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Version: Supplemental Material

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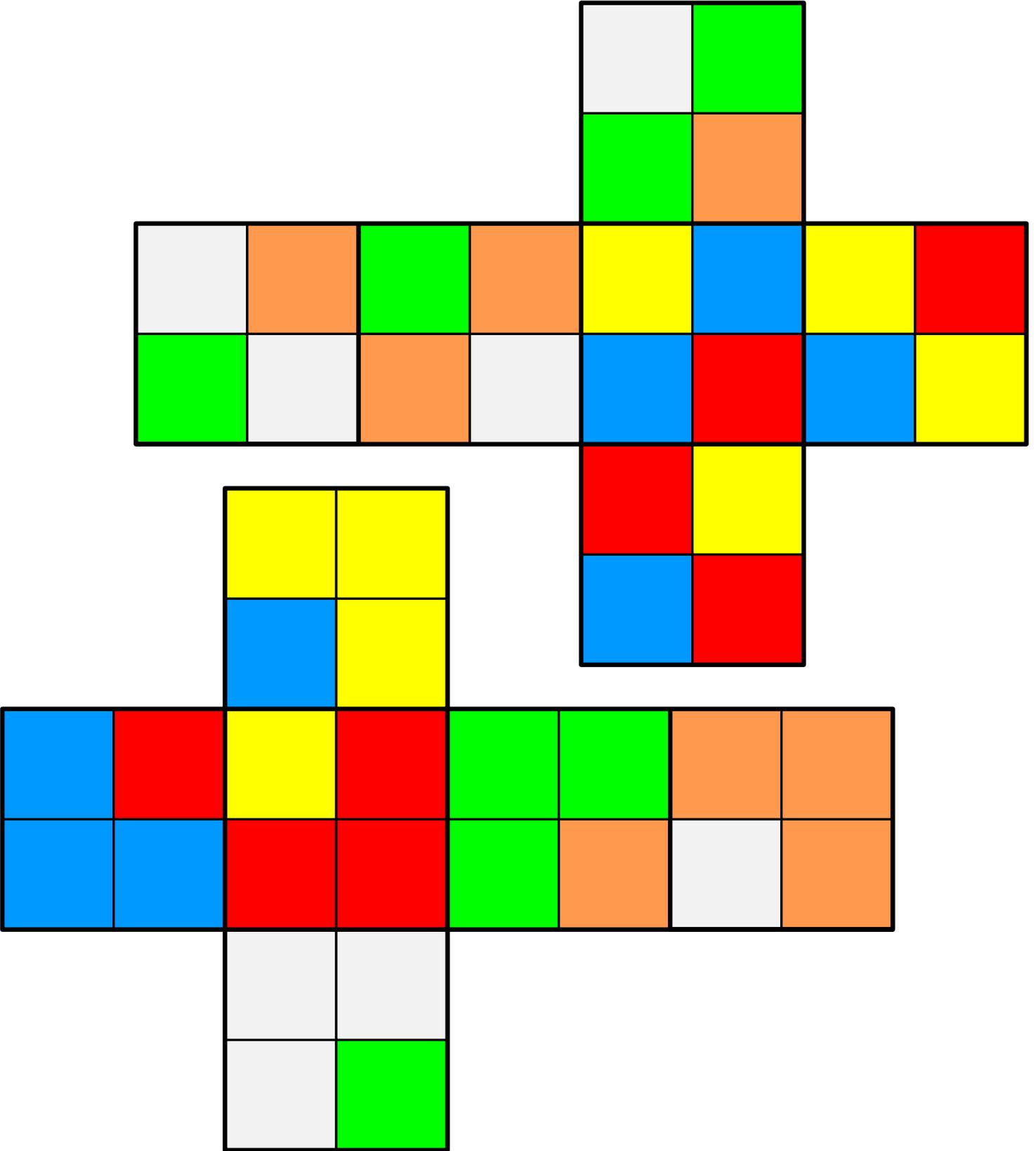


Figure 19: Templates for Figure 1

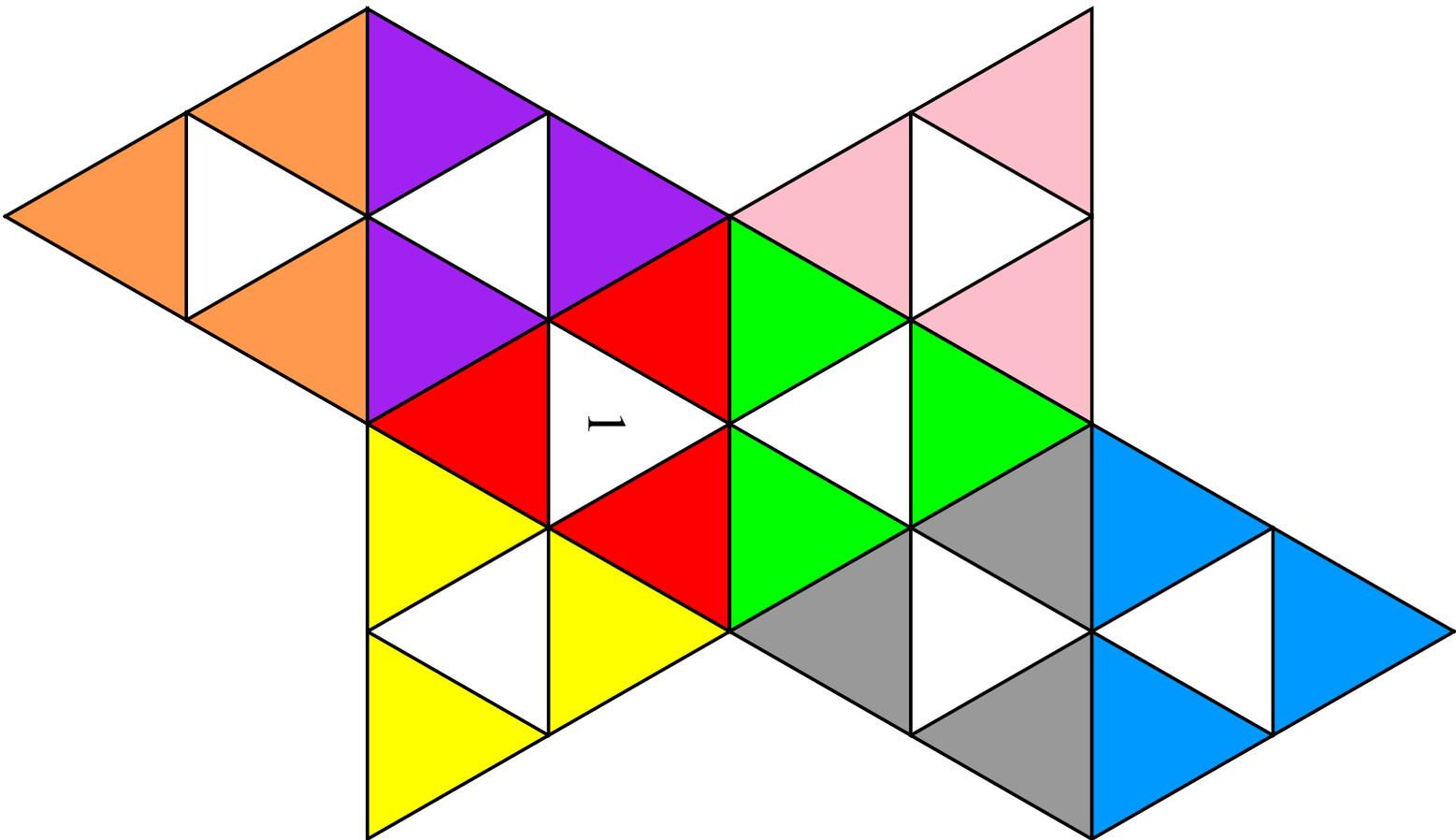
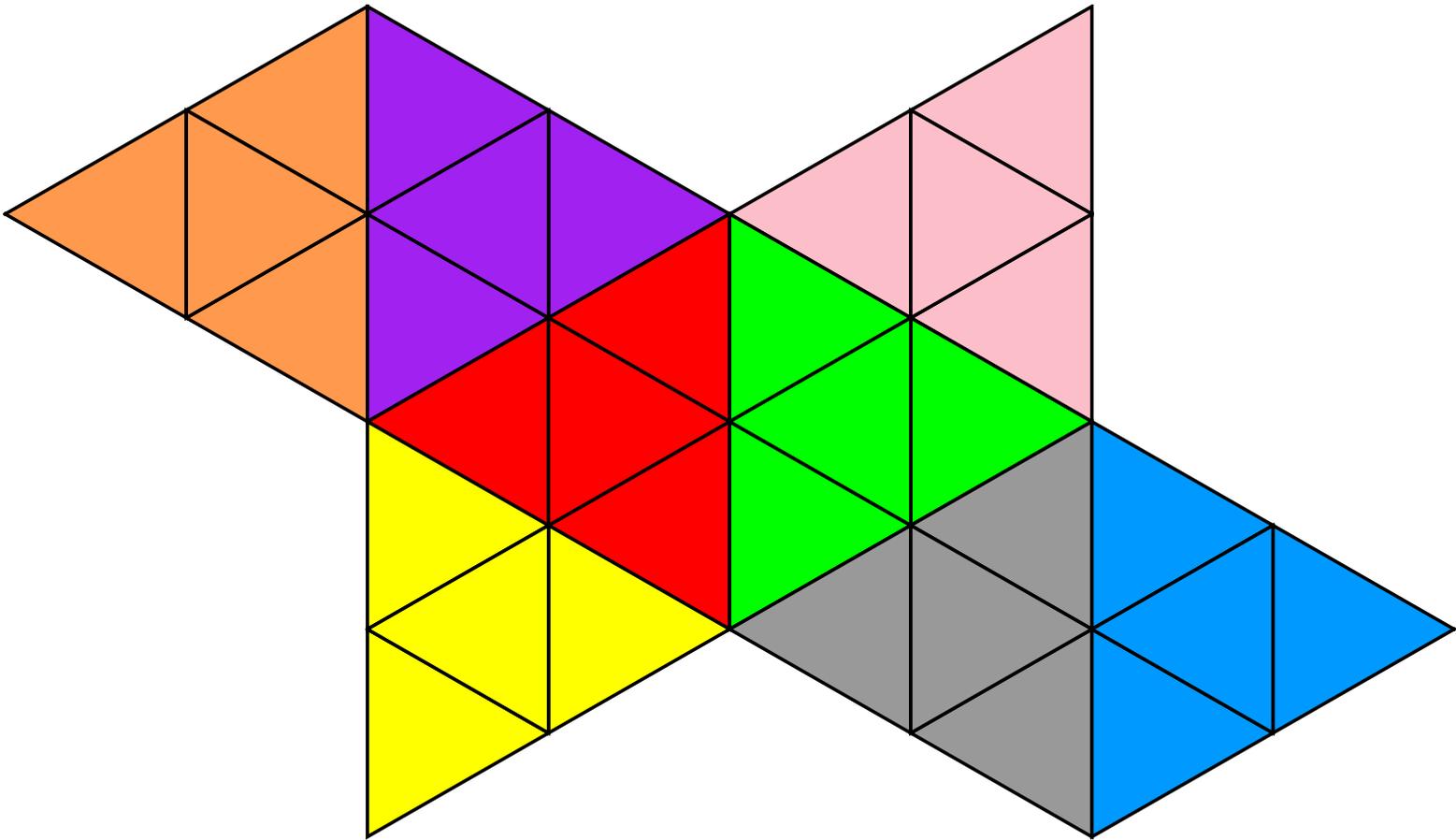


Figure 20: Templates for Pattern 1, once with centers filled in as in Figure 2, and once with centers left blank, as on the other templates below. 2

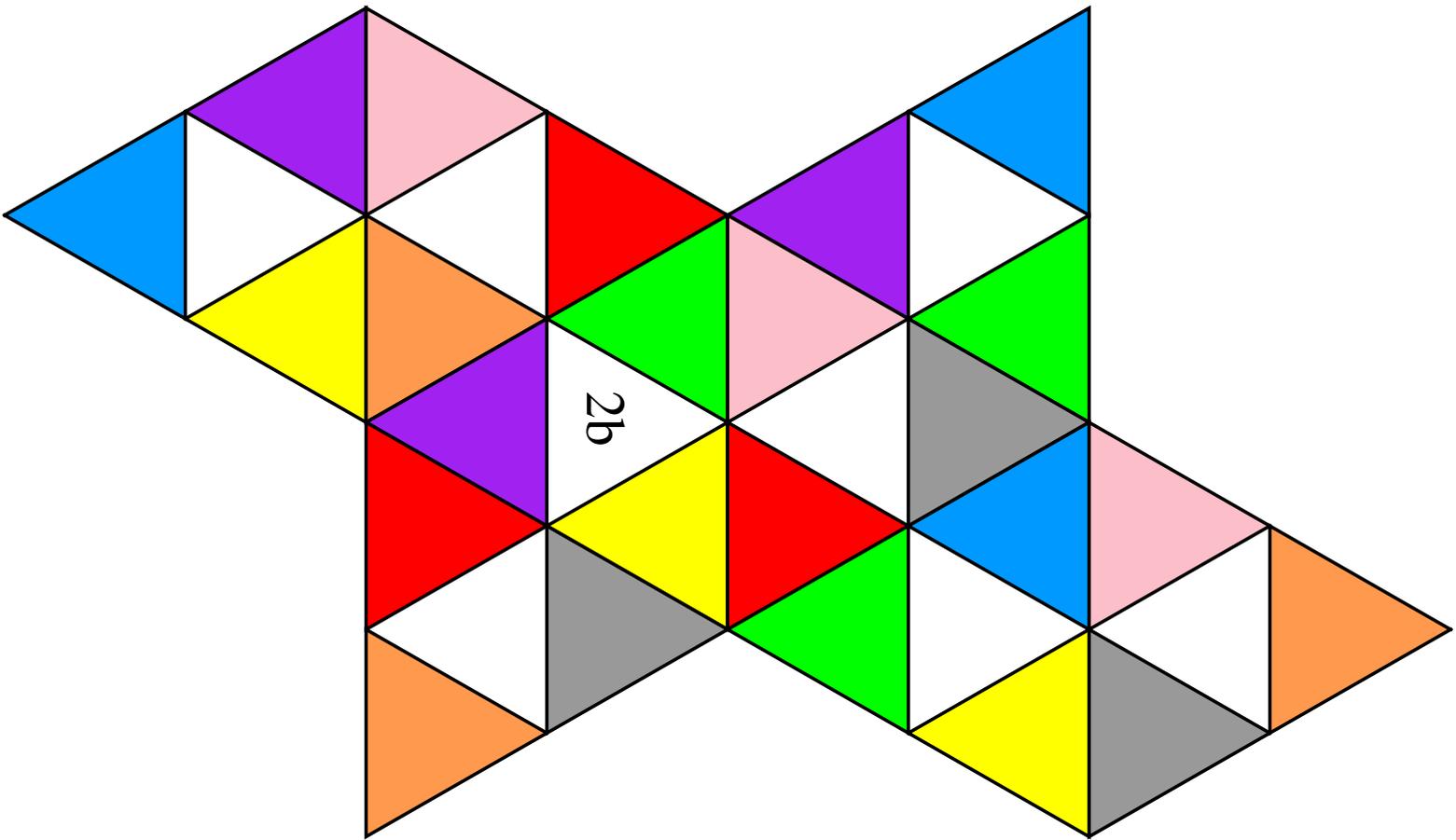
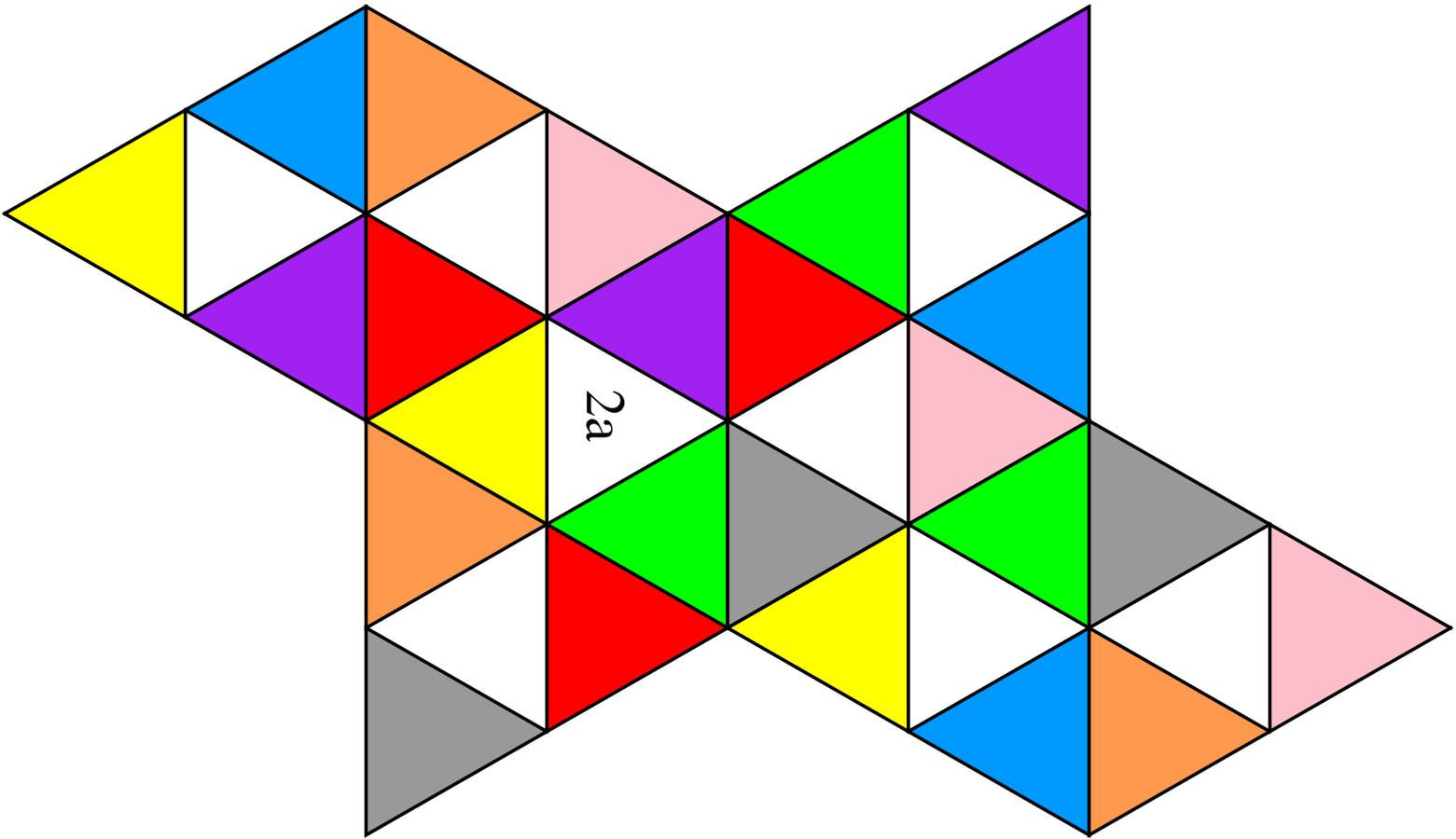


Figure 21: Templates for Patterns 2a,b

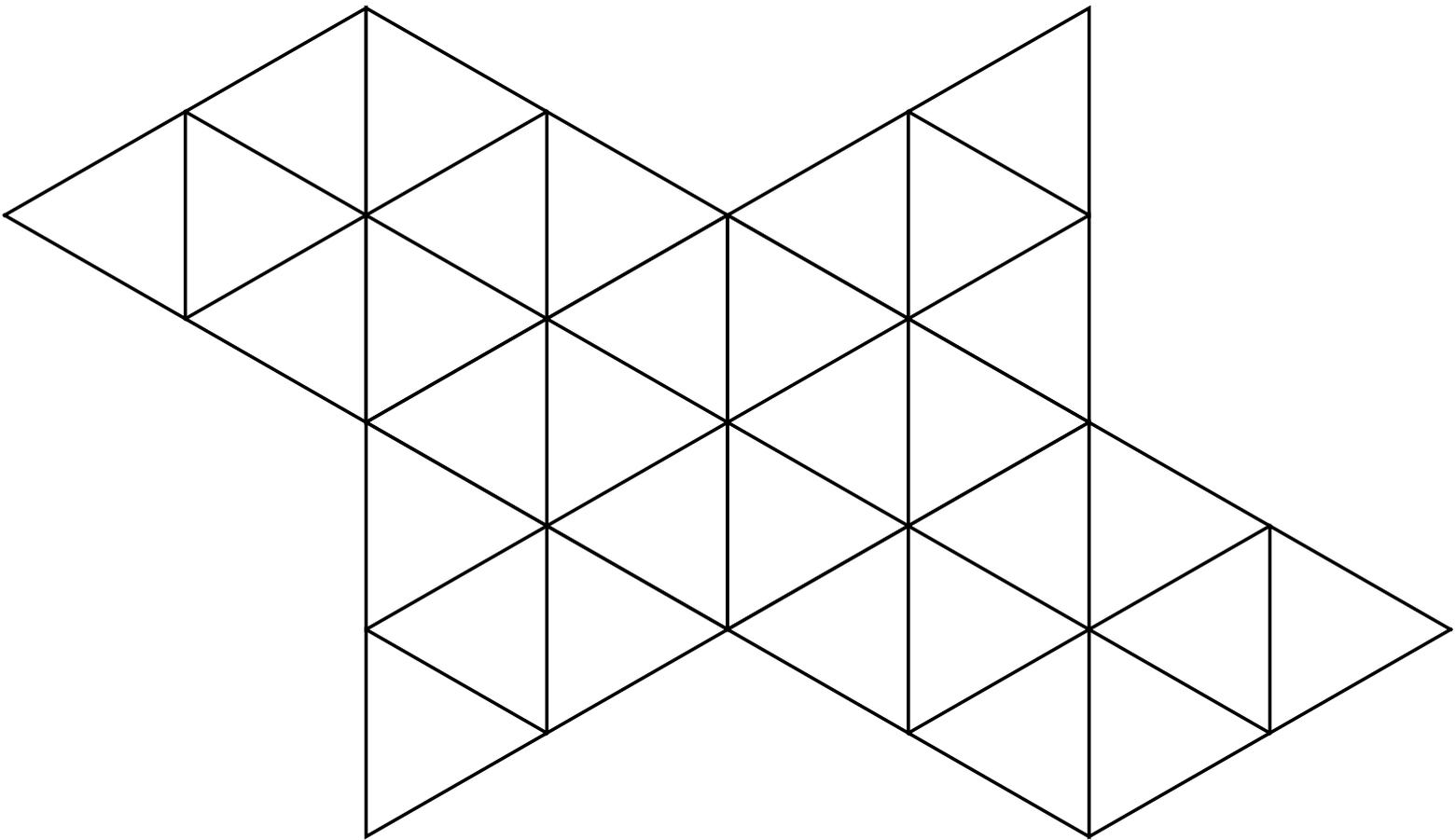
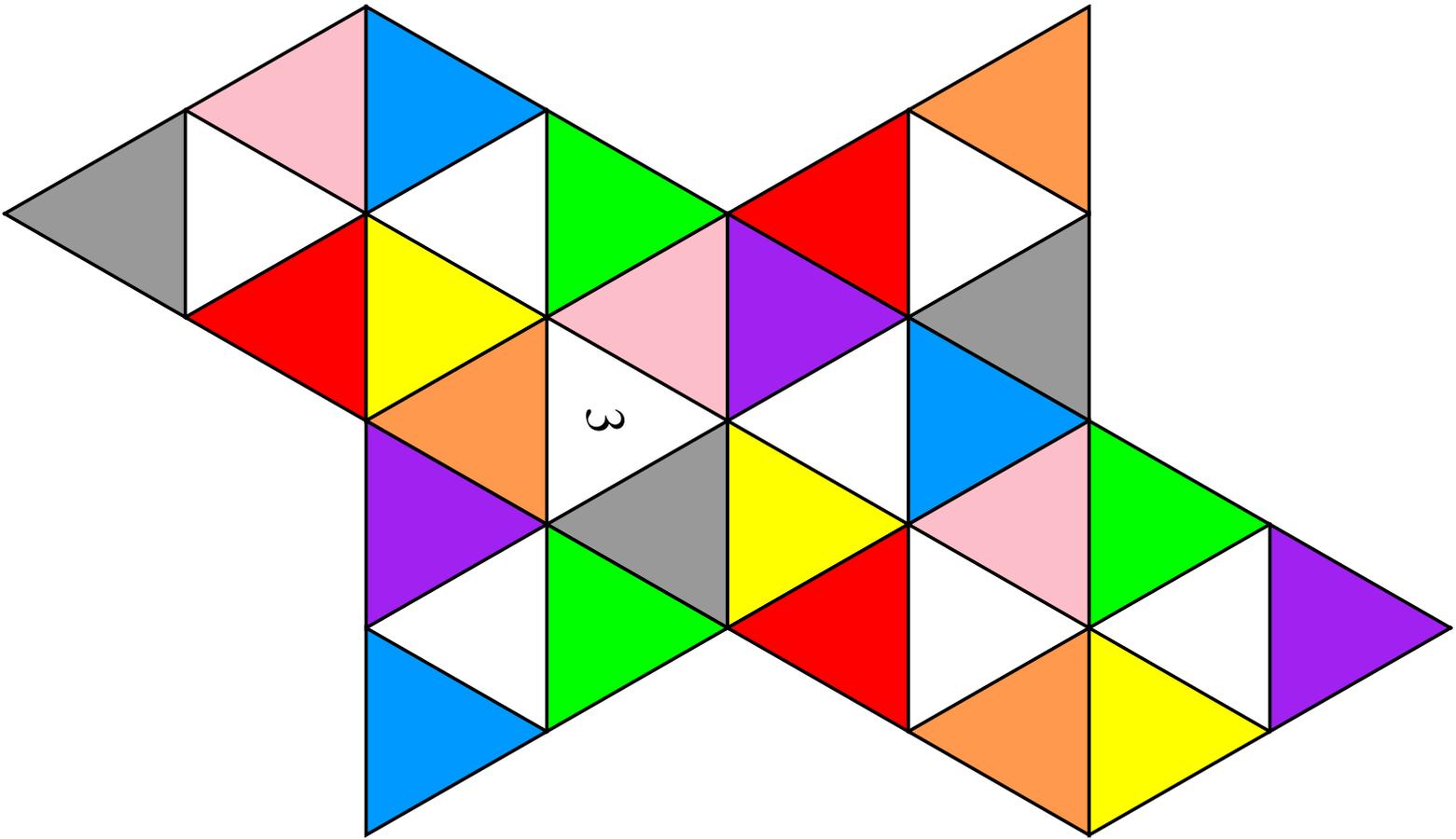


