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Reliability premium: A generic conceptual framework for evaluating the cost of travel time variability ⋆

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ABSTRACT

In this paper, we propose the *reliability premium* as a generic concept that eliminates the utility difference between the expected utility and the utility of a travel choice that is subject to randomness in travel time–without specifying the underpinning utility function nor the travel choice domain to which it relates. Mathematically, the reliability premium quantifies the buffer or additional time a traveller is willing to pay beyond the expected outcome of a travel choice to eliminate the extra disutility due to travel time variability (TTV), thereby conceptualising the cost of TTV directly and intuitively in time units. We then discuss the reliability premium under, first, the Bernoulli approach, which focuses on route choice only, and second, the scheduling delay approach, which encompasses both departure or arrival time choice and route choice. Under the Bernoulli approach, we show that it is convenient to derive the monetary cost of travel time variability based on the reliability premium. In addition, we discuss the preservation of first-order and second-order stochastic dominance (SD) of the reliability premium, which removes the computational concern of using the reliability premium in reliable path-finding problems or related assignment models. Under the schedule delay framework, we derive formulations of the reliability premium for different applications and show the detailed impact of TTV on the resulting valuations. We find that the reliability premium can be effective in capturing the asymmetry and distributional tail of travel times for quantifying the TTV cost, especially for risk-averse users, making it suitable for evaluating the impact of TTV on travellers' route choice decisions. Numerical examples are employed to elucidate the concept of the reliability premium and illustrate its practical application.

1. Introduction

The ubiquitous supply-side and demand-side uncertainties, such as adverse weather conditions, traffic accidents, temporal factors, and special events, make travel time inherently random in transportation networks. This establishes the foundations of the concept of travel time variability (TTV), whose importance has been acknowledged by various stakeholders across the transportation sector, including travellers, planners, and managers. For example, numerous empirical studies have shown that travellers attribute nearly

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equal (or even higher) importance to TTV as compared to the mean travel time (Bates et al., 2001; Hollander, 2006; Asensio and Matas, 2008; Devarasetty et al., 2012; Carrion and Levinson, 2013; Zang et al., 2018b). Moreover, reducing TTV (i.e., improving travel time reliability) can yield substantial benefits, sometimes rivalling the advantages of travel time savings (e.g., see evidence reviewed in Carrion and Levinson, 2012). Consequently, there is a growing need to quantitatively assess the cost of TTV and incorporate it in analyzing travelers' route choice behaviors and appraising transportation projects (Lo et al., 2006; Chen and Zhou, 2010; NZTA, 2010; de Jong and Bliemer, 2015; Fosgerau, 2016; Zang et al., 2024).

To account for the cost of TTV, most existing studies adopt the concept of the *monetary* value of travel time reliability, namely the VOR. Generally speaking, in VOR studies, the standard deviation and variance of travel time are the two most frequently used valuation measures. The monetary VOR is indeed useful for project appraisal of transport infrastructure as it can be directly used in cost-benefit analysis (CBA). Interested readers may refer to reviews by Li et al. (2010), Carrion and Levinson (2012), and Zang et al. (2022) for details. However, despite the widespread application of standard deviation and variance in this context, a crucial practical question is whether these metrics correlate with the actual choice behaviors displayed by travelers. This is especially pertinent to the behaviors of road users, which constitute the focal point of this paper.

For a traveler on a given origin-destination trip, it is conceivable that they might change route, departure and/or arrival times to mitigate the effects of TTV. Yet, in analysing road user behaviour under uncertainty, route choices and associated payoffs, including departure times, arrival times, and travel times, are all defined in time units, which makes monetary VORs cumbersome. That is to say, in many practical and policy contexts, it would appear more natural to quantify the cost of travel time variability in *time* units. It is worthwhile to note that, in the microeconomic theory of choice under uncertainty, the concepts of certainty equivalent and corresponding risk premium have been proposed to intuitively characterise the cost of risk bearing. These are measured in the same units as the payoff, which lend themselves to practical implementation in valuation, appraisal, and understanding of attitudes to risk more generally (Mas-Colell et al., 1995; Kreps, 2013). Relating this to our interest in measuring the TTV cost, the question naturally arises: is it possible to conceptualise the cost of TTV directly and intuitively in time units?

It should be noted that when faced with TTV, road users commonly mitigate its effects (in particular the resulting delay) by imposing 'buffer time' on their travel time - indeed this is the verified behavioral assumption of the safety margin hypothesis (Gaver, 1968; Knight, 1974; Senna, 1994). Note that the summation of mean travel time and safety margin is also interpreted as the travel time budget in Lo et al. (2006), which is conditionally equivalent to percentile travel time (Wu and Nie, 2011; Nie, 2011). In other words, a safety margin-based measure is more behaviourally intuitive for travelers and the public at large as compared to the two key valuation measures in the existing monetary VOR literature, i.e., standard deviation and travel time variance. Because users or the public may find it difficult to understand the statistical concepts of standard deviation and travel time variance (FHWA, 2006; Hollander, 2006; Nevers et al., 2014), so it becomes challenging to communicate the extent of TTV clearly and simply to the public (Engelson and Fosgerau, 2016; Chen et al., 2023). Indeed, there have been considerable efforts to design Stated Preference (SP) questionnaires to communicate the concept of TTV as straightforwardly as possible (Carrion and Levinson, 2012).

Motivated by the above questions and limitations of the extant methods, this paper proposes an alternative valuation approach—namely the reliability premium—as a more intuitive measure of the cost of TTV in *time* units. Based on the theory of choice under uncertainty in micro-economics, we firstly translate this theory to the context of TTV to derive the concept of the reliability premium as a simple but intuitive TTV cost measure. We then show mathematically that the reliability premium is the maximum buffer or additional time that the traveller is willing to pay beyond mean travel time to eliminate the extra disutility due to TTV, making it similar to the safety margin hypothesis, a widely recognized behavioral response to TTV. More importantly, though the reliability premium is naturally expressed in time units, we theoretically prove that it is straightforward to monetise the reliability premium via the value of time (VOT). Specifically, for linear utility function, the cost of TTV is simply the product of reliability premium and the VOT. The above properties of the reliability premium lend it to practical implementation in valuation studies, CBA and analysis of the traveller behaviour under TTV more generally.

In addition, it is shown that the reliability premium can define travellers' risk attitudes without imposing any particular form of utility function, and meanwhile preserve the important property of stochastic dominance. The first property enables revelation of travellers' attitudes to risk (i.e. without assuming any specific attitudes a priori), whilst the latter enables two dimensions of comparability: across lotteries (i.e. across alternative route choices) for a given traveller, and across travellers. It is notable that the mean-variance or standard deviation may not necessarily observe stochastic dominance (Qi et al., 2016).

Underpinned by these properties, the reliability premium provides a unified and integrated framework for evaluating the cost of TTV, bridging the gaps between TTV evaluation in time units and TTV valuation in monetary units which are two separate research lines by scholars from different disciplines and research areas in the existing literature (see a review of the developed theories and models of these two topics in Zang et al. (2022)). For a more specific and definitive illustration, we use the widely adopted schedule delay model with Small's utility function to derive the formulations of reliability premium for two scenarios corresponding to realistic trips, including trips by public transport and commuting trips. Lastly, numerical examples are used to illustrate and demonstrate the validity and appealing properties of the reliability premium.

The remainder of this paper is organized as follows. Section 2 briefly introduces how we apply the theory of individual choice under uncertainty to the context of TTV. Section 3 defines the reliability premium and explores its properties. Section 4 estimates the reliability premium for realistic trips under different scenarios using scheduling model. Section 5 uses empirical examples to demonstrate the advantages of the proposed method whilst Section 6 concludes this paper.

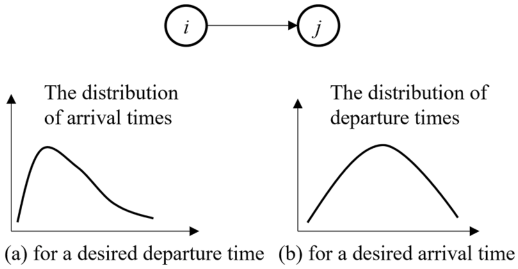

Fig. 1. An illustration of choice and associated payoffs for a trip under travel time variability.

2. Individual choice under travel time variability

Following the formative contributions of Small (1982), Noland and Small (1995) and Bates et al. (2001), this section introduces the theory of individual choice under TTV, by analogy to the theory of individual choice under uncertainty in microeconomics[1]. This will provide a basis of this paper.

2.1. Preliminary definitions

Consider a trip between two places with TTV, i.e., from place i to place j. Then, for an individual who needs to complete this trip, if he/she wants to ensure a particular departure time, the corresponding arrival time would be random due to TTV. Such random arrival times can be represented by a finite set (under a discrete scenario) or a distribution (under a continuous scenario) of all possible arrival times. Results of corresponding departure time for a given arrival time are similar. Fig. 1 provides an illustrative example under the continuous scenario to show the impacts of TTV based on the requirements of (i) a desired departure time and (ii) a desired arrival time.

Let D (A) denote the random departure time (arrival time) with its domain set $S_D = [D_{\min}, D_{\max}]$ ($S_A = [A_{\min}, A_{\max}]$). Then, for a trip with randomness in both D and A, the trip travel time would be $T = A - D$, and thus the travel time T is also random with its domain set $S_T = [T_{\min}, T_{\max}]$.

Now, we can use (i, j, D, A) to represent a trip instance when we focus on the departure time or/and arrival time. Similarly, (i, j, T) represents the trip instance when we focus on the trip travel time only. As this study focuses on a single trip only, we further use (D, A) or (T) for short hereinafter.

2.2. Utility and individual choice

For each departure time choice $D_m \in S_D$ of a trip, the domain set of arrival time S_A contains all possible corresponding pay-offs (i.e., arrival times). Let the probability distribution $\mathbf{p}$ denote a lottery over S_A. Note that for an random event, as a lottery distribution, $\mathbf{p}$ is a specific probability distribution associated with particular outcome(s) in this lottery, whereas the probability distribution of this event contains all possible outcomes and their respective probabilities. In this paper, the lottery distribution is equivalent to travel time distribution used in the context of TTV.

Then, the expected arrival time for the given D_m under lottery $\mathbf{p}$ is given by:

$$\bar{A} = \int_{A_{min}}^{A_{max}} \mathbf{p}(A) A dA \tag{1}$$

Let $\Delta\mathbf{p}$ denote the set of all possible lotteries over the set S_A from which the individual can choose his/her preferred lottery:

$$\Delta\mathbf{p} = \{\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_K\} \tag{2}$$

Then, for continuously and independently rational preferences over the set of all lotteries, there exists a corresponding (von Neumann-Morgenstern, VNM) expected utility for the lottery $\mathbf{p}$ over the set S_A under a given departure time D_m (Mas-Colell et al., 1995),

$$EU_{\mathbf{p}}(D_m, A) = \int_{A_{min}}^{A_{max}} \mathbf{p}(A) U(D_m, A) dA \tag{3}$$

where $U(D_m, A_n)$ is the so-called Bernoulli utility for each $A_n \in S_A$ under a given D_m.

