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1 Modelling the adhesion enhancement at the wheel-rail interface:
2 the role of surface roughness and plastic deformation during rail
3 sanding operation

4
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10
11 **ABSTRACT**

12 Efficient train operation relies on optimal traction at the wheel-rail interface, which can
13 be compromised by factors such as water and/or contamination (e.g. leaf, sand, oil,
14 and surface wear). This study introduces a finite element model to assess adhesion
15 enhancement at the wheel-rail interface, with a focus on the impact of sand particles
16 during the sanding process. Surface roughness is initially introduced to quantify its
17 effect on adhesion, followed by the inclusion of rail plastic deformation. By integrating
18 these two factors, the model provides a comprehensive framework for evaluating the
19 complex mechanisms influencing adhesion at the wheel-rail interface, especially in
20 real-world train operations where surface conditions and contaminants—in this case
21 sand fragments—interact. This approach addresses the existing gap in understanding
22 how rail surface condition and rail plastic deformation contribute to adhesion
23 enhancement during the sanding process, offering new insights for optimising railway
24 maintenance strategies.

25
26 **Keywords:** wheel-rail adhesion; particle breakage; surface roughness; plastic
27 deformation; finite element model; cohesive interface elements.

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29 Manuscript prepared for *Railway Engineering Science*

30 1 Introduction

31 Efficient and safe train operation relies on the adhesion coefficient in the wheel-rail
32 interface. The level of adhesion coefficient in the wheel-rail interface significantly
33 influences the performance and reliability of railway transportation systems [1,2].
34 However, challenges such as wet weather and surface contamination, and variations
35 in rail and wheel conditions can lead to reduced adhesion, posing significant
36 operational risks [3–6]. In particular, low adhesion during traction operation leads to
37 delays and general disruption due to a longer acceleration time. During braking
38 operation, insufficient adhesion results in extended braking distances and may lead to
39 signals passed at danger (SPADs) or collisions in extreme cases [7,8]. When low
40 adhesion is detected, rail sanding from an on-board device is utilised to increase the
41 adhesion level during the wheel-rail contact [9,10].

42 Rail grinding, employed as a maintenance measure to restore the worn rail surface
43 [11–13], results in a very rough rail contact surface which can affect the adhesion
44 coefficient and the influence of contamination. In order to evaluate the restored rail,
45 roughness parameters have been adopted from metrology to quantify the surface after
46 grinding. Mesaritis et al. [14] used a laboratory grinding process to investigate the
47 effect of grinding parameters on the post-grinding roughness of the rail. Later, they
48 utilised a full-scale testing facility to evaluate the performance of three rail grades after
49 the grinding process [15]. Besides the quality control of the grinding process,
50 Lundmark et al. [16] used surface roughness to quantify the tribological performance
51 of the wheel/rail interface in the laboratory by using a two-disc rolling/sliding machine.
52 They found that rougher wheel specimens resulted in lower wear compared to
53 smoother specimens, particularly reducing the wear on the rail disc. Furthermore,
54 Wang et al. [17] investigated the effect of surface roughness on friction and wear of
55 the rail by adding different lubrications at the nanoscale. Their results indicate that
56 reducing surface roughness, up to a critical point, can effectively alleviate friction and
57 wear under mixed lubrication conditions.

58 However, a roughness parameter by itself is not enough to monitor the adhesion level
59 during train operation. Although the rail is made of steel, it suffers from cyclic loading
60 that leads to permanent deformation, which also affects the adhesion level during the
61 wheel-rail contact. Tomlinson et al. [18] used twin disc tests to simulate cyclic loading

62 experienced by rail steel in service, from which a Shear Yield Stress – Plastic Shear
63 Strain (SYS-PSS) relationship can be generated for the tested sample. Later, Zhao
64 and Li [19] used a 3D transient FE approach considering elasto-plasticity to explore
65 the frictional rolling contact between the wheel and rail. They found the contact patch
66 increased in size, shifted forward in the rolling direction, and changed from an ellipse
67 into an asymmetric oval. Recently, Meyer et al. [20] developed a new methodology for
68 finite element simulations of elasto-plastic rolling contact loading. It could provide
69 higher accuracy for a given rolling length due to periodic boundary conditions.

70 Since the wear and plastic deformation of the rail occurs simultaneously during train
71 operation, it is not adequate to investigate the adhesion level at the wheel-rail interface
72 by considering only one of them. As a result, Pletz et al. [21] developed a 3D finite
73 element model of the full-scale test rig. Its result can be transferred to different 2D
74 models to calculate crack tip loading and cyclic deformations of rough surfaces.
75 Following this, they introduced a quasistatic finite element model to calculate the
76 plastic deformations of railway rails caused by rolling/sliding wheels [22]. By employing
77 a 3D dynamic model, Vo et al. [23] found that higher adhesion levels enlarge slip
78 regions and surface damage, while worn profiles increase contact pressure and alter
79 the contact patch geometry. Recently, Spiryagin et al. [24] developed an algorithm to
80 calculate the contact stresses with different surface roughness considering elastic and
81 plastic deformations at the wheel-rail interface.

82 While research on the variation of adhesion during wheel-rail contact has seen
83 advancements with the inclusion of additional parameters to replicate real-world
84 conditions, there remains a scarcity of studies investigating the enhancement of
85 adhesion resulting from the presence of a third body at the wheel-rail interface by
86 incorporating the rail surface roughness and plastic deformation. This gap particularly
87 relates to scenarios encountered during the sanding process in train operation.
88 Studies [25–28] on the characterization of sand particles during the sanding process
89 have highlighted their critical role in adhesion enhancement, demonstrating how
90 particle morphology, such as size and shape, influences the efficiency of adhesion
91 improvement at the wheel-rail interface. Additionally, the FE model proposed by Zhang
92 et al. [29] demonstrated a numerical framework for monitoring the adhesion coefficient
93 at the wheel-rail surface. Therefore, following their study, this paper introduces an
94 enhanced FE model to investigate the adhesion variation at the wheel-rail interface,

95 triggered by sand fragments, surface roughness, and rail plasticity. The surface
96 roughness is introduced to the model first to quantify its effect on adhesion
97 enhancement. Then, the plastic deformation due to cyclic loading is assigned to the
98 rail to investigate its influence on the existing adhesion enhancement contributed by
99 surface roughness.

100

101 2 Methodology

102 The adhesion enhancement triggered by sand particle breakage and fragments is
103 quantified during the wheel-rail contact [29]. The whole operation is simulated through
104 the finite element method (FEM) using Abaqus/Explicit package and the breakage
105 behaviour is achieved numerically using cohesive interface elements (CIEs).

106 2.1 Breakage modelling

107 In FEM modelling, cohesive elements serve a valuable purpose in simulating
108 adhesives, bonded interfaces, and fractures. For each application area, the
109 constitutive response of these elements varies and mainly depends on certain
110 assumptions regarding the related deformation and stress states. In this study, the
111 traction-separation model has been adopted where the intermediate glue element is
112 very thin and for practical purposes could be considered as zero thickness. The failure
113 mechanism (see Fig. 1) of this model has two components: damage initiation criterion
114 and damage evolution law.

115 2.1.1 Damage initiation

116 As the name indicates, the damage initiation marks the start of the degradation that
117 occurs from a point in the material. The procedure of degradation begins when the
118 stresses along the CIE satisfy the selected damage initiation criteria. In this study, the
119 Quadratic nominal stress criterion is used and it assumes the damage will initiate when
120 a quadratic interaction function including the nominal stress ratios reaches a value of
121 one. This criterion can be illustrated as:

$$122 \left(\frac{t_n}{t_n^0}\right)^2 + \left(\frac{t_s}{t_s^0}\right)^2 = 1 \quad (1)$$

123 where t_n and t_s represent the nominal stresses in normal and first shear directions,
124 respectively, while t_n^0 and t_s^0 represent the maximum allowable nominal stresses in
125 normal and first shear directions, respectively.

126 2.1.1 Damage evolution

127 Damage evolution can be characterised by energy dissipation during the damage
128 process, which is also known as fracture energy. Considering the nature of two-
129 dimensional modelling, only two modes of fracture have been involved, the normal
130 mode (mode I) and the shear mode (mode II). The mixed-mode (mode I + mode II)
131 fracture is adopted since it is the dominant failure behaviour of brittle materials

151 accurately represents how the material responds to loading. In this study, the rail is
152 modelled with an elastic/plastic material description that uses a combination of
153 isotropic and kinematic hardening.