Figure 22: Template for Pattern 3, and also a blank

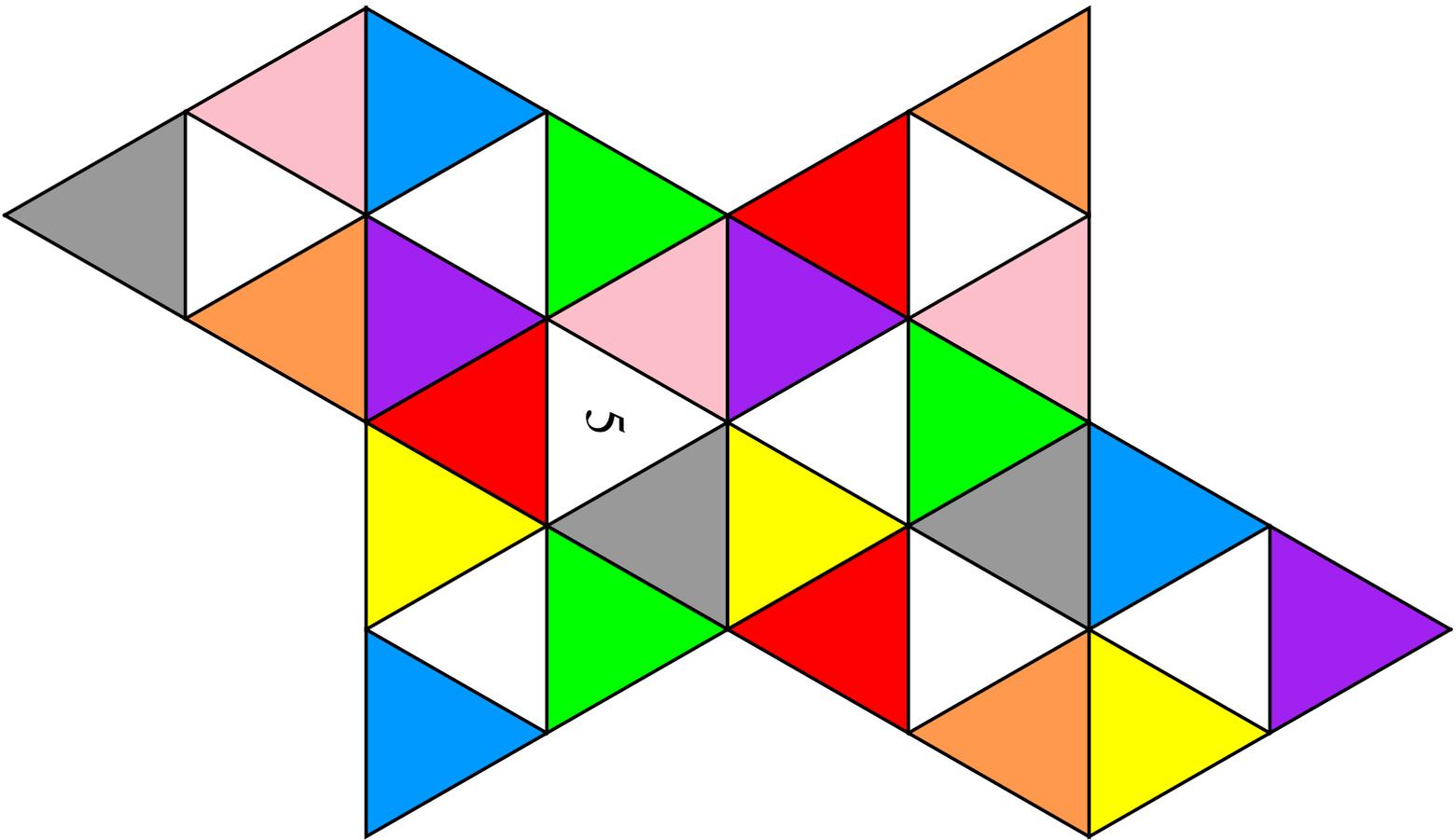
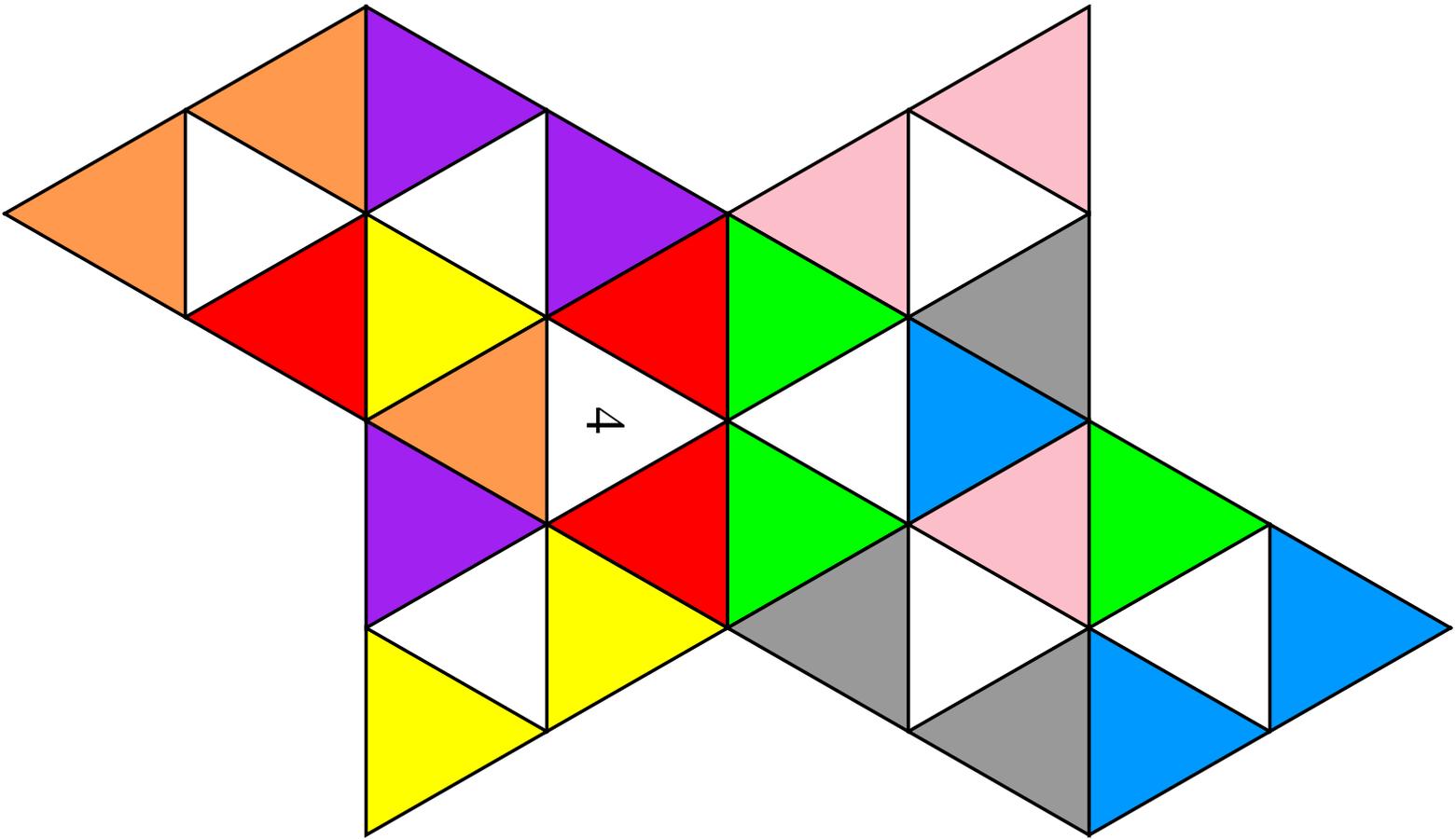


Figure 23: Templates for Patterns 4 and 5

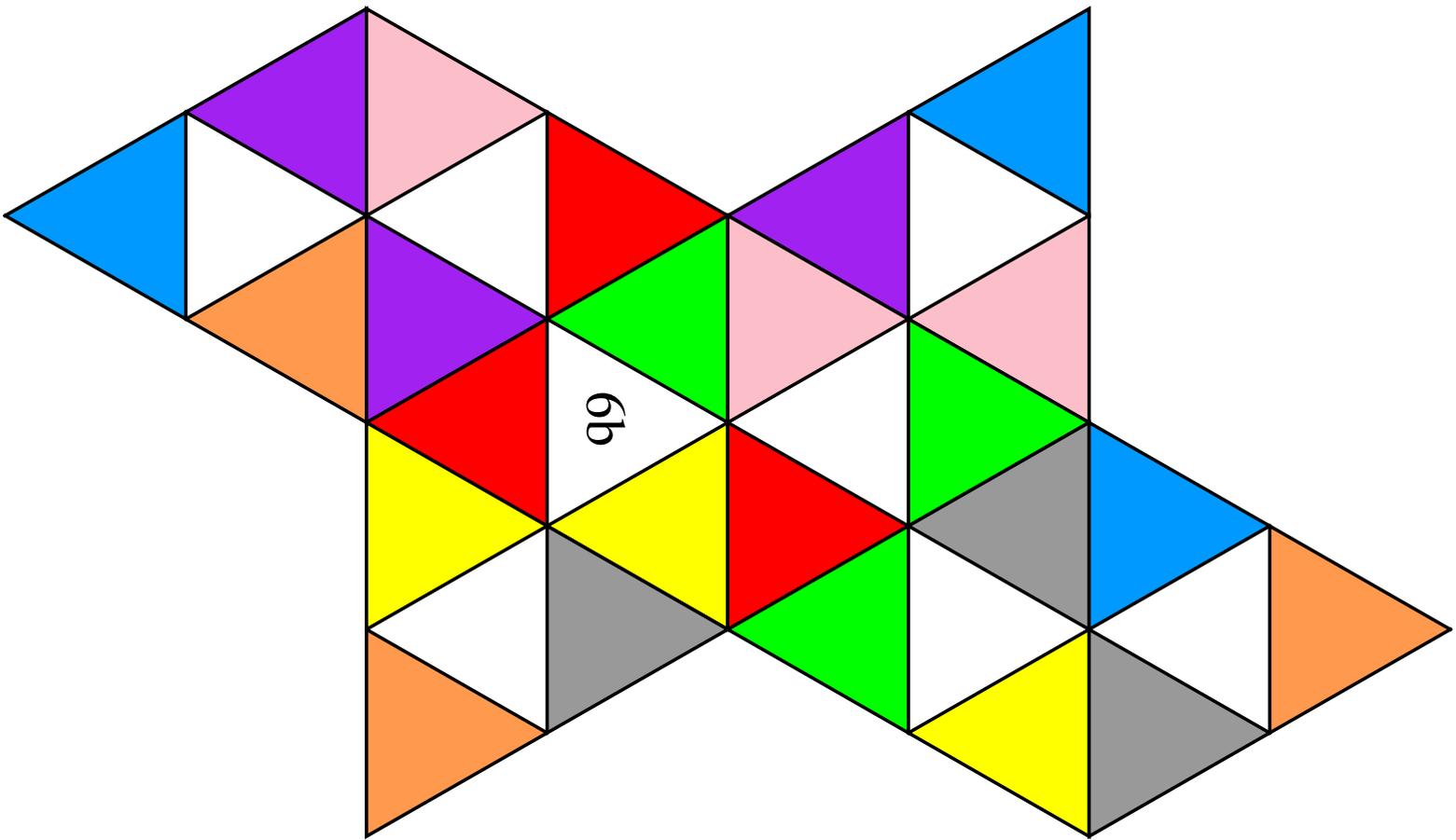
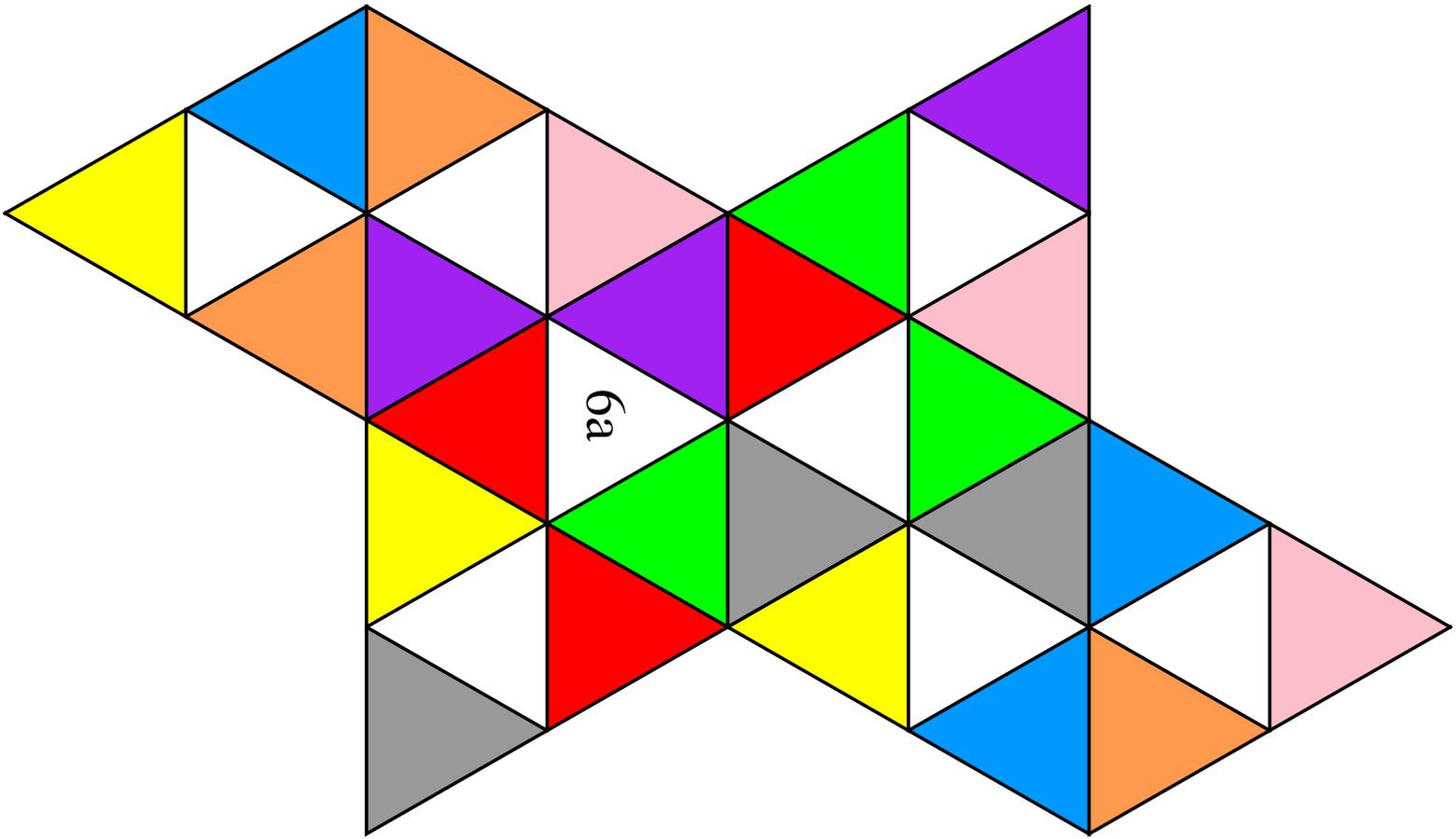


Figure 24: Templates for Patterns 6a,b

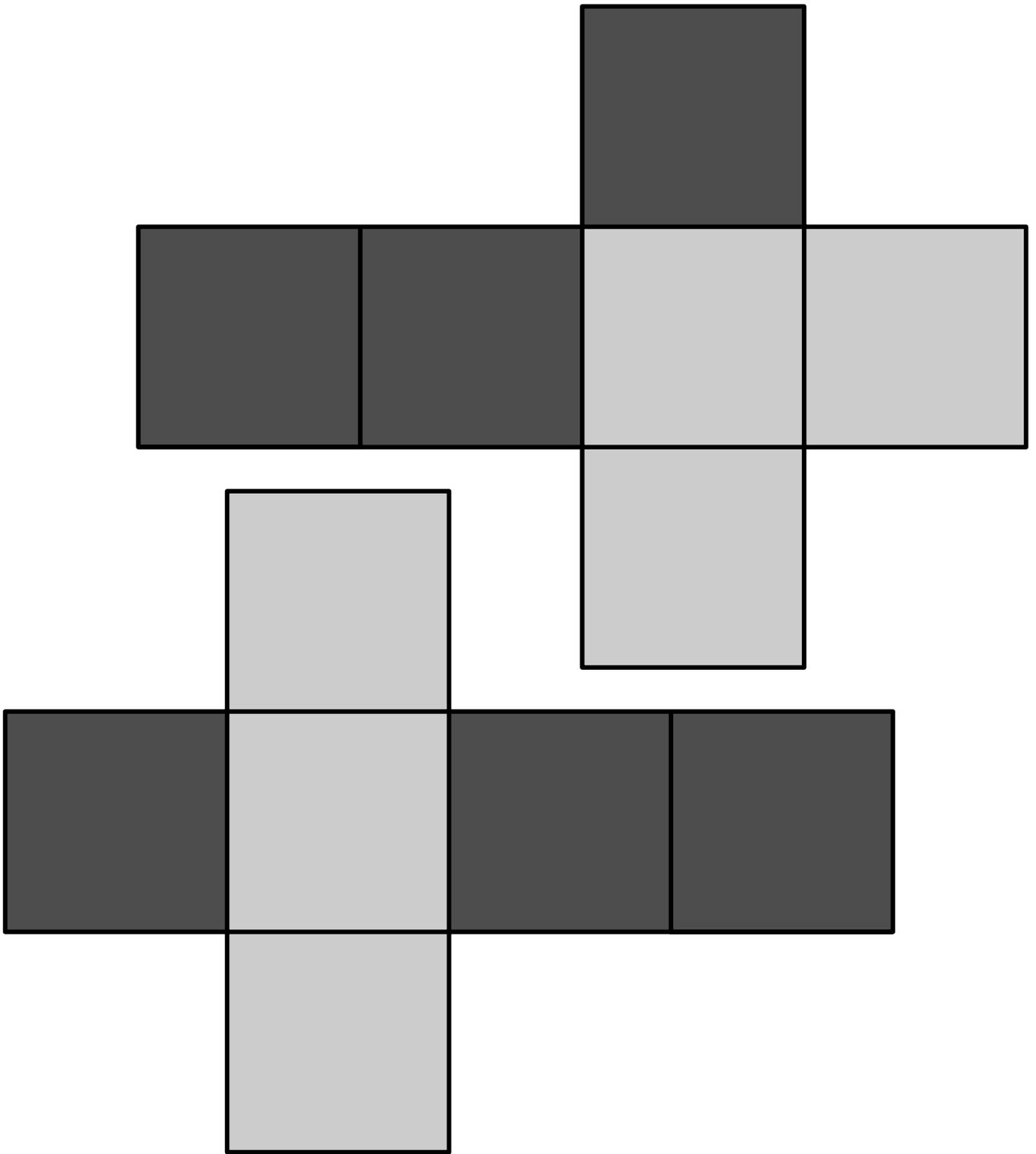


Figure 25: Templates for Figure 6

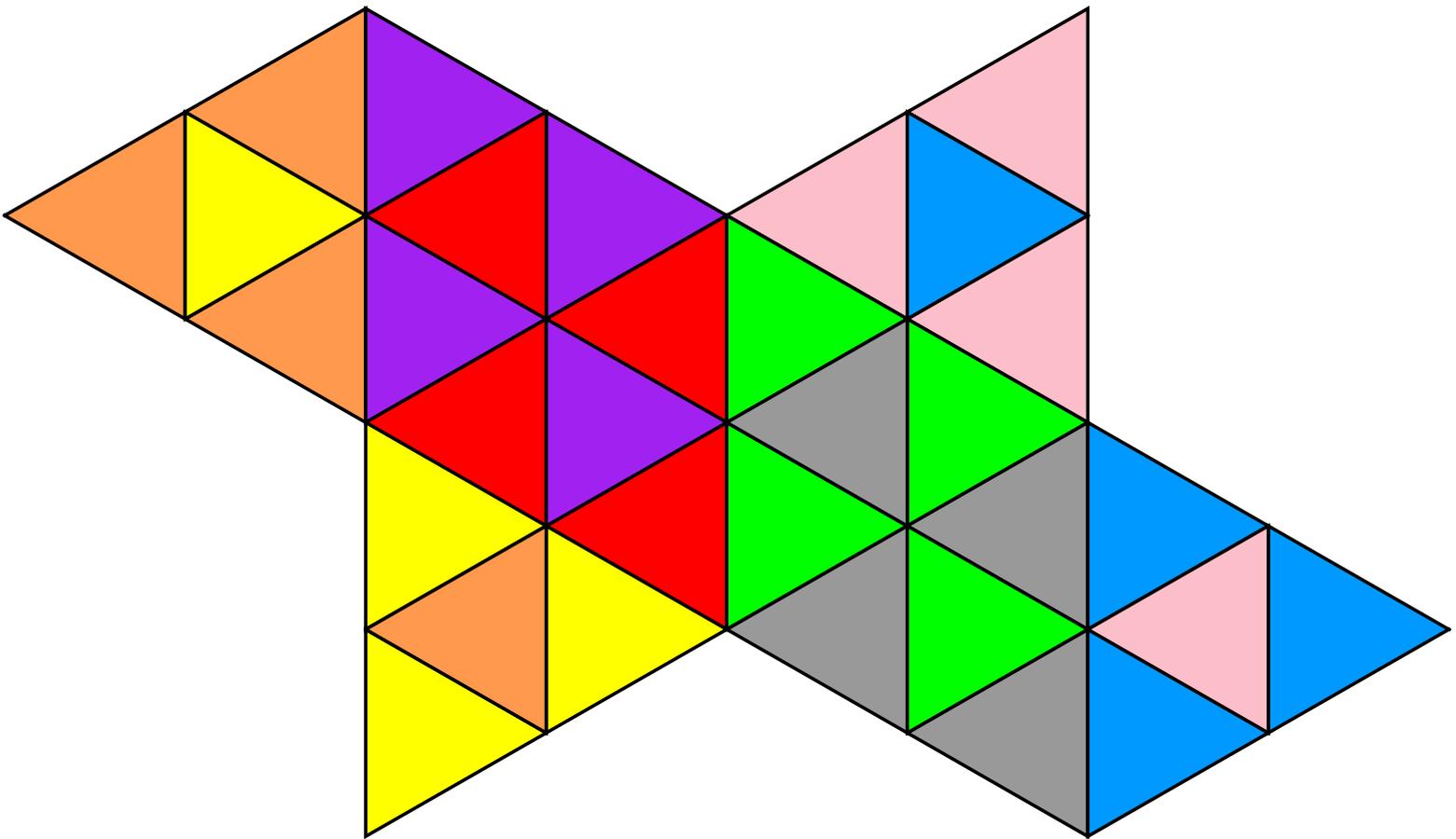
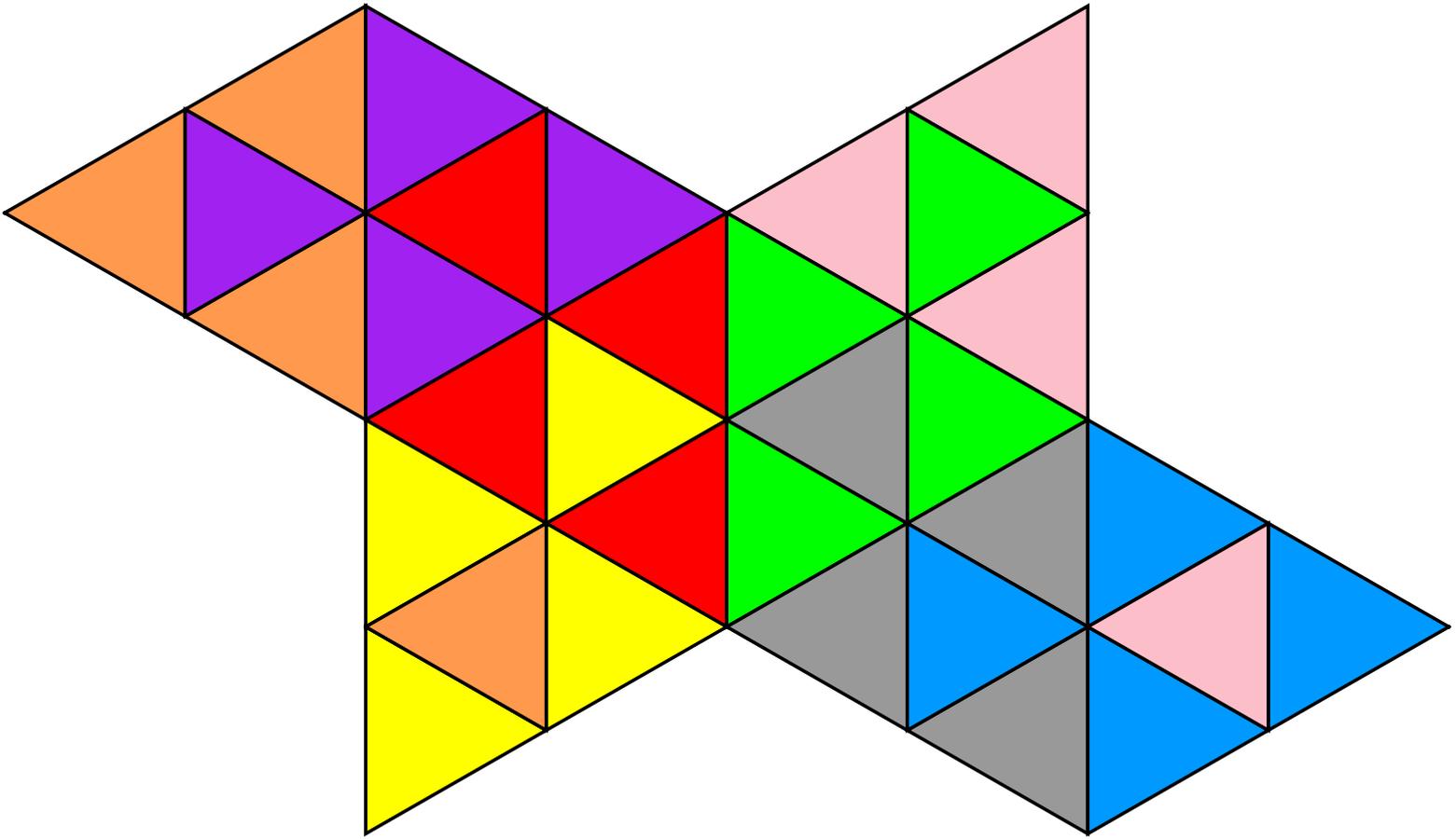


