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THE INTERNAL VALIDATION OF A NATIONAL MODEL OF LONG DISTANCE TRAFFIC

by

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ABSTRACT

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During 1980/81, the Department of Transport developed a model for describing the distribution of private vehicle trips between 642 districts in Great Britain, using data from household and roadside interviews conducted in 1976 for the Regional Highways Traffic Model, and a new formulation of the gravity model, called a composite approach, in which shorter length movements were described at a finer level of zonal detail than longer movements. This report describes the results of an independent validation exercise conducted for the Department, in which the theoretical basis of the model and its the quality of its fit to base year data were examined. The report discusses model specification; input data; calibration issues; and accuracy assessment. The main problems addressed included the treatment of intrazonal and terminal costs, which was thought to be deficient; the trip-end estimates to which the model was constrained, which were shown to have substantial variability and to be biassed (though the cause of the latter could be readily removed), with some evidence of geographical under-specification; and the differences between roadside and household interview estimates. The report includes a detailed examination of the composite model specification and contains suggestions for improving the way in which such models are fitted. The main technical developments, for both theory and practice, are the methods developed for assessing the accuracy of the fitted model and for examining the quality of its fit with respect to the observed data, taking account of the variances and covariances of modelled and data values. Overall, the broad conclusion was that, whilst there appeared to be broad compatibility between modelled and onserved data in observed cells, there was clear evidence of inadequacy in certain respects, such as for example underestimation of intradistrict trips.

This work was done in co-operation with Howard Humphreys and Partners and Transportation Planning Associates, who validated the model against independent external data; their work is reported separately.

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THE INTERNAL VALIDATION OF A NATIONAL MODEL OF LONG-DISTANCE TRAFFIC

1. SUMMARY AND CONCLUSIONS

1.1 INTRODUCTION

1.1.1 This report summarises the work carried out at the Institute for Transport Studies of the University of Leeds to assess the validity of the Department of Transport's National Model (NM) of Long Distance Traffic. Because the commercial vehicle model was not ready for validation, the work was concerned almost exclusively with the private vehicle model, as described in the first draft of Outram (1982).

The Leeds work was primarily concerned with the internal validation of the model, that is, the performance of the model as structured, and judged against the data to which it was fitted. Judgements of model performance against independent data sets (i.e. ones to which the model was not fitted), constituted the external validation, which was the responsibility of Howard Humphreys and Partners (HH&P), working with Transportation Planning Associates (TPA). These consultants also undertook those aspects of the internal validation which were most appropriately handled by the Department of Transport's 'validation and comparison' suite of computer programs, which they had previously developed; the Leeds team provided mathematical and statistical advice to this work, with the links between the two geographically well-separated teams being mainly maintained as a result of Dr. Murchland being based in London.

1.1.2 The internal validation reported here covers four aspects, discussed in succeeding sections of the report, as follows.

1.1.3 (a) Judgements on MODEL SPECIFICATION, including the definition of a composite matrix, composite model, composite costs, multiple deterrence functions, and the effects of changes in intrazonal cost specification.

. . . Section 2.

1.1.4 (b) Comments on INPUT DATA and its adequacy, covering the inter-zonal and intrazonal cost definitions, the treatment of minor road traffic, the correction for inactive households, statistical tests for a cordon-crossings comparison of household and roadside interview data, the method of merging several trip estimates, and the trip-end estimates.

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1.1.5 (c) Comments on the CALIBRATION method, including questions of principle, uniqueness, solution method, calculational economy and the smoothing of the cost functions.

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1.1.6 (d) The making of an ACCURACY ASSESSMENT of the fitted model, including showing how judgements about the extent of appreciable model mis-specification may be made, taking into account the accuracies of the input data; the assessment of the accuracy of the trip-end estimates; the approximate analytic formula for the accuracy of the fitted model; the interpretation of the goodness of fit of the model in intra district cells, and overall.

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1.1.7 The internal validation undertaken here is complementary to that undertaken by Howard Humphreys and TPA, whose final report should also be referred to. (Howard Humphreys and Partners, 1982.)

1.1.8 In the rest of this section we summarise the main findings of our study, and consolidate the conslusions here rather than at the end of the report.

1.1.9 An Appendix contains some statistical summaries of data that are pertinent to our report (see Section 8).

1.1.10 For further technical details, the reader is referred to the Working Notes (WN) produced on this project. These are listed in the contents sections and are available as separate Annexes.

1.2 MAIN FINDINGS

Comments on model specification

1.2.1 Given the choice of a gravity model to describe the distribution of trips, and the existence of the RHTM data base at the 3613 regional zone levels of information, the procedures used to determine the composite approach and define composite cost seem reasonable. (2.1.1)

1.2.2 The composite model itself may be described most simply as a model at the level of a 642 district system*, which differs from conventional models only by having several cost values for nearby district pairs instead of the usual one. (2.2.14)

The model has been structured in such a way as to enable it to proxy the effects of a model constructed at the 3613 regional zone level, but some of the assumptions used in so doing have not been tested. (2.2.4)

1.2.3 The private vehicle and commercial vehicle models represent different ways of attempting to achieve the same goal, of a model fitted at the 642 district level being consistent with that which would have been obtained by aggregation of one fitted at the 3613 zone level. We would expect the private vehicle model to give rather more refined estimates of the cost factors than the commercial vehicle model but have no evidence for assessing how different the two approaches are. (2.3.11, 12)

1.2.4 We have no definite evidence for believing that there is any important bias introduced by the use of RHTM rather than NM cost functions for defining composite costs for remote district pairs, but a number of possible problems have been identified, in which perhaps the main one is that due to using the RHTM HBW cost

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* Note: However, as 2 districts had virtually no trips it was virtually a 640 district system.

functions to define composite cost for all trip purposes. (2.3.14 et seq.)

1.2.5 The definition of different deterrent functions for within-town movements from those elsewhere may be argued on behavioural grounds (2.4.3) and the further distinction between rural/urban/ metropolitan and London distributions was introduced to reflect differences in the strength of the public transport alternative. We are however rather doubtful that this choice has been substantiated because the test bed demonstration was pathological. (2.4.6) Guidelines on how best to define areas in the matrix to which different cost functions apply should be developed (2.4.10)

1.2.5 The adjustments made to intrazonal costs, to make them such as to make the model give better agreement with observed intrazonal trips, complicates the model specification, making it more difficult to analyse the error properties in the fitted model.

Comments on input data

1.2.7 The calculation of O-D generalised cost on the basis of minimum time paths is unlikely to have an adverse influence on model fit (3.1.2). Any adverse effects due to the use of the same value of time for all trip purposes and regions, irrespective of regional variations in income, will be reduced as a consequence of fitting multiple deterrence functions. (3.1.3)

1.2.8 The reasons for the adjustments made to intrazonal costs and the use of terminal cost corrections for movements between zones with towns are obscurely presented and the empirical evidence presented unconvincing. (3.2.3) However, there are sound theoretical reasons for making such changes (2.5.5 - 2.5.7; 3.1.5) and it is urged that these be developed in order to make a case for these (or similar) changes which avoid the charge that the adjustments are made simply in order to improve the fit between model and data (3.2.5 and 2.5.4).

1.2.9 The basis for allocating purpose and trip length characteristics to uninterviewed traffic on minor roads is an improvement on the previous use of corridor factors (3.3.1 - 3.3.5) but, having been carried out on a cordon-wide basis, there may be directional biases in the NM observed flows which should be taken into account when making comparisons with the fitted model (whose parameters should not be affected by these directional biases) or with independent data (3.3.6 - 3.3.9). The assumed magnitude of flows on non-counted roads should be substantiated. (3.3.4)

No comparisons were possible with the alternative more sophisticated corridor expansion procedures developed by Martin and Voorhees Associates (MVA), but it is suggested that the Department consider advising on the use of the MVA procedures in any new O-D travel surveys. (3.3.9 - 3.3.10)

1.2.10 The inactive household correction factor, which was abandoned when providing trip-end estimates, was retained in the observed data set to which the model was fitted, and is a major cause of discrepancies subsequently discovered. (3.4)

1.2.11 The investigation of round trips carried out in the development of the National Model has potentially important implications for data collection and model building strategies, and deserves further investigation. The differences that occur in the proportions and trip-lengths of single-leg trips in the outbound and inbound directions could have a significant influence on the RI trip length characteristics for a particular trip purpose even at the national level, since most roadside interviews were in the outbound direction. (3.5)

1.2.12 Statistical comparisons of the household and roadside interview estimates of cordon crossing trips did not reveal a significant difference between the data sets for HBW and HBEB trips; but HBO trips were significantly different for the data set used, unless the expansion factors had a coefficient variation exceeding 10 percent. The differences were less significant when more weight was given to the HI data, for example by dropping the inactive household correction factor (see 1.2.10). (3.6)

1.2.13 It is surprising that seasonal correction factors had still not been applied to the data, despite previous evidence that their absence could account for some of the differences between household and roadside interviews. (3.7)

1.2.14 The method of merging data sets is satisfactory if there is no bias between the data sets, and is broadly consistent with the calculation of the accuracies of the data sets made in this project (5.3.6), but might require more weight to be given to HI data in the light of the evidence that the data sets are biased with respect to each other.

1.2.15 The synthetic trip end estimates and district totals of observed values in wholly-observed districts are biased with respect to each other. The main bias (for NHB and EB trips) is that due to non-home based trips by non-residents being included in the trip-end estimates but not the observed matrices (3.9.5); the secondary bias is due mainly to the inactive household correction factor being retained in the HI data but omitted from the trip end estimates (3.9.6 - 3.9.8) and partly due to planning data used in the trip end models being a later version to that used in scaling up the HI data (3.9.9 - 3.9.12). For technical reasons, the discrepancies, though large, are not transparent in the comparisons between the model and the data, their effects being propagated to the unobserved cells (3.9.13 - 3.9.14). It is important that the defects be remedied, probably by dropping the inactive household correction factor from the observed data set, and possibly by revising the NHB trip end models to apply to trips by residents only.

1.2.16 The reasons for the new method of balancing attractions to generations are obscure, but if it is retained, the Traffic Appraisal Manual's information on trip attractions may need to be changed. (3.9.15 - 3.9.19)

Comments on calibration

1.2.17 The principle of fitting the model to best estimates of important aggregate quantities - here, trip ends from the trip end model and observed cost band sums - lacks the merits of a best fit method. Methods for the latter should continue to be developed. (4.2)

1.2.18 Whilst it is not known on theoretical grounds whether the solution to a synthetic trip-end model must be unique, empirical evidence, gained from repeated runs in a demonstration data set, have not given evidence of non-uniqueness.

1.2.19 The composite model structure could have been invoked more, to provide a more efficient calculational procedure. (4.5)

1.2.20 Errors due to non-convergence to the desired row and column and cost band constraints are negligible compared to the errors in the trip end estimates. (4.6.3).

1.2.21 It is not recommended that the method of smoothing the cost functions in the National Model be adopted for general use. (4.7)

Comments on accuracy assessment

1.2.22 The error in the fitted model value for a cell has two parts: the error arising from the uncertainty in the data to which the model is fitted, and inherent model bias(or '*misspecification error*'). The former is calculable, at least approximately, from the known data accuracy and the method of fitting. The bias, which is the error that would still be present if the model were fitted to perfectly accurate data, is harder to get at. Each residual is an estimate of it. For most cells the residual has a very large variance, because the observed value depends on such a small or zero count. To assess biases further it seems necessary to suppose a simple statistical description of them - in particular, that they behave as if they were an independent random multiplier in each cell - and attempt to fit this bias model, taking account of the data and model uncertainty.

1.2.23 The accuracy of the observed O-D data was calculated in detail for each cell assuming that there were no errors in the various expansion factors, and then a correction for uncertainty in the expansion factors applied subsequently. (5.3) These were used to provide accuracies for row, column and cost band sums. (5.4) The coefficients of variation were about 3 percent for district totals and (on average) 26 percent for cost band sums.

1.2.24 The inaccuracy of the synthetic trip end estimates (after allowing for the bias between these and the observed row and column sums) was found to be much better than was thought to be the case towards the end of the RHTM project, but still substantial, the coefficient of variation being of the order of $3000/\sqrt{Q}$ percent, where Q is the synthesised trip end value. (5.5) In practice this gives a range of coefficient of variation from about 15 to about 50 percent. (5.8.6)

1.2.25 The errors in district level trip ends are, surprisingly, greater than those for zonal level trip ends, implying that the trip end models are underspecified, with some variable or variables omitted which take similar values in nearby zones. This raises doubts about the extrapolation of the trip end models to the unobserved areas. (5.3.14 - 5.5.16)

1.2.26 An approximate formula has been derived for the accuracy of a gravity model fitted with the NM synthetic trip end technique. (5.6)

1.2.27 Modelled and observed values for a sample of observed cells (all purposes combined) have been examined, together with their accuracies, and the broad conclusion reached that, overall, the modelled values show a strong resemblance to the observed values, with occasional big discrepancies. (5.7)

1.2.28 Similar comparisons for intradistrict cells suggest that the modelled values are lower than the observed values, by about 7 percent, implying that the model is over-estimating the inter-district movements (5.8.5), (and possibly doing so more strongly the smaller the intradistrict modelled value). (5.8.7 - 5.8.8)

1.2.29 The variation in the pattern of residuals over the matrix was examined by categorising them by trip length, size of expansion factor and by type of movement (and by size of modelled value, when appropriate). Neglecting variation with expansion factor, the differences between modelled and observed values are more pronounced for trips less than 25 km, but were not judged to be important, taking into account an approximate standard deviation of the residual. But the differences appear to be statistically significant for all area and trip length categories with low (< 10) expansion factor. Moreover, there are indications that, for trips out of London or between other Areas, the model is performing differently as between cells of low (< 10) expansion factor (where the residuals are always negative) and those of high (> 100) expansion factor, where they are almost always positive. See section 1.3.3 for a comment on the analysis and its implications. (See 5.9; the conclusions are more fully described in 5.9.22.)

1.2.30 The simplest possible descriptions of the biases or misspecification in the distribution model are that the squared biases are haphazard over the cells of the matrix, with an average value which is a constant, or else proportional to the model value, or to its square. These three models of squared bias were fitted to the National Model. No significant biases in these simple overall senses were found, apparently because of the overwhelming number of cells for which the residual was either small or very inaccurate.

1.3 DISCUSSION

1.3.1 Clearly, the data problems affect much of the comparisons, rather than the model specification. Much of this can be corrected easily - for example, the omission of the inactive household correction factor from the O-D data, the revision of NHB trip end models to exclude trips by non-residents.

1.3.2. Of the model specification itself, the most worrying feature is that the longer distance movements are so affected by the intrazonal costs, the determination of which is a complex issue on which very little basic research has been done. Since it is so complex - and since moreover even attempts to choose intrazonal costs to make the intrazonal trips correct led to an oversynthesis of interdistrict trips - it is tempting to think in terms of models which avoid the necessity of estimating intrazonal costs. There could, for example, be intrazonal trip models or; more simply, a model of long distance movements could be developed, in which the synthesised trip ends were those of longer distance movements only.

1.3.3. Concerning our assessment of the adequacy of the fitted model, using the techniques described in Sections 5.7 and 5.9, three points may be made. The first point is that the techniques go well beyond the capabilities of the Department's RDCOSM program, insofar as (i) they take account of variances of both model and data, and their covariances; and (ii) they allow patterns in the residuals to be examined by segmenting the matrix according to the characteristics of the origin-destination pairs. Thus, we hope that the Department will consider providing enhanced software to enable other practitioners to do these sorts of investigations. The second point is that the time scale of the project did not permit us to go as far as we should have liked in developing these techniques. Having received the appropriate data with only about three weeks to go before the end of the contract, we were able to investigate the residuals, taking account of their accuracies, for only a sample of cells (Section 5.7) and able to investigate the variations in the residuals over all cells, initially only by neglecting information on their accuracies (Section 5.9). The third point arises from the second: because we were not able in the time-scale to integrate these two approaches to examining the residuals, nor to carry out further computer runs on the basic data, we were faced with some problems over interpreting the evidence from these two sets of analyses .

Initially, the evidence from the two methods of examining the residuals appeared to conflict, so we scrutinised the analysis more fully subsequently, (including taking into account a rough measure of the accuracy of the residuals when examining their variation over all cells. Our conclusions, summarised in 1.2.29, and given more fully in 5.9.22 mean that though the evidence is not as striking as we at first thought, there still remain indications that the model may be performing differentlyas between cells of low (<10) and high (>100) expansion factor (for trips out of London or between other areas), and this gives rise to the suspicion that this is in part attributable to differences in the HI and RI data sets. To resolve this adequately would require further detailed investigation of the data sets, and of their error structure.

1.3.4. The main thrust of our analysis was directed at the estimates of trips, not of travel (= trips x cost) or trip length. The final report by HH&P, which includes analyses of observed and modelled trip length estimates, should be read in conjunction with this report for a full appreciation of the National Model calibration. (Howard Humphreys and Partners, 1982)

1.3.5. Whilst the evidence for the accuracy of the input data and of the fitted model may appear alarming at first sight, this may be something one has to get used to in transportation modelling. No similar transportation study in this country (and we suspect anywhere else in the world) has been subject to such detailed scrutiny as has the National Model and its predecessor, the Regional Highways Traffic Model. Transportation planning will have to recognise that the kinds and magnitudes of errors presented in this report are likely to arise in very many applications - and greater attention will have to be paid to getting clean data and an appropriate model specification.

2. MODEL SPECIFICATION

2.0.1. The characteristic feature of the composite approach to describing origin-destination movements is that shorter movements are treated at a finer level of aggregation than longer movements.

2.0.2. If the origin-destination data is specified in a composite way, a possible advantage over an entirely fine-level specification is that small amounts of data are grouped together, thereby reducing the effects of sampling variability on the accuracy of the parameter estimates of a trip distribution model.

2.0.3. If the trip distribution model is specified in a composite way, the main advantage is a reduction in computing costs, compared with an entirely fine-zone level of model specification.

It is not necessary to specify both model and data in a composite way. For example, Gunn (1977) showed how a conventional gravity model, specified at a fine zone level of detail, could be fitted to data grouped in a composite way.

2.0.4. The National Model specifies both data and model in a composite way. Since, in transport planning, this is pioneering new techniques, this section seeks to clarify the principles and procedures as well as commenting upon the particular formulation adopted.

2.0.5. The definitions and specifications of, for example, composite matrices, are given in Section 2.1; the model specification is given in Section 2.2; and the cost specification is in Section 2.3.

Note however that both the cost and the model specification have been adjusted in the course of the fitting procedure (discussed in Section 4), so that in Sections 2.2 and 2.3 there is some anticipation of points that arise later.

2.1. COMPOSITE MATRICES

2.1.1 The various ways in which shorter movements could be treated at a finer level of zonal aggregation than longer movements were reviewed in Kirby (1978). The method used in the National Model is probably the simplest and easiest to implement. It has a two level hierarchy of fine zones (the so-called regional zones of RHTM) and coarse zones (called districts*) in which trips are represented as occurring at either the fine-zone/fine zone level or coarse zone/coarse zone level. This avoids the further complexity of representing coarse zone/fine zone interactions explicitly.