154 The equivalent stress σ_v established by Von Mises is employed to check the behaviour
155 of a material in the principal plane stress case. Therefore, the yield stress σ_y can be
156 expressed as follows to predict the plastic deformation.

$$157 \quad \sigma_y = \sigma_v = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2} \quad (3)$$

158 Here, σ_1 and σ_2 are the two principal stresses. When $\sigma_v \leq \sigma_y$, the material is
159 dominated by the elastic deformation. As soon as the σ_v becomes bigger than σ_y , the
160 mechanical behaviour of material transmits from elastic to plastic.

161 The flow rule is another fundamental concept of plastic deformation that defines how
162 the material behaves after the yielding point. It not only describes the relationship
163 between stress and strain, but also specifies the direction and rate of plastic
164 deformation.

165 Two flow rules are adopted in this study: a) isotropic hardening, where the σ_y exists
166 an increment in all directions with the accumulation of plastic strain. It increases
167 uniformly through the entire plastic deformation process; b) kinematic hardening,
168 where the shape of the yield surface remains constant but its centre shifts. It is
169 influenced by the accumulated plastic strain and past loading history. Fig. 2 shows the
170 changes in yield surface when the plastic deformation follows different flow rules.

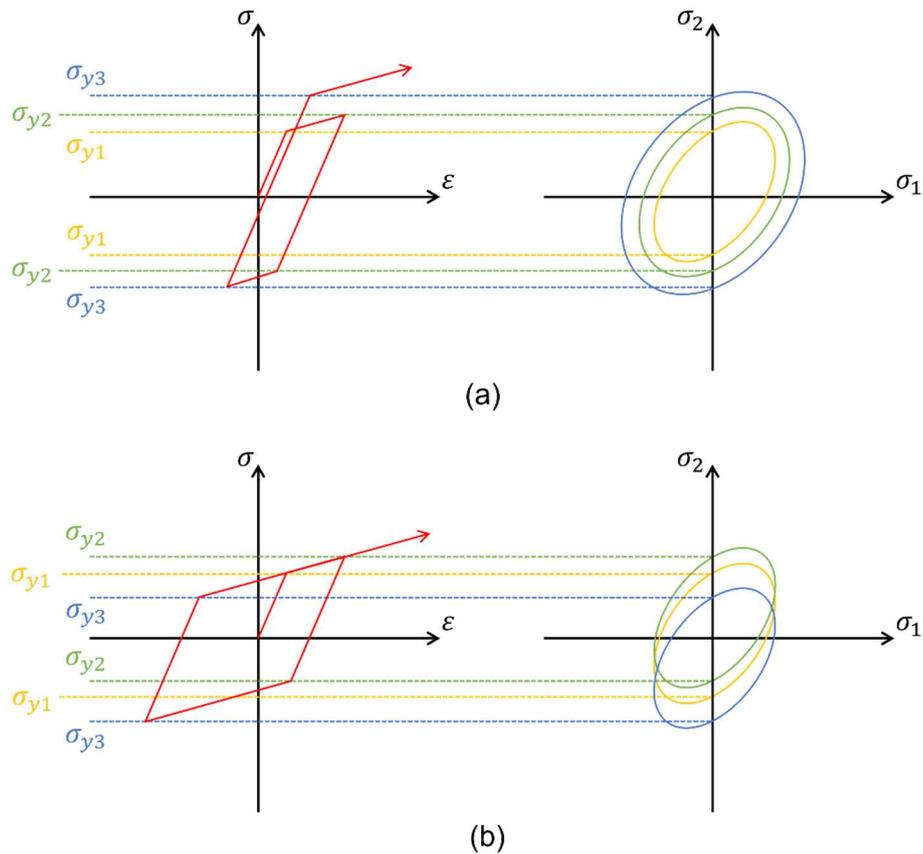


Fig. 2 Two adopted flow rules. a) isotropic hardening and b) kinematic hardening.

171

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173

174 2.3 Roughness

175 Surface roughness constitutes a facet of surface texture, and it is characterised by the
 176 variances in the orientation of a real surface's normal vector from its idealized shape.

177 In railway engineering, the roughness with a broad spectrum of wavelengths is present
 178 on the running surface of the rail. This rail roughness, in turn, triggers the generation

179 of high-frequency wheel-rail contact forces, vibrations, and rolling noise. When the
 180 rails are worn beyond the specific allowable limits, their profiles must be restored to

181 avoid undesired contact conditions in the wheel-rail interface. According to BS
 182 EN13231-3-2012 [31], the arithmetic mean surface roughness (R_a) of the rail should

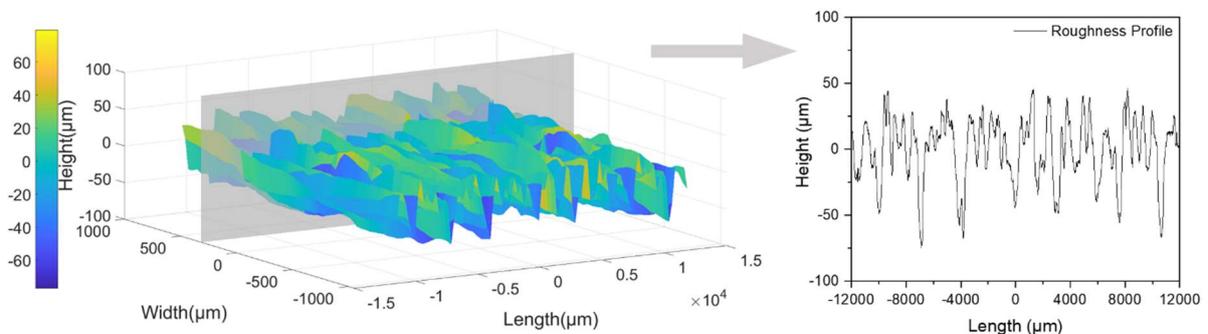
183 not exceed $10 \mu m$ along more than 16% of the measure length after grinding.

184 2.3.1 Data acquisition

185 The roughness of rail surface immediately after the grinding is measured following the
 186 procedure below [14]:

- 187 • Clean the surface using an acetone wipe
- 188 • Apply a layer of Microset 101 Thixotropic replicating compound to the disc's
- 189 surfaces, add a piece of stiff backing paper, and wait 6 minutes for the replica
- 190 to set
- 191 • Use an Alicona InfiniteFocusSL 3D measurement system with a 5X
- 192 magnification lens and a cut-off wavelength of 800 μm to create a digital
- 193 representation of the surface of the replica

194 After the digital surface of the replica has been created, two sets of $20\text{mm} \times 2\text{mm}$ area
 195 are abstracted from the parent surface, one with medium roughness ($R_a < 10 \mu\text{m}$,
 196 labelled A) and one with very high roughness ($R_a \approx 20 \mu\text{m}$, labelled B). A typical
 197 surface profile is plotted in Fig. 3.



198
 199 **Fig. 3** Representation of roughness profile from the extreme case.

200 2.3.2 Data analysis

201 By using a lateral resolution of $4 \mu\text{m}$, five hundred two-dimensional roughness profiles
 202 can be generated from each surface. In order to reduce computational complexity and
 203 make the analysis more manageable while preserving essential information, the
 204 original profiles have been performed a sub-sampling by a factor of 5 to reduce from
 205 five hundred to one hundred. The roughness descriptors listed in Appendix A are used
 206 to quantify each single roughness profile.

207 By selecting the representative values of different descriptors, 2 profiles from case A
 208 and 35 profiles from case B have been chosen to characterise the selected surface
 209 area, and their descriptors are listed in Appendix B.

210 2.3.3 Profile regeneration

211 The next step is to generate the rail roughness in the numerical software. Each
 212 representative profile consists of over six thousand points. Therefore, a progressive

213 data reduction has been conducted on each profile to reduce the geometrical
214 complexity while still maintaining accuracy. The procedure is downsampling the data
215 iteratively by selecting every n th point, where n starts from 2. For each downsampled
216 dataset, R_a of the simplified profile has been calculated to compare with the original
217 one. This downsampling process will continue until the error exceeds 1%, at which
218 point the previous downsampled dataset will be adopted. Now, the representative
219 profiles have been simplified while retaining the essential roughness characteristics
220 within a certain error threshold.

