



A novel analytical approach for evaluating thermally active underground retaining walls

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ABSTRACT

Shallow geothermal energy is a promising renewable technology for sustainable indoor heating and cooling. One method to exploit this energy is through Energy Geostructures, where structural elements embedded in the ground are thermally activated via heat exchange pipes embedded within the structure. These structures function as a specialised form of ground heat exchanger, which can be integrated with ground source heat pump systems. Energy walls (EWs), a specific type of Energy Geostructure, are commonly used in basements, underground parking facilities, and metro stations. Despite their potential, there is currently no simple and reliable analytical method for the thermal analysis of EWs. Instead, their design relies on computationally expensive numerical simulations or oversimplified 'rules of thumb', both of which may lead to inefficiencies in cost and performance. This study presents a novel analytical approach based on the Infinite Plane Source (IPS) model and evaluates its accuracy by comparing it with two-dimensional numerical model data. The results demonstrate that the proposed method provides highly accurate estimates of temperatures at the back of the wall, making it a valuable foundation for future analytical design methodologies. The findings are applicable to EWs having an excavation on one side and ground on the other, as well as fully buried EWs with soil on both sides. This research offers a significant step toward the development of a practical, cost-effective analytical framework for the design and optimisation of EWs, promoting the broader adoption of shallow geothermal energy systems.

1. Introduction

The Paris Agreement for reaching net-zero emissions by 2050 was signed by 195 countries at the 21st Conference of the Parties (COP21) in 2015. Thus, technologies are being developed to harness clean energy sources like biomass, geothermal, hydropower, ocean, solar and wind energy. Of these sources, geothermal energy is highly ranked for both power and heat, especially when considering its low environmental impact beyond carbon intensity.¹ Heating and cooling systems present a significant challenge in the decarbonisation of energy. They constitute approximately 40% of global energy demand and are responsible for 30% of worldwide carbon emissions.¹⁻³ Consequently, shallow geothermal energy used for heating and cooling applications is not restricted to specific geological regions and hence represents a crucial resource for the energy transition.

Shallow Geothermal Energy can be harnessed globally using Ground Heat Exchangers (GHEs) in combination with Ground Source Heat Pumps (GSHPs). These systems facilitate heat exchange with the ground

through a circulating fluid — typically a brine solution or a water-glycol mixture — flowing within heat exchanger pipes, which are commonly made of High-Density Polyethylene (HDPE).⁴⁻⁶ These can also be connected to a district heat network as an energy source or, in some cases, as energy storage, in order to fulfil the energy demands for wider applications and usages in a larger area than a single building.^{7,8} The most common form of GHEs is borehole heat exchangers; however, their implementation involves significant drilling costs. Energy geostructures offer a cost-effective alternative, integrating heat exchange pipes within underground structural elements during construction, thereby reducing both time and drilling expenses.^{4,5} In addition to serving as heat exchangers, energy geostructures function as dual-purpose elements, simultaneously supporting structural loads while facilitating thermal energy transfer.

This study focuses on a specific type of energy geostructure known as Energy Walls (EWs) — thermally activated embedded retaining walls, sometimes referred to as screen walls, used in buildings, underground stations, or other underground infrastructure. Several EWs have been

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constructed and are currently in operation, including a piled retaining wall at Keble College, Oxford,⁹ diaphragm EWs at Shanghai Museum of Natural History,¹⁰ the Bulgari Hotel in Knightsbridge, and Crossrail Stations in London, UK.¹¹ Additionally, Austria hosts various EWs of different types, including those at the Uniqa Tower and the Vienna Metro line U2.¹² Among these, diaphragm Energy Walls are the most common.

Despite their growing application, there is currently no rigorous yet practical method available for engineers to estimate the energy availability from diaphragm EWs. Most projects rely on simplified empirical rules or computationally intensive numerical simulations, both of which present limitations in cost and feasibility for routine engineering practice. An analysis tool that can be coupled to building and heat pump simulations, and predict temperature changes due to small time step changes in thermal demand over decades, is urgently required. Such approaches need to be supported by an underlying physics-based analytical approach to enable efficient and accurate thermal analysis. This paper introduces the first step in developing an easy-to-apply analytical framework for assessing the thermal performance of EWs. Section 2 provides a detailed background on the thermal behaviour of EWs, while Section 3 outlines the numerical and analytical techniques employed. The core analytical approach, based on the Infinite Plane Source (IPS) model, is evaluated for its effectiveness in predicting temperatures at the back of the EW. The results, presented in Section 4, demonstrate the method's accuracy before concluding with key findings and future research directions

2. Thermal analysis of energy walls

Thermal analysis aims to characterise and understand the relationship between the heat transfer within the EW and the corresponding temperature variations. This analysis enables the estimation of available energy while ensuring both the structural integrity and the fluid entering the heat pump remain within appropriate temperature limits. The common thermal design objective is consequently to minimise the temperature gradients/differences between the pipe and the interfaces between the wall and the surrounding ground (as in other forms of ground heat exchangers). This is chiefly dependent on wall thickness, pipe placement (e.g., closer pipe spacing increases the total pipe/fluid surface area available for heat transfer in a given wall area) and wall thermal properties.

Knowledge available from studies of other forms of ground heat exchangers, such as Energy piles^{5,13} and Borehole Heat Exchangers^{14–16} suggests that hybrid modelling approaches using both analytical and numerical (or data derived from numerical calculations) are best suited to the analysis of the wide range of time and spatial scales that need to be considered in problems such as energy wall behaviour. Analytical approaches have been found to be fast and well-suited to the detailed design of system components (i.e., inside the pile or borehole) and relatively short timescales, whilst numerical models may be more generally applicable but at a computational expense. Consequently, analytical approaches have proved to be of greatest value in heat exchanger configuration design and hybrid approaches in overall system design and lifetime modelling. An important question in finding appropriate methods for modelling EW is, according to the range of applicability of analytical methods and their utility in design tasks, some limitations according to timescale, geometric complexity, and boundary conditions are also expected. Advancement of analytical methods has also displaced more demanding numerical approaches as research has progressed in the study of these heat exchanger technologies (e.g.,¹⁷). This is elaborated in the following literature review and is a question considered in this research.

Most analyses of EWs to date have utilised finite element techniques. However, some semi-analytical and analytical approaches have also been developed, as outlined below. Nevertheless, none of these methods provides practical tools that are suitable for routine engineering

applications.

2.1. Numerical approaches

For EWs, numerical analysis is computationally intensive, making it impractical for most projects. However, numerous numerical studies have been conducted to investigate the various parameters influencing EW design. For example, EWs typically feature a large planar surface area, meaning various pipe arrangements can be used. For ease of construction, vertical and horizontal arrangements are mainly constructed in practice. Makasis and Narsilio¹⁸ used COMSOL Multiphysics to show that reducing pipe spacing is the easiest way to extract more heat from the ground through the same thermally active area.

Zannin et al.¹⁹ utilised COMSOL Multiphysics to carry out 3D hydrothermal finite element modelling of EWs, quantifying the energy obtainable under different groundwater flow conditions. Makasis and Narsilio²⁰ studied the long-term behaviour of soldier pile energy walls using COMSOL Multiphysics, demonstrating that increasing pipe length and wall depth enhances performance. Additionally, performance improvements were associated with decreasing the spacing of soldier piles, analogous to reducing pipe spacing in a diaphragm wall. Zannin et al.²¹ conducted experimental tests on a full-scale energy wall within an underground railway station, using the data to validate thermo-hydro-mechanical analysis performed using the COMSOL Multiphysics. Their analysis underscored the impact of temperature on the exposed side of the wall and the potential of energy geostructures for heat storage.

Di Donna et al.²² used FEFLOW to conduct a parametric analysis of EWs, finding that while using a concrete mix with higher thermal conductivity can improve the general performance, pipe spacing remains the most critical factor in the short term. Their study also indicated that the temperature difference between the wall and excavation side plays a key role in the long-term performance of EWs. In a subsequent study, Di Donna et al.²² utilised FEFLOW for numerical analysis of EWs under varying groundwater flow speeds and directions. Their findings demonstrated that the absolute temperature difference between the ground and the circulating fluid is a major influencing factor. Additionally, they developed design charts for energy walls, emphasising the role of ground conditions. These charts, along with those presented by Dornberger et al.²³ represent some of the few available tools for preliminary EW design.

Bourne-Webb et al.²⁴ employed Abaqus to analyse the thermal and mechanical response of EWs forming part of a tunnel structure. Their results showed that seasonal temperature variations within the tunnel had a more pronounced effect on the wall's mechanical response than on its efficiency as a heat exchanger. Similarly, Dong et al.²⁵ developed a small-scale experimental model to investigate the thermo-mechanical behaviour of an EW, which was later used to validate a numerical model in ANSYS. While these studies utilised various software tools to analyse the behaviour of EWs, they were not applied to routine design cases. For such applications, semi-analytical and analytical solutions remain essential.

2.2. Semi-analytical approaches

Semi-analytical approaches for EWs remain limited.^{4,26} pioneered the use of an electrical analogy to develop the first application of thermal resistance models for EWs. These models were integrated with a 3D numerical simulation in the SHEMAT-Suite software for a comprehensive analysis of the wall. In the vertical direction, multiple 2D horizontal planes incorporating 2D resistance models were coupled. The numerical model provided initial values to the 2D resistance models, which then computed thermal resistances, heat flows, and temperatures at all levels. These values were subsequently fed back into SHEMAT-Suite as initial conditions for the next time step. This semi-analytical approach reduced the need for fine discretisation of elements in the horizontal plane,

focusing instead on the vertical direction, thereby lowering computational costs. The approach was validated using physical model data and fully discretised numerical models. However, the validation was limited to a model with a single U- or W-shaped loop, and the experimental tests covered only a short duration (approximately 7–10 h). Given that energy wall systems typically incorporate multiple pipe loops distributed across the structure and operate over significantly longer timescales, this validation remains insufficient for broader application.