For any pair of lotteries $\mathbf{p}_i, \mathbf{p}_j \in \Delta\mathbf{p}$, the individual with a rational preference relation would have

- $\mathbf{p}_i \succsim \mathbf{p}_j$ iff $EU_{\mathbf{p}_i}(D_m, A) \geq EU_{\mathbf{p}_j}(D_m, A)$

[1] The theory of individual choice under uncertainty or risk was well defined in micro-economic theory and interested readers may refer to Mas-Colell et al. (1995) or Kreps (2013) for technical details.

- $\mathbf{p}_i \precsim \mathbf{p}_j$ iff $EU_{\mathbf{p}_i}(D_m,\ A) \le EU_{\mathbf{p}_j}(D_m,\ A)$
- $\mathbf{p}_i \sim \mathbf{p}_j$ iff $EU_{\mathbf{p}_i}(D_m,\ A) = EU_{\mathbf{p}_j}(D_m,\ A)$

Though the above definition and introduction focus on arrival time only in a trip scheduling, it is still valid if we concentrate on departure time or travel time of a trip scheduling. That is, we can have the expected utility for the lottery $\mathbf{q}$ over the set S_D under a given arrival time A_n

$$EU_{\mathbf{q}}(D,\ A_n) = \int_{D_{min}}^{D_{max}} \mathbf{q}(D)U(D,\ A_n)dD \tag{4}$$

For any pair of lotteries $\mathbf{q}_i, \mathbf{q}_j \in \Delta\mathbf{q}$, the individual with a rational preference relation would have

- $\mathbf{q}_i \succsim \mathbf{q}_j$ iff $EU_{\mathbf{q}_i}(D,\ A_n) \ge EU_{\mathbf{q}_j}(D,\ A_n)$
- $\mathbf{q}_i \precsim \mathbf{q}_j$ iff $EU_{\mathbf{q}_i}(D,\ A_n) \le EU_{\mathbf{q}_j}(D,\ A_n)$
- $\mathbf{q}_i \sim \mathbf{q}_j$ iff $EU_{\mathbf{q}_i}(D,\ A_n) = EU_{\mathbf{q}_j}(D,\ A_n)$

Similarly, if we focus on travel time only and $\mathbf{r}$ is a lottery over the set S_T, then we have expected utility for this lottery $\mathbf{r}$:

$$EU_{\mathbf{r}}(T) = \int_{T_{min}}^{T_{max}} \mathbf{r}(T)U(T)DT \tag{5}$$

For any pair of lotteries $\mathbf{r}_i, \mathbf{r}_j \in \Delta\mathbf{r}$, the individual with a rational preference relation would have

- $\mathbf{r}_i \succsim \mathbf{r}_j$ iff $EU_{\mathbf{r}_i}(T) \ge EU_{\mathbf{r}_j}(T)$
- $\mathbf{r}_i \precsim \mathbf{r}_j$ iff $EU_{\mathbf{r}_i}(T) \le EU_{\mathbf{r}_j}(T)$
- $\mathbf{r}_i \sim \mathbf{r}_j$ iff $EU_{\mathbf{r}_i}(T) = EU_{\mathbf{r}_j}(T)$

On this basis, we are suitably equipped with a formal representation of individual's trip choice under TTV in trip scheduling, ready to quantify the impact and associated cost of TTV in the following sections. It is noteworthy that this subsection defines the choice under uncertainty in continuous representations of departure times, arrival times and travel times only. It is, however, straightforward to translate the same definitions to discrete representations, as will be shown in Section 4.2 and indeed was the approach followed by Batley (2007).

3. A unified conceptual reliability premium: definitions and properties

This section adopts the concept of risk premium to our interest in quantifying the cost of travel time variability, linking the notion of uncertainty (or risk) defined by microeconomists with that of variability or reliability used in transport studies.

To be specific, following Section 2, for a lottery $\mathbf{r}_i$ for a trip instance (T) with expected utility $EU_{\mathbf{r}_i}(T)$, let us consider a trip with another lottery $\mathbf{r}_j \in \Delta\mathbf{r}$, wherein the probability of a particular travel time is 1, the probability of all other travel times is 0, and the same expected utility is achieved with lottery $\mathbf{r}_i$, i.e., $EU_{\mathbf{r}_j}(T) = EU_{\mathbf{r}_i}(T)$. These two trips are referred to as "trip I" and "trip J" in Fig. 2. To give some intuition, trip J is actually a trip under certainty whose travel time makes its utility equal to that of trip I. Such a travel time actually identifies an equally acceptable trip under certainty and thus we define it as the "certainty equivalent" of the random outcome of trip I, which is formally given by Definition 1. Below for brevity we use $EU(T)$ to denote $EU_{\mathbf{r}_i}(T)$. Definition 1 customizes the concept of "certainty equivalent" in microeconomic theory to the context of a trip under travel time uncertainty, as was defined in Batley (2007).

Definition 1 (Certainty equivalent). For the random outcome T of trip I (a trip under uncertainty with expected utility $EU(T)$), its certainty equivalent, denoted by T_{CE}, makes trip J (a trip under certainty) yield the same utility as the expected utility of trip I. Mathematically, $EU(T) = U(T_{CE})$.

With Definition 1, let us consider the following three trip instances including the above mentioned trip I and trip J with different outcomes and their utilities as shown in Fig. 2:

- Trip I under uncertainty with its random outcome T with μ as its expected value: $EU(T)$.
- Trip J under certainty with a certain outcome T_{CE}: $U(T_{CE})$ where we have $U(T_{CE}) = EU(T)$.
- Trip K under certainty with expected outcome $\bar{T} = \mu$: $U(\bar{T})$.

One can easily find that trip J is the certainty equivalent trip of trip I, while trip K is the special case of trip I assuming there is no travel time variability or uncertainty. With reference to trip I, trip J has a comparable trip under certainty, i.e., trip K, whose travel time is $\bar{T}$. Furthermore, trip J depends on the risk profile of the traveler, whereas trip K does not. However, the certainty equivalent of trip I actually identifies an equally acceptable trip J whose outcome is not equal to the outcome of trip K. Such adjustment from $\bar{T}$ to T_{CE} ensures trip J to yield the same utility as trip I, which is the result of the impact of travel time variability and thus *the cost*

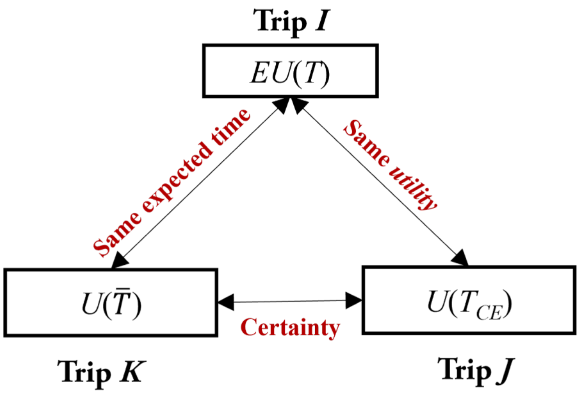

Fig. 2. Comparison of three trip instances.

one traveller must pay to eliminate the TTV. This definition of the certainty equivalent equips us to mathematically quantify the cost of travelers' bearing the TTV in trip scheduling.

Within the microeconomic theory of choice under uncertainty, the risk premium developed by Pratt (1964) is defined as the difference between the expected outcome of a lottery and its certainty equivalent, thereby giving an intuitive characterization of the cost of risk bearing when choosing an uncertain prospect. More importantly, the risk premium is measured in the same units as the pay-off (Mas-Colell et al., 1995; Kreps, 2013). This lends it to practical implementation in studies of the value of reliability and behavioral response to reliability more generally, as all relevant pay-offs (in scheduling including departure times, arrival times, and travel times) are expressed in time units. Based on the concepts of the risk premium and the certainty equivalent, we can define what we call the reliability premium in Definition 2 to measure the cost that a traveller is willing to pay to eliminate the TTV in trip scheduling.

Definition 2 (Reliability premium)**.** The reliability premium measures the willingness-to-pay of an individual, in units of the outcome, to eliminate the extra disutility resulted from travel time variability in trip scheduling.

Mathematically, the reliability premium π is the difference between the expected outcome of a trip under uncertainty and the certainty equivalent of its random outcome. Namely,

$$\pi = T_{CE} - \mu \tag{6}$$

Remark 1. It should be noted that the risk premium is defined as the expected outcome minus the certainty equivalent, whilst the reliability premium is defined as the certainty equivalent minus the expected outcome. The reason is as follows. The pay-offs of goods usually bring positive utilities to consumers such that more goods are preferred to less. On the contrary, the payoffs of trip scheduling usually bring negative utilities to travellers such that less travel time is preferred to more. The widely used schedule delay model, to be introduced in Section 4, exactly models all payoffs of trip scheduling as disutilities. This is also why the Bernoulli utility function is usually assumed to be non-increasing in transportation domain. From this perspective, the sign of the reliability premium should be the opposite of the risk premium. In addition, our definition is consistent with previous studies adopting the risk premium into the field of VOR in transportation domain (Batley, 2007; Beaud et al., 2016).

Remark 2. It is noted that, though the reliability premium is similar to the concept of headstart proposed by Gaver (1968), they are distinct concepts with each other. At the beginning of the paper (Gaver, 1968), it says that 'When a traveller, or other user of a congested system, wishes to be reasonably sure of reaching a destination, or finishing a task, on time he must start early enough to compensate for random delays'. As shown in Definition 2, the reliability premium is slightly different; it seeks to determine the difference between the expected travel time and the travel time at the certainty equivalent. Particularly, based on Fig. 2, the expected travel time corresponds to the trip without time variability (i.e.,trip K); for the same trip with time variability (i.e.,trip I), the travel time at the certainty equivalent identifies the equivalent trip with the same utility but with certainty (i.e.,trip J). Therefore, an important distinguishing feature of the reliability premium is that it is concerned with eliminating the extra disutility (namely $EU(T) - U(\mu)$) resulted from travel time variability - as opposed to eliminating delay per se.

As travellers' choices and associated payoffs in trip scheduling under TTV are all naturally in time units, it would seem intuitive to express the certainty equivalent and reliability premium as measures in *time* units. Theorem 1 shows the relationship between our reliability premium and the existing reliability premiums proposed by Batley (2007) and Beaud et al. (2012), respectively. To be more specific, we generalize Batley (2007)'s definition, illustrate the detailed impact of TTV on outcomes of trip scheduling, and show some appealing mathematical properties not investigated by Beaud et al. (2016).

Theorem 1. *The reliability premium defined in Beaud et al. (2016) is equivalent to Definition 2 and Batley (2007)'s definition of reliability premium is a special case of Definition 2.*

Proof. Recall that trip I and trip J has the same utility, and based on Eq. (6), we further have

$$EU(T) = U(T_{CE}) = U(\pi + \mu) \tag{7}$$

Eq. (7) is exactly the way that Beaud et al. (2016) define reliability premium in their paper,

As travel time is the difference between arrival time and departure time, we have $T_{CE} = A_{CE} - D_{CE}$ and $\mu = \bar{A} - \bar{D}$. With these relationships, we can rewrite Eq. (6) as

$$\pi = (A_{CE} - D_{CE}) - (\bar{A} - \bar{D}) = (A_{CE} - \bar{A}) + (\bar{D} - D_{CE}) \tag{8}$$

Let $D_{CE} = \bar{D}$ in our definition, in which case we would have $\pi = A_{CE} - \bar{A}$. This means that if we focus on arrival time only while keeping departure time fixed, the reliability premium would be defined as the delay in arrival time that an individual is willing to pay in exchange for eliminating TTV. This is exactly the way that Batley (2007) defined reliability premium ($\pi_{Batley} = A_{CE} - \bar{A}$). Therefore, Batley's definition is a special case of our definition. This completes the proof. □

Four observations follow Theorem 1 and its proof immediately.