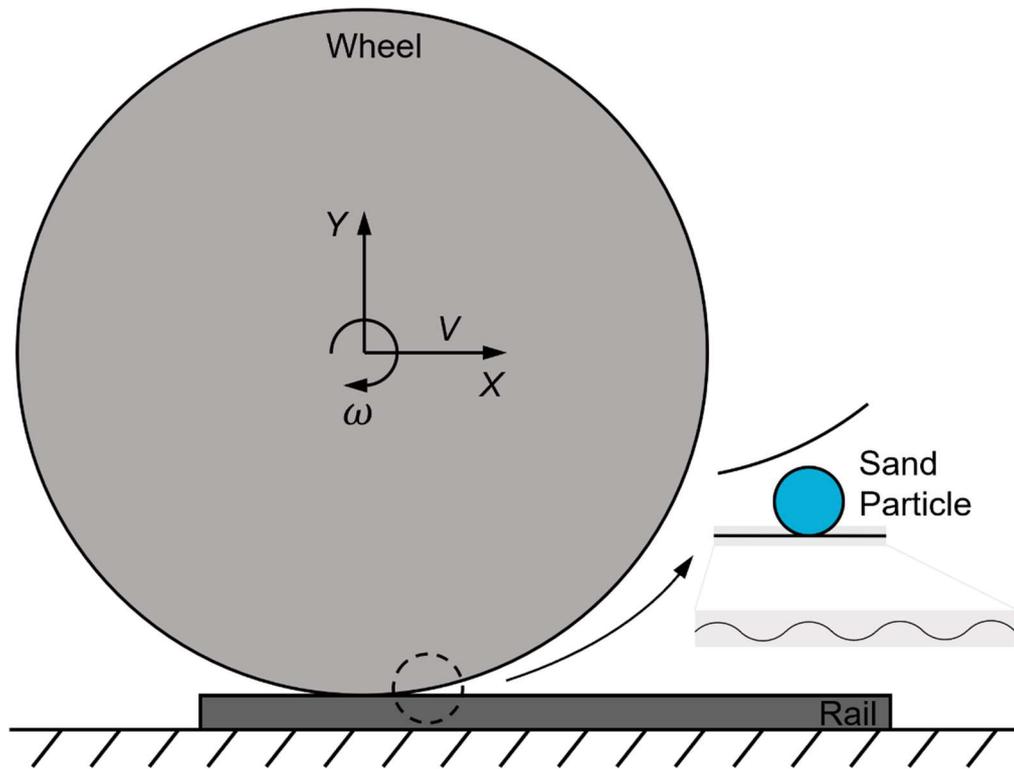
221 From the previous study [29], a rail length of 110mm is required to complete the
222 traction and braking operations during the railway sanding process. Thus, each 20mm
223 representative profile has been duplicated to meet the requirement. A Python script
224 has been developed to create rail parts reflecting the extended roughness profiles in
225 Abaqus.

226 3 Model set-up and materials

227 By utilising an existing two-dimensional wheel-rail model [29], the rail surface now has
228 been amended by considering the roughness to simulate the interactions between
229 sand fragments and uneven rail, which is a typical operating condition at the real
230 wheel-rail interface. Additionally, in order to have a more realistic representation of
231 complex material responses, a combined hardening material property has been
232 assigned to the rail using the Abaqus/Explicit package to reproduce the effect from
233 wear. This could provide a better simulation of the rail interaction under different
234 loading conditions and further reveal the associated adhesion enhancement.

235 3.1 Numerical model

236 As shown in Fig. 4, The wheel-rail model employed in this study comprises three
237 components: a) the wheel functions as a rigid body with displacement in the X direction.
238 An angular velocity is switched on for traction operation and switched off for braking
239 operation; b) the rail is a deformable body that takes into account both elastic and
240 plastic behaviours. It has been fixed at the bottom and the sides; c) the sand particle
241 operates as a deformable body equipped with cohesive interface elements to simulate
242 fragmentation.



243

244

Fig. 4 Schematic diagram of the wheel-rail model

245 In order to compare the adhesion enhancement from previous research [29], the sand
 246 particles used in this study range from 0.71 mm to 2 mm. Taking into account the
 247 average fragment size of 0.1 mm observed from experiments [32], a uniform element
 248 size of 0.1 mm is allocated to all sand particles. This element size effectively captures
 249 the realistic fragmentation process of sand particles at the wheel-rail interface.
 250 Furthermore, according to a mesh sensitivity analysis [33], the element size to mesh
 251 size ratio employed in this study falls within an acceptable range.

252 To achieve an accurate solution while maintaining a reasonable model size,
 253 nonuniform meshing has been applied to the rail. Since the R_a of the representative
 254 roughness profiles are in micrometers, finer mesh size is used at the wheel-rail contact
 255 within the roughness profile length (ca. 110 mm). It enables the capture of stress-strain
 256 behaviour of the rail surface influenced by the roughness on this micro-level. While the
 257 other side of the rail has been meshed using coarser elements. Beyond the roughness
 258 profile range, the rail is modelled as flat to expedite the simulation.

259 All deformable bodies are created using 3-node linear plane strain triangle (CPE3)
 260 elements, which can undergo local deformation based on the current nodal forces. In
 261 the case of sand particles, cohesive elements (COH2D4) are positioned at the

262 interface of CPE3 elements. Since cohesive interface elements (CIEs) are of zero
 263 thickness, the overall mesh geometry remains unaltered, but every element is now
 264 connected by the CIEs. When the normal stress or shear stress at the CIEs reaches
 265 a predefined threshold, the CIEs initiate a process of vanishing, resulting in debonding
 266 between elements. The composition of sand particles used in this study is detailed in
 267 Table 1.

268 **Table 1** Mesh composition of sand particles.

Particle size (<i>mm</i>)	Element size (<i>mm</i>)	CPE3 elements
0.71	0.10	92
1.00	0.10	179
2.00	0.10	733

269 **3.2 Materials**

270 Given that the wheel is a rigid body and does not require any material definition, the
 271 only two materials requiring definition are: a) the sand particle, which employs a
 272 traction-separation model, and b) the rail, which utilizes a combined hardening model.

273 **3.2.1 Sand particle**

274 The material parameters of quartz sand have been carefully chosen to explore the
 275 adhesion enhancement triggered by sand fragmentation at the wheel-rail contact.
 276 Table 2 provides a summary of the material parameters utilised in this study, which
 277 encompass density (ρ), elastic modulus (E), Poisson's ratio (ν), tensile strength
 278 (N_{max}) and shear strength (S_{max} and T_{max}) and CIEs' stiffness (k_n and k_s), fracture
 279 energy (G_n , and G_s) and material parameter (η). These parameters have been
 280 sourced from the previous study [29].

281

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287 **Table 2** Material parameters of sand particle.

	Parameter	Value
Solid Elements (Particle)		
Density	ρ (kg/m^3)	2500
Young's Modulus	E (GPa)	63
Poisson's ratio	ν	0.22
Cohesive interface elements (CIEs)		
Normal stiffness	k_n (N/mm^2)	63000
First shear stiffness	k_s (N/mm^2)	31500
Tensile strength	N_{max} (MPa)	25
First shear strength	S_{max} (MPa)	12
Mode I fracture energy	G_n (N/mm)	0.1
Mode II fracture energy	G_s (N/mm)	0.2
Material parameter	η	2
Contact law		
Particle-to-Structure friction coefficient	μ	0.5

288

289 3.2.2 The rail

290 Since the rail exhibits both isotropic and kinematic hardening when undergoing
 291 external loads, the combined isotropic/kinematic model in Abaqus/Explicit package
 292 has been adopted to represent the realistic mechanical behaviours of the rail. It
 293 provides a more accurate approximation to the stress-strain relation and therefore a
 294 better prediction of the adhesion enhancement. The Young's modulus and the
 295 Poisson's ratio of the rail are 210 GPa and 0.3, respectively. The density of the rail is
 296 7800 kg/m^3 .

297 3.2.2.1 Isotropic parameters

298 According to the work of Chaboche [34], the isotropic hardening can be described by
 299 a change in the yield stress σ_y shifting from its initial value σ_{y0} , corresponding to the
 300 equivalent plastic strain $\bar{\epsilon}_{pl}$ in the following manner:

$$301 \sigma_y = \sigma_{y0} + Q_{\infty}(1 - e^{-b\bar{\epsilon}_{pl}}) \quad (5)$$

302 Here, according to Pletz et al. [8], the two material parameters Q_∞ and b were
 303 established at -40 MPa and 0.04 , respectively. The initial yield stress σ_{y0} was
 304 measured as 320 MPa from the same study.