2.3. Analytical approaches

There have been limited efforts to develop fully analytical approaches for the thermal analysis of EWs. The first such attempt was by Sun et al.¹⁰ who developed a two-dimensional heat transfer model to estimate the heat exchanged through an EW. Their model employed two distinct sets of equations: one for the fully buried section of the wall and another for the excavated section. The approach assumed transient heat transfer in the wall and surrounding ground while considering steady-state heat transfer within the pipes and fluid. Given the disparity in scales, the pipes were modelled as point sources. The analytical solutions derived by Sun et al.¹⁰ were validated against numerical simulations, yielding reasonable accuracy for short durations. However, the model exhibited a relative error of up to 9% over a 48-hour period and has not been tested for longer timescales.

Shafagh et al.²⁷ introduced a response factor approach for diaphragm wall heat exchangers by developing a dynamic thermal network (DTN) model. This method required an initial numerical finite-volume representation of the boundary conditions for the ground and basement to derive weighting factors, which were then applied analytically. The DTN model's predictions were compared against thermal response tests conducted on an EW, demonstrating satisfactory performance in accounting for structural complexities and non-homogeneous material properties. However, a major drawback of this approach is the necessity for pre-calculated weighting factors before application. More recent research has demonstrated, for the first time, that weighting factors can be derived experimentally for ground heat exchange pipes,²⁸ suggesting a potential avenue for application to EWs.

Shafagh and Loveridge²⁹ explored the applicability of the Infinite Plane Source (IPS) method for the thermal analysis of EWs, leveraging the planar geometry of the wall. In this approach, the IPS was conceptualised as a plane at the edge of the repetitive pipe arrangement within the EW and evaluated at a distance corresponding to the concrete cover of the pipes. When tested against field data for a fully buried wall, the IPS method demonstrated an appropriate trend but was unable to predict fluid temperature directly, instead providing only an estimation of the temperature at the rear of the wall. Despite this limitation, the IPS method was recognised as having potential for EW applications. This was further substantiated by Gupta et al.,³⁰ who applied the IPS method to the exposed face of a wall in an excavation. Their results demonstrated that the model effectively captured temperature trends, with acceptable accuracy when compared to a numerically simulated infinite wall.

However, the approaches proposed by Shafagh and Loveridge²⁹ and Gupta et al.³⁰ are constrained by their focus on a single wall type and the omission of both buried and exposed wall geometries. To address these limitations, this paper introduces a novel, analytically precise method for applying the IPS to EWs, termed the Flux and IPS Splitting (FIS) Method. This approach accounts for both buried and exposed portions of an EW, significantly improving accuracy and establishing a foundation for coupling with other analytical solutions to enable fluid temperature prediction for design purposes.³¹

3. Methodology

This study focuses on the analysis of the EWs utilising two commonly used arrangements: (i) when the wall is exposed to a basement or

underground space (e.g. metro station), with ground on one side and an excavation on the other, known as a 'Ground-Excavation' (GE) arrangement, and (ii) the fully embedded wall condition i.e., ground on both sides of the wall or 'Ground-Ground' (GG) arrangement, as illustrated in Fig. 1. The analysis is two-dimensional because it aims to determine, in the first instance, the ability of the simple analytical model (IPS) to account for the primary geometric parameters (e.g. thickness, spacing, pipe position) of EWs.

The performance of the IPS using the FIS method is evaluated against outputs from two-dimensional numerical models to gauge the functionality and precision of the FIS Method. A standard model has been defined based on the typical value of various EW design parameters to achieve this. Following this, a parametric analysis is carried out to understand the factors which increase or decrease the accuracy of the analytical solution. Section 3.1 describes these parameters, Section 3.2 gives details of the numerical model, and Section 3.3 presents the IPS equations and how they are applied using the FIS method. As with many problems in heat transfer, it is convenient to convert the analysis outputs to dimensionless form to better understand the factors affecting the results. Therefore, a new dimensionless approach for thermal analysis of EW is presented in Section 3.4.

3.1. General parameters

Due to the large surface area of energy walls (EWs) and the significant thermal mass of the surrounding ground, EWs exhibit highly transient thermal behaviour. Additionally, heat transfer within the wall is not uniform along the longitudinal plane.³² The extensive surface area of EWs allows for various configurations of embedded piping, with the most common arrangements being parallel, vertical and horizontal rows. From a thermal performance perspective, studies have shown minimal differences between these two configurations.³³ In this study, a vertical pipe arrangement is selected, as illustrated in Fig. 1a. However, the vertical cross-section of a horizontally arranged pipe system (Fig. 1b) would yield an equivalent outcome. The cross-sectional area of the wall features a repetitive pipe arrangement, as shown in Fig. 1c. A single pipe and its surrounding material have been selected for analysis, assuming similar thermal conditions in adjacent pipes, represented by adiabatic lateral boundary conditions. The longitudinal boundary conditions vary based on the wall surface configuration, with ground-exposed (GE) and ground-ground (GG) arrangements, as illustrated in Figs. 1d and 1e, respectively.

In the ground-exposed (GE) arrangement (Fig. 1d), heat exchanger pipes are typically installed closer to the ground and attached to the steel cage on the side opposite the exposed face of the wall. The excavation space can serve various purposes, such as a basement or railway tunnel, and depending on its usage, it may function as a heat source or sink, thereby influencing the boundary conditions. Makasis et al.³⁴ highlight the significant role that excavation space utilisation plays in the analysis and performance of energy walls. They also suggest that assuming a thermally insulated condition provides a conservative estimate in terms of energy design. Consequently, this study adopts an adiabatic boundary condition (ABC) for the excavation side of energy walls (Fig. 2a). For embedded energy walls (GG), heat exchanger pipes are installed on one face of the wall or, in some cases, on both faces. This study focuses on the more common case in practice, where only one side is thermally activated (Fig. 1e, Fig. 2b), as supported by previous research (e.g.²²).

In most cases, the dominant mode of heat transfer in energy walls and the surrounding ground is conduction,³⁵ thus, the influence of groundwater advection is neglected for this first development of energy wall analysis approaches. Additionally, for practical purposes, both the ground and the wall are assumed to be homogeneous and isotropic, meaning that their thermal conductivities are equal in both the x- and y-directions ($\lambda_x = \lambda_y = \lambda_g$). A uniform planar heat flux (q_0) of $2\pi \text{ W/m}^2$ is applied to both the numerical and analytical models. Temperature variations at the ground-wall interface for both GE and GG

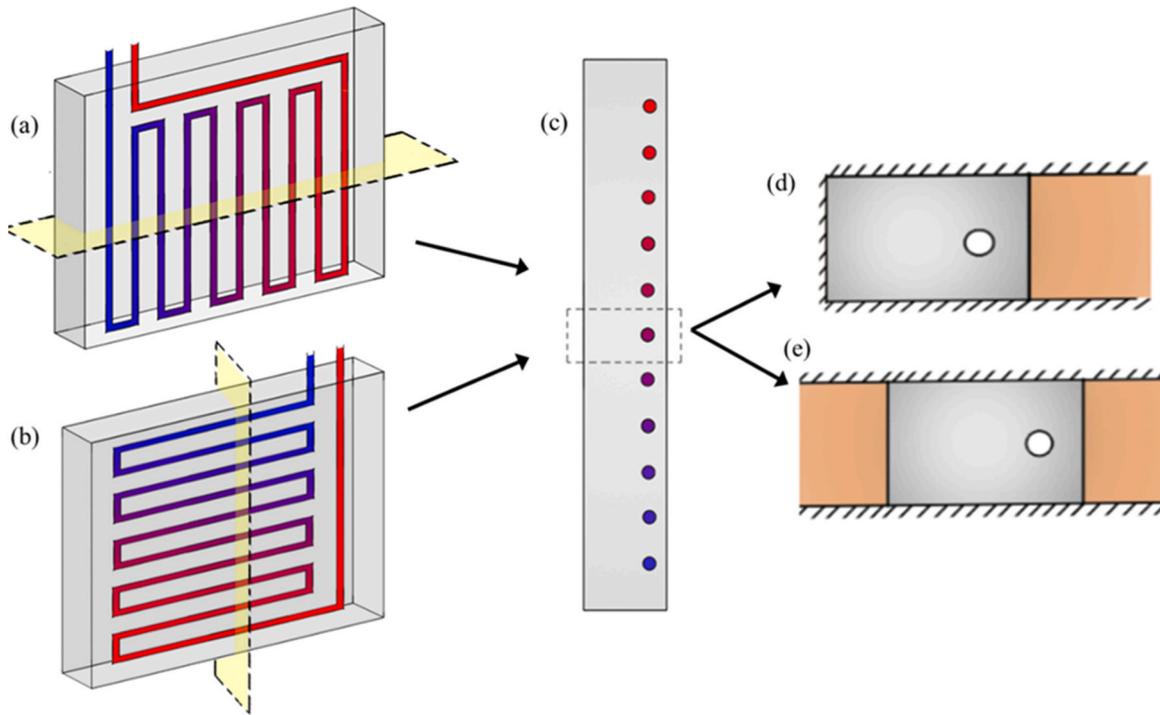


Fig. 1. Schematic representation of (a) energy wall with horizontal and (b) vertical pipe arrangement, (c) Cross-sectional area of an energy wall with repetitive pipe arrangement, (d) Energy Wall with GE, and (e) GG arrangement.