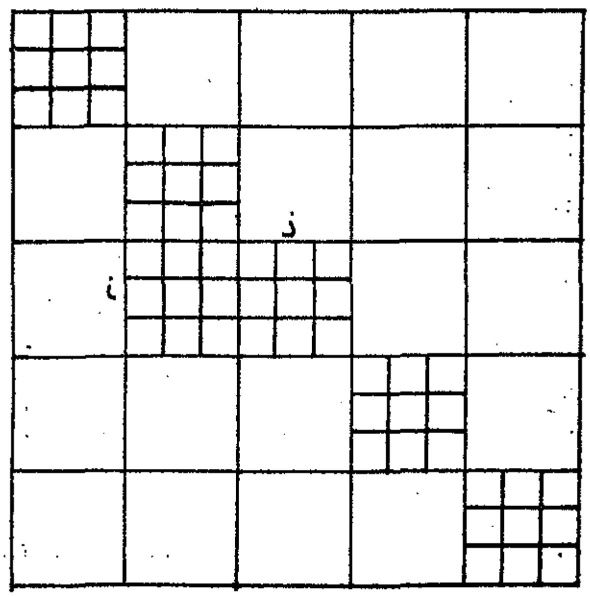
* see footnote on P.13

2.1.2. Thus, if I is a district of origin (or generation), and J is a district of destination (or attraction), and if i, j are regional zones that lie respectively in I and J (the shorthand for which is $i \varepsilon I$, $j \varepsilon J$), the multi-level system of zone-zone movements may be presented roughly in the following form (supposing that the nearby districts have similar numbers).

To district/zone

J

From district or zone I



2.1.3. Some cells of the district-district (I-J) level of interaction are subdivided in the above table, into what we call sub-cells, representing the zone-zone (i-j) level of interaction. A cell that is not subdivided we shall call a simple cell; a cell that is subdivided we shall call a composite cell. An origin-destination matrix that contains only simple cells we call a simple matrix; one that contains a mixture of simple and composite cells we call a composite matrix. For each of the cells or sub-cells there is a known journey cost. See the Appendix (Section 8) for the numbers of such cells.

* NOTE Throughout the internal validation, the term district is used to mean one of the 642 districts used in formulating the model, rather than one of the 447 local authority districts which are amalgamations of these.

2.1.4. In the National Model (Section 2.1) a district to district (I-J) pair was represented as a simple cell if the cost of travel between any regional zone pair included in it exceeded a certain threshold value. Thus, simple cells connect remote district pairs, composite cells connect nearby district pairs. The decision on the cost threshold is a matter of judgement; the value of 100 cost units (assuming an average speed of 60 km/h, this corresponds to a distance of 37 km) was chosen on the grounds that it reduced the total number of (cells and sub-cells) to less than a million (compared with the thirteen million in the RHTM simple matrix of 3613 x 3613 cells). We do not know whether the fit of the model is sensitive to the threshold value, but think it unlikely.

2.2. COMPOSITE MODEL

2.2.1. With observed zone-to-zone movements represented at different levels of spatial detail, the model specification should ideally be such that estimates at one level of detail are consistent in some sense with those at another. The key to the transition is having some information available at the fine level of detail; in the case of the National Model, both synthetic trip-end estimates and zone-zone costs were available at the fine level.

2.2.2. If a gravity model form is required at both fine and coarse levels of detail, then the two forms may be represented as:

$$T_{IJ} = A_I \, B_J \, F^P \, (C_{IJ}) \qquad (2:1)$$

for cells, i.e. remote districts, and

$$t_{ij} = a_i \, b_i \, f^P \, (c_{ij}) \qquad (2:2)$$

for sub-cells, i.e. regional zone-regional zone interactions, in nearby districts, where:

a_i, A_I = generation factors at the fine and coarse levels

b_j, B_J = attraction factors at the fine and coarse levels and $f^P(c_{ij})$, $F^P(C_{IJ})$ = effects of sub-cell costs c_{ij}, or cell costs C_{IJ}, on the interactions between zone pairs ij or district pairs, IJ, where the superscript P denotes the appropriate deterrent function for that part of the matrix in which IJ (or ij) lies.

2.2.3 The consistency question is one of relating a_i to A_I, B_j to B_J, $f^P(c_{ij})$ to $F^P(C_{IJ})$.

For a fully consistent fine zone/coarse zone specification of trips, one would require that :

$$\sum_{ij \text{ in } IJ} t_{ij} = T_{IJ} \qquad (2:3)$$

In the National Model, the first requirement that this led to was that the zonal parameters at the fine zone level were related to those at the coarse level (which are the ones to be estimated) by:

$$a_i = \frac{q_i}{Q_I} A_I \qquad \text{for i in I} \qquad (2:4)$$

and

$$b_j = \frac{r_j}{R_J} B_J \qquad \text{for j in J} \qquad (2:5)$$

where q_i, Q_I = trip generations synthesised in fine zone i, coarse zone I, and are such that $\sum_{i \text{ in } I} q_i = Q_I$

r_j, R_J = trip attractions synthesised in fine zone j, coarse zone J, and are such that $\sum_{j \text{ in } J} r_j = R_J$

2.2.4. Many other variants could have been taken. Whilst we have no evidence to suggest that the relationships (2:4; 2:5) are inadequate, we should point out that, so far as we know, no-one has demonstrated that, for a model fitted to fine zones, the parameters (a_i), (b_j) are such that

$a_i/q_i \approx$ constant for nearby zones

and $b_j/r_j \approx$ constant for nearby zones

The RHTM parameter estimates for the 3613 zone system could have been used to demonstrate this.

2.2.5. Any relationship between $F^P(C_{IJ})$ and $f^P(c_{ij})$ may be entirely subsumed within the relationship between coarse zone costs C_{IJ} and fine zone costs c_{ij} (for ij in IJ, assuming only one cost function is included) by setting:

$$f(x) = F(X) \qquad (2.6)$$

(see Note *).
Further discussion of the cost relationships needed to satisfy (2.3) is in Section 2.3

2.2.6 In fact, the relationship (2.6) is fundamental, rather than a supposition, since the district-district costs were not available from a coarse zone network, but have had to be constructed from the zone to zone costs. This is discussed in section 2.3.

* Note Because district to district interactions are modelled only for costs above the chosen threshold, there is, strictly speaking, no value of cost C above this threshold which applies to fine zone-fine zone interactions. However, whilst it may be natural to require that $F(c) = f(C)$ for all $C>0$ (or the equivalent with c), the requirement is rather abstract.

2.2.7 If there is no further requirement imposed to meet the condition (2:3), then, for a given array of costs, the model may be represented as:

for simple cells LM (i.e. remote districts)

$$T_{LM} = A_L B_M F^P (C_{LM}) \qquad (2{:}7)$$

for subcells ij within a composite cell IJ

$$t_{ij} = A_I B_J \frac{q_i}{Q_I} \frac{r_j}{R_J} F^P (C_{ij}) \qquad (2{:}8)$$

(This assumes the costs C_{LM} to be given; actually they are constructed, as in section 2.3)

2.2.8 The subscripts LM are introduced here to reinforce the distinction between simple and composite cells, but later we use IJ throughout.

2.2.9 A simpler mathematical description of the gravity model The mathematical form of the model given in Section 2.2 of the NMLDTM report reduces to the expressions in (2:7) and (2:8). However, as Murchland (in a note dated 24th Feb. 1981) and Gunn (in WN 10) have pointed out, it is possible to express it even more simply. Before doing so however it is best to express the separation function in (2:7, 8) in a different way.

2.2.10. Since the separation function $F^P(C)$ is defined differently in different parts of the matrix but is such that, in each part, a parameter is estimated for a given interval of cost, intervals k can be defined corresonding to both the cost-interval and function definition such that $F^P(C) = F_K$ if cost C and part P correspond to interval K. Thus (2:7) becomes :

2.2.11 <u>For remote districts LM</u>

$$T_{LM} = A_L B_M F_k \quad D_{LMk} \qquad (2:9)$$

where D_{LMk} = 1 if C_{LM} lies in interval K
= 0 otherwise.

and (2:8) becomes ;

2.2.12 <u>For subcells ij in nearby districts IJ</u>

$$t_{ijk} = A_I \; B_J \frac{q_i}{R_J} \; F_k \quad d_{ijk} \qquad (2:10)$$

where d_{ijk} = 1 if c_{ij} lies in the k^{th} interval
= 0 otherwise.

2.2.13 The main simplification arises by adding the models estimates for the composite cell as a whole. Thus, for (2:10) for <u>nearby districts</u> (cells)

$$T_{IJK} = \sum_{i \text{ in } I} \; \sum_{j \text{ in } J} \; t_{ijk}$$

$$= A_I \; B_J \; F_k \quad D_{IJK} \qquad (2:11)$$

which is the same form as (2:9), but here

$$D_{IJK} = \sum_{ij \text{ in } IJ} \frac{q_i}{Q_I} \; \frac{r_j}{R_J} \quad d_{ijk} \qquad (2:12)$$

Obviously $0 < D_{IJk}$ and $\sum_k D_{IJk} = 1$.

Note that since all the quantities on the right hand side of (2:12) are dependent only on the trip-end estimates and costs, the value of D_{IJk} is known in advance of and is unaffected by the fitting process. It is thought that this simplification enables the fitting procedure to be greatly simplified; a point which will be taken up again in Section 4.5.

2.2.14 Summary : The model form may be most simply represented as providing district-district estimates everywhere. These all have the form

$$T_{IJk} = A_I \; B_J \; F_k \; D_{IJk} \qquad (2{:}13 \text{ - as } 2{:}9 \text{ or } 2{:}11)$$

For remote districts there is only one non-zero D_{IJk} value. For nearby districts there are several D_{IJk} values (given by (2:12)).

2.2.15 This representation of the model will be used in the rest of this report. In both cases the summation notation

$$T_{IJ+} = \sum_k T_{IJk}$$

applies, although it must be remembered that if the cell IJ is composite, the trips T_{IJ+} are associated with several cost bands.

2.2.16 Note that, so far, the condition (2:3) for consistency in the two levels of modelling is not fully met. The way in which costs were defined in order to achieve this in certain respects is discussed in 2.3.

2.2.17 Note also that, for convenience, the functions will be described as having a categorised form (F_k rather than $F^p \; (c_{ij})$) throughout, despite the fact that the functions were eventually smoothed. As already noted, the categorised notation conveniently indicates not only the cost value but also the function type.

2.2.18 In section 2.5, the way in which the model specification is affected by adjusting intra-zonal costs is discussed.

2.3 COMPOSITE COSTS

2.3.1. The National Model introduces a further relationship between the fine and coarse levels of modelling through the costs c_{ij} and C_{IJ}. This is done in two quite distinct ways.

2.3.2. (a) For remote districts, for both private and commercial vehicle models, the principle is essentially that the costs between remote districts should be such that the trips given by the coarse model would be equal to that given by a fine model (were that to be applied to such cells).
That is, in a simple cell (LM), where (2:7) applies, if (2:8) applied there too then one would have

$$T_{LM} = \sum_{lm \text{ in } LM} t_{lm} = \frac{A_L \, B_M}{Q_L \, R_M} \sum_{lm \text{ in } LM} q_l r_m \; F(c_{lm})$$

By the definitions in section 2.4, one function F applies to all pairs lm within a given district pair LM. Hence is defined the composite cost for remote cells:

$$C_{LM} = F^{-1} \left\{ \sum_{lm \text{ in } LM} q_l r_m \; F(c_{lm}) \, / \, Q_L R_M \right\} \qquad (2{:}14)$$

(The term composite cost, and its definition, are equivalent to those in the modal split literature). The operation (2:14) is also called a quasi-average.

2.3.3. The right hand side of (2:14) contains known quantities, but also the function F(C), which is to be estimated. In principle, this implies an iterative procedure. In practice, the quantities F(C) were not those estimated in the National Model calibration, but those previously estimated in the RHTM calibrations, denoted by $F_o(C)$ say.
Hence, the composite costs for simple cells were such that :

$$C_{LM} = F_0^{-1} \{ \sum_{lm \text{ in } LM} q_l r_m F_o (c_{lm}) / Q_L R_M \} \quad (2{:}15)$$

2.3.4. Whether the use of RHTM cost functions as opposed to National Model cost functions makes much difference is discussed in 2.3.14 et seq. (The three RHTM cost functions for HBW were used to produce the composite costs by (2:14), and these same costs were also used for the other three purposes).

2.3.5. (b) For nearby districts the private vehicle and commercial vehicle models have taken different approaches so far as the use of composite costs are concerned.

2.3.6. In the private vehicle model, each nearby district-district pair IJ is treated as a composite cell: the costs between regional zone pairs ij within IJ are represented explicitly, as shown in Section 2.2.

2.3.7. In the commercial vehicle mode, nearby district-district pairs IJ are treated as simple cells, but with a composite cost that represents implicitly the several regional zone pair costs within IJ. The composite cost is defined as

$$C_{IJ} = F^{-1} \sum_{ij \text{ in } IJ} \frac{q_i}{Q_I} \frac{r_j}{R_J} F(c_{ij}) \qquad (2{:}16)$$

and, in this case, the function 'F' is that being fitted to the National Model, and thus C_{IJ} is updated as part of the iterative procedure that estimates (A_I), (B_J) and (F_k). In contrast to the non-iterative use of the RHTM cost function $F_o(C)$ in calculating composite costs for remote districts, iterative calculations of composite cost for nearby districts may be appropriate.

2.3.8 The questions are, whether one method is preferable to another; and would they give very different results?

2.3.9 The first point to note is that, given the solution (that is, the A_I, B_J and F_K values) to a model of the private vehicle kind, it is possible to represent that solution in the form of a model of the commercial vehicle kind, by appropriate choice of composite costs for nearby districts. Thus there is an equivalence between the two forms.

2.3.10 However, this does not mean to say that the (A_I), (B_J) and (F_K) values derived by fitting the model of the private vehicle kind are the same as those derived by fitting the model of the commercial vehicle kind. The former, as it represents fine zone-zone movements explicitly, includes O-D data at this finer level; the latter includes O-D data only at the coarser level. For a given observed district-district cell, the commercial vehicle model will allocate all the trips to a single interval of trip cost (that corresponding to the composite C_{IJ}), whereas the private vehicle model will allocate the trips to several intervals of cost (those corresponding to the c_{ij}).

2.3.11 Hence one would expect the private vehicle model to give rather more refined (F_k) estimates than the commercial vehicle model, essentially for costs below the 100 pence threshold, for the same fitting method (i.e. synthetic trip end or partial matrix method).

2.3.12 There has however been no direct evaluation of the two model forms, so there is no quantitative evidence for how different the two approaches are.

2.3.13 (Note that the fitting methods used in the two cases were different - that for the private vehicle model constrained the model's row and column totals to synthesised trip-ends, that for the commercial vehicle model did not. Since the private vehicle model is the main concern of this study, there will be little further discussion of the different approaches.)

2.3.14 The use of the RHTM cost functions RHTM had 3 cost functions for each purpose, but in the National Model the three HBW functions were used to derive the costs used for all four purposes. The RHTM functions had been manually smoothed, and were monotonically decreasing, so that there was no ambiguity as to what the inverse function value was in taking the quasi-average.

2.3.15 The question discussed is, does it matter that the old RHTM cost functions (or rather, time functions adjusted to a cost basis) were used in (2:15) for remote cells, as opposed to using cost functions obtained in the National Model? The questions is particularly apposite for cells near the 100 pence cost threshold, because, below this threshold, trips are estimated in a way which corresponds to using the National Model function to define a composite cost, as in 2:14 or 2:16, and above it to the use of RHTM function values, as in 2:15.

2.3.16 Although we have no evidence, this may not matter, despite the fact that the old functions were obtained for 3 different 'areas' of the HBW matrix, as opposed to 9 in the National Model for each purpose (see Section 2.4). The reason is that the averaging represented by 2:15 is being done for districts that are far apart. All the costs c_{lm} for remote districts LM will (by definition of the simple cell) exceed the cost threshold of 100 pence. It seems unlikely that the relative variation of $F(c_{lm})$ over all the subcells within a given LM will make the quasi-average given by (2:15) very different from a more straightforward average cost, and hence it is unlikely that inaccuracies in the relative values of F_0 will have much effect on the quasi-average. (By relative variation, we mean that due to the slope.)

2.3.17 Moreover, the main difference between the three function types used in RHTM (urban, rural, London) and the nine used in the National Model (see Section 2.4), is that the latter distinguished intra-town movements from the rest; but the relative values for F(C) curves for these two types of movements for each of the National Model suburban/rural/metropolitan/London categories were broadly similar.

2.3.18 The fact that the trip-ends used in forming the quasi-average with the RHTM function (in 2:15) were not the same as those used in fitting the National Model (and so appropriate to 2:16) is likely to have only a negligible effect.

2.3.19 The main inconsistencies that are likely to arise are for journeys other than home-based work. This is because only the HBW RHTM cost function was used to define composite costs for remote cells.

2.3.20 For a few cells, there may be inconsistencies due to the various sub-cells within it having different RHTM cost functions applied to them (i.e., a mixture say of urban and rural). Note that the situation does not arise with the National Model function, which is of the same type for all sub-cells within a given district-district cell.

2.3.21 Empirical evidence for the differences that are likely is available from NATDEF output (reproduced in WN 19). This shows that the quasi-averages given by (2:15) are almost always less than the simple unweighted average cost. (Theory given in WN 22 confirms that, for a convex cost function, the quasi-average 2:15 will always be less than the corresponding simple weighted average. For a rapidly decreasing function, F_0, the quasi-average will be close to the least of the costs in the sub-cells.) Most of the quasi-averages are within 10 pence of the unweighted average.

2.3.22 For the important region near the 100 pence threshold, Table 2.3(1) summarises some of the WN 19 data. It is unlikely that the use of National Model function values rather than RHTM function would change the value of the quasi-average by as much as the difference between the RHTM-based quasi-average and the unweighted average given in Table 2.3(1). This would affect the composite cost value by no more than about $\pm$ 5 pence in 100, which is a difference of no more than $\pm$ 1 in the cost band.

2.3.23 If one ignores the effect of any change to the calibrated values of the cost function F_k, then a change in cost in a particular cell IJ that implies a change from F to F + ΔF in the deterrent function value will generate a proportional change in the model's estimates of trips in that cell given by, approximately,

$$\frac{\Delta T_{IJ}}{T_{IJ}} \approx \frac{\Delta F}{F} \; / \; (1 + \alpha_{IJ})$$

where α_{IJ} takes account of the row and column constraints, and is given approximately by

$$\alpha_{IJ} \approx \frac{T_{IJ}}{T_{++}-Q_I-R_J+T_{IJ}} + (1 + \frac{\Delta F}{F})(\frac{T_{IJ}}{Q_I-T_{IJ}} + \frac{T_{IJ}}{R_J-T_{IJ}}) \quad (2:17)$$

(Kirby, 1973). In many cases, α_{IJ} will be negligible near the 100 pence threshold. The proportional changes in the numbers of trips for a one-band shift in cost at the 100 pence threshold are given in Table 2.3(2).

2.3.24 As a general point, we note that, since the composite costs for remote districts are so close to the simple unweighted average zone-zone costs (see WN 19), it seems possible that a simple cost, from district centroid to district centroid, may have been adequate for these districts. In practice though, since district centroids were undefined and zone-zone costs had to be used to calculate an average cost between districts, there is virtue, and very little extra computational effort, in calculating the composite costs (rather than say the average cost) for all district pairs.

2.3.25 The main virtue of calculating composite costs for remote districts is that it reduces the risk of discontinuity arising in the treatment of cells near the 100 pence threshold.