305 3.2.2.2 Kinematic parameters

306 The kinematic component of the hardening is represented through the yield function
 307 Y . In this function, the backstress tensor α is deducted from the stress tensor σ to
 308 simulate the kinematic hardening $Y(\sigma - \alpha)$. The α is calculated as the sum of each
 309 backstress α_i . The incremental of α_i during plastic deformation is computed using the
 310 backstress rate $\dot{\alpha}_i$, which is derived from σ and the equivalent plastic strain rate $\dot{\epsilon}_{pl}$
 311 according to:

$$312 \quad \dot{\alpha}_i = C_i \frac{1}{\sigma_y} (\sigma - \alpha) \dot{\epsilon}_{pl} - \gamma_i \alpha_i \dot{\epsilon}_{pl} \quad (6)$$

313 Here, the values of the material parameters C_i and γ_i are listed in Table 3. A final
 314 hardening curve is generated by combining the listed six backstress curves to provide
 315 a better capture of the shape of the plastic behaviour.

316 **Table 3** Backstress parameters of the railhead part [21].

i	C_i (GPa)	γ_i (1)
1	350	5000
2	80	1000
3	15	150
4	10	50
5	8	20
6	0.19	0.01

317

318 3.3 Parametric study

319 Three particles with diameters of 0.71 mm , 1 mm , and 2 mm are used to investigate
 320 adhesion enhancement on a selected rough surface, with results compared to those
 321 on a flat surface previously reported by Zhang et al. [29]. Both traction and braking
 322 operations are simulated to replicate the actual sanding process. For this analysis,
 323 only the elastic deformation of the rail is considered, ensuring a focused examination
 324 of the particle size effect due to surface roughness.

325 3.3.1 Traction operation

326 The change of adhesion during traction operation is plotted in Fig. 5. Using Fig. 5(a)
327 as an example, displacement smaller than 45 (40 for the flat surface) is defined as the
328 prior-to-fracture stage and bigger than 70 (65 for the flat surface) is defined as the
329 post-fracture stage. For these two stages, the wheel is only interacting with the rail.
330 From the displacement of 45 to 70 (40 to 65 for the flat surface), the wheel is rolling
331 on top of the sand fragments, which is considered as the fracture stage. By comparing
332 the two profiles using the same particle size, although the roughness case takes a
333 longer time to achieve the fracture stage due to the uneven surface contact, the
334 durations of the fracture stage are the same for both flat and rough surfaces. However,
335 when comparing fracture stages for simulations using different particle sizes, the
336 duration becomes longer as a result of particle size increment.

337 It is noteworthy that the adhesion enhancement on rough surfaces during prior-to-
338 fracture and post-fracture stages are identical for all simulations as no particle is
339 involved. However, the peak value during the fracture stage varies from particle size
340 to particle size. In order to reveal the effect of particle size on adhesion enhancement,
341 the data are normalised based on their respective fragment counts, as shown in Fig.
342 5(d). During the prior-to-fracture and post-fracture stages, since no fragments are
343 involved, the plots from small particles show higher values than the large particles due
344 to the normalisation. However, the participation of fragments during the fracture stage
345 harmonizes this difference, as the plots from different particle sizes converge. This
346 indicates the new surface area generated in the contact due to fragmentation is similar
347 after the normalization.

348 Since the same roughness profile is used to investigate adhesion enhancement due
349 to the changing of particle size, the agreements found here also validate the numerical
350 model in terms of repeatability and reliability.

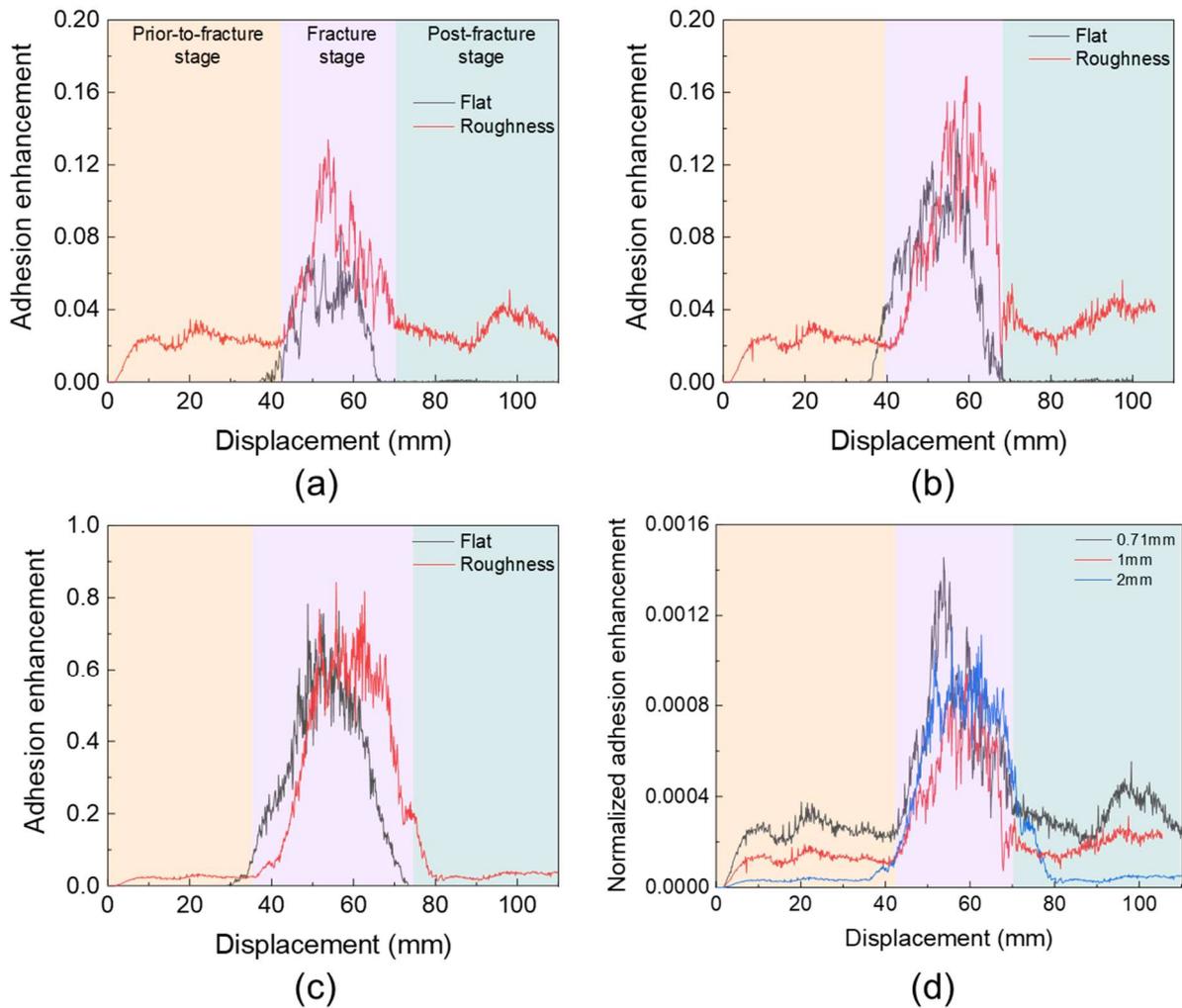
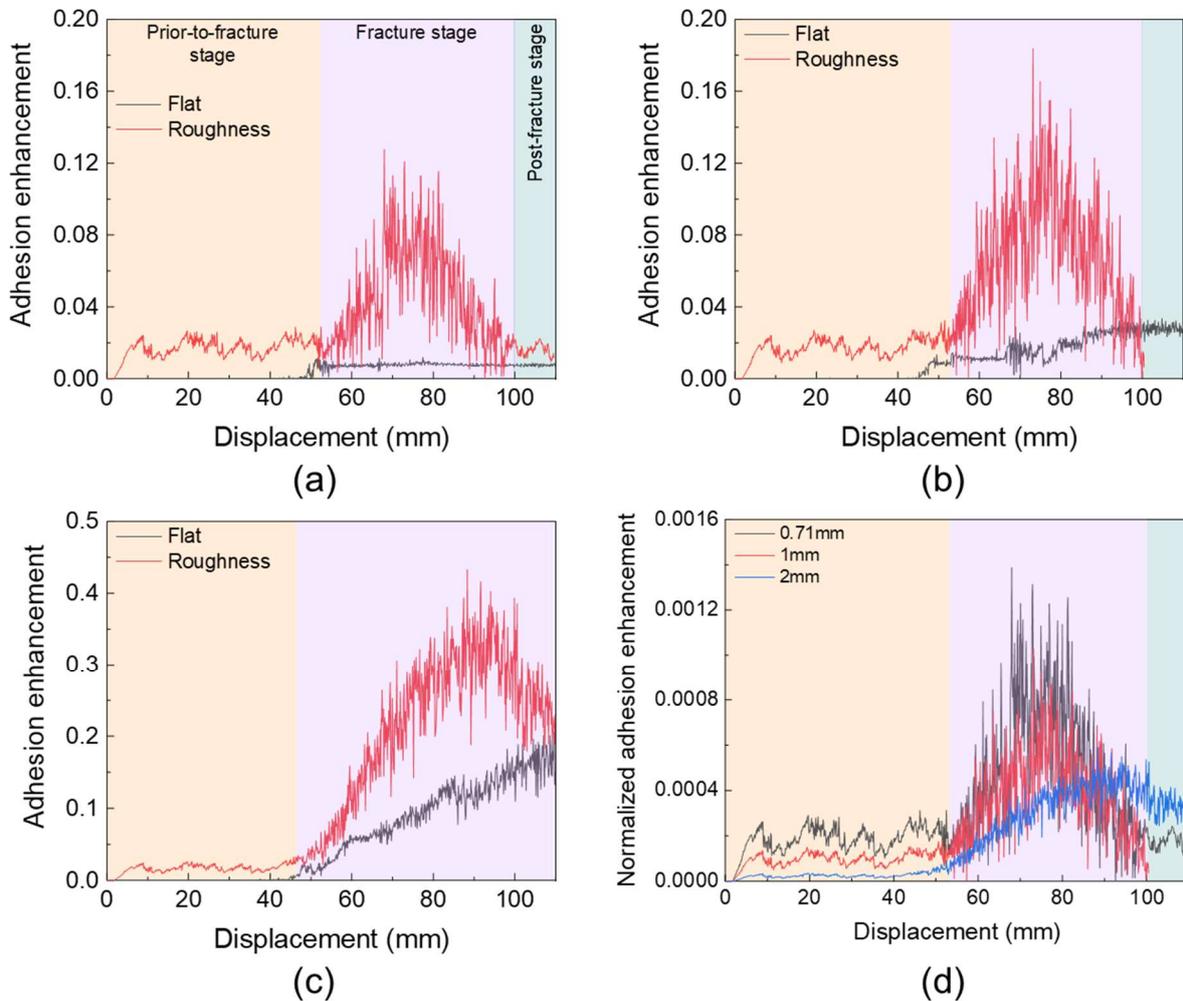