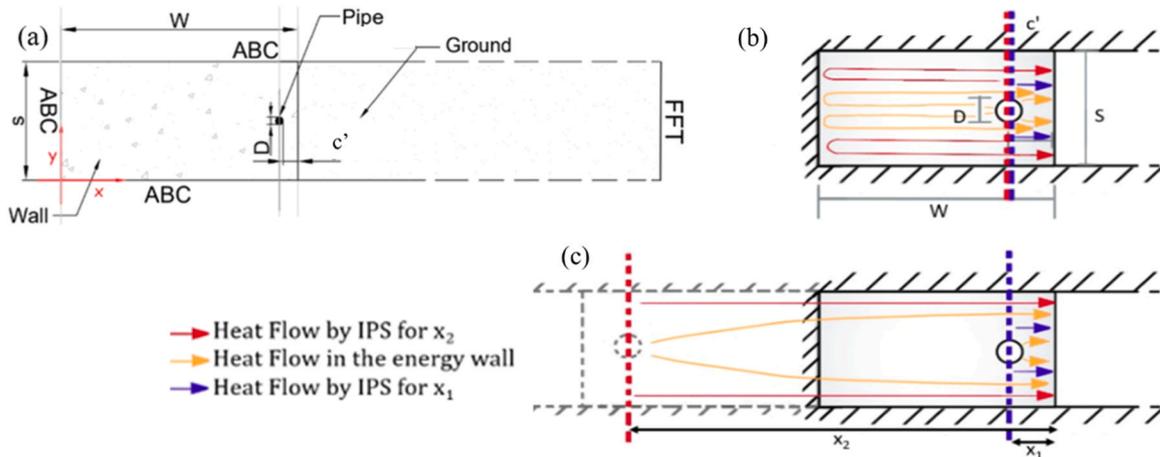


Fig. 2. Schematic representation of GE arrangement of Energy wall (a) for numerical modelling (ABC-Adiabatic Boundary Condition and FFT-Far-Field Temperature), (b) for placement of dual IPS in the FIS method and (c) also representing the mirror geometry through the ABC with a fake source on the excavated side.

configurations are analysed over one year, using an hourly time step. The initial ground and wall temperatures are set at 16°C.

The standard model assumes a fixed wall thickness (W) of 800 mm, with pipe spacing (s), pipe diameter (D), and pipe cover (c) set at 500 mm, 25 mm, and 75 mm, respectively. The thermal conductivity of both the wall and ground is fixed at 1 W/m/K. Table 1 summarises these parameters, along with the higher and lower values of wall thickness, pipe spacing, pipe cover, and thermal conductivity used in subsequent sensitivity analyses. These parameters, which have the greatest influence on the accuracy of the Flux and IPS Splitting (FIS) Method, are varied one at a time to assess their impact.

3.2. Numerical modelling

This study employs finite element analysis (FEA) conducted using

COMSOL Multiphysics,³⁶ a software widely utilised in previous research on energy geostructures (e.g.^{10,18}). A single, isolated pipe, representative of the repetitive geometry of an infinite EW (Fig. 1), is modelled. The pipes are represented as hollow cut-out sections, with a constant heat flux (q , W/m²) applied to these boundaries, as described by Eq. 1. Since q_0 is specified in terms of π , Eq. 1 yields a whole number result.

$$q = \frac{sq_0}{\pi D} \quad (1)$$

where s is the pipe spacing (m), and D is the pipe diameter (m). The mesh is designed for both accuracy and computational efficiency. The minimum and maximum element sizes are 0.001 m and 0.5 m, respectively, with a growth rate of 1.25 between successive elements. A refined mesh is applied near the wall-ground interface. The number of elements varies depending on the arrangement, pipe spacing, wall thickness,

Table 1

Values and formulas used for the standard model in numerical analysis, along with the parametric values(in brackets).

Property	Annotation (unit)	Values /Formulas
Diameter of pipe	D (mm)	25
Pipe Spacing	S (mm)	500 (300, 800)
Wall Thickness	W (mm)	800 (500, 1000)
Cover to pipes	c (mm)	75 50 100
Planar Heat flux	q_0 (W/m ²)	6.283185 or 2π
Initial Temperature	T_i (°C)	16
Far-Field Temp	FFT (°C)	16
Thermal Conductivity of the Ground	λ_g (W/m/K)	1 (1.5, 2)
Thermal Conductivity of the Wall	λ_w (W/m/K)	1 (2)
Thermal Conductivity of IPS	λ (W/m/K)	1 (1.5, 2)
Density of the ground/ soil	ρ_g (kg/m ³)	2000
Density of the wall/ concrete.	ρ_w (kg/m ³)	2000
Heat capacity/Specific heat of the ground/soil	C_{pg} (J/kg/K)	800
Heat capacity/Specific heat of the wall/ concrete	C_{pw} (J/kg/K)	800
Thermal diffusivity	α_g or α_w	$\frac{\lambda}{\rho C_p}$

cover, and pipe diameter; however, it typically ranges from 1750 to 3300 elements.

An adiabatic boundary condition (ABC) is applied to the excavation interface (Fig. 2a) for ground-exposed (GE) geometries. For both GE and ground-ground (GG) arrangements, a repetitive pipe geometry is reduced to a single isolated pipe section, with additional ABCs applied to the top and bottom portions of the model where symmetry exists (Fig. 2a, Fig. 3a). The ground on the right side of the GE arrangement, as well as both sides of the GG arrangement, is extended sufficiently to ensure that temperature effects at the far end do not influence the results. The temperature at the far end is fixed at the initial temperature (Fig. 2a, Fig. 3a, Table 1).

3.3. Analytical model - infinite plane source

The concept of the Infinite Plane Source (IPS) was first introduced by Carslaw and Jaeger.³⁷ In this model, the yz-plane (analogous to the energy wall) extends infinitely along both the positive and negative y-axes, while the solid material extends infinitely in the positive x-direction, as depicted in Fig. 4.

This infinite plane is assumed to act as a source or sink, and the movement of heat is one-dimensional. The temperature change (ΔT)

under an applied constant flux q_0 , with respect to the initial temperature at any distance, x (m), from the plane, and at any time, t (s), can be calculated by Eq. 3 :³⁸

$$\Delta T(x, t) = \frac{q_0}{\lambda} \left(\sqrt{\frac{4\alpha t}{\pi}} e^{-x^2/4\alpha t} - x \operatorname{erfc} \frac{x}{\sqrt{4\alpha t}} \right) = \frac{q_0}{\lambda} G_{IPS}(x, t) \tag{3}$$

where α (m²/s) and λ (W/mK) are the diffusivity and thermal conductivity of the semi-infinite solid, respectively, and erfc is the complementary error function. The main terms of the IPS within brackets can be referred to as the G-function, termed as G_{IPS} . In this formulation, the thermal properties of the ground and wall are assumed to be equal to each other; thus $\lambda_g = \lambda_w = \lambda$ and $\alpha_g = \alpha_w = \alpha$. In this study, the thermal properties in Eq. 3 are assumed to be those of the soil, and the sensitivity analysis presented in Section 4.3 examines the limitations of this assumption.

Shafagh and Loveridge²⁹ and Gupta et al.³⁰ attempted to predict the temperature at the wall-soil interface by evaluating Eq. 3, assuming $x = c$ and $x = W$, respectively (Fig. 5). However, neither approach accurately captures the heat flow within an energy wall (EW) nor accounts for both ground-exposed (GE) and ground-ground (GG) geometries. This study addresses these limitations by splitting the heat flux and applying half of the total flux (q_0) to each of the two planar heat sources. This method, referred to as the Flux and IPS Splitting (FIS) Method, positions the two planar heat sources back-to-back along the centreline of the pipes (Fig. 5), with each source generating heat that flows in opposite directions.

In simple terms, in the assumptions for the analytical model, the pipe domain is divided into two symmetric subdomains through the two planes, assuming an equal heat flux split. This approximation is clearly valid for near-symmetric configurations and comparable thermal conductivities of the wall and surrounding ground. However, there may be some discrepancies when the pipe is off-centre or when thermal properties differ significantly as may be the case for energy walls, and these will be investigated in Section 4. In practice this shows that while the flux will not be divided exactly equally between these two theoretical planes; this assumption is acceptable for the case studied.

In the FIS method, both IPSs are used to calculate temperature changes at the single ground-wall interface for the GE arrangement. Based on Fig. 2b, the half flux applied on the blue IPS exits directly from the interface; therefore $x_1 = c + D/2$, is applied. Meanwhile, the other half of the flux applied to the red IPS and travels towards the excavation side, is reflected by the ABC, and exits from the interface. This gives $x_2 =$

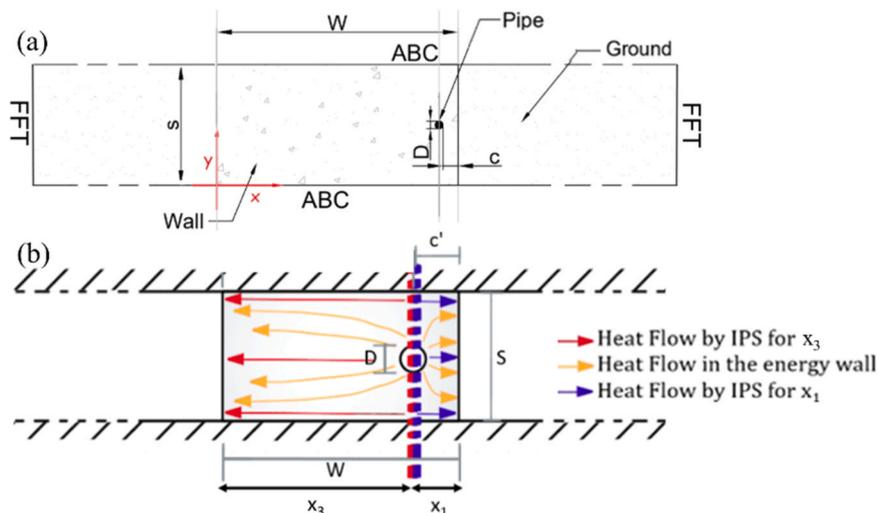


Fig. 3. Schematic representation of Energy Wall with GG arrangement (a) for numerical models(ABC - Adiabatic Boundary Conditions and FFT - Far-Field Temperature), (b) Placement of dual IPS in the FIS Method.

