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Coordinating transfers between bus and metro train in real time: A logic-based branch-and-bound method

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Abstract

The concept of Mobility-as-a-Service and on-demand public transit increasingly depends on integrating multiple transportation modes to offer seamless door-to-door travels. However, coordinating these modes in real time to address fluctuating demand and disturbances remains a major technical challenge. We tackle this issue by formulating a mixed-integer quadratic programming model to generate coordinated bus and train adjustment strategies. The model captures key factors such as traffic dynamics, passenger loads, and vehicle overtaking. To address the computational challenge of the mixed-integer property, we incorporate logic-based concepts into a branch-and-bound framework, analyzing the logical relationships among variables to improve solution efficiency. This logic-based branch-and-bound method exploits the distinct strengths of discrete and continuous components. It employs logical inference to guide the search, incorporates domain reduction to accelerate computation, and constructs reduced continuous optimization problems to efficiently update bounds and estimate logical values. Computational results demonstrate that the proposed coordinated adjustment strategy effectively improves vehicle punctuality and headway regularity, while reducing the number of stranded non-transfer and transfer passengers. The solution method demonstrates desirable computational efficiency in real-world settings, which is suitable for real-time applications.

Keywords: Multi-modal public transport network; Transfer coordination; Mixed-integer non-linear programming; Logic-based branch-and-bound

1. Introduction

1.1. Background and motivations

With increasing urbanization and the expansion of public transport networks, multimodal travel has become a common part of daily life. Coordinating different modes, such as bus and metro systems to ensure smooth and convenient passenger transfers is becoming increasingly important (Cheng and Tseng, 2016; Liu et al., 2021). Well-coordinated transfers can significantly reduce waiting times and missed connections, thereby enhancing the overall user experience and improving the attractiveness of public transportation.

Traditional bus and metro coordination approaches typically rely on static and pre-planned schedules to manage transfers (Dou et al., 2015; Takamatsu and Taguchi, 2020; Zheng et al., 2025). While effective under stable conditions, such methods are unable to well adapt to dynamic operating environments. Frequent real-world disturbances, including adverse weather, driver behaviors, or large events, often cause demand surges

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or vehicle delays, rendering even well-designed schedules less effective and resulting in long waiting times and poor transfer experiences for passengers. In contrast, real-time coordination can flexibly adjust vehicle operations based on continuously updated data, enabling better adaptation to dynamic conditions (Daganzo and Anderson, 2016; Liu et al., 2021). This can mitigate delays, improve punctuality and reliability, enhance intermodal coordination, and ultimately elevate overall operational efficiency and service quality. The real-time coordination problem (RTCP) involves dynamically adjusting vehicle operations (e.g., timetables) to ensure stable and efficient transfers across urban transit networks in real time. The availability of real-time data from vehicle sensors and automatic fare collection systems, together with improved communication between drivers and control centers, further supports advancements in this area.

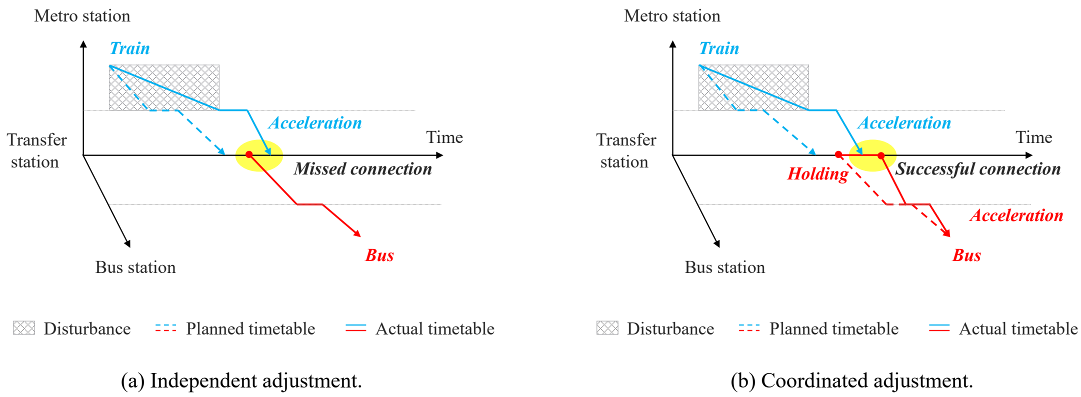


Figure 1: Comparison of the independent and coordinated adjustment strategies.

To further illustrate the practical implications of RTCP, Figure 1 compares independent and coordinated timetable adjustments, with a transfer hub linking two lines. A disturbance causes a train (blue solid lines) to deviate from its planned timetable (blue dotted lines), delaying its arrival at the transfer hub. As a result, transfer passengers from the train miss the connecting bus (red lines) and must wait for the next one. In the case of independent adjustments (Figure 1 (a)), the train accelerates to recover from the delay, but the bus continues as scheduled. While this reduces the train delay, it fails to restore the original transfer connection between the train and bus, making effective transfer coordination difficult. In contrast, the coordinated adjustment scheme (Figure 1 (b)) not only reduces train delays by accelerating the train but also successfully connect two vehicles by holding the bus. Besides, after leaving the hub, the bus accelerates to reduce the impact of holding. This coordinated adjustment approach effectively restores transfer connections in disturbed environments. Notably, Figure 1 is a simplified illustration to show the coordination mechanism at the transfer hub, where both bus stations before the transfer station, and metro stations after it are omitted. In this study, the transfer station can be an intermediate station on either the bus or the metro line.

Recent advances in Mobility-as-a-Service (MaaS), on-demand public transport, and vehicle automation further highlight the practical importance and feasibility of RTCP. Specifically, MaaS platforms integrate various transport modes into a platform, allowing passengers to book combined tickets for all their trip legs. The detailed door-to-door travel information creates opportunities to coordinate public transport services (Gkiotsalitis, 2022). On-demand public transport operates dynamically based on real-time user requests, creating variable transfer patterns with other services. This variability makes real-time coordination essential to ensure smooth and efficient transfers (Steiner and Irnich, 2020; Vansteenwegen et al., 2022). Besides, advances in sensors, communication technologies, automatic fare collection systems, and computational capabilities allow for the monitoring and control of public transport systems in real time (Elassy et al., 2024; Nie et al., 2024), thereby enabling the implementation of real-time transfer coordination. However, addressing RTCP involves significant challenges: Modeling transfer connections inevitably introduces binary variables, thus leading to the computationally challenging mixed-integer property. Additionally, the real-time nature of RTCP demands high computational efficiency to quickly adapt to changing conditions. In this work, we propose a mixed-integer quadratic programming (MIQP) model for RTCP. By analyzing the model properties, we find that the logical relationships among discrete components help in reducing the search space, while continuous components facilitate the construction of optimization problems that can be solved quickly. By rethinking the roles of discrete and continuous components in the solution process, we incorporate logic-based ideas into the branch-and-bound framework, to develop a logic-based branch-and-bound (LBB) method for solving RTCP. LBB effectively separates discrete and continuous components, leveraging their unique strengths for faster computation, enabling efficient real-time coordination.

1.2. Literature review

We review three areas of literature relevant to this study: (1) bus and metro service coordination, (2) real-time timetable adjustment methods, and (3) solution approaches for mixed-integer optimization problems.

1.2.1. Bus and metro service coordination.

Increasing studies have incorporated the impact of transfer demands into the formulation of vehicle operation strategies (e.g., timetables) from strategic or tactical perspectives, with the aim of enhancing transfer coordination (involving reducing passenger transfer waiting times or failed transfers) within urban public transport systems. For example, at the strategic level, Wong et al. (2008) optimized train timetables in metro networks to reduce transfer waiting times, Yuan et al. (2023) optimized train timetables and stopping patterns for metro networks to reduce waiting times, while Petersen et al. (2013) combined timetabling and vehicle dispatching to balance reduced waiting times with operational costs. For bus and metro systems, Dou et al. (2015) adjusted bus timetables to minimize missed transfers between buses and trains, and Takamatsu and Taguchi (2020) tackled similar issues between infrequent bus and train services, to shorten transfer waiting times. Zheng et al. (2025) focused on bus timetable optimization to reduce passenger transfer waiting time between metro and bus systems. At the operational level, researchers have explored the delay management problem (DMP), which involves deciding whether a vehicle should wait for a delayed feeder vehicle or depart on time at a transfer station. Schöbel (2001) modeled this as a mixed-integer linear program (MILP) using the event-activity network, aiming to minimize missed connections and delays. Subsequent formulations and improvements were presented by Ginkel and Schöbel (2007), while Schöbel (2009) further considered the limited track capacity in the DMP. Another extension of DMP allowed passenger rerouting to reach their destinations during delays (Dollevoet et al., 2011; Schmidt, 2013), and Dal Sasso et al. (2019) introduced advanced MILP models with valid inequalities to enhance DMP.

Despite growing interest in transfer coordination, the existing works remain limited in several critical aspects: (a) Most existing studies on real-time transfer coordination focus on a single-mode system, typically either metro or bus, and only address internal coordination within that system (Wong et al., 2008; Fouilhoux et al., 2016; Hu et al., 2022; Ansarilari et al., 2024). These works only consider decisions within a single system and do not consider cross-mode transfer coordination. (b) A few studies address bus-metro transfer coordination (Dou et al., 2015; Dal Sasso et al., 2019; Takamatsu and Taguchi, 2020; Zheng et al., 2025), but always ignore key practical considerations, such as vehicle overtaking, limited vehicle capacity, and passenger loading dynamics. Among them, Dou et al. (2015); Takamatsu and Taguchi (2020); Zheng et al. (2025) operate at a tactical level, focusing on schedule design rather than real-time control. They only adjust bus operations while keeping train operations fixed, without involving integrated optimization. Besides, Dal Sasso et al. (2019) consider real-time adjustments but only optimize the dwell time of connecting vehicles at transfer stations, without adjusting the feeder vehicle operations. In contrast, our contribution lies in the following aspects. (a) Compared to approaches that only adjust operations within a single system, we propose a framework that jointly optimizes running times and dwell times for both buses and metro trains at transfer and non-transfer stations. This allows for more comprehensive and effective coordination across modes, enhancing overall service reliability. (b) Our model incorporates more crucial real-world operational factors: it explicitly accounts for bus overtaking, passenger loading and limited vehicle capacity. Including these practical considerations enables the model to produce control strategies adapting to more realistic operating conditions, ultimately leading to more efficient coordination strategies.

1.2.2. Real-time timetable adjustments for bus and metro.

Given the role of real-time timetable adjustments in enhancing transfer coordination, we conduct a literature review on this topic, particularly in the context of bus and metro operations. Real-time timetable adjustments for buses and trains, mainly by adjusting vehicle running and dwell times based on dynamically updated disturbance and other relevant information, can effectively mitigate the impact of disturbances on vehicle operations. Numerous researchers have explored this field from the perspective of transit lines or networks. For bus systems under disturbances, Eberlein et al. (2001) optimized bus holding times at stations in a rolling horizon framework to minimize passenger waiting times for loop lines. Cao et al. (2019) investigated real-time autonomous bus adjustment problems. Their holding and speed-changing strategies effectively reduce timetable deviations for bus lines. He et al. (2022) used Q-learning to address the dynamic bus holding problem for single lines in real time. By considering bus overtaking, Seman et al. (2022) optimized bus holding strategies for single-line bus systems, to minimize headway deviation and passenger waiting times. Considering transfer passenger flows between lines, Dessouky et al. (2003) examined real-time bus control for networks, evaluating with average passenger travel times and total delays. Hadas and Ceder (2010) developed optimization models for bus holding and skip-stop patterns using real-time data to reduce transfer times. Daganzo and Anderson (2016) also proposed a dynamic holding strategy for transfer coordination based on control theory. Manasra and Toledo (2019) optimized bus holding and inter-stop control strategies for bus service with transfers, to minimize the passenger waiting times. For metro systems, Van Breusegem et al. (1991) studied real-time adjustments for metro lines, modifying dwell and running times to minimize deviations in departures and headways. Chen et al. (2024) designed transfer-coordinated train adjustment schemes in response to frequent disturbances, reducing timetable deviation and stranded passengers.

The existing literature on real-time timetable adjustments has the following limitations: (a) Most existing studies investigate dwell time and running time adjustments for single-mode systems and do not account for decisions involving intermodal transfers (Van Breusegem et al., 1991; Eberlein et al., 2001; Cao et al., 2019). (b) Compared to studies on transfer coordination, single-system bus or metro adjustment literature indeed considers a broader range of operational decisions, such as passenger loading and limited capacity. However, to the best of our knowledge, despite its significant impact on modeling transfer connections, bus overtaking has not been addressed at the network level in existing bus adjustment research (Wu et al., 2017; Seman et al., 2022). In contrast, our study extends the scope of real-time dwell and running time adjustments to a multi-modal setting, jointly optimizing the operations of both bus and metro systems. Our model explicitly incorporates bus overtaking and captures its influence on modeling transfer connections, thereby addressing an underexplored aspect in the existing literature.

In the existing literature on real-time timetable adjustments, it is common to use dwell and running time adjustments as the primary decision variables. Consistent with most prior studies (Daganzo and Pilachowski, 2011; Moaveni and Najafi, 2018; Manasra and Toledo, 2019; Bian et al., 2020; Ma et al., 2021; Seman et al., 2022), our model also focuses on adjusting dwell and running times, and we extend the decision scope to include stations and segments across both bus and metro networks. Regarding metro train adjustments, our model is mainly inspired by Chen et al. (2024), which addresses real-time train adjustment problems within metro networks. We adopt a similar train transfer connection modeling framework and follow their objective function design that minimizes departure deviations, headway deviations, and the number of stranded passengers. For bus adjustments, we build upon the work of Seman et al. (2022), who models bus adjustments along a single line while allowing for vehicle overtaking. We extend their model by incorporating overtaking into a multimodal, network-wide coordination framework that integrates both metro and bus systems. Notably, the introduction of bus overtaking leads to variable vehicle departure sequences, which introduces new challenges for modeling transfer connections. To the best of our knowledge, this aspect has not been explored in previous literature. In summary, based on these two most relevant studies, we integrate metro and bus adjustments into a unified optimization framework for real-time transfer coordination. Moreover, we address the modeling challenges related to transfer connections both between different modes and within the same mode under conditions allowing bus overtaking, which is typically not covered in existing works.

1.2.3. Solution methods tailored to mixed-integer properties.

The RTCP discussed in this paper is a computationally challenging optimization problem with discrete components for transfer connections and overtaking activities, along with continuous components for traffic operations and passenger loading. This mix of discrete and continuous components leads to the well-known difficulties of mixed-integer problems. The real-time demands of RTCP further intensify these challenges, requiring highly efficient solution methods. To address this, the branch-and-bound (BB) method is recognized as an effective framework (Lawler and Wood, 1966; Demeulemeester and Herroelen, 1992; Zhang et al., 2024). BB employs a tree search-based concept to implicitly enumerate potential solutions while utilizing pruning rules to eliminate less promising areas of the search space. Researchers have successfully applied the BB method to various challenges, such as robust schedule coordination for transit networks (Wu et al., 2016), real-time train scheduling for railway networks (D'Ariano et al., 2007), and train platoon scheduling for metro networks (Chai et al., 2024). Additionally, integrating logic-based ideas into the solution process has proven effective for optimizing problems with both discrete and continuous components (Hooker, 1994; Hooker and Osorio, 1999). These logic-based ideas leverage the relationships among discrete variables (also called logical variables in logic-based methods), to accelerate the solution process. They have been applied to various optimization problems, including the optimal structural design (Bollapragada et al., 2001), optimal control for hybrid systems (Bemporad and Giorgetti, 2006), and synthesis and optimization of refrigeration systems (Matovu et al., 2022). Building on previous studies, we apply the BB scheme to separate the discrete and continuous components of RTCP. By incorporating logic-based ideas, we develop a logic procedure within the BB framework that leverages logical relationships to guide the search and implement domain reduction, accelerating the process. Additionally, we exploit the optimization's role in efficiently solving continuous programs, to promptly provide logic estimates and update upper and lower bounds.

Totally, existing research on transfer coordination between buses and trains at the operational level often uses simplified models that either prohibit overtaking, ignore passenger demand, or assume unlimited capacity. These simplifications, though intended to balance computational complexity with real-time requirements, inevitably limit the effectiveness and practicality of adjustment methods. First, restricting overtaking, delayed vehicles can cause a chain reaction, further delaying subsequent vehicles (Wu et al., 2017). This can lead to bus bunching at transfer stations, negatively impacting passenger transfer experience. In contrast, allowing overtaking provides vehicles with greater flexibility, enabling better coordination improvements. Second, to maximize the overall system performance, it is essential to prioritize transfer connections with higher passenger demand. This is because coordinating one transfer connection may negatively affect other connections and non-transfer passengers. Ignoring actual demand can result in under-coordination of high-demand connections, which impacts the travel experience of primary transfer passengers and potentially harm the interests of non-transfer passengers. Ultimately, this would reduce the overall system efficiency (Liu et al., 2021). Besides, unlimited capacity implies that as long as the connecting vehicle departs after transfer passengers arrive at the platform, all passengers can board. However, during peak hours, limited capacity often prevents some transfer passengers from boarding, causing delays and overcrowding. While this is fully explored in metro and bus systems (Niu and Tian, 2013; Wu et al., 2021; He et al., 2022), it is often neglected in more complex multimodal models, leading to suboptimal solutions during peak times. In contrast, this paper makes significant contributions by addressing these challenges with a more accurate model and an effective solution method, i.e.,

- We formulate a more realistic and detailed modeling framework for RTCP, by integrating vehicle traffic dynamics, overtaking activities, passenger loads, transfer flows, and capacity limits, to generate the coordinated bus and train timetable adjustment strategies.
- We develop a LBB method incorporating the logic-based idea to separate and fully exploit the distinct strengths of discrete and continuous components, thus effectively solving RTCP with the mixed-integer nature and facilitating real-time and embedded applications in MaaS.