2.3.26 The calculation of average costs It must be stressed in conclusion that the composite costs are used in order to get the coarse model's estimates of trips consistent with a finer level of specification; they will not simultaneously achieve consistency in the estimation of average or total trip cost. For this, one needs to sum the products of the trips T_{IJ} calculated by the model (using composite costs C_{IJ}), with a simple average cost C'_{IJ} say. For the case considered here, $C'_{IJ} > C_{IJ}$.

2.4 THE DEFINITION OF COST FUNCTIONS

2.4.1 The way in which different cost-functions were defined is obscure in the report (WN 15); see instead Table 2.4(1). The relative amounts of data in each function area are given in Table 2.4(2).

2.4.2 The question is: why choose to define cost-functions in this way?

2.4.3 The distinction between intra town and other movements might be argued on the grounds that one is more likely to be familiar with the opportunities for undertaking certain kinds of activity in the community in which one lives, than one is with opportunities elsewhere. This is the kind of argument advanced in the GMC Transportation Model (Greater Manchester Council, 1981, section 3.5.11), which led to the definition of 'self-contained areas'.

2.4.4 This argument is reasonable because it is often not appreciated sufficiently that models of trip distribution are really modelling two quite distinct distance-related phenomena: one is the tendency for the number of opportunities one knows about to decrease with distance; the other is the tendency for the frequency with which one visits these known contacts to decrease. Thus, a refinement of model specification that reflects this distinction should be an improvement for some purposes.

2.4.5 Of course, the distinction could be taken further, and perhaps should have been in a National Model: it does not enhance model credibility if, as is the case, all opportunities, no matter how far away, are candidates for a destination. (In the disaggregate model literature, the definition of 'choice sets' plays an important role; and some attempts have been made at modifying the conventional models of trip distribution, by excluding zones beyond a certain cut-off point from each origin zone. See for example Benson, 1977.)

2.4.6 The division of the National Matrix into a number of areas in which different deterrent functions apply (ignoring for a moment the ninth function, which applies to the trips crossing the Welsh/Scottish screenlines), has not however been based on such behaviourally based arguments, but on attempts to find a set of definitions which, with a syntehtic trip end method of estimation, reduced the level of oversynthesis in the observed cells. It is said in the NM Report (Section 4.3) that, for two 'test bed' study areas, the oversynthesis was reduced considerably by using two deterrent functions (intra-town/other); and removed completely when the intra town/other distinction was extended to incorporate town type.

Because of the way the test beds were defined, we can accept that the intra town/other distinction, on its own, indicates an improvement in model performance overall; but we cannot accept the same conclusion for the introduction of the town type distinction (viz, metropolitan, urban, rural). This is because the test beds were such that, in fitting the model so as to give agreement with the trip cost frequency distribution (and hence overall numbers of trips) in each of the six different areas of the test bed matrices, the fitted model was virtually bound to reproduce the observed number of trips in the observed areas.

2.4.7 Hence this particular test-bed result is not a valid basis for concluding that this definition of cost functions would improve the model specification in the National Matrix as a whole.

(For, if one so adjusts a model specification as to match exactly some previously used simple aggregate goodness-of-fit criterion, one then has to choose different criteria before one can really judge the adequacy of the model.)

2.4.8 In application to the National Model, the extension of the intra town/other distinction to cover town type (giving in this case 8 function areas, with London treated as another category) does not necessarily mean that the oversynthesis in the observed cells is removed completely. This is because, unlike the test beds, there will in general be several study areas contributing to each of the eight function areas. Nevertheless, this method of defining the function areas does exert a powerful constraining influence on the level of oversynthesis; and hence the level of oversynthesis is not a useful indication of the adequacy of the performance of the model even in the observed cells, let alone the unobserved cells.

2.4.9 The main argument for the distinction between rural/urban/ Metropolitan/London distributions is the varying richness of the public transport alternative. From the above, we are somewhat sceptical that this choice has been demonstrated to be a good one, and other behaviourally-based arguments might have been put up in support of distinghishing between different functions on the basis of attraction-end characteristics.

2.4.10 As a general comment, it may be noted that at present there are no accepted guidelines for determining how best to define function areas in the matrix; and the issue is in any case bound up with the question of the adequacy of the gravity model specification itself. It is possible that a better model specification would emerge if many function areas were defined, with few parameters in each, than the present combination of a few function areas with many parameters (= cost factors) in each. But any such approach would have to define the function areas in a behaviourally meaningful way. The key problem is to produce relations that are sound enough not only to explain the distribution in the observed cells, but in the unobserved cells as well!

2.4.11 Finally, it should be noted that the results actually obtained with the 9 function areas given in Table 2.4(1) show that, to a very good approximation, the intra town/other functions are virtually parallel to each other, within each town type. That is, for a given trip purpose and town type, approximately:

$$\frac{\text{intra-town cost function}}{\text{'other' cost function}} = \text{constant}$$

2.4.12 The average value of the ratio is given in the report (being 2 to 3 for HBW and HBO trips, 1½ for EB trips and 3 to 8 for NHBO trips). This suggests that the intra-town/other distinction could have been expressed more simply as the determination of a single factor (a so called 'K-factor'?!) for each town type, rather than the determination of a whole new range of separation function values. Were this to be done, the accuracy of the model's estimates would be increased (due to having fewer parameters).

(Not that we advocate a K-factor based approach, which tends to be arbitrary and difficult to extend to unobserved cells).

2.4.13 The tabular nature of the cost functions The initially defined tabular sets of functions (with a total of 964 parameters) for the private vehicle model were eventually replaced by smoothed values. Smoothing is discussed in Section 4. (The commercial vehicle model adopted the analytic function - the gamma function - at the outset.)

2.5 INTRAZONAL MODEL ADJUSTMENTS

2.5.1 In the proceeding sections, the model has been described as if the costs in each cell and sub-cell were independent data (although with a composite cost treatment for the remote district pairs).

2.5.2 In fact, the intrazonal costs (that is, the costs in the sub-cells) were adjusted in order to give better agreement between the fitted model and the data for intrazonal movements. A relationship was established between intrazonal distance and zone size. In order to embrace the zone sizes encountered in the unobserved areas as well as the observed areas, data and modelled estimates were included at both the regional zone and district and cordoned-area levels. The interzonal movements within towns were also modified, with a terminal cost correction, in a manner which related to the changes made in intrazonal costs. The procedures used are described in WN 14 (Section 7 and the Appendix).

2.5.3 This means that we are no longer dealing with a clear cut model specification in which the dependent variable (trips) is (in the fitting process) a function only of independent variables (costs, trip-ends and observed trips). One of the independent variables (cost) has now become a function not only of trip ends, and observed trips, but also of the model being fitted. In consequence, it becomes very much more difficult to analyse the error structure in the fitted model or to deduce the properties of the model.

2.5.4 Model adjustments of this kind, which appear to strive to force the model to give the right amount of intrazonal trips, do not increase one's confidence in the adequacy of the fitted model for prediction in either the unobserved cells, or for a future year.

2.5.5 It is therefore desirable that the reasons for such adjustments are brought out.

2.5.6 The appropriate principle would be that the values taken by the intrazonal cost or indeed interzonal cost should be those appropriate to averaging the cost-function F(c) over all possible interactions within the zone(s) in question (using for this purpose subdivisions of a zone that are similar in size). This principle is related to that used in defining composite costs for remote districts.

2.5.7 Whilst this principle is briefly acknowledged in the report in the discussion on intrazonal costs, it requires substantial elaboration.

2.5.8 In this section we have discussed the implication of the intrazonal cost adjustments for model specification. In the next, on input data, the way cost is defined, and the empirical evidence for the adjustments, is discussed. In Section 5, on empirical validation, we show that in fact the intrazonal cost adjustments were not very successful in producing agreement with the observed data: on average, the modelled intrazonals were 7 percent too low.

3. INPUT DATA

3.0.1 This section comments on the changes made to the data used as input to the fitting of the model. Costs are first discussed, in Sections 3.1 and 3.2, and should be taken in conjunction with the comments on model specification in the Sections 2.3 and 2.5. The post-RHTM changes in origin-destination trip data are described in Section 3.3, and the trip-ends used are commented on in Section 3.4.

3.1 INTERZONAL COSTS

3.1.1 The origin-destination journey costs are based on:

(i) the minimum time paths between RHTM regional zones, using the network times in Update 22;

(ii) using the O-D times, distances and tolls encountered on these paths, to deduce a generalised cost of the form c(pence) = 1.44 dist(km) + 1.28 time(mins) + toll(pence).

3.1.2 The use of the already-available RHTM minimum time paths rather than a costly re-calculation of minimum cost paths is unlikely to have any adverse influence on model fit. (The true minimum path cost will always be lower, but the form of model, with a factor for each cost band in each function, really only needs a consistent ranking of costs for each cost function.)

3.1.3 The main concern is that the same 'value of time' has been applied everywhere, and to all trip purposes. Since most empirical studies suggests that the value of time is proportional to wage rate, it is conceivable that the model could have fitted the data better had regional variations in income been allowed for in the value of time. However, the effect of not doing so is lessened by the fact that different deterrent functions have been applied to trips from the London, metropolitan, urban and rural areas. Thus, a trip from one area which had a given generalised cost (using the average value of time) will not be grouped with a trip of the same cost from another area.

3.1.4 The way in which the costs between pairs of regional zones were averaged to give costs between districts was discussed in Section 2.3.

3.1.5 <u>Terminal cost corrections</u> The costs of travelling from a regional zone within a town to another regional zone within the same town were displaced from their centroid to centroid values by a terminal cost correction at each end of the movement. The reason for this is that, with origins and destinations spread over quite large zones, and not concentrated at the centroids, the centroid to centroid cost would in general be an overestimate of the average cost. In principle, in order to achieve consistency with a finer level of gravity model specification (one in which zones are homogeneous in size), it is a quasi-average cost measure that is needed, defined in a similar way to the quasi-average for composite costs. The principle is alluded to in the report as being the reason for adjusting the costs for interzonal movements within towns. The adjustment was carried out using a terminal cost correction at each end of the trip (see WN 14). The adjustment was related to the intrazonal cost adjustment. The arguments for doing the adjustment in this way are not explained in the report, but have been described to us. In order to be convincing, though, we would recommend that the theoretical basis for estimating these corrections to intrazonal costs should be established much more strongly.

3.2 INTRAZONAL COSTS

3.2.1 No network times exist for movements within regional zones (i.e. the subcells of the composite matrix). The National Model initially based intrazonal costs on the RHTM relationship between average intra-survey-area times and zone size. As mentioned in Section 2.5, these were then replaced by a relationship between intrazonal distance and zone size that was such as to give good agreement with the observed numbers of trips. Our detailed comments on the method are in WN 14 and 16, and in Section 2.5 we urged that fresh attention be given to the principles for the intrazonal (and nearby intrazonal) costs. Here we simply draw attention to certain empirical matters.

3.2.2 In Fig 3.2(1) intrazonal times are shown as a function of the effective radius of the zone (Z_i km) using the previous RHTM curves, and the new NM intrazonal time curve. This latter takes the form

$$\text{time} = (1.26 \ln Z_i + 0.57) / v$$

where v = 60 km/h. (rural areas)
30 km/h. (urban areas)

(This would give negative values for $Z < 0.63$ km).

It is clear that the changes have substantial implications for the estimates in the larger unobserved zones.

3.2.3 The empirical evidence which led to the revision of the intrazonal costs was based on plots of the ratio of synthesised to observed trips in intra zonal cells as a function of effective zone radius. Those for home-based work are given in Figs. 3.2 (2 and 3) (others are in WN 16). These were held to show that the oversynthesis decreased as zone size increased, so implying the need to change the intrazonal time/zone-size relationship. Taking the graphs and their statistics as a whole however, we do not find the evidence convincing.

3.2.4 Although clearly desirable, a detailed examination of the effect of the changes made to intrazonal costs, and the sensitivity of the intrazonal distance/zone size relationship to the input data, has not been possible in the time scale of this project. However, in section 5, we report the empirical evidence for the adequacy of the model's fit to observed intrazonal movements.

3.2.5 As with the terminal cost corrections (section 3.1.5), we recommend that the basis for estimating intrazonal costs be established more firmly, as that which estimates the quasi-average costs for all movements within a zone. With a suitable theoretical basis, it would be possible to avoid the charge that one was simply adjusting the costs in order to improve the agreement between the model and data.

3.3 MINOR ROAD TRAFFIC

3.3.1 The earlier RHTM roadside interview (RI) data were such that:

(i) no estimate was made of traffic on non-counted roads;

(ii) traffic on MCC-only roads was allocated the purpose distribution, trip length and origin-destination characteristics of nearby RI roads by including a 'corridor factor' in grossing-up the traffic on interviewed roads.

3.3.2 The new NM roadside interview data were such that:

(i) an estimate of traffic on non-counted roads was made, equal to the lower quartile of the distribution of MCC, for different types of road;

(ii) traffic on (MCC only and non-counted) roads may be represented as having been allocated to the interviewed traffic in the following way: it was split up by purpose according to a modified purpose distribution of the cordon as a whole; by trip length, according to a modified trip length distribution for the cordon as a whole, for that trip purpose; and then, by origin-destination, according to the proportion of cordon's traffic at that trip length and purpose which had the stated origin-destination.

3.3.3 The three-stage process we have just described reduced to the two stage treatment discussed in Section 3.3.3 of the NM report. (Details are given in the Appendix to WN 17.)

3.3.4 The inclusion of estimates of non-counted traffic remedies a previous deficiency. It would be helpful if the Department could supply figures from other studies, perhaps based only on ATC data, which would substantiate the assumed figure for the order of magnitude of flows on non-counted roads.

3.3.5 We suspected that the various factors applied to allocate non-interviewed traffic by purpose and by trip length would vary, in the first case, by region of the country, and in the second case by trip purpose. However the ratios of what we denote as

$$\alpha^h = \frac{\text{proportion of minor road traffic of purpose } h}{\text{proportion of major road traffic of purpose } h}$$

and

$$R_\ell = \frac{\text{proportion of minor road traffic of trip length in range } \ell}{\text{proportion of major road traffic of trip length in range } \ell}$$

were based on comparing figures from just 7 or 8 minor road (='C' class) roadside interview sites with those for the 1000 or so major road sites.

Hence :

a) no disaggregation of α^h by, say, region was possible

b) an attempted disaggregation of R_ℓ by trip purpose resulted in 'too noisy' a picture.

We therefore accept that no improvement is likely in these estimates for the time being.

3.3.6 Our main concern is that MCC-only and non-counted roads might in practice have some directional bias on a given cordon, which a cordon-wide as opposed to corridor basis for adjustment does not reflect.

3.3.7 The directional bias, if it exists, should only marginally affect the estimates of the distribution model parameters, since the bias would have no effect on the allocation of trips to cost bands, and only a minor effect on the row and column sums of observed trips. However, where it might be important is in a comparison of the N M data against external O-D data, or a comparison of the fitted model with the O-D data to which it has been fitted.

3.3.8 Those cordons which may be particularly affected by such a directional bias may be judged from Table 3.3, which shows the proportion of non-interviewed traffic as a fraction of the total (interviewed, counted and estimated) traffic across the cordon. It is suggested that, for those cordons where the fraction is high, a map showing the incidence of interviewed roads, counted-only roads and non-counted roads, be inspected to judge whether the non-interviewed traffic is more or less evenly spaced around the cordon. If it is not, then the NM data will have directional biases which affect one's judgement of how the NM O-D data compares with independent estimates of O-D flows, or with the fitted model, and so some adjustment to the NM O-D data may be desirable.

3.3.9 The methods used in the National Model for attributing trip purpose and trip length characteristics to traffic on uninterviewed roads may be contrasted with those proposed earlier in the Trip End Consolidation Project (Martin & Voorhess Associates, undated, section 3) subsequently explored further in the Trip End Model Research Project (Martin & Voorhees Associates, 1981, Working Paper 2). The alternative roadside interview expansion process that MVA considered required a much more detailed assessment, for each cordon, of the zones which trips crossing a cordon on uninterviewed roads might be expected to be coming from and going to, and some association with the characteristics of traffic on nearby interviewed roads. In addition, the expansion procedures would have reflected the differing proportions of traffic in peak and off-peak hours, by allocating the differing traffic proportions in each hour of the day at each interview site to the hourly traffic flows at the MCC sites.

3.3.10 It is somewhat surprising that the National Model report makes no reference to this work, because the methods proposed seem superior in principle to those that were done in either RHTM or the National Model. It is presumed that the MVA procedures were rejected on grounds of the processing cost involved. Yet, as will be clear later on in the report, the extent to which one can judge the adequacy of the fitted model depends fundamentally on the goodness of the data to which it is fitted. It is hoped that the Department will consider advocating the use of the more detailed MVA expansion procedures, or something akin to them, in other studies.

3.3.11 With the available information, no direct comparisons have been possible between the results of applying the N M re-expansion procedures and the MVA re-expansion procedures.

3.4 INACTIVE HOUSEHOLDS

3.4.1 The household interview data incorporated in the National Model data base OB17 was taken unchanged from the last RHTM data base OB13. This differs from its predecessor, OB6, by reducing all HI trips by a factor of 0.935 (for London) and by a factor of 0.96 (elsewhere). These factors had been introduced in the later RHTM runs because the previously used expansion factors were held not to have allowed adequately for the fact that some households in the Planning Data file would be 'inactive' on a travel day, the household members being absent and hence not contactable during the survey. (Alastair Dick and Associates, 1979a and 1979b, Paras. 1.5, 1.6.)

3.4.2 Although reference to this was not included in our final report on RHTM Trip Distribution Investigation, we had there concluded (TDI-WN 33) that, although the correction factor might be justified in principle, the magnitude seemed too high, and indeed each household interview area should have been corrected for the effect individually (rather than using only a London/non-London distinction). A more detailed examination undertaken by MVA led to the stronger conclusion that the true magnitude of the effect was likely to be very much smaller than the basis on which ADA had estimated it, and recommended that the use of the factors be abandoned. (Martin and Voorhees Associates, 1981, Working Paper 1, Revised expansion factors and inactive households.)

3.4.3. We understand that, in the light of the MVA work, the Department did abandon the application of these factors to the trip end model estimates. The failure to abandon them in the observed data set does much to explain the inconsistencies that we have subsequently found in the validation: see section 3.9 (and also 3.6).

3.5. ROUND TRIPS

3.5.1. The National Model report includes an investigation of the assumption that all observed movements in one direction at a roadside interview station are accompanied by an unobserved movement in the opposite direction. Using cordon-crossing trips from household interview data, it was shown that large differences occurred in the proportions of single leg trips in the outbound and inbound directions, and in their average trip lengths.

3.5.2. We consider this finding to have potentially important implications for data collection and model building strategies, and suggested that further work be done on this to advise other studies on the best way of proceeding. We agree with the report's conclusion that 'at the individual survey area level these differences could generate significant problems'; but are not convinced that at a large area or national level the differences are less significant. This is because most roadside interviews on a cordon are carried out in the outbound direction, and thus there would be a tendency overall for, say, HBO other trips to be underestimated in their number and their average trip length.