Fig. 5 Size effect during traction operation: (a) 0.71mm, (b) 1mm, (c) 2mm and (d) normalization.

3.3.2 Braking operation

When the train is approaching a station or designated stop, the wheel is sliding (worst case scenario) to decelerate the train from its current speed to a complete halt. Fig. 6 shows the observed adhesion enhancements within this operation. Depending on the participation of sand fragments, the braking operation can also be separated into three stages: the prior-to-fracture stage, the fracture stage, and the post-fracture stage. The adhesion enhancement during the prior-to-fracture stage is comparable for particles of different sizes. Compared to traction operation, for rough surface, the duration of the fracture stage increases tremendously for bigger particles during the braking operation, and therefore the post-fracture stage is not well observed due to the predefined run-length of the model. However, based on the plots from small particles and the stage definition, the post-fracture stage for bigger particles will be similar to its

366 prior-to-fracture stage and only require a longer time to be achieved. This observation
 367 introduces a significant difference from the result using the flat surface where the
 368 adhesion enhancement increases sharply to its peak value during the fracture stage
 369 and then carries the value over into the post-fracture stage till the end of the simulation.



370

371 **Fig. 6** Size effect during braking operation: (a) 0.71mm, (b) 1mm,

372

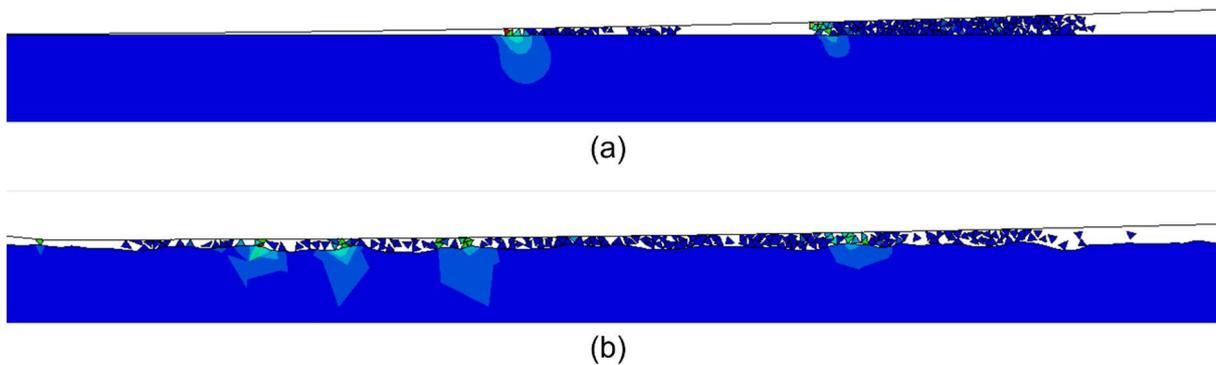
(c) 2mm and (d) normalization.

373 Similarly, a normalization is conducted on the data and plotted in Fig. 6(d). The initial
 374 difference in plots during the prior-to-fracture stage has been discussed in the previous
 375 section. Although the fracture stage still lasts longer for bigger particles, its peak value
 376 now is comparable to results from small particles.

377 3.3.3 Fragment distribution

378 During the braking operation, the adhesion enhancement behaves differently on flat
 379 and rough surfaces. This requires a further investigation of the fragment distribution

380 during the wheel-rail contact to reveal the underlying facts. The observation has been
381 plotted in Fig. 7. In the flat case, the sand fragments have been held and pushed away
382 by the sliding wheel till the end of the simulation. Thus, the number of fragments
383 entering the wheel-rail interface starts to increase till a certain value and then this value
384 will be maintained. This explains the reason why adhesion enhancement increases
385 sharply at the beginning and then becomes stable later on. In contrast, the interlocks
386 between the fragments and the uneven surface generate reaction forces which causes
387 the deformation of the rail. Hence, instead of being pushed away, this deformation
388 enables the sand fragments to be passed by the rolling wheel. Consequently, the plots
389 in Fig. 6 illustrating initial increases in adhesion enhancement, followed by a
390 subsequent decline, have been generated.



391

Fig. 7 Fragment distribution during braking operation:

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(a) flat surface and (b) rough surface.

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In summary, the magnitude of adhesion enhancement in the rough case is depending on the number of fragments entering the wheel-rail interface rather than the particle size. This aligns with the earlier findings using a flat rail [29]. Furthermore, the rough surface permits the wheel to roll over the fragments during braking operation, a feature distinct from the flat surface.

399

4 Results and discussion

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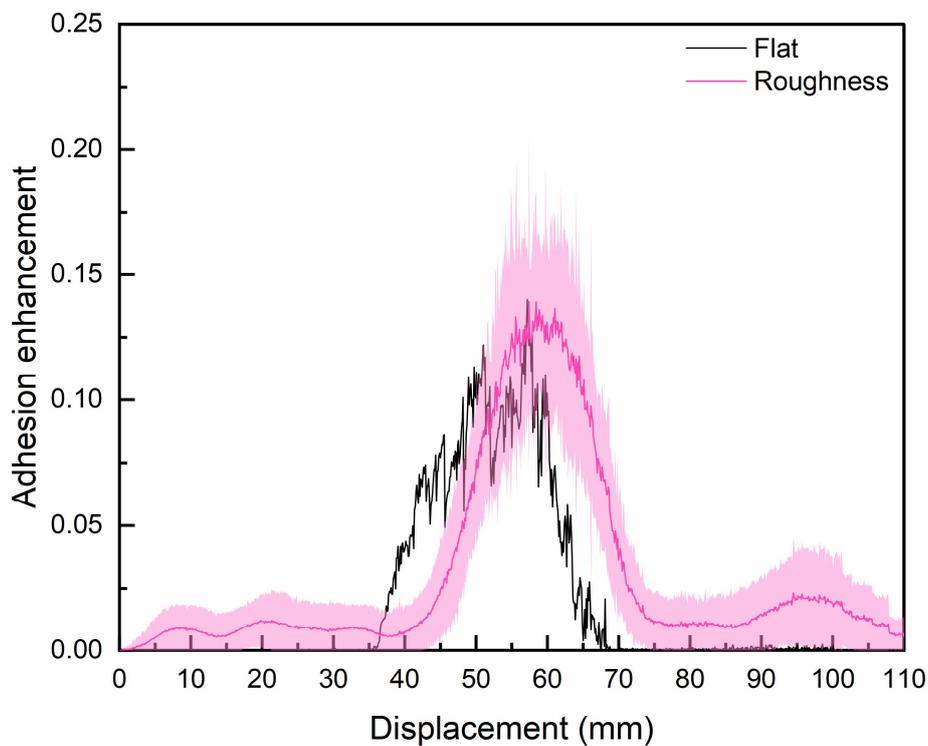
To ensure consistency and comparability, particles with a 1 mm diameter are selected for further investigation. A total of 37 representative roughness profiles from cases A and B are analysed to examine the effect of surface roughness and rail plastic deformation on adhesion enhancement at the wheel-rail interface during traction and braking operations.