The remainder of this paper is organized as: Section 2 describes the real-time coordination problem. Section 3 constructs a mathematical model for RTCP. Section 4 states an efficient LBB method. Section 5 presents computational experiments to assess the performance of the proposed model and LBB. Finally, Section 6 offers conclusions.

2. Problem statement

In this paper, we focus on an urban public transport network with multiple bus and metro lines connected at transfer hubs. Under ideal conditions, buses and trains operate according to their planned schedules, which specify arrival and departure times at each station. As scheduled, vehicles arrive at each station, allow passengers to board and alight during the dwell time, and then depart to travel to the next station. However, bus and train operations are always subject to disturbances in practice. We consider disturbances on dwell times and on running times. The disturbances can bring increased dwell times and extended running times (the travel time between adjacent stations). They may be caused by factors such as extreme weather, signal failures, and can result in actual vehicle departure and arrival times deviating from planned schedules.

We consider passenger load dynamics in the urban public transport network, involving passenger arriving, waiting, boarding, alighting, and transferring. Specifically, passengers begin waiting after arriving at stations, then board vehicles subject to capacity constraints. At each station, partial onboard passengers alight and exit the public transport system, while partial passengers alight to transfer to other lines. These transfer passengers need to walk to the platforms of their connecting lines and wait for connecting vehicles. In practice, the passenger demand information is often estimated and forecast using historical smart card and AFC systems through short-term forecasting or other advanced methods (Yap et al., 2018; Nagaraj et al., 2022). It can also be dynamically updated using real-time APC (automatic passenger count) data when available. These obtained data provides detailed travel information such as stop locations and times, enabling the estimation of origin-destination matrices. From these matrices, arrival rates and boarding/alighting counts at each station are calculated, allowing us to determine alighting ratios.

In the context of daily operations with minor disturbances, existing literature commonly considers dwell and running time adjustments as the primary and effective real-time control strategies (Lin and Sheu, 2011; Moaveni and Najafi, 2018; Manasra and Toledo, 2019; Ma et al., 2021; He et al., 2022). Dwell time adjustments involve extending or shortening station stops within operational bounds. In practice, vehicles can hold at stations to maintain headway regularity or depart immediately after passenger boarding and alighting to mitigate delays. This control strategy has been widely validated in bus and metro operations. Running time adjustments involve extending or shortening travel times between stations within safe limits. Metro systems implement this through different speed profiles, In bus operations, there is also typically operational flexibility in adjusting driving speeds within safe limits on uncongested segments. Drivers can slightly increase cruising speed to recover from minor delays. This practice has been observed and investigated in the real-time bus control area (Daganzo and Pilachowski, 2011; Bian et al., 2020). These two strategies are widely adopted due to their practicality, ease of implementation, and less impact on passengers. Therefore, we adopt these two strategies to adjust bus and metro operations. Besides, bus overtaking is allowed, offering more flexibility in bus movements. For example, if a bus experiences severe delays, the subsequent bus can overtake, preventing buses from being constrained by the disturbances of other buses. These adjustments apply not only at the transfer hub but also at non-hub stations within the network, accommodating the needs of non-transfer passengers as well.

We consider the following objective function terms: (1) departure deviation, (2) headway deviation, and (3) the number of stranded passengers. The departure and headway deviations indicate the difference between actual and scheduled departure times and headways, respectively. Minimizing these deviations helps reduce delay and maintain regular service intervals. We employ squared deviation terms, a widely used formulation in timetable adjustment studies (Van Breusegem et al., 1991; Seman et al., 2022; Jin et al., 2022). It can penalize large individual delays and prevent extreme deviations of a vehicle at a station. Stranded passengers are those who cannot board their intended vehicle and need to wait for the next one to continue their journey, including both non-transfer and transfer passengers. This is also commonly adopted in the literature to reflect service quality (Jiang et al., 2019; Hou et al., 2019; Chen et al., 2024; Wang et al., 2024). In this work, we formulate the objective function as a weighted sum to handle multiple conflicting objectives. Since different objective terms are measured on different scales, we set weights to normalize them accordingly (Samà et al., 2016). This normalization can be achieved by estimating the maximum value that each term can take (Altazin et al., 2017). Beyond normalization, the weights also reflect practical trade-offs between objectives. For instance, if system stability is prioritized, weights for departure and headway deviations can be set higher. Conversely, if service quality is prioritized, the weights related to the number of stranded passengers can be set higher.

To facilitate modeling and analysis, we consider the following main assumptions: First, the transfer time is assumed to be known, representing the duration for passengers to exit the feeder vehicle and walk to the boarding area of the transfer station for the connecting vehicle. This can be estimated from the passenger walking speed and the length of the transfer channel. Second, overtaking is not allowed for metro trains due to safety concerns and the limited track infrastructure. However, overtaking is allowed for buses, as roadways provide multiple lanes. Besides, infrastructure and regulations can accommodate such maneuvers. Third, we assume passengers arrive with a known passenger arrival rate during the vehicle headway. Actually, the use of arrival rates implicitly assumes a Poisson arrival process for passengers with a constant average rate, where the given parameter represents the average passenger arrival intensity over each headway. Fourth, buses and trains have different door configurations. Metro passengers alight before boarding, while bus passengers board and alight simultaneously where they typically board at the front and exit through the rear door.

To adapt to dynamic environments, we adopt a real-time decision-making mechanism implemented through rolling horizon (RH). It divides the entire study horizon into multiple time windows, known as decision stages or stages. Each decision stage involves two critical time intervals: the prediction horizon (PH) and the control horizon (CH). At each decision stage, we formulate an optimization problem over PH of a longer time horizon to generate adjustment solutions for vehicles involved in PH. Then, only the solutions within the CH of a shorter time horizon are implemented, while those beyond the CH but within the PH are discarded. The passenger demand and disturbance information are treated as fixed and known inputs within each prediction horizon, but they can be dynamically updated across decision stages. RH integrates long-term optimization to account for future trends, with short-term implementation to maintain flexibility, strongly supporting effective real-time decision making in dynamic environments. As new information on disturbances and passenger demand emerges, we roll forward the time window and progress to the next decision stage, updating the adjustment schemes.

3. Mathematical model

This section presents a mathematical model for RTCP. In Section 3.1, we define the notations and symbols used throughout the formulation. In Section 3.2, we describe the detailed mathematical formulation of the problem, including the objective function and constraints.

3.1. Notations

In this work, we formulate a mathematical model for RTCP at each decision stage under a rolling horizon scheme. To describe the topological structure of the network, we introduce sets $\mathcal{R}_{\text{metro}}$ and $\mathcal{R}_{\text{bus}}$ of metro and bus lines in the network, respectively. Each line $r \in \mathcal{R}_{\text{metro}} \cup \mathcal{R}_{\text{bus}}$ contains $|\mathcal{S}_r|$ stations. For each transfer station i on line r, $\mathcal{C}_{ri}$ represents the set of connecting bus and metro lines with line r at this station. To model vehicle operations at each decision stage k, we define the set $\mathcal{J}_{ri}^k$ to include vehicles that require decisions on dwell time adjustments at station i and running time adjustments at the segment between stations i and $i+1$ of line r for the stage.

For each vehicle j, the planned schedule specifies its departure time D_{ij}^r, dwell time T_i^r, and headway H_r at each station, as well as the running time L_i^r between consecutive stations. Under ideal conditions, vehicles would adhere to the schedule. However, in practice, train and bus operations are affected by disturbances W_{ij}^r and $\tilde{W}_{ij}^r$, which influence dwell and running times, respectively, and inevitably lead to deviations from the schedule. To mitigate these effects, we introduce adjustment decisions $g_{ij}^r \in [G_{\min}^r, G_{\max}^r]$ and $u_{ij}^r \in [U_{\min}^r, U_{\max}^r]$ for dwell and running times, respectively. Both adjustments are constrained within reasonable operational ranges. Here, $G_{\min}^r$, $G_{\max}^r$, $U_{\min}^r$, and $U_{\max}^r$ denote the corresponding minimum and maximum allowable adjustments. For operational safety, the headway between consecutive trains is greater than a minimum headway $H_{\min}^r$. The vehicle dwell time at each station should be long enough to ensure that passengers can safely board and alight, where we use B_{rj} to denote the time required per passenger for boarding or alighting.

To capture vehicle traffic dynamics, we define d_{ij}^r and a_{ij}^r as the actual departure and arrival times of vehicle j at station i on line r, respectively. Considering overtaking may change the bus departure order, we introduce binary variable $z_{j'j}^{ri}$ to indicate whether bus j' departs before bus j at station i on line r, and binary variable $x_{j'j}^{ri}$ to identify whether bus j' is the immediate predecessor of bus j. Based on them, we define $\hat{d}_{ij}^r$ as the actual departure time of the bus that departs immediately before bus j at station i on line r. This variable enables the determination of actual headways between vehicles with overtaking.

To model passenger flow and its interaction with vehicle dynamics, we assume passengers arrive at each station at a known rate λ_{ij}^r, and define p_{rij}^{wait} as the number of passengers waiting for vehicle j at station i on line r. Given the vehicle capacity limit V_r, only p_{rij}^{board} passengers can board, while the remaining passengers are stranded. After boarding, the vehicle carries p_{rij}^{load} passengers onboard. At each station, a proportion β_{ij}^r of onboard passengers alight. To model transfer passengers, we introduce binary variable $y_{rij}^{r'j'}$ to denote whether the arrival time of vehicle j' on line r' plus passenger transfer walking time $E_{ri}^{r'}$ is earlier than the departure time of vehicle j on line r. We use $\rho_{ri}^{r'j'}$ to represent the transfer ratio, and $q_{rij}^{r'j'}$ to denote the number of passengers transferring from vehicle j' on line r' who are waiting for vehicle j on line r at transfer station i.

With the above parameters and variables, we formulate corresponding system constraints related to vehicle dynamics, bus overtaking, passenger loading, and transfer flows, and a weighted-sum objective function to reduce departure deviation, headway deviation and stranded passengers. The notations used for RTCP modeling are listed in Table 1.

3.2. Formulation

Based on the notation introduced above, we formulate optimization model M1 for RTCP at each decision stage k under a rolling horizon scheme, as follows.

$$\textbf{[M1]} : \min \quad F^k = F_1^k + F_2^k + F_3^k \tag{1}$$

Objective function

$$F_1^k = \zeta_1 \sum_{r \in \mathcal{R}_{\text{bus}} \cup \mathcal{R}_{\text{metro}}, i \in \mathcal{S}_r, j \in \mathcal{J}_{ri}^k} (d_{ij}^r - D_{ij}^r)^2 \tag{2}$$

$$F_2^k = \zeta_2 \sum_{r \in \mathcal{R}_{\text{bus}}, i \in \mathcal{S}_r, j \in \mathcal{J}_{ri}^k} (d_{ij}^r - \hat{d}_{ij}^r - H_r)^2 + \sum_{r \in \mathcal{R}_{\text{metro}}, i \in \mathcal{S}_r, j \in \mathcal{J}_{ri}^k} \zeta_2 (d_{ij}^r - d_{i(j-1)}^r - H_r)^2 \tag{3}$$

$$F_3^k = \zeta_3 \sum_{r \in \mathcal{R}_{\text{bus}} \cup \mathcal{R}_{\text{metro}}, i \in \mathcal{S}_r, j \in \mathcal{J}_{ri}^k} (p_{rij}^{\text{wait}} - p_{rij}^{\text{board}}) \tag{4}$$

Table 1: Notations used in modeling RTCP.

Notations	Definition
Indices and sets	
k	The decision stage index.
$\mathcal{R}_{\text{bus}}/\mathcal{R}_{\text{metro}}$	Set of bus lines / metro lines.
$\mathcal{S}_r$	Set of stations on line r.
$\mathcal{Q}_r$	Set of transfer stations on line r.
$\mathcal{J}_{ri}^k$	Set of buses or trains involved at station i on line r at stage k.
$\mathcal{C}_{ri}$	Set of connecting lines with line r at transfer station i.
Parameters	
W_{ij}^r	Dwell time disturbance for vehicle j at station i on line r.
$\tilde{W}_{ij}^r$	Running time disturbance for vehicle j between stations i and $i+1$ on line r.
L_i^r	Planned running time between stations i and $i+1$ on line r.
T_i^r	Planned dwell time at station i on line r.
D_{ij}^r	Planned departure time for vehicle j at station i on line r.
H_r	Planned headway for line r.
S	Time required for vehicle door operations (opening and closing).
B_{rj}	Average boarding/alighting time per passenger for vehicle j of line r.
$H_{\min}^r$	Minimum train headway.
$U_{\min}^r/U_{\max}^r$	Minimum/maximum running time adjustments.
$G_{\min}^r/G_{\max}^r$	Minimum/maximum dwell time adjustments.
$E_{ri}^{r'}$	Transfer time at station i from line r' to r.
V_r	Vehicle capacity on line r.
λ_{ij}^r	Arrival rate at which passengers arrive at station i during the headway between departures of vehicles $j-1$ and j on line r.
β_{ij}^r	Proportion of passengers on vehicle j who alight at station i on line r.
$\rho_{ri}^{r'j'}$	Transfer ratio to line r among those from vehicle j' of line r' at station i.
Variables	
u_{ij}^r	Running time adjustment for vehicle j between stations $i-1$ and i on line r.
g_{ij}^r	Dwell time adjustment for vehicle j at station i on line r.
d_{ij}^r	Actual departure time of vehicle j from station i on line r.
$\hat{d}_{ij}^r$	Actual departure time of the bus immediately ahead of bus j from station i on line r.
a_{ij}^r	Actual arrival time of vehicle j at station i on line r.
$z_{j'j}^{ri}$	Binary variable, $=1$ if bus j departs after bus j' at station i on line r; $=0$ otherwise.
$x_{j'j}^{ri}$	Binary variable, $=1$ if bus j' is immediately ahead of bus j at station i on line r; $=0$ otherwise.
p_{rij}^{wait}	Number of waiting passengers for vehicle j at station i on line r.
p_{rij}^{board}	Number of passengers boarding vehicle j at station i on line r.
p_{rij}^{load}	Number of onboard passengers in vehicle j at station i on line r.
$q_{rij}^{r'j'}$	Number of transfer passengers from vehicle j' of line r' to vehicle j of line r at station i.
$y_{rij}^{r'j'}$	Binary variable, $=1$ if the arrival time of vehicle j' on line r' plus transfer time is earlier than the departure time of vehicle j at station i on line r; $=0$ otherwise.