First, the equivalence between our defined reliability premium and Beaud et al. (2016)'s definition provides a convenient way to calculate the reliability premium, as per Eq. (7). Based on such equivalence, Section 4 derives the reliability premium using a schedule delay model for practice.

Second, the reliability premium quantifies the **maximum buffer time** that an traveler is willing to pay beyond the mean travel time to eliminate TTV in trip scheduling. This makes the proposed reliability premium similar to a widely recognized behavioral assumption for travellers' actual response to TTV—namely the safety margin hypothesis (Gaver, 1968; Knight, 1974; Senna, 1994; Lo et al., 2006). Real surveys and industrial applications do support this behavior assumption. For example, a previous survey conducted by Lo et al. (2006) showed that most of students reserved buffer time beyond mean travel time for important examination trips considering travel time uncertainty. Lots of performance indicators used by providers or agencies of public services follow the same assumption. For example, Association of Train Operating Companies (ATOC, 2009) used the mean lateness at departure and/or arrival time to measure the reliability of train services in the UK, in which our Section 4.2 and Section 4.3 explore the adjustment in departure and arrival time based on reliability premium to improve trip reliability. FHWA (2006) used buffer time as the performance metric to monitor road networks, while TRB's second Strategic Highway Research Program SHRP (2009) used buffer time index to support the collaborative decision-making framework for additions to highway capacity. This guarantees reliability premium a realistic and reasonable behavior evidence.

Third, as defined, the reliability premium π, i.e., $U(\mu + \pi) = EU(T)$, does not involve any specific form of the utility function. This means that the reliability premium should be readily applicable to models under the random utility model (RUM) framework, i.e., $U(T) = V(T) + \epsilon$, including logit or mixed logit models.

Lastly, Batley (2007)'s definition restricts the reliability premium to adjusting arrival times only for a given departure time of a trip under uncertainty. This paper generalizes Batley (2007)'s definition, by admitting adjustments to both arrival time or departure time as shown in Section 4.

3.1. Defining risk attitudes without assuming types of utility functions

The proposed reliability premium is ready to define travellers' risk attitudes based on Jensen's inequality without assuming a certain type of the utility function, which is given by Theorem 2 based on the analysis in Batley (2007).

Theorem 2. *For an individual making a trip decision under travel time variability with an nonincreasing Bernoulli utility function, this individual is (1) risk averse iff $\pi > 0$, (2) risk neutral iff $\pi = 0$, and (3) risk prone iff $\pi < 0$.*

From its definition, the idea of risk aversion is that compared to accepting a risky lottery $\mathbf{p}_l$ with the expected arrival time $\bar{A}$, an individual prefers a non-risky lottery $\mathbf{p}_k$ with $\bar{A}$ as a certain outcome. That is, if an individual is risk averse, then this individual will have $\mathbf{p}_l \prec \mathbf{p}_k$. Section 3.2 will use second order stochastic dominance to discuss the relationship between reliability premium and risk aversion in detail. By contrast, if an individual is risk prone, then this individual will have $\mathbf{p}_l \succ \mathbf{p}_k$. As for a risk neutral individual, he/she will be indifferent between any two lotteries as long as they have the same (expected) outcome.

It should be pointed out that the traditional means of estimating VOR is using Stated Preference (SP) methods, but communicating the concept of VOR in this context is notoriously challenging, and can lead to bias in the resulting estimates. Theorem 2 means that we do not need to assume travellers' risk attitudes in advance for analyzing traveler's behavior and thus is quite useful for in current rich or big data environment. Put differently, we can simply analyze their risk attitudes according to collected GPS data set (e.g., Carrion and Levinson (2013), Zeng et al. (2018)), reducing the bias brought by travellers themselves due to inexact report or even wrong cognition of their risk attitudes. Theorem 2 also allows us to show the detailed impact of TTV as well as travellers' risk attitudes on outcomes of trip scheduling. Hence, reliability premium and certainty equivalent, though received limited attention, has been applied in the literature to show or capture different risk attitudes, including risk-aversion, risk neutral and risk-taking such as Senna (1994), Illenberger et al. (2011), and Beaud et al. (2012, 2016).

4. Reliability premium in Bernoulli model

4.1. Comparability of reliability premium in the small and in the large

Another key property of the reliability premium is that it preserves the first-order and second-order stochastic dominance (FSD and SSD) to be introduced subsequently in Theorem 3.

Before proceeding to Theorem 3, it is necessary to first present the definition of stochastic dominance in the context of trip scheduling. As introduced before, compared to decision-makers who always prefer more payoff to less (e.g. the return of an investment) in finance or economics, travellers generally prefer less pay-off rather than more for a trip. Therefore, our Definition 3 is slightly different from its definition in economic or finance context (Mas-Colell et al., 1995; Kreps, 2013) but is consistent with Wu and Nie (2011) as we both focus on route choice context. Obviously, the FSD compares the CDFs of two lotteries at every point, while the SSD compares the cumulative sums over travel time of the CDFs of two lotteries at every point. With Definition 3 and the reliability premium, we can conclude Theorem 3 and Corollary 1 as follows. Proofs of this theorem and corollary are based on Beaud et al. (2016).

Definition 3 (Stochastic dominance). Let **p** and **q** denote two lotteries over the payoffs of a trip scheduling with $F_{\mathbf{p}}(T)$ and $F_{\mathbf{q}}(T)$ as their CDFs. Then, we have the following definitions.

(I) Lottery **p** first order stochastically dominates lottery **q**, denoted as $\mathbf{p} \geq_1 \mathbf{q}$, if $F_{\mathbf{p}}(T) \geq F_{\mathbf{q}}(T)$ for any $T \in [T_{\min}, T_{\max}]$.

(II) Lottery **p** second order stochastically dominates lottery **q**, denoted as $\mathbf{p} \geq_2 \mathbf{q}$, if $\int_T^{T_{\max}} F_{\mathbf{p}}(\omega)d\omega \geq \int_T^{T_{\max}} F_{\mathbf{q}}(\omega)d\omega$ for any $T \in [T_{\min}, T_{\max}]$.

Theorem 3. *The reliability premium preserves FSD and SSD between lotteries over payoffs of trip scheduling.*

Corollary 1. *The reliability premium of lottery* **p** *is greater than that of lottery* **q** *if* $\mathbf{p} \geq_1 \mathbf{q}$ *or* $\mathbf{p} \geq_2 \mathbf{q}$.

Corollary 1 enables any given traveller to compare alternative risky route choices for a given trip and the subsequent Theorem 4 further allows comparison of the different risk attitudes of different travellers. A preliminary to Theorem 4 is the definition of comparative risk aversion in the context of trip scheduling, which is given by Definition 4 based on Pratt's groundbreaking paper (Pratt, 1964) and recent work of Beaud et al. (2016) in the transportation domain.

Definition 4 (Comparative risk aversion). Let $A_U(T)$ denote the absolute risk aversion function: $A_U(T) = \frac{U''(T)}{U'(T)}$. Traveler i is more risk averse than traveler j if the absolute risk aversion function of traveler i is greater than or equal to that of traveler j. Namely, $A_{U_i}(T) \geq A_{U_j}(T)$

Theorem 4. *Traveler i is more risk averse than traveler j if and only if the reliability premium of traveler i is greater than or equal to that of traveler j.*

Referring to the terminology of Pratt (1964) and Beaud et al. (2016), Corollary 1 defines comparability in the small (i.e., for an individual to compare risky choices) and Theorem 4 defines comparability in the large (i.e., for comparing risk averse degree between different individuals). Put differently, we could simply state that *a more risk averse traveller has a greater reliability premium*. In addition, as stochastic dominance provides a coherent framework of modeling risk preferences (Wu and Nie, 2011; Li et al., 2015), this property of the reliability premium gives it a distinct advantage over the mean-variance approach. This is due to the fact that the mean-variance approach may violate stochastic dominance under certain conditions (interested readers can refer to a small example in Qi et al. (2016)) whilst the reliability premium always obeys it. Most importantly, as stated by Wu and Nie (2011), the preserve of stochastic dominance makes the concept of SD-admissible path sets ready for solving the proposed reliability premium-based route choice models without enumerating all paths. This is a great computational advantage of using reliability premium for route choice and related assignment models! In addition, Section 5.3.1 and Section 5.4 further show that the reliability premium can effectively capture the asymmetry and distribution tail of travel times, exactly adapting empirically reported long fat tail of TTDs. This addresses the needs of risk-averse decision-making under the context of TTV, highlighting again the suitability of reliability premium for evaluating the TTV for the use of reliability-based route choice and related assignment models.

4.2. Monetising reliability premium via the VOT

This section proves that we can straightforwardly monetise the reliability premium via the VOT and hence easily calculate the cost of travel time variability in monetary units. In other words, we have successfully responded to the criticism that measurement in time units hinders the application of the reliability premium to cost-benefit analysis (e.g., Beaud et al. (2016)).

For a trip, a traveler's utility including trip cost and utility of his/her choice can be formulated as:

$$Y = \varphi c + U(T) \tag{9}$$

where c is the fixed cost of a trip (e.g., ticket fee) and φ is the marginal utility of income.

With reference to Eq. (9), if the travel time is marginally increased, i.e., from T to $T + dT$, then the trip utility would become $Y' = \varphi c + U(T + dT)$. Then, the monetary VOT per time unit can be defined as follows:

$$VOT(T) = \lim_{dT\to 0} \frac{Y' - Y}{dT}\frac{1}{\varphi} = \lim_{dT\to 0} \frac{U(T + dT) - U(T)}{dT}\frac{1}{\varphi} = U'(T)\frac{1}{\varphi} \tag{10}$$

When the traveler adjusts trip travel time to the certainty equivalent, the utility change is $(U(\mu + \pi) - U(\mu))$. Such utility change is due to the existence of TTV. Therefore, with reference to Eq. (9), the cost of TTV in monetary unit, denoted by C_{TTV}, can be defined as the following Eq. (11). Note that the cost of TTV is also called as the VOR in the literature and in this paper these two terminologies has the same meaning.

$$C_{TTV} = (U(\mu + \pi) - U(\mu)) * \frac{1}{\varphi} \tag{11}$$

The following Theorem 5 provides a exact mathematical formulation to compute the cost of variability in monetary unit via simply monetising the reliability premium based on VOT at the point of mean travel time. The subsequent observation is Corollary 2 for the linear utility function, which indicates that the variability cost is simply equal to the reliability premium times VOT. This important property makes the monetisation of the reliability premium simply and directly applicable for the appraisal of transport projects or policies no matter they require monetary cost or non-monetary cost. Numerical examples will demonstrate the advantages of this property in detail.

Theorem 5. *The cost of TTV in monetary unit can be easily obtained by monetising the benefits of the reliability premium. Mathematically, we have*

$$C_{TTV} = \pi\left(VOT(\mu) + \frac{1}{\varphi}\frac{\pi}{2}U''(\mu)\right) + \frac{1}{\varphi}o(\pi^2) \tag{12}$$

where $o(\pi^2)$ is the Peano form of the remainder and the higher order infinitesimal of π^2.