Figure 26: Templates for Figure 7

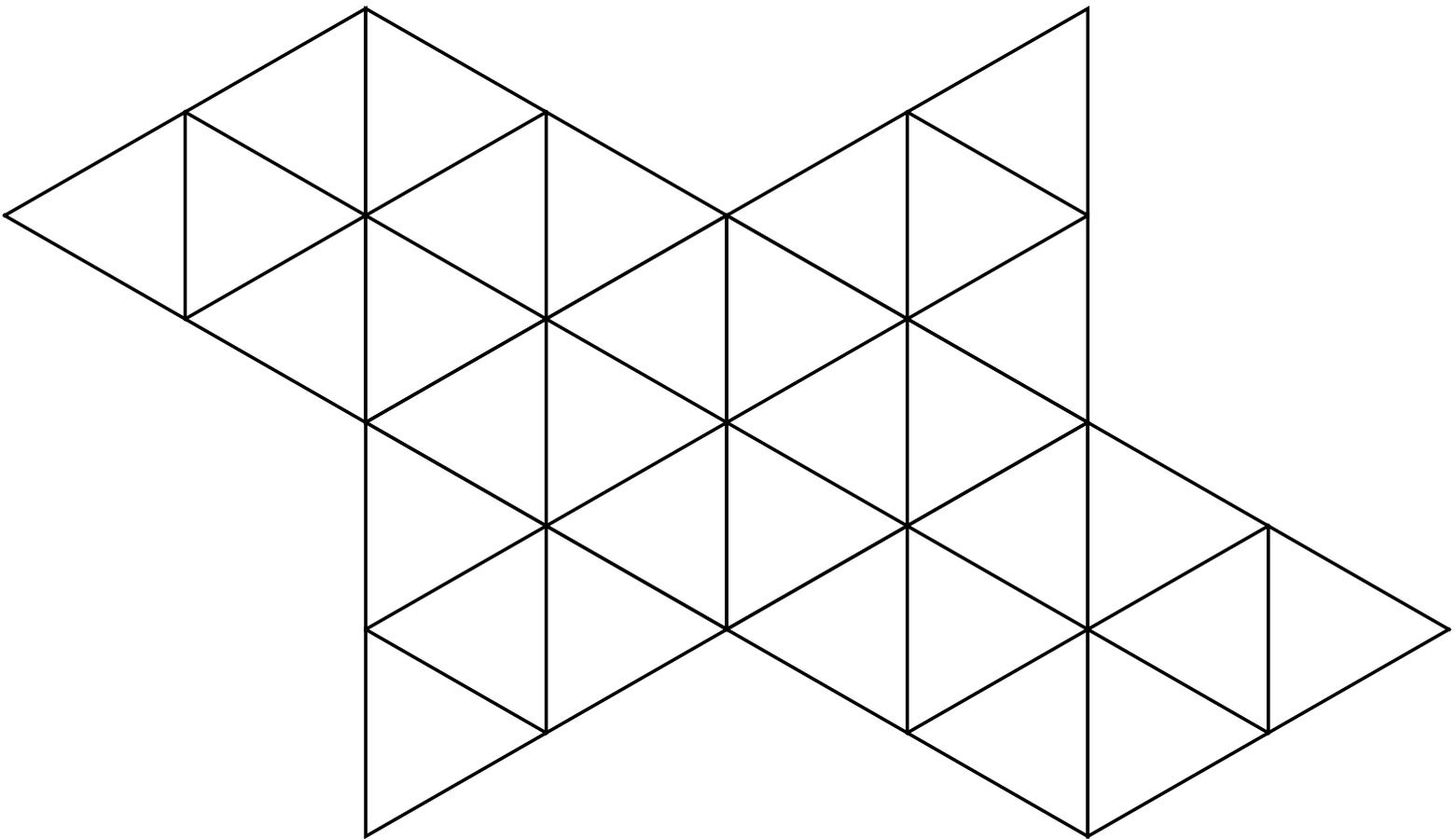
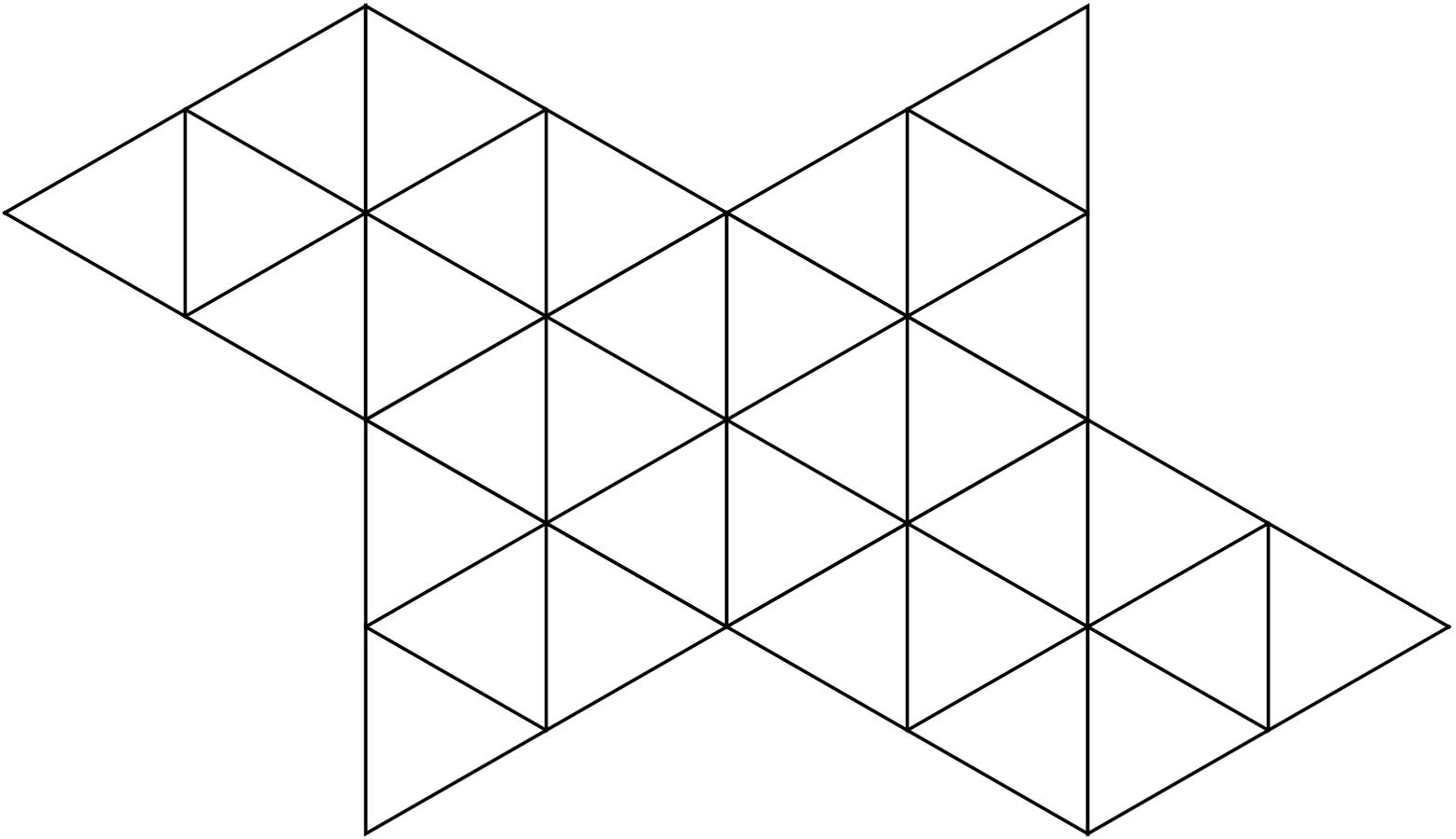


Figure 27: Two blank templates, so you can try out various combinations yourself.

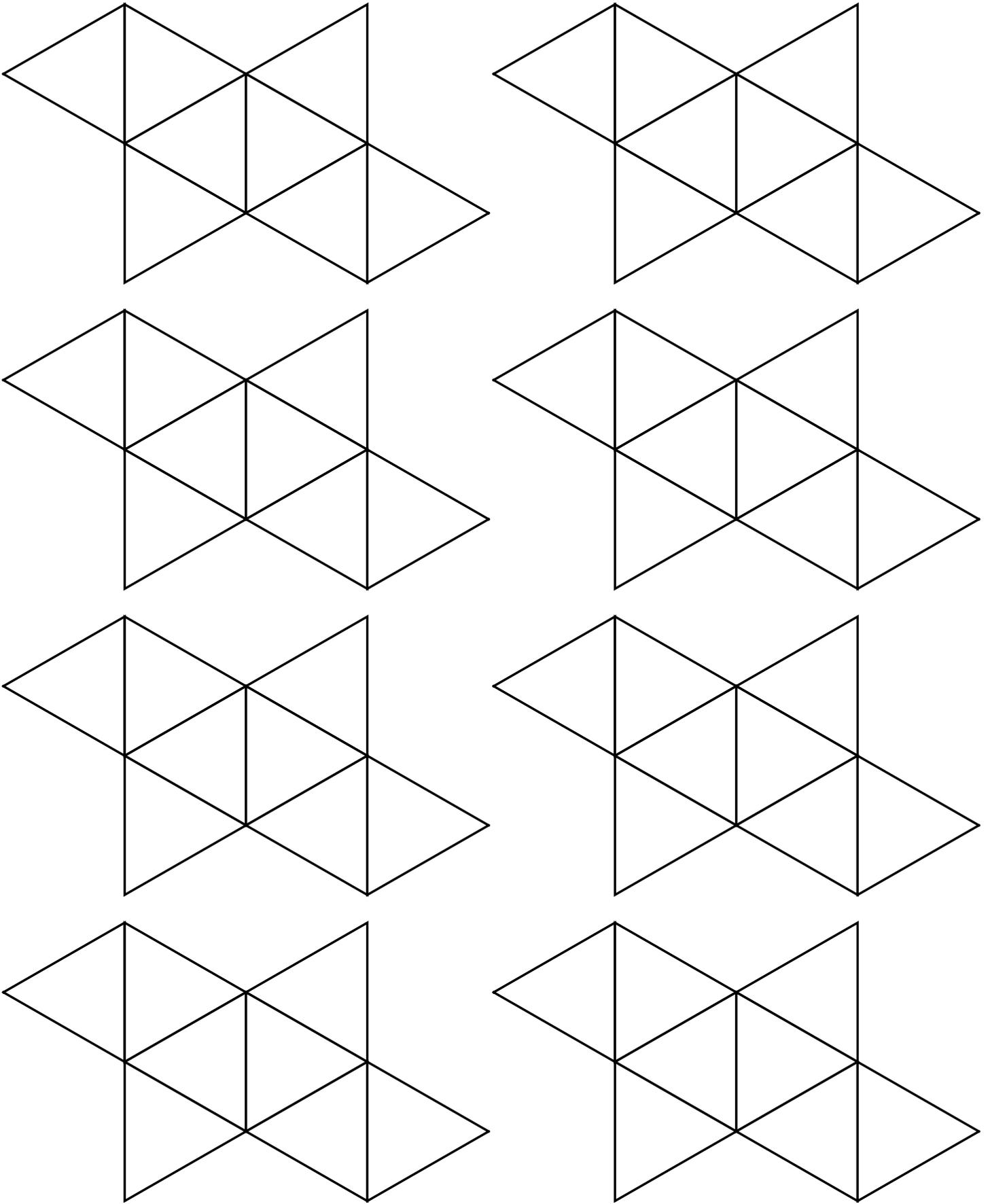


Figure 28: Recall that one of the main challenges was to construct the equivalent of Figure 6 for an octahedron. How many geometrically different ways are there of dividing the eight faces on an octahedron into two sets of four each? Play around with these blanks here, and work out the various possibilities. The answer is less than eight, so printing out this one page is sufficient.

As noted also in the Supplementary Materials, the article presented a number of challenges:

1. Can you guess the eight equality patterns that weren't given in the article, and thereby obtain the – surprisingly simple – rules governing all 16 of them?
2. Can you guess at least a few of the equivalence patterns?
3. As the first step in systematically classifying all possible equivalence patterns, what is the equivalent of Figure 6 on an octahedron?
4. Given at least some of the possible corner arrangements, play around a bit with different arrangements of the centers, such as those in Figure 7.

All templates after this page relate to some of the answers to these challenges, so please try to work out at least some of the answers yourself first before moving on and looking at the Supplementary Material, and the templates following this page.



Figure 29: As Professor Lira is always telling her students Moosie, Chuchi and Chichu, the best way to learn math is to try to work it out yourself before looking at someone else's answers!

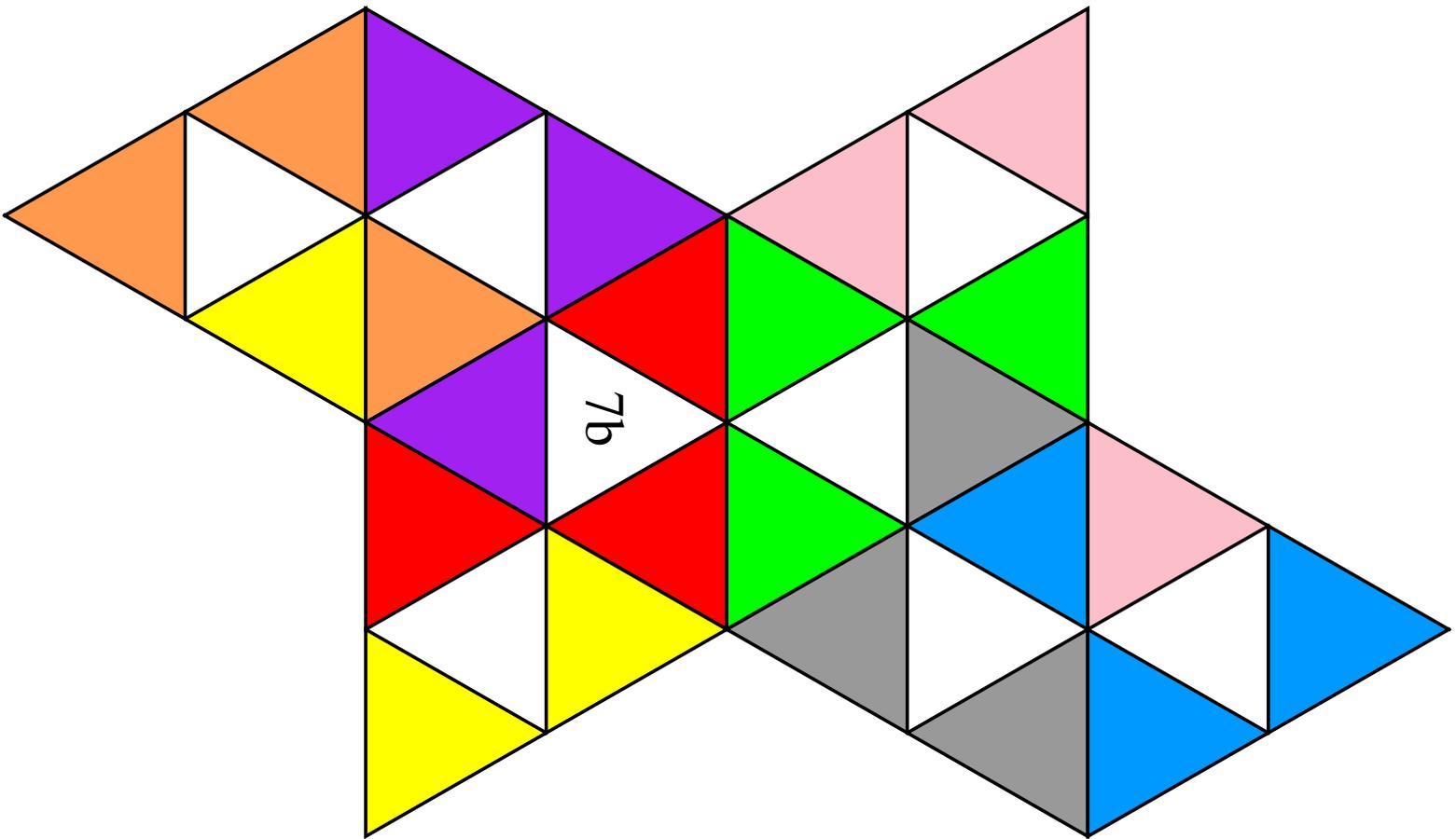
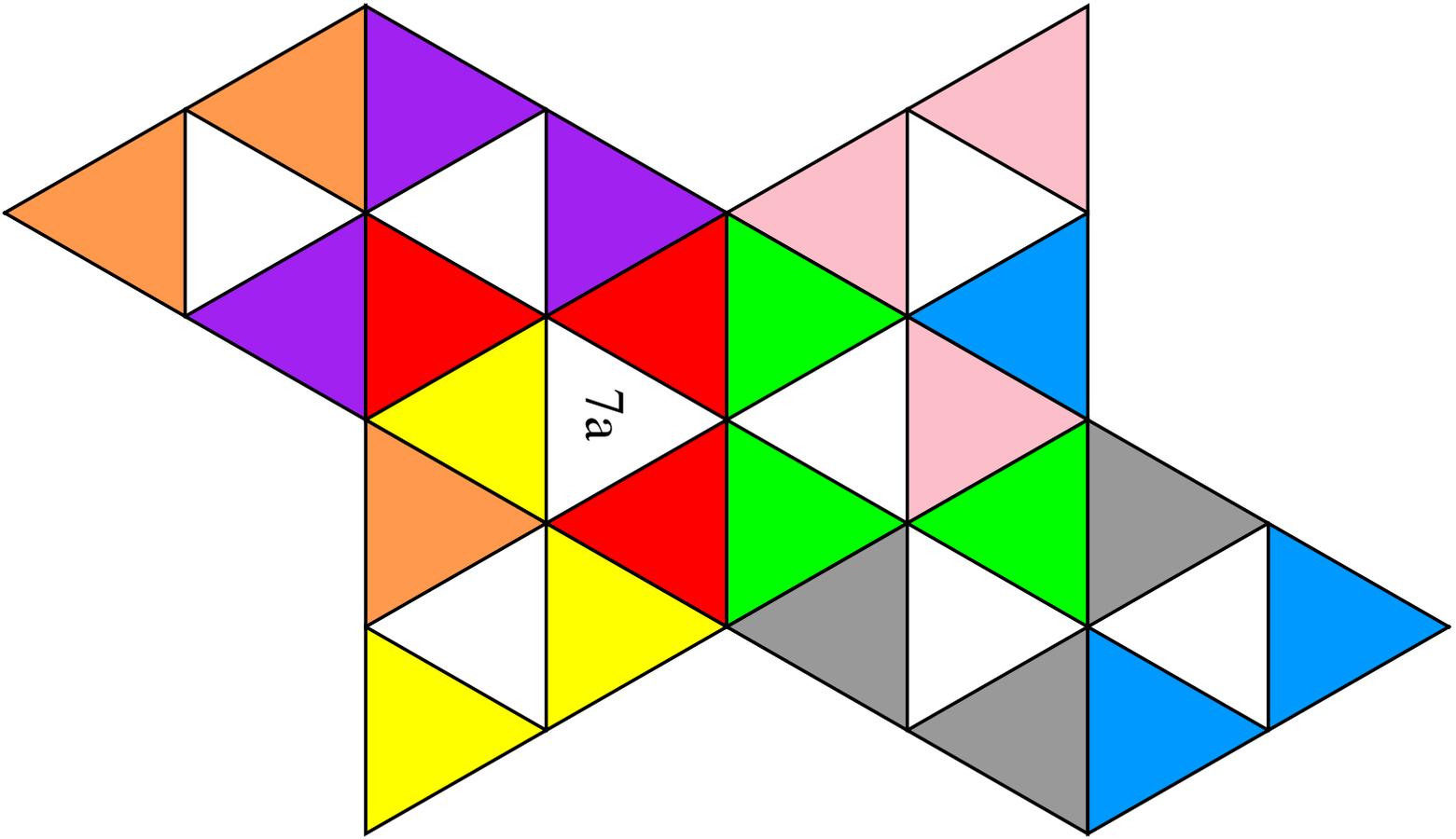


Figure 30: Templates for Patterns 7a,b

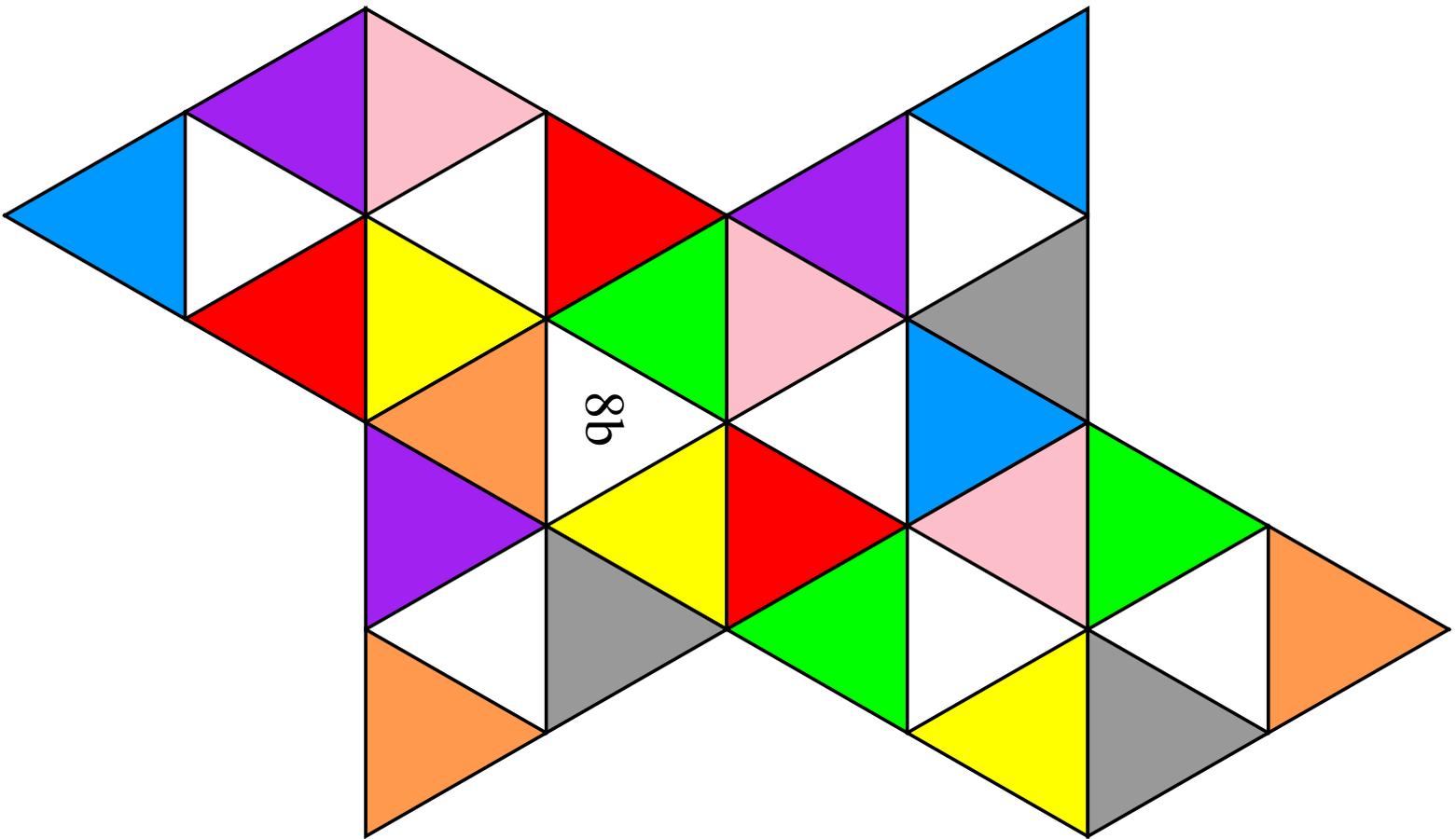
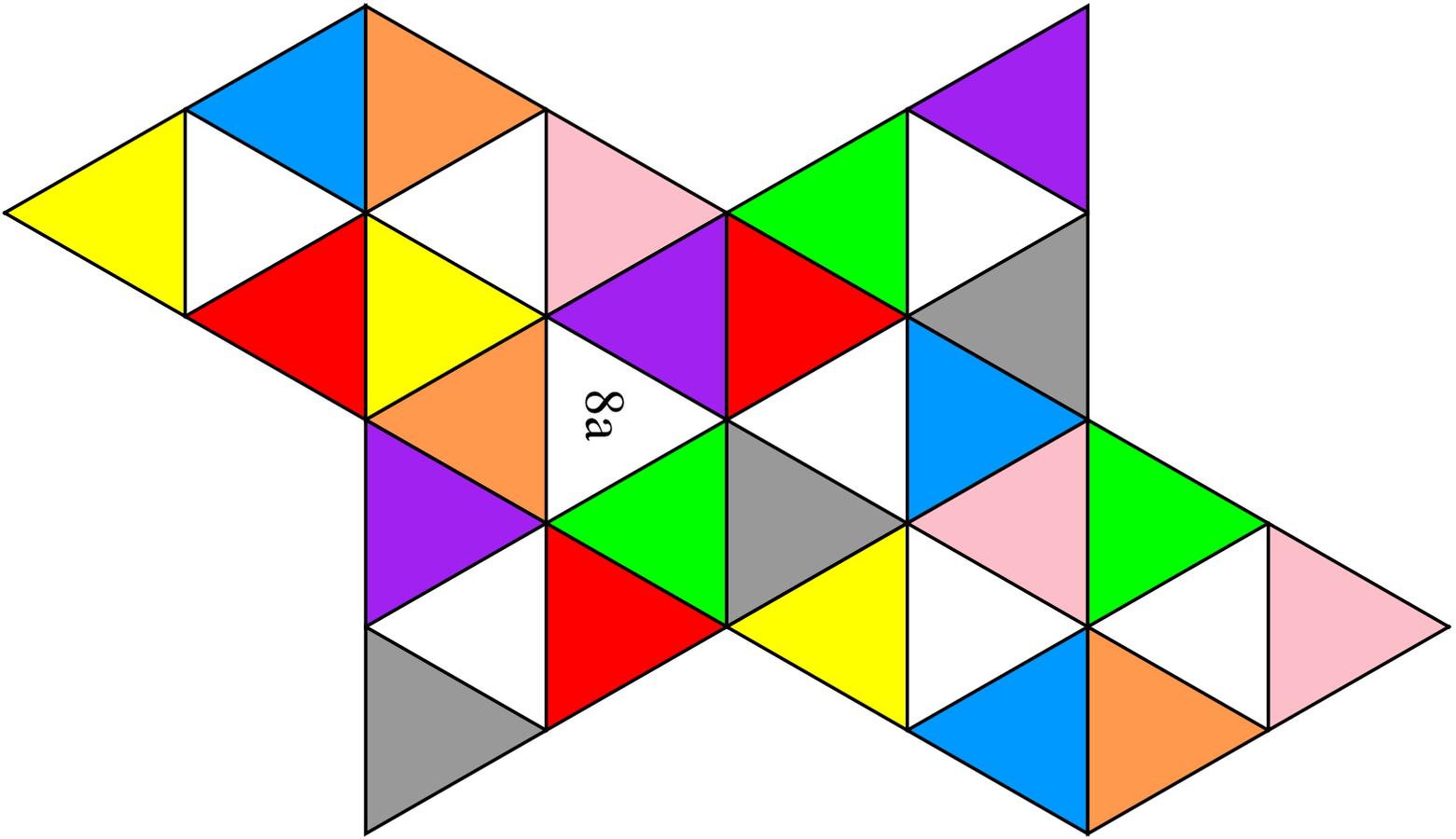


Figure 31: Templates for Patterns 8a,b

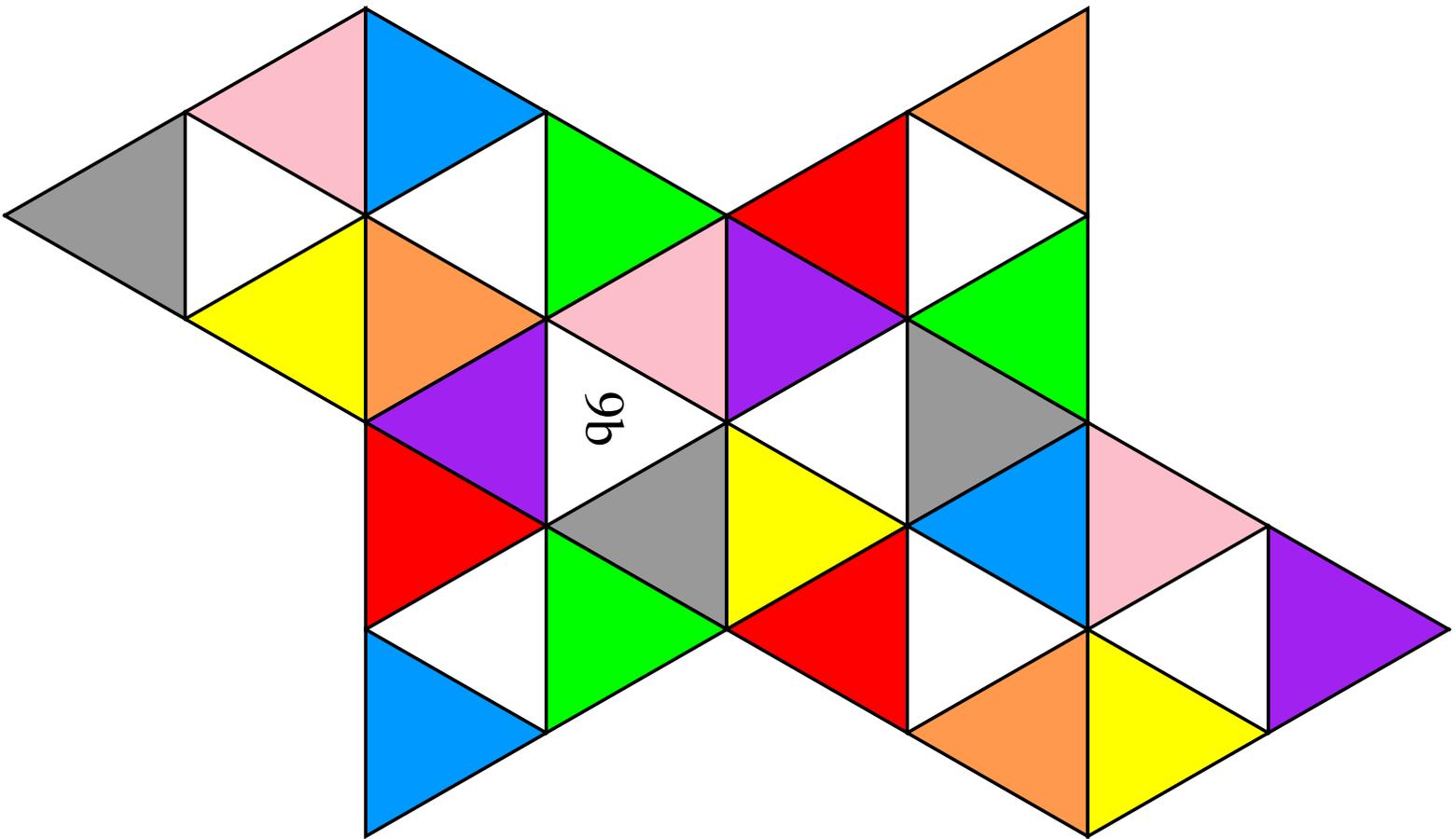
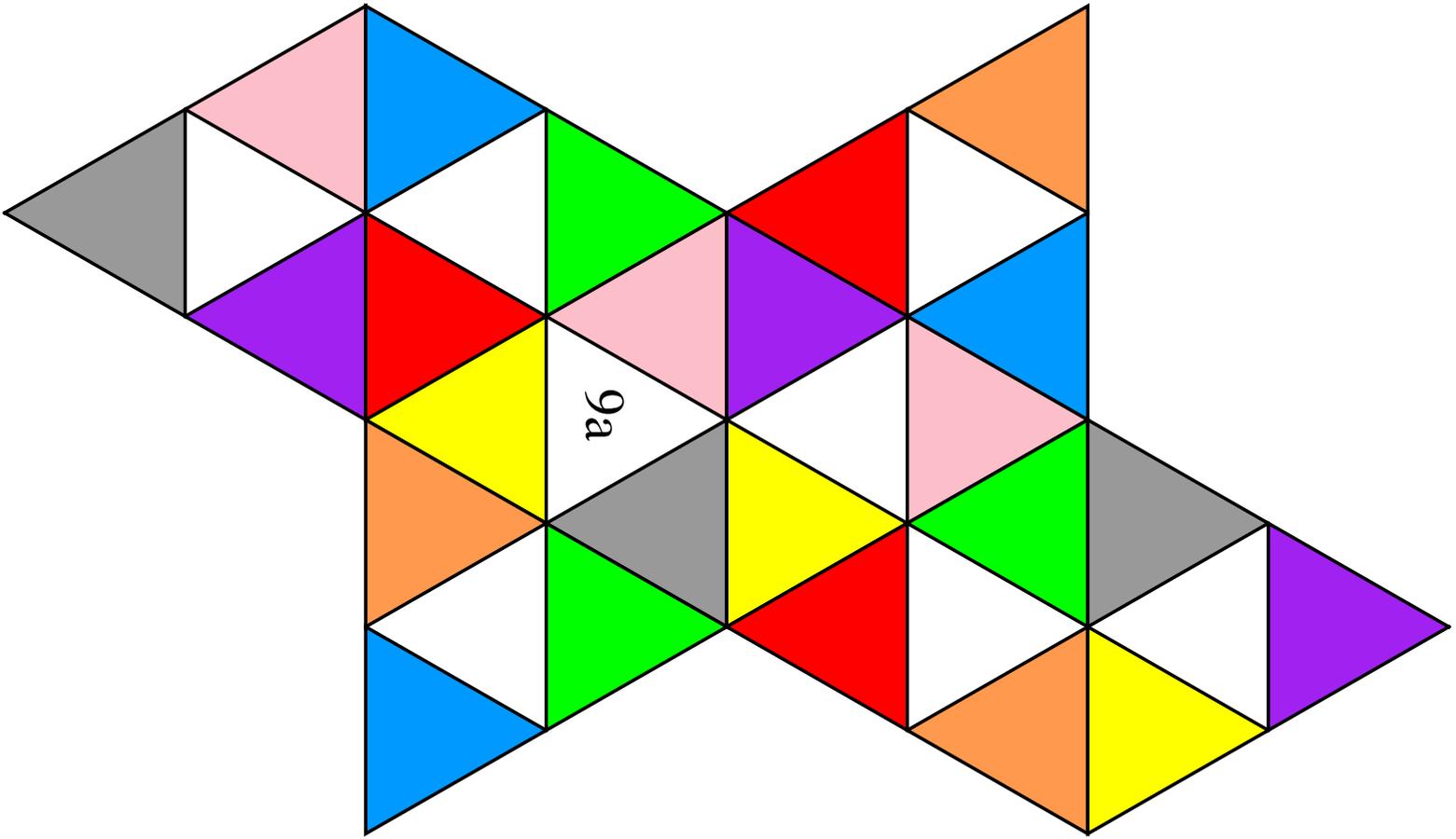