Modeling vehicle traffic dynamics

$$d^r_{ij} = a^r_{ij} + T^r_i + W^r_{ij} + g^r_{ij}, \quad j \in \mathcal{J}^k_{ri}, i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{bus}} \cup \mathcal{R}_{\text{metro}} \tag{5}$$

$$a^r_{ij} = d^r_{(i-1)j} + L^r_{i-1} + \tilde{W}^r_{(i-1)j} + u^r_{(i-1)j}, \quad j \in \mathcal{J}^k_{ri}, i \in \mathcal{S}_r \setminus \{1\}, r \in \mathcal{R}_{\text{bus}} \cup \mathcal{R}_{\text{metro}} \tag{6}$$

$$d^r_{ij} - a^r_{ij} \geq S + B_{rj}(p^{\text{board}}_{rij} + \beta^r_{ij} p^{\text{load}}_{r(i-1)j}), \quad j \in \mathcal{J}^k_{ri}, i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{metro}} \tag{7}$$

$$d^r_{ij} - a^r_{ij} \geq S + B_{rj} \max\{p^{\text{board}}_{rij}, \beta^r_{ij} p^{\text{load}}_{r(i-1)j}\}, \quad j \in \mathcal{J}^k_{ri}, i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{bus}} \tag{8}$$

$$d^r_{ij} - d^r_{i(j-1)} \geq H^r_{\min}, \quad j \in \mathcal{J}^k_{ri}, i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{metro}} \tag{9}$$

Modeling bus overtaking

$$d^r_{ij} - d^r_{ij'} \geq M(z^{ri}_{j'j} - 1), \quad j, j' \in \mathcal{J}^k_{ri}, i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{bus}} \tag{10}$$

$$d^r_{ij} - d^r_{ij'} + \varepsilon \leq M z^{ri}_{j'j}, \quad j, j' \in \mathcal{J}^k_{ri}, i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{bus}} \tag{11}$$

$$\hat{d}^r_{ij} \leq d^r_{ij'} + M(1 - x^{ri}_{j'j}), \quad j, j' \in \mathcal{J}^k_{ri}, i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{bus}} \tag{12}$$

$$\hat{d}^r_{ij} \geq d^r_{ij'} - M(1 - z^{ri}_{j'j}), \quad j, j' \in \mathcal{J}^k_{ri}, i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{bus}} \tag{13}$$

$$x^{ri}_{j'j} - z^{ri}_{j'j} \leq 0, \quad j, j' \in \mathcal{J}^k_{ri}, i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{bus}} \tag{14}$$

$$\sum_{j' \in \mathcal{J}^k_{ri} \setminus \{j\}} x^{ri}_{j'j} = 1, \quad j \in \mathcal{J}^k_{ri}, i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{bus}} \tag{15}$$

Modeling passenger load dynamics

$$p^{\text{wait}}_{rij} = \lambda^r_{ij}(d^r_{ij} - d^r_{i(j-1)}) + \sum_{r' \in \mathcal{C}_{ri}, j' \in \mathcal{J}^k_{r'i}} q^{r'j'}_{rij} + (p^{\text{wait}}_{ri(j-1)} - p^{\text{board}}_{ri(j-1)}) \quad j \in \mathcal{J}^k_{ri}, i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{metro}} \tag{16}$$

$$p^{\text{wait}}_{rij} = \lambda^r_{ij}(d^r_{ij} - \hat{d}^r_{ij}) + \sum_{r' \in \mathcal{C}_{ri}, j' \in \mathcal{J}^k_{r'i}} q^{r'j'}_{rij} + \sum_{j' \in \mathcal{J}^k_{ri} \setminus \{j\}} x^{ri}_{j'j}(p^{\text{wait}}_{rij'} - p^{\text{board}}_{rij'}), \quad j \in \mathcal{J}^k_{ri}, i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{bus}} \tag{17}$$

$$p^{\text{load}}_{rij} = p^{\text{load}}_{r(i-1)j} + p^{\text{board}}_{rij} - \beta^r_{ij} p^{\text{load}}_{r(i-1)j}, \quad j \in \mathcal{J}^k_{ri}, i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{bus}} \cup \mathcal{R}_{\text{metro}} \tag{18}$$

$$0 \leq p^{\text{board}}_{rij} \leq p^{\text{wait}}_{rij}, \quad j \in \mathcal{J}^k_{ri}, i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{bus}} \cup \mathcal{R}_{\text{metro}} \tag{19}$$

$$p^{\text{board}}_{rij} \leq V_r + (\beta^r_{ij} - 1) p^{\text{load}}_{r(i-1)j}, \quad j \in \mathcal{J}^k_{ri}, i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{bus}} \cup \mathcal{R}_{\text{metro}} \tag{20}$$

Modeling transfer passengers

$$d^r_{ij} - (a^{r'}_{ij'} + E^{r'}_{ri}) \geq M(y^{r'j'}_{rij} - 1), \quad j \in \mathcal{J}^k_{ri}, j' \in \mathcal{J}^k_{r'i}, r' \in \mathcal{C}_{ri}, i \in \mathcal{Q}_r, r \in \mathcal{R}_{\text{metro}} \cup \mathcal{R}_{\text{bus}} \tag{21}$$

$$d^r_{ij} - (a^{r'}_{ij'} + E^{r'}_{ri}) + \varepsilon \leq M y^{r'j'}_{rij}, \quad j \in \mathcal{J}^k_{ri}, j' \in \mathcal{J}^k_{r'i}, r' \in \mathcal{C}_{ri}, i \in \mathcal{Q}_r, r \in \mathcal{R}_{\text{metro}} \cup \mathcal{R}_{\text{bus}} \tag{22}$$

$$q^{r'j'}_{rij} = \rho^{r'j'}_{ri}(y^{r'j'}_{rij} - y^{r'j'}_{ri(j-1)}) \beta^{r'}_{ij'} p^{\text{load}}_{r'(i-1)j'}, \quad j \in \mathcal{J}^k_{ri}, j' \in \mathcal{J}^k_{r'i}, r' \in \mathcal{C}_{ri}, i \in \mathcal{Q}_r, r \in \mathcal{R}_{\text{metro}} \tag{23}$$

$$q^{r'j'}_{rij} = \rho^{r'j'}_{ri}(y^{r'j'}_{rij} - \sum_{j'' \in \mathcal{J}^k_{ri} \setminus \{j\}} x^{ri}_{j''j} y^{r'j'}_{rij''}) \beta^{r'}_{ij'} p^{\text{load}}_{r'(i-1)j'}, \quad j \in \mathcal{J}^k_{ri}, j' \in \mathcal{J}^k_{r'i}, r' \in \mathcal{C}_{ri}, i \in \mathcal{Q}_r, r \in \mathcal{R}_{\text{bus}} \tag{24}$$

Decision variables

$$u^r_{ij} \in [U^r_{\min}, U^r_{\max}], g^r_{ij} \in [G^r_{\min}, G^r_{\max}], \quad j \in \mathcal{J}^k_{ri}, i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{bus}} \cup \mathcal{R}_{\text{metro}} \tag{25}$$

$$d^r_{ij}, a^r_{ij}, p^{\text{wait}}_{rij}, p^{\text{board}}_{rij}, p^{\text{load}}_{rij} \geq 0, \quad j \in \mathcal{J}^k_{ri}, i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{bus}} \cup \mathcal{R}_{\text{metro}} \tag{26}$$

$$\hat{d}^r_{ij} \geq 0, \quad j \in \mathcal{J}^k_{ri}, i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{bus}} \tag{27}$$

$$z^{ri}_{j'j}, x^{ri}_{j'j} \in \{0, 1\}, \quad j, j' \in \mathcal{J}^k_{ri}, i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{bus}} \tag{28}$$

$$y^{r'j'}_{rij} \in \{0, 1\}, q^{r'j'}_{rij} \geq 0, \quad j \in \mathcal{J}^k_{ri}, j' \in \mathcal{J}^k_{r'i}, r' \in \mathcal{C}_{ri}, i \in \mathcal{Q}_r, r \in \mathcal{R}_{\text{bus}} \cup \mathcal{R}_{\text{metro}} \tag{29}$$

Objective function: Objective function (1) minimizes the departure deviation, headway deviation and accumulative number of stranded passengers, as detailed in equations (2)–(4), respectively. ζ_1, ζ_2 and ζ_3 denote the weights.

Modeling vehicle traffic dynamics: Equation (5) defines the departure time d^r_{ij} as the sum of the arrival time a^r_{ij}, and the actual dwell time, which consists of the planned dwell time T^r_i, adjustment g^r_{ij}, and disturbance W^r_{ij}. Equation (6) represents the arrival time a^r_{ij} as the departure time from the previous station plus the actual running time between stations, where the actual running time includes the planned running time L^r_{i-1}, adjustment $u^r_{(i-1)j}$, and disturbance $\tilde{W}^r_{(i-1)j}$. Constraints (7) and (8) ensure passenger safety during boarding and alighting, accounting for different door configurations in metro and bus systems. For completeness of the model, we set $p^{\text{load}}_{r0j} = 0$. The maximum function is equivalently transformed into a linear form in Appendix A. Additionally, constraint (9) enforces a minimum train headway of $H^r_{\min}$.

Modeling bus overtaking: Constraints (10) and (11) describe the departure sequence of buses at each station, which signify that if bus j departs later than bus j' at station i on line r, then $z^{ri}_{j'j} = 1$; otherwise, $z^{ri}_{j'j} = 0$. M and ε are a large and a small positive number, respectively. Constraints (12) and (13) determine the departure time $\hat{d}^r_{ij}$ of the immediate predecessor of bus j based on $x^{ri}_{j'j}$ and $z^{ri}_{j'j}$. Constraint (14) indicates that a bus is the immediate predecessor of bus j only if it departs before bus j. Constraint (15) ensures that each bus has exactly one immediate predecessor at each station. For completeness of the formulation, a virtual predecessor is introduced for the first bus at each station. For bus systems, the bus immediately ahead of bus j is not necessarily equal to $j-1$ due to potential overtaking. The introduced $z^{ri}_{j'j}$ and $x^{ri}_{j'j}$ in modeling bus overtaking can be considered logical variables, which are suitable for effective processing by logic-based methods to facilitate rapid solution.

Modeling passenger load dynamics: Equations (16) and (17) formulate the number of waiting passengers, which includes new arrivals, passengers stranded by previous vehicles, and transfer passengers at transfer stations. For metro passengers waiting for train j, stranded passengers are those left behind by train $j-1$, while for bus passengers, the stranded may not be left behind by bus $j-1$ due to possible overtaking. Thus, we model the number of waiting passengers p^{wait}_{rij} separately for metro trains and buses in equations (16) and (17), respectively. $(p^{\text{wait}}_{ri(j-1)} - p^{\text{board}}_{ri(j-1)})$ indicates the number of passengers stranded by train $j-1$, while $\sum_{j' \in \mathcal{J}^k_{ri} \setminus \{j\}} x^{ri}_{j'j}(p^{\text{wait}}_{rij'} - p^{\text{board}}_{rij'})$ represents the stranded passengers left by the bus immediately ahead of bus j. This nonlinear term is linearized as (A.2a)–(A.2d) in Appendix A. Equation (18) defines the number of onboard passengers p^{load}_{rij} as the sum of onboard passengers from the previous station, plus boarding passengers, minus alighting passengers. Constraints (19) and (20) ensure that the number of boarding passengers is positive and does not exceed the number of waiting passengers or the remaining vehicle capacity.

Modeling transfer passengers: Constraints (21) and (22) indicate that if the arrival time $a^{r'}_{ij'}$ plus the transfer time $E^{r'}_{ri}$ is earlier than the departure time d^r_{ij}, then $y^{r'j'}_{rij} = 1$ (i.e., feeder vehicle j' forms a transfer connection with connecting vehicle j); otherwise, $y^{r'j'}_{rij} = 0$. Similar to $z^{ri}_{j'j}$ and $x^{ri}_{j'j}$, the introduced variable $y^{r'j'}_{rij}$ for modeling transfer connections is treated as a logical variable, which can be effectively handled by logic-based methods. Equations (23) and (24) model the number of transfer passengers waiting for trains and buses, respectively. In cases where multiple connecting vehicles form transfer connections ($y^{r'j'}_{rij} = 1$) with a feeder vehicle, transfer passengers will board the earliest available vehicle if there is sufficient capacity. When the connecting vehicle is a train, the first available vehicle can be identified using $y^{r'j'}_{rij} - y^{r'j'}_{ri(j-1)}$. When the connecting vehicle is a bus, however, overtaking may cause $y^{r'j'}_{rij} - y^{r'j'}_{ri(j-1)}$ to incorrectly identify the first available vehicle, since bus $j-1$ may not be the one immediately preceding bus j. Therefore, we use $y_{rij}^{r'j'} - \sum_{j'' \in \mathcal{J}_{ri}^{k} \setminus \{j\}} x_{j''j}^{ri} y_{rij''}^{r'j'}$ to identify the first available bus for transfer passengers. Equations (23) and (24) are linearized as (A.3a)–(A.4f) in Appendix A.

In M1, there are discrete variables concerning transfer connections and bus overtaking, as well as continuous variables concerning vehicle traffic and passenger loading dynamics. The mixed-integer nature of M1 makes it challenging to solve quickly, which poses difficulties for real-time implementation in practical implementations. To address this, we explore the theoretical properties of M1 in Section 4.1, to motivate the development of an efficient logic-based solution method.

4. Solution method

This section entails our solution method LBB. We begin by outlining the motivation of LBB in Section 4.1. Sections 4.2–4.5 present the four main components: the continuous relaxation, problem-specific branching, domain reduction, and a problem-specific primal heuristic. Finally, Section 4.6 provides a detailed algorithmic procedure of the complete LBB method.

4.1. Motivation of LBB

As mentioned earlier, M1 is a MIQP model, minimizing an objective function with convex quadratic terms subject to linear constraints. Theoretically, M1 can be solved to optimality using the standard BB method. However, due to the numerous coupling constraints and integer variables, obtaining high-quality solutions quickly for real-world cases is challenging, rendering it impractical for real-time operational requirements. Given that the main computational challenge of M1 lies in the mixed-integer property, let Γ be a combination of binary variables $z_{j'j}^{ri}, x_{j'j}^{ri}, y_{rij}^{r'j'}$ and $b_{rij}^{r'j'j''}$ (introduced in Appendix A), we explore its theoretical properties associated with this aspect to motivate the development of efficient solution methods.

Proposition 4.1. *If Γ is fixed or relaxed to be continuous, a continuous quadratic program can be derived from M1. The former corresponds to an upper bound, while the latter corresponds to a lower bound.*

Proposition 4.1 is straightforward, as fixing or relaxing the binary variables transforms the original model M1 into one without discrete variables, making it easier to solve efficiently. This insight motivates the use of a BB framework to address the MIQP model M1, where binary variables are used for branching while continuous variables are optimized efficiently. Notably, we notice that the binary variables in M1 exhibit strong logical relationships facilitating accelerating computation, as presented in the following Propositions 4.2–4.5.

Proposition 4.2. *For each station $i \in \mathcal{S}_r$ on bus line $r \in \mathcal{R}_{\text{bus}}$, once the values of variables in the set $\{z_{j'j}^{ri} | j, j' \in \mathcal{J}_{ri}^{k}\}$ are known, we can uniquely determine the values of variables in the set $\{x_{j'j}^{ri} | j, j' \in \mathcal{J}_{ri}^{k}\}$.*

Proof. For each station i on bus line r, with known values of variables in the set $\{z_{j'j}^{ri} | j, j' \in \mathcal{J}_{ri}^{k}\}$, the relative departure order between any two buses can be determined. If it has been determined that bus j' departs before bus j (i.e., $z_{j'j}^{ri} = 1$), and all other buses that depart before j also depart before j', then we can infer that there is no other vehicle departing between them, and identify that j' is the immediate predecessor of j (i.e., $x_{j'j}^{ri} = 1$). Meanwhile, no other vehicle $j'' \neq j'$ can be the immediate predecessor of j (i.e., $x_{j''j}^{ri} = 0$ for $j'' \in \mathcal{J}_{ri}^{k} \setminus \{j'\}$), and j' cannot be the immediate predecessor of any other vehicle $j'' \neq j$

(i.e., $x^{ri}_{j'j''} = 0$ for $j'' \in \mathcal{J}^k_{ri} \backslash \{j\}$). This logic is formalized as follows:

$$z^{ri}_{j'j} \wedge (\bigwedge_{j'' \in \mathcal{K}^{\text{pred}}_{rij} \backslash \{j'\}} z^{ri}_{j''j'}) \rightarrow x^{ri}_{j'j} \wedge (\bigwedge_{j'' \in \mathcal{J}^k_{ri} \backslash \{j'\}} \neg x^{ri}_{j''j}) \wedge (\bigwedge_{j'' \in \mathcal{J}^k_{ri} \backslash \{j\}} \neg x^{ri}_{j'j''}), \quad j', j \in \mathcal{J}^k_{ri}, i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{bus}} \quad (30)$$

where $\mathcal{K}^{\text{pred}}_{rij}$ represents the set of all buses that depart before j from station i of line r. By applying this logic, for any vehicle j, its immediate predecessor j' can be uniquely identified: the corresponding variable $x^{ri}_{j'j} = 1$ for the immediate predecessor j', while $x^{ri}_{j''j} = 0$ for any other vehicle $j'' \in \mathcal{J}^k_{ri} \backslash \{j'\}$. Therefore, the values of variables $\{x^{ri}_{j'j} | j, j' \in \mathcal{J}^k_{ri}\}$ can be uniquely determined. □

Proposition 4.3. *For each station $i \in \mathcal{S}_r$ on bus line $r \in \mathcal{R}_{\text{bus}}$, once the values of variables in the set $\{x^{ri}_{j'j} | j, j' \in \mathcal{J}^k_{ri}\}$ are known, we can uniquely determine the values of variables in the set $\{z^{ri}_{j'j} | j, j' \in \mathcal{J}^k_{ri}\}$.*

Proof. For each station i on bus line r, if the immediate predecessor of each bus is known (i.e., $\{x^{ri}_{j'j} | j, j' \in \mathcal{J}^k_{ri}\}$ are determined), we can construct a departure sequence of all buses at this station by linking these consecutive predecessor-successor pairs. This yields an ordered list $(j_1, j_2, ..., j_{|\mathcal{J}^k_{ri}|})$ of all buses, where bus j_1 departs first, followed by j_2, and so on. $|\mathcal{J}^k_{ri}|$ denotes the number of buses at station i on line r within the decision stage. From this sequence, the relative departure order of any pair of buses can be inferred: if bus j_m appears before bus j_n in the sequence (i.e., $m < n$), then $z^{ri}_{j_m j_n} = 1$; otherwise, $z^{ri}_{j_m j_n} = 0$. This logic is formalized as follows:

$$\bigwedge_{m=1}^{|\mathcal{J}^k_{ri}|-1} x^{ri}_{j_m j_{m+1}} \rightarrow (\bigwedge_{(j',j) \in \mathcal{K}^{\text{prec}}_{ri}} z^{ri}_{j'j}) \wedge (\bigwedge_{(j',j) \in \mathcal{K}^{\text{succ}}_{ri}} \neg z^{ri}_{j'j}), \quad i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{bus}} \quad (31)$$

where $\mathcal{K}^{\text{prec}}_{ri} = \{(j_m, j_n) \in \mathcal{J}^k_{ri} \times \mathcal{J}^k_{ri} | m < n\}$ denotes the set of all bus pairs where the first bus j_m precedes the second j_n in the departure sequence, and $\mathcal{K}^{\text{succ}}_{ri} = \{(j_m, j_n) \in \mathcal{J}^k_{ri} \times \mathcal{J}^k_{ri} | m > n\}$ denotes the set of all bus pairs in reverse order. □