405 4.1 Roughness effect

406 To isolate and quantify the effect of surface roughness on adhesion, only the elastic
407 deformation of the rail is considered in this section, allowing for a better understanding
408 of how roughness alone contributes to the overall adhesion enhancement at the
409 wheel-rail interface.

410 4.1.1 Traction operation

411 The results from 37 simulations using rough surfaces under traction operation have
412 been plotted in Fig. 8 to compare with the flat surface. The red solid line indicates the
413 average value of the 37 simulations and the pink shade demonstrates its standard
414 deviation. The black line shows the result of adhesion enhancement from the flat
415 surface.



416

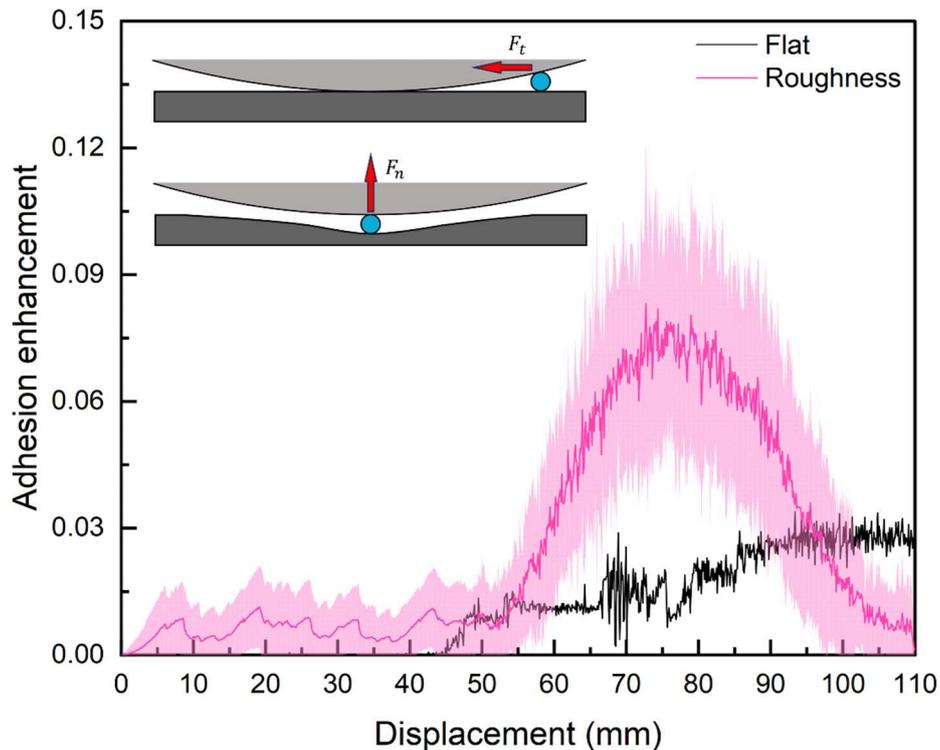
417 **Fig. 8** Roughness effect during traction operation.

418 By comparing the two solid lines, the observation from the previous section using the
419 single roughness profile has been confirmed. Furthermore, the smooth portion of the
420 red line during the prior-to-fracture stage indicates a stable improvement of adhesion
421 due to rough surfaces. It begins to rise shortly when the fracture stage occurs. After a
422 sharp curve, the adhesion begins to decrease in a pattern similar to how it had
423 previously increased. It ends with an adhesion enhancement level comparable to that

424 of the prior-to-fracture stage. This variation trend is identical to the black line from the
425 flat surface. The difference in peak values between the two solid lines can be
426 correlated to the initial improvement due to roughness.

427 4.1.2 Braking operation

428 The simulations are repeated for the braking operation. Similarly, as illustrated in Fig.
429 9, the average value of adhesion enhancements from 37 simulations is represented
430 by the red solid line, with the pink-shaded area indicating its standard deviation. The
431 adhesion enhancement obtained from the flat surface is depicted by the black line.
432 The hill-shaped plot of average value proves the observation of fragment distribution
433 in the previous section that the mechanical interaction of sand fragments gets changed
434 when switching from the flat surface to the rough surface. Rather than being repelled,
435 the fragments can pass through the wheel-rail interface.



436

437 **Fig. 9** Roughness effect during braking operation.

438 During the prior-to-fracture stage, the plot of the red line indicates the adhesion can
439 be improved during braking operation as well. Although its value is comparable to the
440 enhancement during traction operation, the zigzag shape shows the unstable reaction
441 between the sliding wheel and uneven rail. This phenomenon persists throughout the
442 entire operation, even when sand fragments are present, leading to significant noise
443 in the hill-shaped plot during the fracture stage. Due to the horizontal force from the

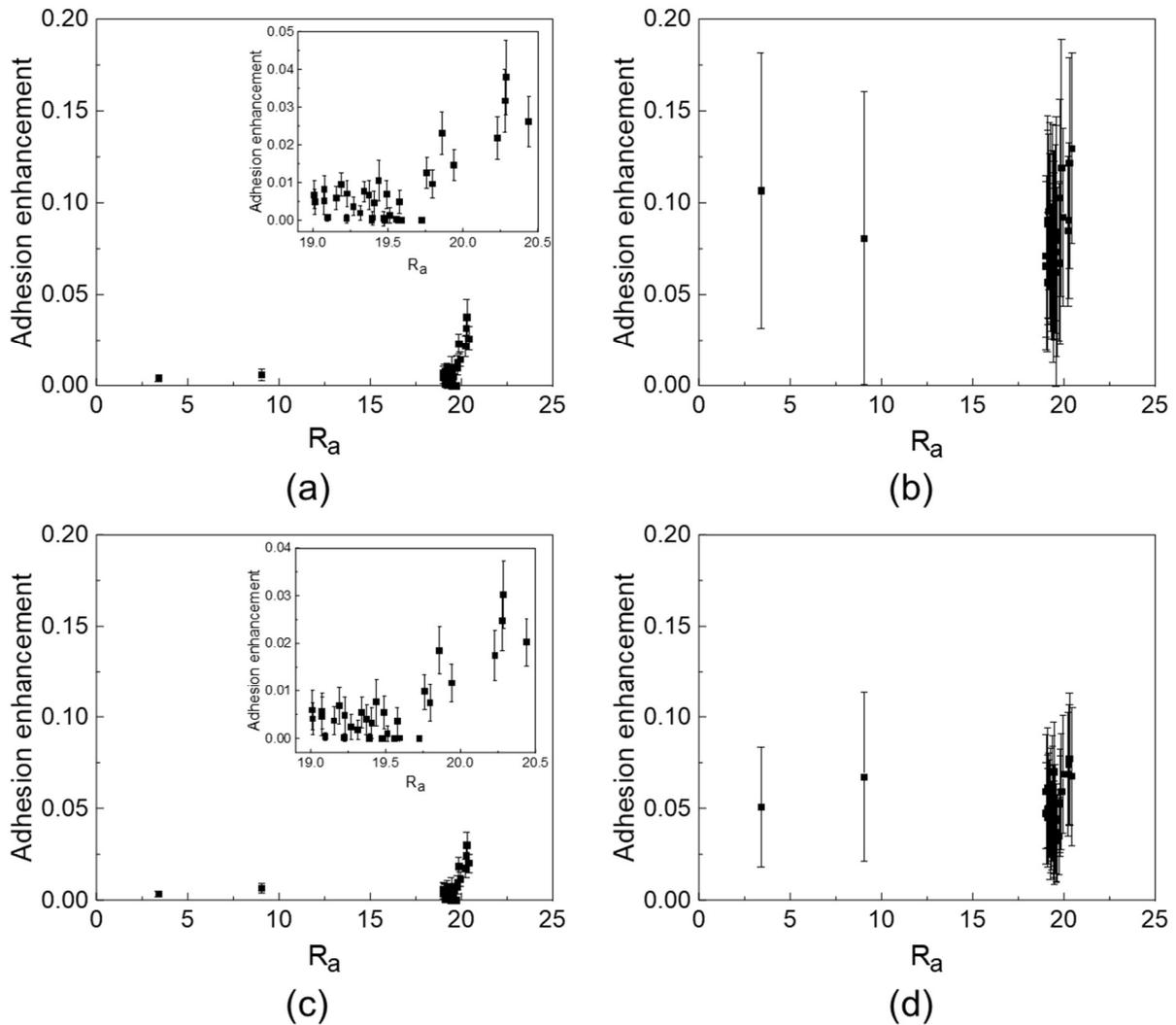
444 sliding wheel, it takes longer to complete the fracture stage compared to traction
445 operation. The peak value generated from rough surfaces is about three times higher
446 than the one from the flat surface. This is owing to the total normal force has been
447 reduced by the reaction force generated from the fragments entering the wheel-rail
448 interface, leading to a higher adhesion level compared to the result from the flat
449 surface where no normal force reduction exists, as illustrated by the inset of Fig. 9.
450 However, this peak value is still lower than the one from the rough surface during
451 traction operation as the rolling wheel contributes to a higher total tangential force.