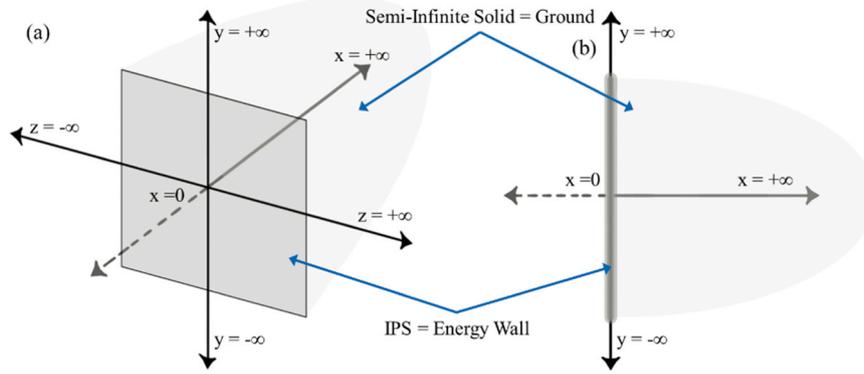


Fig. 4. Schematic representation of the Infinite Plane Source (IPS) in (a) 2D and (b) 3D.

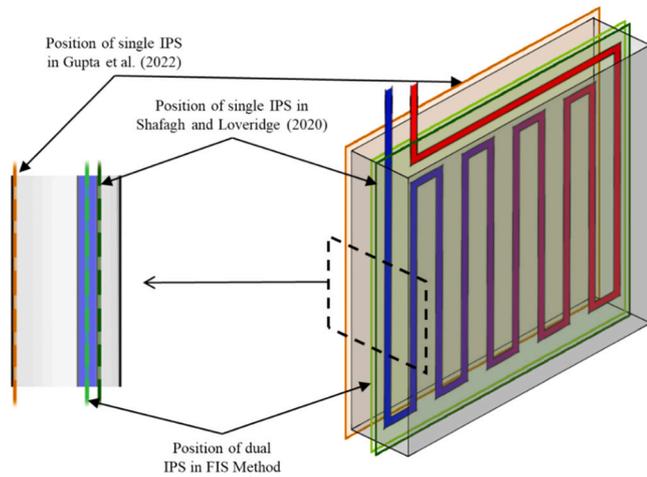


Fig. 5. Positions of infinite plane source (IPS) used in different methods – Shafagh and Loveridge²⁹ and Gupta et al.³⁰ and the FIS Method.

$2W - x_1$. This approach can also be conceptualised by expanding the ABC boundary on the left and assuming an imaginary source on the other side, which is responsible for the movement of heat in that direction, as seen in Fig. 2c. Since the flux from each IPS influences the temperature change at the interface, they can be added to the initial temperature, T_0 , to get the final temperature at the interface, T_{W-GE} , at any time t .

$$T_{W-GE}(t) = T_0 + \frac{\left(\frac{q_0}{2}\right)}{\lambda} \left(\sqrt{\frac{4at}{\pi}} e^{-x_1^2/4at} - x_1 \operatorname{erfc} \frac{x_1}{\sqrt{4at}} \right) + \frac{\left(\frac{q_0}{2}\right)}{\lambda} \left(\sqrt{\frac{4at}{\pi}} e^{-x_2^2/4at} - x_2 \operatorname{erfc} \frac{x_2}{\sqrt{4at}} \right) \quad (4)$$

$$T_{W-GE}(t) = T_0 + \frac{q_0}{2\lambda} [G_{IPS}(x_1, t) + G_{IPS}(x_2, t)] \quad (5)$$

In the GG arrangement, the thermal assessment is relatively straightforward. The two IPS are again placed at the pipe centre line. However, in this case, each is used to determine the temperature change of one side of the wall where it interfaces with the ground. Hence, at time t , the temperature at the interface adjacent to the pipes, T_{W-GG} , can be calculated using Eq. 6 and $x_1 = c + D/2$. Meanwhile, the temperature for the opposite interface, T'_{W-GG} , can be calculated using Eq. 7 with $x_3 = W - x_1$.

$$T_{W-GG}(t) = T_0 + \frac{q_0}{2\lambda} [G_{IPS}(x_1, t)] \quad (6)$$

$$T'_{W-GG}(t) = T_0 + \frac{q_0}{2\lambda} [G_{IPS}(x_3, t)] \quad (7)$$

3.4. Non-dimensionalisation of results

The temperature changes with time under constant thermal load at the wall ground interfaces, calculated using the numerical model and by the FIS Method, are non-dimensionalised to reduce the number of dependent variables. Non-dimensional time is calculated using the Fourier number (F_0) (Eq. 8), and the non-dimensional temperature, ϕ , is determined using Eq. 9. For this application, the characteristic length for the non-dimensionalisation was determined to be the wall thickness, W .

$$F_0 = \frac{\alpha t}{W^2} \quad (8)$$

$$\phi = \frac{\lambda(T_0 - T)}{q \cdot W} \quad (9)$$

The results in Section 4 are presented as either absolute temperature (T, ϕ) or the difference between the numerical model and analytical model ($\Delta T, \Delta \phi$). The results from the two models are compared non-dimensionally at a Fourier number of 0.1 and 10 to understand the wall's short-term and long-term behaviour. $F_0=0.1$ corresponds to a short period of about 1–2 days, and depending on thermal diffusivity, α and thickness of wall, showcasing the short-term behaviour. Whereas, $F_0=10$ represents the behaviour at 111–185 days, depending on the wall dimensions and thermal properties used. In the current study, since the values of non-dimensionalized temperature i.e. ϕ would have much smaller values as the flux used is on the higher side, and also, in most cases, the difference between the values is studied, which might make them even smaller.

4. Results and discussion

4.1. Validity of assumptions

Figs. 6 and 7 present temperature contours for the standard GE and GG Energy Walls, respectively, at the end of the simulation period. These allow examination of the applicability of some of the key assumptions behind the analytical method. In both cases, it's evident that for infinitely long walls, heat transfer exhibits a planar behaviour at a distance from the pipe of only a few centimetres. This suggests that in general the IPS model, which is based on planar behaviour, should serve as a suitable analytical approach for thermal analysis.

Looking more closely at the contours of temperature at the back of the energy wall (at x coordinate zero in Figs. 6 and 7) does however show that the heat transfer behaviour is not perfectly planar at this locations. To examine this region in more detail, Figs. 8 and 9 present the temperature profile with distance from the back of the wall for three

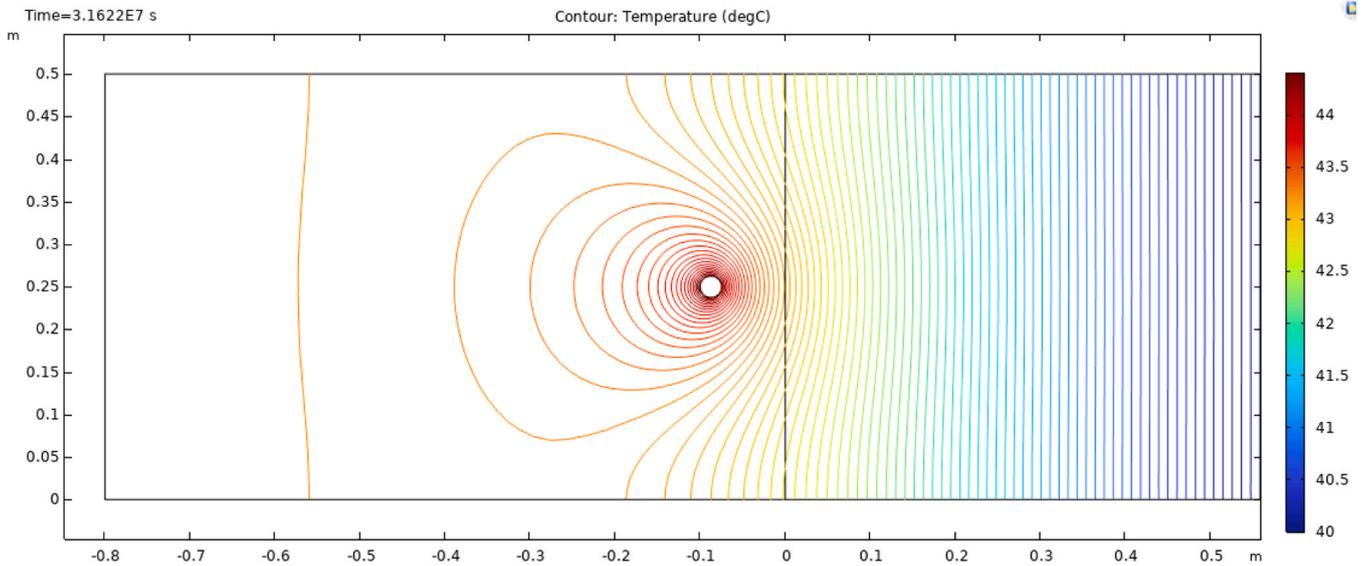


Fig. 6. Isothermal contour extracted from the numerical analysis via COMSOL of the region of the GE Energy Wall and surrounding soil.

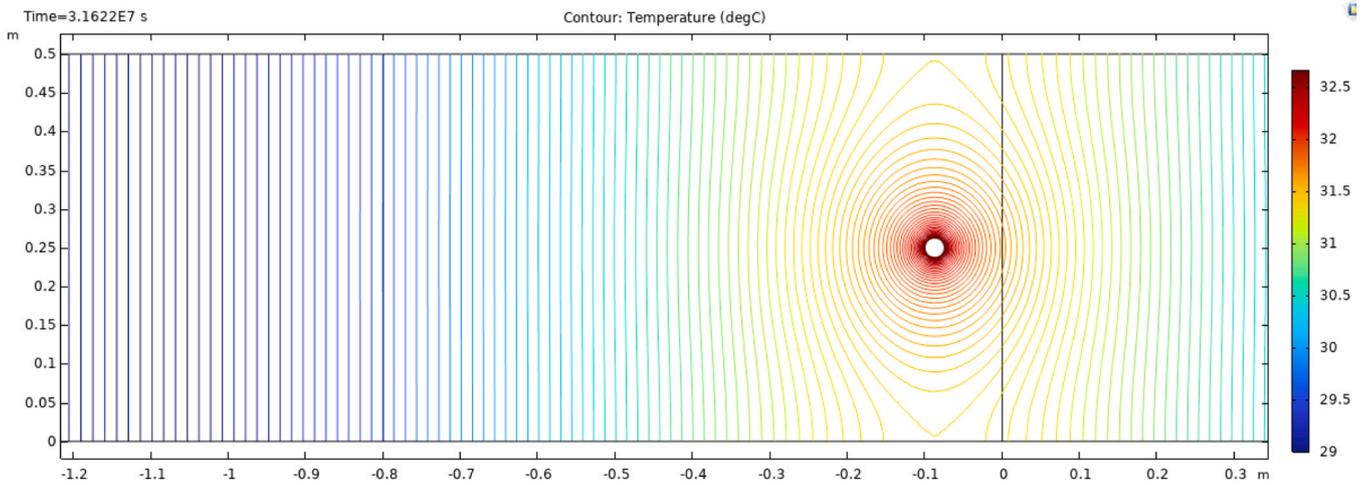


Fig. 7. Isothermal contour extracted from the numerical analysis via COMSOL of the standard model of GG Energy Wall and its surrounding soil after one year.