Proposition 4.4. *If the values of variables in the sets $\{y^{r'j'}_{rij} | r \in \mathcal{R}_{\text{bus}}, r' \in \mathcal{C}_{ri}, i \in \mathcal{Q}_r, j \in \mathcal{J}^k_{ri}, j' \in \mathcal{J}^k_{r'i}\}$ and $\{x^{ri}_{j'j} | r \in \mathcal{R}_{\text{bus}}, i \in \mathcal{Q}_r, j, j' \in \mathcal{J}^k_{ri}\}$ (or $\{z^{ri}_{j'j} | r \in \mathcal{R}_{\text{bus}}, i \in \mathcal{Q}_r, j, j' \in \mathcal{J}^k_{ri}\}$) are known, we can uniquely determine the values of variables in the set $\{b^{r'j'j''}_{rij} | r \in \mathcal{R}_{\text{bus}}, r' \in \mathcal{C}_{ri}, i \in \mathcal{Q}_r, j, j'' \in \mathcal{J}^k_{ri}, j' \in \mathcal{J}^k_{r'i}\}$.*

Proof. According to (A.4a)–(A.4f), the variable $b^{r'j'j''}_{rij}$ is essentially equivalent to the logical conjunction $x^{ri}_{j''j} \wedge y^{r'j'}_{rij''}$. It is straightforward to infer that if $x^{ri}_{j''j} = y^{r'j'}_{rij''} = 1$, then $b^{r'j'j''}_{rij} = 1$; conversely, if either $x^{ri}_{j''j} = 0$ or $y^{r'j'}_{rij''} = 0$, then $b^{r'j'j''}_{rij} = 0$. This logic can be expressed as:

$$x^{ri}_{j''j} \wedge y^{r'j'}_{rij''} \rightarrow b^{r'j'j''}_{rij}, \quad j, j'' \in \mathcal{J}^k_{ri}, j' \in \mathcal{J}^k_{r'i}, r' \in \mathcal{C}_{ri}, i \in \mathcal{Q}_r, r \in \mathcal{R}_{\text{bus}} \quad (32a)$$

$$\neg x^{ri}_{j''j} \vee \neg y^{r'j'}_{rij''} \rightarrow \neg b^{r'j'j''}_{rij}, \quad j, j'' \in \mathcal{J}^k_{ri}, j' \in \mathcal{J}^k_{r'i}, r' \in \mathcal{C}_{ri}, i \in \mathcal{Q}_r, r \in \mathcal{R}_{\text{bus}} \quad (32b)$$

Hence, with known $\{y^{r'j'}_{rij} | r \in \mathcal{R}_{\text{bus}}, r' \in \mathcal{C}_{ri}, i \in \mathcal{Q}_r, j \in \mathcal{J}^k_{ri}, j' \in \mathcal{J}^k_{r'i}\}$ and $\{x^{ri}_{j'j} | r \in \mathcal{R}_{\text{bus}}, i \in \mathcal{Q}_r, j, j' \in \mathcal{J}^k_{ri}\}$, we can uniquely determine the set $\{b^{r'j'j''}_{rij} | r \in \mathcal{R}_{\text{bus}}, r' \in \mathcal{C}_{ri}, i \in \mathcal{Q}_r, j, j'' \in \mathcal{J}^k_{ri}, j' \in \mathcal{J}^k_{r'i}\}$. Furthermore, from Proposition 4.2, once the values of variables in $\{z^{ri}_{j'j} | j, j' \in \mathcal{J}^k_{ri}\}$ are known, we can uniquely determine the values of variables in $\{x^{ri}_{j'j} | j, j' \in \mathcal{J}^k_{ri}\}$. Therefore, when $\{y^{r'j'}_{rij} | r \in \mathcal{R}_{\text{bus}}, i \in \mathcal{Q}_r, r' \in \mathcal{C}_{ri}, j \in \mathcal{J}^k_{ri}, j' \in \mathcal{J}^k_{r'i}\}$ and $\{z^{ri}_{j'j} | r \in \mathcal{R}_{\text{bus}}, i \in \mathcal{Q}_r, j, j' \in \mathcal{J}^k_{ri}\}$ are determined, the values of variables in $\{b^{r'j'j''}_{rij} | r \in \mathcal{R}_{\text{bus}}, i \in \mathcal{Q}_r, r' \in \mathcal{C}_{ri}, j, j'' \in \mathcal{J}^k_{ri}, j' \in \mathcal{J}^k_{r'i}\}$ can be also derived. □

Proposition 4.5. *For each train $j' \in \mathcal{J}^k_{r'i}$ at transfer station $i \in \mathcal{Q}_{r'}$ on metro line $r' \in \mathcal{R}_{\text{metro}}$, if there exists $j \in \mathcal{J}^k_{ri}$ such that $y^{r'j'}_{rij} = 1$ while $y^{r'j'}_{ri(j-1)} = 0$, then the values of variables in the set $\{y^{r'j'}_{rij''} | j'' \in \mathcal{J}^k_{ri}\}$ can be uniquely determined.*

Proof. Since trains are not allowed to overtake on metro lines, for any train j on line r, all subsequent trains $j'' > j$ depart after train j. Hence, we can infer that if connecting train j of line r can form a transfer connection with train j' of line r' (i.e., train j departs after the arrival of train j' plus the transfer time, $y^{r'j'}_{rij} = 1$), then any subsequent train $j'' > j$ on line r can also form a transfer connection with train j' (i.e., $y^{r'j'}_{rij''} = 1$ for all $j'' > j$). Conversely, if train j cannot form a transfer connection (i.e., $y^{r'j'}_{rij} = 0$), then no train $j'' < j$ can form such a connection either (i.e., $y^{r'j'}_{rij''} = 0$ for all $j'' < j$). Therefore, $y^{r'j'}_{rij}$ is non-decreasing in j for $j \in \mathcal{J}^k_{ri}$. Consequently, once we identify that $y^{r'j'}_{rij} = 1$ while $y^{r'j'}_{ri(j-1)} = 0$, we can infer: For any train $j'' \geq j$: $y^{r'j'}_{rij''} = 1$; For any train $j'' < j-1$: $y^{r'j'}_{rij''} = 0$. This inference is formalized as:

$$y^{r'j'}_{rij} \wedge \neg y^{r'j'}_{ri(j-1)} \rightarrow (\bigwedge_{j'' \geq j} y^{r'j'}_{rij''}) \wedge (\bigwedge_{j'' < j-1} \neg y^{r'j'}_{rij''}), \quad j \in \mathcal{J}^k_{ri}, j' \in \mathcal{J}^k_{r'i}, r' \in \mathcal{C}_{ri}, i \in \mathcal{Q}_r, r \in \mathcal{R}_{\text{bus}} \tag{33}$$

Thus, the values of all variables in $\{y^{r'j'}_{rij''} | j'' \in \mathcal{J}^k_{ri}\}$ are uniquely determined if there exists $y^{r'j'}_{rij} = 1$ while $y^{r'j'}_{ri(j-1)} = 0$. □

The progressive fixing of binary variables in the BB framework provides a natural idea to leverage the logical relationships shown in the above propositions for accelerating the solution process. Propositions 4.2–4.5 motivate the incorporation of logic-based inference within the BB framework: as binary variables are fixed during branching, additional variable values can be inferred through their logical relationships, thus reducing the number of required branches. Additionally, Proposition 4.6 motivates quick optimization by using the continuous components of M1, as follows.

Proposition 4.6. *A feasible timetable adjustment scheme that satisfies constraints (5)–(9) corresponds to a unique vector Γ, and the problem associated with Γ is always feasible. A fixed vector Γ corresponds to multiple timetable adjustment schemes.*

Based on Proposition 4.6, within the BB framework, we can establish a reduced optimization problem with a finite set of continuous constraints and variables to quickly generate an adjustment scheme. Based on the obtained adjustment scheme, we can determine a unique Γ. Then, fixing it to solve the complete continuous optimization problem can potentially yield a better upper bound in a short time.

Motivated by the above theoretical properties of M1, we develop an LBB method by incorporating logic-based idea into the BB framework. Specifically, inspired by Proposition 4.1, LBB separates discrete and continuous variables to enable faster computations. Continuous variables are used to construct efficiently solvable continuous programs. The strong logical relationships among the discrete variables (e.g., Proposition 4.2–4.5) allow for computationally inexpensive logical inferences to guide the optimization process. Leveraging these logical relationships, LBB determines search variables and employs domain reduction to reduce the number of required branches. Additionally, based on Proposition 4.6, LBB constructs reduced optimization problems to derive better upper bounds, further accelerating convergence.

Totally, LBB follows a branch-and-bound framework. It begins by constructing a continuous relaxation of M1 to obtain a lower bound and employs feasibility recovery to generate upper bounds. Problem-specific branching strategies exploit the logical relationships in M1, while domain reduction, by interacting with branching, progressively fixes logical variables during the iterative process. Additionally, problem-specific heuristics based on reduced optimization problems rapidly generate high-quality upper bounds. The detailed algorithmic components are presented in Sections 4.2–4.5.

4.2. Continuous relaxation

By relaxing logical variables to be continuous and preserving all mixed-integer constraints in continuous forms, we construct the following relaxed optimization problem M2:

$$\textbf{[M2]}:\quad \begin{aligned} &\min F_1^k + F_2^k + F_3^k \\ &\text{s.t.} \begin{cases} (2)-(7), (9)-(16), (18)-(22), (\text{A.1a})-(\text{A.4f}) \\ u_{ij}^r \in \left[U_{\min}^r, U_{\max}^r\right], g_{ij}^r \in \left[G_{\min}^r, G_{\max}^r\right], \quad j \in \mathcal{J}_{ri}^k, i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{bus}} \cup \mathcal{R}_{\text{metro}} \\ d_{ij}^r, a_{ij}^r, p_{rij}^{\text{wait}}, p_{rij}^{\text{board}}, p_{rij}^{\text{load}} \geq 0, \quad j \in \mathcal{J}_{ri}^k, i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{bus}} \cup \mathcal{R}_{\text{metro}} \\ \hat{d}_{ij}^r \geq 0, \quad j \in \mathcal{J}_{ri}^k, i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{bus}} \\ z_{j'j}^{ri}, x_{j'j}^{ri} \in [0,1], \quad j, j' \in \mathcal{J}_{ri}^k, i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{bus}} \\ y_{rij}^{r'j'} \in [0,1], q_{rij}^{r'j'} \geq 0, \quad j \in \mathcal{J}_{ri}^k, j' \in \mathcal{J}_{r'i}^k, r' \in \mathcal{C}_{ri}, i \in \mathcal{Q}_r, r \in \mathcal{R}_{\text{bus}} \cup \mathcal{R}_{\text{metro}} \\ b_{rij}^{r'j'j''} \in [0,1], \quad j, j'' \in \mathcal{J}_{ri}^k, j' \in \mathcal{J}_{r'i}^k, r' \in \mathcal{C}_{ri}, i \in \mathcal{Q}_r, r \in \mathcal{R}_{\text{bus}} \end{cases} \end{aligned} \tag{34}$$

which is a convex QP model allowing for fast solutions. Let $z_{j'j}^{ri*}, x_{j'j}^{ri*}, y_{rij}^{r'j'*}, b_{rij}^{r'j'j''*}$ denote the optimal binary solution to M2, which may be fractional, then the corresponding optimal objective function value provides a lower bound $\mathcal{F}_{\underline{\ell}}$ for M1. As branching continues, more logical variables are fixed, progressively increasing the lower bound $\mathcal{F}_{\underline{\ell}}$. If all logical variables reach 0 or 1, the current solution is a feasible solution for M1. If it matches the optimal solution, $\mathcal{F}_{\underline{\ell}}$ is the optimal value; otherwise, it is an upper bound for M1.

Since achieving $z_{j'j}^{ri*}, x_{j'j}^{ri*}, y_{rij}^{r'j'*}, b_{rij}^{r'j'j''*}$ all as 0 or 1 often requires multiple branches, we design feasibility recovery methods to update the upper bound. One method is to fix the optimal solutions g_{rij}^*, u_{rij}^*, and $p_{rij}^{\text{board}^*}$ of M2. These fixed values are then used to reload timetables and passenger flows, followed by recalculating the objective function. Specifically, logical variables related to transfer connections and bus overtaking, if left continuous, can lead to inaccuracies in passenger flow descriptions. To correct this, we recalculate the actual number of boarding passengers as: either the maximum boarding passengers (the smaller of waiting passengers and remaining capacity), or the smaller of this maximum boarding numbers and the optimized boarding numbers. We then reload passenger flows using (16)–(18), (23), and (24). The second method calculates logical variable values using (10)–(15), (21), (22), and (A.4a)–(A.4c) based on the reloaded timetables in the first method, then resolves M2 with these fixed values to obtain the upper bound.

4.3. Problem-specific branching

M1 involves logical variables $z_{j'j}^{ri}$ and $x_{j'j}^{ri}$ with respect to bus overtaking, as well as $y_{rij}^{r'j'}$ and $b_{rij}^{r'j'j''}$ (introduced from linearization) concerning transfer coordination. Based on logical relationships, we propose a problem-specific branching strategy as follows.

- Branching concerning bus overtaking.

  According to Propositions 4.2 and 4.3, once any one of the sets $\{z_{j'j}^{ri} | r \in \mathcal{R}_{\text{bus}}, i \in \mathcal{S}_r, j, j' \in \mathcal{J}_{ri}^k\}$ and $\{x_{j'j}^{ri} | r \in \mathcal{R}_{\text{bus}}, i \in \mathcal{S}_r, j, j' \in \mathcal{J}_{ri}^k\}$ is assigned, the other will be uniquely determined. Therefore, we can choose one of the two sets to branch on, and gradually determine the value of variables in the other set with branching. As for the choice of variables, $z^{ri}_{j'j}$ is preferred since it promotes a more direct and effective application of the domain reduction technique presented in Section 4.4. For clarity, we introduce an example to show the advantage to choose $z^{ri}_{j'j}$ for branching, as follows.

Example 4.1. *Consider a case of three buses 1, 2 and 3 at station i of line r, and suppose we can obtain that bus 1 is ahead of both buses 2 and 3, through (38) in domain reduction of Section 4.4. Then, we can deduce that $z^{ri}_{12} = 1, z^{ri}_{13} = 1, z^{ri}_{21} = 0, z^{ri}_{31} = 0$. In this case, when branching on $z^{ri}_{j'j}$, only z^{ri}_{23} and z^{ri}_{32} require branching. However, when branching on $x^{ri}_{j'j}$, no logical variables can be predetermined, potentially necessitating more branching.*

In this case, the value of $x^{ri}_{j'j}$ can be gradually determined with branching on $z^{ri}_{j'j}$ using conditional constraint (30), as detailed in Proposition 4.3. Notably, constraint (30) is only formulated when all elements in the set $\{z^{ri}_{j'j} | j, j' \in \mathcal{J}^k_{ri}\}$ are fixed, so we derive logical constraint (35) based on constraint (14), to allow more flexibility in deriving the value of $x^{ri}_{j'j}$ with branching (this can be considered a part of domain reduction). Specifically, if bus j' does not depart before bus j, then bus j' cannot be the immediate predecessor of bus j, i.e.,

$$\neg z^{ri}_{j'j} \to (\neg x^{ri}_{j'j}), \quad j, j' \in \mathcal{J}^k_{ri}, i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{bus}} \tag{35}$$

- Branching concerning transfer connection.

Denote $s^{r'j'}_{rij} = y^{r'j'}_{rij} - y^{r'j'}_{ri(j-1)}$ if the connecting vehicle j is a train, while $s^{r'j'}_{rij} = y^{r'j'}_{rij} - \sum_{j'' \in \mathcal{J}^k_{ri} \setminus \{j\}} x^{ri}_{j''j} y^{r'j'}_{rij''}$ if it is a bus. Here, $s^{r'j'}_{rij} = 1$ indicates that vehicle j on line r is the first vehicle to arrive after passengers from vehicle j' on line r' arrive at the transfer platform; $s^{r'j'}_{rij} = 0$ otherwise. Actually, in Section 3.2, the essential role of the transfer connection variable $y^{r'j'}_{rij}$ is to identify the first available vehicle for transfer passengers, i.e., $s^{r'j'}_{rij}$, thus calculating the number of transfer passengers and their corresponding waiting times.

As established in Proposition 4.5, for metro systems, $y^{r'j'}_{rij}$ is non-decreasing in $j \in \mathcal{J}^k_{ri}$. This monotonicity implies that $\{s^{r'j'}_{rij} | j \in \mathcal{J}^k_{ri}\}$ forms a special ordered set of type 1 (SOS1) structure, i.e., at most one variable is nonzero. In other words, at most one vehicle in $\mathcal{J}^k_{ri}$ can be the first available vehicle for passengers transferring from bus j' of line r'. Rather than branching on individual variables, we leverage this SOS1 structure to branch by partitioning the variable set. Specifically, the branching strategy creates $|\mathcal{J}^k_{ri}| + 1$ branches: (i) For each vehicle $\bar{j} \in \mathcal{J}^k_{ri}$, one branch sets $s^{r'j'}_{ri\bar{j}} = 1$ and $s^{r'j'}_{rij} = 0$ for any $j \neq \bar{j}$, indicating that $\bar{j}$ is selected as the first available vehicle for transfer passengers. (ii) An additional branch sets $s^{r'j'}_{rij} = 0$ for $j \in \mathcal{J}^k_{ri}$, indicating that no vehicle is available (i.e., all vehicles depart before transfer passengers arrive).