3.6 CORDON-CROSSINGS COMPARISON

3.6.1 In the RHTM Trip Distribution Investigation (3.2.9-3.2.10) the numbers of household interview (HI) and roadside interview (RI) trips crossing the HI cordons were compared using various statistical procedures, and it was concluded that, under the assumptions made, there were strong grounds for supposing that the HI and RI data were biased with respect to each other for the HBW purpose, but not over all HB purposes; and that the differences between the estimates were much too large to be accounted for by the assumptions made about the sampling distribution. There were also survey differences between different areas of the country, which seemed largely accounted for by survey differences between spring and autumn, and between high and low traffic peaks. The differences were too large to be accounted for by the uncertainty the scaling-up factors (Gunn, Kirby, Murchland and Whittaker,1981).

3.6.2. This kind of analysis was repeated using the new NM RI cordon-crossing data given in Figs 6,7 and 8 of the NM report, and the unexpanded HBW trip data given in the RHTM TDI report. Note however that whilst the HI data given in Figs 6,7 and 8 of the NM report reflected that which was intended to be used, the HI data set actually used in the NM was that from RHTM data set OB 13, which incorporated inactive household correction factors (see Section 3.4). Consequently, the cordon-crossings analysis was done for both the data set as used (with HI data scaled by 0.935 for London, 0.96 elsewhere), and the data set as intended.

3.6.3. This time the analysis was extended to show the effects of different assumptions about the amount of uncertainty in the scaling up factors (expressed as a coefficient of variation), and used a somewhat different test, the Watson 'U^2' test, to those used before. The Watson modified U^2 statistic is a test that each of a sample of n independent variates comes from a normal distribution with zero mean and unit variance. Assuming this is the case, the variates are transformed to uniform variates on (0,1), sorted, and a measure of discrepancy calculated between the empirical cumulative distribution and the theoretical one (a straight line). A small modification to reflect sample size is made to this U^2 value. The higher the modified U^2 value the less likely the standard normal assumption is to be true. The test is sensitive to departure of the mean from zero and also to departure of the variance from unity. The normal distribution assumption is not in doubt in this case, because the samples are large.

3.6.4 The values of the modified U^2 statistic are given in the Table 3.6 for both the used and intended HI data sets. (Thurrock was omitted as no sample count was available.) The first conclusion is that, whatever the coefficient of variation of the expansion factor, the HBW and HBEB estimates of trips by the RI data sets cannot be regarded as significantly different from the estimates given by the HI data sets. The second conclusion is that, for the intended data set, HBO trips are significantly different only for expansion factors with coefficients of variation of 5% or less: but that for the data set actually used, the significant difference remains even up to coefficients of variation of 10%. That is to say, giving less weight to HI data than was intended, made the cordon-crossing discrepancies (for HBO trips) worse. The discrepancy in HBO trips largely accounted for discrepancies in the total of home based trips, which were significantly different for all coefficients of variation tested in the used data set, but were not significant for coefficients of variation above 2½% in the intended data set.

3.6.5. The fact that the significance of the discrepancies in HBO trips got worse if HI trips had less weight than the intended value leads to the thought that, if one were to give more weight to HI trips, the significance of the discrepancies would disappear altogether (at a given level of error in the expansion factors). (Of course, whether one would be justified in giving more weight to HI data would have to be argued on other grounds.) A further series of cordon-crossing comparisons was therefore made, with HI data scaled up by a further factor, in the range 1.05 up to, in some cases, 1.25. In principle, an optimum combination of coefficient of variation and scaling factor could be found, that minimised the U^2 value for a given trip purpose; but it was more appropriate to find scaling factors that were as close as possible to one without making the U^2 value too improbable for any trip purpose. (Note that increasing the weight of HI data actually makes the discrepancies in HBW trips worse, not better, so a balance has to be struck across all trip purposes.)

It was found, for example, that a scaling factor of slightly more than 1.05 (on the intended data) would be needed to make the HBO discrepancies not significant, for a coefficient of variation of 5%.
Results of other variations in scaling factors are in WN 24.

3.6.6 Note that these comparisons used the same basis for calculating variances as was used with the RHTM comparisons; namely, the assumption that the sample data had a Poisson distribution. The variances could instead have been obtained from the calculations done in the course of establishing the accuracy of the O-D matrices (see section 5.3); but, apart from the fact that these calculations had not been completed when we did this work, it would have in any case required further computation to establish the variance of cordon-crossing trips for each cordon. This did not seem worthwhile (given that the true average sampling fractions for cordon-trip crossing trips had already been established in the RHTM Trip Distribution Investigation).

3.5.7 It should be expressly noted that these tests for consistency of the HI and RI data are for cordon-crossing trips only. They do not cover the possibly more important comparison of trip lengths for the two surveys.

3.7 SEASONAL CORRECTION FACTORS

3.7.1 The one adjustment to RHTM data that is conspicuous by its absence in the National Model is that for seasonal correction factors in the two data sets. Those for RHTM were taken to be unity everywhere; yet we had earlier shown (see 3.6.1) that differences between spring and autumn and high and low traffic periods might account for some of the differences in the RHTM cordon crossings comparison. It is therefore to be regretted that these factors were not investigated further.

3.8 MERGING OF ESTIMATES

3.8.1 The National Model report (section 3.3.3) shows that, where several data sets provided an estimate of the whole origin-destination (ij) movement, they were merged by taking the average as:

$$\frac{\text{sum of (average sampling fraction x trip estimate)}}{\text{sum of average sampling fractions}}$$

3.8.2 Compared with the approach now adopted in the Department's validation and comparison programs, RDMVAR and RDMERGE, the assumptions implied are:

i) that the sampling fraction for all trips in a given survey period is homogenous;

ii) that the effects of uncertainty in these sampling fractions (or rather the component scaling-up factors) are the same in all data sets;

iii) that each data set is providing an unbiased estimate of the true number of trips.

3.8.3. Departures from assumption (i) are likely to have little effect on the calibration. Assumption (ii) probably means that the merged estimate is weighted rather more towards the roadside interview estimate than it would have been with the RDMVAR and RDMERGE procedures (since in practice the uncertainty in scaling-up factors is rather greater for RI data than it is for HI data).

3.8.4 The method of merging was certainly an improvement on the previous procedure (in which an unweighted average of RI estimates was taken and HI estimates of cordon-crossing trips discarded).

The main deficiency with the method arises if, as is implied by the cordon-crossings comparison discussed in Section 3.6, assumption (iii) concerning unbiased estimates does not hold. The comparisons in Section 3.6 seem to suggest that rather more weight might be given to HI rather than RI data, at least for HBO trips.

3.9 TRIP END ESTIMATES

3.9.1 The trip-end estimates (q_i or Q_I; r_j or R_J) used in the fitting of the gravity model were:

a) for wholly-observed rows or columns, observed trip-ends derived by summing the trips in the trip matrix;
b) elsewhere, synthesised trip-ends, derived eventually by utilising the models described in the Traffic Appraisal Manual (TAM), together with the planning information included in PDUP16.

3.9.2 For district-level estimates, the zonal trip end estimates were appropriately aggregated.

3.9.3. It was not within the terms of reference of this project to enquire into the trip-end models used. However, our assessment of the accuracy of the synthetic trip-end estimate (WN 11, 12, 13), reported in Section 5, led us to conclude that there were serious problems with those estimates.

3.9.4 The main problem is that, for the wholly-observed rows or columns, the zonal or district totals of trips and the corresponding synthetic trip end estimates are biased with respect to each other. The differences are given in Table 3.9, and they appear to be due to two main contributory factors.

3.9.5 Unobserved non-home-based trips. The startlingly large differences in col. 3 for non-home-based and employer's business trips were shown, after investigation in APM Division, to be due to the fact that trips within HI areas by non-residents could not be observed by either HI or RI, but the NHB trip-end models included trips by both residents and non-residents. Martin and Voorhees Associates reported that those non-observed trips accounted for 24% of NHB trips. Since, of the EB trips, 59.9% are NHB, this implies that a total of 14.4% additional trips were added to all EB trips. Thus, after allowing for this factor, one finds the discrepancy between the two estimates reduces to about + 7% for each of the purposes (Col. 4 of Table 3.9). This suggests that there may be a factor common to all trip purposes that explains the discrepancy.

3.9.6 Planning data/expansion factor changes. It seems that the most likely 'common factor' to account for much of the remaining 7 per cent over-synthesis by the trip-end model of the observed numbers of trips is the inconsistency in the application of the 'inactive household correction factor, discussed in Section 3.4. These inactive household factors, if applied to the data, should be applied to the trip end models as well: see, for example, the RHTM calibration and validation report of Aug-Oct 1979 (Alastair Dick & Associates, 1979c, Section 1.6), where it is said that: "When the trip end model is used, it is applied to 93.5 per cent and 96.0 per cent of the Planning Data households for London and the Rest of the Country respectively.

3.9.7 We understand that, following the recommendation by Martin and Voorhees Associates (1981) to drop the use of these factors, the Department ceased to apply them to the trip-end models. Unfortunately, the observed matrices in the NM have been built using a data-set (OB17) which had taken its HI data from OB13, which had had these factors applied to them.

3.9.8. If the household interview data have to be revised by a factor P (= 1/0.935 for London, 1/0.96 elsewhere), and recalling that cordon-crossing trips were merged with RI data (Section 3.8), the effect on the zonal totals of observed trips would be to increase these to approximately:

$$\text{Px(intra-area HI trips)} + \frac{\text{Px (Cordon crossing HI trips)} \times \frac{1}{F_H} + \text{(Cordon crossing RI trips} \times \frac{1}{F_R}}{\frac{1}{F_H} + \frac{1}{F_R}}$$

where F_H and F_R are the expansion factors for household and roadside interview data respectively. Hence, dropping the inactive household factor from the observed data set would reduce the overestimation of synthetic trip-end estimates from about 7% to about 4%. This explanation of the bulk of the discrepancy, although simple, was not immediately obvious, because of the complex sequence of stages which the observed data and trip end estimates go through.

3.9.9 The discrepancy that remains may be mainly attributable to the fact that the planning data used as input to the trip generation models, PDUP16, was not the same as that used to expand the household interview data (PDUP 14A, which had the same household information as PDUP12). (The parameters of the trip generation models would not however have been affected by the change in planning data). We recommend that the changes in expansion factors implied by the changes in the PDUP information be investigated.

3.9.10 For trip attractions, changes in 'PDUP' input data are not expected to explain the difference between synthetic trip ends and observed trip ends. PDUP16 used the same employment information as PDUP14A, and the attraction models were fitted to zonal trip-end estimates based on the data set (OB13) that utilised PDUP14A.

3.9.11 However, due to the internal balancing between trip attractions and generation that goes on inside the trip end program REGTRIP (see 3.9.15), any over-synthesis in trip generation in a given 'balancing area' will be reflected in the trip attractions for those areas. Hence, there is little need to look for reasons why these synthetic trip attractions as a whole are overestimating the total O-D data.

3.9.12 It should however be noted that, if the data set is again revised (by dropping the inactive household correction factor), the trip attraction models might best be revised also, to take account not only of the dropping of the inactive household factor, but also of the revised method of treating minor road traffic (which is the main difference between the OB17 and OB13 data sets).

3.9.13 <u>The effects of the discrepancies</u> The discrepancies between observed and synthesised trip end estimates in wholly-observed zones (or districts) is clearly important to correct, preferably by ensuring compatibility in the data used for the two estimates. It is a little surprising though that the systematic nature of the discrepancies and their magnitude had not been noted before; the National Model report (Section 4.3) refers only to 'small differences in the magnitudes of the synthetic trip ends and the row and column totals', which led to the decision to replace the synthetic trip ends by the observed trips ends in order to improve the fit given by the synthetic trip end method of calibration (see Section 4). We think that the calibration should have been cured by correcting the incompatibility rather than by adjusting the calibration procedure.

3.9.14 It is at first sight more surprising though that the magnitude of the HBO and EB discrepancies did not manifest themselves more obviously

A possible explanation lies in the fact that much of this difference is concentrated within HI areas (within which no NHB trips could be surveyed), and hence within towns. Since within-town movements had different deterrent functions from elsewhere, the effect of having too few data within the town will be to displace the within town function relative to the other function. One might therefore appear to get a good fit by each type of function to the data in the corresponding part. But the functions will be useless for making estimates in the unobserved cells, because they imply an incorrect balance between data within the towns and data outside them.

3.9.15 Trip end balancing. It is usual in transport modelling to adjust the total of synthesised trip attractions to be equal to the total of synthesised trip generations, so that $\Sigma_J R_J = \Sigma_I Q_J$. In the National Model, as in its predecessors (the Regional Highways Traffic Model and the National Traffic Model), the balancing process was applied not to all zones together, but to all zones lying within certain 'balancing areas' so that

$$\sum_{J \varepsilon X} R_J = \sum_{I \varepsilon X} Q_I \quad \text{for each } X$$

where X = set of zones in a balancing area. For RHTM, which defined 52 balancing areas for the HBW matrix, the balancing areas were defined by the concept that

traffic generated inside and attracted outside = traffic generated outside and attracted inside

If, indeed, one could define such balanced communities from previous knowledge (e.g. using Census data), then this requirement on the estimated trip-ends may be reasonable. But there seems little evidence to support the choice of balancing areas on those grounds.

3.9.16 There is a more general argument to support the idea of balancing attractions to generations. This is, that the attraction equations are not responsive to certain variables such as car ownership which vary across the country, but the trip generations are; so one should adjust the level of the former to the level of the latter.

3.9.17 The RHTM attraction balancing process is usually automatically applied within REGTRIP, so that the level of synthesised attractions put out by REGTRIP for a particular zone has already had a factor applied to them that depends on the RHTM balancing area within which they lie.

3.9.18 However, we understand that, for the National Model, the use of these 52 balancing areas (for HB trips; or 23 balancing areas for NHB trips) was not satisfactory (some of the areas were very small). Since the National Model fitted the gravity model to observed trip ends in wholly observed rows and columns, and synthesised trip-ends elsewhere, it was decided that the trip attractions in wholly observed zones should be unaltered, and adjustments made to trip attractions only in the partially observed zones. Eventually, only zones in 'Area F' were modified. As area F was crossed by six MCC-only screenlines, it was decided to apply the trip attraction balancing process only within the 7 subdivisions of Area F that these created.

3.9.19 The reason for this choice is obscure, and the consequences difficult to interpret. But if indeed it has been found appropriate to dispense with the balancing to the original 52 (or 23) balancing areas, this suggests that the advice in . TAM on the use of the attraction equation may need to be modified.

4. CALIBRATION

4.0.1 This section first describes the method adopted for estimating the parameters, then the principles that might apply, then questions of uniqueness, solution method, efficiency, convergence and smoothing.

4.1 METHOD

4.1.1. For the private vehicle model, the parameters (A_I), (B_J) and (F_k) in

$$T_{IJk} = A_I B_J F_k D_{IJk} \qquad (4:1)$$

were in principle estimated by the so-called synthetic trip end method. This chooses the parameters so that total model trips from each district, to each district, and summed over observed cells in each cost band, agree with the known totals Q_I, R_J and S_k^*, respectively. Formally,

$$T_{I++} = Q_I \text{ , each origin } I \qquad (4:2)$$

$$T_{+J+} = R_J \text{ , each destination } J \qquad (4:3)$$

$$T^*_{++k} = S^*_k/\lambda\text{, each cost band } k, \qquad (4:4)$$

where $S^*_k = N_{++k}$ =number of trips in cost band k which were found in the survey, in total over observed calls only, the superscript * denoting sums over observed cells only.

4.1.2. As shown, an additional parameter λ has been introduced. This is necessary since otherwise there is one more independent constraint than there are parameters for fitting. It is necessary that

$$Q_+ = R_+ \text{ ,} \qquad (4:5)$$

of course. This is achieved in advance, through the trip-end balancing process. The above is one way of inserting the additional parameter - the other obvious way is to adjust each Q_I and each R_J.

4.1.3 In practice, for the wholly observed districts, the National Model replaced the synthesised trip ends by the row and column sums of observed data, i.e. replaced Q_I by Q^*_I, R_J by R^*_J. This aspect is discussed in 4.8.

4.1.4 The factors (F_k) were subsequently smoothed (see Section 4.4), but the method of fitting initially proceeded by finding unsmoothed factors.

4.1.5 The size of the adjustment factor λ is summarised in Table 4.1.

4.2 PRINCIPLES

The principle adopted is clearly one of fitting to best estimates of important aggregate quantities. Also, this is consistent with the usual methods of forecasting - the trip end estimates provided by the trip distribution model agree exactly with those provided by the trip end models. However, there is already one difficulty in the fact that there is one more important aggregate quantity than there are adjustable parameters. So far as is known there is no sense of best fit which it can be considered to achieve. Unlike a best fit method, the procedure is rigid and can not easily be adapted to use more information, more constraints (smoothing), detailed knowledge of data accuracy or fitting accuracy preferences. Whilst accepting that, in the time scale within which the National Model was meant to have been developed, it might have been unwise to have taken on board entirely novel fitting methods, we think it should be emphasised again that, if the best-fitting model is to be found, this requires the use of a consistent statistical estimation criterion, which will utilise information about not only the estimates of the mean numbers of trips, trip-ends etc., but also the estimates of their standard errors. The combined calibration procedure developed in the RHTM Trip Distribution Investigation (Gunn, Kirby, Murchland and Whittaker, 1980), was a tool to do this job: we still think the Department should consider it for practical use.

4.3 UNIQUENESS

4.3.1 Although fitting by synthetic trip ends resembles partial matrix fitting, there are distinct differences in principle.

4.3.2 Partial matrix fitting with empirical deterrence functions fits so that the model numbers of trips in observed cells in each row, column and cost band agree exactly. The resulting model values will be uniquely determined for each observed cell. (The proof of uniqueness follows directly from a minimization formulation of the problem, since the objective function is strictly convex.) Partial matrix estimates for the unobserved cells are made by using the fitted factors of the model.

Unfortunately it can happen that such estimates for unobserved cells are indeterminate despite the unique estimates for observed cells (Day and Hawkins, 1979, Murchland, 1979). Aside from obvious cases in which a cost band was not observed at all, this 'non-identifiable' can arise when the observed cells are too few or not well placed. In detail, this happens when the equations, one for each observed cell,

$$A_i B_j F_k = \text{a specified value, for an observed cell } ijk ,$$

fail to ensure that

$A_i B_j F_k$ is uniquely determined for each unobserved cell ijk.

Of course this possible indeterminancy is quite sensitive to the choice made for the cost functions and cost bands.

4.3.3. Synthetic trip end model fitting, as employed in the National Model, fits so that model trip numbers in all cells in a row or column sum to particular row and column totals, and so that model trips in observed cells in each cost band agree with the observed number. In order that the number of independent constraints is the same as the number of independent variables it is necessary to introduce one extra variable, in the form actually used, by applying a factor to the observed number of trips in each cost band. (If the model is reasonably well-fitting this factor will end up very close to unity.)

4.3.4. As the model value for every cell is included in the constraints, and there is no extrapolation from fitted observed cells to the unfitted ones, there is no question of non-identifiability as may perhaps occur with the partial matrix method. However, there may be a question that the solution is not unique. It is generally believed that more than one solution cannot occur. This has not been definitely proved. Unlike the partial matrix method for observed cells, the synthetic trip end method does not have a known reformulation as a convex minimization problem, making this question harder to resolve.

4.3.5. Empirical evidence about the uniqueness of the solution, obtained by repeating the calibration process from many different starting values, would be too costly on the National Model. For this and other reasons, we constructed a small demonstration example, with an 18 x 18 matrix, and did not find any evidence of non-uniqueness from the limited number of runs undertaken.