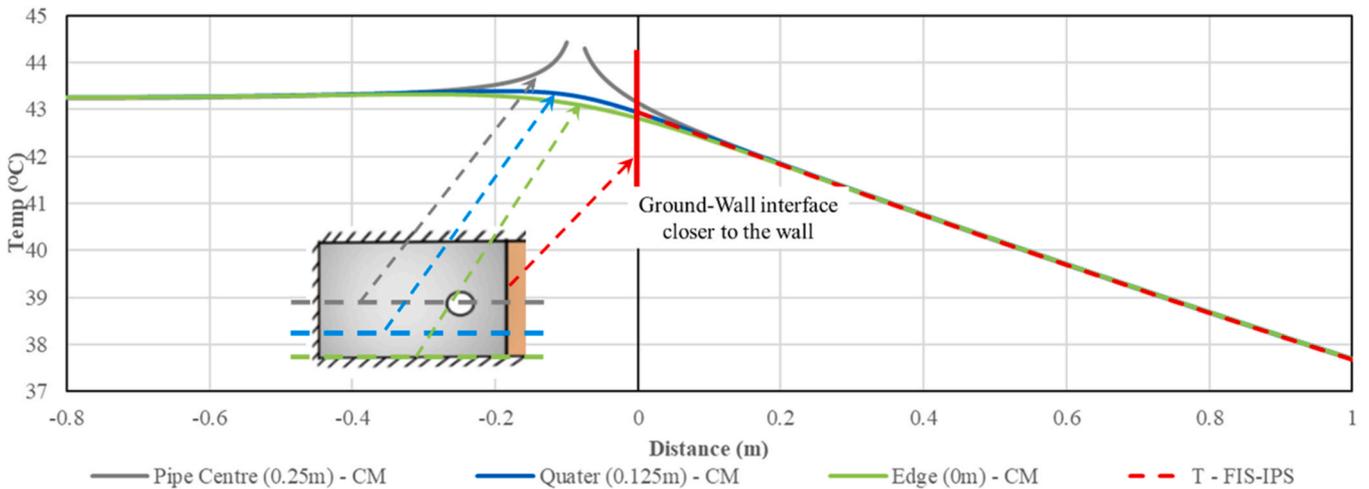


Fig. 8. Simulated temperature profiles measured at varying distances (x) from the GE energy wall after one year of thermal analysis.

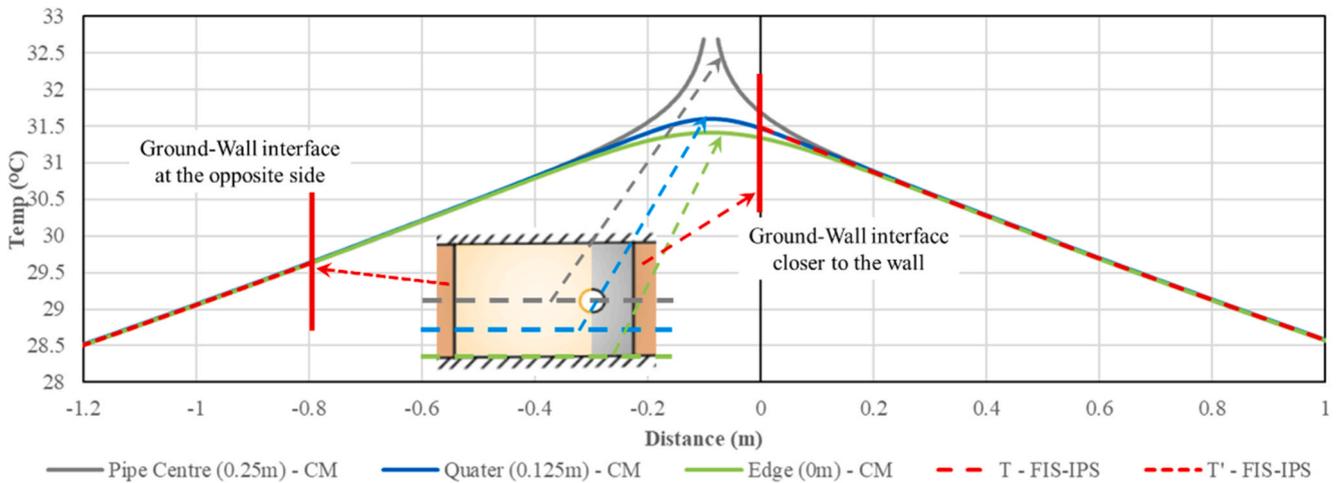


Fig. 9. Temperature distribution along various distances (x) after one year of simulation for the GG energy walls.

different points in the numerical model, and as predicted using the infinite plane source implemented by the FIS approach. This shows that the small temperature variation on the plane at the back of the wall does not persist on parallel planes within the ground beyond a few centimetres, again pointing to the validity of the planar heat transfer assumption.

The assumption of equal flux splitting is also examined using the numerical model outputs. For fully buried walls (GG), the heat flux through both sides of the wall at the end of the simulation was also extracted. Since this is a transient analysis, not all flux passes through the wall, which is only at the pseudo-steady state. Approximately 49.5% (3.11 W/m²) of the total flux is observed on the wall side closer to the pipes, while around 45.6% (2.86 W/m²) is recorded on the opposite side. This result suggests that the assumption of equally splitting the flux is reasonable, although the results would vary when the thermal conductivities of the ground and concrete differ.

4.2. Temperature prediction

The temperature of the ground wall interface for one year of constant heating for GE and GG is shown in Figs. 10 and 11, respectively, for the standard case. In addition to the numerical and FIS results, Fig. 10 also includes temperature predictions with the IPS according to the earlier approaches of Shafagh and Loveridge²⁹ and Gupta et al.³⁰ It can be seen

that the FIS method gives a much-improved fit to the numerical simulation for both short and long time periods due to accounting for heat flow in both directions from the pipes. The FIS method is always within 0.005°C of the numerical model, while the other approaches can lead to more significant errors. The approach of Shafagh and Loveridge²⁹ matches well in the short term, but neglecting the full wall thickness leads to 2 °C errors in the long term. Meanwhile, the results, according to Gupta et al.³⁰ give better long-term results due to capture flow throughout the concrete's thermal mass, but have shorter-term errors (up to 1.5°C) due to the delay in the thermal front reaching the back of the wall.

For fully embedded walls (GG arrangement), Fig. 10 shows that the ground-wall interface temperature calculated using the FIS Method differs from the numerical results by a maximum of approximately 0.002°C regardless of which side of the wall is being assessed. Therefore, the FIS method shows excellent agreement for the selected standard models for exposed and embedded cases. The following sections present the impact of different wall geometries and thermal properties, with Fig. 10 and Fig. 11 giving the non-dimensional temperature differences between the analytical and numerical models at $F_0=0.1$ and $F_0=10$, from the GE and GG arrangements, respectively.

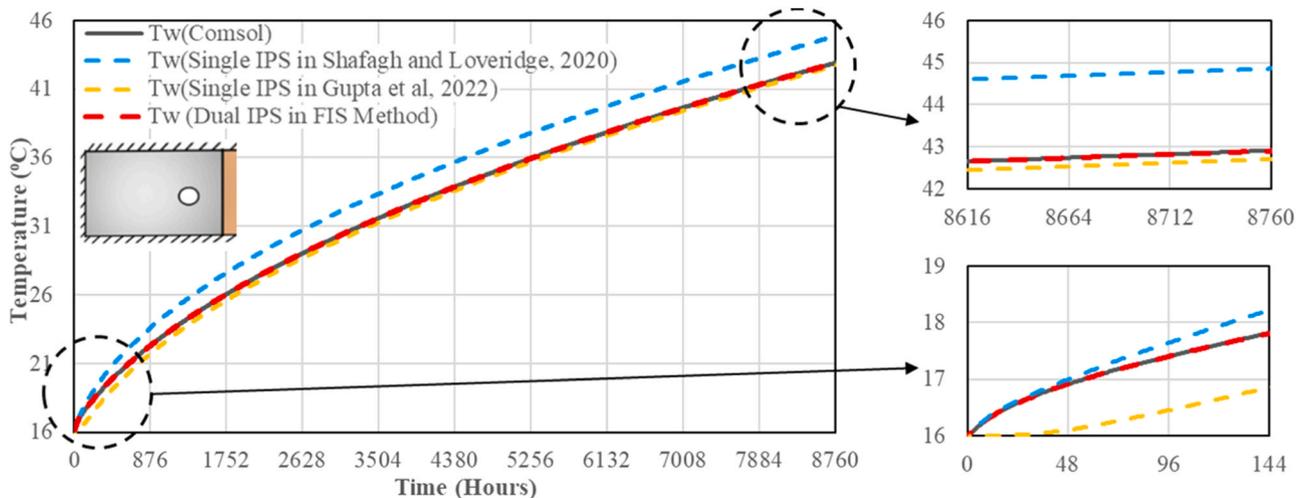


Fig. 10. Testing temperature changes at the ground wall interface(T_w) of the GE Energy wall calculated through the FIS method and the methods suggested by Shafagh and Loveridge²⁹ and Gupta et al.³⁰ against COMSOL results.