Figure 32: Templates for Patterns 9a,b

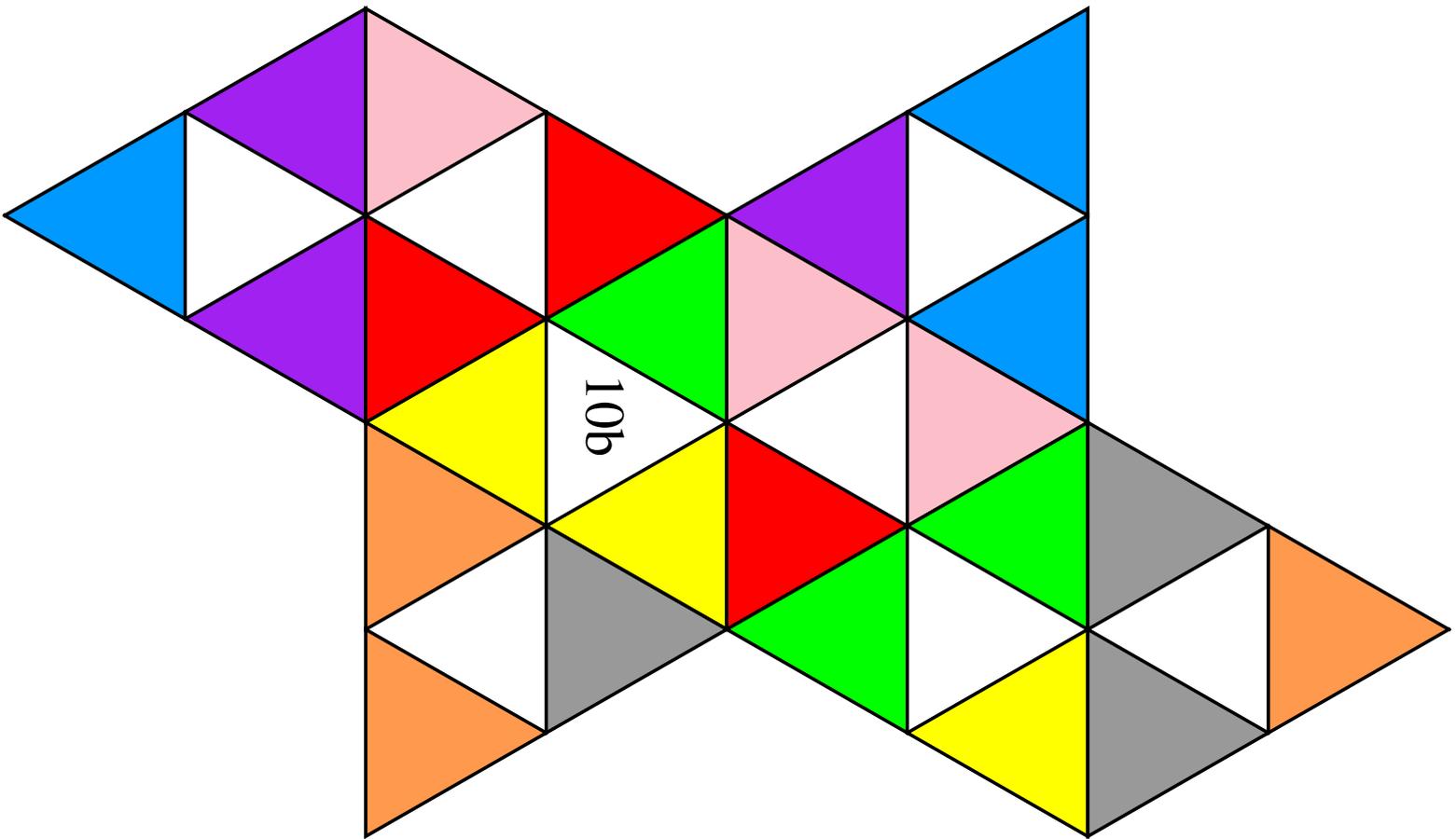
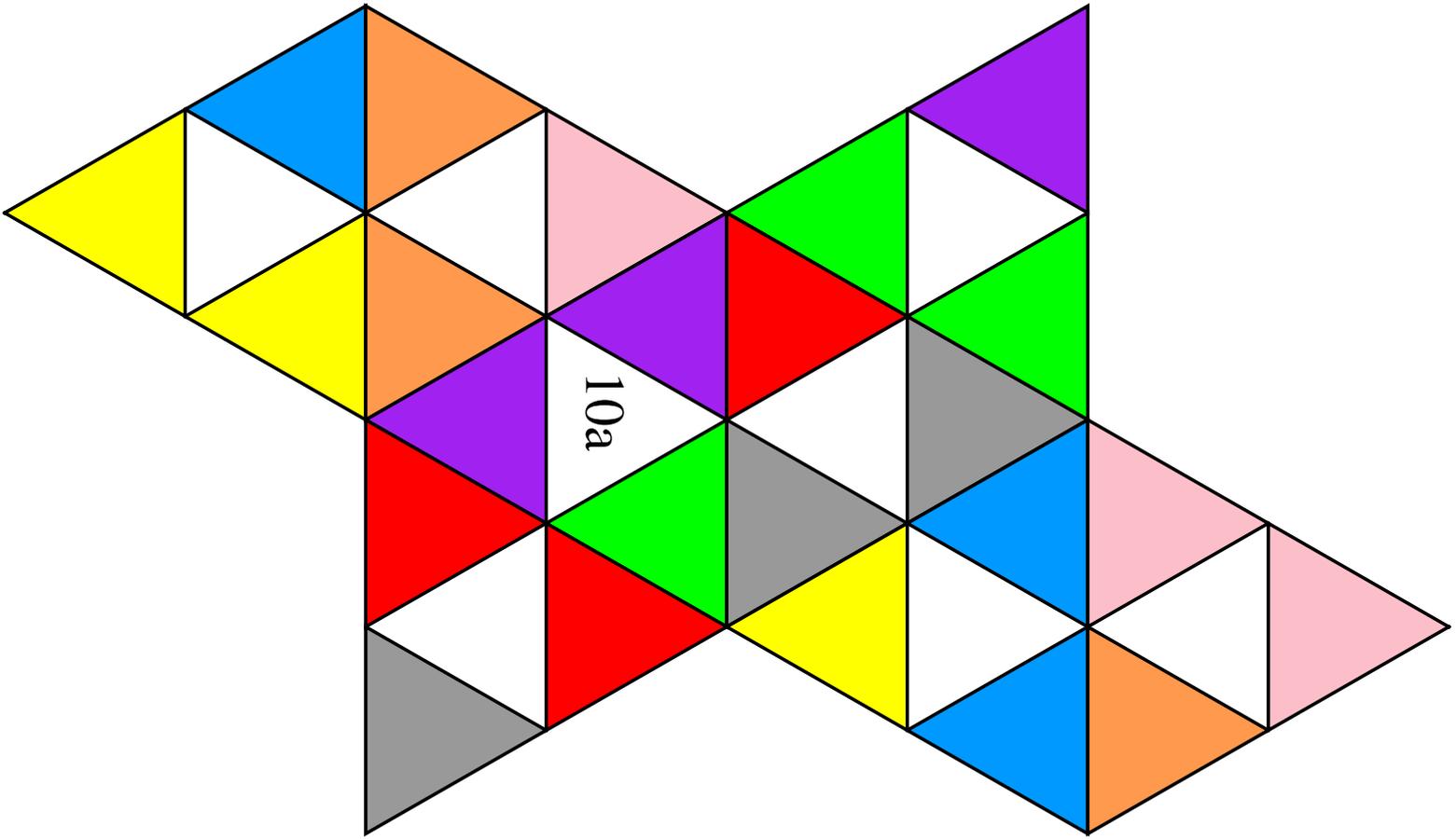


Figure 33: Templates for Patterns 10a,b

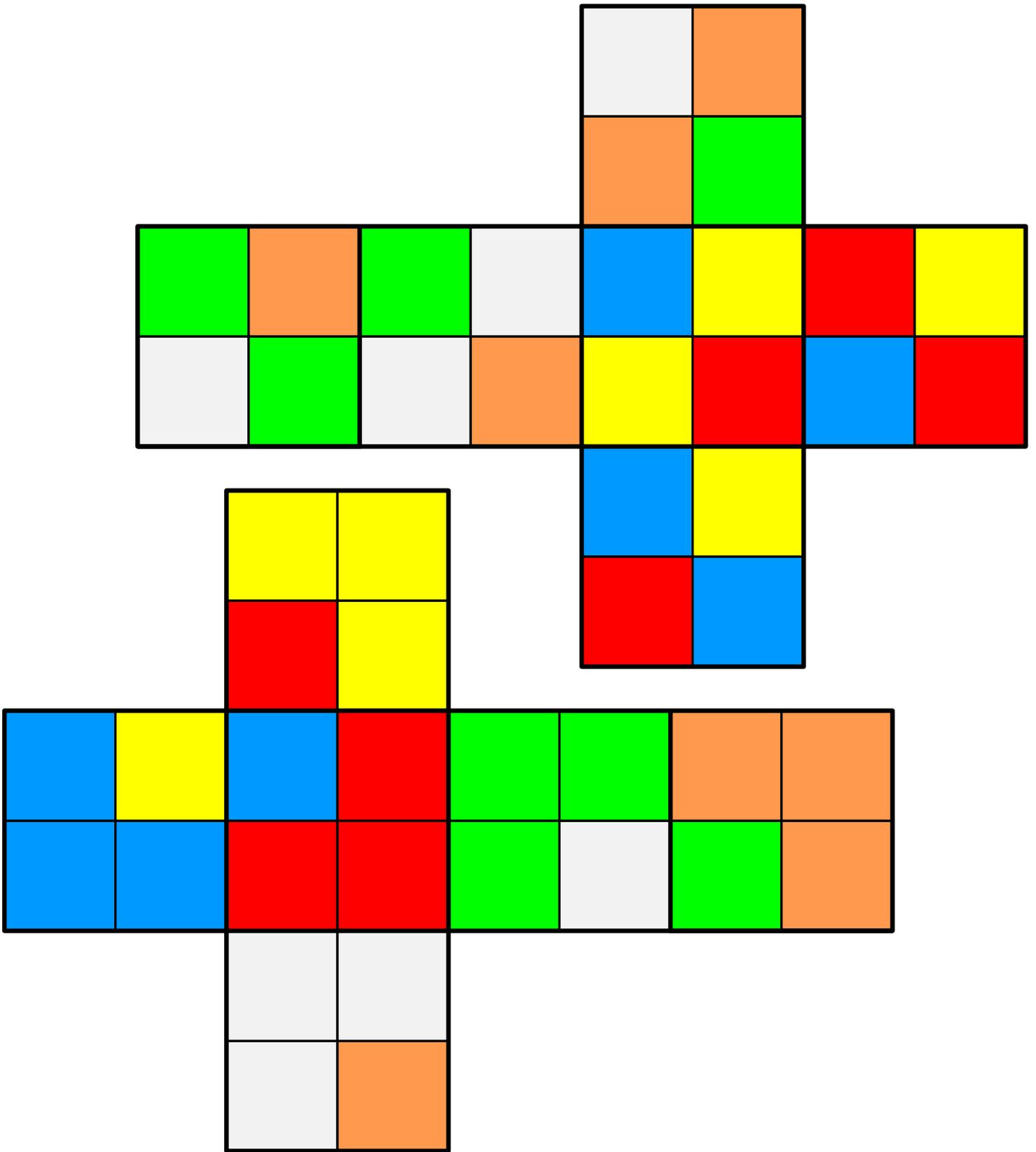


Figure 34: Templates for Figure 9

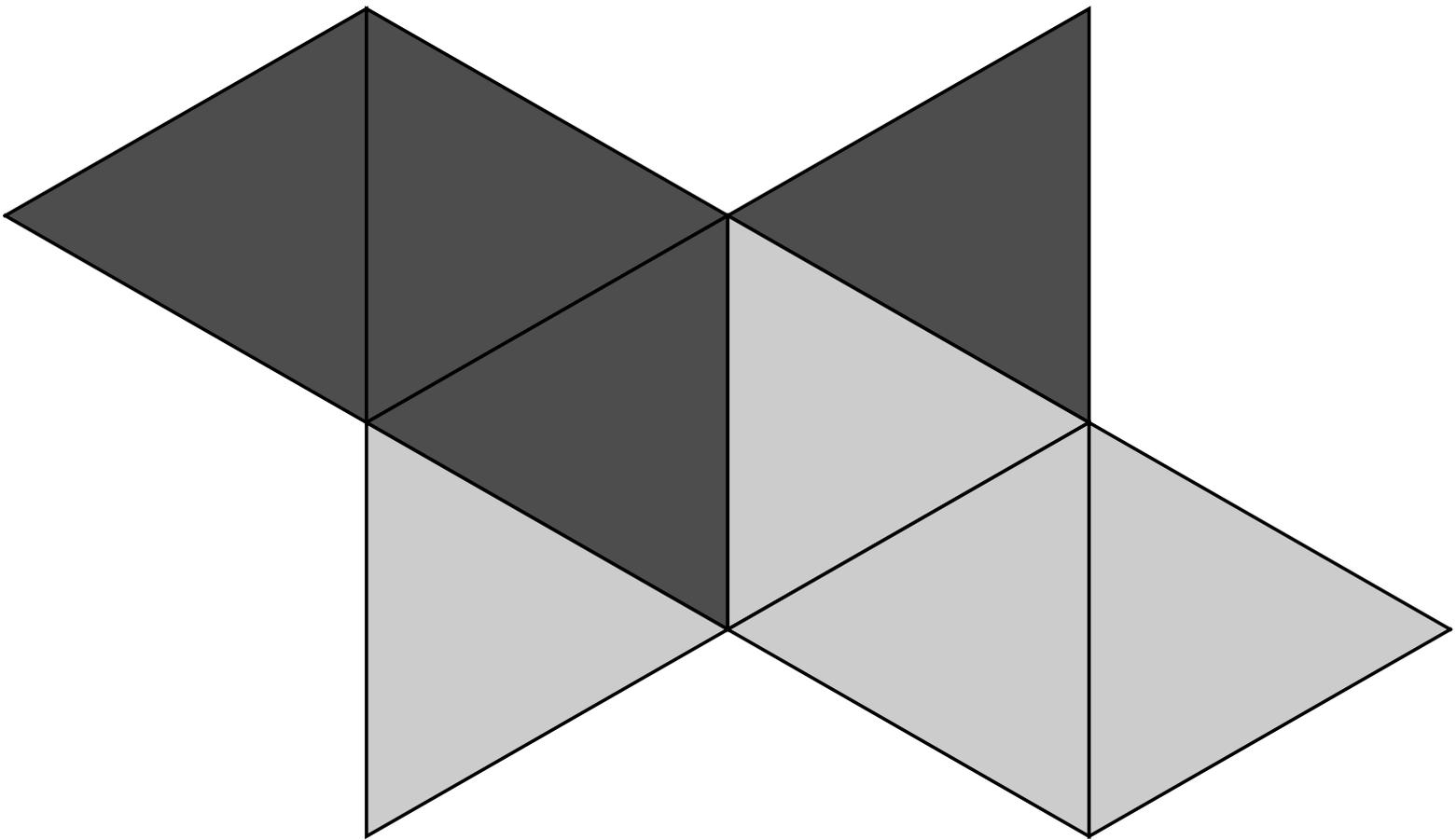
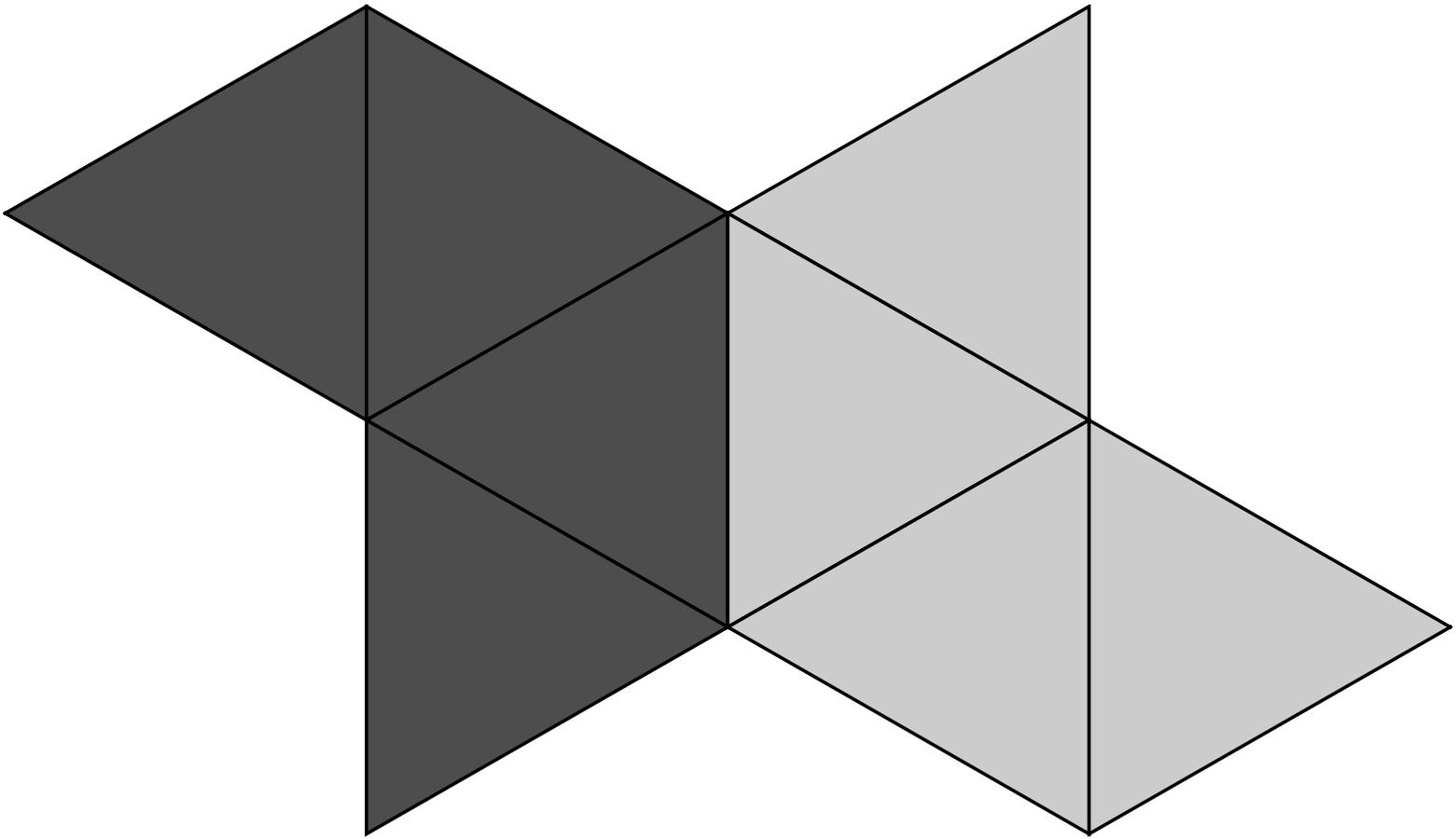


Figure 35: Templates for Figure 10a,b. a also gets relabelled as the Type I equivalence.

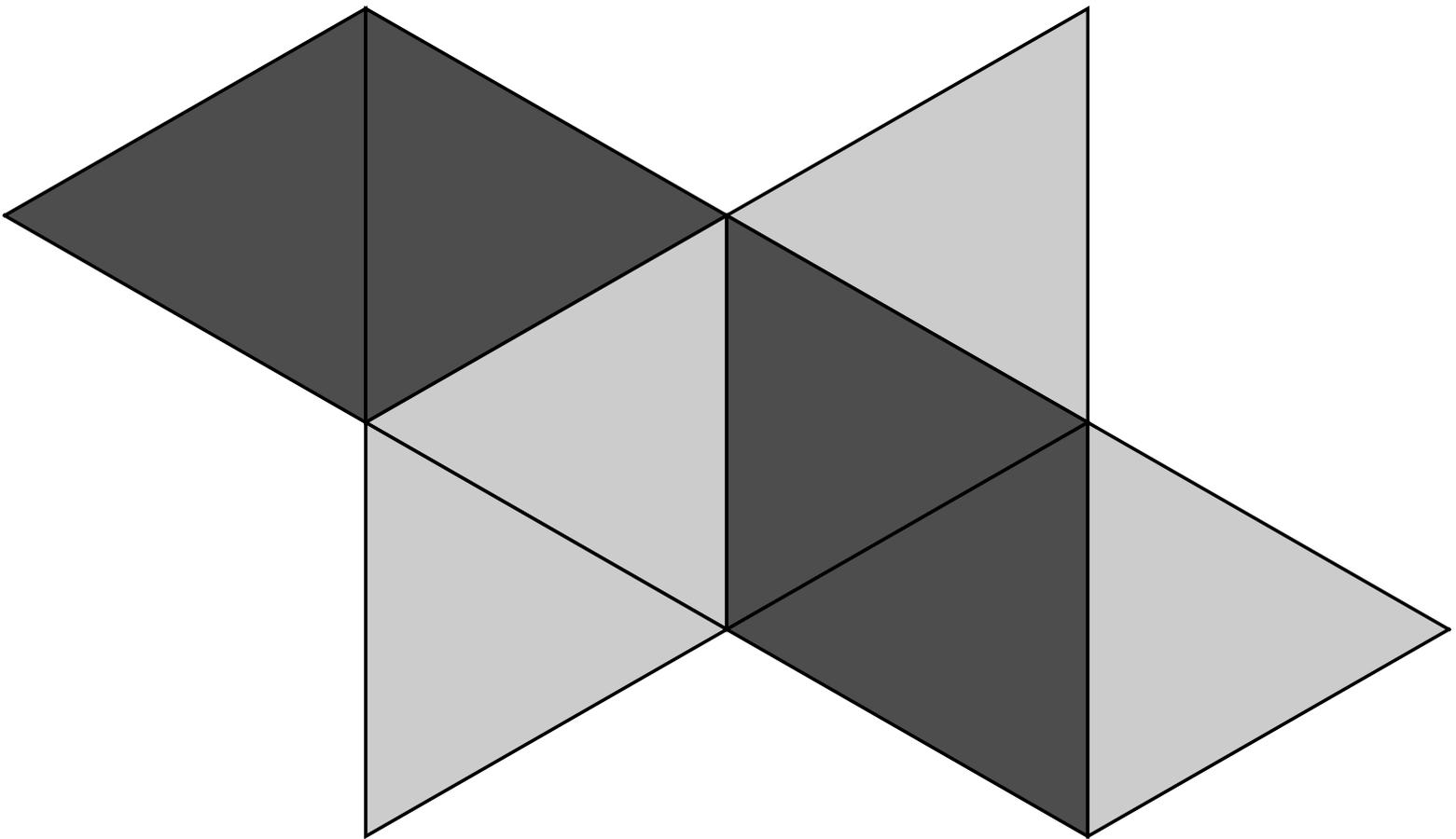
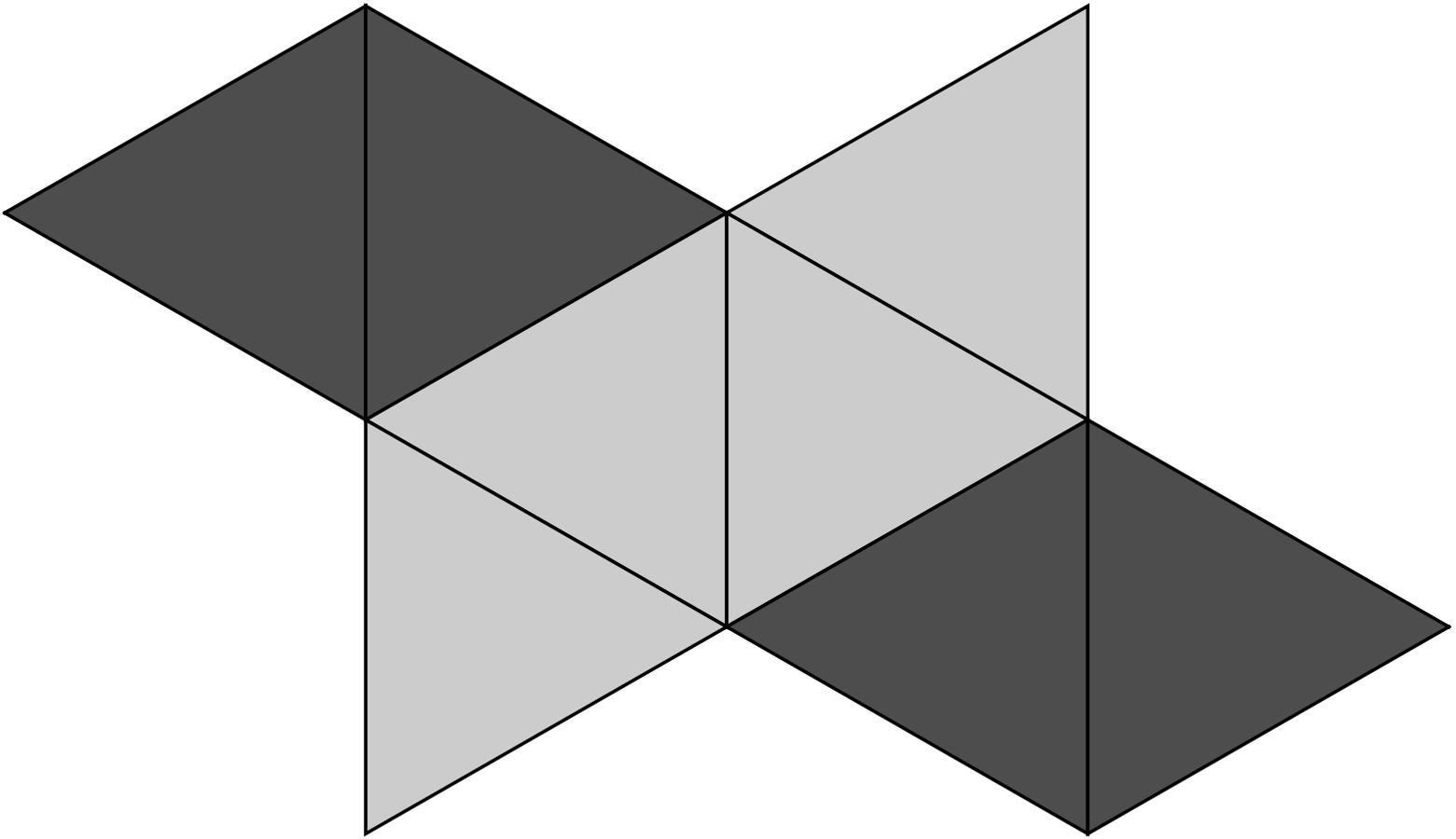


Figure 36: Templates for Figure 10c,d. d also gets relabelled as the Type II equivalence.