4.4 SOLUTION METHOD

4.4.1 The solution method is essentially a variant of the well known iterative method of scaling rows, columns and cost bands in turn. A two stage algorithm may be adopted. Details given in the two stages described below are those which we suggest could have been used for the National Model. These are compared with the method actually used in Section 4.5.

4.4.2 After selecting initial values for $B_J^{(o)}$ and $F_k^{(o)}$, starting with $n = 0$ successive values would be given by the following.

4.4.3 First stage For each row I in turn form

$$T_{IJk}^{(n+1)} = B_J^{(n)} F_k^{(n)} D_{IJk} \text{ for each Jk for which } D_{IJk} > o,$$

$$T_{I++}^{(n+1)} = \sum_J \sum_k T_{IJk}^{(n+1)} \text{, and}$$

$$A_I^{(n+1)} = R_I / T_{I++}^{(n+1)} .$$

Then, for each Jk form, or accumulate over rows,

$$T_{IJk}^{(n+1)} = A_I^{(n+1)} T_{IJk}^{(n+1)} ,$$

$$T_{++k}^{*(n+1)} = \sum_{\substack{I \\ IJ \text{ observed}}} \sum_k T_{IJk}^{(n+1)} \text{, summed over observed cells only,}$$

$$T_{IJ+}^{(n+1)} = \sum_k T_{IJk}^{(n+1)} \text{, the model value for IJ, and}$$

$$T_{+J+}^{(n+1)} = \sum_I T_{IJ+}^{(n+1)} .$$

Second stage Form

$$T_{+++}^{*(n+1)} = \sum_k T_{++k}^{*(n+1)} \text{ and}$$

$$\lambda^{(n+1)} = \sum_k S_k^* / T_{+++}^{*(n+1)} ,$$

$$B_J^{(n+1)} = B_J^{(n)} Q_J / T_{+J+}^{(n+1)} \text{ for each J, and}$$

$$F_k^{(n+1)} = F_k^{(n)} S_k^* / \lambda^{(n+1)} T_{++k}^{*(n+1)} \text{ for each k}$$

4.4.5 The number of multiplications needed by this method is about $2\ nr^2 + nsc + 2nsco$, where nr is the number of rows and nsc the number of composite sub-cells for which D_{IJK} is non-zero, of which nsco are observed. This assumes avoidance of the multiplication by a writ D_{IJK} for a simple cell, and only one multiplication by $A_I^{(n+1)}$ for a composite cell which is not observed. (Further similar economies are possible if the number of cost bands is appreciably less than the number of rows). The storage needed for one row is nr plus the (worst case) number of non-zero sub-cell D_{IJK} in a row, if the $T_{IHK}^{(N+1)}$ overwrite the D_{IJK}.

4.5 <u>CALCULATIONAL ECONOMY</u>

4.5.1 While not doubting that the implementation of the National Model fitting procedure achieves the same effect, it appears that the above would require substantially less calculational effort.

4.5.2 The main difference is that the D_{IJk} are calculated once and for all at the beginning, according to the formula (2:12) in section 2.2.13 above. The National Method method used a so-called C-file technique, in which the C-file contains the data needed for each row in stage 1. This included the information equivalent to a recalculation of the D_{IJk} for each iteration (they did not appear explicitly). Thus, the conventional C-file technique uses one pointer for each sub-cell; our proposed (D-file?) technique would use several pointers for each composite cell (but with only as many pointers as there are different cost bands within the composite cell).

4.5.3 The explicit use of the D_{IJk} was included in our demonstration project with an 18 x 18 example (using the data known as 'Beulah' - see WN 10).

4.5.4. A second difference of the algorithm of Section 4.4. from the National Model method is the use of $T_{IJ+}^{(u+1)}$ or $T_{IJK}^{(n+1)}$ for the sums needed in Stage 2. Without this the term $2\ nr^2$ in the number of multiplications would be $3\ nr^2$.

4.5.5 An untried variant is to solve for a new row factor and corresponding column factor simultaneously. This may reduce the number of iterations needed, by directly dealing with the marked interaction that occurs between a row and its corresponding column factor when the intrazonal cell is a high proportion of all trips in its row or column. This variant is also the natural algorithm if the purpose matrix is required to be, or is taken to be, symmetric.

4.5.6 Since the main argument for adopting the composite approach was to reduce computational cost, it is suggested that any other applications of the approach should adopt such cost-saving features.

4.6. *CONVERGENCE*

4.6.1. The convergence criterion used in the NM was that each of the district trip-end totals were within 1 per cent of the constraints applied, and that the average cost over observed cells was within 1 per cent of the observed average. For most of the districts, the criterion was relatively quickly achieved; but those districts in Cornwall, Wales and Scotland held up the convergence overall, largely because of their high proportion of intra-district trips. In order to reduce this difficulty, the Scottish and Welsh cordons were represented by a separate deterrence function.

4.6.2 Convergence was not good, especially for HBW. In the early stages of the validation, we advised on ways of accelerating the rate of convergence, apparently with some benefit. The obvious strategy of separately estimating and removing those intrazonal trips which were a high fraction (say over .95) of their row and column sum was not followed.

4.6.3. The one per cent convergence criterion is quite acceptable for this sort of model: errors due to non-convergence to the given trip-end totals are going to be negligible compared with the errors in the trip end estimates themselves.

4.7. SMOOTHING

4.7.1. The unsmoothed factors (F_k) having been established, we urged at the outset of this project that the factors should be smoothed for use in forecasting, or for predicting trips in the unobserved cells, since otherwise one would have the anomaly that, for some ranges of cost, travel becomes more likely as cost increases. However, the methods we suggested for smoothing were not followed, and this gave us some difficulty subsequently in deciding how to estimate the accuracy of the fitted model. Our preferred strategy was to amalgamate adjacent cost bands until one had a continuously decreasing function: this would have had the benefit of providing a new set of precisely defined cost bands, for which the accuracy could have been readily calculated.

4.7.2. The surprising thing about the method of smoothing adopted was that, when first tried, it led to the smoothed values over-estimating trip length compared with the unsmoothed values. This was subsequently corrected by an iterative procedure; but it does suggest that the principle used in the method of smoothing is not one to be advocated for general use.

4.7.3. Another difficulty apparent in the adopted method would lie in the treatment of observed cost bands for which no trips were found. Here the program used interpolated or extrapolated values for all unobserved bands - 3.4 per cent of the bands.

4.7.4. Comment on various smoothing methods is summarised in WN 25.

5. ACCURACY ASSESSMENT

5.0.1 There are two main aspects of model accuracy: that due to variability in the input data, and that due to model misspecification.

5.0.2 In the Traffic Appraisal Manual (Section 12.4.3), it was contended that model specification error could not be quantified, so that accuracy measures for the modelled matrix should be based on the errors of measurement and sampling in the observed matrix. However, it was also said that the model's value should be compared with

i) information derived from observations to which the model was fitted but not constrained;

ii) information from independent observations.

5.0.3 Since the National Model Validation is virtually the first detailed study of what is involved in validating the matrices produced by gravity models of trip distribution, there were a number of statistical issues arising which had not been confronted fully before, and so the details of the procedures and the reasons for them are breaking new ground, and in particular will probably lead to some modifications of the advice in TAM. For example, ways of investigating the extent of model misspecification were established.

5.0.4 However, it should be stressed that the time scale in which this work was accomplished, and the very late availability of appropriate data, means that there is considerable scope for refining the arguments and analyses presented here.

5.1 THE COMPONENTS OF MODEL ACCURACY

5.1.1 The ways in which different components of model accuracy are related to each other are reviewed here.

5.1.2 (A) The accuracy due to variability in the input data
The components are as follows:

5.1.3 (a) The accuracy due to sampling variability and to errors in the expansion factors in the observed O-D data.

5.1.4 (b) The accuracy due to sampling variability etc. in the wholly-observed row and column totals of O-D data (i.e. the district trip-ends), which act as constraints in the calibration procedure. (This uses the results in (a)).

5.1.5 (c) The accuracy due to sampling variability etc. in the cost-band sums.(This uses the results in (a)).

5.1.6 (d) The accuracy of the trip-end estimates synthesised by the trip generaion and attraction models. (This uses the results in (b)).

5.1.7 (B) The adequacy of the fitted model
All the above aspects of accuracy assessment come together in estimating

5.1.8 (a) the accuracy of the fitted gravity model due to variability in the input data to which it is fitted.

5.1.9 However, it may be that the more important source of error is due to biases in the model values. The extent of model biases can be assessed, by

5.1.10 (b) examining the residuals, between the observed data in each cell and the fitted model values (requiring the use of Aa, Bb), and by

5.1.11 (c) comparing the fitted model values with the values obtained from independent data sets. The latter comparison is especially useful, of course, for assessing the adequacy of the model in the unobserved cells.

5.1.12 In the validation project as a whole, in which Howard Humphreys and Partners were primarily responsible for the external validation (B(c) above), and ITS for the internal validation, there was a good deal of interdependence, since ITS was providing mathematical and statistical advice to HH&P at various stages, and HH&P provided the software for calculating the accuracies of the observed data (at A(a)) and, with RDCOSM, were also able to undertake comparisons of the kind (b) which are parallel to but different from the analyses we undertook for B(b).

5.1.13 The present report is however reporting only the ITS work; for a full picture, the HH&P final report should also be referred to.

5.1.14 The rest of this section is structured as follows:

5.2 On distinguishing model error and data error.
5.3 The accuracy of observed O-D data.
5.4 The accuracy of marginal totals.
5.5 The accuracy of trip end estimates.
5.6 The accuracy of fitted models values.
5.7 An overall view of model fit.
5.8 The model fit in intra district cells.
5.9 The examination of residuals overall.

5.1.15 Note that all empirical examinations were conducted for a 642 x 642 simple matrix, in which observed trips and fitted model values in the composite cells of the composite matrix were added together.

5.2 ON DISTINGUISHING MODEL ERROR AND DATA ERROR

5.2.1 At the outset of the project, it was hoped that one outcome of the assessment of model accuracy might be some kind of 'error law', which could be used to provide an estimate of the effects of both data error and (gravity) model error on the modelled estimates.

5.2.2 Our examination of the statistical issues involved showed that a number of the difficulties arise in the interpretation of the residuals, defined as the differences between the expanded calibration data set and the fitted model. (See WN 3, 4 and 7 for the details.) The conclusions reached were as follows.

5.2.3 (a) The residuals can be usefully scanned for patterns of persistent bias, but the statistical significance of any particular discrepancy must be judged in relation to the expected variance of the residual, which is a complex function involving both variances of model and data and their possibly non-negligible co-variance. (unless the comparison data is independent.)

5.2.4. The expected value of the squared residual for an observed cell is the sum of the square of the model bias for the cell plus the variance of the residual. (The latter, of course, is the variance of the observed value plus the variance of the model value minus twice the covariance between the two, which is always positive here). Both the squared residuals and their estimated variances vary very much from cell to cell. Bias in a single cell is only detectable if the residual is serveral multiples of its standard deviation. Examination of the squared sum of residuals over sets of cells would give a more sensitive indication of the presence of model biases, but this would require rather complex calculations of the variance and covariances of sums of model values.

5.2.5. (c) The extrapolation of simple rules (i.e. descriptions) of model biases, were any to be found, from the calibration data set to the unobserved regions of the matrix will be rather an act of faith, since, although the synthetic trip end method utilises all cells in the row and column constraints, the O-D data set (which contains the cost band constraints) was not assembled with any such problem in mind, and the accuracy of the synthetic trip ends in unobserved rows and columns is not confirmed.

5.3 THE ACCURACY OF OBSERVED O-D DATA

5.3.1 It would have been too costly to reprocess the RHTM data sets through the RDMVAR programs to get variances of the observed data, especially as this would have involved re-writing RDMVAR to accommodate a data set of this scale.

5.3.2 Instead, the variance estimation was conducted in two stages:

i) the calculation of the variance under the assumption that there was no uncertainty in the scaling-up factors;

ii) the subsequent application of a common factor to allow for the effects of uncertainty in the scaling up factors.

5.3.3 The variance calculation (i) was carried out by HH&P by processing the trip records in TRIFILA with a program which did what RDMVAR would have done with zero coefficients of variation for the expansion factors (except that the 'finite population correction factor' for hourly RI samples was not included).

5.3.4 The uncertainty factor (ii) was investigated by processing one RHTM data set (that for W. Oxford) through RDMVAR, both with and without the appropriate coefficients of variation in the scaling up factors, and looking at the values of:

$$\frac{\text{variance (observed trips with appropriate coefficients of variation in the scaling up factors)}}{\text{variance (observed trips with zero coefficients of variation in the scaling up factors)}}$$

It was decided to adopt a simple average of 1.05^2 for this factor, to be applied subsequently to the variance derived from 5.3.3 for any cell or row, column or cost band sum.

5.3.5 Since, for a given data set, all data elements in the subcells of a composite cell could be regarded as independent, there was no problem in working out the variances of the data for the simple 642 x 642 matrix, which was the level at which the validation was carried out.

5.3.6 The method of merging that was actually used (see Section 3.8) did reflect, approximately, the 'optimal' merging that would have been performed in RDMERGE using the variances calculated in accordance with these procedures (except that the NM method substituted a non-zero default factor value for any zero observed RI movement that was being combined with a non-zero HI estimate).

5.3.7 For those cells observed in more than one data set, a method was devised for combining the variances of the several estimates, in accordance with the way in which the estimates were combined in the National Model.

5.3.8. Subsequently, some of the index of dispersion values seemed remarkably high or low; values below 1 occurred.

5.4 THE ACCURACY OF MARGINAL TOTALS

The variances of a row sum, column sum or cost band sum of observed data were taken to be the sum of the variances of the cells contributing to these sums. (Note here though that, for cost bands, the sums were taken over data values in the form of a composite matrix.) Tables 5.4(1) and 5.4.(2) illustrate the orders of magnitude of variances provided by these procedures.

5.5 THE ACCURACY OF TRIP END ESTIMATES

5.5.1 The estimation error properties of the National Model depend on the accuracy of the control totals, including the trip ends. The accuracy of the synthetic trip ends should ideally be determined during the fitting of the trip end models. Although the accuracy of the parameters of the trip end models were calculated (and given in TAM), this is not sufficient for validation purposes. Since the row and column sums of the NATGRAV O-D matrices are being constrained (for the partially observed districts) to the TAM-related synthesised trip ends, one needs to assess, using data for the wholly observed districts, the accuracy of one set of estimates with respect to the other. For this purpose, the synthetic and observed trip ends may be treated as if they were independent (though this assumption is later relaxed).

5.5.2 The first examination of these data (WN 11) showed that the two estimates were biased with respect to each other. The nature and extent of the bias and possible reasons for it are reported in Section 3.

5.5.3 It was also shown that the assumptions made about the way the standard errors are related to the size of the estimated numbers of trips ends (that is the treatment of heteroscedasticity) has a considerable effect on the estimated error.

5.5.4 Thus, if the variance of the numbers of synthesised trips were constant, then there should be a constant scatter about the 45^{o} line on a trip end/trip end plot; if it varied as the square of the observed numbers of trips, the scatter should be constant on a log-log plot; if it varied as the observed numbers of trips, the scatter should be constant on a square rooted/square rooted plot. The three situations are demonstrated for HBW, for regional zone trip ends, in Figures 5.5 (1 - 3). The scatter tends to increase in the first case, decrease in the second; only with the square root transform does one seem to satisfy the hypothesis of constant variance. The same pattern persists for all trip purposes, and generations (or origins) as well as attractions (or destinations). The square root transform also appears to stabilise the variance for trip ends at district level: see Figure 5.5(4). (The scatter diagrams for other trip purposes are given in WN 11 at regional zone level and WN 13 at district level.) The bias is also evident in these plots.

5.5.5 To estimate the accuracy of the trip end estimates, the usual approach is to regress the synthetic trip ends on the observed trip ends, assuming both are independent estimates and the latter are free from error. This was the approach taken in the RHTM Status Report (Alastair Dick and Associates, 1979c), in which regressions were performed on both untransformed and log-transformed data.

5.5.6 However, in fact, the observed values are estimates formed from HI data (and, for trip attractions, from RI data as well), and so have an error (whose calculation was described in 5.4). Thus, taking the residual scatter as a measure of the synthetic model accuracy seems bound to result in underestimation of the model performance.

5.5.7 Moreover, the independence assumption, which seems reasonable to apply for HBW and HBO generations (since the synthetic trip generation models used HI data at household rather than zonal level), is not very reasonable for HBW and HBO trip attractions, or for NHB or EB trip ends, since their trip end models were formed by regression at zonal level.

5.5.8 Hence, approaches were devised for estimating the accuracies of the synthetic trip end estimates taking into account the bias in trip ends, the errors in observed trip ends, and the lack of independence for zone based trip end models.

5.5.9 Details of the methods are in WN 11 and WN 12.

5.5.10 At the time the analyses were done, the variances of the observed trip end estimates calculated by the methods described in 5.4, were not available, and so the simplifying assumption was made that the observed trip end totals were derived from uniform 1/40 HI surveys and 1/10 RI surveys. Since it was found that the results were not sensitive to these assumptions, (the variance in the observed trip ends is relatively small, so the results are not very different from straightforward regression of synthesised trip ends on observed trip ends - see WN 11), the analyses were not repeated when the more appropriate estimates of observed trip end variances became available.

5.5.11 The results of the analyses at district and zonal level are given in Table 5.5 (1).

5.5.12 The 95 per cent confidence intervals about the mean number of regional zone trip ends are given in Table 5.5 (2).

5.5.13 From Table 5.5(2) we see that the confidence intervals at regional zone level are much less wide than those given in the RHTM Status Report. This is mainly due to the improved treatment of the varying amounts of scatter in the estimates (i.e. using a square rooted rather than logarithmic transform). Accounting for the lack of independence between zone-based models and zonal observed totals has also reduced the confidence intervals. However, the intervals are still quite substantial.

5.5.14 The most surprising result is in the comparison between district level and zonal level estimates of accuracy, given in Table 5.5 (1). We had expected the district level variances to be less, since the effect of uncertainties in the planning data would tend to introduce negative correlations between regional zone members of the same district, in that the district totals are usually much better determined than the regional zone portions (indeed, it is at district level that the planning data is primarily estimated).

5.5.15 The results imply instead that there is a degree of positive correlation between errors in regional zones within districts. This is clear evidence of underspecification in the trip end models; some variable or variables are omitted which take similar values in 'near' zones.

5.5.16 If geographical underspecification is apparent even within the observed zones which contained the data to which the trip end models were fitted, one must be even more dubious about the biases that might then occur in using these trip end models to estimate trip ends for the partially observed zones/districts. Indeed, the fact that the partially observed zones are in some respects (e.g. size) very different from the observed zones may mean that there is an extrapolation problem in applying the trip end models nationwide. This aspect should be assessed in any re-examination of the trip end models.

5.5.17 The method of estimating the variances of the synthetic trip end estimates with respect to the observed estwmates (assumed to be independent), taking into account the sampling and measurement errors in the latter, allows an overall 'best estimate' to be formed of the numbers of trips in each district. Because the variances of the observed trip ends are so much smaller than the variances of the synthetic model values, the best estimate lies much closer to the observed value than it does to the synthesised value. This lends some support to the use of observed trip-ends rather than synthesised trip ends in the wholly observed districts when fitting the National Model.

5.6 THE ACCURACY OF THE FITTED MODEL'S VALUES

5.6.1 The variances of the wholly-observed row and column and cost-band totals, and of the synthetic trip-end totals for partially-observed rows and columns, have been used as input to an approximate analytic expression for the accuracy of the fitted model.