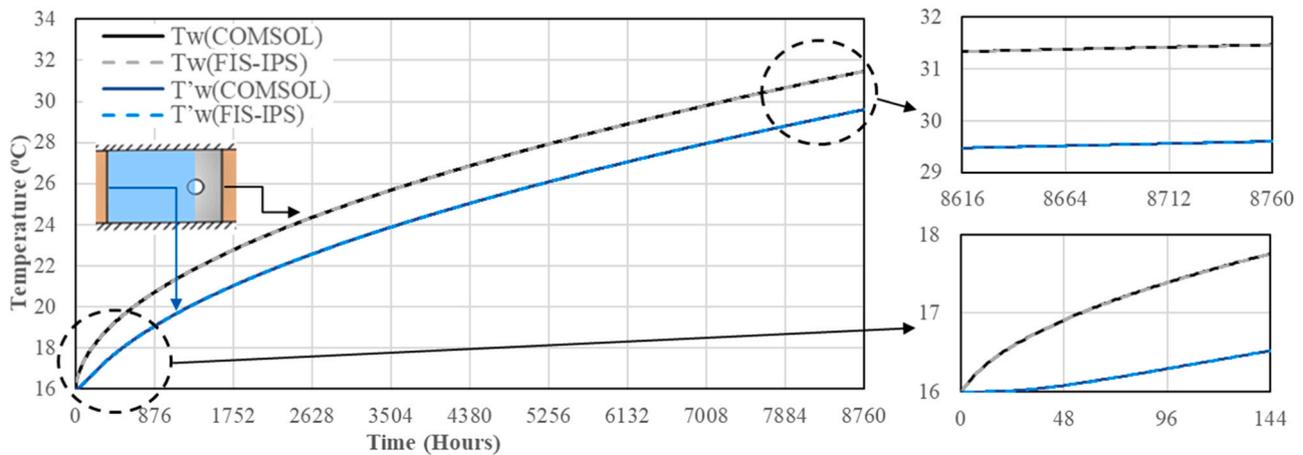


Fig. 11. Testing temperature changes calculated at interfaces of GG arrangement of Energy Wall through FIS method against the COMSOL results, where T_w is for the interface adjacent to the pipes and T'_w is for the opposite interface.

4.3. Effect of pipe spacing (s)

Pipe spacing is one of the factors that the designer can freely alter to suit the thermal demands of an EW. Since Di Donna et al.²² suggested a minimum spacing of 300 m; hence, in this parametric analysis, the lower pipe spacing is selected as 300 mm, whereas 800 mm is selected as the higher value.

As the pipe spacing is not a characteristic length for non-dimensionalisation, $F_0=0.1$ and $F_0=10$ are the same for all values of pipe spacing, being 1.2 and 118.5 days, respectively, and the results for GE and GG energy walls are presented in Figs. 12 and 13, respectively. In

the case of GE, with the increase in pipe spacing, the non-dimensionalised ground-wall interface temperature difference between the numerical and analytical model ($\Delta\phi_w$) decreases, as shown in Fig. 12. This remains true for both $F_0=0.1$ and $F_0=10$, as seen in. This behaviour can be explained by the isothermal contour lines in the numerical model, presented in Fig. 13. The isothermal lines are not perfectly straight but curved at the ground wall interface. The smaller the pipe spacing, the further the heat flow behaviour is from linear, whereas at larger pipe spacing, the contour lines are more planar. The temperature changes extracted from the numerical model are the average values of the whole interface. Since the IPS is based on planar

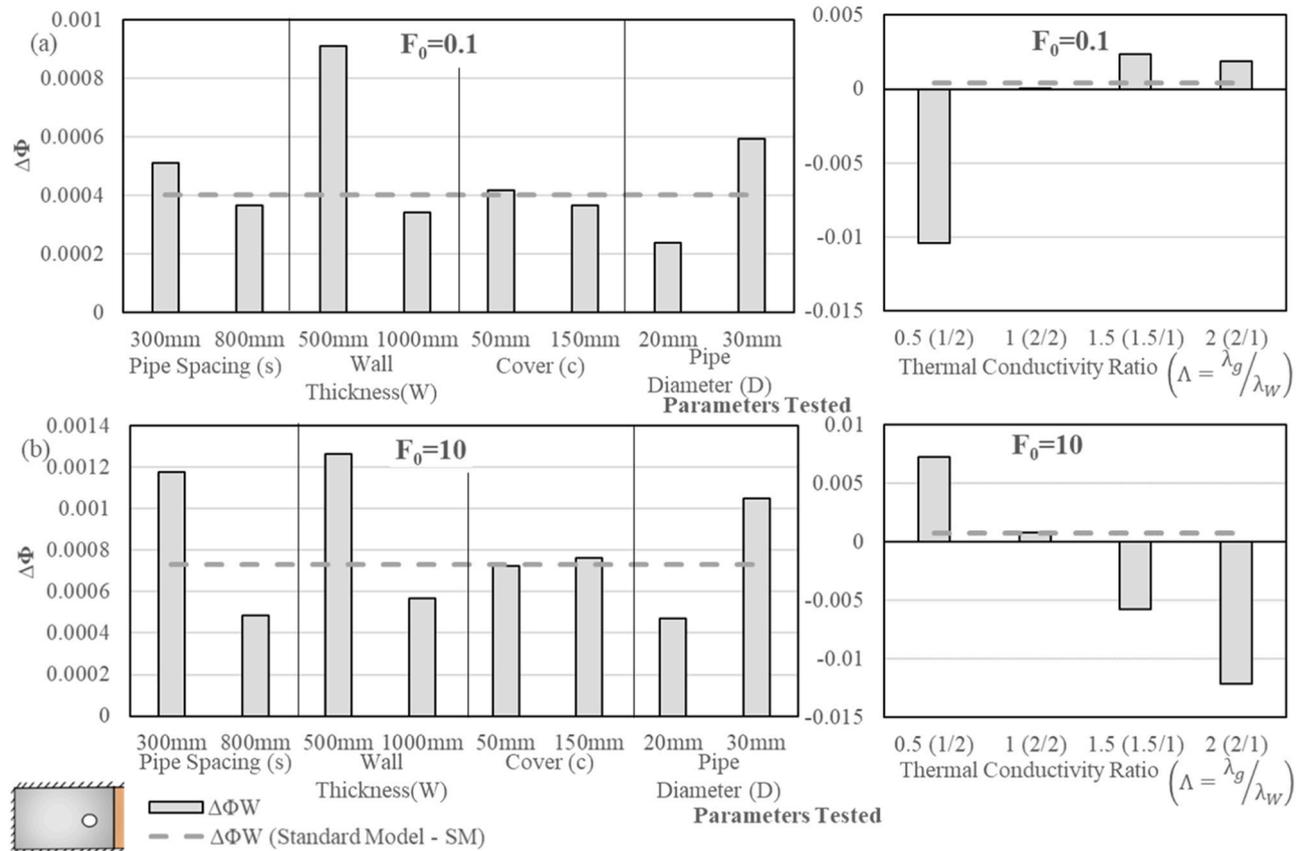


Fig. 12. Non-dimensionalised temperature differences between numerical and FIS Method ($\Delta\phi_w$) at the ground-wall interface for GE energy wall, compared against the standard models - SM (dotted lines) at (a) $F_0=0.1$ and (b) $F_0=10$.

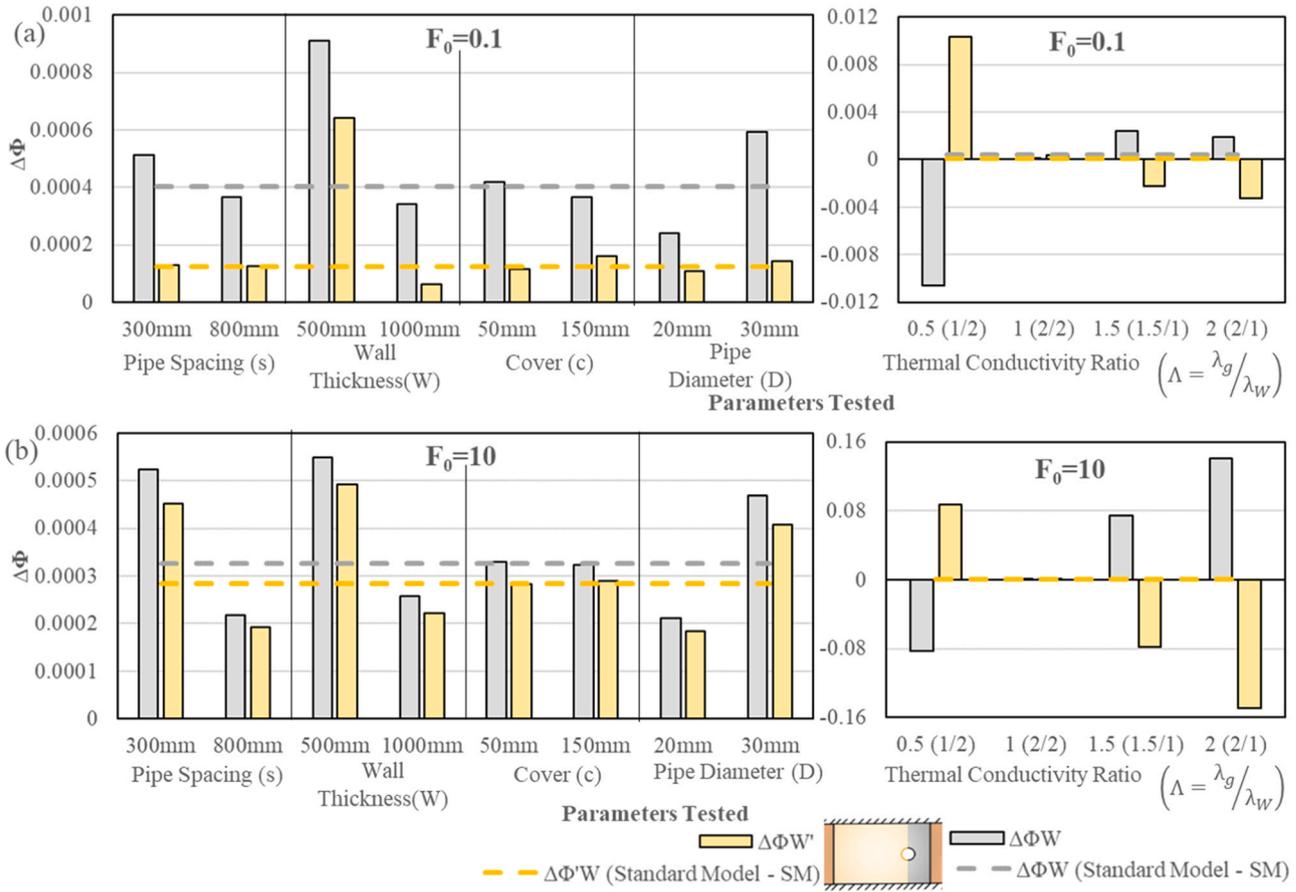


Fig. 13. Non-dimensionalised temperature differences between numerical and FIS Methods for GG type energy wall at the wall surface adjacent to pipe ($\Delta\phi_w$) and further away from pipe ($\Delta\phi_w'$), compared against the standard models (dotted lines) at (a) $F_0 = 0.1$ and (b) $F_0 = 10$.

heat distribution, larger pipe spacings have a better match to the IPS, and $\Delta\phi_w$ is lower at larger pipe spacing.