Corollary 2. *The TTV cost is equal to the reliability premium times the VOT for a linear utility function. Namely,*

$$C_{TTV} = \pi * VOT \tag{13}$$

Theorem 5 and Corollary 2 make sense as (I) the defined VOT actually plays the role of transforming time into money and thus VOT(μ) provides a reference point to transform time around mean travel time to money, and (II) the reliability premium plays a similar role as VOT(μ) by transforming variability into time based on mean travel time as the reference point. Therefore, even for **a non-linear utility function**, the local approximation of TTV cost can be simply the product of the reliability premium π and VOT(μ) according to Eq. (12) if we do not consider Peano form of the remainder (i.e., we only use second-order Taylor series expansion).

Remark 3. It should be noted that reliability ratio is a well-known approach for the translation from time units of TTV cost to monetary units of TTV cost under scheduling framework (i.e., the alpha-beta-gamma case to be introduced in Section 5.1), as reviewed by related studies (See e.g., Li et al., 2010; Carrion and Levinson, 2012; Fosgerau, 2016; Taylor, 2017; Zang et al., 2022). Our works of Theorem 5 and Corollary 2 contribute to converting the reliability premium into monetary units under the Bernoulli framework, which is indeed the question highlighted by (Beaud et al., 2016) for cost-benefit analysis. In addition, even under the scheduling framework, our derived TTV cost does not belong to the class of scheduling cost measure defined by Engelson and Fosgerau (2016) (the reason for which is explained in Section 5) and thus is useful for the case of not pursuing expected utility minimisation. Finally, for deriving the monetary cost of TTV, reliability ratio uses standard deviation as the valuation measure which depends on travel times only and is a fixed value across different users, while the present paper use the reliability premium as the valuation measure which depends on both travel times and users' risk preferences and its value may vary across different users. We do not argue that one measure necessarily outperforms the other, but they seem applicable to different situations. For example, if a user wants to know the rate of return of the additional time that he or she pays for a trip, the reliability premium could be a good choice to quantify its value of per unit of additional time.

5. Reliability premium in scheduling model

This section uses the scheduling model with Small's utility function to evaluate reliability premia for realistic trips under different scenarios.

In practice, for a trip scheduling, individuals can adjust either their arrival times or their departure times to hedge against TTV. In other words, as shown by Fig. 3, we consider the following two scenarios travellers may consider to protect them against TTV. Section 2.2 has already equipped us with individual's trip choice theory under TTV in trip scheduling for these two scenarios.

- ***Scenario* 1**: Delayed but certain arrival time for a given departure time is accepted. This is commonly observed for trips or services with fixed departure time, such as trips carried out by scheduled buses or trains.
- ***Scenario* 2**: Earlier departure time for a given arrival time is accepted. This is commonly observed for trips or services with fixed arrival time, such as commuting trips.

Below we will derive the formulations of reliability premium with different applications using a simple schedule delay model and show the detailed impact of TTV on trip scheduling.

5.1. A simple schedule delay model

Although there are few restrictions on the form of utility functions in microeconomic theory, Small's utility function has received considerable support from empirical studies and has been widely used in practice for trip scheduling. In Small (1982), an individual is assumed to have a preferred arrival time (PAT) and thus holds different preferences for being late or early compared to PAT. Based on previous works (Small, 1982; Noland and Small, 1995; Fosgerau and Karlström, 2010; Zang et al., 2018b), a simple model of the trip utility under certainty is given by

$$U(D,\ A) = \alpha T + \beta SDE + \gamma SDL \tag{14}$$

where *SDE* is schedule delay early: $SDE = \max[(PAT - A), 0]$; and *SDL* is schedule delay late: $SDL = \max[(A - PAT), 0]$. The preference parameters α, β, and γ are all negative and represent the marginal (dis)utilities of travel time, SDE and *SDL*, respectively.

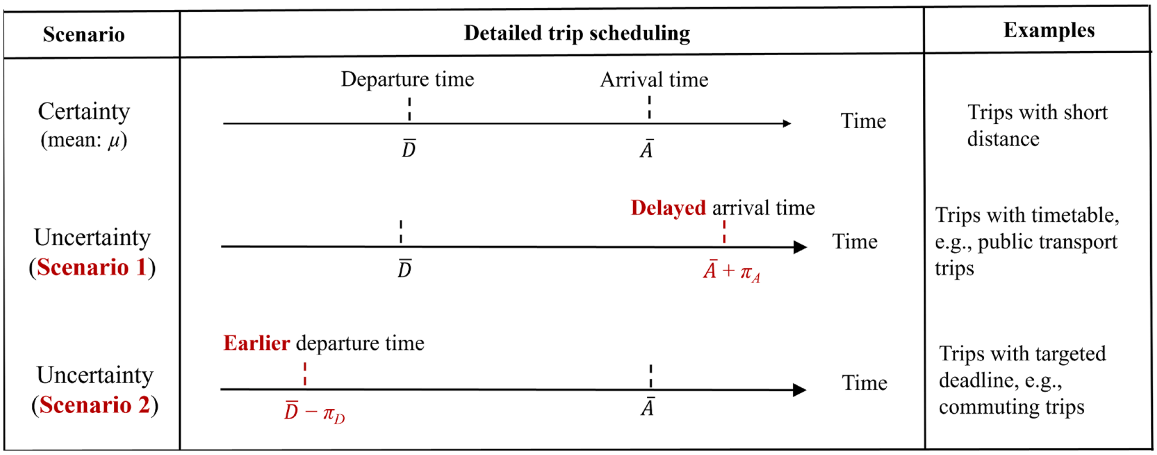

Fig. 3. Two scenarios of adjustments in pay-offs in terms of trip scheduling.

It is worthwhile to note that the last three pay-offs are measured in time units under the scheduling model in Small (1982). This property lays the foundation of defining and developing associated reliability premia to capture the cost of eliminating the TTV in trip scheduling in time units, to be discussed in Sections 5.2, 5.3, and 5.4. To simplify mathematical deductions, without loss of generality, an individual's PAT can be normalized to time zero, i.e., $PAT = 0$. As a result, $D < 0$ because travel time must be positive. Then, one can easily find that (a) if $A < 0$, he/she will be early; (b) if $A > 0$, he/she will be late; and (c) if $A = 0$, he/she will arrive at exactly the PAT. On this basis, Eq. (14) can be specified as follows:

$$U(D,\ A) = \begin{cases} (\alpha - \beta)A - \alpha D, & A \le 0 \\ (\alpha + \gamma)A - \alpha D, & A > 0 \end{cases} \tag{15}$$

Notice that Eq. (15) is expressed based on D and A. As $T = A - D$, we can easily get an alternative expression with the focus on T as follows:

$$U(D,\ T) = (\alpha - \beta)T + \beta D + (\beta + \gamma)(T - D)^{+} \tag{16}$$

where $^{+}$ is a function operation such that $(T - D)^{+} = T - D$ if $T - D > 0$ and 0 otherwise.

Let random trip travel time $T = \mu + \sigma X$, where $X = (T - \mu)/\sigma$ is a standardised travel time with mean 0, variance 1, probability density function ϕ, and CDF Φ. By further assuming that X's distribution is independent of departure time D, we can derive the expected utility:

$$EU(D,\ A) = -\beta D + (\alpha - \beta)\mu + (\beta + \gamma)\int_{\frac{-D-\mu}{\sigma}}^{\infty} (\mu + \sigma x + D)\phi(x)dx \tag{17}$$

where we use Θ to denote the last integral part and thus we further have

$$\Theta = (\mu + D)(1 - \Phi((-D - \mu)/\sigma)) + \sigma H(\phi, \Phi((-D - \mu)/\sigma)) \tag{18}$$

The H in Eq. (18) is equivalent to mean lateness factor of Fosgerau and Karlström (2010), which could capture the distribution tail of travel times to be discussed in detail in Section 5.4.

Remark 4. It should be noted that typically in the literature, $U(T)$ in previous Section 3 is usually referred to Bernoulli model. For $U(T)$, travel time T is an exogenous variable; but for the above scheduling model with $U(D, A)$, if D and A are random variables, travel time T should be an endogenous variable. It is therefore necessary to pay attention to this inconsistency between the scheduling and Bernoulli models. The reader should however note that when this study focuses on the $U(D, A)$ formulation in Section 4, we will held the departure time or arrival time *fixed* and thus the resulting scheduling model can be considered equivalent to the Bernoulli model (Engelson and Fosgerau, 2016). In addition, the D in scheduling model is not required to be the typical optimal departure time to minimize the expected utility. Therefore, the defined reliability premium under scheduling models does not belong to defined class of scheduling cost measures in Engelson and Fosgerau (2016), thereby not violating the conclusion of *"there is no scheduling measure that is a Bernoulli measure, and there is no Bernoulli measure that is a scheduling measure"* proved in Engelson and Fosgerau (2016).

5.2. Reliability premium for a fixed departure time

This section derives the reliability premium for trips with a fixed departure time, e.g., trips carried by public transport, train or airplane.

In practice, it is not difficult or even easy for service providers to ensure that the departure times of their services punctual. This has two reasons: (1) no TTV is involved before the start of service, and (2) the on-time departure is usually required by related

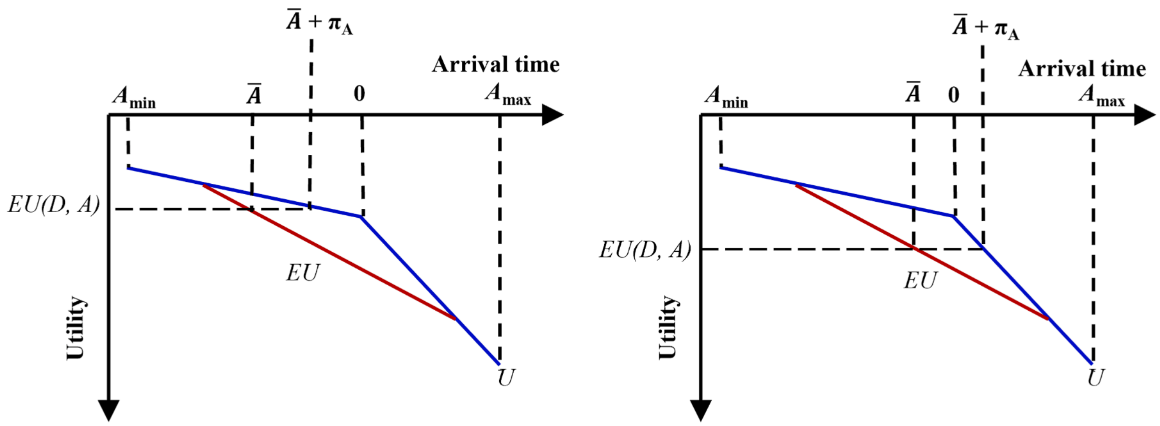

Fig. 4. Reliability premium and utility function for a given departure time with $\alpha < \beta$: (a) the left for $EU(D_m, A) \le U(D_m, 0)$; (b) the right for $EU(D_m, A) > U(D_m, 0)$.