5.6.2 The expression obtained for the variance of the synthetic trip-end model's estimates is from WN 20,

$$\frac{\mathrm{var}\,(T_{IJk})}{T_{IJk}^{2}} = \{ \frac{\mathrm{var}(Q_I)}{Q_I^2} + \frac{\mathrm{var}(R_J)}{R_J^2} + \frac{\mathrm{var}(N_{++k})}{N_{++k}^2} + $$

$$+ \; 2\,\delta_I \frac{\mathrm{var}(N_{I+k})}{N_{I++}N_{++k}} + 2\,\varepsilon_J \frac{\mathrm{var}(N_{+Jk})}{N_{+J+}N_{++k}} + $$

$$+ \; 2\,\delta_I\,\varepsilon_J \frac{\mathrm{var}N_{IJ+}}{N_{I++}N_{+J+}} \} \qquad (5{:}1)$$

where δ_I, ε_J = 1 if the district is wholly observed, and 0 otherwise; and for the wholly observed districts, in which the observed rather than synthesised trip-ends are used, that is

$$Q_I = N_{I++} \;, \quad \text{or} \quad R_J = N_{+J+} \;,$$

the variances of the observed trip end totals will be used, that is, we set

$$\mathrm{var}\,(Q_I) = \mathrm{var}\,(N_{I++}) \;, \quad \text{or } \mathrm{var}\,(R_J) = \mathrm{var}\,(N_{+J+})$$

5.6.3. Expression (5:1) thus constitutes the error law due to inaccuracies in data input for the composite model when the cost factors are not smoothed. The expression in (5:1) is of course just the square of the coefficient of variation of the modelled value, sometimes known as the relative variance.

5.6.4 For cells not in wholly-observed rows or columns, (5:1) reduces to

$$\frac{\text{var }(T_{IJk})}{T_{IJk}^{2}} = \frac{\text{var } Q_I}{Q_J^2} + \frac{\text{var } R_J}{R_J^2} + \frac{\text{var }(N_{++k})}{N_{++k}^2} \qquad (5:2)$$

5.6.5 From the results for district level trip-end accuracy, (Section 5.5 and Table 5.5(1))

$$\text{var } Q_I = \alpha\, Q_I \quad , \quad \text{var } R_J = \beta\, R_J$$

where α, β depend on the trip purpose. Also, from the accuracy of the cost-band sums, it would appear that, for movements not within towns (i.e. for function types 5-9), the index of dispersion (= variance to mean ratio) is approximately constant, giving

$$\text{var }(N_{++k}) = \gamma \; N_{++k}$$

Hence, for cells not in wholly observed rows and columns or within towns, (5:2) becomes

$$\frac{\text{var }(T_{IJk})}{T_{IJk}^{2}} \simeq \frac{\alpha}{Q_I} + \frac{\beta}{R_J} + \frac{\gamma}{N_{IJk}} \qquad (5.3)$$

For home based work trips, α =800 β =850, γ = 9.

5.6.6 The order of magnitude may be crudely judged for a district with the average number of district trip ends.

With about 12 million non-HI Area HBW trip-ends, and 642 - 104 =538 non HI Area districts, this gives $\bar{Q}_I \simeq \bar{R}_J \simeq$ 22,300 trip-ends per district. With smoothed cost functions, assuming that the smoothing is equivalent to having 11 cost bands, the average value of N_{++k} (for non-intra town function types 5-9) is $\bar{N}_{++k} \simeq$ 2289199/11 =208000. Hence, substituting in (5:2),

$$\text{coefficient of variation of } T_{IJk} \simeq \sqrt{\left\{\frac{800}{22300} + \frac{850}{22300} + \frac{9}{208000}\right\}}$$

$$\simeq \sqrt{\{ .03587 + .03812 + .00004 \}}$$

$$\simeq 0.27.$$

But it would be misleading to attach any importance to this - or any other - simple average.

5.6.7 For cells in wholly-observed rows or columns or both, the expression (5:1) allows for the correlation between the marginal totals of observed trip-ends (since the observed N_{IJk} contributes to each of N_{I++}, N_{+J+} and N_{++k}).

5.6.8 Because the variance of the observed trip-ends is much less than that of the synthesised inputs, the accuracy of the fitted model due to input data variability will be better in the cells with one or other or both ends of the trips from a wholly observed district.

5.6.9 Some typical values for the accuracies calculated by (5:1) for a sample of cells are given in the tables in the next section.

5.7 AN OVERALL VIEW OF MODEL FIT

5.7.1 A visual inspection of two random samples of observed and modelled value was made, to help acquire a feel for the way the model was performing. From the first sample of about 600 observed cells, the ability of the model to give the right orders of magnitude of the estimates in the cells, with magnitudes varying from 0.01 to several thousand, came over strongly.

5.7.2 For illustrative purposes, a second sample, of just 170 observed cells, over all the purposes interleaved was taken, from which Tables 5.7(1) and 5.7(2) have been prepared. This sample was of every 6143rd cell, so giving equal importance to each cell, regardless of the number of trips in it. As the sample contained only 15 non-zero observed cells, results are summarised for those separately. Table 5.7(1) gives the observed and modelled values and their calculated coefficients of variation for the non-zero observed cells, for each of the purposes. Table 5.7(2) gives the modelled values and their coefficients of variation for the 'observed zero' cells for the HBW purpose only. Also given are standardised differences, discussed below.

5.7.3 Similar comparisons were made for intradistrict cells separately. These are reported in 5.8.

5.7.4 Note that there is a persistent difficulty with estimating the variance (and coefficient of variation) of the observed data. This is calculated by multiplying the index of dispersion by the 'mean' number of trips. But what value should be taken as the mean? If the value is taken as the observed value, that certainly does not work for zero observed values, and seems very unreliable when the count is only one or two. What was done here, was to form a best estimate from the observed (O) and modelled (E) value, using the inverse of the indices of dispersion as weights:

$$X = (id(E) \times O + id(O) \times E) / (id(E) + id(O))$$

so var (O) = X id (O).
This amounts to using the null hypothesis that there is no bias. (Note that values of indices of dispersion that were absurdly high or low - see 5.3.8 - were deemed implausible; values greater than 500 were set to 350, and values less than 4 set to 4.) These difficulties in turn affect the computed variance of the observed row sums (eg. N_{I++}) and thus the variance of the fitted model's values in wholly-observed cells.

5.7.5 The modelled (E) and observed (O) values may best be compared by examining the standardised difference

$$Z = (E-O) / \sqrt{\{ var (E) + var (O) - 2 \, cov (E,O) \}}$$

5.7.6 For the non-zero cells in the comparison, Table 5.9(1) shows that there are just two (starred) cells which appear to have observed values significantly different from modelled ones. The model values are low, and although the observed values are based on counts of only 1 (incidentally one of the strange cases where the i.d equals the value) and 3, because of this lowness the data values are considered to have small variances. If the observed value is used to calculate the observed variance, the latter increases by a large factor - and the standardized differences change from

-7.1 to about -.9 and -5.6 to about -1.45. The significance disappears. This seems to be a real paradox: the hypothesis that they are not significantly different leads to the conclusion that they are, the opposite hypothesis that they are not! When the model value is higher than the observed the paradox is reversed: each hypothesis confirms itself.

5.7.7 For the observed-zero cells, Table 5.7(2) shows that the model values are very nearly zero, but with one apparently outstanding exception. But is the modelled estimate of 48 trips really significantly different from zero? As we have demonstrated in 5.7.6 above, this depends on what one takes as the 'mean value' for the cell when multiplying this by the index of dispersion. Substituting the observed value (zero) gives a zero standard deviation for the observed (zero) count, and the modelled value is indeed significantly different from the observed.

But the index of dispersion for the observed-zero count was 378, and taking a weighted average of observed and modelled values as the best estimate of the mean (see 5.7.4), gave a standard deviation of the observed data of 140. This, taken with the standard deviation of the modelled value of 10, gives a standardised difference of 0.34; clearly not significant.

5.7.8 From these limited comparisons one can conclude that broadly speaking, the model is in the right 'ball-park'. Whilst it occasionally produces a value substantially different from the observed value, the differences must be judged in the light of the variances of the modelled and observed value and their covariances; and it is clear from 5.7. that even substantially different values may be not significant, given these accuracies.

5.7.9 The approach outlined in this section provides a way of understanding the significance of the residuals between the model and data, taking account of the accuracies and dependencies in them. The approach complements that described in Section 5.9, where the emphasis is on finding factors that help to explain the variation in the magnitude of the residuals across the whole data set, but without information on their variances and covariances. It would have been desirable for the methods of analysis described here to have been extended to the whole data set.

5.8 THE MODEL FIT IN INTRA-DISTRICT CELLS

5.8.1 A listing has been made of the most important trip-related quantities for each of the 642 districts, for each purpose. This is for the National Model values aggregated to simple cells. The items tabulated are district number, row sum, column sum, observed intradistrict trips if observed (and zero if not), and its relative variance, modelled intra district trips and its relative variance, modelled intra district trips as a fraction of the row sum, the cost band number (which combines function number and cost band), the cost band sum, and the relative variances of the row sum, column sum and cost band sum. (Relative variance is just a short name for the square of the coefficient of variation.)

5.8.2 The row and column sum relative variances use the approximatios of Section 5.5 (WN 13) for the district-level synthetic trip ends, and the calculated values for the observed trip ends. The latter are very much more accurate (usually having coefficients of variation of under 3 per cent, while the synthetic values are seldom less than 15 per cent and range up to 50 per cent). In both cases the smaller the total the less accurate. The model relative variance was calculated using the more elaborate approximation which takes account of covariance between the observed cell value and the row column and cost band sums (that is eqn. 5:1). This effect is important for intra-district trips since they are likely to be such a large fraction of the sums.

5.8.3 Some descriptive statistics for observed and modelled intra district movements are included in Section 8.

5.8.4 Since the intrazonal costs and the within-town inter-zonal costs (in the subcells of the composite matrix) were taken from a relationship with zone size that had been adjusted to make the synthesised intrazonal/intra-district/intra-area trips agree with those observed, we expected that the modelled intradistrict trips (which will be the sum of the intrazonal trips for, and the interzonal trips between, the zones that are members of a district), would be in good agreement with the observed intradistrict values.

5.8.5 By eye, however, the modelled values appear to be mostly lower than the observed values. This is confirmed by a Watson U^2 test on the differences, standardised by division by the standard deviation of the differences calculated from the two relative variances. (This standard deviation is too large, because it ignores the covariance between the modelled and observed values which, as noted above, will be appreciable). The U^2 test strongly rejects the hypothesis that the modelled and observed intradistrict trips are estimates of the same thing.

A factor which, applied to the model values, reduces the mean standardised difference value to zero, is 1.09 - that is, the model values seem to be too small by about 9 per cent. (Since the model is fitted to row, column and cost band sums, the total remaining trips in each row, column and cost band must be correspondingly too great.) This value of 1.09 is not the value which minimises U^2; it is too great for that and a minimising value is more like 1.05. The factor which makes the totals of each equal is 1.068.

5.8.6 The worst individual cells in terms of standardised differences are listed in Table 5.8.

5.8.7 For exploratory purposes, a simple relationship between the modelled and observed values has been fitted, by doing a regression of the log model values on the log observed, ignoring the relative variances. For what it is worth, this fit is

$$\text{modelled} = .56\ (\text{observed})^{1.046}$$

suggesting that the larger the modelled value the smaller the factor that should be applied to it.

5.8.8 Since intradistrict movements account for over 50 per cent of the synthesised trip ends (see Section 8.2.1 for the details), it is disturbing that there appears to remain a tendency to underestimate intradistrict trips, and thus over-estimate inter district trips.

5.9 THE EXAMINATION OF RESIDUALS OVERALL

5.9.1 As noted in Section 5.2, the residuals can be scanned for patterns of persistent bias, and the residual sum of squares can provide useful information as to the presence of appreciable model misspecification.

5.9.2 This section reports the results of an analysis of the residuals for the National Model, for all trip purposes combined. The residuals have been grouped into 216 categories according to 'area type', size of 'effective expansion factor', 'trip length' and 'size of modelled flow'. The categories that were used were as follows.

'Area type'	Intra-London, extra-London, intra-rest, inter-rest.
'Effective expansion factor'	Low (less than 10), medium (10 100), high (over 100).
'Trip length'	Less than 25 km, 25 to 100 km, over 100 km.
'Modelled flow'	Less than 1 trip, 1-5, 5-10, 10-100, 100-500, over 500 trips.

5.9.3 The 'effective expansion factor' is given by the indices of dispersion (that is, the variance to mean ratios) of the observed trips, whose calculation was described in Section 5.3.

5.9.4 For each of the 216 categories, the following statistics were calculated:

Σ (O-E)	being the sum of the expanded residuals;
Σ $(O-E)^2$	being the sum of squared residuals;
Σ O	being the sum of observed trips;
Σ E	being the sum of modelled trips;
M	being the number of cells falling into that category (= $\Sigma(1)$)

5.9.5 Preliminary examination of the results showed that the sum of residuals and the sum of squares of residuals, were each strongly related to the sum of modelled trips (Σ E). The residual sum of squares, Σ $(O-E)^2$, also consisted of very large numbers in certain cells. Other statistics based on these were accordingly used in the first interpretation of the pattern of residuals, described in WN 23. In that way, all trip purposes were considered together; here we consider the residuals for HBW trips only.

5.9.6 The interpretation of the residuals (or, rather, the pattern of their sums in the various categories) should ideally take into account the variances of modelled and observed values and the covariance between them, in a manner similar to that described, for a sample of cells, in Section 5.7, and also in the attempt to fit a squared bias. However, the timescale of the project did not permit a proper examination of this, and this should be borne in mind when forming an overall judgement as to the adequacy of the fitted model.

5.9.7 The first analyses of the residuals, reported in WN 23, led to the conclusion that there were strong indications that the model was performing differently according to whether the data came from HI-only cells, or (mainly) RI cells; and thus that the model was interpolating between two data sets which presented distinctly different patterns of trip making.

5.9.8 However, in the light of the evidence discussed in Section 5.7, we softened this conclusion in the draft final report, and have since examined further the patterns that emerged in the analysis. Our further examination is however for HBW trips only, whereas the original tabulations in WN were for all purposes togethe

5.9.9 This further examination had to be restricted to the categories in which the data were first classified, and had to be conducted in our own time, the contract having ended and the team dispersed soon after the residuals were first investigated.

5.9.10 The rest of this section therefore presents the analysis of residuals in a different manner to that presented in WN 23 (and in the first draft of this report).

5.9.11 Table 5.9(1) shows how the mean residual per cell varies between distance bands and from one part of the country to another, both in absolute terms (col. 4), as $\Sigma(O-E)/M$; and in relative terms (col. 5), as $\Sigma(O-E)/M \div \Sigma E/M$, $= \Sigma(O-E)/\Sigma E$. If both model and data are estimating the same quantity, the expected behaviour of these statistics is that the mean residual should be small, and approximately constant from one category to another (and, correspondingly, the relative mean residual, $\Sigma(O-E)/\Sigma E$, would vary inversely as ΣE).

5.9.12 The second and third columns of the table ($\Sigma O/M$, $\Sigma E/M$) suggest that the modelled and observed values are in broadly close agreement. The constraints applied to modelled trip-ends are such that one should expect close agreement over all trip lengths combined. Although trips of length < 25 km are underestimated by the model over all categories, the mean residual is small relative to the modelled value, except for the out-of-London movement, where it exceeds 10 percent. The most apparent discrepancy is for the medium distance (25-100 km) movements within areas other than London, where both the absolute and relative residuals are large (-419 and -18% respectively), although in this case only 29 cells contribute to the average. Note too that, for the longer distance (over 100 km) movements between areas other than London, the mean residual, though small in absolute terms (-0.04), is a high proportion (-20%) of the modelled value, and applies to a high proportion (75%) of the 198738 non-zero observed and modelled* cells.

* Note The cells for which the modelled value was rounded to zero, and for which the observed value was also zero, were excluded from this analysis of residuals. Since 81.9% of the 642x642 (= 337562) cells in the simple matrix were observed, that means that about 138824 cells, = 33.7% of all cells or 41.1% of observed cells, had a zero observed and zero modelled value. These were called "double-zero" cells.

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5.9.13 The variation in mean residual by trip length band within area type is further disaggregated by the low/medium/high categorisation of cells by expansion factor, in Table 5.9(2).

5.9.14 It would appear from Table 5.9(2) that the discrepancies associated with trip length commented upon in 5.9.12 are more pronounced in each category of expansion factor - but are generally of opposite sign, according as the expansion factor is < 100 or > 100. However, this effect is to be expected for all within-area movements (both for London and elsewhere), and for trips out of London less than 25 km in length, since, for the given choice of categories, the low expansion factor category happens in each case to contain zero observed trips, for which the model is bound to give positive values. It should be remembered that, for cells for which the observed value is zero, the observed value is inevitably biassed downwards from the true value. Because the model is constrained to reproduce observed totals of trips, and gives positive values everywhere, this means that the model's estimates for the non-zero observed cells will tend to be biased below their true values. For those categories (starred) in Table 5.9(2), in which theobserved values are zero, the pattern of '+'s and '-'s as between high and low expansion factor ranges in thus largely explained; and of course judgements about whether the model (or data) is poorly specified are difficult to make here. But for the remaining categories (extra-London journeys longer than 25 km, and inter-rest journeys of all lengths), it is more surprising that the pattern of '+'s and '-'s remains the same, with modelled estimates overpredicting trips of low expansion factor and underpredicting trips of high expansion factor. This is commented upon further in 5.9.22.

5.9.15 For the 'intra-rest', '25 - 100 km' category, there is no tendency for under-prediction in the high expansion factor range to compensate for over-prediction in the low expansion factor range. A high negative residual in each explains the high value overall noted in Table 5.9(1).

5.9.16 The question arises as to whether the differences apparent so far are significant.

Ideally, this requires standardisation of the mean residual, $(\Sigma O - \Sigma E)/M$ by its standard deviation, $\sqrt{\{\text{var}(\Sigma O) + \text{var}(\Sigma E) - 2\,\text{cov}(\Sigma O, \Sigma E)\}}/M$, but this being infeasible in the time available (and being a much more difficult calculation than the corresponding one for individual cells discussed in Section 5.7.5) we standardised by the standard deviation of the mean observed value only, $\sqrt{\{\text{var}\,\Sigma O\}}/M$.

As the data had been categorised with respect to the index of dispersion (or effective expansion factor) of the observed values, I, the standardised residuals were calculated on the presumption that they were approximately constant within each such category, taking the following values :

	Index of Dispersion		
Category	low	medium	high
Range	<10	10–100	>100
Mean I	9.5	40	373

For each ID category, the standard deviation of the mean observed value may be estimated either by

$$\sqrt{\{I\,\Sigma O\}}/M \qquad \text{or} \qquad \sqrt{\{I\,\Sigma E\}}/M$$

depending whether we take the observed or modelled value as the most appropriate estimate of the true mean number of observed trips in a cell. (cf. the discussion in 5.7.4).
The two estimates of the standardised mean residual,

$$A = \frac{\Sigma O - \Sigma E}{\sqrt{\{I\,\Sigma O\}}} \qquad \text{and} \qquad B = \frac{\Sigma O - \Sigma E}{\sqrt{\{I\,\Sigma E\}}}$$

are both given in Table 5.9(3).