452 4.1.3 Roughness parameter

453 Based on the earlier discussions, it is evident that rough surface significantly
454 influences the adhesion in both traction and braking operations. Hence, the widely
455 used descriptor, R_a , is employed here to examine the contribution of roughness to
456 adhesion enhancement. It is a general indicator of surface finish by specifying the
457 average height deviations. Since adhesion enhancement during the post-fracture
458 stage in both traction and braking operations behaves similarly to the prior-to-fracture
459 stage, the only first two stages (i.e., prior-to-fracture stage and fracture stage) are
460 considered here.

461 As shown in Fig. 10, the black square indicates the average value of adhesion
462 enhancements during the stage affected by different R_a and the flat end bars represent
463 its standard deviation. By comparing Fig. 10(a) and (c), although the adhesion
464 enhancement does not show a significant rise, it is clear that the standard deviation
465 increases with the rise of R_a , suggesting that a rougher surface tends to induce greater
466 fluctuations in adhesion. Additionally, the values exhibit comparability between traction
467 operation and braking operation during the prior-to-fracture stage. It signifies the
468 adhesion level is not affected by the mode of operation when sand fragments are not
469 involved. However, this relationship is changed when the fracture stage begins, as
470 illustrated in Fig. 10(b) and (d). While the adhesion enhancement is stable during both
471 operations, the average value and standard deviation of adhesion enhancement are
472 bigger during traction operation than the ones during braking operation. This indicates
473 that adhesion experiences a tremendous fluctuation at the fracture stage during
474 traction operation. In addition, the values of adhesion enhancement are levelled up
475 during the fracture stage compared to the prior-to-fracture stage, as the same amount
476 of fragments were generated during particle breakage. This observation suggests a

477 bigger R_a contributes to a higher adhesion enhancement during the prior-to-fracture
 478 stage where no sand particles are involved, and its influence becomes less
 479 pronounced compared to the effect of sand fragments during the fracture stage.



480
 481 **Fig. 10** Adhesion enhancements influenced by R_a :
 482 (a) prior-to-fracture stage and (b) fracture stage during traction operation,
 483 and (c) prior-to-fracture stage and (d) fracture stage during braking operation.

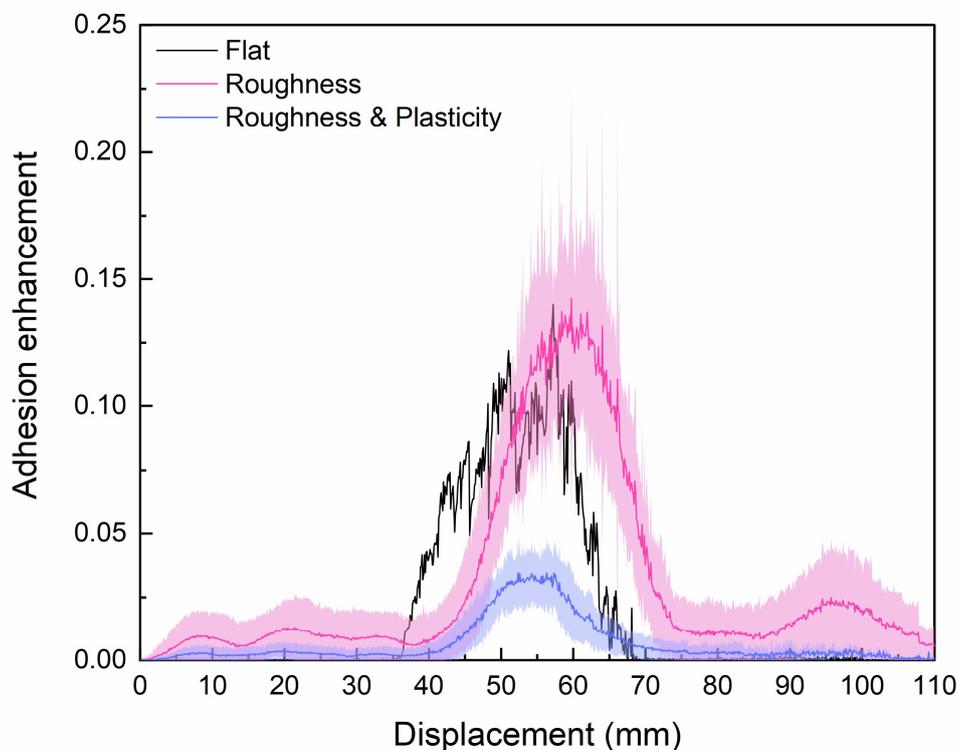
484 **4.2 Plasticity effect**

485 The interlocking between sand fragments and the rough surface generates forces that
 486 deform the rail profile, allowing more fragments to enter the wheel-rail interface. When
 487 only elastic deformation is considered, the recovery of rail deformation introduces
 488 reaction forces that increase the adhesion level. However, in practice, plastic
 489 deformation, such as wear, typically results in a lower adhesion level. Therefore, it is
 490 crucial to use a numerical model that accounts for plastic deformation to further

491 investigate the effects of roughness. Therefore, after defining the plastic parameters
492 of the rail, the previous model now can be used to further investigate the adhesion
493 level in a more realistic way.

494 4.2.1 Traction operation

495 As shown in Fig. 11, the blue shaded area represents the standard deviation of results
496 considering both elastic and plastic deformation, while their mean value is depicted as
497 a continuous blue line. Although the trends of variation are similar, the values from
498 roughness cases that account for plasticity are significantly smaller than those
499 considering only elasticity, with a reduction of approximately 60%. This causes the
500 values to be even smaller than those observed in the flat case. The reduction is
501 attributed to the permanent deformation in the rail caused by its plastic properties,
502 which diminishes the reaction force generated by the recovery of elastic deformation,
503 thereby reducing the adhesion enhancement. Moreover, compared to the red plots,
504 the width of the blue shaded area becomes narrower, and the noise in the blue solid
505 line appears smoother, with a plateau forming at the peak during the fracture stage.
506 All these observations suggest that the enhanced model, which considers plasticity,
507 results in a more stable adhesion level.