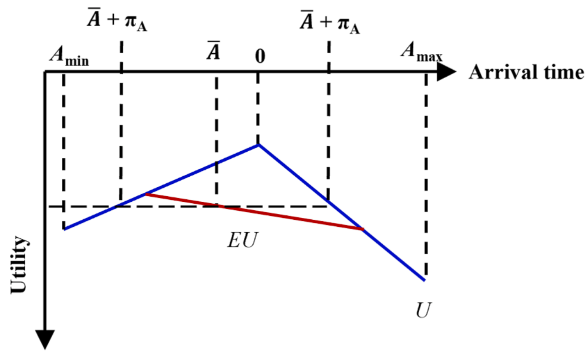

Fig. 5. Reliability premium and utility function for a given departure time with $\alpha > \beta$.

authorities and thus possible measurements can be taken to guarantee the punctuality. However, it is quite challenging for them to guarantee punctuality of the scheduled arrival time due to the existence of TTV. Such experiences may lead passengers to consider the service unreliable, possibly with adverse consequences for patronage as reliability is a key performance metric of public transport service, e.g., de Oña et al. (2016) and Soza-Parra et al. (2019, 2022). The reliability premium provides a simple solution for these service providers to improve their service reliability by simply *delaying* their original scheduled arrival time to account for TTV as shown in scenario 2 of Fig. 3. This practice is sometimes referred to in the literature as 'recovery time' as introduced in Ojeda-Cabral et al. (2021).

For a given departure time, we need discuss the relationship among preference parameters, risk attitudes, and the existence of reliability premium. Based on equations for utility and expected utility, it stands to reason that if all possible arrival times are less than or greater than PAT (i.e. there is no TTV), then the utility of expected arrival time will be equal to the expected utility. For $A_{\min} < PAT < A_{\max}$, Theorem 6 shows that the existence of reliability premium depends on the relationship between α and β.

Theorem 6. *If $\alpha \le \beta$, only one reliability premium exists for risk-averse travellers. If $\alpha > \beta$, two reliability premia exist: one for risk prone travellers and another for risk averse travellers.*

Proof. First, if $\alpha \le \beta$, there will be only one reliability premium and the traveller's risk attitudes must be risk-averse. As can be observed from Fig. 4, the slopes before and after PAT $= 0$ are all non-positive since $\alpha - \beta \le 0$ and $\alpha + \gamma < 0$. This means that the utility of the expected arrival time is monotonically decreasing, and consequently there is a unique reliability premium satisfying $EU(D_m, A) = U(D_m, \bar{A}_m + \pi_A)$. Furthermore, the reliability premium must be positive as the utility function is concave, i.e., $EU(D_m, A) < U(\bar{A})$, indicating that the traveller's risk attitudes must be risk-averse according to Theorem 2.

Second, Fig. 5 plots utility functions for $\alpha > \beta$ with a given departure time. The slope (i.e., $\alpha - \beta > 0$) before PAT is positive and the slope after PAT is negative (i.e., $\alpha + \gamma < 0$), which means that the utility of expected arrival time is first monotonically increasing and then decreasing and consequently there will be two reliability premia satisfying $EU(D_m, A) = U(D_m, \bar{A}_m + \pi_A)$. Clearly, according to Theorem 2, $\bar{A}_m + \pi_A < 0$ is for risk prone as $\pi_A < 0$ and another $\bar{A}_m + \pi_A > 0$ is for risk averse as $\pi_A > 0$. In contrast to increased travel time, travellers dislike increased travel time variability regardless of earlier or late arrival. This is consistent with Beaud et al. (2016) and validated by considerable empirical evidence summarized in Fig. 3 of Carrion and Levinson (2012)'s review paper. This completes the proof. □

From Eq. (15), we know that the expression for utility when $\bar{A}_m + \pi_A > 0$ is different from that when $\bar{A}_m + \pi_A \le 0$. Due to the decreasing monotonicity of the utility function, it is easy to know that if $EU(D_m,\ A) > U(D_m,\ 0)$, $A_{CE} < 0$; if $EU(D_m,\ A) < U(D_m,\ 0)$, $A_{CE} > 0$. Besides, even though we do not concern ourselves with the risk attitudes and just focus on possible adjustments, an earlier arrival time corresponding to $\pi_A < 0$ may be infeasible due to practical limitations such as limited vehicle speed, heavy traffic or safety considerations. In contrast, a delayed arrival time would seem more practicable.

Hence, in what follows we only derive the reliability premium for delayed arrival time using the schedule delay model with $\alpha > \beta$. As for $\alpha \le \beta$, it would be straightforward to follow the same process. Mathematically, based on Definition 2 and Theorem 1, the reliability premium for a trip with a fixed departure time D_m can be simply defined and calculated by Theorem 7, in which $EU(D_m,\ A) = U(D_m,\ \bar{A}_m + \pi_A)$.

Theorem 7. *For a trip instance $(D_m,\ A)$ with a given departure time D_m, the reliability premium π_A is defined as the maximum amount of delayed adjustment in the arrival time A compared to the expected arrival time $\bar{A}_m$. Mathematically, π_A measures the cost a traveller is willing to pay for eliminating TTV, and its formulation based on the schedule delay model with $\alpha > \beta$ is given by*

$$\pi_A = -\frac{\beta+\gamma}{\alpha+\gamma}(D_m+\mu)\Phi\big((-D_m-\mu)/\sigma\big) + \frac{\beta+\gamma}{\alpha+\gamma}\sigma H\big(\phi, \Phi\big((-D_m-\mu)/\sigma\big)\big) \tag{19}$$

Proof. Since delayed arrival time $\bar{A}_m + \pi_A > 0$ and $EU(D_m,\ A) = U(D_m,\ \bar{A}_m + \pi_A)$ in which $\bar{A}_m = D_m + \mu$, according to Eq. (15), we have

$$EU(D_m,\ A) = (\alpha+\gamma)(D_m+\mu+\pi_A) - \alpha D_m = \gamma D_m + (\alpha+\gamma)(\mu+\pi_A) \tag{20}$$

Based on Eq. (17), the expected utility of the trip instance $(D_m,\ A)$ is

$$EU(D_m,\ A) = -\beta D_m + (\alpha-\beta)\mu + (\beta+\gamma)\int_{\frac{-D_m-\mu}{\sigma}}^{\infty}(\mu+\sigma x+D_m)\phi(x)dx \tag{21}$$

For Eq. (20), we replace its $EU(D_m,\ A)$ by Eq. (21) and consequently we can derive

$$\begin{aligned}\pi_A &= \frac{EU(D_m,\ A) - (\alpha+\gamma)\mu - \gamma D_m}{\alpha+\gamma}\\ &= -\frac{\beta+\gamma}{\alpha+\gamma}(D_m+\mu) + \frac{\beta+\gamma}{\alpha+\gamma}\int_{\frac{-D_m-\mu}{\sigma}}^{\infty}(\mu+\sigma x+D_m)\phi(x)dx\end{aligned} \tag{22}$$

Namely, we have

$$\pi_A = -\frac{\beta+\gamma}{\alpha+\gamma}(D_m+\mu)\Phi\big((-D_m-\mu)/\sigma\big) + \frac{\beta+\gamma}{\alpha+\gamma}\sigma H\big(\phi, \Phi\big((-D_m-\mu)/\sigma\big)\big) \tag{23}$$

This completes the proof. □

Here, we make some observations on the resultant reliability premium. Firstly, Theorem 7 gives a detailed expression for calculating the reliability premium for continuous representation of travel time, as compared to Batley's discrete representation (Batley, 2007). Secondly, it consists of two parts as shown by Eq. (19): (1) for a trip with a given departure time, the first part is a constant value; and (2) the second part includes the mean lateness factor to account for travel time variability. From its formulation, we know that the mean lateness factor accounts for the shape of standardized travel time and its distribution tail above $(-D_m - \mu)/\sigma$. Consequently, the mean lateness factor means that our proposed reliability premium explicitly captures the impact of distribution tail, which is vital due to the typically long and fat tail of travel time distributions. The forthcoming Section 5.3 and Section 5.4 will further elaborate on this property.

For public services, it is more common to adjust the scheduled timetable by a fixed interval such as five or ten minutes. Under such circumstances, the discrete representation used by Batley (2007) may be preferred to the continuous representation used in the present paper. Although the previous sections use continuous representations, all definitions and propositions are valid within the discrete representation also. Below we briefly re-define the individual's trip choice under TTV in trip scheduling with the discrete representation. Specifically, let S_D and S_A denote the finite and exhaustive sets of all possible departure times and arrival times in trip scheduling under TTV, respectively: $S_D = \{D_1, D_2, \dots, D_M\}$ and $S_A = \{A_1, A_2, \dots, A_N\}$ where $D_1 \le D_2 \le \dots \le D_M$ and $A_1 \le A_2 \le \dots \le A_N$. The random trip travel time $T = A - D$ with μ as the mean travel time. Namely, $S_T = \{T_1, T_2, \dots, T_L\}$. For each departure time $D_m \in S_D$, the set S_A contains all corresponding pay-offs (i.e., arrival times). Lottery $\mathbf{p}$ is now the probability vector over the set S_A, and we have:

$$\mathbf{p} = \{p_1, p_2, \dots, p_N\},\ \text{where}\ \sum_{n=1}^{N} p_n = 1\ \text{and}\ 0 \le p_n \le 1 \tag{24}$$

Then, the expected utility for lottery $\mathbf{p}$ over the set S_A under a given D_m (Mas-Colell et al., 1995):

$$EU_{\mathbf{p}}(D_m,\ A) = \sum_{n=1}^{N} p_n U(D_m,\ A_n) \tag{25}$$

All the working thus far can be reconciled with the discrete representation of the reliability premium, and Section 6.1 in the numerical examples will verify this.

5.3. Reliability premium for a given arrival time

This section derives the reliability premium for trips with a fixed arrival time, e.g., commuting trips of employees with a deadline, trips for important conferences/examinations, delivery trips with a required arrival time, etc.

In practice, for travellers with these types of trips, they need to depart earlier to avoid being late because late arrival usually incurs some form of penalty. For example, workers may lose money due to not coming to work on time and students may miss important examinations will be to their detriment. The reliability premium provides a simple answer to the question "how much earlier should I depart" under such circumstances. In other words, we aim to determine the maximum amount of earlier adjustment in the departure time to account for TTV. As shown in scenario 2 of Fig. 3, based on Definition 2 and Theorem 1, the reliability premium for a trip with a given arrival time A_n can be simply defined and calculated by the following Theorem 8, which satisfies $EU(D,\ A_n) = U(\bar{D}_n - \pi_D,\ A_n)$.