Table 5.9(4) gives the number of cells in each category.

5.9.17 What value of the ratio should be taken to indicate a 'significant' difference between the observed and modelled values is not however clear, since we have neglected the variance of the modelled values in this standardisation (and the covariance). For this reason, a ratio of, say, 6 should not be regarded as unsurprising. Highlighted in Table 5.9(3) and those categories with a ratio in excess of 10 for both 'A' and 'B'. Particularly discrepant, across all trip lengths, are the cells with low expansion factors (<10), although, the (small number of cells in the 'intra-rest' category are much less discrepant than other categories.

From Table 5.9.(8) it can be seen that the 'low expansion factor' categories account for a high proportion (92%) of the observed and modelled cells.
Trips out of London, or between other areas, that have effective expansion factors in excess of 100, do not appear to have modelled values that are significantly different from the observed ones. They are however few in number (4% of observed and modelled cells).

5.9.18 The tendency for the residuals to be 'not significantly different from zero' in the high expansion factor ranges may of course simply reflect the notion that (by definition, given the high expansion factor) there is relatively less data with which to reject the model in these categories.

5.9. It should also be noted that the apparently very high discrepancy of 646 in the middle distance, mid-expansion factor category for 'intra-rest' trips in Table 5.9(2) does not appear to be significantly different from zero, judging by Table 5.9(3).

5.9.20 Finally, we examine in Tables 5.9(5) and (6) the residual statistics for (a) six categories with the largest number of cells, (b) five categories with the largest number of trips. Table 5.9(5) also shows that the second must commonly occurring category is one omitted from the preceding analysis: those in which the observed value is zero and the modelled value, being less than 0.01, has been set to zero. The standardised residuals in both tables 5.9(5) and 5.9(6) have been approximated by :

$$\frac{(\Sigma O - \Sigma E)/M}{\sqrt{\{\text{ var } (\Sigma O/M)\}}} \approx \frac{\Sigma O - \Sigma E}{\sqrt{\{M \text{ var } O\}}}$$

assuming the variance within each category to be approximately constant, where

$$\text{var } O \approx I(\Sigma E/M) ,$$

where the index of dispersion $I \approx 9.5$, 40 or 373 according to the effective expansion factor is low, medium or high, as in 5.9.16 . Note that, had we used the mean observed value ($\Sigma O/M$) to multiply the index of dispersion by, the standardised residual would in general have b een higher (r cf. Table 5.9(3), where both kinds of calculation are given.)

5.9.21 Bearing in mind that the variance of the modelled value, and the covariance term, have not been included in the estimation of the approximate standardised residual, it would appear from Table 5.9(6) that certainly these and possibly all of the five categories which account for the largest numbers of trips have modelled values which do not differ significantly from the observed ones. It should however be noted that (a) the relative difference is of the order of 14 per cent for category E which is relatively important for trunk road planning, having trips in the 25 - 100 km range; (b) those trip categories account for 83.7% of the observed trips, but occupy only 0.8% of the observed cells; (c) all five categories are for cells with effective expansion factors (= index of dispersion) in excess of 100, suggesting they are predominantly based on household interview data.

5.9.22. Our conclusions from this analysis are as follows:
Differences between modelled and observed values are biggest in absolute terms for trips less than 25 km in length, but are small in

relative terms (10 per cent or less) (Table 5.9(1), The largest relative discrepancy (about 20 per cent) occurs with trips in the range 25 - 100 km, but for the trips between districts other than London the absolute difference of -0.04 is small enough to be negligible, and for trips within such districts, whilst the absolute difference is large (-419) (Table 5.9(1), it seems to be largely accounted for by sampling variability in the data (Table 5.9(3), Categories containing cells with expansion factors greater than 100 have average residuals which are not significantly different from zero (Table 5.9(3),and these include those five sub-categories with the largest numbers of trips (Table 5-9(6), Categories containing cells with expansion factors less than 10 have average residuals which appear to be significantly different from zero, and are always negative. (Table 5.9(2), The negative bias is to be expected for those categories which contain only cells with zero observations but for the remaining categories, this is more surprising (Section 5.9.14). One would have expected the negative bias associated with cells of zero observations to be largely compensated by a positive bias for other cells within the same category. Instead, most of the categories with expansion factors greater than 100 have a positive bias, accounting for most of the compensation. There then remains the suspicion first noted in WN 23 , that the model is behaving differently in the two extreme ranges of expansion factor (for extra-London and intra-rest categories, other than trips <25 km out of London), and since expansion factors <10 will almost invariably imply that they are based on roadside interview only data, and expansion factors >100 suggest that they are based on a mixture of roadside and household interview data, this may be indicative of discrepant data sets, which the model is interpolating between. To be certain of such a conclusion would however require more detailed analysis of the modelled and observed data sets and their error distributions.

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8. APPENDIX: SOME DESCRIPTIVE STATISTICS

The Appendix includes some pertinent descriptive statistics which were summarised in the course of various stages of the project.

8.1 ZONE AND CELL STATISTICS

RHTM regional zones	3613
RHTM local authority districts	447
NM districts	642
HI areas	21
Towns	218
NM districts within HI areas	104
NM districts within towns	381
Regional zones within towns	
Towns within HI areas	

Regional zones within districts: min = 1; max = 22; mean = 5.

Simple 642 x 642 matrix has	412164	cells
of which	81.9%	were observed.

Composite 642 x 642 matrix has	399515	simple cells (for remote district pairs)
	12649	composite cells (for near-by district pairs)
and the composite cells contain	583333	sub-cells
gives a total of	982848	cells or sub-cells in all
and an average of	46	sub-cells per composite cell.

8.2 OBSERVED INTRADISTRICT TRIPS

94 of the 642 districts had observations of the total number of intradistricts trips for home-based work. Since there are 104 districts within the Household Interview Areas, this implies that there are 10 observed districts, for which no intradistrict movement was observed.

The cost functions involved in the 94 districts are 1, 2, 3, 4 and one instance of 5, with about two-thirds in function 4. The cost bands in the functions range up to 65; they are not concentrated in the very lowest bands in each function.

Their percentage intrazonals vary from 4.5 to 89.3, and do not appear to be much different from the unobserved zones.

8.3 MODELLED INTRADISTRICT TRIPS

Unless otherwise stated, the following are for the home-based work purpose only. The others seem to be quite similar.

8.3.1 Intradistricts, as fractions of row sums

The modelled intradistrict trips as a fraction of the (synthesised) row sum are very variable - the minimum is zero per cent and the maximum just over 100 per cent. (The minimum is for district 554 in Area B (Exeter), the maximum for district 599, in Area BJ (Scotland).)

The median intradistrict fraction, estimated from a sample of 22, comes out to just 50 per cent. Summation over districts give the following percentages of trips as intradistrict:

HBW 57, HBO 68, EB 50, NHB 72.

8.3.2 Percentage of sythesised trips in observed cells

These percentages were:

HBW 32.2, HBO 28.3, EB 36.2, NHB 24.7

This gives an indication of the extent to which the model is being relied upon to extrapolate to unobserved cells.

8.3.3 Very large intradistrict movements

For HBW, 42 of the 642 districts have intradistrict fractions over 90 percent. The 17 districts exceeding 98 percent are listed in Table 8.3. (None were observed.)

TABLE 2.3(1) QUASI-AVERAGE AND UNWEIGHTED AVERAGE COSTS NEAR THE COST THRESHOLD

QUASI-AVERAGE COST	UNWEIGHTED AVERAGE COST	DIFFERENCE
119	132	- 13
133	138	- 5
144	153	- 9
153	160	- 7
154	161	- 7
157	166	- 9
166	173	- 7
169	175	- 6
166	176	- 10
174	181	- 7
174	184	- 10
177	184	- 7
185	194	- 9

TABLE 2.3 (2)

PROPORTIONAL CHANGES IN COST FUNCTION VALUE FOR A ONE-BAND SHIFT IN COST NEAR THE 100 PENCE THRESHOLD

HBW, Iteration 20

Function	Numbers synthesised in 100 - 105 range	Percentage change	Maximum observed cost band
1	323	-34	131 - 135
2	57	-32	121 - 125
3	0	- 2	89 - 90
4	8841	-31	176 - 180
5	14262	-22	1281 - 1300
6	12716	-18	1281 - 1300
7	6543	-23	1281 - 1300
8	1917	-17	1281 - 1300
9	362	-25	1281 - 1300

Values quoted are the percentage change in smoothed cost function from the 101 - 105 band to the 106 - 110 band. (Below 100 pence, bands are in 1 pence units and the comparison is therefore more difficult.)

TABLE 2.4 (1) THE DEFINITION OF MULTIPLE DETERRENCE FUNCTIONS

Origin district type	District type number	No. of districts	% of generations (NHB)	Associated cost function		
				intra town	other	
					E	S & W
Rural E	1	261	29	1	5	5
Urban E	2 & 3	173 + 1	29 + 0.3	2	6	6
Metropolitan E	4	73	18	3	7	7
London	5	51	11	4	8	8
Rural S & W	6	61	7	1	9	5
Urban S & W	7	21	5	2	9	6
Metropolitan S & W	8	1	0.7	3	9	7

Notes : E denotes England, S & W Scotland and Wales.

The cost function for a cell is 'intra' if the two zones are within the same one of 203 towns, otherwise 'other'.

TABLE 2.4(2)

THE DISTRIBUTION OF HBW TRIPS AND TRAVEL AMONGST THE NINE FUNCTION AREAS

observed cells only

Cost function type	% trips	% travel	average cost (pence)
4	36	27	35
5	16	24	72
6	14	21	70
7	12	14	57
8	2	4	88
3	7	3	24
2	18	3	18
1	5	3	24
9	.2	.8	155
TOTAL	5 206 217	245 213 000	47.1

Notes: See Table 2.4(1) for definition of function type.

'Travel' means trips times cost (in pence). The above are synthesised values, taken from HBW iteration 20. The values are for observed cells only. For each function the synthesised trip values are about 1 per cent more, and the average costs about 1 per cent less, than the observed values (except for function 1 trips) - this is the extra adjustment factor.

TABLE 3.3

FLOWS ON INTERVIEWED, COUNTED AND UNCOUNTED ROADS BY CORDON

Cordon boundary		Flows across boundary				% uninterviewed flows	Roads		
No.	Name	RI	MCC	Uncounted roads	Total		With interview	With MCC only	Without counts
1	Cornwall	12582		1830	14412	12.70	5	0	30
2	Somerset	23054	2319	2318	27691	16.75	9	6	38
3	Plymouth	31762		4272	36034	11.86	5	0	16
4	Exeter	52102	2950	3870	58922	11.57	18	6	43
5	Yeovil	35636	5117		40753	12.56	24	15	6
6	Bournemouth	76141		1869	78010	2.40	13	0	7
7	Bristol	108978	21542	1869	132389	17.68	22	10	7
8	Gloucester	48888		2679	51567	5.20	11	0	10
9	Swindon	37697		2403	40100	6.00	8	0	9
10	Southampton	89864	1325	270	91459	1.74	24	4	3
11	Portsmouth	73416	7404	1335	82155	10.64	18	5	5
12	Brighton	57107	507	801	58415	2.24	8	1	3
13	Lewes	83754	4981	915	89650	6.58	25	9	15
14	South Kent	64412	4214	1890	70516	3.66	26	14	21
15	Reading	73799	5003	1335	80137	7.91	16	5	5
16	London	697372	25971	7232	730575	4.54	123	34	33
17	Thurrock	62309	2703		65012	4.16	13	2	1
18	Southend	66901	737		67638	1.09	13	1	1
19	Oxford	74268	11565		85833	13.47	12	9	2
20	West Oxford	81709	5035		86744	5.80	32	25	30
21	Luton	67165	1533	1335	70033	4.10	17	3	5
22	Cambridge	42306	502	1530	44338	4.58	16	3	17
23	Ipswich	48490	3603		52093	6.92	13	5	2
24	East Anglia	85105	5964	1350	92419	7.91	39	29	15
25	Norwich	61352	5427	1602	68381	10.28	18	13	6
26	Wales	61327	1688	3102	66117	7.24	34	7	47
27	Hereford	43247	2328	3600	49175	12.05	28	6	40
28	Northampton	50235	7371		57606	12.80	12	6	1
29	Fens	40122	7250	1464	48836	17.84	30	35	24
30	Wolverhampton	93890	19650	1045	114585	18.06	17	17	5
31	Birmingham	259280	45800	4499	309579	16.25	38	20	11
32	Coventry	70223	20043	2136	92402	24.00	16	7	8
33	Leicester	83551	20319	990	104860	20.32	18	16	11
34	Stoke	72624	14701	2403	89728	19.06	21	17	9
35	Derby	72038	6260	1068	79366	9.23	16	10	4
36	Nottingham	97840	10574	450	108864	10.13	15	5	5
37	Wirral	71179	1582	2454	75215	5.37	7	1	6
38	Chester	51546	2952		54498	5.42	9	2	0
39	Chesterfield	62962	6322	1080	70364	10.52	27	11	12
40	Lincoln	30611	10553	1068	42232	27.52	11	8	4
41	Liverpool	120026	9772		129798	7.53	15	4	0
42	Manchester	281027	23626		304653	7.76	39	12	0
43	Sheffield	93364	9130	2863	105357	11.38	20	11	7
44	Grimsby	15523	3389		18912	17.92	4	4	0
45	Preston	69099	12444	1869	83412	17.16	10	7	7
46	Blackburn	52317	3347		55664	6.01	12	4	2
47	Rossendale	57545	2572	801	60918	5.53	17	5	3
48	Burnley	23576	4181	2136	29893	21.13	9	7	8
49	Bradford	88614	10539	2045	101198	12.44	14	12	5
50	Leeds	132646	21137		153783	13.74	17	15	2
51	Scunthorpe	19365	4372	630	24367	20.53	10	10	7
52	Hull	35796	5324		41120	12.95	8	8	1
53	York	28449	4432	1350	34231	16.89	8	4	15
54	Allerdale	8784	1223	976	10983	20.02	5	2	16
55	Carlisle	21638	3487		25125	13.88	8	6	2
56	North York. Moors	33885	3405	2970	40260	15.83	16	12	36
57	Teesside	65741	4521	1260	71522	8.08	17	3	14
58	Sunderland	44959	4340		49299	8.80	11	5	1
59	Newcastle	110268	26024	1227	137519	19.82	24	16	3
60	Wansbeck	31009	5973	1636	38618	19.70	12	5	5
61	Scottish	16624		976	17600	5.55	9	0	18
62	Felixstowe	7925		366	8291	4.41	2	0	6
63	Harwich	5071			5071		2	0	1
	Total	4680095	459033	87169	5226297		1116	519	676

TABLE 3.6. CORDON-CROSSINGS COMPARISON FOR THE USED AND INTENDED DATA SETS

Assumed coefficient of variation of expansion factors	HBW	Value of modified U^2 test statistic HBO	EB	Total
HI and RI data sets as used				
2.5%	.081	.460	.108	.378
5.0%	.015	.312	.053	.240
7.5%	.046	.212	.064	.172
10.0%	.133	.188	.127	.185
HI and RI data sets as intended				
2.5%	.138	.436	.082	.260
5.0%	.065	.240	.026	.113
7.5%	.088	.121	.042	.056
10.0%	.166	.096	.108	.086

Underlined figures exceed the critical value of the modified U^2 statistic at the 5% level of significance.

The critical value of the modified U^2 statistic are as follows:

Level of significance	15%	10%	5%	2½%	1%
Critical U^2 value	.131	.152	.187	.221	.267

TABLE 3.9

MEAN TRIP RATES, OBSERVED AND SYNTHESISED

	MEAN TRIPS PER ZONE		% DIFFERENCE TO OBSERVED	% DIFFERENCE TO OBSERVED AFTER ADJUSTMENT
	FROM OBSERVED O-D MATRIX	SYNTHESISED TRIP-ENDS		
HBW: Gen	6461	6952	+ 7.6	+ 7.6
Att	7021	7531	+ 7.3	+ 7.3
HBO: Gen	7638	8118	+ 6.7	+ 6.3
Att	7741	8402	+ 8.5	+ 8.5
NHB: Gen/Att	2244	2941	+31.1	+ 7.1
EB: Gen/Att	1738	2078	+19.6	+5.2

TABLE 4.1 SIZE OF THE TRIP ADJUSTMENT FACTOR

Although not explicitly obtained in the NM calibration, the value for λ (defined in Section 4.1) is estimated from:

$$\lambda \simeq \frac{\text{observed trips in observed cells}}{\text{modelled trips in observed cells}}$$

As the value varies from iteration to iteration, the average over the last five iterations gave the following estimates of λ , for the composite matrix.

Purpose	λ
HBW	0.9895
HBO	0.9955
EB	0.9740
NHB	0.9808

The proportions of modelled trips that occur in the observed cells are given in Section 6.3.2.

TABLE 5.4 (1) ROW AND COLUMN SUMS OF OBSERVED DATA AND THEIR ACCURACIES

District	Row Sum	% coeff. varn.	Column Sum	% coeff. varn.
4	3668	46.7	5154	40.6
28	435	135.6	885	98.0
52	20038	20.0	17876	21.8
76	1463	74.0	966	93.8
100	4769	41.0	4367	44.1
124	23354	18.5	21351	20.0
148	2287	59.1	4754	42.3
172	12426	25.4	6826	35.3
196	23352	18.5	22907	19.3
220	5757	37.3	2237	61.7
244	49786	12.7	40211	14.5
268	49325	12.7	42115	14.2
292 *	77212	6.2	51001	6.9
316 *	44068	10.9	41165	10.2
340	32841	15.6	27725	17.5
364	39848	14.2	30208	16.8
388	27835	16.9	34485	15.7
412	30063	16.3	21214	20.0
436	10285	27.9	6901	35.1
460 *	20395	5.8	24635	5.2
484	8256	31.1	8250	32.1
508	3956	45.0	3941	46.4
532	31223	16.0	35264	15.5
556 *	154	55.8	92	63.1
580	12651	25.1	8665	31.3
604	47571	13.0	55133	12.4
628	24897	17.9	18353	21.5
138 *	94569	2.2	135503	1.7
482 *	26	554.9	0	

NOTE: This is a selection of some of the row and column sums for Home Based Work, together with, at the end, the most and least accurate row sums.

* denotes that this zone was in a home-interview area.

TABLE 5.4(2) COST BAND SUMS AND ACCURACIES

observed cells only

Purpose	Function	Total trips	%	Average id	Average cv
HBW	1	267 160	5.2	30.2	.158
	2	424 276	8.2	44.1	.239
	3	335 979	6.5	77.7	.197
	4	1 876 576	36.4	372.8	.188
	5	800 807	15.5	10.7	.261
	6	735 319	14.3	9.0	.270
	7	598 302	11.6	9.9	.308
	8	99 718	1.9	9.5	.352
	9	12 436	.2	7.3	.455
	All	5 150 577	100.0	-	.231
HBO	1	401 702	7.4	30.8	.166
	2	524 885	9.6	44.6	.272
	3	376 200	6.9	79.1	.250
	4	2 164 700	39.7	355.0	.251
	5	730 173	13.4	9.4	.112
	6	660 706	12.1	8.4	.124
	7	460 288	8.4	9.4	.193
	8	119 797	2.2	8.6	.210
	9	18 168	.3	6.9	.268
	All	5456 612	100.0	-	.201
EB	1	72 273	4.3	30.7	.273
	2	87 557	5.2	41.2	.356
	3	62 108	3.7	82.8	.406
	4	412 041	24.6	358.4	.394
	5	295 410	17.7	8.8	.172
	6	366 930	21.9	6.7	.153
	7	286 032	17.1	7.1	.197
	8	78 383	4.7	7.3	.226
	9	12 085	.7	5.4	.272
	All	1672 815	100.0	-	.252

Cont/d.,

TABLE 5.4(2) (Cont/d.)