Actually, when all $s^{r'j'}_{rij}$ in M1 are fixed, $y^{r'j'}_{rij}$ and $b^{r'j'j''}_{rij}$ will be uniquely determined. Additionally, during the branching process, as more $s^{r'j'}_{rij}$ is determined, $y^{r'j'}_{rij}$ can be determined using conditional constraint (36). Specifically, if vehicle j is the first vehicle to arrive after transfer passengers from vehicle j' reach the platform (i.e., $s^{r'j'}_{rij} = 1$), then: (i) If vehicle j is a train ($r \in \mathcal{R}_{\text{metro}}$), then vehicle j and all subsequent vehicles can form transfer connections with vehicle j' ($y^{r'j'}_{rij''} = 1$ for any $j'' \geq j$), while all preceding vehicles cannot form transfer connections with vehicle j' ($y^{r'j'}_{rij''} = 0$ for any $j'' < j$).

(ii) If vehicle j is a bus ($r \in \mathcal{R}_{\text{bus}}$), vehicle j can form a transfer connection with vehicle j' ($y_{rij}^{r'j'} = 1$).

$$s_{rij}^{r'j'} \rightarrow \begin{cases} y_{rij}^{r'j'} \wedge \neg y_{ri(j-1)}^{r'j'}, & \text{if } r \in \mathcal{R}_{\text{metro}} \\ y_{rij}^{r'j'}, & \text{if } r \in \mathcal{R}_{\text{bus}} \end{cases}, \quad j \in \mathcal{J}_{ri}^{k}, j' \in \mathcal{J}_{r'i}^{k}, r' \in \mathcal{C}_{ri}, i \in \mathcal{Q}_r, r \in \mathcal{R}_{\text{bus}} \cup \mathcal{R}_{\text{metro}} \tag{36}$$

As discussed, $z_{jj''}^{ri}$ and $y_{rij}^{r'j'}$ (i.e., $s_{rij}^{r'j'}$) are search variables used for branching, and other logical variables like $x_{rij}^{r'j'}$ can be implicitly determined through branching. In LBB, by relaxing logical variables to be continuous, a QP model (M2 in Section 4.2) is constructed to provide a lower bound. LBB then performs branching on $z_{jj''}^{ri}$ and $y_{rij}^{r'j'}$, with domain reduction to generate subnodes. This process repeats for subnodes, using bounds to prune infeasible branches. Branching involves two key decisions: selecting the node and variables for branching. In LBB, we choose the node with the lowest lower bound and the variables with the largest logical error (i.e., the difference between the optimal solution of logical variables from relaxations and their boundary values, 0 or 1).

4.4. Variable fixing and domain reduction.

In M1, the coupled relationships among variables through constraints motivate the use of computationally efficient variable fixing and domain reduction, working by logical inference. By leveraging constraints, variable bounds, and problem-specific knowledge, domain reduction narrows the domains of certain variables, which, in turn, propagates to other constraints, further contracting other variable domains.

- Domain reduction concerning bus overtaking.

  (1) First, by exploiting the variable bounds, we can determine certain values of $z_{j'j}^{ri}$ at the start of LBB. Specifically, based on constraints (5) and (6), we derive the upper and lower bounds of vehicle departure times, by accounting for the planned schedules, disturbances, and adjustments. The upper bound $\bar{d}_{ij}^{r}$ assumes the maximum adjustments occur, while the lower bound $\underline{d}_{ij}^{r}$ assumes the minimum values, computed as:

$$\bar{d}_{ij}^{r} = \bar{d}_{(i-1)j}^{r} + L_{(i-1)j}^{r} + \tilde{W}_{(i-1)j}^{r} + U_{\max}^{r} + T_i^r + G_{\max}^{r} + W_{ij}^{r}, \quad j \in \mathcal{J}_{ri}^{k}, i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{bus}} \cup \mathcal{R}_{\text{metro}} \tag{37a}$$

$$\underline{d}_{ij}^{r} = \underline{d}_{(i-1)j}^{r} + L_{(i-1)j}^{r} + \tilde{W}_{(i-1)j}^{r} + U_{\min}^{r} + T_i^r + G_{\min}^{r} + W_{ij}^{r}, \quad j \in \mathcal{J}_{ri}^{k}, i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{bus}} \cup \mathcal{R}_{\text{metro}} \tag{37b}$$

  Then, based on constraints (10) and (11), we can determine the values of certain variables $z_{j'j}^{ri}$. If the lower bound of the departure time of vehicle j exceeds the upper bound of the departure time of vehicle j' at station i, i.e., $\underline{d}_{ij}^{r} > \bar{d}_{ij'}^{r}$, then vehicle j' must depart before vehicle j, and thus we can get $z_{j'j}^{ri} = 1$. Conversely, if the upper bound of the departure time of vehicle j is less than the lower bound of the departure time of vehicle j', i.e., $\bar{d}_{ij}^{r} < \underline{d}_{ij'}^{r}$, then vehicle j must depart before vehicle j', and thus we can get $z_{j'j}^{ri} = 0$. Formally, we determine $z_{j'j}^{ri}$

$$(\underline{d}_{ij}^{r} > \bar{d}_{ij'}^{r}) \rightarrow z_{j'j}^{ri}, \quad j, j' \in \mathcal{J}_{ri}^{k}, i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{bus}} \tag{38a}$$

$$(\bar{d}_{ij}^{r} < \underline{d}_{ij'}^{r}) \rightarrow \neg z_{j'j}^{ri}, \quad j, j' \in \mathcal{J}_{ri}^{k}, i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{bus}} \tag{38b}$$

  (2) Second, as more logical variables are fixed through the aforementioned bound information or branching, additional logical variables can be derived. Specifically, if bus j departs after bus j' at

station i on line r (i.e., $z^{ri}_{j'j} = 1$), it is evident that bus j cannot depart before bus j' (i.e., $z^{ri}_{jj'} = 0$). This logical relationship is captured by the following constraint:

$$z^{ri}_{j'j} \to \neg z^{ri}_{jj'}, \quad j, j' \in \mathcal{J}^k_{ri}, i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{bus}} \tag{39}$$

For conditional constraints such as (39), the validity of the consequents depends on whether the antecedents are satisfied (Hooker, 2000). In our algorithm, when certain variables are fixed through branching or domain reduction (i.e., specific antecedents become satisfied), the corresponding consequents are activated, enabling to deduce values of additional variables. Similarly, constraint (35), introduced earlier, also facilitates the derivation of additional logical variable values as the branching process progresses. Besides, let $\bar{z}^{ri}_{j'j}$ denote the fixed value of $z^{ri}_{j'j}$ determined during the algorithm process. We define the set $\bar{\mathcal{B}}^{ri}_j = \left\{ j' \,\middle|\, \bar{z}^{ri}_{j'j} = 1 \right\}$ to contain all vehicles that are determined to depart before vehicle j at station i on line r. We then define the set $\hat{\mathcal{B}}^{ri}_j = \left\{ j' \in \bar{\mathcal{B}}^{ri}_j \,\middle|\, \exists j'' \in \bar{\mathcal{B}}^{ri}_j \setminus \{j'\} \text{ satisfies } \underline{d}^r_{ij''} > \bar{d}^r_{ij'} \right\}$, to identify vehicles that cannot be the immediate predecessor of vehicle j. This is because for each vehicle j' in $\hat{\mathcal{B}}^{ri}_j$, there exists at least one other vehicle j'' (also departing before vehicle j) whose minimum departure time exceeds the maximum departure time of vehicle j', implying that vehicle j'' must depart after vehicle j'. Therefore, vehicle j' cannot be the immediate predecessor of vehicle j. Hence, we have

$$x^{ri}_{j'j} = 0, \quad j' \in \hat{\mathcal{B}}^{ri}_j, j \in \mathcal{J}^k_{ri}, i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{bus}} \tag{40}$$

(3) To further accelerate the algorithm and enhance its scalability to large-scale cases, we design a rule-based mechanism. This mechanism follows the principle that buses typically maintain their order without overtaking when departure deviations remain within adjustable ranges.

Specifically, let $\tilde{i}$ denote the station corresponding to the largest fixed departure time of bus j in the previous stage. We define the possible cumulative departure deviation as the fixed departure time of bus j at station $\tilde{i}$ in the previous stage, minus its scheduled departure time at station $\tilde{i}$, plus the cumulative disturbances from station $\tilde{i}$ to i. Then, we introduce a logical indicator $e^r_{ij} = 1$ if the possible cumulative departure deviation of vehicle j at station i on line r is within the adjustable range; $= 0$, otherwise. e^r_{ij} is defined as

$$e^r_{ij} \leftrightarrow (d^r_{\tilde{i}j} - D^r_{\tilde{i}j} + \sum_{i'=\tilde{i}}^{i-1} \tilde{W}^r_{i'j} + \sum_{i'=\tilde{i}+1}^{i} W^r_{i'j} \leq (i - \tilde{i})(|U^r_{\min}| + |G^r_{\min}|)),$$
$$j \in \mathcal{J}^k_{ri}, i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{bus}} \cup \mathcal{R}_{\text{metro}} \tag{41}$$

At each station $i \in \mathcal{S}_r$ of line $r \in \mathcal{R}_{\text{bus}}$, if the possible cumulative departure deviation of all buses remain within adjustable ranges, we can infer that they maintain their originally planned sequence, i.e., no overtaking occurs. Specifically, for any pair of buses j and j' at this station: if j' is originally scheduled before j (i.e., $j' < j$), then j' will still depart before j ($z^{ri}_{j'j} = 1$); conversely, if j' is originally scheduled after j (i.e., $j' > j$), then j' will not depart before j ($z^{ri}_{j'j} = 0$). Moreover, the immediate predecessor of each bus j remains the bus immediately ahead of it in the original sequence, namely bus $j-1$ (i.e., $x^{ri}_{(j-1)j}=1$ while $x^{ri}_{j'j}=0$ for all $j'\neq j-1$). This logic is formalized as:

$$(\bigwedge_{j\in\mathcal{J}^k_{ri}} e^r_{ij}) \rightarrow (\bigwedge_{j,j'\in\mathcal{J}^k_{ri},j'<j} z^{ri}_{j'j}) \wedge (\bigwedge_{j,j'\in\mathcal{J}^k_{ri},j'>j} \neg z^{ri}_{j'j}) \wedge (\bigwedge_{j\in\mathcal{J}^k_{ri}} x^{ri}_{(j-1)j}) \wedge (\bigwedge_{j,j'\in\mathcal{J}^k_{ri},j'\neq j-1} \neg x^{ri}_{j'j}), \quad i\in\mathcal{S}_r, r\in\mathcal{R}_{\text{bus}} \tag{42}$$

Additionally, in cases where the cumulative departure deviations of not all buses remain within the adjustable range, we can still determine the departure order for certain pairs of buses. Specifically, if both buses j and j' maintain their cumulative deviations within the adjustable range (i.e., $e^r_{ij}=1$ and $e^r_{ij'}=1$), then their relative departure order $z^{ri}_{j'j}$ can be determined based on their originally scheduled order, even though their immediate predecessors (i.e., $x^{ri}_{j'j}$) may remain undetermined due to potential overtaking of other buses. Formally, this relationship is expressed as:

$$e^r_{ij}\wedge e^r_{ij'} \rightarrow \begin{cases} z^{ri}_{j'j}, & \text{if } j'<j \\ \neg z^{ri}_{j'j}, & \text{if } j'>j \end{cases}, \quad j,j'\in\mathcal{J}^k_{ri}, i\in\mathcal{S}_r, r\in\mathcal{R}_{\text{bus}} \tag{43}$$

- Domain reduction concerning transfer connection.

  (1) By utilizing the variable bounds of d^r_{ij} and a^r_{ij}, we can determine specific values of $y^{r'j'}_{rij}$. Specifically, based on constraints (21) and (22), if the minimum departure time of vehicle j on line r is not earlier than the maximum arrival time of vehicle j' on line r' plus the transfer time $E^{r'}_{ri}$, then the two vehicles form a transfer connection, i.e., $y^{r'j'}_{rij}=1$. Conversely, if the maximum departure time of vehicle j is earlier than the minimum arrival time of vehicle j' plus the transfer time, then they cannot form a transfer connection, i.e., $y^{r'j'}_{rij}=0$. Formally, these relationships are expressed as:

$$\left(\underline{d}^r_{ij} - \left(\bar{a}^{r'}_{ij'} + E^{r'}_{ri}\right) \geq 0\right) \rightarrow y^{r'j'}_{rij}, \quad j\in\mathcal{J}^k_{ri}, j'\in\mathcal{J}^k_{r'i}, r'\in\mathcal{C}_{ri}, i\in\mathcal{Q}_r, r\in\mathcal{R}_{\text{bus}}\cup\mathcal{R}_{\text{metro}} \tag{44a}$$

$$\left(\bar{d}^r_{ij} - \left(\underline{a}^{r'}_{ij'} + E^{r'}_{ri}\right) + \varepsilon \leq 0\right) \rightarrow \neg y^{r'j'}_{rij}, \quad j\in\mathcal{J}^k_{ri}, j'\in\mathcal{J}^k_{r'i}, r'\in\mathcal{C}_{ri}, i\in\mathcal{Q}_r, r\in\mathcal{R}_{\text{bus}}\cup\mathcal{R}_{\text{metro}} \tag{44b}$$

  (2) According to the non-decreasing property of $y^{r'j'}_{rij}$ with respect to j, if a transfer connection can be established from train j' on line r' to train j on line r, then transfer connections to all subsequent trains (i.e., $j''\geq j$) on line r can also be established. Conversely, if a transfer connection from train j' to train j cannot be formed, then transfer connections to all preceding trains (i.e., $j''<j$) are also impossible. Formally, these relationships are expressed as:

$$y^{r'j'}_{rij} \rightarrow \bigwedge_{j''\in\mathcal{J}^k_{ri},j''\geq j} y^{r'j'}_{rij''}, \quad j\in\mathcal{J}^k_{ri}, j'\in\mathcal{J}^k_{r'i}, r'\in\mathcal{C}_{ri}, i\in\mathcal{Q}_r, r\in\mathcal{R}_{\text{metro}} \tag{45a}$$

$$\neg y^{r'j'}_{rij} \rightarrow \bigwedge_{j''\in\mathcal{J}^k_{ri},j''<j} \neg y^{r'j'}_{rij''}, \quad j\in\mathcal{J}^k_{ri}, j'\in\mathcal{J}^k_{r'i}, r'\in\mathcal{C}_{ri}, i\in\mathcal{Q}_r, r\in\mathcal{R}_{\text{metro}} \tag{45b}$$

Based on constraints (21) and (22), if a transfer connection can be established from vehicle j' on line r' to vehicle j on line r at station i, then the reverse transfer connection from vehicle j to vehicle j'

may not be possible, i.e.,

$$y_{rij}^{r'j'} \rightarrow (\neg y_{r'ij'}^{r,j}), \quad j \in \mathcal{J}_{ri}^k, j' \in \mathcal{J}_{r'i}^k, r' \in \mathcal{C}_{ri}, i \in \mathcal{Q}_r, r \in \mathcal{R}_{\text{bus}} \cup \mathcal{R}_{\text{metro}} \tag{46}$$

Note that the validity of constraint (46) is conditional and depends on the relationship between vehicle dwell times and passenger transfer times, as detailed in Proposition 4.7.

Proposition 4.7. *Conditional constraint (46) is valid when* $E_{r'i}^r + E_{ri}^{r'} \geq \bar{s}_{ij'}^{r'} + \bar{s}_{ij}^r$*, where* $\bar{s}_{ij}^r = T_i^r + W_{ij}^r + G_{\max}^r$.

