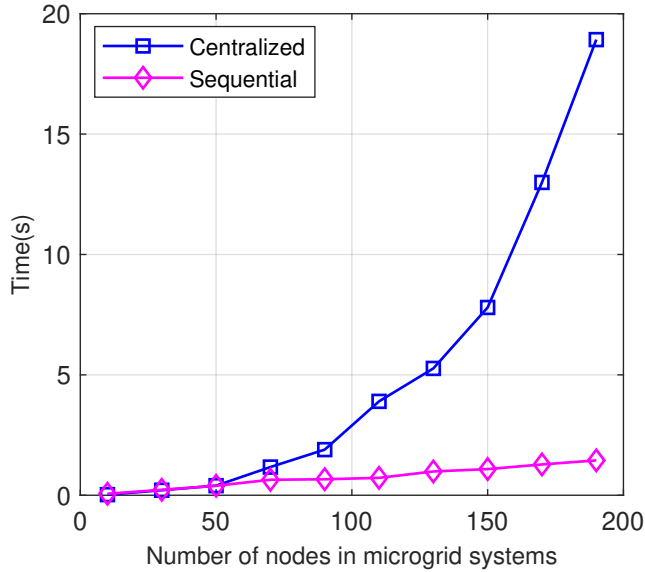


Fig. 6: The time consumption of a microgrid system with 100 nodes using centralized and sequential ways.

6 Conclusions

This paper investigates the stability issue of microgrid systems under actuator faults using chordal decomposition and a sequential approach based on clique trees. This approach involves information exchange solely with neighboring subsystems, contributing to the privacy protection of microgrid system data. Two simulation examples indicate significant improvements in reducing computational complexity and enhancing scalability with the chordal decomposition approach.

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