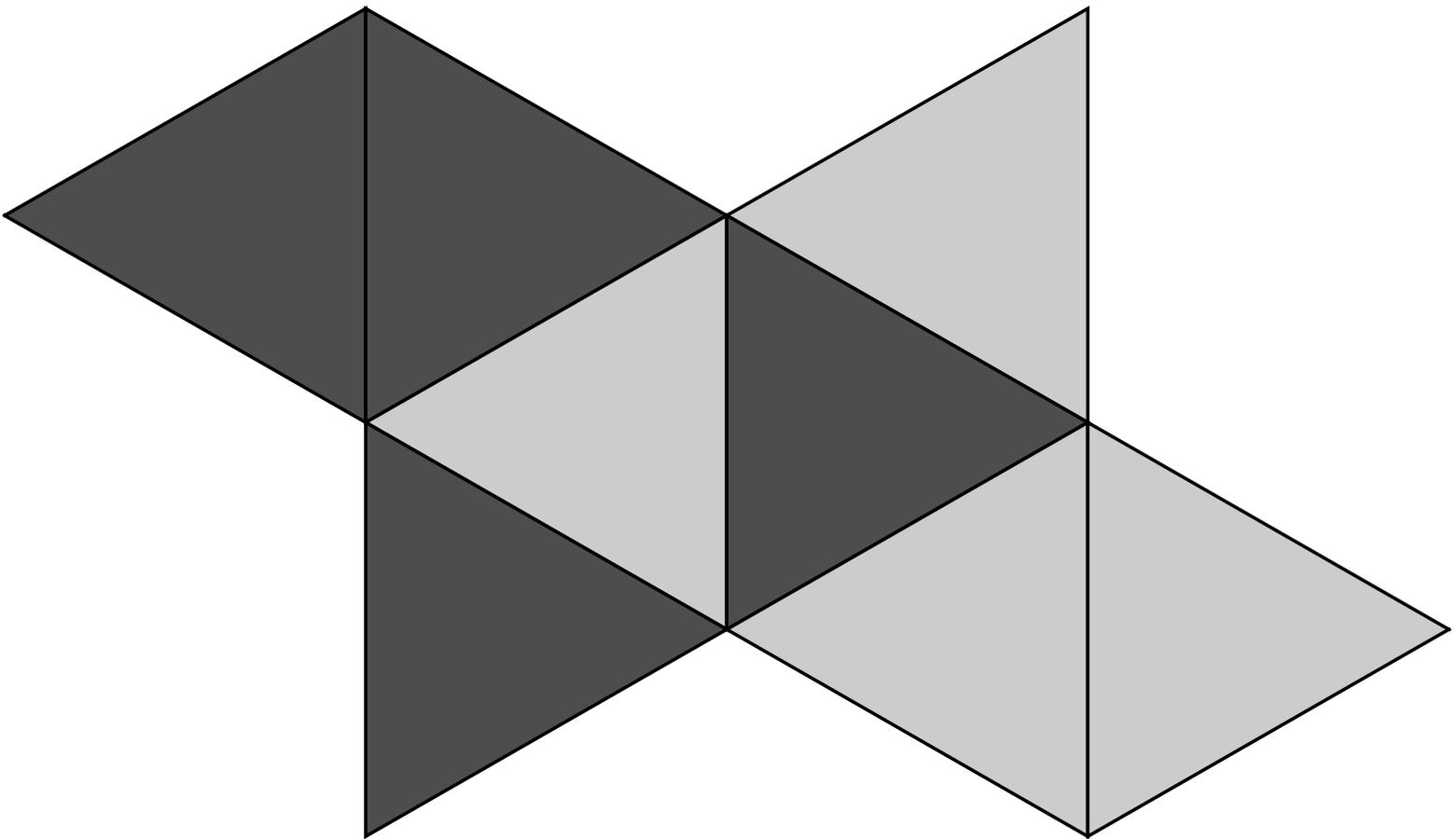
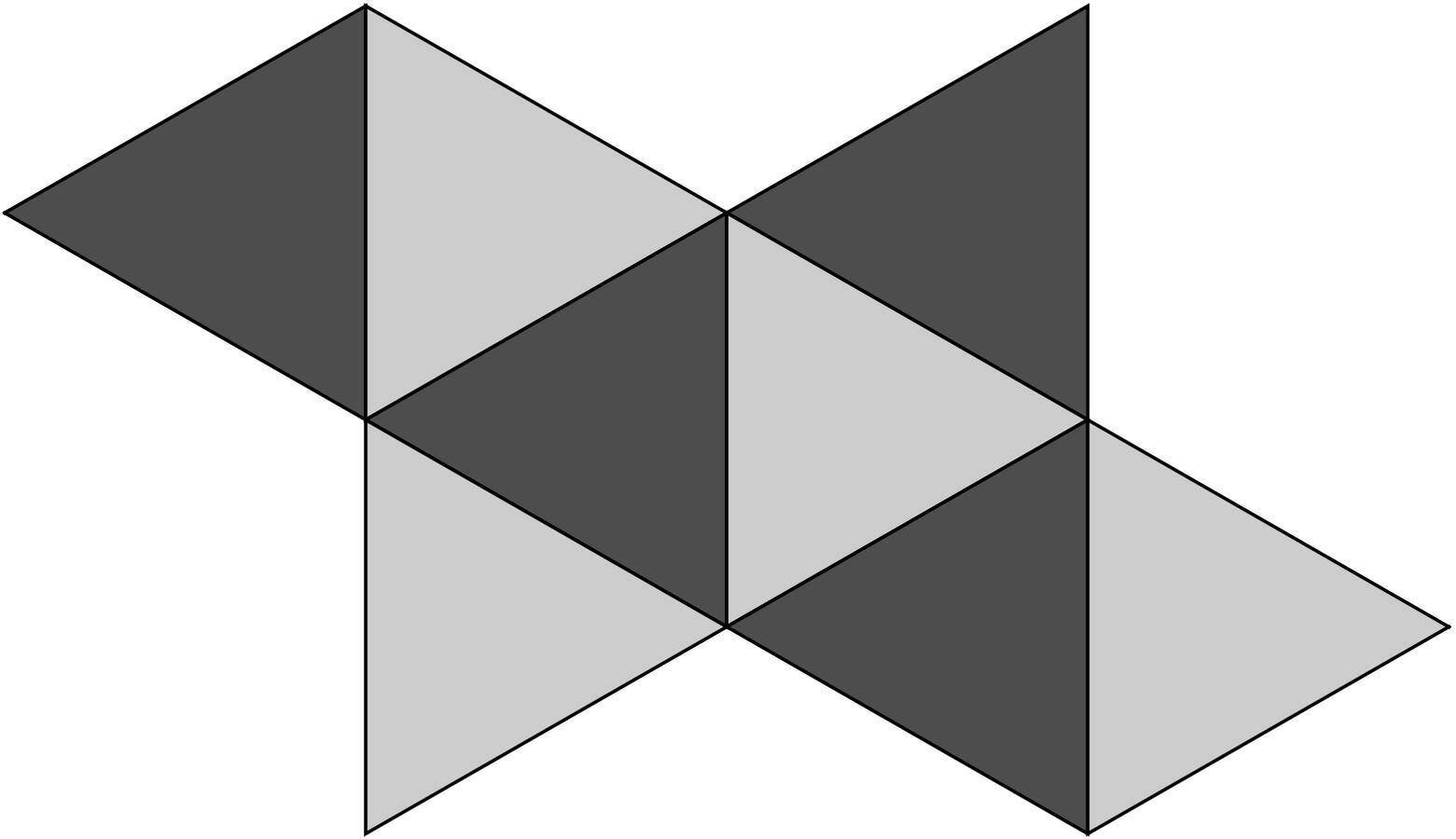


Figure 37: Templates for Figure 10e,f. e also gets relabelled as the Type III equivalence.

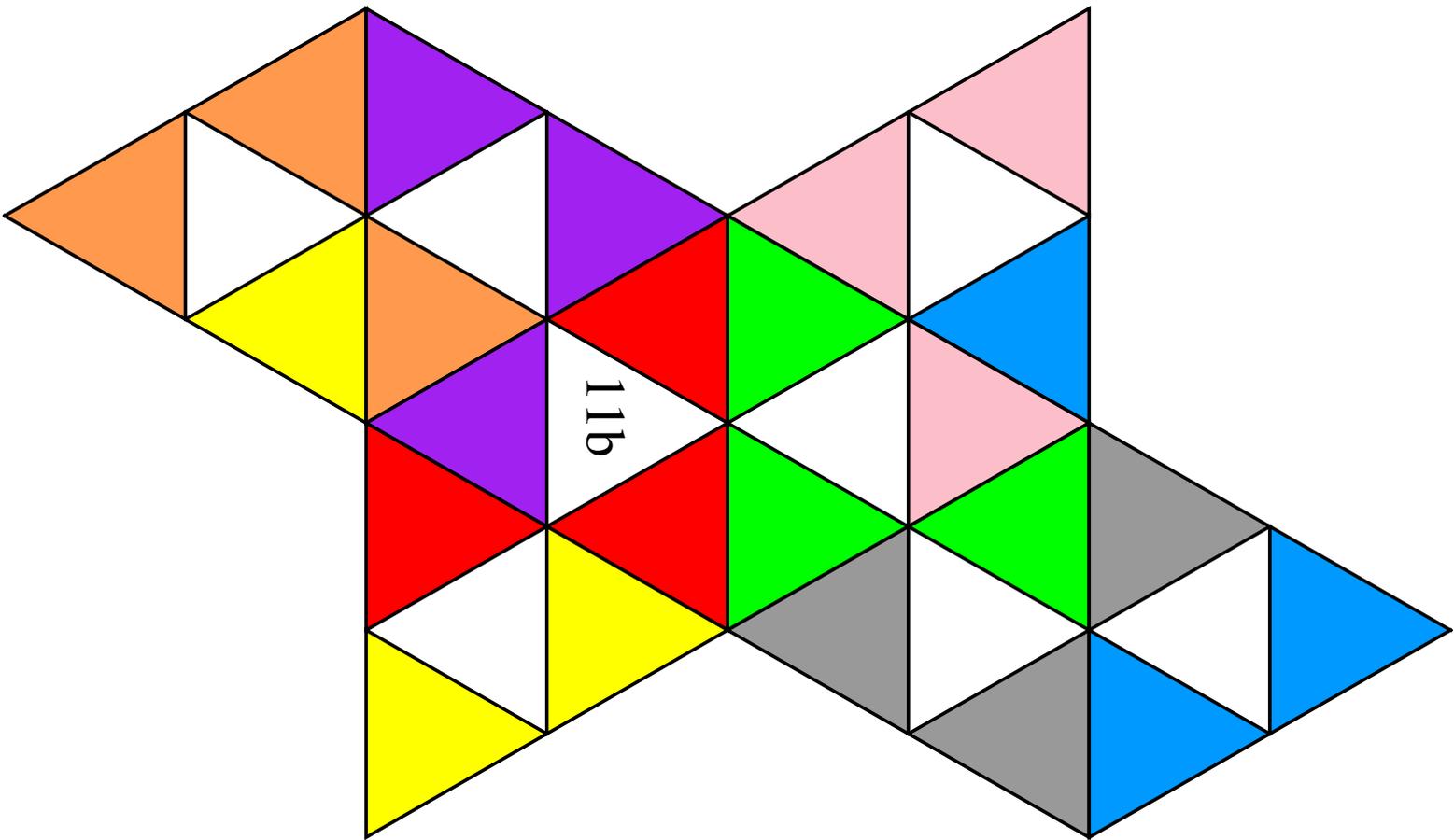
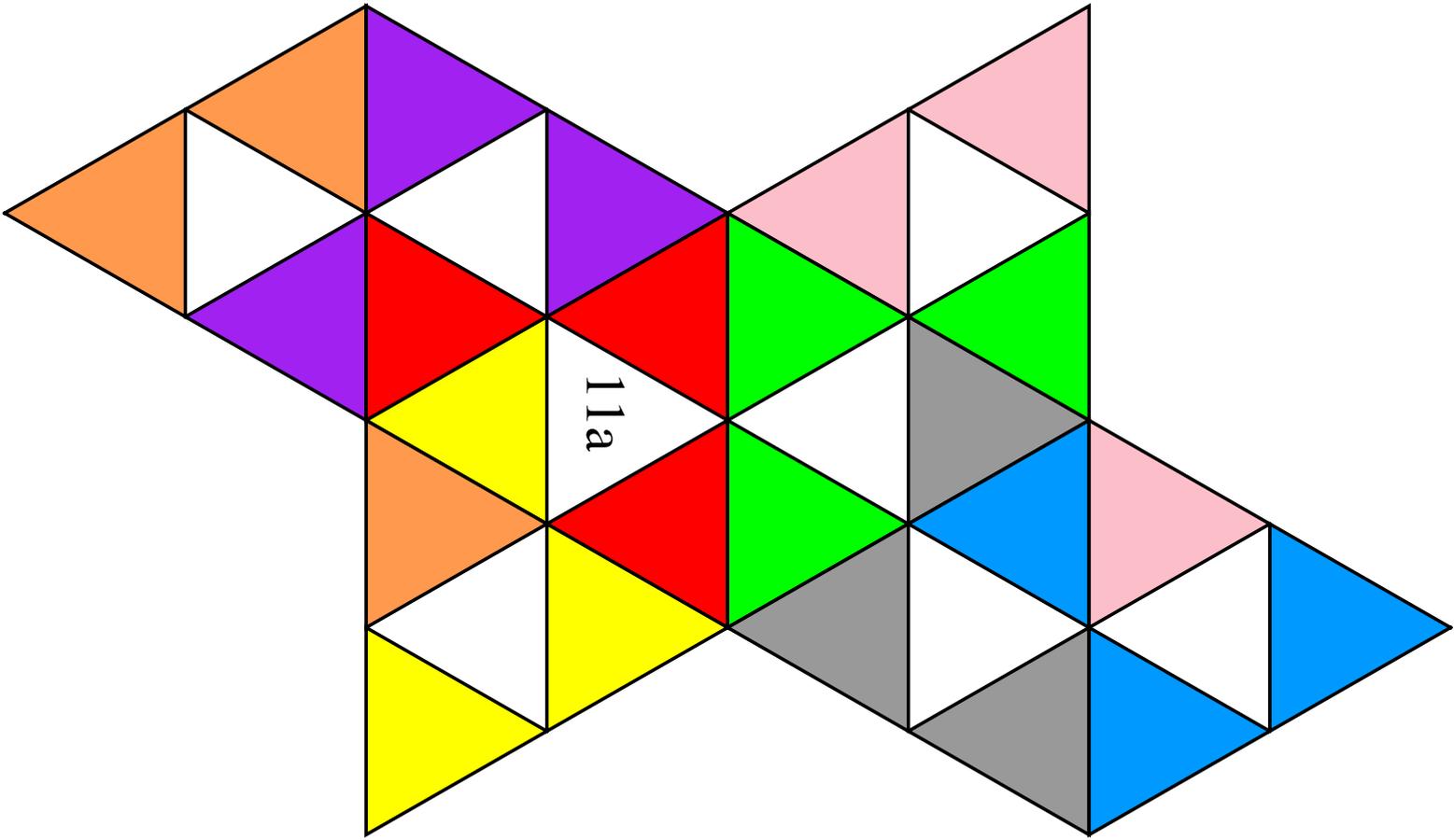


Figure 38: Templates for Patterns 11a,b

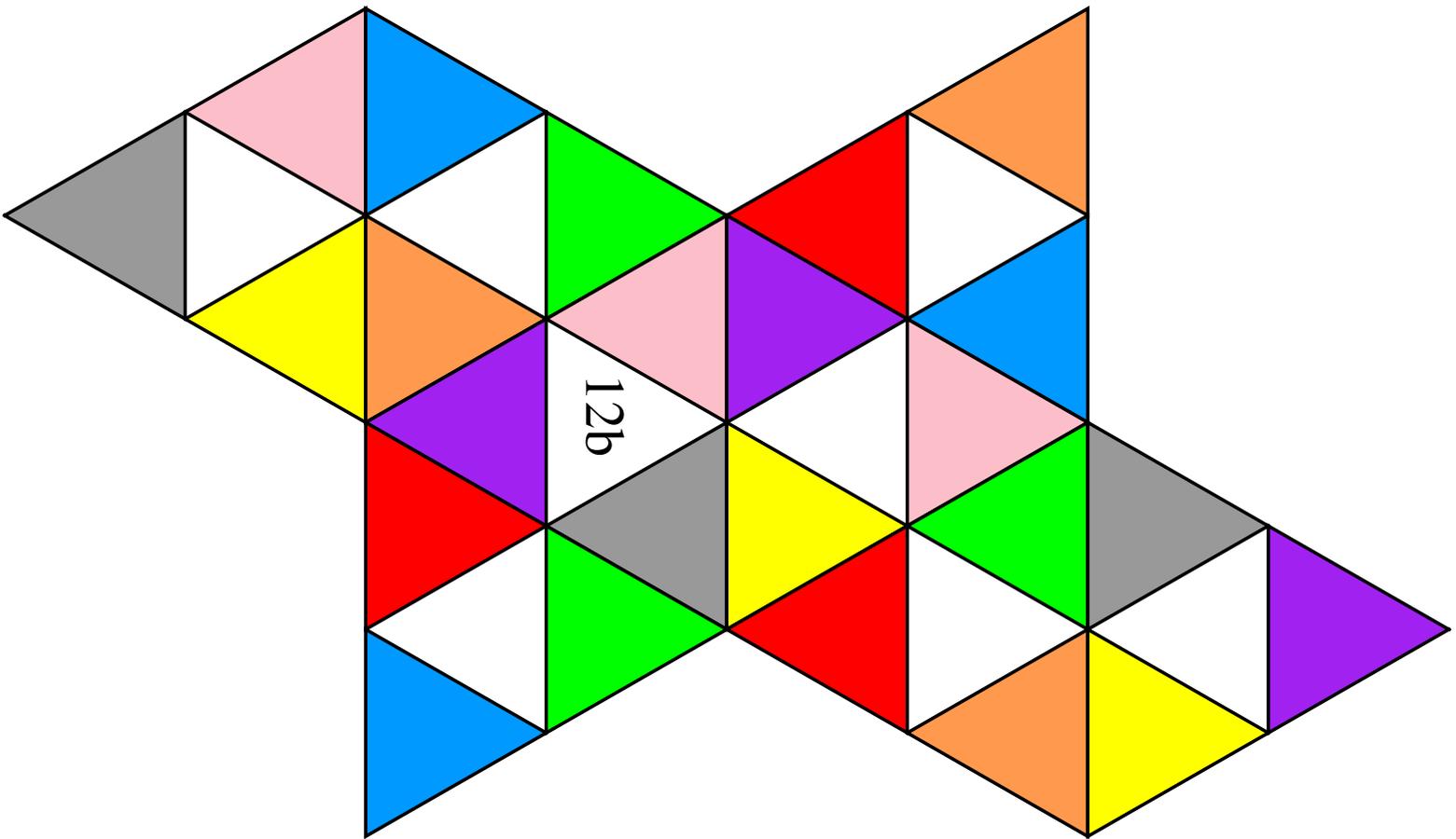
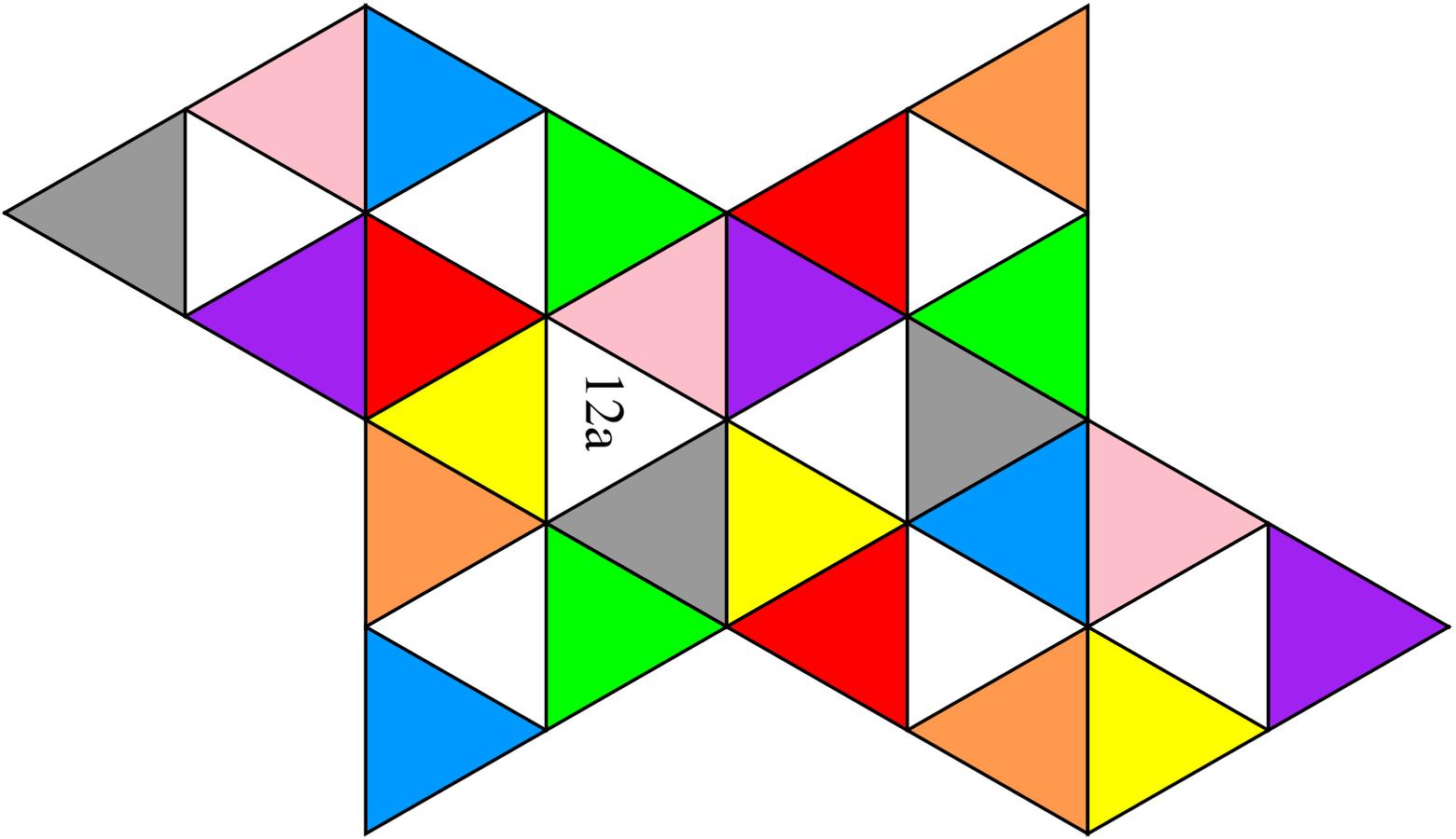