Proof. First, we denote $\bar{s}_{ij}^r = T_i^r + W_{ij}^r + G_{\max}^r$ as the upper bound on the dwell time. According to (21), the departure time is no greater than the arrival time plus the upper bound on the dwell time, i.e., $d_{ij'}^{r'} \leq a_{ij'}^{r'} + \bar{s}_{ij'}^{r'}$. Similarly, the arrival time is no less than the departure time minus the upper bound on the dwell time, i.e., $a_{ij}^r \geq d_{ij}^r - \bar{s}_{ij}^r$. Then, it is easy to obtain that

$$\begin{aligned} d_{ij'}^{r'} - (a_{ij}^r + E_{r'i}^r) \leq & a_{ij'}^{r'} + \bar{s}_{ij'}^{r'} - (d_{ij}^r - \bar{s}_{ij}^r + E_{r'i}^r) \\ & = -(d_{ij}^r - a_{ij'}^{r'}) + \bar{s}_{ij'}^{r'} + \bar{s}_{ij}^r - E_{r'i}^r, \\ & \quad j' \in \mathcal{J}_{r'i}^k, j \in \mathcal{J}_{ri}^k, r' \in \mathcal{C}_{ri}, i \in \mathcal{Q}_r, r \in \mathcal{R}_{\text{metro}} \cup \mathcal{R}_{\text{bus}} \end{aligned} \tag{47}$$

Then, according to the definition of variable $y_{rij}^{r'j'}$ as in (21), if $y_{rij}^{r'j'} = 1$, then $d_{ij}^r - a_{ij'}^{r'} \geq E_{ri}^{r'}$ holds. That is, vehicle j departs after vehicle j' arrives plus the transfer time. Thus, we can obtain

$$-(d_{ij}^r - a_{ij'}^{r'}) + \bar{s}_{ij'}^{r'} + \bar{s}_{ij}^r - E_{r'i}^r \leq -E_{ri}^{r'} + \bar{s}_{ij'}^{r'} + \bar{s}_{ij}^r - E_{r'i}^r, \quad j' \in \mathcal{J}_{r'i}^k, j \in \mathcal{J}_{ri}^k, r' \in \mathcal{C}_{ri}, i \in \mathcal{Q}_r, r \in \mathcal{R}_{\text{metro}} \cup \mathcal{R}_{\text{bus}}$$

Combining with (47), we obtain:

$$d_{ij'}^{r'} - (a_{ij}^r + E_{r'i}^r) \leq -E_{ri}^{r'} + \bar{s}_{ij'}^{r'} + \bar{s}_{ij}^r - E_{r'i}^r, \quad j' \in \mathcal{J}_{r'i}^k, j \in \mathcal{J}_{ri}^k, r' \in \mathcal{C}_{ri}, i \in \mathcal{Q}_r, r \in \mathcal{R}_{\text{metro}} \cup \mathcal{R}_{\text{bus}}$$

Therefore, if $E_{r'i}^r + E_{ri}^{r'} \geq \bar{s}_{ij'}^{r'} + \bar{s}_{ij}^r$, then $d_{ij'}^{r'} - (a_{ij}^r + E_{r'i}^r) \leq 0$, $y_{r'ij'}^{r,j} = 0$. Consequently, we obtain that, for any $j' \in \mathcal{J}_{r'i}^k, j \in \mathcal{J}_{ri}^k, r' \in \mathcal{C}_{ri}, i \in \mathcal{Q}_r, r \in \mathcal{R}_{\text{metro}} \cup \mathcal{R}_{\text{bus}}$, if $y_{rij}^{r'j'} = 1$, then $y_{r'ij'}^{r,j} = 0$ holds when $E_{r'i}^r + E_{ri}^{r'} \geq \bar{s}_{ij'}^{r'} + \bar{s}_{ij}^r$, where $\bar{s}_{ij}^r = T_i^r + W_{ij}^r + G_{\max}^r$. □

Besides, according to the definition of $s_{rij}^{r'j'}$, if $y_{rij}^{r'j'} = 0$, then $s_{rij}^{r'j'} = 0$ must hold. If vehicles j' and j cannot form a transfer connection (i.e., vehicle j departs before transfer passengers from vehicle j' arrive at the platform), then vehicle j cannot be the first available vehicle for these transfer passengers, i.e.,

$$\neg y_{rij}^{r'j'} \rightarrow \neg s_{rij}^{r'j'}, \quad j' \in \mathcal{J}_{r'i}^k, j \in \mathcal{J}_{ri}^k, r' \in \mathcal{C}_{ri}, i \in \mathcal{Q}_r, r \in \mathcal{R}_{\text{metro}} \cup \mathcal{R}_{\text{bus}} \tag{48}$$

(3) Similar to the domain reduction for overtaking, we adopt a rule-based mechanism, following the principle that vehicles typically maintain the scheduled transfer connections when their departure and arrival deviations remain within adjustable ranges. We introduce a logical indicator ${e'}_{ij}^r = 1$ if the possible cumulative arrival deviation of vehicle j at transfer station i on line r is within the adjustable

range; = 0, otherwise, i.e.,

$$
{e'}^{r}_{ij} \leftrightarrow (a^{r}_{\tilde{i}j} - A^{r}_{\tilde{i}j} + \sum_{i'=\tilde{i}}^{i-1} \tilde{W}^{r}_{i'j} + \sum_{i'=\tilde{i}}^{i-1} W^{r}_{i'j} \leq (i-\tilde{i})(|U^{r}_{\min}| + |G^{r}_{\min}|))
$$

$$
j \in \mathcal{J}^{k}_{ri}, i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{bus}} \cup \mathcal{R}_{\text{metro}} \tag{49}
$$

Then, for lines r and r' that connect at station i, if the possible cumulative arrival deviations of all vehicles of line r' and the possible cumulative departure deviations of all vehicles of line r remain within adjustable ranges, we can infer that they will maintain their originally planned transfer connection relationships.

$$
(\bigwedge_{j' \in \mathcal{J}^{k}_{r'i}} {e'}^{r'}_{ij'} \wedge \bigwedge_{j \in \mathcal{J}^{k}_{ri}} e^{r}_{ij}) \rightarrow \left[y^{r'j'}_{rij} = Y^{r'j'}_{rij}, j \in \mathcal{J}^{k}_{ri}, j' \in \mathcal{J}^{k}_{r'i} \right], \quad r' \in \mathcal{C}_{ri}, i \in \mathcal{Q}_r, r \in \mathcal{R}_{\text{metro}} \cup \mathcal{R}_{\text{bus}} \tag{50}
$$

where $Y^{r'j'}_{rij}$ denotes the value of $y^{r'j'}_{rij}$ calculated by the planned schedule.

Domain reduction is applied at the start of LBB and during branching, to deduce logical variable values. It reduces branching and thus accelerates the search process. There is a positive interaction between domain reduction and branching: domain reduction narrows variable domains, reducing the number of branches needed, while branching specifies more variable values, allowing for additional domain reductions.

4.5. Problem-specific primal heuristic

As mentioned earlier, keeping logical variables continuous in M2 may lead to inaccurate passenger loading. To improve computational efficiency, we design a problem-specific primal heuristic approach. Specifically, we introduce a reduced optimization model M3, which provides estimates for the optimal values of logical variables that can then be passed to M2, thus generating feasible solutions and upper bounds for M1. M3 is formulated as

$$
\textbf{[M3]}: \quad \begin{aligned} & \min F^{k}_{1} + F^{k}_{2} + F^{k}_{4} \\ & \text{s.t.} \begin{cases} (2), (3), (5), (6), (9) \\ F^{k}_{4} = \sum_{r \in \mathcal{R}_{\text{bus}} \cup \mathcal{R}_{\text{metro}}, i \in \mathcal{S}_r, j \in \mathcal{J}^{k}_{ri}} \eta^{r}_{ij} h^{r}_{ij} \\ u^{r}_{ij} \in [U^{r}_{\min}, U^{r}_{\max}], g^{r}_{ij} \in [G^{r}_{\min}, G^{r}_{\max}], \quad j \in \mathcal{J}^{k}_{ri}, i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{bus}} \cup \mathcal{R}_{\text{metro}} \\ d^{r}_{ij}, a^{r}_{ij} \geq 0, \quad j \in \mathcal{J}^{k}_{ri}, i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{bus}} \cup \mathcal{R}_{\text{metro}} \end{cases} \end{aligned} \tag{51}
$$

which eliminates passenger flow-related logical variables and constraints from M1. Solving M3 can generate a timetable adjustment scheme, followed by calculating the values of logical variables using (10)–(15), (21), (22), (A.4a)–(A.4c). M2 can then offer an upper bound for M1 with these values, if feasible. To ensure that these logical values obtained from M3 are close to the optimal solution of M1, we introduce penalty term F^{k}_{4} to capture the impact of passenger flows on adjustment decisions. Here, $\eta^{r}_{ij} = \mu(\tilde{p}^{\text{wait}}_{rij} + (\tilde{p}^{\text{wait}}_{rij} - \tilde{p}^{\text{board}}_{rij}))$ updates passenger flow estimates during branching, where $\tilde{p}^{\text{wait}}_{rij}$ and $\tilde{p}^{\text{board}}_{rij}$ represent numbers of waiting and boarding passengers from the best feasible solution found so far. The coefficient μ acts as a weight for vehicle headways in M3, adjusting how strongly the model penalizes headway deviations based on estimated passenger demand. A larger value of μ results in shorter headways when there are more waiting or stranded passengers, while a smaller value of μ leads to longer headways. If the solution from M3 produces headways that are too long, we can increase μ; conversely, if headways are too short, we decrease μ to obtain longer headways. The variable h^r_{ij} represents the departure headway. For metro lines, since trains cannot overtake, the headway is the interval between vehicles j and $j-1$. For bus lines, the calculation depends on whether the actual immediate predecessor has been determined. We define $\mathcal{A}_{ri}$ as the set of buses whose immediate predecessors are fixed at station i on line r. Let $\bar{x}^{ri}_{j'j}$ denote the fixed value of $x^{ri}_{j'j}$. For buses with fixed predecessors (i.e., $j \in \mathcal{A}_{ri}$), the headway is calculated based on the determined predecessor: $h^r_{ij} = d^r_{ij} - \sum_{j' \in \mathcal{J}^k_{ri} \setminus \{j\}} \bar{x}^{ri}_{j'j} d^r_{ij'}$, where the summation identifies the departure time of the actual predecessor. For buses without fixed predecessors (i.e., $j \notin \mathcal{A}_{ri}$), the headway is calculated as: $h^r_{ij} = d^r_{ij} - d^r_{i(j-1)}$. Formally, this is expressed as:

$$h^r_{ij} = d^r_{ij} - d^r_{i(j-1)}, \quad j \in \mathcal{J}^k_{ri}, i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{metro}} \tag{52a}$$

$$h^r_{ij} = \begin{cases} d^r_{ij} - d^r_{i(j-1)}, & \text{if} \quad j \notin \mathcal{A}_{ri} \\ d^r_{ij} - \sum\limits_{j' \in \mathcal{J}^k_{ri} \setminus \{j\}} \bar{x}^{ri}_{j'j} d^r_{ij'}, & \text{otherwise} \end{cases}, \quad j \in \mathcal{J}^k_{ri}, i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{bus}} \tag{52b}$$

Remark 4.1. *The constraint matrix in M3 has a desirable block structure, allowing it to be partitioned into* ($|\mathcal{R}_{\text{bus}}| + |\mathcal{R}_{\text{metro}}|$) *line-level subproblems that can be solved in parallel. Each subproblem retains a continuous QP form, ensuring rapid solutions.*

Given that M3 discards logical variables and corresponding constraints, we introduce conditional constraints to dynamically add continuous constraints to M3. As more logical variables are fixed, we can construct the following conditional constraints. Once the antecedent is satisfied, the consequent is added to M3. Constraint (53) indicates that if $z^{ri}_{j'j} = 1$, then $d^r_{ij} \geq d^r_{ij'}$ holds, i.e., vehicle j' departs before vehicle j. Constraint (54) represents the converse: if $z^{ri}_{j'j} = 0$, then $d^r_{ij} < d^r_{ij'}$ holds, i.e., vehicle j' does not depart before vehicle j. Constraint (55) ensures that if $x^{ri}_{j'j} = 1$ (vehicle j' is the immediate predecessor of vehicle j), then $\hat{d}^r_{ij}$, representing the departure time of the immediate predecessor of vehicle j is not later than the departure of vehicle j'.

$$z^{ri}_{j'j} \rightarrow (d^r_{ij} - d^{r'}_{ij'} \geq 0), \quad j, j' \in \mathcal{J}^k_{ri}, i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{bus}} \tag{53}$$

$$\neg z^{ri}_{j'j} \rightarrow (d^r_{ij} - d^{r'}_{ij'} + \varepsilon \leq 0), \quad j, j' \in \mathcal{J}^k_{ri}, i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{bus}} \tag{54}$$

$$x^{ri}_{j'j} \rightarrow (\hat{d}^r_{ij} - d^r_{ij'} \leq 0), \quad j, j' \in \mathcal{J}^k_{ri}, i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{bus}} \tag{55}$$

Besides, we have

$$y^{r'j'}_{rij} \rightarrow (d^r_{ij} - (a^{r'}_{ij'} + E^{r'}_{ri}) \geq 0), \quad j \in \mathcal{J}^k_{ri}, j' \in \mathcal{J}^k_{r'i}, r' \in \mathcal{C}_{ri}, i \in \mathcal{Q}_r, r \in \mathcal{R}_{\text{metro}} \cup \mathcal{R}_{\text{bus}} \tag{56a}$$

$$\neg y^{r'j'}_{rij} \rightarrow (d^r_{ij} - (a^{r'}_{ij'} + E^{r'}_{ri}) + \varepsilon \leq 0), \quad j \in \mathcal{J}^k_{ri}, j' \in \mathcal{J}^k_{r'i}, r' \in \mathcal{C}_{ri}, i \in \mathcal{Q}_r, r \in \mathcal{R}_{\text{metro}} \cup \mathcal{R}_{\text{bus}} \tag{56b}$$

which indicate that, if $y^{r'j'}_{rij} = 1$, then vehicle j departs after vehicle j' arrives plus the transfer time; conversely, if $y^{r'j'}_{rij} = 0$, then vehicle j departs before vehicle j' arrives plus the transfer time.

Actually, M3 initially removes constraints related to logical variables. As the logical variables become progressively fixed during branching, the constraints corresponding to the satisfaction of these logical conditions are added back (Hooker, 2000). Although the added constraints differ in form from the original ones, being simpler and avoiding big-M formulations, they are fundamentally derived from the original constraints.

This process reflects the outcome of branching and aims to preserve the logical relationships inherent in the original constraints. In LBB, with continuization and removing logical components, we develop M2 and M3 for optimization. M2 provides lower bounds for M1 and upper bounds when combined with feasibility recovery. M3 helps to generate high-quality feasible solutions by offering optimal estimates of logical variables as inputs for M2. By continuously updating fixed logical values in M2 and dynamically adding constraints to M3, the optimization and logic inference are effectively integrated.

4.6. The overall approach

Based on the above descriptions, Algorithm 4.1 details the overall LBB procedure.

Algorithm 4.1. *Procedure of LBB for solving M1.*

Step 1. *Initialize the sets of lower and upper bounds, i.e.,* $\mathcal{L} \leftarrow \emptyset$, $\mathcal{U} \leftarrow \emptyset$. *Initialize the set of indices of search variables fixed to 0,* $\mathcal{I}_{\bar{\ell}} \leftarrow \emptyset$; *the set of indices of search variables fixed to 1,* $\mathcal{I}_{\bar{u}} \leftarrow \emptyset$; *the set of indices of fixed search variables,* $\mathcal{T} \leftarrow (\mathcal{I}_{\bar{\ell}}, \mathcal{I}_{\bar{u}})$; *the set of* $\mathcal{T}$, $\mathcal{P} \leftarrow \{\mathcal{T}\}$.

Repeat

Step 2. *Choose an index set* $\mathcal{T}$ *from* $\mathcal{P}$ *with the minimum lower bound, and take it out. Set* $\mathcal{P} \leftarrow \mathcal{P} \setminus \{\mathcal{T}\}$, $\mathcal{L} \leftarrow \mathcal{L} \setminus \{\min(\mathcal{L})\}$.

Step 3. *For the following logic and optimization procedures,* ***do***

Step 3.1 *Logic procedure.*

Step 3.1.1 *Select search variables and construct corresponding subnodes.*

Step 3.1.2 *For each subnode from branching,* ***do***

(1) Update $\mathcal{T}$ *with the current fixed values of logical variables. Perform domain reduction (as Section 4.4). If detecting infeasibility, then delete this subnode.*

(2) Solve M2 with fixed logical variables. If feasible, obtain the optimal value $\mathcal{F}_{\bar{\ell}}$ *and the optimal solutions.*

(3) If $\mathcal{F}_{\bar{\ell}} < \min(\mathcal{U})$, $\mathcal{P} \leftarrow \mathcal{P} \cup \{\mathcal{T}\}$, *calculate the upper bounds* $\mathcal{F}_{\bar{u}}$ *using the feasibility recovery methods referred in Section 4.2 with the optimal solutions of M2, and set* $\mathcal{U} \leftarrow \mathcal{U} \cup \{\mathcal{F}_{\bar{u}}\}$ *and* $\mathcal{L} \leftarrow \mathcal{L} \cup \{\mathcal{F}_{\bar{\ell}}\}$.

Step 3.2 *Optimization procedure.*

Step 3.2.1 *With* $\mathcal{T}$, *perform domain reduction (as Section 4.4). Formulate continuous constraints according to conditional constraints* (53)–(56).

Step 3.2.2 *With continuous constraints formulated in Step 3.2.1, and the values of* $\widetilde{p}_{rij}^{\text{wait}}$ *and* $\widetilde{p}_{rij}^{\text{board}}$ *from the feasible solution with the current minimum upper bound, construct and solve M3.*

Step 3.2.3 *With logic estimates from solving M3 in Step 3.2.2, solve M2. If feasible, obtain the optimal value* $\mathcal{F}_{\bar{u}}$ *and the optimal solutions, update* $\mathcal{U} \leftarrow \mathcal{U} \cup \{\mathcal{F}_{\bar{u}}\}$.