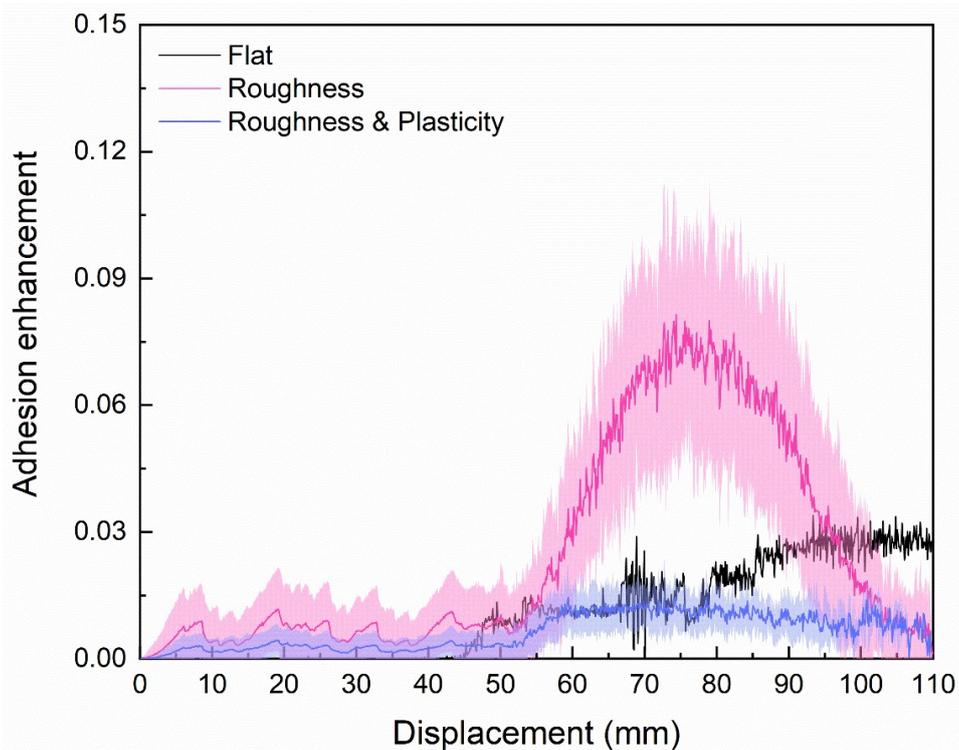
508

509

Fig. 11 Plasticity effect during traction operation.

510 4.2.2 Braking operation

511 The plasticity also affects the adhesion level during braking operation, as shown in Fig.
512 12. Due to the sliding wheel during braking operation, the plots from all cases exhibit
513 a persistent vibration throughout the entire simulation. However, their variation trends
514 behave differently. The great divergence starts during the fracture stage. Although all
515 the plots begin to increase at the same time, the slope and peak values vary between
516 cases. For the flat case, the plot starts with a gentle slope and transitions into a plateau
517 that continues until the end. For the roughness case, which only considers elasticity,
518 the plot rises rapidly and falls sharply, reaching a higher peak value compared to the
519 flat case. When plasticity is included in the rough case, the plot ascends with a gentle
520 slope, transitions into a prolonged plateau at half the peak value of the flat case, and
521 then descends with an even more gradual slope.



522

523 **Fig. 12** Plasticity effect during braking operation.

524 Overall, this enhanced model reveals the significant impact of plastic deformation on
525 adhesion enhancement during both traction and braking operations. While roughness
526 without plasticity produces higher peak adhesion, it exhibits greater variability and
527 instability, as indicated by the wider shaded regions and sharper transitions. In
528 contrast, the inclusion of plastic deformation in the model leads to a more stable
529 adhesion response, with smoother transitions and prolonged plateaus at lower

530 adhesion levels. However, the reduction in peak adhesion due to plasticity highlights
531 the importance of routine rail grinding to mitigate the effects of wear and maintain the
532 desired surface roughness for optimal adhesion during sanding. These findings
533 emphasize the need for numerical models that incorporate both elastic and plastic
534 behaviours to accurately predict adhesion performance and the importance of
535 consistent rail maintenance to ensure safe and efficient traction and braking in railway
536 operations.

537 5 Conclusions

538 By considering the surface roughness and plastic deformation of the rail, this study
539 developed a finite element model to investigate adhesion enhancement at the wheel-
540 rail interface, triggered by fragments from sand particles. The effects of roughness and
541 plastic deformation on adhesion were observed and analysed, leading to the following
542 key conclusions:

- 543 • Adhesion enhancement triggered by sand particles is size-independent and
544 can be correlated to the number of generated fragments, regardless of the rail
545 surface finish.
- 546 • Surface roughness significantly influences adhesion levels during the pre-
547 fracture stage but becomes less impactful during the fracture stage, where the
548 presence of fragments dominates adhesion behaviour.
- 549 • Plastic deformation in the rail reduces the reaction forces from fragments during
550 wheel-rail contact, ultimately leading to a lower overall adhesion level.

551 The methodology employed in this study offers valuable new insights into adhesion
552 behaviour at the wheel-rail interface by incorporating surface roughness and plastic
553 deformation parameters. This approach provides a more realistic simulation
554 environment, paving the way for future studies to refine the understanding of wheel-
555 rail interactions and adhesion mechanisms.

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560 RAILSANDING - Modelling Particle Behaviour in the Wheel-Rail Interface.

561 **Data availability**

562 Data will be made available on request.

563 **Appendix A** Supported rail roughness descriptors.

Roughness descriptor	Formula	Range	Comments
Arithmetical mean height (R_a)	$R_a = \frac{1}{\ell} \int_0^\ell Z(x) dx$	$[0, +\infty)$	Units of length
Root mean square height (R_q)	$R_q = \frac{1}{\ell} \int_0^\ell Z^2(x) dx$	$[0, +\infty)$	Units of length
Root mean square gradient (R_{dq})	$R_{dq} = \sqrt{\frac{1}{\ell} \int_0^\ell \left[\frac{d}{dx} Z(x) \right]^2 dx}$	$[0, +\infty)$	Unitless
Skewness (R_{sk})	$R_{sk} = \frac{1}{R_q^3} \left[\frac{1}{\ell} \int_0^\ell Z^3(x) dx \right]$	$(-\infty, +\infty)$	Unitless
Kurtosis (R_{ku})	$R_{ku} = \frac{1}{R_q^4} \left[\frac{1}{\ell} \int_0^\ell Z^4(x) dx \right]$	$[0, +\infty)$	Unitless

564

565

566 **Appendix B** List of representative rail head profiles.

Profile ID	R_a (μm)	R_q (μm)	R_{dq} (-)	R_{sk} (-)	R_{ku} (-)
A-1	3.384	4.314	0.004	-0.119	3.662
A-2	9.037	12.092	0.011	-1.103	-1.103
B-1	20.228	25.399	0.019	-0.554	3.107
B-2	20.438	25.328	0.022	-0.563	3.058
B-3	20.279	25.595	0.019	-0.571	3.128
B-4	20.286	25.682	0.014	-0.584	3.181
B-5	20.229	25.267	0.019	-0.568	3.054
B-6	20.105	25.052	0.007	-0.554	2.998
B-7	19.858	24.672	0.013	-0.514	2.971
B-8	19.939	24.977	0.027	-0.606	3.025
B-9	19.758	24.804	0.018	-0.627	3.266
B-10	19.437	24.466	0.029	-0.742	3.688
B-11	19.795	24.765	0.018	-0.636	3.224
B-12	19.489	24.449	0.013	-0.626	3.247
B-13	19.406	24.243	0.018	-0.636	3.142
B-14	19.576	24.637	0.013	-0.623	3.220
B-15	19.510	24.260	0.018	-0.622	3.095
B-16	19.394	24.206	0.022	-0.699	3.220
B-17	19.224	24.113	0.013	-0.680	3.222
B-18	19.391	24.565	0.018	-0.659	3.235
B-19	19.557	24.488	0.013	-0.674	3.168
B-20	19.723	24.743	0.019	-0.612	3.067
B-21	19.592	24.569	0.013	-0.649	3.052
B-22	19.479	24.466	0.022	-0.617	2.971
B-23	19.470	24.397	0.018	-0.683	3.072
B-24	19.095	24.044	0.022	-0.620	2.972
B-25	19.314	24.064	0.018	-0.700	3.082
B-26	19.269	24.047	0.022	-0.668	3.016
B-27	19.073	23.836	0.017	-0.662	3.084
B-28	19.154	23.752	0.021	-0.675	3.065
B-29	19.011	23.797	0.017	-0.636	3.040
B-30	19.372	24.366	0.022	-0.741	3.156
B-31	19.340	24.307	0.017	-0.652	3.060
B-32	19.225	24.374	0.024	-0.735	3.046
B-33	19.075	23.889	0.021	-0.712	3.054
B-34	19.188	23.947	0.012	-0.664	3.041
B-35	19.008	23.763	0.018	-0.631	3.073

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