Figure 39: Templates for Patterns 12a,b

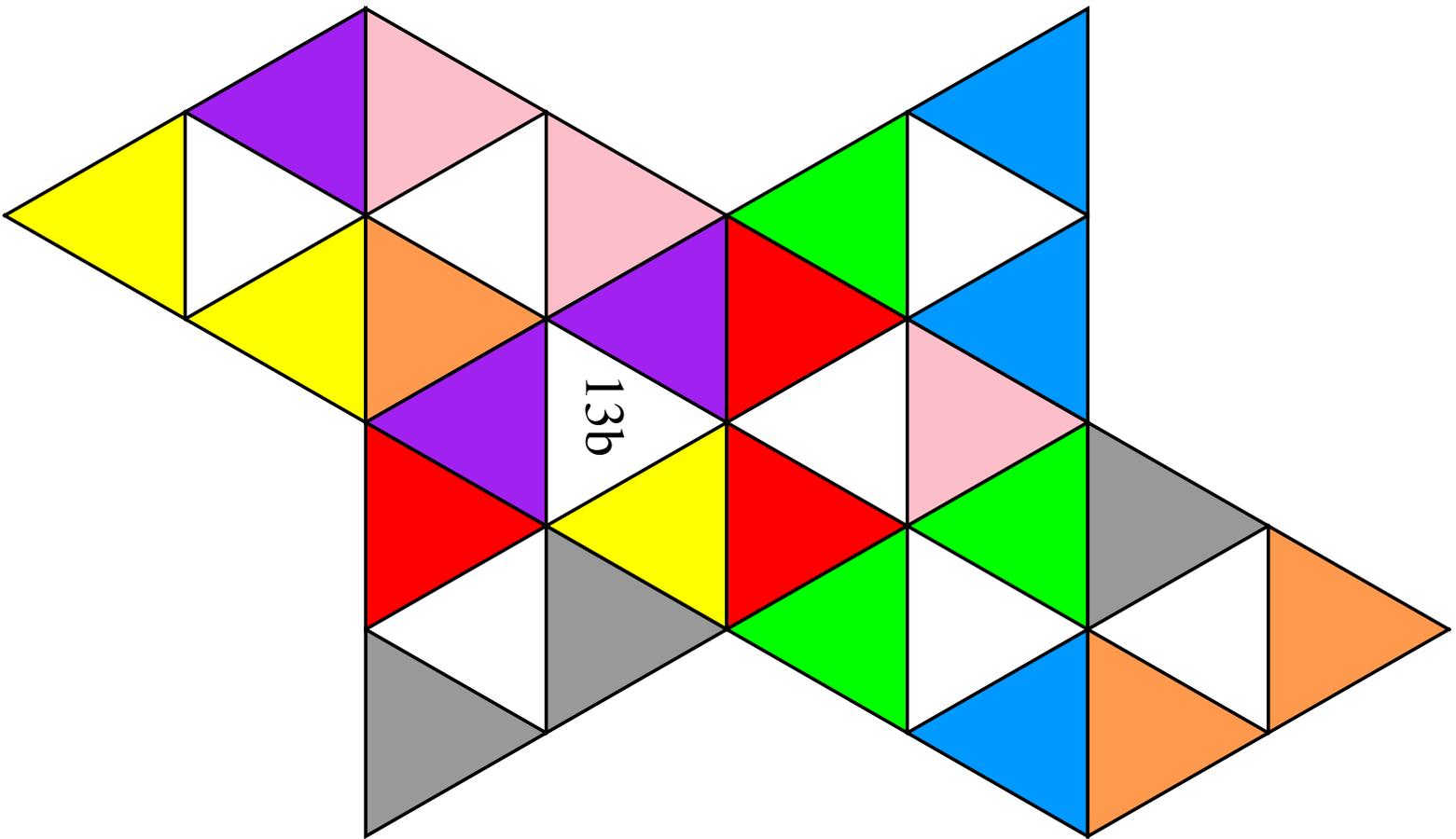
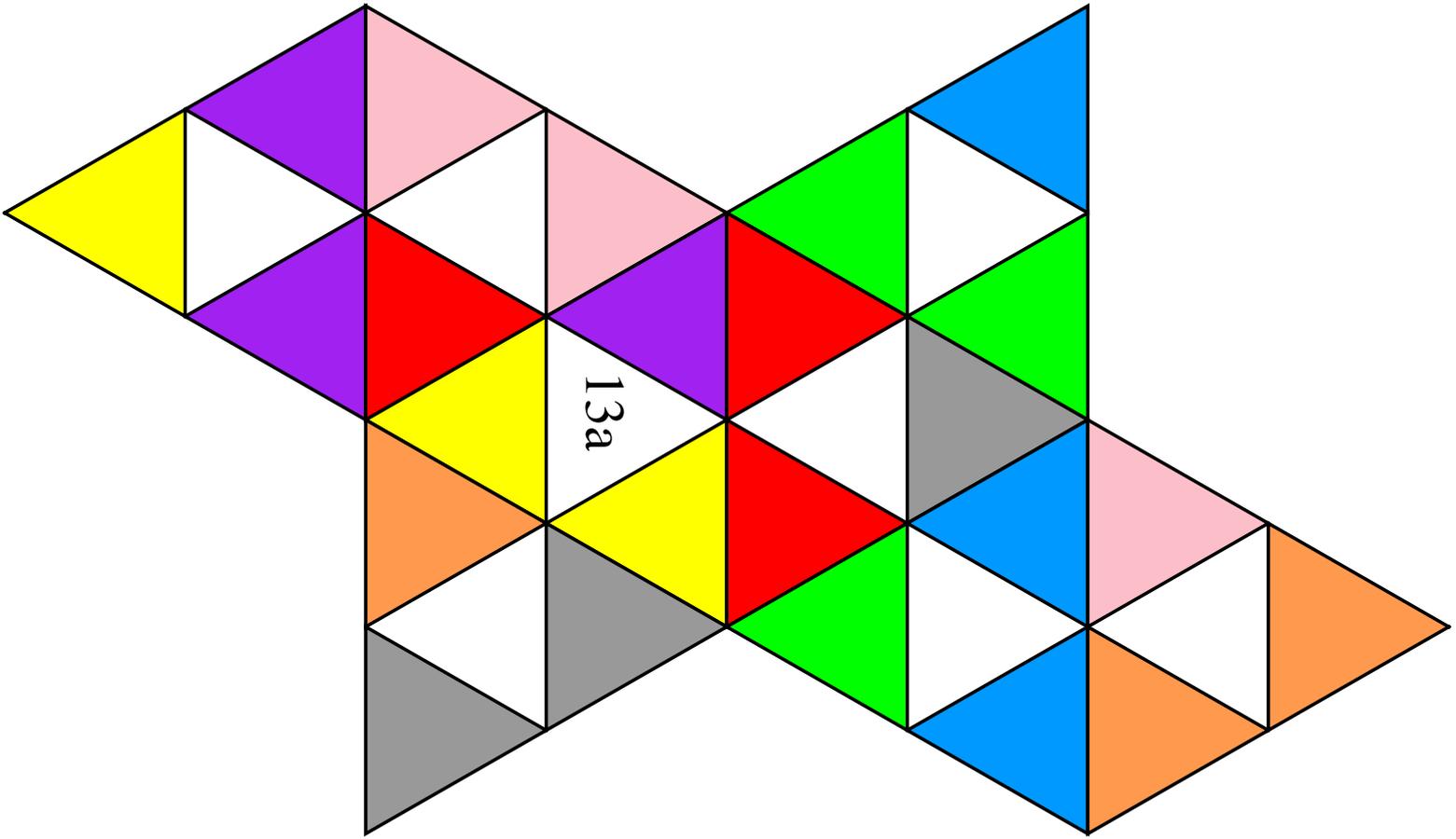


Figure 40: Templates for Patterns 13a,b

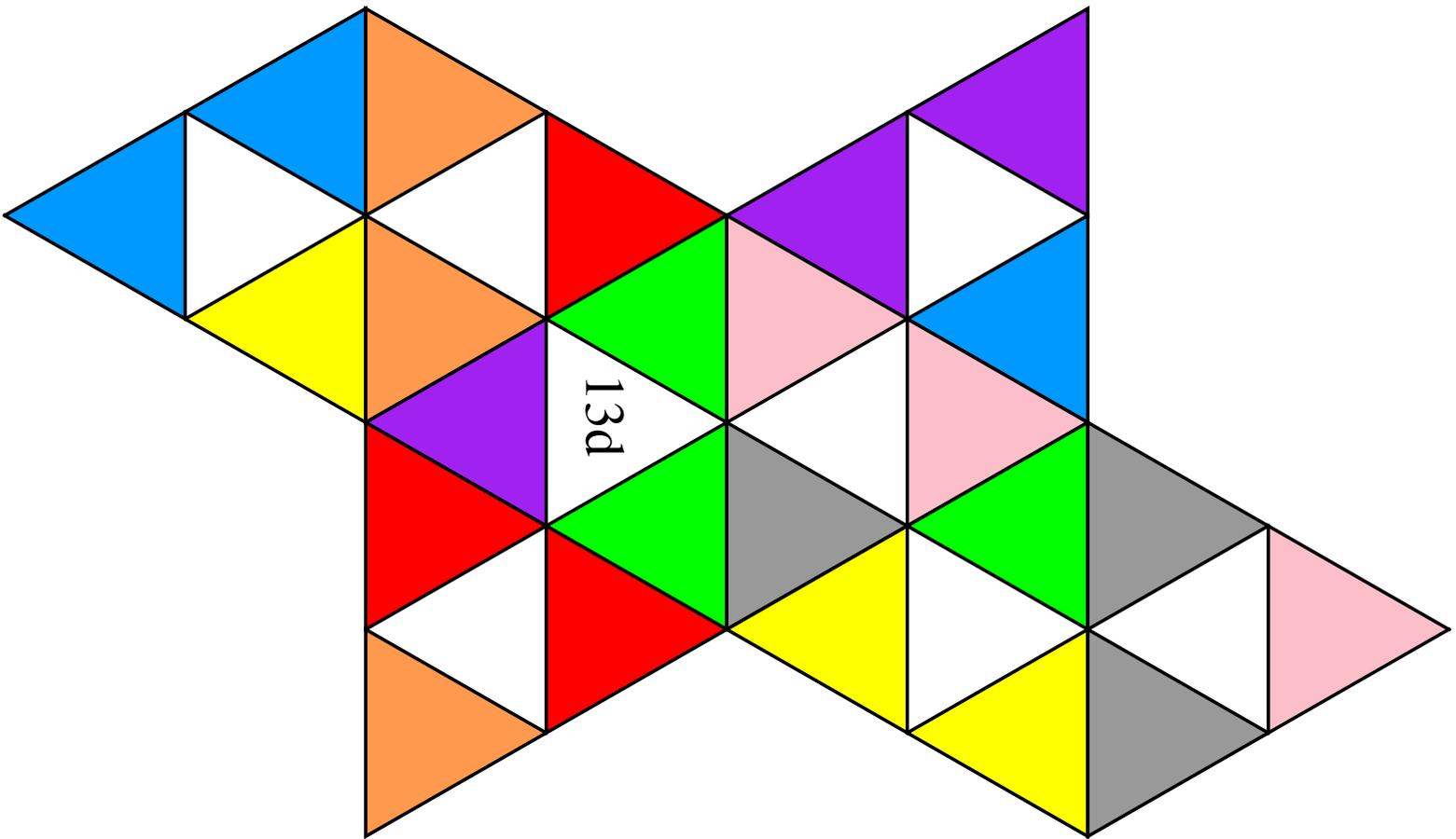
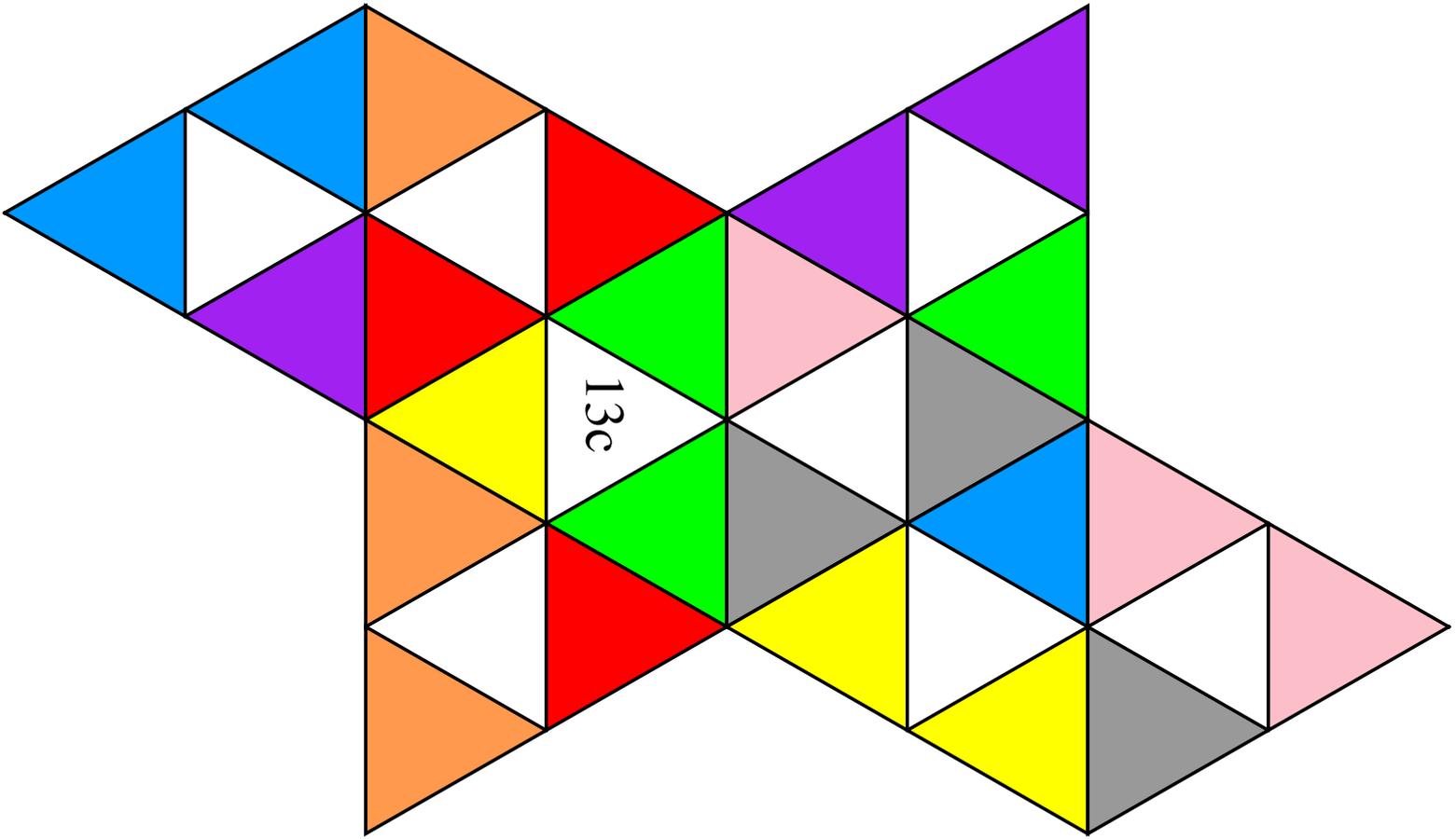


Figure 41: Templates for Patterns 13c,d

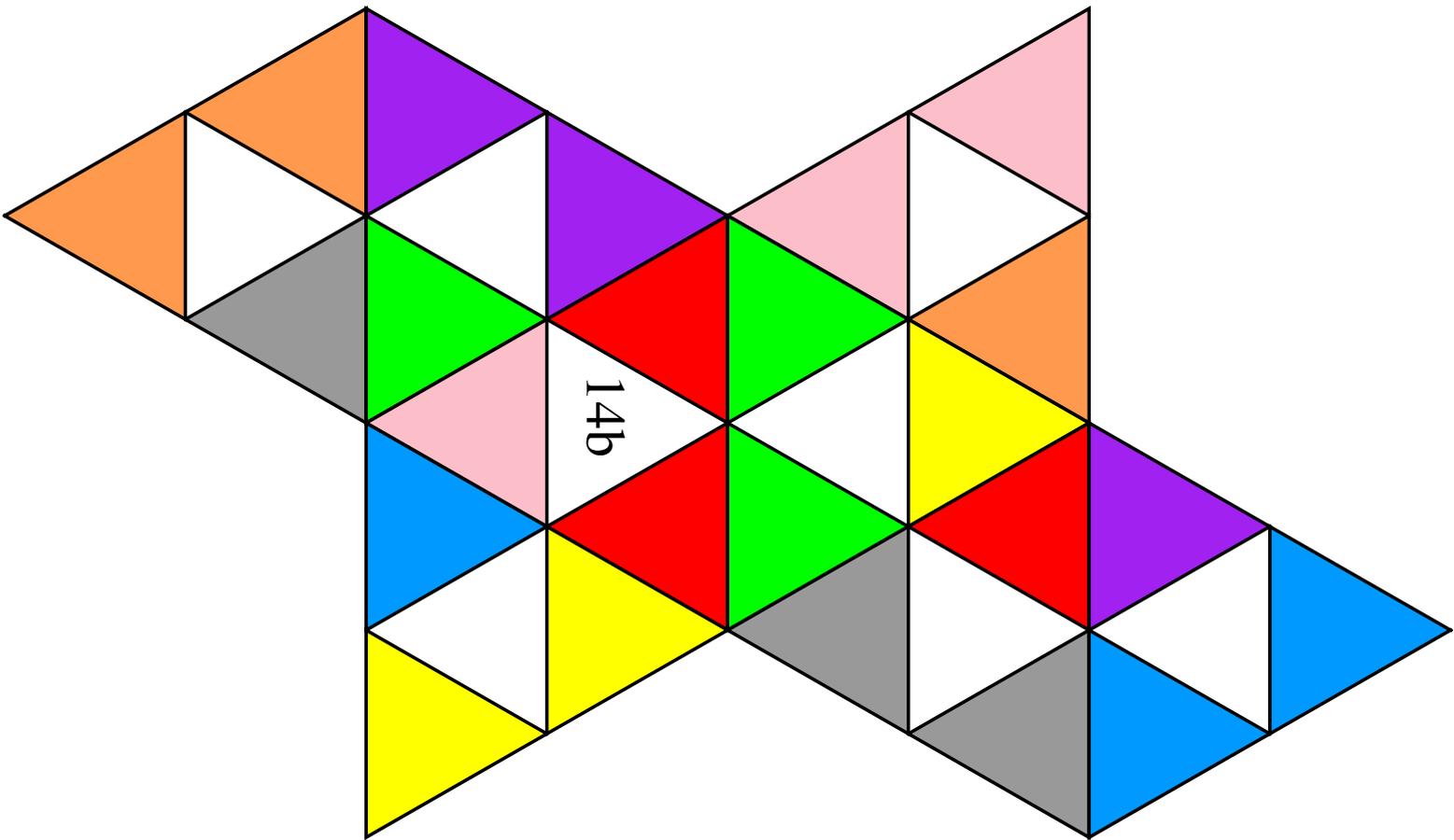
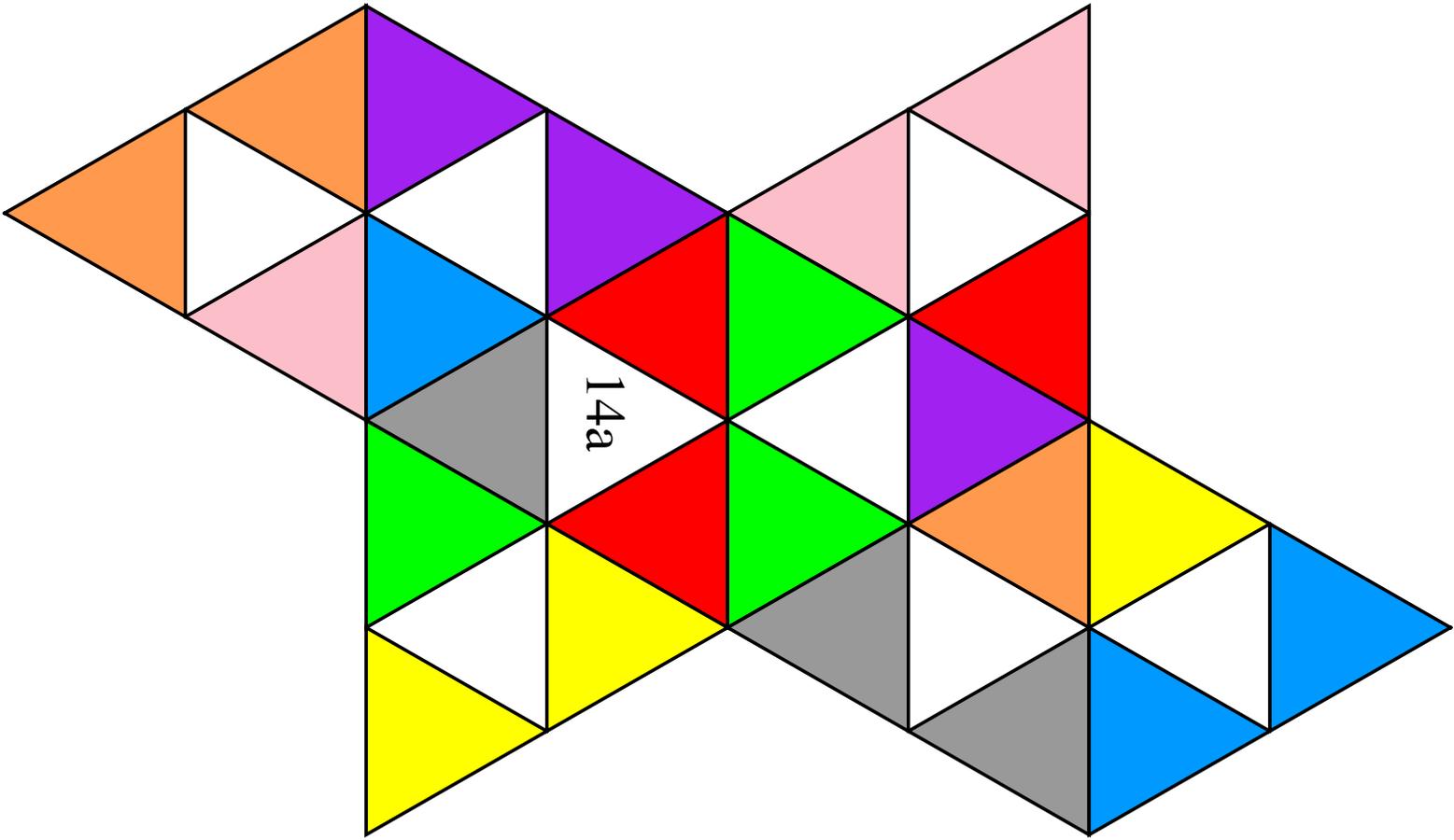


Figure 42: Templates for Patterns 14a,b

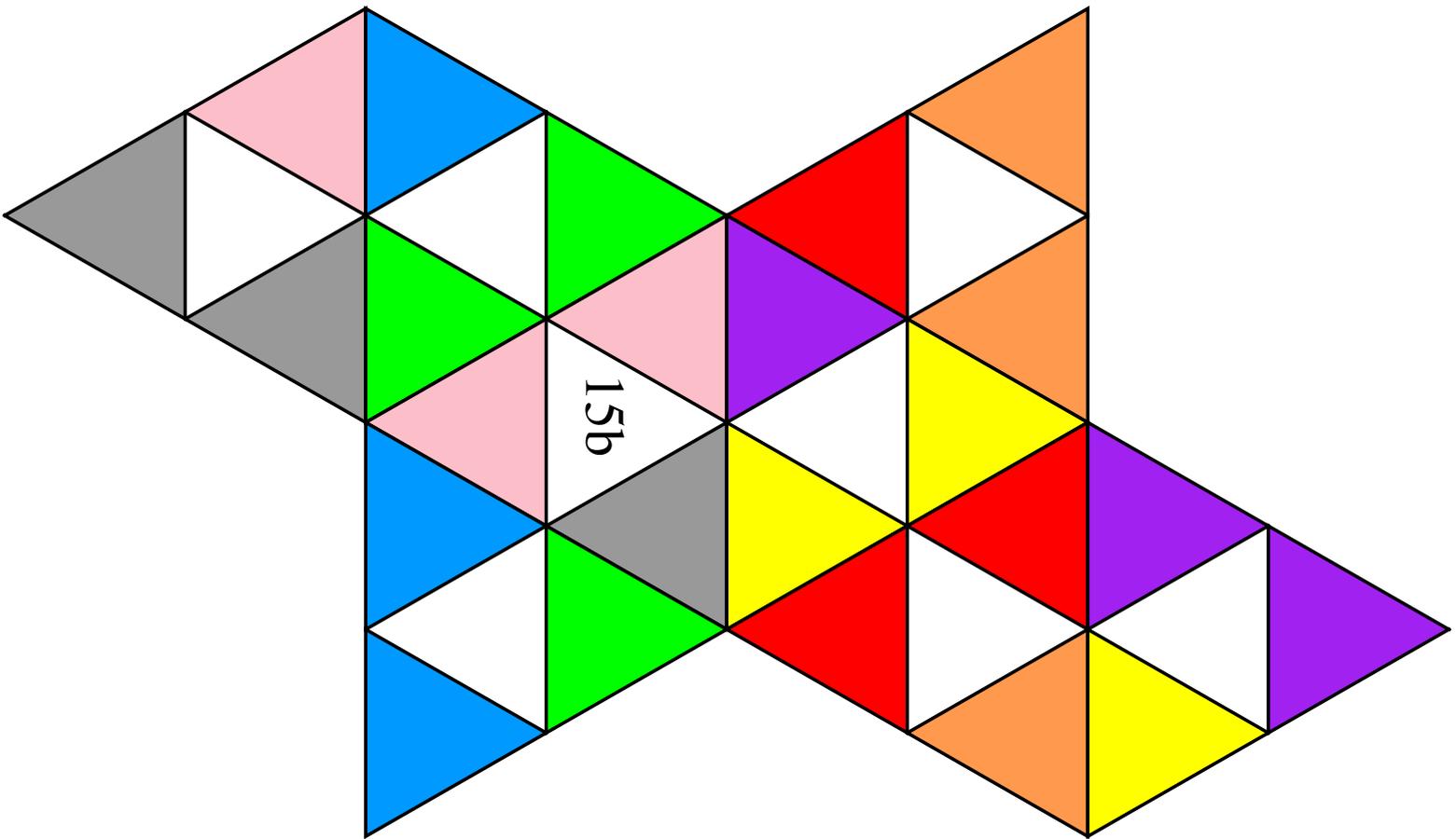
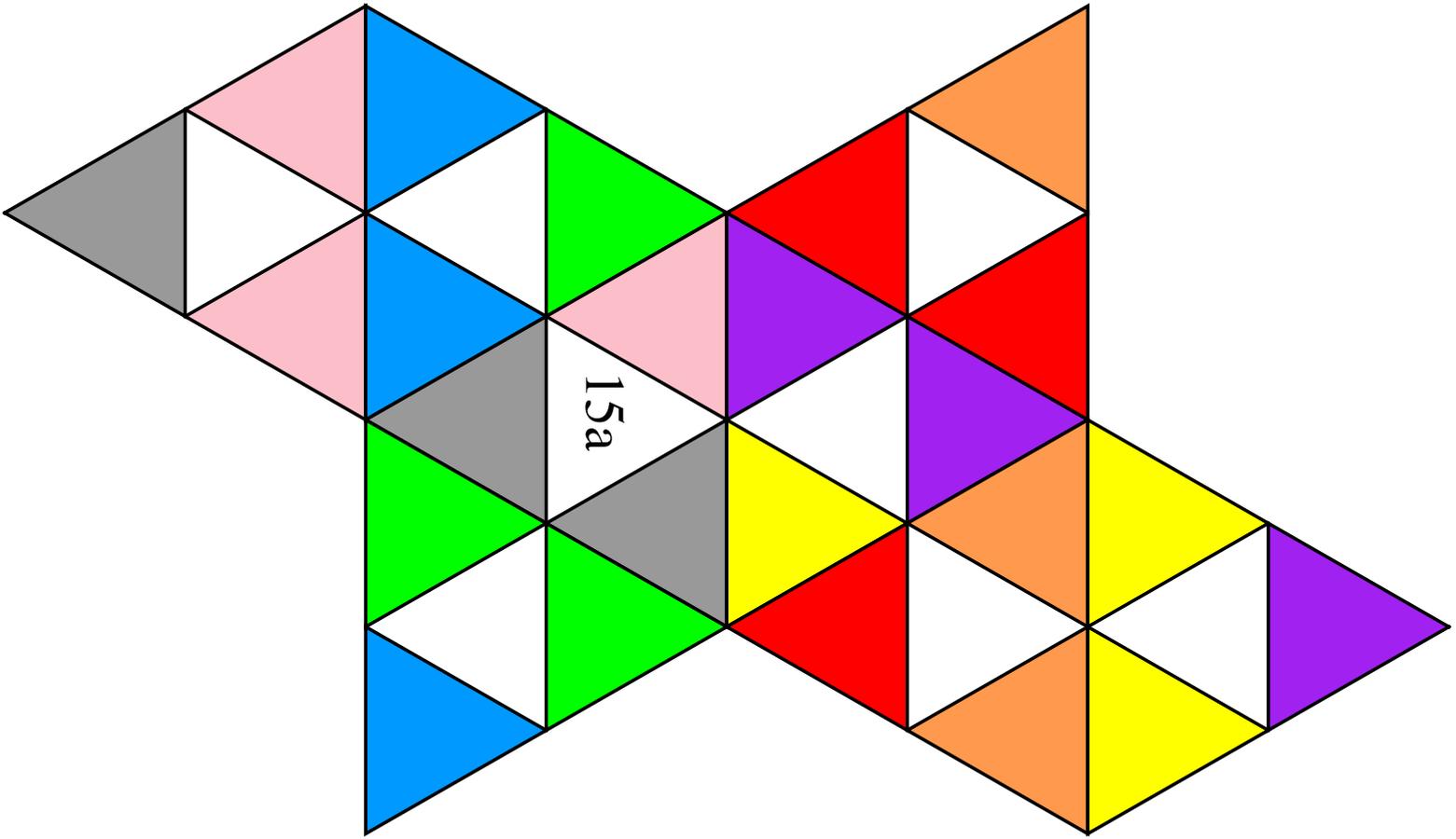