Theorem 8. *For a trip instance $(D,\ A_n)$ with a given arrival time A_n, the reliability premium π_D is defined as the maximum amount of early adjustment in the departure time D compared to the expected departure time $\bar{D}_n$. Mathematically, π_D measures the cost a traveller is willing to pay for eliminating TTV, and its formulation using the schedule delay model is expressed as*

$$\pi_D = \frac{\beta+\gamma}{\alpha}(\mu + D)(1 - \Phi((-D-\mu)/\sigma)) + \frac{\beta+\gamma}{\alpha}\sigma H(\phi, \Phi((-D-\mu)/\sigma)) \tag{26}$$

Proof. Obviously, the given arrival time satisfies $A_n < 0$. This means $EU(D,\ A_n) = U(\bar{D}_n - \pi_D,\ A_n)$ in which $\bar{D}_n = A_n - \mu$. Then, according to Eq. (15), we have

$$EU(D,\ A_n) = (\alpha - \beta)A_n - \alpha(A_n - \mu - \pi_D) = \beta A_n + \alpha(\mu + \pi_D) \tag{27}$$

Based on Section 2.2 and Eq. (14), the utility of a particular arrival time $A_n \in A$ for a given departure time D_m and the expected utility for a fixed D_m are:

$$EU(D,\ A_n) = -\beta\bar{D}_n + (\alpha - \beta)\mu + (\beta + \gamma)\int_{\frac{-D-\mu}{\sigma}}^{\infty}(\mu + \sigma x + D)\phi(x)dx \tag{28}$$

Combining the above two formulations, we can finally derive that:

$$\pi_D = \frac{EU(D,\ A_n) + \beta A_n - \alpha\mu}{\alpha} = \frac{\beta+\gamma}{\alpha}\int_{\frac{-D-\mu}{\sigma}}^{\infty}(\mu + \sigma x + D)\phi(x)dx \tag{29}$$

This completes the proof. □

Specifically, π_D must be greater than 0, which means that under this scenario, all travellers are risk-averse according to Theorem 2. Indeed, the literature widely demonstrates the attitude of risk aversion of travellers toward travel time variability for trips under uncertainty (e.g., Knight (1974), Senna (1994), Lo et al. (2006), Chen and Zhou (2010), Li et al. (2010), Sikka and Hanley (2013), Alonso-González et al. (2020), Zang et al. (2018a,b, 2024)). Below we will explore the reliability premium for risk-averse decision-making in detail.

5.3.1. Application to risk-averse trip decision-making

The topic of risk averse trip decision-making research under the schedule delay model framework has attracted considerable interest in the literature(see recent reviews of Li et al. (2010), Carrion and Levinson (2012), Shams et al. (2017), Zang et al. (2022)). One standard proposition in the literature is that the objective of a risk-averse traveller is to identify the optimal departure time D^* to maximise his/her expected utility. Corollary 3 presents an interesting finding if we have $D = D^*$ that shows the proportional relationship between the reliability premium and two well-defined tail-based reliability measures, namely the expected excess delay δ_{EED} in the mean-excess travel time of Chen and Zhou (2010) and the unreliability area S_u in Zang et al. (2024).

Corollary 3. *If $D = D^*$ in Theorem 8, then (1) $\pi_D = \frac{\beta+\gamma}{\alpha}S_u$ and (2) $\pi_D = \frac{\beta}{\alpha}\delta_{EED}$.*

Proof. The expected utility of Eq. (17) is maximized at its first-order condition where D is the optimal departure time D^*. That is to say, $D = D^* = -\mu - \sigma\Phi^{-1}(\gamma/(\beta+\gamma))$. For detailed derivation, interested readers can refer to Fosgerau and Karlström (2010) or Zang et al. (2018b, 2024). If $D = D^*$, the reliability premium given by Eq. (26) would be

$$\begin{aligned}
\pi_D &= \frac{\beta+\gamma}{\alpha}\int_{\frac{-D^*-\mu}{\sigma}}^{\infty}(\mu + \sigma x + D^*)\phi(x)dx \\
&= \frac{\beta+\gamma}{\alpha}\sigma\int_{\Phi^{-1}(\gamma/(\beta+\gamma))}^{\infty}(x - \Phi^{-1}(\gamma/(\beta+\gamma)))\phi(x)dx \\
&= \frac{\beta+\gamma}{\alpha}\cdot\sigma\int_{\gamma/(\beta+\gamma)}^{1}\left(\Phi^{-1}(s) - \Phi^{-1}(\gamma/(\beta+\gamma))\right)ds
\end{aligned} \tag{30}$$

Since $S_u = \sigma\int_{\gamma/(\beta+\gamma)}^{1}\left(\Phi^{-1}(s) - \Phi^{-1}(\gamma/(\beta+\gamma))\right)ds$, so $\pi_D = \frac{\beta+\gamma}{\alpha}S_u$. Besides, as $\delta_{EED} = \frac{\beta+\gamma}{\beta}S_u$ (see Zang et al. (2024) for details), so $\pi_D = \frac{\beta}{\alpha}\delta_{EED}$. This completes the proof. □

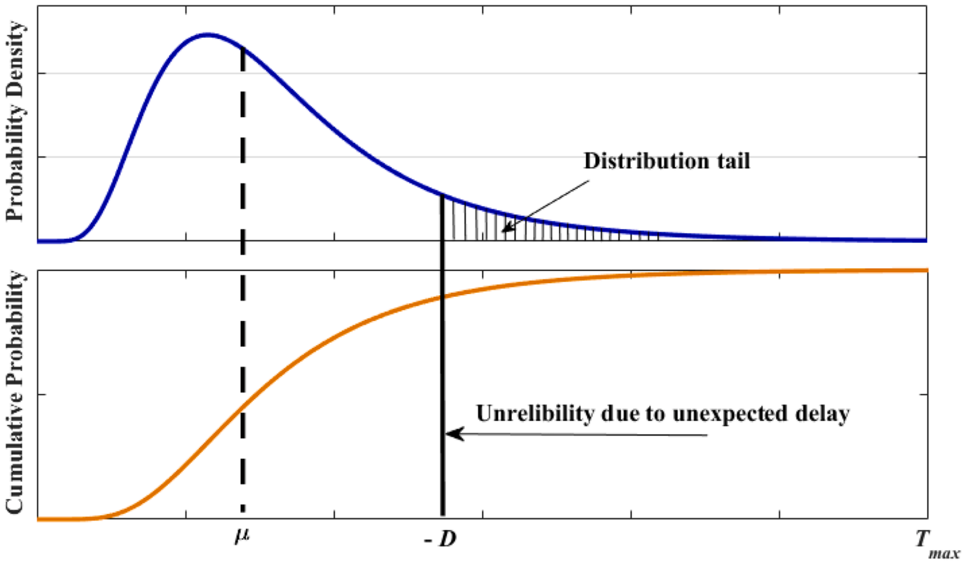

Fig. 6. Illustration the distribution tail captured by the integral Θ (modified based on Chen and Zhou (2010) and Xu et al. (2014)).

The unreliability area is also a tail-based reliability measure illustrated in Zang et al. (2024) and determined by the upper tail of the travel time percentile function. Meanwhile, Chen and Zhou (2010) used the expected excess time to answer the question: how bad should a traveller expect from the distribution tail? Both measures are capable of quantifying the unreliable aspects of travel time variability. Corollary 3 demonstrates that the reliability premium is effective in capturing the asymmetric travel time distributions with long/fat tails and can quantify both the reliable and unreliable aspects, exactly adapting the widely reported long/fat tails of travel time distributions (Van Lint et al., 2008; Fosgerau and Fukuda, 2012; Kim and Mahmassani, 2015; Zang et al., 2018a; Li, 2019).

5.4. A unified framework for evaluating the cost of TTV

Let us re-examine the integral part Θ contained in all calculated reliability premia. Recalling that $T = \mu + \sigma X$, the integral part Θ has an equivalent expression based on T not X (see Appendix A of Zang et al. (2018b) for details). Namely,

$$\Theta = \int_{\frac{-D-\mu}{\sigma}}^{\infty} (\mu + \sigma x + D)\phi(x)dx = \underbrace{\int_{-D}^{T_{max}} T f(T)dT}_{\text{Distribution tail}} + \underbrace{D(1 - F^{-1}(-D))}_{\text{Fixed value}} \tag{31}$$

Note that $D < 0$ since we normalize $PAT = 0$, so $-D$ is positive. As shown in Eq. (31), the second part of the integral Θ is able to capture the distribution tail of trip travel time beyond $-D$. Section 5.3.1 has already proved that when $D = D^*$, the second part is exactly the mean lateness factor defined in Fosgerau and Karlström (2010). We use Fig. 6 to visually illustrate it more clearly, in which the upper panel is the PDF of trip travel time T, and the lower panel is the CDF of T. As pointed out by the upper figure of Fig. 6, the shaded area is the area of the distribution tail beyond the travel time of $-D$. If $T > -D$, all travel times in the shaded area given by the vertical solid line can make this trip unreliable and thus travelers will experience unexpected delay. This is the source of travel time variability for travellers and the reliability premium fully captures this using the integral Θ. This illustrates why the reliability premium measures the TTV cost.

The ability to explicitly capture the distribution tail makes reliability premium outperform the existing valuation measures, such as standard deviation and variance, percentiles or ranges between percentiles[2]. The standard deviation and variance may not capture the distribution tail of travel time especially well, as they can only quantify the extent of variation or dispersion from the mean value. Similarly, the percentiles tend to focus on the mass of the travel time distribution, but omits consideration of the tail. It should be noted that the VOR defined in Fosgerau and Karlström (2010) could imexplicitly consider the impact of the distribution tail as its expression included the mean lateness factor, while the reliability premium explicitly considers the distribution tail and its impact on traveler's pay-offs of trip choices and then clearly quantifies the cost resulting from the distribution tail. As pointed out by Zang et al. (2024), the cost resulting from distribution tail of travel times has not been explicitly considered in previous VOR studies, which may result in bias in policy-making or decision-making as it could account for up to 12% of the total cost of TTV. Specifically, the simple

[2] Interested readers may refer to Li et al. (2010), Carrion and Levinson (2012), Zang et al. (2022) for comprehensive reviews of theories and evidence.

Table 1
Pay-off matrix with discrete representations in which $PAT = 525$.

Pay-offs		*A*	*A*	*A*	*A*	*A*	*A*	*A*	*A*	*A*	*A*	*A*	*A*	*A*	*A*	*A*
		480	485	490	495	500	505	510	515	520	525	530	535	540	545	550
D	445	0.20	0.60	0.10	0.10											
D	450		0.05	0.50	0.30	0.15										
D	455				0.20	0.50	0.20	0.10								
D	460					0.10	0.70	0.10	0.10							
D	465							0.30	0.50	0.10	0.05	0.05				
D	470								0.05	0.60	0.20	0.10	0.05			
D	475									0.20	0.40	0.20	0.10	0.10		
D	480											0.30	0.50	0.10	0.10	
D	485												0.50	0.30	0.10	0.10

monetisation of the reliability premium further makes it to be a ideal valuation method to be used in the appraisals of transport projects or policies regardless of the requirements of quantifying the TTV in monetary unit or/and in time unit.

In addition, the reliability premium, as a time-based measure, remains consistent with the unit of payoffs involved in trip decision-making including departure times or arrival times, making it behaviourally reasonable and easily understood by travellers. This property enables its practical implementation in valuation studies and policy-making concerning travellers' behaviours. The ease of application of the reliability premium has already been justified by Wijayaratna and Dixit (2016) who study how traveller information impact on risk attitudes and the valuation of reliability based on the reliability premium. Discussion in Section 4.3 has already proved the feasibility and superiority of the reliability premium as a reliability measure used for route choice and related traffic assignment models.

In short, the reliability premium provides a unified framework for evaluating TTV in time units and valuing the cost of TTV in monetary units, bridging the existing gaps between these two topics. In the literature, TTV evaluation measures are usually expressed in time units and thus not applicable for project appraisal; meanwhile, the existing derived VORs for TTV valuation are in monetary unit, making it lack of insights/guidance in travellers' realistic decision-making.

5.5. Calibration issues related to the reliability premium

As the reliability premium is defined based on the utility function of travel times, its calibration process should be similar to those associated with the reliability ratio. The most common way is to conduct Revealed Preference (RP) or Stated Preference (SP) surveys, which have received substantial attention in the existing literature. Li et al. (2010) and Carrion and Levinson (2012) undertook a comprehensive review of such surveys. In addition, as discussed in the Introduction part, considerable efforts have been invested in designing SP questionnaires to communicate the concept of TTV as straightforwardly as possible with the interviewees. Since the reliability premium reflects the natural response of travellers to travel time variability, it may indeed reduce the difficulties of communicating TTV to survey respondents.