The results for the GG arrangement of the energy wall follow the same trend as the GE arrangement. The $\Delta\phi_w$ data decreases with the increase in pipe spacing, as seen in Fig. 13. Again, this is because the real behaviour is not perfectly planar, as seen in the isothermal contours extracted from numerical models (Fig. 13). The curved contours are more significant for the interface adjacent to the pipes than on the opposite interface, which is further away from the heat source. Therefore, in the short term, the non-dimensional temperature difference

between the numerical model and the FIS method is almost the same at all pipe spacings.

While the maximum difference between the ground-wall interface temperature predicted by the FIS method compared with the numerical model has increased due to smaller pipe spacings than the standard model, those increases are small both in non-dimensional and absolute terms. The absolute temperature differences for GE and GG geometries are given at the end of the one-year analysis period in Figs. 14 and 15, respectively. This is the time when the largest absolute temperature differences are seen for the standard model. It can be seen that for

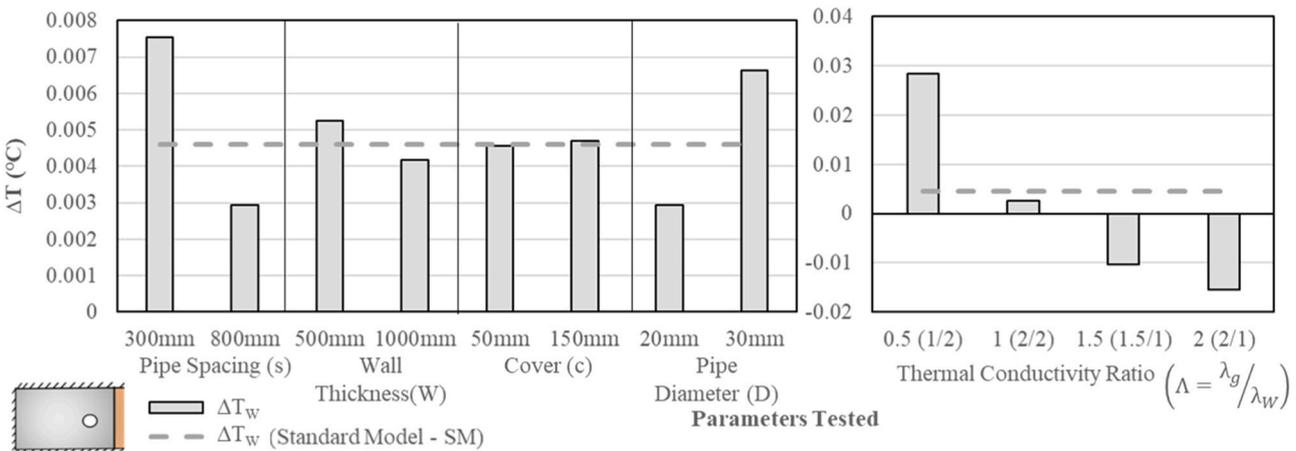


Fig. 14. Temperature differences between numerical and FIS Method (ΔT_w) at the ground-wall interface for GE energy wall, compared against the standard models - SM (dotted lines) after a year.

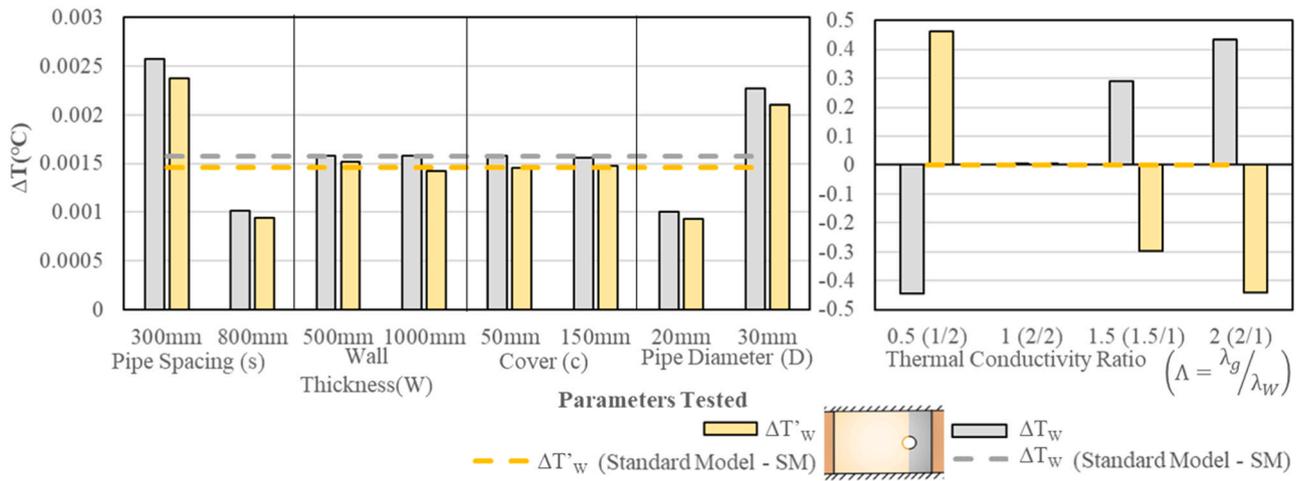


Fig. 15. Temperature differences between numerical and FIS Methods for GG-type energy wall at the wall surface adjacent to pipe (ΔT_w) and further away from pipe ($\Delta T'_w$), compared against the standard model (dotted lines) at the end of the year.

different pipe spacings, the maximum error is always within 0.008°C (for GE, Fig. 14) and is, therefore well within acceptable limits.

4.4. Effect of wall thickness (W), concrete cover (c) and pipe diameter (D)

The thickness of the wall is a parameter that an engineer cannot freely manipulate in the thermal design process as it is significantly dependent on the structural loads and geotechnical properties. Since the wall thickness is the characteristic length (Eqs. 8 and 9) for the EWs, the Fourier numbers 0.1 and 10 represent different absolute time values in each case. $F_0 = 0.1$ corresponds to 0.5 days and 2 days for EW of the wall thickness of 500 mm and 1000 mm, respectively. Meanwhile, $F_0 = 10$ is equivalent to 46 days and 185 days for EW of the wall thickness of 500 mm and 1000 mm, respectively. With an increase in the wall thickness, $\Delta\phi_w$ decreases for both GE and GG cases (as seen in Figs. 12 and 13). This is because increasing the thickness of the energy wall increases x_2 and x_3 (as seen in Figs. 2 and 3), thereby increasing the distance of the observation plane from the heat source. However, these changes are relatively small. Although in non-dimensional terms, thinner walls can have larger model errors up to $\Delta\phi_w = 0.013$ later in the analysis period, in absolute terms, the maximum temperature difference is still less than 0.006°C in all cases (as seen in Figs. 14 and 15).

Concrete cover (c) refers to the concrete thickness provided to the heat exchanger pipes that are embedded within the energy walls. The cover (c) and pipe diameter (D) do not contribute to non-dimensionalisation. Therefore, the absolute times for $F_0 = 0.1$ and $F_0 = 10$ remain the same as for the different pipe spacings, being 1.2 days and 118.5 days, respectively. When considering the different concrete cover values, these do change the value x_1 , but the amounts of this change are very small, and therefore, the results in terms of non-dimensional and absolute temperature differences are considered non-significant for overall behaviour.

Meanwhile, in the case of pipe diameters (D), when this is decreased, the magnitude of heat flux applied to the pipe boundaries (q) in the numerical model increases (refer to Eq. 1). There is also a very small increase in the end-to-end spacing between the pipes. Moreover, since the cover is fixed, in the GE arrangement, decreasing the diameter moves the heat plane slightly towards the ground wall interface. Together, these changes lead to an overall increase in $\Delta\phi_w$ (and also ΔT_w), as seen in Fig. 12. On the other hand, in the GG arrangement, decreasing the pipe diameter moves the plane of heat towards the interface adjacent to the pipes and shifts it away from the opposite interface. This leads to an increase in $\Delta\phi_w$ (also ΔT_w) with the increase in pipe diameter, as shown in Fig. 13. However, again, these changes are

very small, and the model errors due to changes in pipe diameter are smaller than those due to changes in pipe spacing (as seen in Figs. 14 and 15).