Figure 43: Templates for Patterns 15a,b

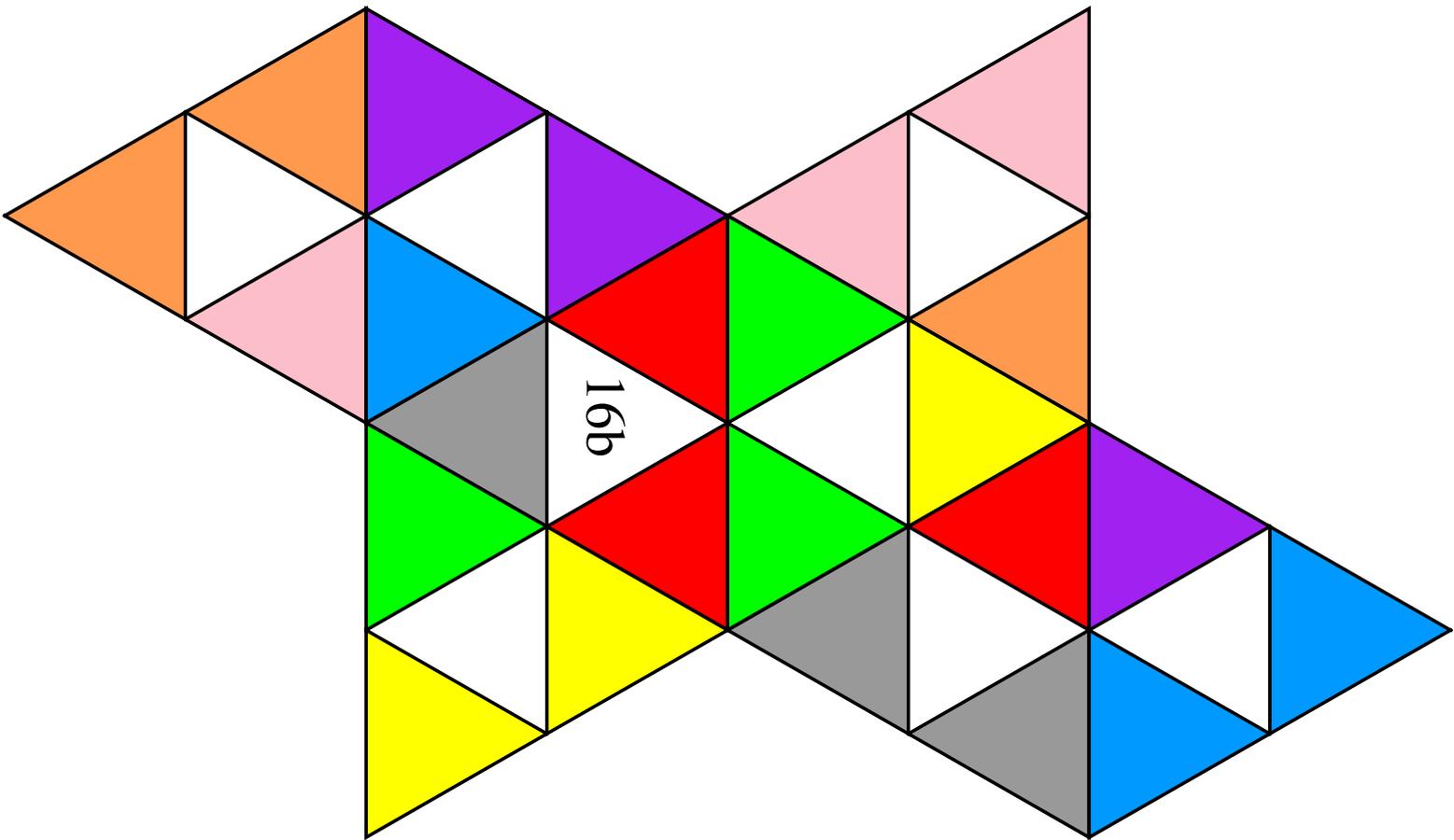
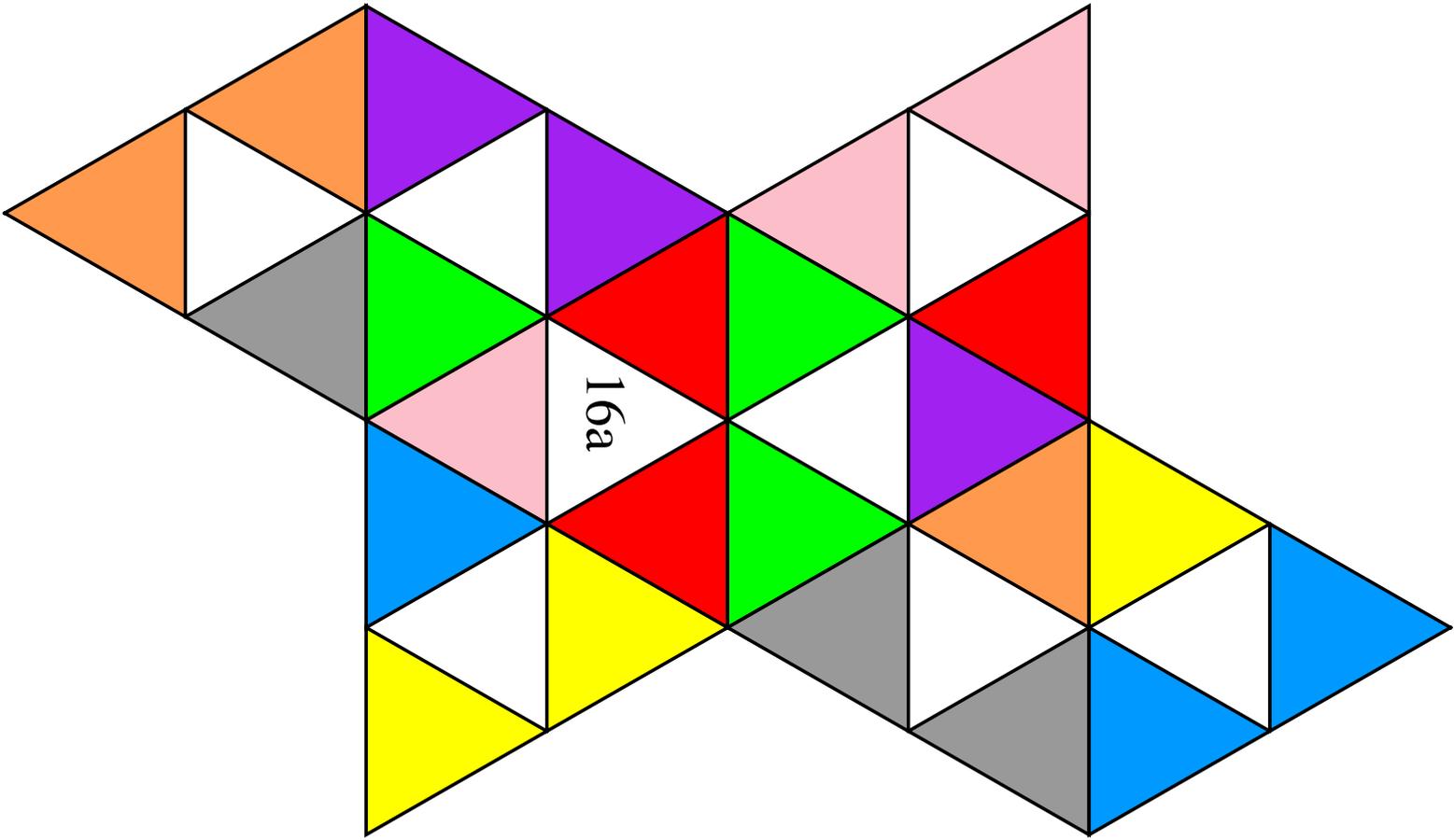


Figure 44: Templates for Patterns 16a,b

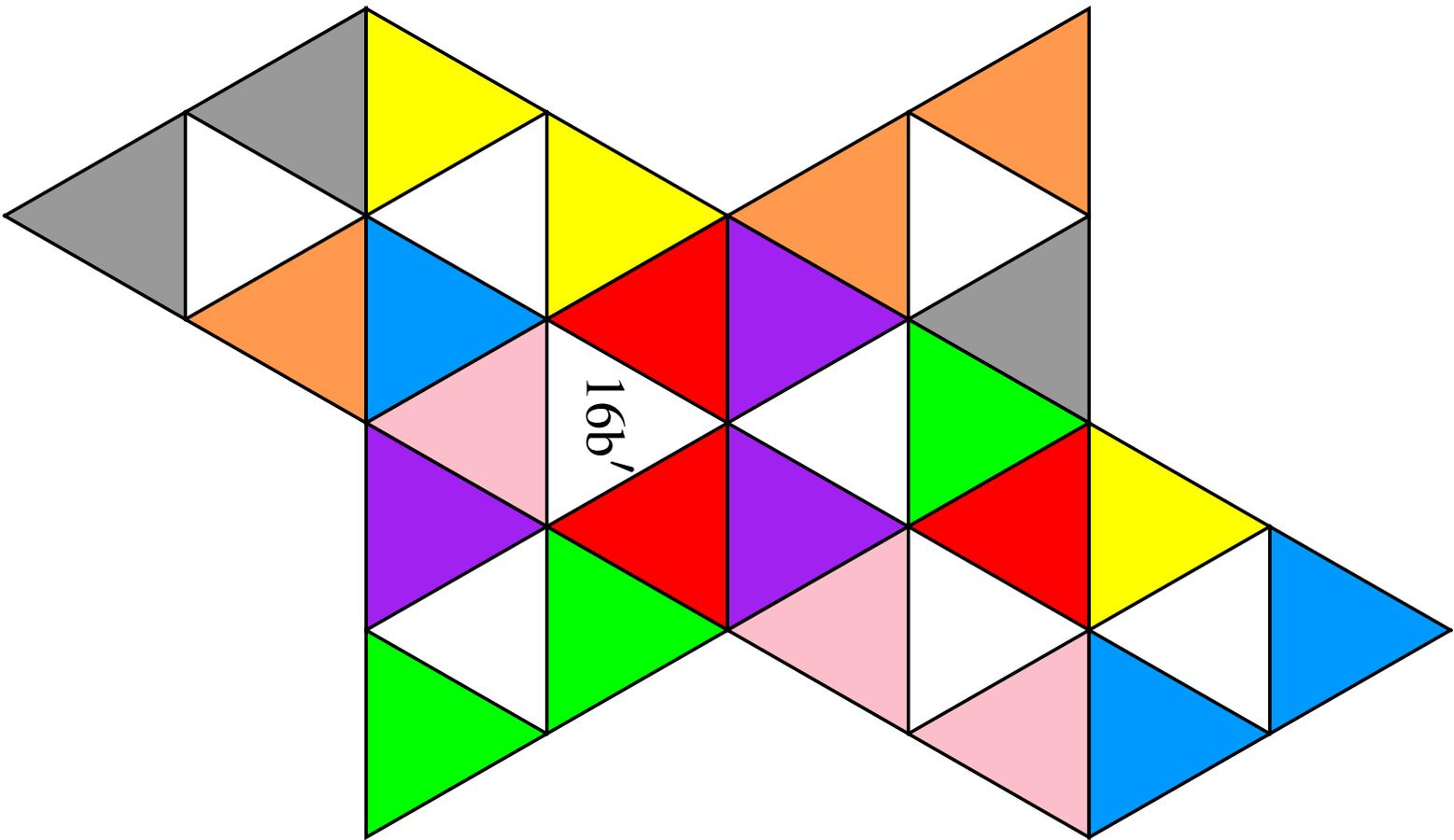
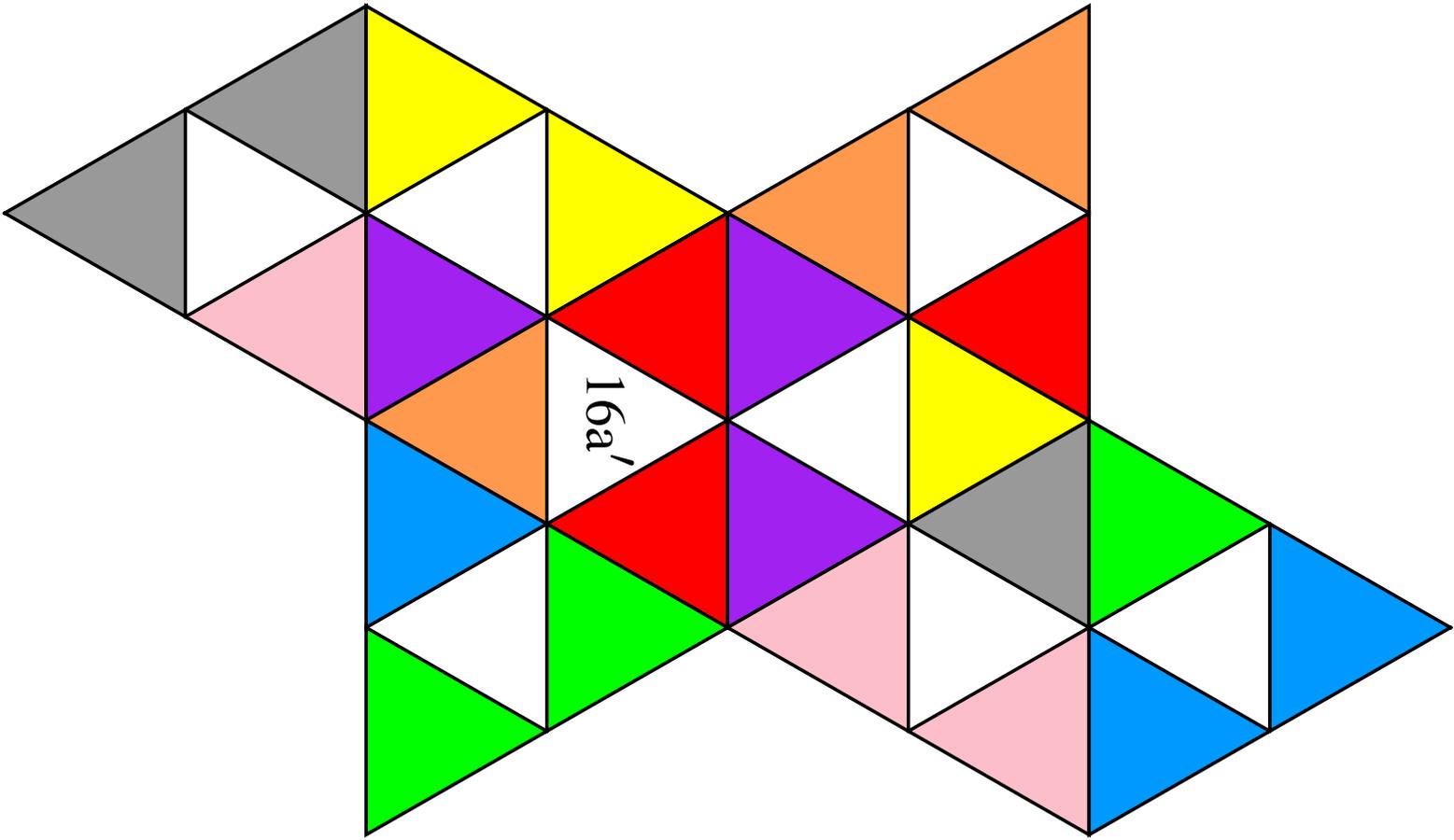


Figure 45: Templates for Patterns 16a',b'

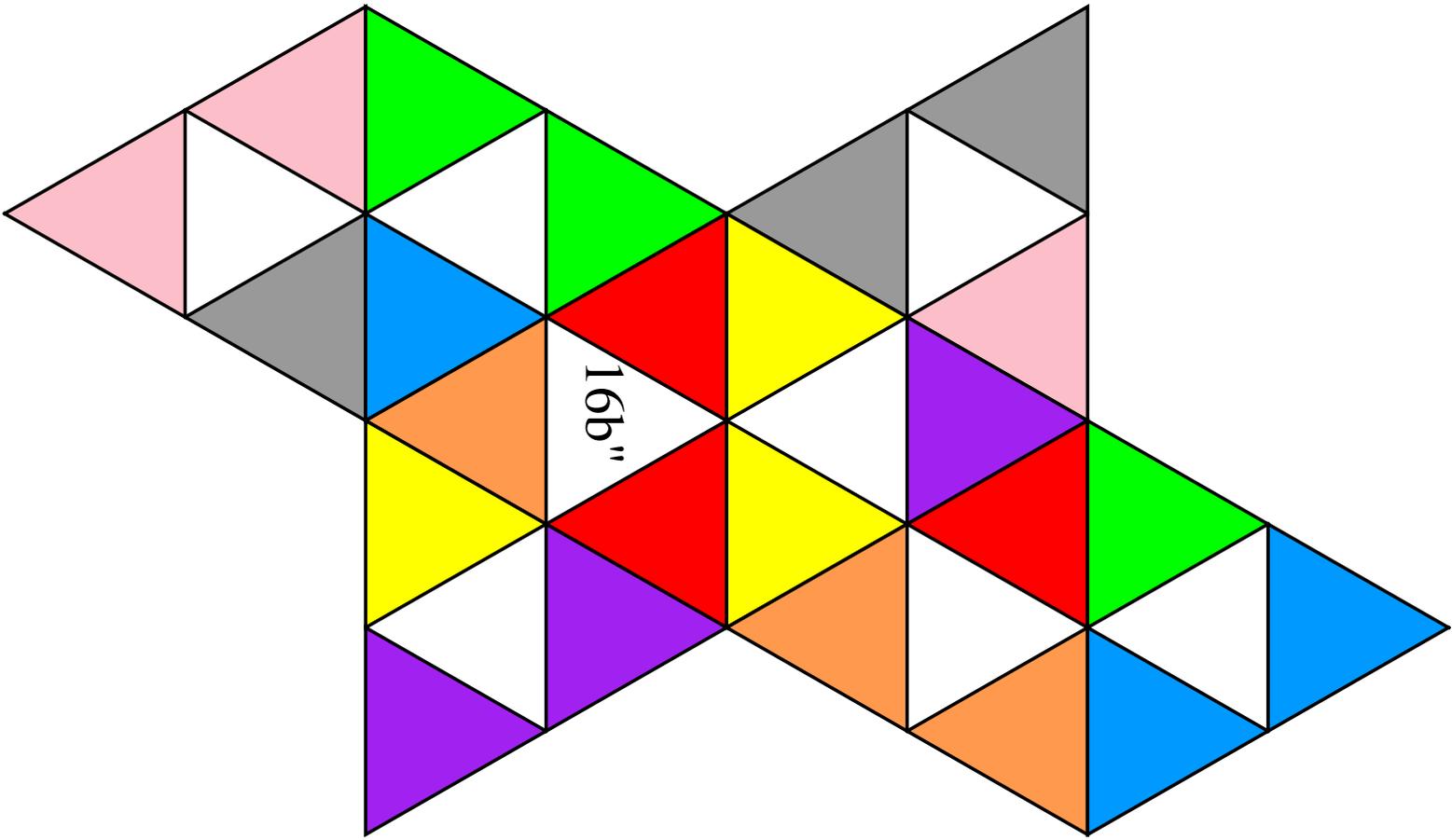
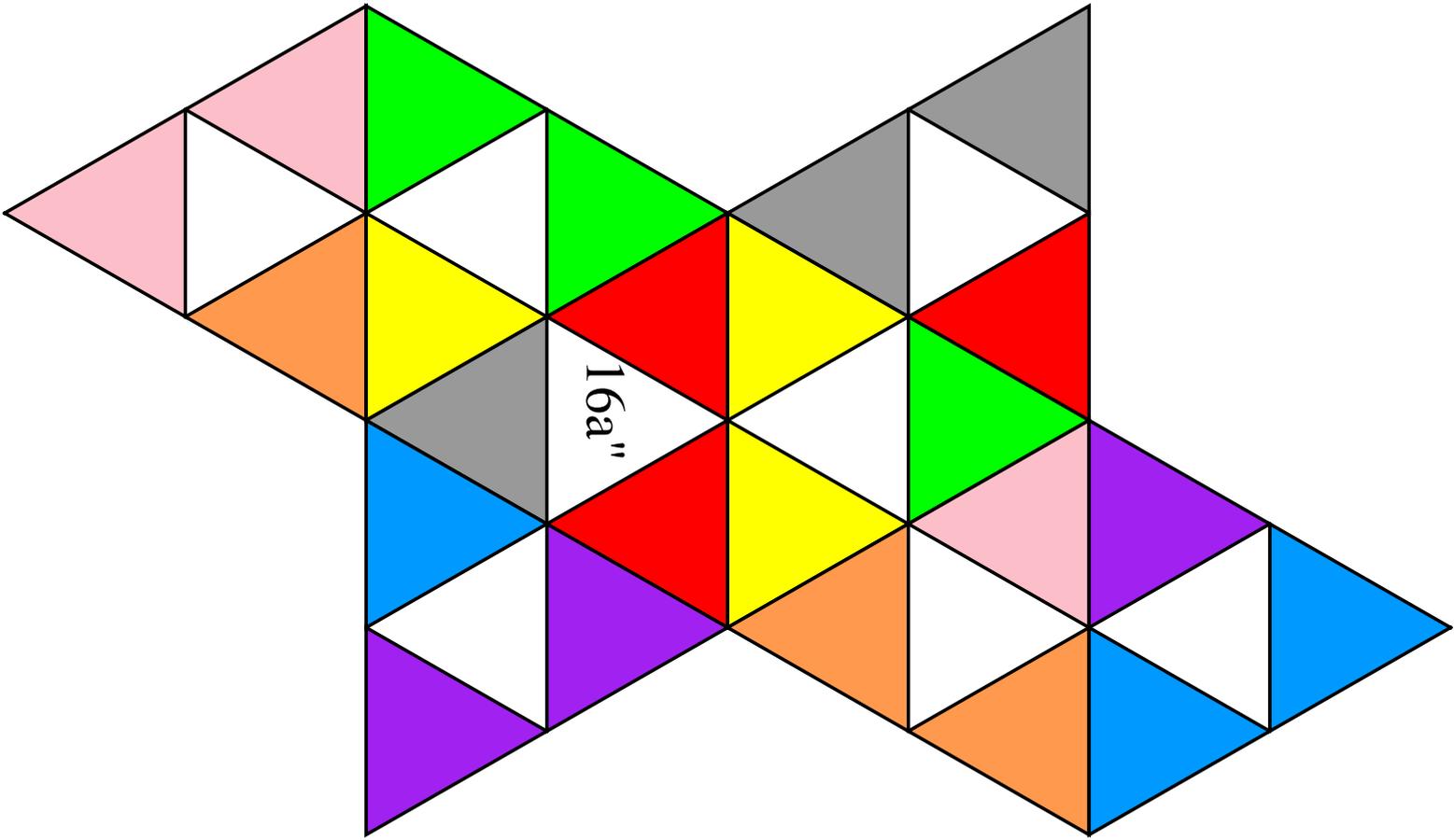


Figure 46: Templates for Patterns 16a",b"

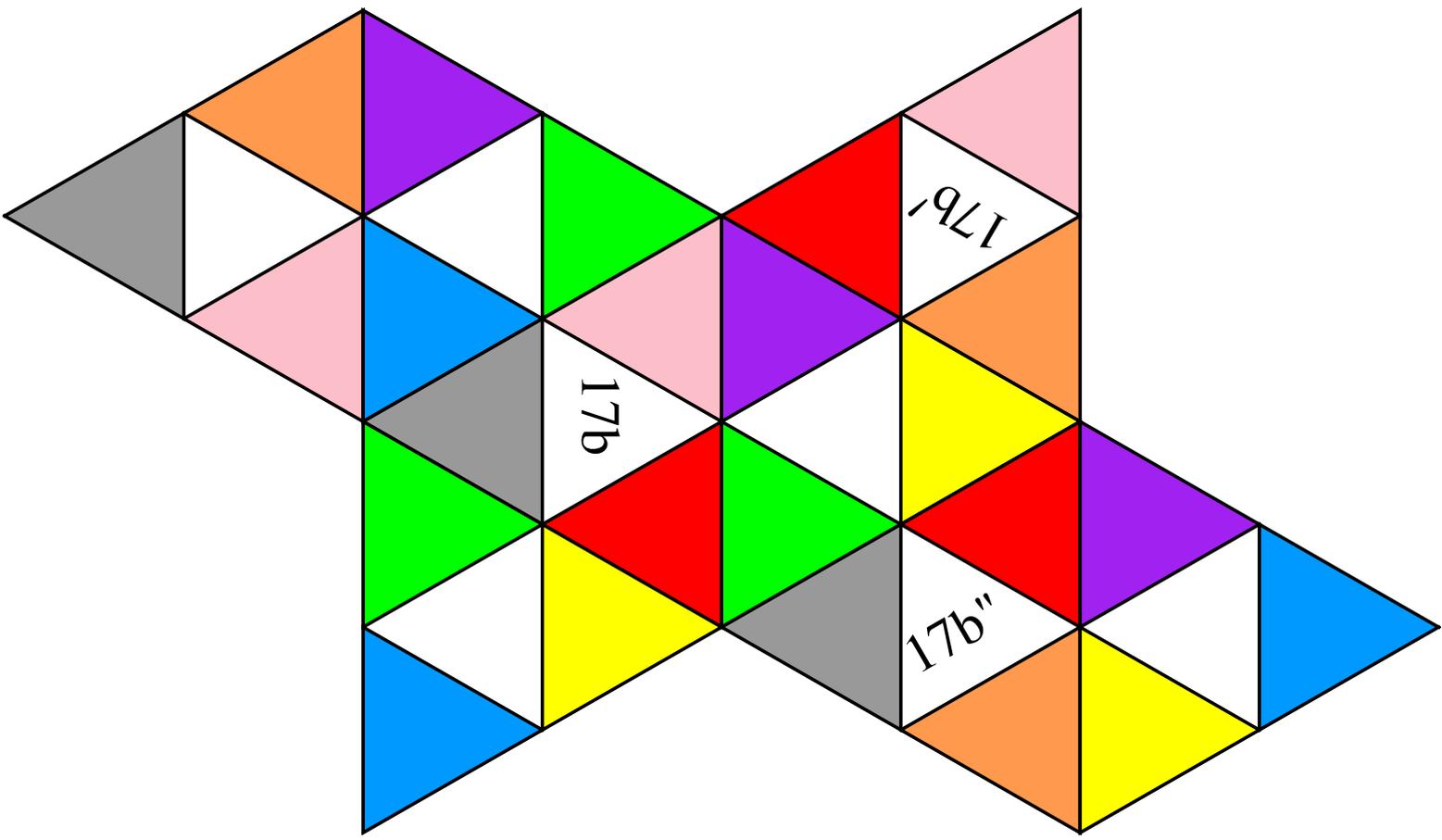
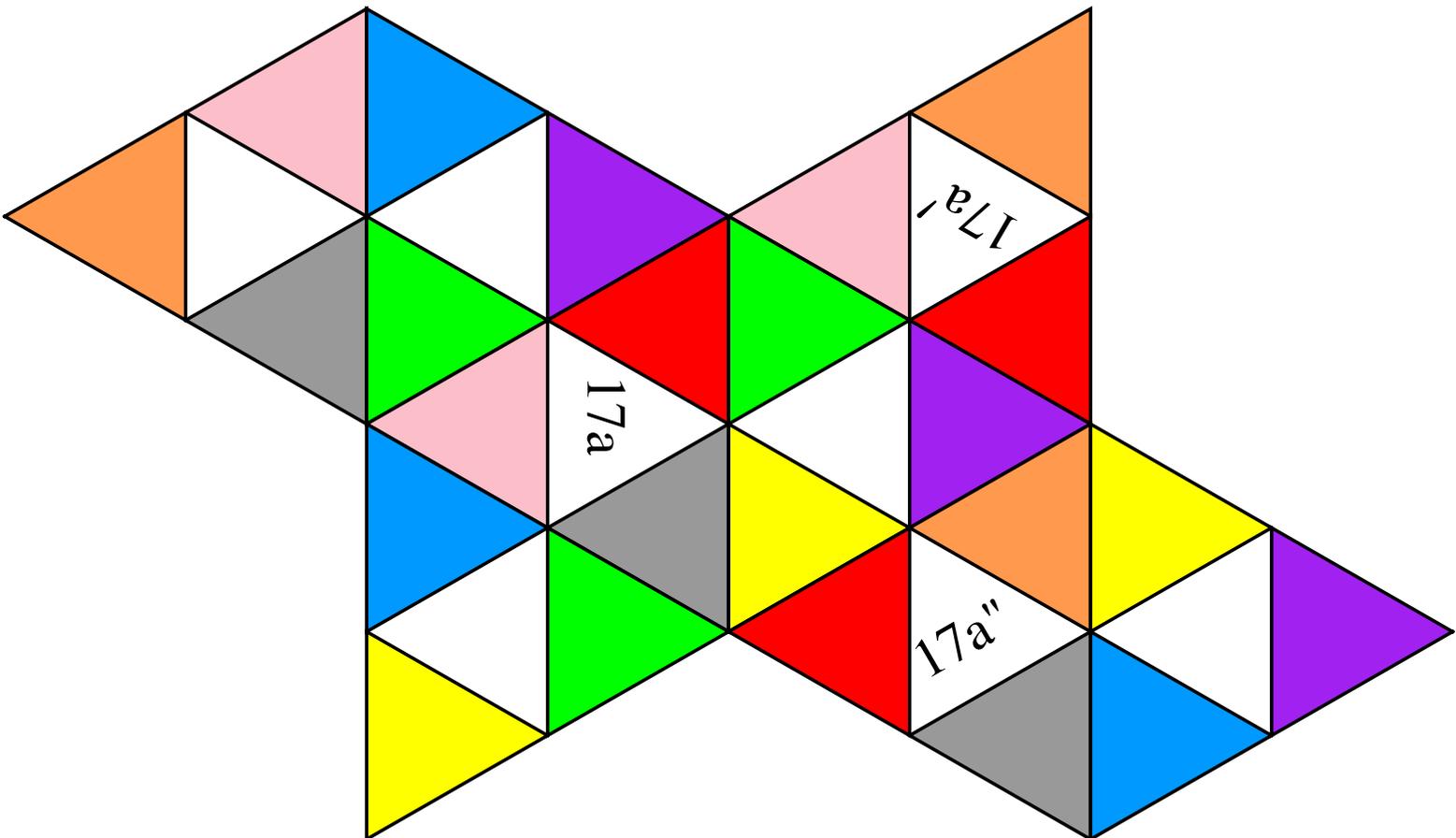