Besides, the concept of the reliability premium makes it possible to use passive ways to estimate the values of these parameters. For example, we can collect long-term massive datasets for the same commuting trip of an individual through data acquisition techniques, such as GPS, Bluetooth, and cellular data. That is, for each trip record, we know the departure and arrival times, allowing us to calculate the expected travel time and infer the reliability premium. In addition, we can obtain the empirical travel time distribution and its mean and standard deviation from long-term massive data. Then, classical parameter estimation methods, such as maximum-likelihood estimation, can be used to estimate the risk preference parameters. This is quite useful as travellers may not know their own values and these parameters may be flow-dependent rather than fixed.

6. Numerical examples

This section illustrates the theoretical exposition of the reliability premium, and thereby demonstrate its benefits to both travellers' decision-making and economic project appraisal.

6.1. Public transport

This section uses data extracted from the example of Batley (2007) with slight modifications to show the benefits of the reliability premium for public transport.

Table 1 shows data pertaining to the same journey carried out at different departure times together with their corresponding arrival times and their probabilities. The time of 0:00 am is set as reference point and we use 0 minutes to denote 0:00 am. The PAT = 525 means that we use 8:45 am as preferred arrival time in our model. Each departure time typically has four or five possible arrival times. In accordance with the analysis of Section 5.2, we consider two sets of preference parameters. One is based on Small (1982)'s empirical estimates to investigate the case of $\alpha < \beta$. Namely, we set $\alpha = -0.106$, $\beta = -0.065$ and $\gamma = -0.254$. Another set slightly changes β from -0.065 to -0.165 while keeping other parameters unchanged for the case of $\alpha > \beta$. As for its ticket fare, we assume $\varphi = 0.5$ dollar.

Table 2
Reliability premiums for different departure times with $\alpha < \beta$ and $\alpha > \beta$.

D	$\bar{A}$	$\alpha < \beta$				$\alpha > \beta$	
		$U(\bar{A})$	EU	$\pi_A\vert_{A_{CE}<PAT}$	$\pi_A\vert_{A_{CE}>PAT}$	$\pi_A\vert_{A_{CE}<PAT}$	$\pi_A\vert_{A_{CE}>PAT}$
465	515.25	-5.96	-6.04	1.95	**N.A. [524.11]** [a]	-1.78	11.64
470	522.50	-5.73	-6.05	**N.A. [530.28]** [a]	3.10	-7.10	4.07
475	527.50	-6.20	-6.52	**N.A. [554.73]** [a]	0.89	-24.86	1.16

[a] Notes: (1) $PAT = 525$; (2) for columns of π, N.A. means not applicable due to the nonexistence of associated certainty equivalent arrival time A_{CE}

Table 3
Reliability premia for different late probabilities and associated earlier adjustments.

D	γ	ρ[a]	π_D	μ	$-\mu - \pi_D$	Earlier?[a]	μ'	$-\mu' - \pi_D$	Earlier?	μ''	$-\mu'' - \pi_D$	Earlier?
-65	-4.00	20%	3.95	57.08	-61.03	No	60	-63.95	No	62	-65.95	Yes
-65	-5.66	15%	5.25	57.08	-62.33	No	60	-65.25	Yes	62	-67.25	Yes
-65	-9.00	10%	7.89	57.08	-64.97	No	60	-67.89	Yes	62	-69.89	Yes
-65	-19.00	5%	15.78	57.08	-72.86	Yes	60	-75.78	Yes	62	-77.78	Yes
-65	-99.00	1%	78.90	57.08	-135.98	Yes	60	-138.90	Yes	62	-140.90	Yes

[a] Notes: (1) ρ is the late probability and $\rho = 1 - \gamma/(\gamma + \beta)$ in which $\beta = -1$;
(2) "Earlier?" means if we need an earlier departure time. If $D > -\mu - \pi_D$, yes; no otherwise.

Only three departure times (i.e., 465, 470, 475) have different utilities and expected utilities. This is because for each of these three departure times, the associated arrival times as shown in Table 1 are not all less than or greater than PAT. Obviously, the utility function in our schedule delay model is linear and thus the second-order derivatives of the schedule delay model with respect to D, A, and T are all 0. Therefore, according to Corollary 2, the TTV cost for a given departure time (or equivalently, the benefit of eliminating travel time variability via delaying arrival time) should be estimated based on Eq. (13) and Eq. (10). Specifically,

$$Benefits = C_{TTV} = \frac{\alpha - \beta}{\varphi}\pi_A \tag{32}$$

Table 2 gives the corresponding reliability premia for these three departure times, i.e., 465, 470, and 475, under the cases of $\alpha < \beta$ and $\alpha > \beta$. As reported by Table 2, for $\alpha < \beta$, the reliability premia for delayed arrival times are 1.95, 3.10, and 9.89. With reference to Eq. (32), the $VOT = -0.082$ and their benefits are -0.16, -0.25, and -0.07, respectively. Similarly, for $\alpha > \beta$, the reliability premiums for delayed arrival times (the last column) are 11.64, 4.07, and 1.16. Accordingly, the $VOT = 0.118$ and their benefits are 1.37, 0.48, and 0.14, respectively.

The above analysis is valid for both agencies and travellers. With this detailed information, travellers could re-schedule their arrival times for planned activities in advance to avoid being late. Agencies or service providers can simply delay the scheduled arrival time to improve the reliability of arrival time for their services, yielding additional benefits to some users as well as themselves due to increased attractiveness of public services. The further analysis of the possible benefits to all users depends on the number of beneficiaries, and their individual preferences and associated the reliability premiums, to be shown in Section 6.3.

6.2. Commute trip

This section utilizes the school bus data obtained from Tongji University, the same as Zang et al. (2024), to explore the application of the reliability premium to bus services with required arrival times. It is important to note that the school bus data serves as an illustrative instance only of similar on-demand transportation systems with fixed arrival times.

The data comprises GPS records capturing the routing of the school bus from Jiading Campus to Siping Campus in Shanghai, encompassing both expressways and urban streets. The total length of the route is approximately 37.10 km, with data collection from 18 September 2017 to 17 January 2018. The school bus starting at 6:40 am is mainly provided for students and teachers who take or teach the first class in the morning at 8:00 am. As there is some distance between car park for the terminus of school bus and classrooms, it is necessary for this bus to arrive by 7:45 am for on-time class. In other words, we set 7:45 am as the PAT and normalize the PAT to zero, in which case the scheduled departure time of 6:40 am will be -65 mins.

Since a punctual start to class is critical to both teachers and students, it seems reasonable to assume that most of travellers of the school bus are risk-averse. In fact, the previous survey by Lo et al. (2006) showed that most students are risk averse for such trips and will reserve buffer time beyond mean travel time. According to previous studies (e.g., Bates et al., 2001; Fosgerau and Karlström, 2010; Zang et al., 2024), the value of $1 - \gamma/(\gamma + \beta)$ can be interpreted as late probability. The subsequent Table 3 presents reliability premia for different late probabilities and associated earlier departure times based on Eq. (26) of Theorem 8.

Table 3 explores the impact of a small change of mean travel time on the school bus service while keeping all others unchanged. We can see that only a 3-minute increase in mean travel time, i.e., from 57.08 to 60, prompts users with $\rho \leq 15\%$ to switch to an earlier departure time, and only a 5-minute increase in mean travel time prompts all users with $\rho \leq 20\%$ to switch to an earlier

Table 4
Resultant benefits of earlier adjustment of departure time from −65 to −75 for different mean travel times with fixed trip travel time distribution and reliability premia.

μ	Group Percentage Late probability	Teachers 1 20% 5%	Teachers 2 40% 10%	Students 1 20% 15%	Students 2 20% 20%	Total 100%
$\mu = 57.08$	Benefits	3.93	-5.00	-2.00	-2.00	-5.07
$\mu' = 60.00$	Benefits	5.00	1.45	0.05	-2.00	4.50
$\mu'' = 62.00$	Benefits	5.00	2.45	0.45	0.19	8.09

Table 5
Resultant benefits of earlier adjustment of departure time from −65 to −75 for different late probabilities with fixed travel time distribution and mean travel time.

Cases	Group Percentage	Teachers 1 20%	Teachers 2 40%	Students 1 20%	Students 2 20%	Total 100%
Case 1	Late probability	5%	10%	15%	20%	
	Benefits	3.93	-5.00	-2.00	-2.00	-5.07
Case 2[a]	Late probability	5%	5%	10%	15%	
	Benefits	3.93	3.93	-2.00	-2.00	3.86

[a] Note. The last three groups in Case 2 have smaller late probabilities compared to that of Case 1.

departure time. In other words, the current departure time $D = -65$ may not satisfy users' requirements if road traffic becomes a little worse. In reality, the university officially adjusted the departure time from 6:40 am to 6:30 am (i.e., 10 minutes earlier than before) on November 23, 2020. Interested readers can refer to the website (https://see.tongji.edu.cn/info/1144/8381.htm) for the official notice in Chinese. The university stated that this adjustment was due to road traffic congestion and the needs of the university's teaching arrangements, which is consistent with our analysis.

From Table 3, the change in the reliability premium becomes quicker with smaller late probability and there is a sudden change, nearly 5 times, in the reliability premium for reducing late probability $\rho = 5\%$ to $\rho = 1\%$. Such sudden change means that blindly pursuing on-time arrival (i.e., $1 - \rho$) is economically inefficient, which is consistent with the diminishing marginal effect in Xu et al. (2014) and Zang et al. (2024).

6.3. *Project appraisal*

This section continues to use the school bus data to explore inclusion of the reliability premium as a measure of TTV cost in project appraisal. It is worth acknowledging that adjustments to either departure times or arrival times may generate benefits for some users but costs for others.

For simplicity, assume that α and β are the same for all users and only γ is different. Set $\alpha = -2$ and $\beta = -1$. The realistic ticket fare for students is 5 Chinese Yuan (CNY) and for teachers is 2 CNY, so the VOT for students $= -0.2$ CNY and the VOT for teachers $= -0.5$ CNY. As the benefits embodied by the reliability premium will depend on the number of beneficiaries and their individual preferences, we assume that there are two groups of students and two groups of teachers, accounting for 20%, 40%, 20%, 20%, respectively. Risk preference parameters are the same for users within each group.

We conduct two experiments to examine the total additional benefits of adjusting the departure time from −65 to −75: (i) one assumes that only mean travel time increases while the distribution of trip travel time and associated reliability premium remain unchanged; (ii) another assumes that only risk preference parameters change (i.e., the late probability) while the distribution of trip travel time and associated mean travel time remain unchanged. Table 4 shows the results of the first experiment, while the results of the second experiment are summarized in Table 5.

As shown in Table 4, for current mean travel time μ, the total benefits are negative and only the Teachers 1 group can gain benefits from the earlier departure time but the remaining three groups lose interests. But with increase in mean travel time, the total benefits become positive. Specially, for an increase of 5 minutes in mean travel time, all groups can gain positive benefits from the policy of earlier departure time. This supports decision to reschedule the 6:40 am departure.