4.5. Effect of thermal conductivity ratio (Λ)

In general, the thermal properties of the energy wall and the ground inevitably differ in most cases. However, when the IPS is used, it can only account for one thermal conductivity. This is why, in the standard model, ground and wall thermal conductivity are kept equal at 1 W/m/K. Therefore, the thermal conductivity of the wall and the ground and their ratio are important for determining the performance of the FIS method. The thermal conductivity ratio (Λ) is defined as the ratio of the thermal conductivity of the ground (λ_g) to the thermal conductivity of the wall or concrete (λ_c):

$$\Lambda = \frac{\lambda_g}{\lambda_c} \quad (10)$$

Two cases are presented where $\Lambda = 1$, but with different absolute thermal conductivity values. Due to the role of thermal conductivity and applied heat flux in the non-dimensionalisation process, these will not show different performances in Figs. 12 and 13, and hence only one case is shown. However, there is a small difference in absolute temperature since higher conductivities will always lead to lower absolute temperature changes and, hence, lower differences (Figs. 14 and 15). Again, this is not shown for simplicity.

In the cases when $\Lambda \neq 1$, since the specific heat is unchanged in the sensitivity analysis, the movement of heat will be faster in the material with a higher thermal conductivity due to the materials' thermal diffusivity also increasing. In the case of GG energy walls, two interfaces are available for heat exchange with the ground, and pipes at which thermal load is applied are much closer to one of the interfaces (Fig. 3). In the current thermal loading, heat is injected into the ground. So, the heat is applied to the pipe boundaries, moving from the wall to the ground. When $\Lambda < 1$, the movement of heat is comparatively higher within the concrete; initially, the heat would try to escape from the interface closer to the pipe; however, due to lower conductivity and diffusivity, heat flow would then become more significant towards the opposite interface. When these temperature changes are compared with the models whose $\Lambda = 1$, the results show slightly lower temperatures at the interface adjacent to the pipes and higher temperatures at the opposite interface. As the FIS method can not accommodate different thermal properties, it ultimately underestimates the temperature changes at the adjacent interface, giving a negative $\Delta\phi'_w$, and overestimates the temperature changes on the opposite interface, as seen in

Fig. 13.

On the other hand, when $\Lambda > 1$, the thermal conductivity of the ground is higher. The behaviour observed in this case is opposite to that of $\Lambda < 1$. In this scenario, the movement of heat within the ground is much higher. Since $x_2 > x_1$, the wall heats up more in the direction of x_2 than in x_1 , essentially showing the opposite behaviour to that of the previous case. Therefore, an increased and positive $\Delta\phi_w$ is obtained in the direction of x_1 . Whereas for its opposite interface, for x_2 , since less heat moves through a heated central region, due to which temperature at the interface is lower than its counterpart at $\Lambda = 1$, a negative $\Delta\phi_w$ is obtained, as seen in Fig. 13.

In the GE arrangement of the energy walls, there is one interface from where the heat can be transferred to the ground (Fig. 2). When $\Lambda < 1$, at the initial stages, at $F_0 = 0.1$, when the heat load is applied, due to the lower conductivity and diffusivity of the ground, heat transfer within the wall is dominant. This means that the interface temperature is lower than predicted by the FIS assuming one material. Therefore, a negative $\Delta\phi_w$ is obtained (Fig. 12). However, the heat accumulates at the interface with time, increasing the temperature, and when $F_0 = 10$, there is a positive value of $\Delta\phi_w$. The opposite behaviour is shown by the wall when $\Lambda < 1$, as the higher diffusivity of the soil leads to faster dissipation and, therefore, a lower interface temperature.

Generally, changes to the thermal conductivity ratio have the largest impact on the overall performance of the FIS, compared with all other parameters in the sensitivity analysis. The greater the divergence of the two thermal conductivities and hence the larger the ratio when $\Lambda > 1$ (or the smaller when $\Lambda < 1$), then the greater the model errors. Compared with the standard model, where absolute temperature differences between the FIS and the numerical model were limited to less than 0.005°C , these can now reach 0.5°C in extreme cases. While it is acknowledged that these values could also be larger when greater heat flux magnitudes are applied, this is generally still within an acceptable range for routine application by analytical means. This aspect is further discussed in Section 4.6. below.

4.6. Model application and its limitations

The development of a precise approach for applying the IPS model to predict the temperature change on the back of EWs is significant, as it is the first step to developing a fully analytical tool for the thermal design of these energy geostructures. The approach can already be used to make observations about design choices. For example, the IPS deals in a planar heat flux, and the relationship between that and the heat flux at any pipe is given in Eq. 1. It can be seen that the same planar flux can be extracted through different spacing; however, the thermal load on individual pipes increases with the increase in pipe spacing. In reverse, if pipes have reached their maximum flux capacity and the area is limited, the performance can be increased by decreasing the pipe spacing and vice versa. This is one of the similar conclusions given by Di Donna et al.,²² Makasis and Narsilio¹⁸ and Sterpi et al.³⁹ that an increase in thermal performance can be achieved by reducing the pipe spacing. It can also be understood from a perspective that if a sufficient area is available, pipes could be spread more widely to thermally activate a larger area, thereby inducing lower planar heat flux and lesser load on the pipes while still achieving the thermal demands.

Of course, the approach derived so far comes with limitations. The final design objective is to determine the fluid temperature. Consequently, the back-of-wall temperatures calculated by the FIS method will require pairing with an additional model for the thermal performance of the wall itself. Such models are under development and are the next step for design method development (e.g.^{40–42}). For the specific part of an EW that is embedded (GG Energy Wall), it is seen that there are two independent routes to calculating the wall-ground interface temperature and that these values may not be the same (see Figs. 7 and 11). When determining the fluid temperature, since the FIS approach assumes equal splitting of the flux, it may be possible to use whichever of

Eq. 6 or Eq. 7 is most appropriate in each specific situation.

The absolute temperature differences presented in this study all fall within an acceptable range, considering a constant planar thermal load of 6.28 W/m^2 . Since typical energy wall heat transfer rates can be higher than this, with typically up to $20\text{--}40 \text{ W/m}^2$ in operation,⁴³ maximum absolute errors may reach a few degrees. However, in reality, heat transfer rates will not be constant at these values. In practice, the thermal loads applied to EWs will vary with time depending on the energy demand of the building as well as the season of operation. While not tested explicitly in this study, since the FIS Method is an application of a solution to the diffusion equation, and this is linear, then the response to variations in thermal demand can be found through temporal superposition, as is done for many other types of ground heat exchangers (e.g.^{44,45}) and is routinely implemented in ground heat exchanger design software. Consequently, application of the FIS for specific cases should be evaluated in terms of expected thermal demand and factors such as the conductivity ratio which will influence its accuracy. However, in most typical operating scenarios and conditions, typical errors are expected to be acceptable.

The other most notable limitation of the work presented in this study is that verifying the FIS method of IPS deployment has been carried out with respect to EW simulations, where the wall is effectively of infinite length. In practice, EWs are finite, and their length may vary from a few tens of metres to hundreds of metres and their depth to a few tens of metres. From experience with borehole heat exchangers and energy piles, it is known that the finite depth of ground heat exchangers will limit the temperature response under constant thermal load or periodic loads in the long term.^{20,46,47} Therefore, under long-term constant flux conditions, large overestimations of temperature will occur by analytical models assuming an infinite heat exchanger. However, it is also known that the fluctuating nature of real thermal demands in buildings with balanced heating and cooling demands over the lifetime of ground heat exchangers will also act to minimise that effect. This is because, when temporal superposition⁴⁴ is applied, the time-step at which the thermal demand changes becomes important as well as the total time elapsed. Consequently, the actual temperature change that occurs is limited, and therefore, two-dimensional models have been shown to be effective for the analysis of energy piles¹³ and walls.³² For these reasons, while additional errors in temperature prediction should be expected for particularly short or shallow walls, the FIS approach is still expected to be useful in practice. Further work is also required to quantify these effects, with initial assessments presented in Gupta.³¹

Finally, it is acknowledged that the approach presented relies upon the assumption of homogeneous and isotropic ground conditions in the absence of groundwater advection. These assumptions are often made for the design of ground heat exchangers to allow for fast computation times, despite not always being appropriate. Therefore, if heat transfer conditions are expected to be especially heterogeneous, e.g. due to the presence of fast groundwater movement, then alternative analysis approaches would be required.

5. Conclusions

This study applies the Infinite Plane Source (IPS) as an analytical model for the thermal design of energy walls. A new Flux and IPS Splitting (FIS) Method offers an analytically precise approach to IPS implementation. Splitting heat flux into two accommodates the flow of heat on both sides of the pipes. Two energy wall arrangements are discussed: the exposed portion of the energy wall with excavation on one side and ground on the other (GE) and the fully embedded portion of the energy wall with the ground on both sides of the wall (GG).

Comparing the performance of the FIS method to the results of two-dimensional numerical modelling, it can be concluded that the FIS method is an excellent match for numerical results for a range of different wall thicknesses, pipe spacing, pipe covers, and pipe diameters in predicting energy wall thermal responses for both GE and GG

arrangements with Fourier Numbers in the range 0.1 – 10. Because the IPS, which underpins the FIS method, assumes only one material, where the ratio of thermal conductivity of the soil and wall deviates from unity, agreement with numerical results is reduced but remains within an acceptable range. Differences are slightly more for the GG case than the GE case. Absolute temperature errors will also depend on the magnitude of the conductivity and flux applied in specific cases.

In reality, energy walls are not of infinite length, so further work is required to confirm whether discrepancies in three dimensions are significant and whether the impact of fluctuating thermal demand in operation may at least partially offset them, as is seen for other ground heat exchangers.

Overall, it can be concluded that the IPS as an analytical model is a viable approach for the development of methods for the design and analysis of the energy walls, and splitting the flux applied to the IPS has increased the accuracy of the approach significantly.

CRedit authorship contribution statement

Aakash Gupta: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Methodology, Investigation, Formal analysis, Conceptualization. **Fleur Loveridge:** Writing – review & editing, Visualization, Supervision, Resources, Methodology, Conceptualization. **Ida Shafagh:** Writing – review & editing, Visualization, Supervision, Methodology, Conceptualization. **Simon J Rees:** Writing – review & editing, Supervision, Conceptualization.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

Key data associated with this publication are openly accessible through the University of Leeds Research Data Repository ⁴⁸.

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