Step 4 *Update the gap* $\mathcal{G} = (\min(\mathcal{U}) - \min(\mathcal{L})) / \min(\mathcal{L})$.

until *the time limit is reached or the gap falls below a certain tolerance.*

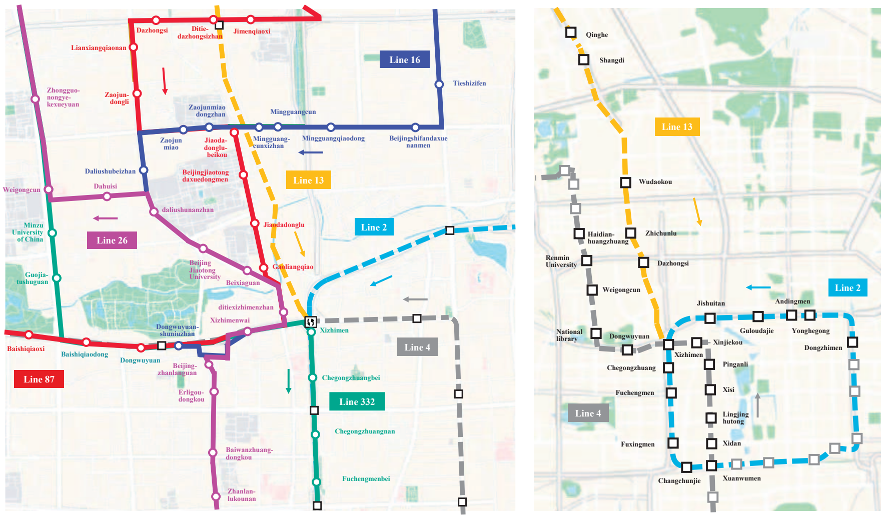
The main algorithmic contributions of the proposed LBB method can be summarized as follows: We design a feasibility recovery technique, based on the relaxation solution, to construct high-quality upper bounds. By exploiting the logical structure of M1, we develop a problem-specific branching strategy and an effective domain reduction method. A synergistic integration of domain reduction and branching enables efficient variable fixing throughout the search process. Finally, we introduce customized primal heuristics based on reduced optimization problems to accelerate the generation of high-quality feasible solutions. Essentially, LBB exploits problem properties to naturally separate logical and continuous variables, leveraging their respective advantages in logical inference and rapid solution processes. It enables efficient computation and conformance to a real-time implementation.

5. Numerical experiments

This section presents computational experiments to evaluate the performance of the proposed MIQP model and LBB method. Section 5.1 introduces the instance generation. Section 5.2 presents the computational results, providing detailed visualization and analysis. Section 5.3 analyzes the computational efficiency of LBB. Section 5.4 discusses independent adjustment strategies, further demonstrating the necessity of coordinated adjustment. Section 5.5 presents experiments under different weight settings. Section 5.6 assesses the impact of different modeling and methodological components on performance.

5.1. Network description and data preparation

We focus on the urban public transport network surrounding the transfer hub of Xizhimen in Beijing. As depicted in Figure 2, the network comprises four bus lines: 16, 26, 87, 332, and three metro lines: 2, 4, 13, involving eighty stations. The operating direction analyzed in this case is indicated. As a key transfer hub, Xizhimen station intersects substantial vehicle and passenger flows between bus and metro systems. There are bus-bus, metro-metro, metro-bus, and bus-metro transfer passenger flows.



(a) Partial Beijing bus network around station Xizhimen. (b) Partial Beijing metro network around station Xizhimen.

Figure 2: Illustration of the partial Beijing public transport network around station Xizhimen.

In this case study, the vehicle capacity is set as 85 for buses and 1450 for trains. The boarding and alighting delay rates for dwell times are specified as 0.50 s/pax for buses and 0.05 s/pax for trains. The time required for vehicle door operations is 3 s. The transfer time is 120, 50, 150, 150 s for metro-metro, bus-bus, metro-bus, bus-metro passengers. Disturbances for vehicle running and dwell times range from 0 to 180 s. The maximum values for adjustments of dwell and running times are set as 15 and 20 s, while the minimum values are set as -15 and -20 s, respectively. Regarding the line frequencies, the departure headway for bus lines is set to 480 s. For metro lines, the headway is 240 s for lines 2 and 4, and 270 s for line 13. The average passenger arrival rates for each line are illustrated in Figure 3. In our implementation, the coefficients for departure punctuality and headway regularity are set to 0.02, while the coefficient for the number of stranded passengers is set to 25. This setting balances the contribution of each term based on their relative magnitudes. Regarding the scale of the optimization problem, M1 is solved over a 30-minute prediction horizon at each decision stage. We consider a total of 10 decision stages under the rolling horizon scheme. All numerical experiments are conducted by MATLAB R2021a, with quadprog employed as the quadratic programming solver. To ensure the applicability of our method in real-time settings, we impose a 3-second time limit for LBB. This choice is motivated by standards commonly adopted in the real-time optimization-based transit control literature. For example, Seman et al. (2019) report an average solution time of 1.81 s, and Bian et al. (2023) demonstrate that their method typically completes within 3 s in real-world scenarios. Our time limit ensures that our approach remains practical for real-time applications.

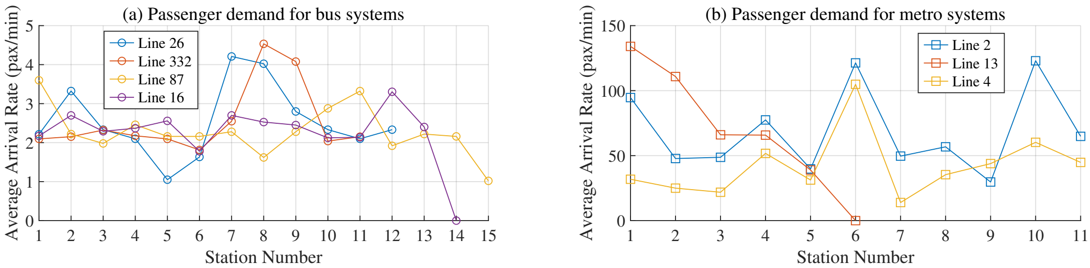


Figure 3: Illustration of passenger demands.

5.2. Computational results

Applying the parameter settings described in Section 5.1, we implement the proposed method in the studied network. To evaluate the performance of our approach, we compare our proposed strategy (PS) with a reference strategy (RS). RS combines the commonly adopted schedule-based bus control (Wu et al., 2017) and automatic train regulation (Chen et al., 2022). It involves adjustments to dwell and running times to align vehicle operations as closely as possible with the planned timetable (specifying planned dwell, running, arrival, and departure times). The implementation procedure of RS is illustrated in Appendix B. In practice, when a vehicle experiences delays, RS allows it to depart promptly while ensuring the safety of passenger boarding and alighting, thereby reducing delays. Conversely, when a vehicle operates ahead of schedule, RS allows for vehicle holding to maintain closer adherence to the schedule. Therefore, RS is an effective adjustment method and can be considered a relevant benchmark. In RS, the planned dwell, running, arrival, and departure times are set from the planned vehicle timetables.

Table 2: Comparison of computational performances under RS and PS.

Infrastructure	Strategy	Departure deviation (s^2)	Headway deviation (s^2)	Stranded passengers
Bus	RS	$3.29 \cdot 10^6$	$2.75 \cdot 10^6$	$2.87 \cdot 10^2$
	PS	$2.97 \cdot 10^6$	$9.49 \cdot 10^5$	$1.44 \cdot 10^2$
Metro	RS	$1.21 \cdot 10^5$	$8.99 \cdot 10^4$	$3.34 \cdot 10^3$
	PS	$1.34 \cdot 10^5$	$1.00 \cdot 10^5$	$1.84 \cdot 10^3$
Total	RS	$3.41 \cdot 10^6$	$2.84 \cdot 10^6$	$3.63 \cdot 10^3$
	PS	$3.10 \cdot 10^6$	$1.05 \cdot 10^6$	$1.99 \cdot 10^3$

The performance of PS and RS in terms of departure deviation, headway deviation, and stranded passengers are compared in Table 2. The results show that RS underperforms PS across all three metrics for the urban transit network. Specifically, RS leads to higher departure and headway deviations, reaching $3.41 \cdot 10^6$ and $2.84 \cdot 10^6$ s^2, respectively, causing more vehicle delays and stranding $3.63 \cdot 10^3$ passengers. In contrast, PS actively adjusts vehicle operations based on optimization objectives, taking future disturbances and passenger flows into account to make more "forward-looking" decisions through a RH scheme. Compared to RS, PS improves delay adjustments by reducing departure and headway deviations by 9.17% and 63.02%, respectively, and decreases the number of stranded passengers by 45.24%. Notably, in the metro system, PS results in fewer stranded passengers but higher timetable deviations compared to RS. This is because PS, guided by the objective function, prioritizes improving passenger service quality and reducing stranded passengers, at the cost of some punctuality and regularity in the metro system. To further analyze the results, Figures 4 and 5 compare partial bus and train timetables under RS and PS. The blue dashed lines represent planned timetables, red lines show actual timetables, and blue squares mark the serial numbers of specific vehicles.

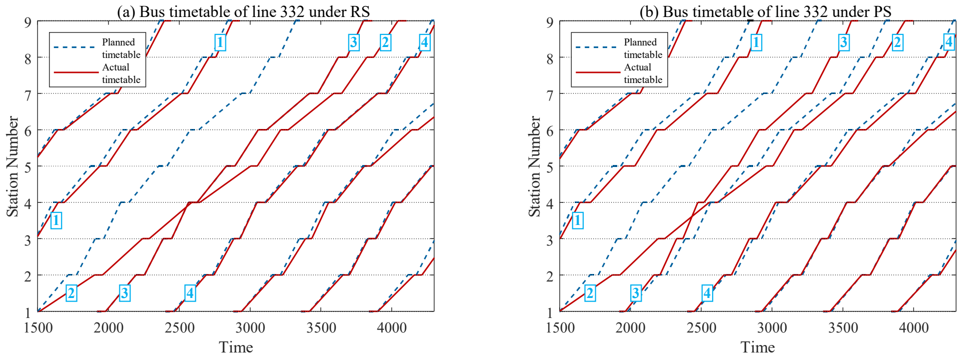


Figure 4: Illustration of bus timetables with RS and PS.

As shown in Figure 4, bus 3 overtakes bus 2 before station 4 due to the significant delays experienced by bus 2. This overtaking prevents further delays for the subsequent buses, reducing the negative effects on operational efficiency and service quality. Under PS, buses 1 and 4 are intentionally delayed, while bus 3 operates ahead of schedule, to maintain more regular headways between buses 1 and 3, 3 and 2, 2 and 4 after the overtaking event. Compared to RS, these adjustments under PS result in increased departure deviations but reduced headway deviations, ultimately reducing bus bunching and improving overall service quality.

The performance of RS and PS for the metro system is illustrated in Figure 5. Both approaches effectively mitigate train delays, with PS demonstrating superior delay recovery performance for trains 2, 3, and 4. This is achieved by anticipating disturbances through the RH framework, allowing preemptive adjustments, such as early departures for trains 2 and 3 at station 4 and train 4 at station 2. For train 1, however, PS intentionally introduces timetable deviations, with early arrivals and departures at transfer stations, thus reducing stranded passenger at the busy station. Train 1 then slows down in subsequent segments to realign with the planned timetable at less busy stations 7 and 8.

Figure 6 provides a comparison of departure deviation, headway deviation and stranded passengers between RS and PS at each decision stage. It demonstrates the same pattern as Figures 4 and 5. In metro systems, the primary advantage of PS lies in reducing stranded passengers. In bus systems, PS exhibits

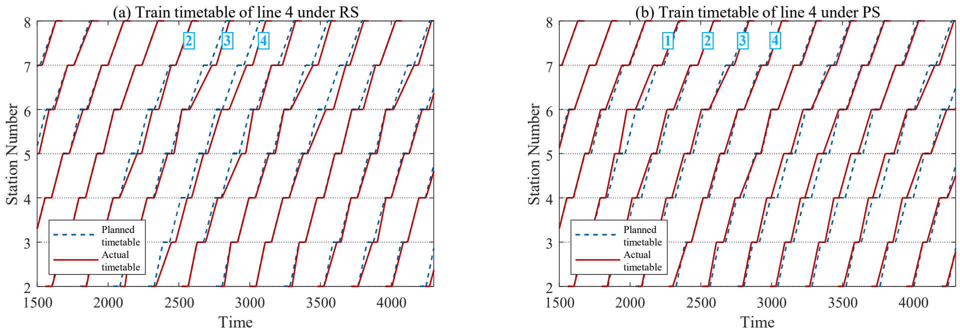
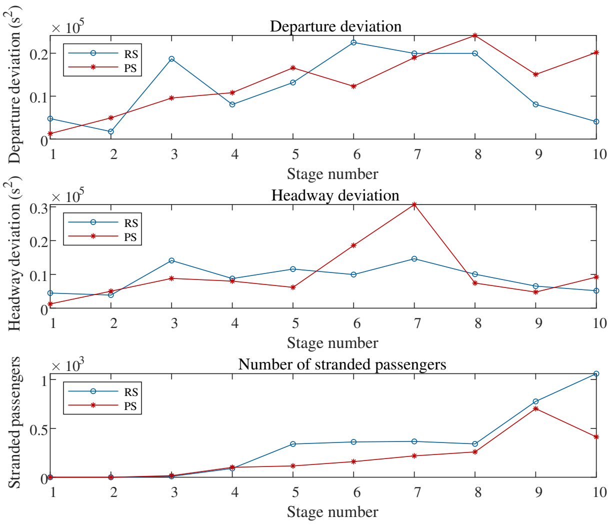
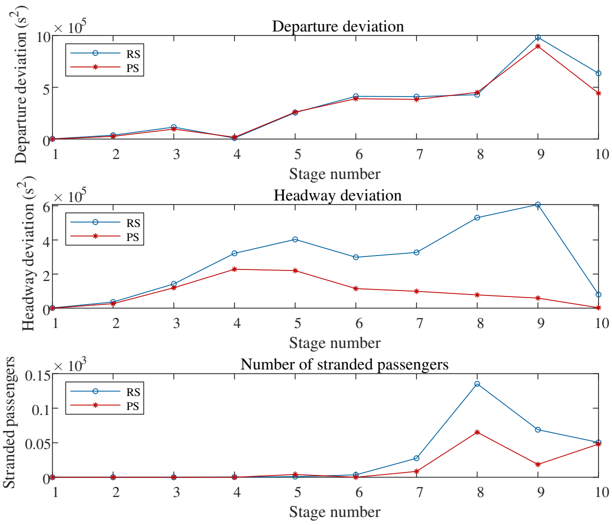


Figure 5: Illustration of train timetables with RS and PS.



(a) Performance indicators for metro trains in different stages.



(b) Performance indicators for buses in different stages.

Figure 6: Illustration of performance indicators for different decision stages.

the most notable advantage in headway regularity, consequently reducing stranded numbers and enhancing service quality.

5.3. Computational efficiency

RTCP demands rapid generation of bus and train adjustment strategies within few seconds in response to dynamic inputs and conditions. The ability to respond quickly within a short time limit is crucial. To demonstrate the computational efficiency of LBB, we compare its performance with the commercial solver Gurobi in solving M1 for RTCP. For the Gurobi settings, we use the default configuration, with the following stopping criteria: a default MIPGap of $1 \cdot 10^{-4}$ and a time limit of 3 s. Additionally, when measuring the time required for Gurobi to reach a solution of the same quality as that of LBB, we remove the maximum time limit and instead set the termination condition based on the objective function value achieved by LBB. Furthermore, we incorporate problem-specific knowledge by applying domain reduction techniques during the Gurobi implementation. Table 3 presents the comparison results, with row 1 indicating the stage number. Rows 2 and 3 show the optimality gap achieved by Gurobi and LBB within three seconds. "-" indicates no feasible solution is found within the time limit. Row 4 presents the time it takes for Gurobi to find a solution with the same objective function value as LBB.

Table 3: Comparison of computational efficiency between Gurobi and LBB.

Stage	1	2	3	4	5	6	7	8	9	10
Gurobi gap	34.14%	10.59%	9.54%	39.10%	-	-	4.94%	7.23%	5.04%	0.56%
LBB gap	1.17%	1.46%	4.67%	3.38%	1.84%	2.27%	3.18%	2.56%	2.59%	2.01%
Gurobi time (s)	15.46	13.46	8.37	9.66	21.23	7.30	5.95	6.29	4.71	1.41

From Table 3, the average optimality gap for solutions generated by LBB is 2.51% across all stages, with the maximum gap not exceeding 5%. This indicates that LBB is capable of quickly producing high-quality solutions in a short time, making it suitable for real-time applications. In contrast, Gurobi delivers inferior results, with an average optimality gap of 13.89%, and fails to find feasible solutions for 2 stages. This highlights the limitation of Gurobi in meeting real-time requirements. When tasked with finding solutions with objective values matching those of LBB, Gurobi requires longer computational times, averaging 9.39 s. Overall, LBB demonstrates superior computational efficiency and is well-suited for addressing real-time coordination challenges in dynamic operational environments.

5.4. Comparison with independent adjustment strategies

To further demonstrate the effectiveness of coordinated adjustment, we compare the proposed strategy (PS) to other optimization-based strategies: (1) Line-level adjustment strategy (LS): adjustments for each line are made independently. (2) System-level adjustment strategy (SS): adjustments for bus and metro systems are made separately.

The comparative results are shown in Table 4. All three optimization-based strategies improve upon RS in terms of departure punctuality and headway regularity, to varying degrees. Among them, LS achieves the lowest departure deviation (11.36% reduction compared to RS) and headway deviation (64.79% reduction compared to RS), but at the cost of service quality, with the highest number of stranded passengers ($3.59 \cdot 10^3$). Since LS overlooks transfer flows in its decision-making process, it fails to accurately capture the coupling between vehicles and passenger flows, leading to the highest number of stranded passengers among

Table 4: Computational results under different strategies.