Table 5 indicates that even though road traffic congestion remains unchanged, the earlier adjustment of the school bus can bring positive total benefits as teachers and students become more conservative on late probability. To be specific, for case 1, only the first group of Teachers 1 can gain benefits from earlier adjustment while other three groups. Compared to case 1, in case 2 the last three groups except the first group all decrease their late probability by 5%, indicating a more conservative attitude on late probability. Such 5% decrease only has make the benefits gained by the earlier adjustment for all these three group change from negative to positive.

7. Concluding remarks

The VORs derived in existing studies are typically expressed in monetary units, which is useful for project appraisal of transport infrastructure. However, the VOR-related payoffs associated with trip choices are defined in time units, making the derived VORs (in monetary units) in the existing literature conceptually unclear and challenging to be applied in travellers' decision-making of trip choice. Therefore, this paper proposes an alternative approach to intuitively measure the cost of travellers eliminating TTV in the context of trip scheduling, using time units. Firstly, we establish individuals' trip choices under TTV in trip scheduling based on the theory of choice under uncertainty in microeconomics. Then, we formally and comprehensively define the concept of reliability premium and prove their appealing mathematical properties, including defining travellers' risk attitudes without assuming the type of utility function, comparability in the small and in the large, and simple monetisation via the VOT. Such properties guarantee the reliability premium to be not only solidly theory grounded but also behaviorally realistic, thereby enabling the reliability premium to provide a unified framework for both evaluating the TTV in time units, ready for individual traveler's routing choice decisions; and valuing TTV in monetary units for cost-benefit analysis in project appraisal. Then, a simple schedule delay model was used to derive the formulations of reliability premiums for three scenarios to promote its application in practice, corresponding to realistic trips in travellers' daily lives, including trips by public transport, commuting trips, and recreation trips, etc. Numerical examples verified and illustrated the above mentioned appealing properties of the proposed reliability premium.

For future research, it is natural to extend this work to quantify the effects of other factors in traveler's decision-making based on our proposed method. For example, to quantify the value of provided travel time information, we could derive the reliability premium for the cases with and without provided information using developed methodological framework in the literature (Engelson and Fosgerau, 2020). Besides, even for real-time condition, the proposed reliability premium could bring additional benefits to online platforms such as delivery apps and ridesharing apps by incorporating reliability premium into the estimation of travel time or waiting time displayed by these platforms. Specifically, the online platforms could display a range of possible values of waiting time to the customer due to stochastic nature of customers' waiting time; however, the setting of the magnitude of the estimated waiting time information could have a significant impact on customer abandonment (Yu et al., 2022). Therefore, the reliability premium and associated certainty equivalent provides a reliable waiting time reporting way which is not only easily understandable to the public but also trustworthy/reliable due to its a certain risk consideration of distribution tail beyond mean travel time (Chen et al., 2023). In addition, the reliability premium can be regarded as a new reliability measure with appealing properties as demonstrated in this manuscript. Consequently, the methods of modelling perceived reliability measures, such as the perceived travel time budget (Shao et al., 2006) and perceived mean-excess travel time (Chen et al., 2011; Xu et al., 2013), are directly applicable for modelling the perceived reliability premium. In other words, based on Chen et al. (2011) and Xu et al. (2013), we can calculate the perceived reliability premium π_p based on the perceived travel time distribution $T_p = T + \epsilon|T$, where users' stochastic perception error ϵ depends on the actual travel time distribution T.

CRediT authorship contribution statement

Zhaoqi Zang: Conceptualization, Methodology, Formal analysis, Writing – original draft; **Richard Batley:** Methodology, Writing – review & editing, Conceptualization; **David Z.W. Wang:** Funding acquisition, Methodology, Writing – review & editing, Supervision, Conceptualization; **Hong K. Lo:** Methodology, Writing – review & editing, Validation.

Declaration of competing interest

None.

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Appendix A. Appendix of Proofs

Proof of Theorem 2

Let us first focus on (1) risk averse iff $\pi = T_{CE} - \mu > 0$. If an individual is risk averse, according to Jensen's inequality, we must have $EU(T) < U(\mu)$. Since the Bernoulli utility function is nonincreasing, we could easily have

$$EU(T) < U(\mu) \Longrightarrow U(T_{CE}) < U(\mu) \Longrightarrow T_{CE} > \mu \Longrightarrow \pi > 0 \tag{A.1}$$

Conversely, if $\pi = T_{CE} - \mu > 0$, we have

$$T_{CE} > \mu \Longrightarrow U(T_{CE}) < U(\mu) \Longrightarrow EU(T) < U(\mu) \tag{A.2}$$

This means that this individual is risk averse according to Jensen's inequality. In summary, an individual is risk averse iff $\pi = T_{CE} - \mu > 0$. The proof of risk neutrality or risk proneness follows the same logic, in which we only need to change the initial condition.

Proof of Theorem 3

Let $\delta(T) = F_{\mathbf{p}}(T) - F_{\mathbf{q}}(T)$. If lottery **p** first order stochastically dominates lottery **q**, then we have $F_{\mathbf{p}}(T) \geq F_{\mathbf{q}}(T)$, which means $\delta(T) \geq 0$ for all T. For the expected utility, we have

$$\begin{aligned} EU_{\mathbf{p}}(T) - EU_{\mathbf{q}}(T) &= \int_{T_{min}}^{T_{max}} U(T) d(F_{\mathbf{p}}(T)) - \int_{T_{min}}^{T_{max}} U(T) d(F_{\mathbf{q}}(T)) \\ &= \int_{T_{min}}^{T_{max}} U(T) d(F_{\mathbf{p}}(T) - F_{\mathbf{q}}(T)) \\ &= \int_{T_{min}}^{T_{max}} U(T) d\delta(T) \\ &= -\int_{T_{min}}^{T_{max}} U'(T)\delta(T) dT \geq 0 \text{ (Integrating by parts)} \end{aligned} \tag{A.3}$$

As $EU_{\mathbf{p}}(T) = U(\mu + \pi_{\mathbf{p}})$ and $EU_{\mathbf{q}}(T) = U(\mu + \pi_{\mathbf{q}})$, replacing $EU_{\mathbf{p}}(T)$ and $EU_{\mathbf{q}}(T)$ in Eq. (A.3) using these relationships, we have $U(\mu + \pi_{\mathbf{p}}) \geq U(\mu + \pi_{\mathbf{q}})$ and thus $\pi_{\mathbf{p}} \leq \pi_{\mathbf{q}}$. So, the reliability premium of lottery **p** is always less than that of lottery **q** and traveler would prefer **p** to **q**.

Let $\eta(T) = \int_T^{T_{\max}} \delta(\omega) d\omega$. If lottery **p** second order stochastically dominates lottery **q**, then we have $\int_T^{T_{\max}} F_{\mathbf{p}}(\omega) d\omega \geq \int_T^{T_{\max}} F_{\mathbf{q}}(\omega) d\omega$, which means that $\eta(T) \geq 0$ for all T. Then, for the expected utility of lottery **p** and lottery **q**, we have

$$\begin{aligned} EU_{\mathbf{p}}(T) - EU_{\mathbf{q}}(T) &= -\int_{T_{min}}^{T_{max}} U'(T)\delta(T) dt \\ &= \int_{T_{min}}^{T_{max}} U'(T)\eta'(T) dT \text{ (since } \delta(T) = -\eta'(T)) \\ &= -\int_{T_{min}}^{T_{max}} U''(T)\eta(T) dT \geq 0 \text{ (Integrating by parts)} \end{aligned} \tag{A.4}$$

Similarly, we know that $U(\mu + \pi_{\mathbf{p}}) \geq U(\mu + \pi_{\mathbf{q}})$ and thus $\pi_{\mathbf{p}} \leq \pi_{\mathbf{q}}$. Put differently, for $\mathbf{p} \succ_2 \mathbf{q}$, reliability premium of lottery **p** is always less than that of lottery **q**, in which case the traveler would prefer **p** to **q**. This completes the proof.

Proof of Theorem 4

Let $U_i(T)$ and $U_j(T)$ represent the utility functions of traveler i and traveler j, respectively. We know $U_i(T)' \leq 0$ and $U_j(T)' \leq 0$. According to the rule of monotonicity of function composition, there always exists a non-decreasing function $\psi : \mathbb{R} \to \mathbb{R}$, which can be defined as:

$$\psi(U_j(T)) = U_i(T) \text{ where } U_i' = \psi(U_j)' U_j' \text{ and } U_i'' = \psi(U_j)''(U_j')^2 + \psi(U_j)' U_j'' \tag{A.5}$$

According to Definition 2 and Eq. (7), we have

$$E\psi(U_j(T)) = EU_i(T) = U_i(\mu + \pi_i) \text{ and } \psi(EU_j(T)) = \psi(U_j(\mu + \pi_j)) = U_i(\mu + \pi_j) \tag{A.6}$$

For the sufficiency of Theorem 4. if traveler i is more risk averse than traveler j, we have,

$$A_{U_i}(T) - A_{U_j}(T) = \frac{U_i''(T)}{U_i'(T)} - \frac{U_j''(T)}{U_j'(T)} = \frac{\psi(U_j)'' U_j'}{\psi(U_j)'} \geq 0 \tag{A.7}$$

Since $\psi(U_j)' \geq 0$ and $U_j' \leq 0$, we must have $\psi(U_j)'' \leq 0$ and the function ψ is concave, leading to $E\psi(U_j(T)) \leq \psi(EU_j(T))$. Substituting this into Eq. (A.6), we finally get $U_i(\mu + \pi_i) \leq U_i(\mu + \pi_j)$ and therefore $\pi_i \geq \pi_j$. This completes the proof of sufficiency.

As for necessity of Theorem 4, we use proof by contradiction. Suppose that when $\pi_i \geq \pi_j$, we have traveler i is not more risk averse than traveler j, namely $A_{U_i}(T) - A_{U_j}(T) < 0$. On the basis of Eq. (A.7), now we must have $\psi(U_j)'' > 0$ and consequently the function ψ is convex. The convexity of ψ means $E\psi(U_j(T)) > \psi(EU_j(T))$, which further results in $\pi_i < \pi_j$ based on Eq. (A.6). The final result contradicts the given condition $\pi_i \geq \pi_j$. As a result, the necessity condition is true. This completes the proof of necessity.

Proof of Theorem 5

According to Taylor series expansion, $U(\mu + \pi)$ at the point μ is given by

$$U(\mu + \pi) = U(\mu) + U'(\mu) * \pi + \frac{U''(\mu)}{2} * \pi^2 + o(\pi^2) \tag{A.8}$$

Substituting Eq. (A.8) into Eq. (11), we have

$$\begin{aligned} C_{TTV} &= \left(U(\mu) + U'(\mu) * \pi + \frac{U''(\mu)}{2} * \pi^2 + o(\pi^2) - U(\mu) \right) * \frac{1}{\varphi} \\ &= \left(U'(\mu) * \pi + \frac{U''(\mu)}{2} * \pi^2 + o(\pi^2) \right) * \frac{1}{\varphi} \\ &= U'(\mu) * \frac{1}{\varphi} * \pi + \frac{U''(\mu)}{2} * \pi^2 * \frac{1}{\varphi} + o(\pi^2) * \frac{1}{\varphi} \\ &= \pi \left(VOT(\mu) + \frac{1}{\varphi} \frac{\pi}{2} U''(\mu) \right) + \frac{1}{\varphi} * o(\pi^2) \end{aligned} \tag{A.9}$$

This completes the proof.

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