Figure 47: Templates for Patterns 17a,b

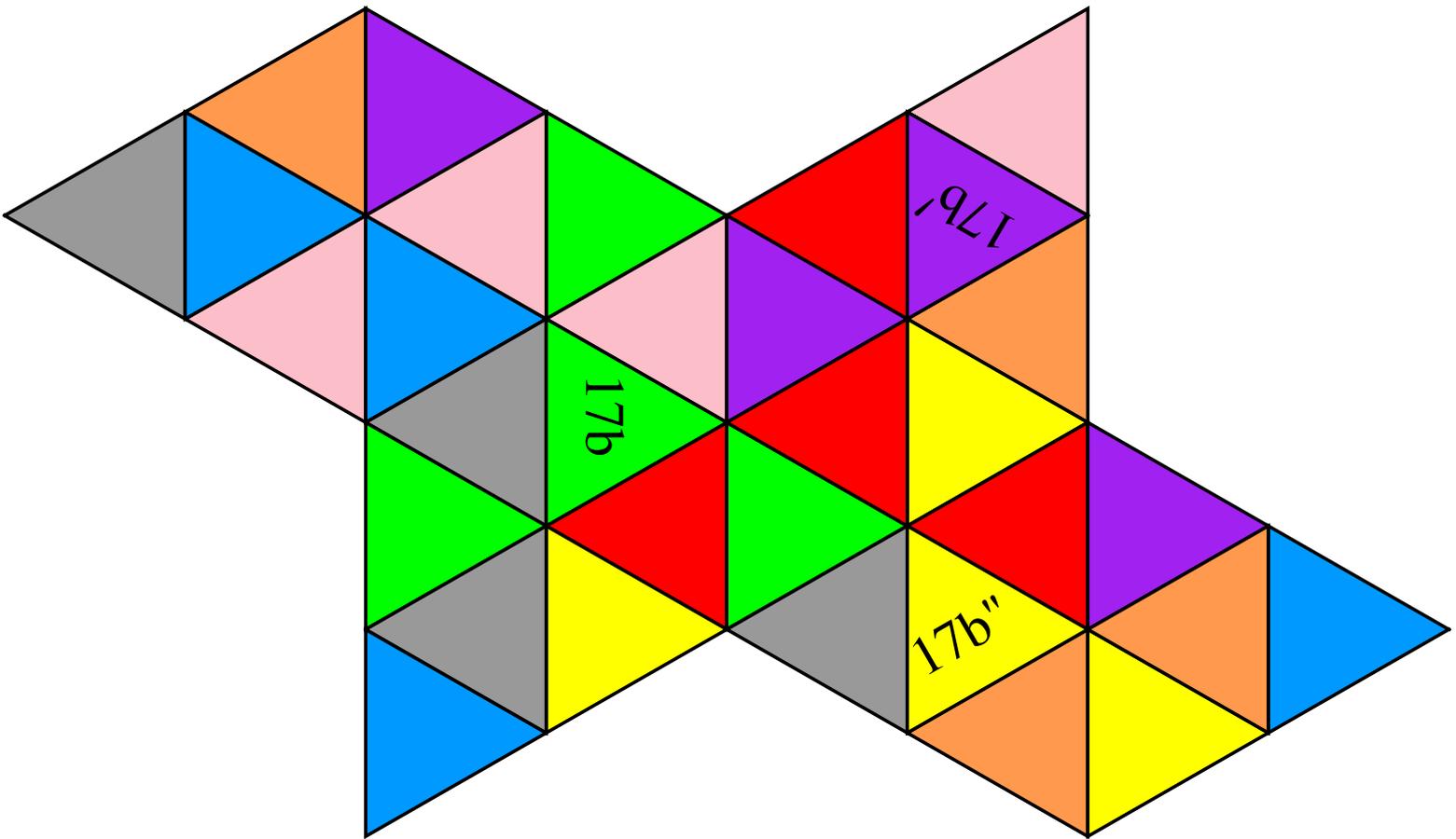
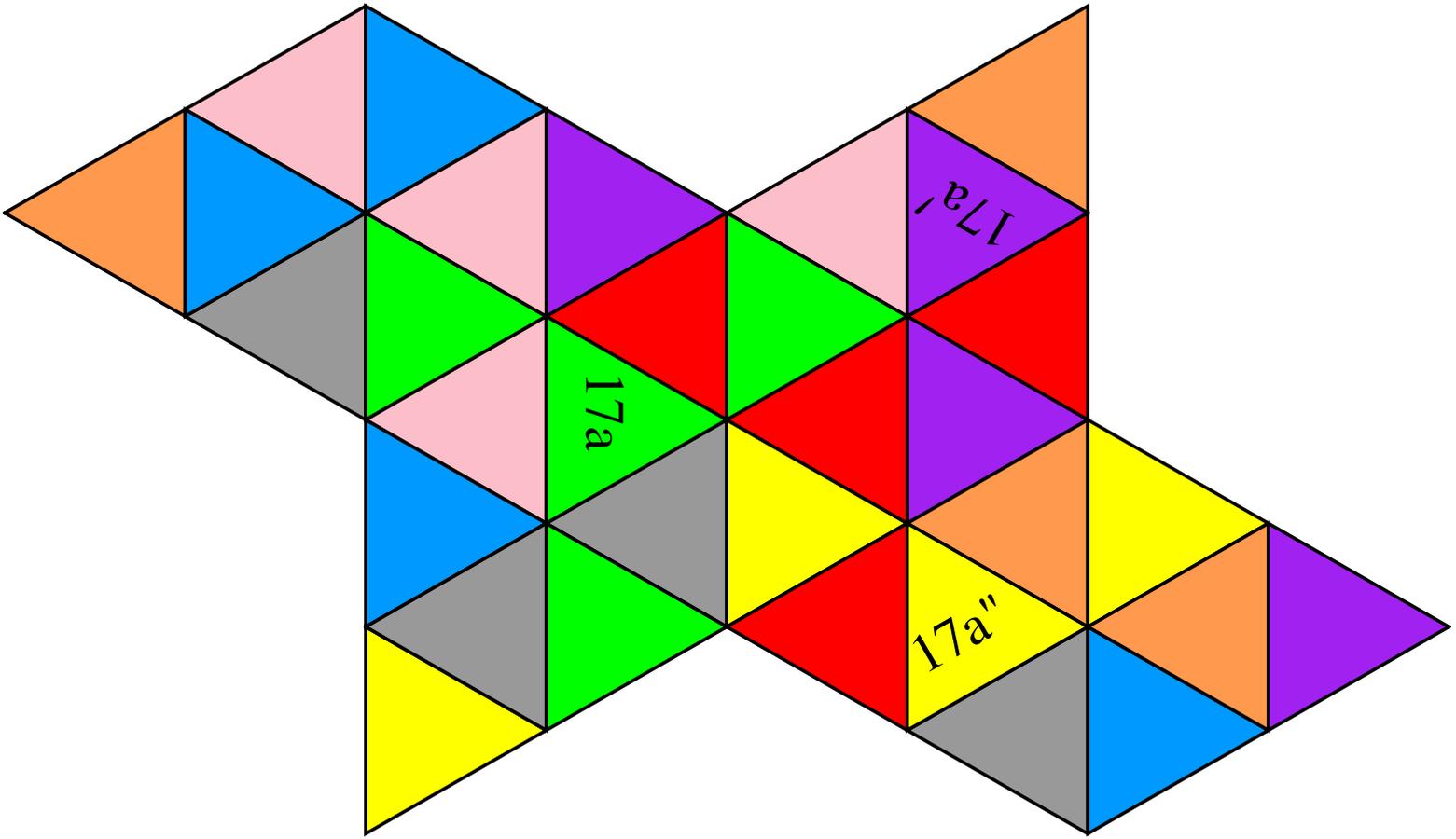


Figure 48: Templates for Figure 17

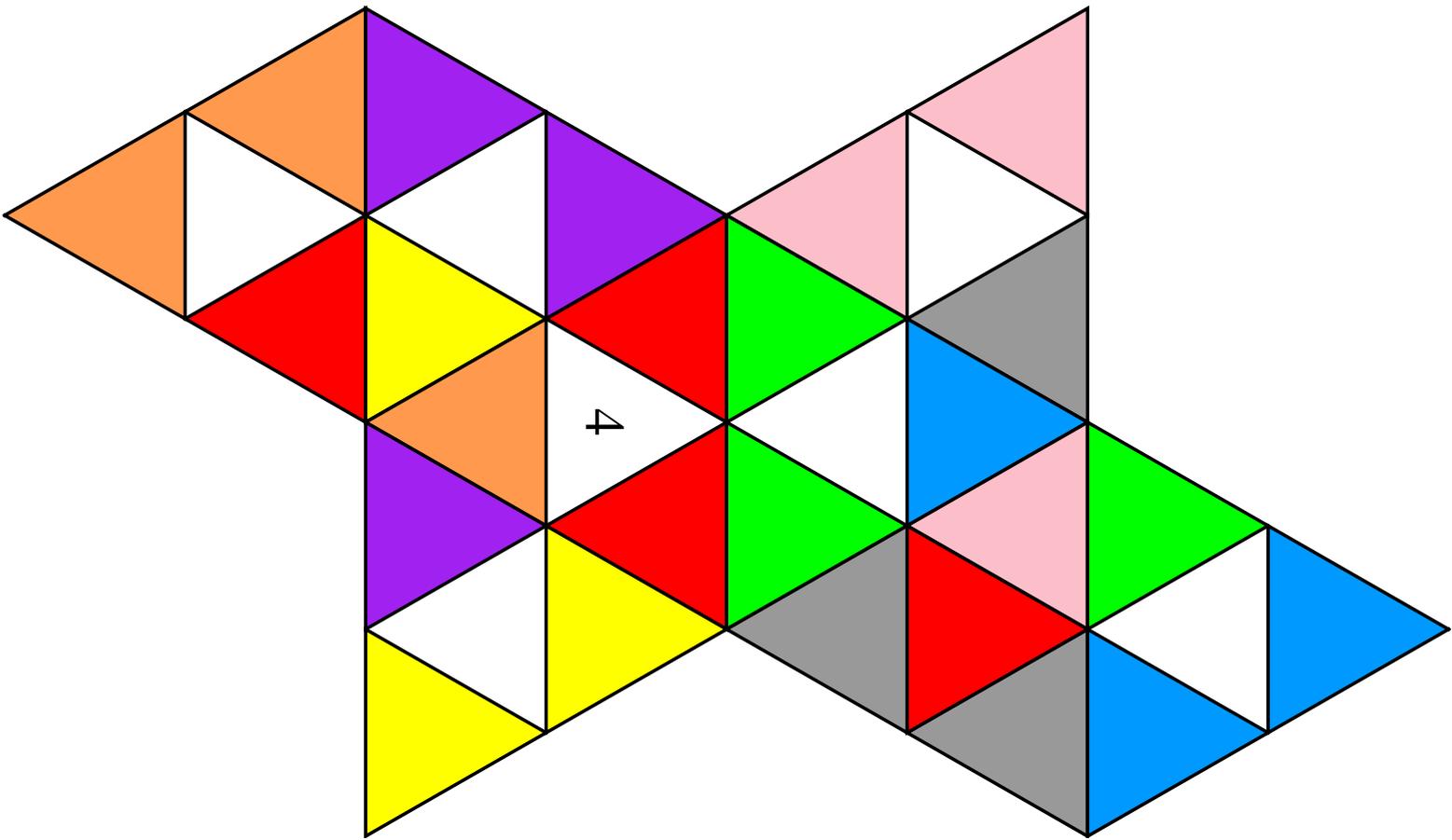
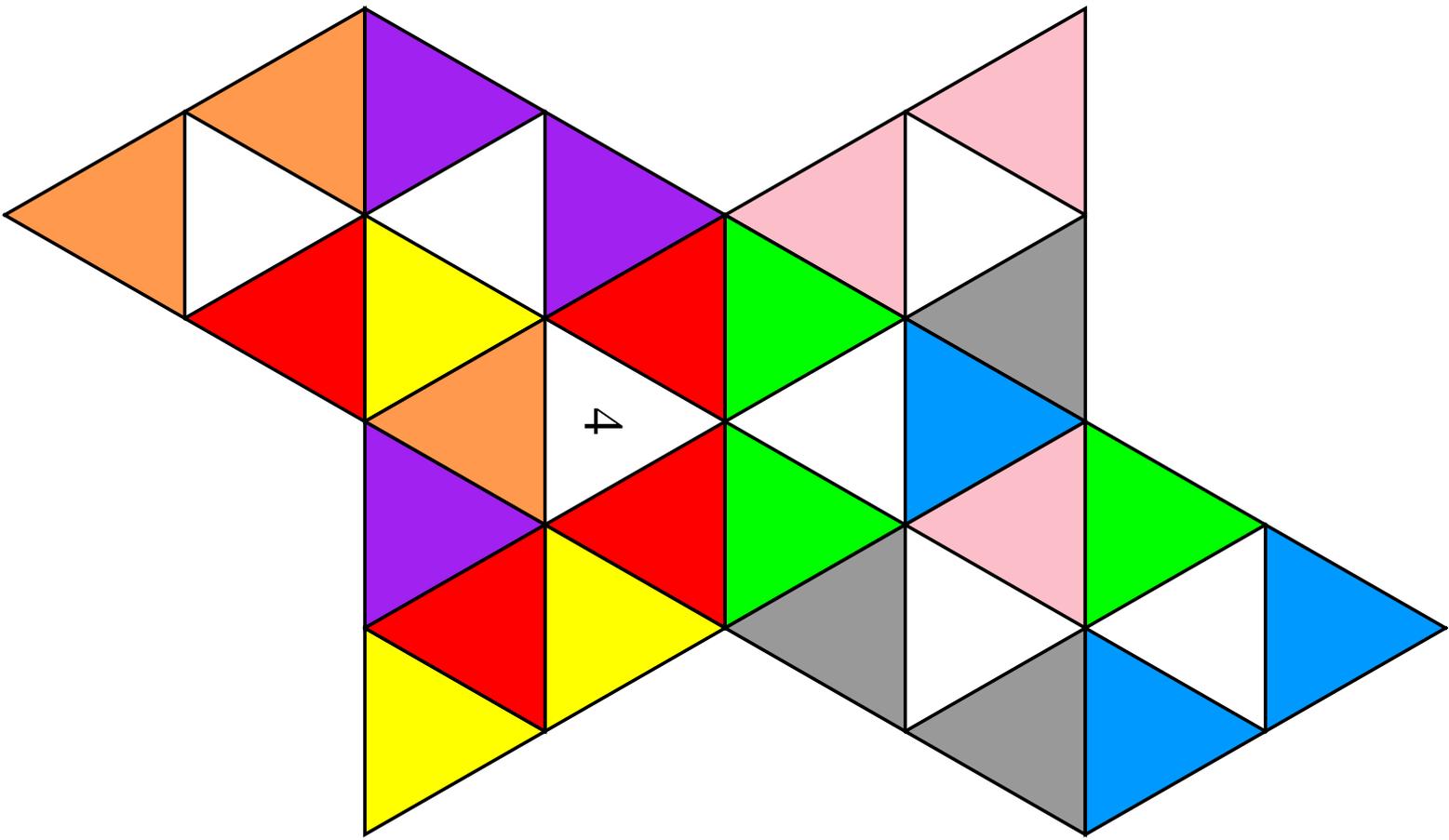


Figure 49: Pattern 4, with two possible choices for where to put the red center. These two choices, together with their mirror images, are the only possible ways to add the center to create a handed distribution for red. Then try adding the orange center to have either the same distribution as red (which works), or the mirror image distribution as red (which doesn't work).

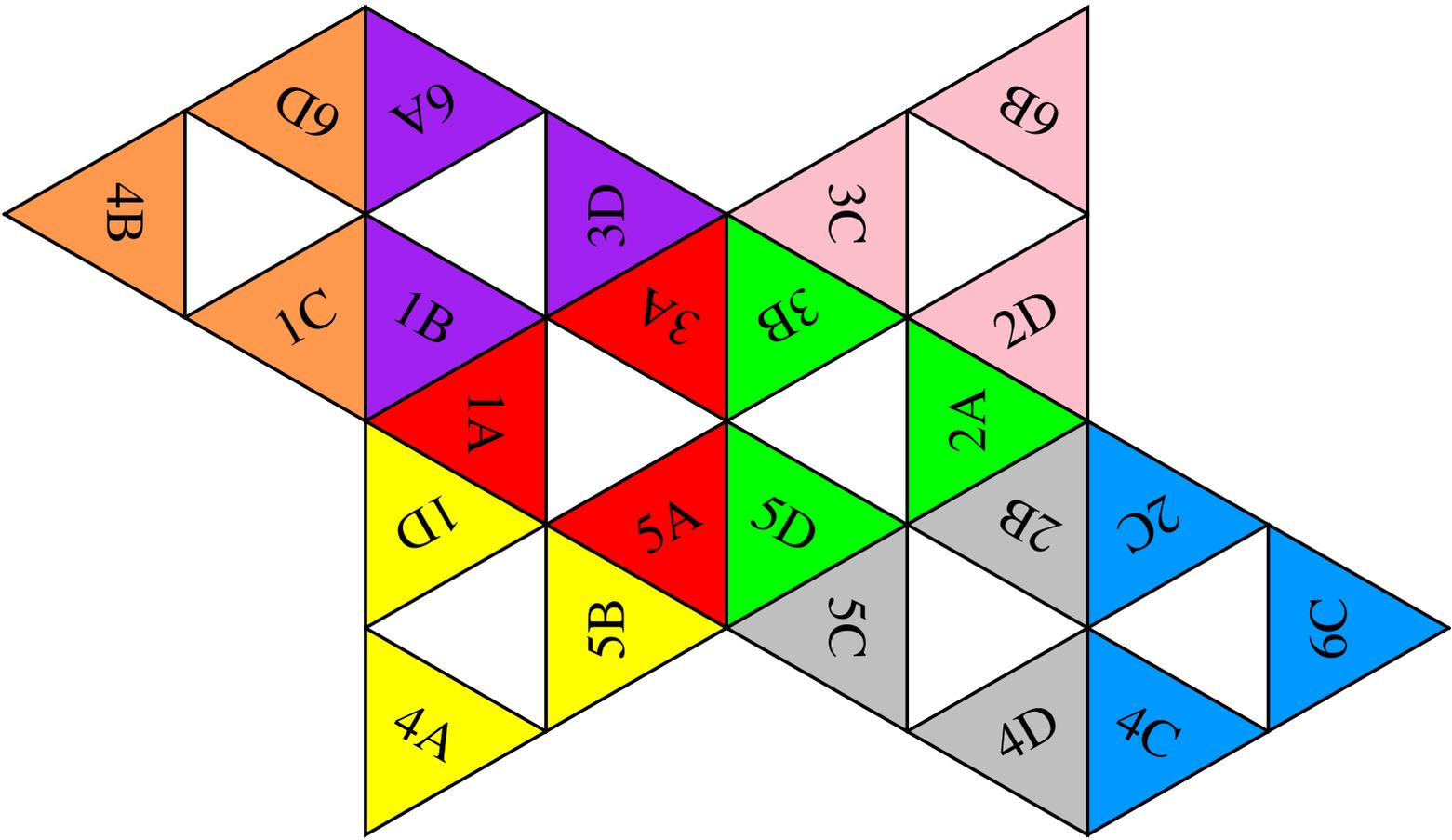


Figure 50: The pattern here is just the original configuration, but the six pieces have now been labelled as 1-6, and the four color patches on each piece are further labelled as A-D. If you compare these labels with the rows and columns of Table 6, you should be able to see how everything matches up. If you similarly consider the templates on the next page, you should be able to match them up with Tables 7 and 9. In all cases, both here on the templates and in the tables, the labels 1A-6D remain in their original locations, and it is the pieces that move on the templates, and similarly the colors move in the tables. That is, by meticulously labelling everything in this way, three-dimensional motions of the pieces become encoded into just rules for how the colors can move in the tables, which is something a computer can easily deal with. For each of the $5! \cdot 4^5 = 122,880$ possible tables, the ‘equal squared-distances’ test is then applied to decide whether the pattern should be rejected (as for Table 7), or needs to be examined further by hand (as for Table 9).

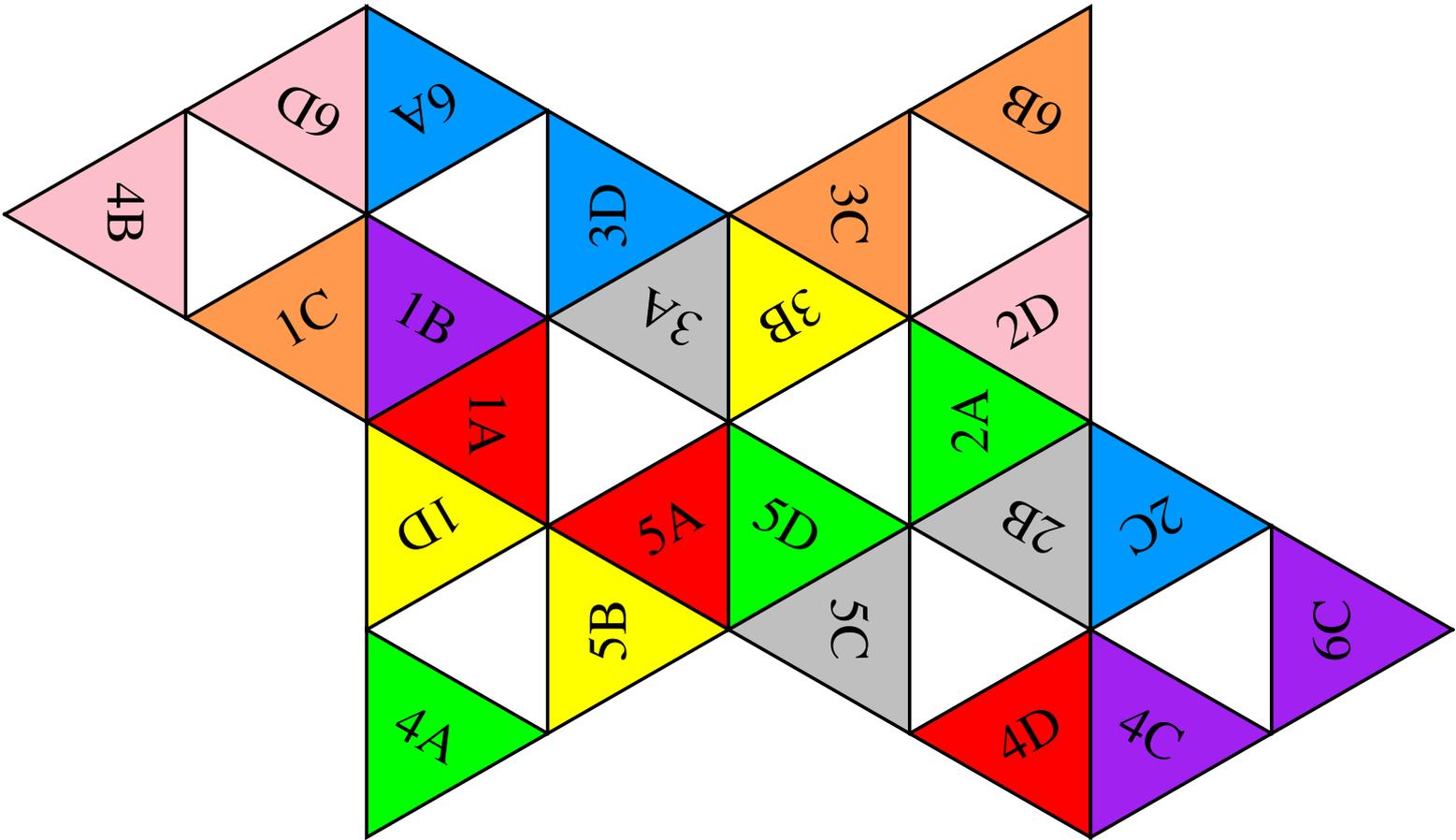
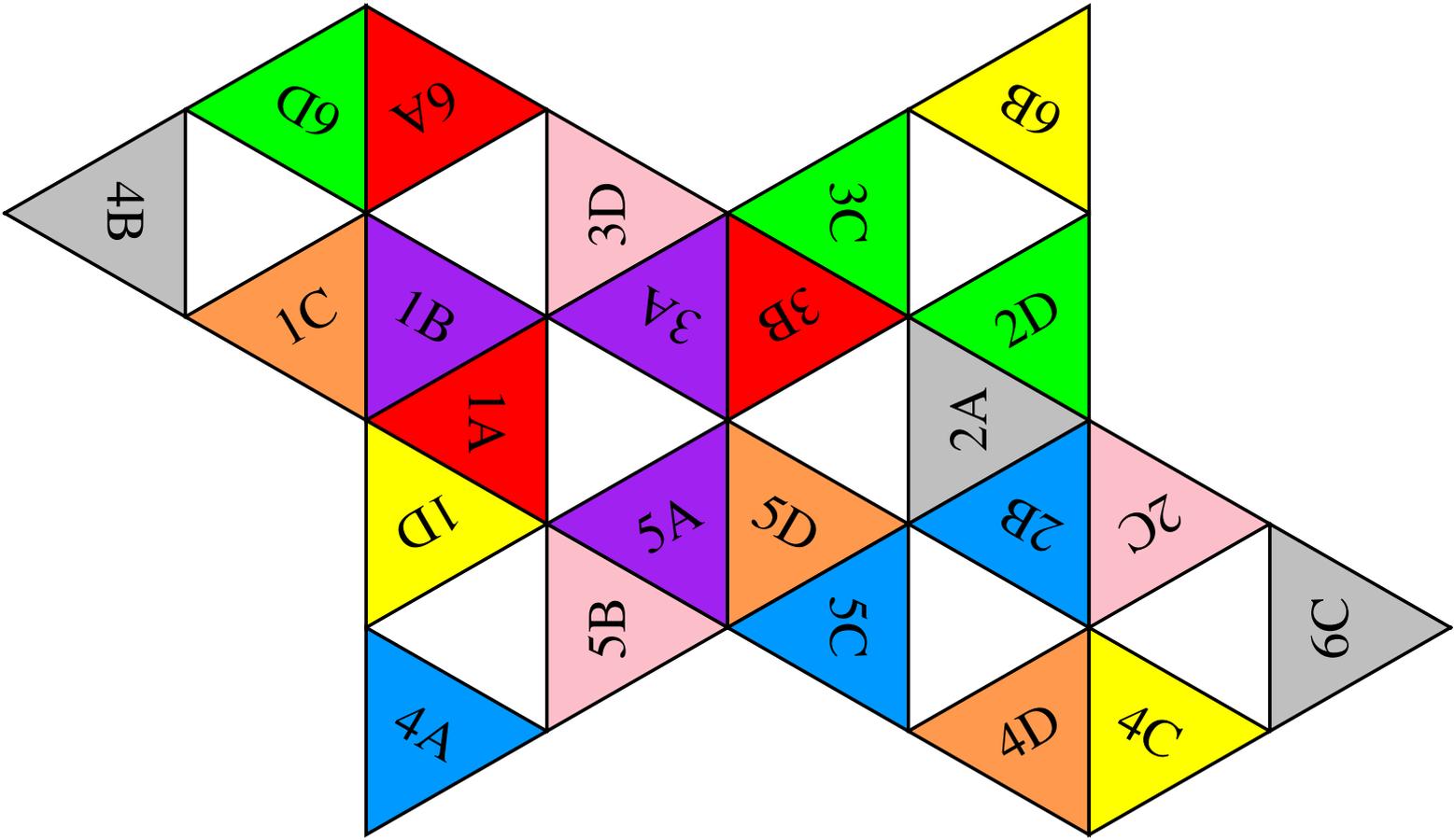


Figure 51: Templates for the patterns denoted by Tables 7 and 9.