Strategy	Departure deviation (s^2)	Headway deviation (s^2)	Stranded passengers
RS	$3.41 \cdot 10^6$	$2.84 \cdot 10^6$	$3.63 \cdot 10^3$
LS	$3.02 \cdot 10^6$ (−11.36%, −2.48%)*	$1.00 \cdot 10^6$ (−64.79%, −5.02%)	$3.59 \cdot 10^3$ (−1.01%, +44.68%)
SS	$3.09 \cdot 10^6$ (−9.47%, −0.33%)	$1.04 \cdot 10^6$ (−63.46%, −1.19%)	$2.33 \cdot 10^3$ (−35.89%, +14.58%)
PS	$3.10 \cdot 10^6$ (−9.17%, —)	$1.05 \cdot 10^6$ (−63.02%, —)	$1.99 \cdot 10^3$ (−45.24%, —)

* Each value is followed by its percentage difference from RS and PS, respectively.

the three strategies. SS incorporates passenger transfers within each system, reducing stranded passengers by 35.23% compared to LS. PS further considers transfers between metro and bus systems, thus resulting in the fewest stranded passengers (44.68% fewer than LS, while 14.58% fewer than SS). These results show that PS achieves a balance between operational efficiency and service quality. Compared to independent adjustment for each line or system, coordinated adjustment reduces stranded passengers and improves service quality, contributing to a more efficient urban public transport system under disturbances.

5.5. Experiments under different weight configurations

To further investigate the impact of weight settings in the objective function, we conduct a series of experiments by adjusting the weights associated with departure deviation, headway deviation, and stranded passengers. Based on the reference weight setting ($\zeta_1 = 0.02$, $\zeta_2 = 0.02$, $\zeta_3 = 25$), we construct six groups of comparative experiments by individually increasing each weight coefficient to 10 times and 50 times its original value. The computational results under each weight configuration are summarized in Table 5.

Table 5: Computational results under different weight configuration.

	Departure deviation (s^2)	Headway deviation (s^2)	Stranded passengers
Reference	$3.10 \cdot 10^6$	$1.05 \cdot 10^6$	$1.99 \cdot 10^3$
$\zeta_1 \cdot 10$	$2.46 \cdot 10^6$	$1.72 \cdot 10^6$	$2.08 \cdot 10^3$
$\zeta_1 \cdot 50$	$2.29 \cdot 10^6$	$2.68 \cdot 10^6$	$2.15 \cdot 10^3$
$\zeta_2 \cdot 10$	$3.95 \cdot 10^6$	$8.40 \cdot 10^5$	$1.98 \cdot 10^3$
$\zeta_2 \cdot 50$	$4.02 \cdot 10^6$	$7.99 \cdot 10^5$	$2.27 \cdot 10^3$
$\zeta_3 \cdot 10$	$3.95 \cdot 10^6$	$1.23 \cdot 10^6$	$1.58 \cdot 10^3$
$\zeta_3 \cdot 50$	$3.41 \cdot 10^6$	$2.96 \cdot 10^6$	$1.19 \cdot 10^3$

From the computational results, increasing the weight on departure deviation (ζ_1) effectively improves punctuality but leads to higher headway deviations and more stranded passengers, indicating a trade-off between punctuality and service regularity. Similarly, increasing the weight on headway deviation (ζ_2) reduces headway irregularity. A tenfold increase in ζ_2 slightly lowers the number of stranded passengers, suggesting that improving regularity can also enhance service quality. However, excessive emphasis (e.g., a 50-fold increase) leads to service degradation. Increasing the weight on stranded passengers (ζ_3) significantly reduces stranding but at the cost of greater departure and headway deviations. These results highlight the need to balance weights carefully to achieve a good compromise between punctuality, regularity, and passenger service quality according to operator priorities. Additionally, we compare the performance of Gurobi and our LBB method under various weight combinations. The results show that Gurobi has an average optimality gap of 14.32%, and in 18 out of 60 instances, it fails to find any feasible solution within the time limit. In contrast, our LBB method consistently finds feasible solutions within 3 s for all stages, achieving an average optimality gap of 4.90%. These results further confirm the robustness and efficiency of our approach across diverse problem settings.

5.6. Evaluation of model and methodological components

To evaluate the impact of key modeling flexibilities introduced in our model, we conduct two simplified model variants for comparison: (1) considering unlimited capacity (UC) and (2) prohibiting overtaking (PO). The computational results are compared with PS, which is obtained from the proposed model incorporating both overtaking and capacity limits. Table 6 summarizes the corresponding computational results.

Table 6: Computational results under different model flexibilities.

Strategy	Departure deviation (s^2)	Headway deviation (s^2)	Stranded passengers
UC	$3.01{\cdot}10^6$	$1.01{\cdot}10^6$	$2.83{\cdot}10^3$
PO	$3.06{\cdot}10^6$	$1.58{\cdot}10^6$	$2.08{\cdot}10^3$
PS	$3.10{\cdot}10^6$	$1.05{\cdot}10^6$	$1.99{\cdot}10^3$

As shown in Table 6, UC achieves the lowest departure and headway deviations. However, due to ignoring real-world vehicle capacity limits, it results in the highest number of stranded passengers. In contrast, PS slightly compromises punctuality and regularity but significantly reduces passenger stranding, thereby improving service quality. For PO, departure deviation remains moderate among the three strategies. However, without overtaking, a delayed bus cannot be bypassed by a faster one. This can lead to uneven headways, bus bunching, thus increasing the number of stranded passengers. Compared with PO, PS introduces slight sacrifices in schedule adherence but achieves much better headway regularity and passenger service by accounting for overtaking flexibility.

To evaluate the effects of the methodological components in our solution approach, we conduct a series of comparative experiments. Specifically, we compare the following four variants: (1) BB: the standard branch-and-bound method; (2) BB_BR: BB with the incorporation of our branching strategies; (3) BB_BD: BB_BR with domain reduction; (4) LBB: the complete proposed method that integrates all methodological components; (5) Gurobi. The optimality gap of these solutions from the four methods within the 3-second time limit is reported in Table 7.

Table 7: Comparison among optimality gaps of methods with different methodological components.

Stage	1	2	3	4	5	6	7	8	9	10
BB	18.96%	29.76%	32.38%	49.34%	13.17%	23.14%	15.82%	19.42%	21.86%	5.20%
BB_BR	16.15%	22.42%	31.97%	45.24%	9.58%	23.14%	11.38%	16.34%	17.69%	5.20%
BB_BD	6.30%	4.52%	7.22%	9.30%	5.66%	7.93%	7.50%	11.83%	7.11%	3.33%
LBB	1.17%	1.46%	4.67%	3.38%	1.84%	2.27%	3.18%	2.56%	2.59%	2.01%
Gurobi	34.14%	10.59%	9.54%	39.10%	-	-	4.94%	7.23%	5.04%	0.56%

As shown in Table 7, Gurobi exhibits unstable performance, yielding the highest optimality gap among all methods in stage 1, and even failing to obtain a feasible solution within the time limit in stages 5 and 6. In contrast, although the four BB-based methods produce solutions of different quality, all successfully obtain feasible solutions within the time limit, demonstrating superior practical applicability. Among all BB-based methods, the standard BB method yields the largest optimality gaps, with an average of 22.91%

across all stages. Incorporating the proposed branching strategy (BB_BR) leads to moderate improvements, reducing the average gap by 2.99% compared to BB and improving solutions in 8 out of 10 stages. Building on this, BB_BD applies domain reduction to further tighten the feasible region during branching, resulting in a more substantial improvement with a 12.84% reduction in the optimality gap. In comparison, the proposed LBB approach, which additionally integrates a primal heuristic, achieves the best performance, producing the smallest gaps across all stages with an average gap of just 2.51%.

6. Conclusions

This paper presents an MIQP model to coordinate transfers between buses and metro trains under disturbances by adjusting timetables in real time. The model captures the dynamics of bus and train traffic, passenger loads, overtaking, and capacity limits, offering a more precise and realistic representation of interactions between vehicles, transfer, and non-transfer passengers. To address the computational challenges posed by the mixed-integer nature of the problem and real-time requirements, we design an efficient LBB method. The novelty of LBB lies in its ability to leverage the respective advantages of discrete and continuous components in logical inference and rapid solution processes. LBB integrates logic-based ideas for branching and domain reduction, alongside tailored optimization procedures for accelerated computation.

The numerical experiments, based on a practical case study of a central area with a key transfer hub, validate the practicality of the proposed methods. The computational results demonstrate that the coordinated adjustment strategy, derived from the integrated decision-making process for both bus and metro operations, significantly reduces stranded passengers compared to independent adjustment strategies, thereby improving service quality. Additionally, the solution method demonstrates strong computational performance, enabling efficient real-time implementation in real-world settings. Overall, our coordination adjustment method provides valuable insights and support for the practical application of real-time decision-making for MaaS within urban public transport systems.

The proposed strategies are well-suited for managing vehicle delays within a certain range, but real-time recovery from more severe disruptions requires further exploration. Future research could focus on enhancing bus and train coordination in response to disruptions, by incorporating strategies such as skip-stopping and partial cancellations. Furthermore, in future work, we will consider the uncertainty of passenger arrival rates, alighting rates, etc. We can formulate the bus and metro coordination problem as a stochastic optimization problem through chance-constraint, scenario-set or other methods, to generate more stable and reliable coordination effects in practice.

Appendix A. The linearization process

The linearization process for M1 is described as follows. Constraint (8) is equivalent as the following constraints:

$$d_{ij}^r - a_{ij}^r \geq S + B_{rj} p_{rij}^{\text{board}}, \quad j \in \mathcal{J}_{ri}^k, i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{bus}} \tag{A.1a}$$

$$d_{ij}^r - a_{ij}^r \geq S + B_{rj} \beta_{ij}^r p_{r(i-1)j}^{\text{load}}, \quad j \in \mathcal{J}_{ri}^k, i \in \mathcal{S}_r, r \in \mathcal{R}_{\text{bus}} \tag{A.1b}$$

By introducing auxiliary real variables $c^{ri}_{j'j}$, equation (17) can be reformulated as

$$p^{\text{wait}}_{rij} = \lambda^r_{ij}(d^r_{ij} - \hat{d}^r_{ij}) + \sum_{r'\in\mathcal{C}_{ri}, j'\in\mathcal{J}^k_{r'i}} q^{r'j'}_{rij} + \sum_{j'\in\mathcal{J}^k_{ri}\setminus\{j\}} c^{ri}_{j'j}, \quad j\in\mathcal{J}^k_{ri}, i\in\mathcal{S}_r, r\in\mathcal{R}_{\text{bus}} \tag{A.2a}$$

$$0 \le c^{ri}_{j'j} \le Mx^{ri}_{j'j}, \quad j, j'\in\mathcal{J}^k_{ri}, i\in\mathcal{S}_r, r\in\mathcal{R}_{\text{bus}} \tag{A.2b}$$

$$c^{ri}_{j'j} \le p^{\text{wait}}_{rij'} - p^{\text{board}}_{rij'}, \quad j, j'\in\mathcal{J}^k_{ri}, i\in\mathcal{S}_r, r\in\mathcal{R}_{\text{bus}} \tag{A.2c}$$

$$c^{ri}_{j'j} \ge (p^{\text{wait}}_{rij'} - p^{\text{board}}_{rij'}) - M(1 - x^{ri}_{j'j}), \quad j, j'\in\mathcal{J}^k_{ri}, i\in\mathcal{S}_r, r\in\mathcal{R}_{\text{bus}} \tag{A.2d}$$

Similarly, equation (23) is equivalent to:

$$0 \le q^{r'j'}_{rij} \le \rho^{r'j'}_{ri}\beta^{r'}_{ij'}p^{\text{load}}_{r'(i-1)j'}, \quad j\in\mathcal{J}^k_{ri}, j'\in\mathcal{J}^k_{r'i}, r'\in\mathcal{C}_{ri}, i\in\mathcal{Q}_r, r\in\mathcal{R}_{\text{metro}} \tag{A.3a}$$

$$q^{r'j'}_{rij} \ge \rho^{r'j'}_{ri}\beta^{r'}_{ij'}p^{\text{load}}_{r'(i-1)j'} - \beta^{r'}_{ij'}\rho^{r'j'}_{ri}V_{r'}[1 - (y^{r'j'}_{rij} - y^{r'j'}_{ri(j-1)})],$$
$$j\in\mathcal{J}^k_{ri}, j'\in\mathcal{J}^k_{r'i}, r'\in\mathcal{C}_{ri}, i\in\mathcal{Q}_r, r\in\mathcal{R}_{\text{metro}} \tag{A.3b}$$

$$q^{r'j'}_{rij} \le \beta^{r'}_{ij'}\rho^{r'j'}_{ri}V_{r'}(y^{r'j'}_{rij} - y^{r'j'}_{ri(j-1)}), \quad j\in\mathcal{J}^k_{ri}, j'\in\mathcal{J}^k_{r'i}, r'\in\mathcal{C}_{ri}, i\in\mathcal{Q}_r, r\in\mathcal{R}_{\text{metro}} \tag{A.3c}$$

For equation (24), introducing auxiliary binary variable $b^{r'j'j''}_{rij}$ renders it equivalent to

$$b^{r'j'j''}_{rij} \le x^{ri}_{j''j}, \quad j, j''\in\mathcal{J}^k_{ri}, j'\in\mathcal{J}^k_{r'i}, i\in\mathcal{Q}_r, r'\in\mathcal{C}_{ri}, r\in\mathcal{R}_{\text{bus}} \tag{A.4a}$$

$$b^{r'j'j''}_{rij} \le y^{r'j'}_{rij''}, \quad j, j''\in\mathcal{J}^k_{ri}, j'\in\mathcal{J}^k_{r'i}, i\in\mathcal{Q}_r, r'\in\mathcal{C}_{ri}, r\in\mathcal{R}_{\text{bus}} \tag{A.4b}$$

$$b^{r'j'j''}_{rij} \ge x^{ri}_{j''j} + y^{r'j'}_{rij''} - 1, \quad j, j''\in\mathcal{J}^k_{ri}, j'\in\mathcal{J}^k_{r'i}, i\in\mathcal{Q}_r, r'\in\mathcal{C}_{ri}, r\in\mathcal{R}_{\text{bus}} \tag{A.4c}$$

$$0 \le q^{r'j'}_{rij} \le \rho^{r'j'}_{ri}\beta^{r'}_{ij'}p^{\text{load}}_{r'(i-1)j'}, \quad j, j''\in\mathcal{J}^k_{ri}, j'\in\mathcal{J}^k_{r'i}, i\in\mathcal{Q}_r, r'\in\mathcal{C}_{ri}, r\in\mathcal{R}_{\text{bus}} \tag{A.4d}$$

$$q^{r'j'}_{rij} \ge \rho^{r'j'}_{ri}\beta^{r'}_{ij'}p^{\text{load}}_{r'(i-1)j'} - \beta^{r'}_{ij'}\rho^{r'j'}_{ri}V_{r'}[1 - (y^{r'j'}_{rij} - \sum_{j''\in\mathcal{J}^k_{ri}\setminus\{j\}} b^{r'j'j''}_{rij})],$$
$$j, j''\in\mathcal{J}^k_{ri}, j'\in\mathcal{J}^k_{r'i}, i\in\mathcal{Q}_r, r'\in\mathcal{C}_{ri}, r\in\mathcal{R}_{\text{bus}} \tag{A.4e}$$

$$q^{r'j'}_{rij} \le \beta^{r'}_{ij'}\rho^{r'j'}_{ri}V_{r'}(y^{r'j'}_{rij} - \sum_{j''\in\mathcal{J}^k_{ri}\setminus\{j\}} b^{r'j'j''}_{rij}), \quad j, j''\in\mathcal{J}^k_{ri}, j'\in\mathcal{J}^k_{r'i}, i\in\mathcal{Q}_r, r'\in\mathcal{C}_{ri}, r\in\mathcal{R}_{\text{bus}} \tag{A.4f}$$

Appendix B. Procedure of RS

Algorithm Appendix B.1. *Procedure of RS.*

For *each vehicle j at each station i of each line r* ***do***

Step 1. *Adjustment to dwell times for vehicles.*

Step 1.1. *Calculate the expected departure time $\tilde{d}^r_{ij}$ as the arrival time plus the passenger boarding and alighting time (if there is remaining capacity, boarding is always allowed).*

Step 1.2. *Check: If $\tilde{d}^r_{ij}$ is earlier than the planned departure time $\check{d}^r_{ij}$, then set the holding time as $g^r_{ij} = \min\{\check{d}^r_{ij} - \tilde{d}^r_{ij}, G^r_{\max}\}$; otherwise, set $g^r_{ij} = 0$ for immediate departure.*

Step 2. *Adjustment to running times for trains.*

Step 2.1. *Calculate the expected arrival time $\tilde{a}^r_{ij}$ as the time the train is anticipated to arrive at the station using the planned running time.*

Step 2.2. *Check: If $\tilde{a}^r_{ij}$ is earlier than the planned arrival time $\check{a}^r_{ij}$, then set $u^r_{ij} = \min\{\check{a}^r_{ij} - \tilde{a}^r_{ij}, U^r_{\max}\}$ for deceleration; if it is later, set $u^r_{ij} = \max\{\check{a}^r_{ij} - \tilde{a}^r_{ij}, U^r_{\min}\}$ for acceleration; otherwise, $u^r_{ij} = 0$.*

Step 3. *Maintain safety headway for trains.*

Check: If the current train and the preceding one do not meet the safety constraint (9)*, adjust* u_{ij}^r *and* g_{ij}^r *until the constraint is satisfied.*

end for

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