Purpose	Function	Total trips	%	Average id	Average cv
NHB.	1	102 167	6.8	31.5	.296
	2	171 466	11.4	39.2	.345
	3	104 205	6.9	79.1	.348
	4	654 754	43.5	352.1	.353
	5	138 396	9.2	5.2	.228
	6	164 419	10.9	4.1	.225
	7	132 061	8.8	4.7	.303
	8	33 134	2.2	5.1	.309
	9	4 093	.3	3.3	.368
	All	1 504 686	100.0	–	.317
Overall		13 784 690			.231

NOTES: The values are for cost band sums over observed cells only. id denotes index of dispersion (variance to mean ratio), and may be regarded as an average expansion factor. Clearly function 4 is mainly London HI data, functions 1, 2 and 3 other HI data, 5,6,7,8 and 9 RI data. The average is a simple unweighted average across all observed cost bands for the function concerned.

cv denotes coefficient of variation (standard deviation to mean ratio). The simple average is included only to give a rough impression of the accuracy. Unlike the index of dispersion, the coefficient of variation varies markedly with cost band, becoming much larger in the highest cost bands.

TABLE 5.5(1)

APPROXIMATE STANDARD DEVIATIONS IN SYNTHETIC TRIP END ESTIMATES AT DISTRICT AND ZONAL LEVEL

PURPOSE	DIRECTION	APPROXIMATE STANDARD DEVIATION OF SYNTHESISED TRIP-ENDS (Q or R)	
		DISTRICT LEVEL	ZONAL LEVEL
HBW	Gen	$28\sqrt{Q}$	$21\sqrt{Q}$
	Att	$29\sqrt{R}$	$25\sqrt{R}$
HBO	Gen	$38\sqrt{Q}$	$24\sqrt{Q}$
	Att	$37\sqrt{R}$	$28\sqrt{R}$
EB	O&D	$28\sqrt{Q}$	$17\sqrt{Q}$
NHB	O&D	$30\sqrt{Q}$	$21\sqrt{Q}$

Notes: The bias between observed/synthetic trip-end estimates has been taken out.

TABLE 5.5(2)

APPROXIMATE 95 PERCENT CONFIDENCE INTERVALS ABOUT THE MEAN NUMBERS OF TRIP-ENDS IN A REGIONAL ZONE

PURPOSE	DIRECTION	MEAN SYNTHESISED TRIP-ENDS	95% CONF. INTERVAL FROM TABLE 5.5(1)	95% CONF. INTERVAL FROM RHTM STATUS REP. (JUNE 1979)
HBW	Gen	6952	3485 - 10419	2110 - 21616
	Att	7531	3195 - 11867	1917 - 27687
HBO	Gen	8118	3792 - 12444	2330 - 28897
	Att	8402	3320 - 13434	2799 - 23536
EB	O&D	2078	530 - 3626	546 - 5943
NHB	O&D	2941	686 - 5196	655 - 8489

Note: The mean number of synthesised trip-ends about which the confidence interval was calculated was different in the RHTM Status Report (Table 4), since the trip-end models have been revised since. The RHTM confidence intervals relate to the logarithmic regression.

TABLE 5.7(1)

ILLUSTRATIVE VALUES FOR NON-ZERO OBSERVED AND MODELLED VALUES AND THEIR ACCURACIES

Trips		Percent coefficient of variation		Standardised difference.
Observed (O)	Modelled (E)	Observed	Modelled	Z
Purpose 1 (HBW)				
49.80	97.00	39	15	1.63
6.17	11.45	77	30	0.58
60.50	21.83	44	22	-2.19
3.24	0.87	225	21	-1.19
Purpose 2 (HBO)				
62.00	4.23	233	20	-5.55
498.80	381.10	14	27	-0.94
Purpose 3 (EB)				
4.59	2.10	152	31	-0.75
5.82	0.09	843	37	-7.09
2.02	4.38	100	30	-0.53
4.71	0.42	348	27	-2.83
9.00	1.26	280	81	-1.74
7.03	2.43	135	50	-1.19
2.87	0.29	389	26	-2.23
2.34	3.16	118	31	0.21
Purpose 4 (NHB)				
2.99	0.23	437	19	-2.71

Taken from random sample of 170 observed cells from all purposes interleaved, of which 155 had zero observed value (91 per cent). Of the 15 non-zero observed values, 10 (67 per cent of the non-zeros, 6 per cent of observed cells) were based on a single count. The standardised difference

$$Z = (E - O)/\sqrt{\{\text{var}(E) + \text{var}(O) - 2\,\text{cov}(E,O)\}}$$

TABLE 5.7(2)

MODELLED ESTIMATES AND THEIR ERRORS FOR OBSERVED CELLS WITH ZERO OBSERVATION (HBW)

Modelled	% cv (Modelled)	Standardised difference Z	Modelled	% cv (Modelled)	Standardised difference Z
0.12	79		0.02	40	
0.01	40		0.03	30	
0.01	29		0.04	29	
1.73	29		47.76	21	
0.96	28		0.37	27	
0.07	31		0.01	32	
0.03	17		0.02	55	
0.07	22		0.09	26	
0.01	49		0.01	28	
0.01	37		0.02	30	
0.03	30		0.01	51	
0.01	41		0.02	27	
0.12	28		0.13	27	
0.50	36		0.40	136	
0.07	24		0.04	24	
0.01	39		0.06	21	
0.01	73		0.05	30	
1.47	28		0.08	32	

Taken from random sample of 170 observed cells from all purposes interleaved, of which 155 had zero observed value.

TABLE 5.8

DISTRICTS WITH THE WORST-FITTING INTRA-DISTRICT ESTIMATES (HBW)

District	Area, name	Intradistrict trips		z
		Observed	Modelled	
5	BI Wansbeck/Blyth	2320	1465	-3.06
8	BI Wansbeck/Blyth	8330	6280	-3.47
111	AW Burnley	16300	12510	-3.20
112	AV Rossendale	10070	7800	-3.57
251	Q London	906	144	-3.86
283	Q London	21720	13600	-2.55
295	Q London	11770	6800	-2.06
302	P Reading	923	207	-3.41
328	Q London	13210	5570	-3.23
363	N Lewes	15940	13980	-2.10
460	AF Birmingham	10400	4623	-5.24
549	D Exeter	18160	13220	-3.82
558	D Exeter	24790	20630	-2.32

Note 'z' is the standardised difference, defined as:

$$\frac{\text{modelled} - \text{observed}}{\sqrt{\text{var(modelled)} + \text{var(observed)}}}$$

TABLE 5.9(1)

THE MEAN RESIDUAL AND RELATIVE MEAN RESIDUAL CATEGORISED BY TRIP-LENGTH AND AREA TYPE

for home based work trips

AREA TYPE	Trip length band (km)	Number of Cells M	Mean observed trips per cell ΣO/M	Mean Modelled trips per cell ΣE/M	Mean residual (ΣO-ΣE)/M	Relative Mean Residual (ΣO-ΣE)/ΣE
INTRA	<25	891	1979	1960	19	0.01
LONDON	25-100	1609	71	82	11	0.1
	all	2500	751	752	-1.3	-0.002
INTRA	<25	152	6664	6658	6.6	0.001
REST	25-100	29	1884	2304	-419	-0.2
	all	181	5900	5961	61	0.01
EXTRA	<25	139	322	287	35	0.1
LONDON	25-100	4732	10	10	-0.6	-0.06
	>100	16922	0.1	0.1	-0.004	-0.04
	all	21793	4.2	4.2	-0.09	-0.02
INTER	<25	1482	797	777	20	0.02
REST	25-100	23878	38	41	-3.1	-0.04
	>100	148904	0.2	0.2	-0.04	-0.2
	all	174264	12.1	12.4	-0.3	-0.02
ALL	<25	2664	1487	1482	4.7	0.003
	25-100	30248	37	41	-0.02	-0.0004
	>100	165826	0.2	0.2	-0.02	-0.1
	all	198738	25.9	26.2	-0.3	-0.01

TABLE 5.9(2)

THE MEAN RESIDUAL CATEGORISED BY TRIP LENGTH, AREA TYPE, AND EXPANSION FACTOR.

for home-based work trips

AREA TYPE	EFFECTIVE EXPANSION FACTOR	TRIP LENGTH BAND <25 km	25-100 km	>100 km	ALL
INTRA LONDON	<10	-364*	-62*	-	-125*
	10-100	-	-	-	-
	>100	+319	+573	-	+372
INTRA REST	<10	-35*	-99*	-	-53*
	10-100	-	-	-	-
	>100	+20	-646	-	-65
EXTRA LONDON	<10	-37*	-2.5	-0.1	-0.5
	10-100	-6	-0.9	+6	-2.7
	>100	+46	+7.7	+13	+13
INTER REST	<10	-22	-3.7	-0.1	-0.5
	10-100	-28	-5.1	+5.6	-1.4
	>100	+27	-0.3	+16	5.7

* All cells in these categories have zero observed trips.

TABLE 5.9(3)

ESTIMATES OF STANDARDISED MEAN RESIDUALS IN EACH CATEGORY OF TRIP LENGTH, AREA TYPE AND EXPANSION FACTOR.

for home-based work trips

AREA TYPE	EFFECTIVE EXPANSION FACTOR	TRIP LENGTH BAND <25 km	25-100 km	>100 km	ALL
INTRA LONDON	<10	*/-123	*/-98	-/-	*/-157
	10-100	-/-	-/-	-/-	-/-
	>100	6 / 6	11/19	-/-	9 / 9
INTRA REST	<10	*/-11	*/-11	-/-	*/-15
	10-100	-/-	*/-	-/-	-/-
	>100	0.1/0.1	-2.4/-2.2	-/-	0.4/-0.4
EXTRA REST	<10	*/-7.6	-210/-30	-39/-13	-175/-33
	10-100	-0.9/-0.7	-1.2/-1.1	5.4/17	1.7/1.8
	>100	1.3/1.4	1.6/1.7	1.1/5.5	2.1/2.3
INTER REST	<10	-221/-19	-398/-76	-115/-42	-359/-89
	10-100	-11/-6	-14/-11	14/37	-5.3/-4.9
	>100	1.6/1.7	-0.1/-0.1	4.7/14.8	1.2/1.2

Note: The two values supplied, a/b, correspond to two different estimates of the mean number of observed trips, with which to multiply the index of dispersion by to give the variance. (a) corresponds to using the observed value, (b) to using the modelled value.

TABLE 5.9(4)

NUMBERS OF OBSERVED CELLS IN EACH CATEGORY OF TRIP LENGTH, AREA TYPE AND EXPANSION FACTOR.

for home based work trips

AREA TYPE	EFFECTIVE EXPANSION FACTOR	TRIP LENGTH BAND <25 km	25-100 km	>100 km	ALL
INTRA LONDON	<10	392	1481	0	1873
	10-100	0	0	0	0
	>100	499	128		627
INTRA REST	<10	30	12	0	42
	10-100	0	0	0	0
	>100	122	17	0	139
EXTRA LONDON	<10	15	3410	16652	20077
	10-100	7	509	232	748
	>100	117	813	38	968
INTER REST	<10	164	15462	146602	162228
	10-150	63	2792	1715	4570
	>100	1255	5624	587	7466
ALL	ALL	2664	30248	165826	198738
'DOUBLE ZEROS'		Distribution not known			138824
TOTAL OBSERVED CELLS					337562

TABLE 5.9(5)

RESIDUAL STATISTICS FOR THE 6 CATEGORIES WITH LARGEST NUMBERS OF CELLS home based work trips

Label	Category: Area	Category: Exp. Fac.	Category: Dist.	Category: Size	Number of Cells M	% obs. cells.	% cum. cells	Average obs. val $\Sigma O/M$	Average modelled val. $\Sigma E/M$	Mean Residual $\Sigma(O-E)/M$	Relative Mean Residual $\Sigma(O-E)/\Sigma E$	Approx. Standardised resid. $\frac{\Sigma(O-E)}{\sqrt{(I\Sigma E)}}$
a	Inter R	<10	>100	< 1	142146	42.1	42.1	.02	.09	-107	-.80	-29.0
-	'Double Zeroes'				138824	41.1	83.2	.00	<.01	-	-	-
b	Extra L	<10	>100	< 1	16364	4.8	88.1	.01	.10	-.09	-.87	-11.3
c	Inter R	<10	25-100	< 1	7801	2.3	90.4	.05	.32	-.27	-.85	-13.8
d	Inter R	<10	25-100	1-5	4805	1.4	91.8	.17	2.41	-2.24	-.93	-32.4
e	Inter R	<10	>100	1-5	4172	1.2	93.0	.11	1.90	-1.79	-.94	-27.2
f	Inter R	>100	25-100	10-100	2800	0.8	93.9	49.4	40.6	+8.80	+.22	+3.8
	Sum				316912		93.9	.46	.46			
	Total observed cells				337562		100					

* L = London, R = Rest.

** See text for explanation.

I = Index of dispersion in given category of expansion factor.

TABLE 5.9(6)

RESIDUAL STATISTICS FOR THE 5 CATEGORIES WITH LARGEST NUMBERS OF TRIPS   home based work trips

Label	Area	Category Exp. Fac.	Dist.	Size	Number of Cells M	Numbers of Trips Observed ΣO	Modelled ΣE	Average Obs. Val. $\Sigma O/M$	Mod. Val. $\Sigma E/M$	Mean residual $\Sigma(O-E)/\Sigma E$	Relative Mean Residual $\Sigma(O-E)/\Sigma E$	Approx. Standardised Residual $\frac{\Sigma(O-E)}{\sqrt{(I\Sigma E)}}$
A	Intra L	>100	<25	>500	412	1693000	1578000	4109	3831	278	.07	4.8
B	Inter R	>100	<25	>500	577	1047000	1014000	1814	1758	56	.03	1.7
C	Intra R	>100	<25	>500	94	1007000	1004000	10714	10683	31	.00	0.2
D	Inter R	>100	25-100	>500	375	460838	463886	1229	1237	-8	-.01	-0.2
E	Inter R	>100	25-100	100-500	354	257818	299803	190	221	-31	-.14	-4.0
			Sum		2812	4466000	4360000	1588	1550			

* L = London, R = REst

** See text for explanation. I = Index of dispersion for the given category of expansion factor.

TABLE 8.3

DISTRICTS WITH HIGH PROPORTIONS OF INTRA DISTRICT MOVEMENT.

District	Area, name	Synthesised row sum	Intradistrict fraction
599	BJ, Scotland	54 251	1.001
601	BJ, Scotland	53 772	1.000
590	AA, Wales	20 058	1.000
642	BJ, Scotland	13 803	.999
604	BJ, Scotland	47 571	.999
592	AA, Wales	16 653	.999
589	AA, Wales	19 982	.998
585	AA, Wales	8 356	.998
615	BJ, Scotland	18 110	.997
586	AA, Wales	14 181	.996
614	BJ, Scotland	21 554	.997
607	BJ, Scotland	36 718	.990
612	BJ, Scotland	121 716	.987
608	BJ, Scotland	22 268	.986
624	BJ, Scotland	133 274	.985
622	BJ, Scotland	21 237	.985
498	AA, Wales	87 152	.981

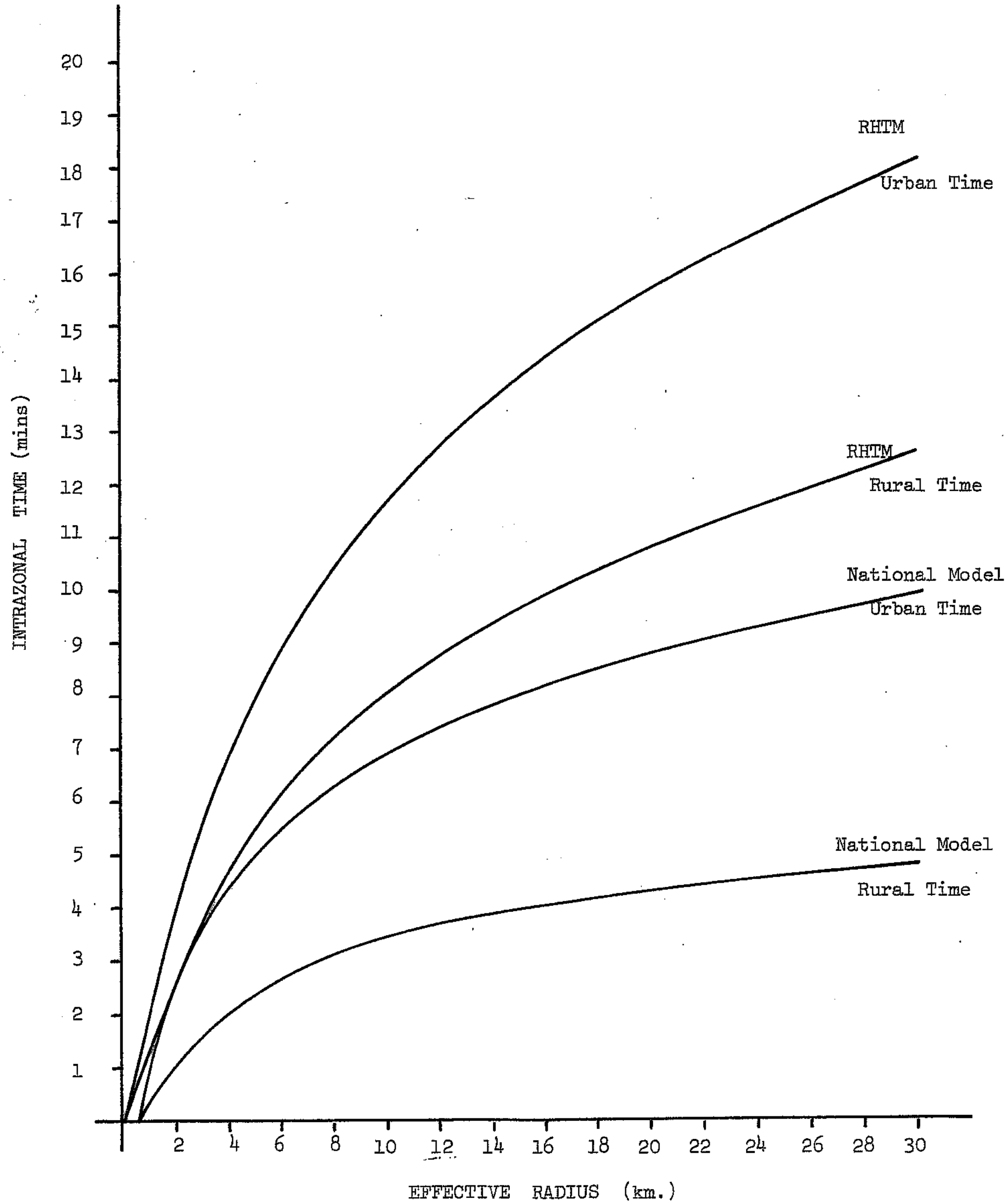

Fig. 3.2(1) Intrazonal time relationships used in the National Model, and the previous RHTM Relationships.

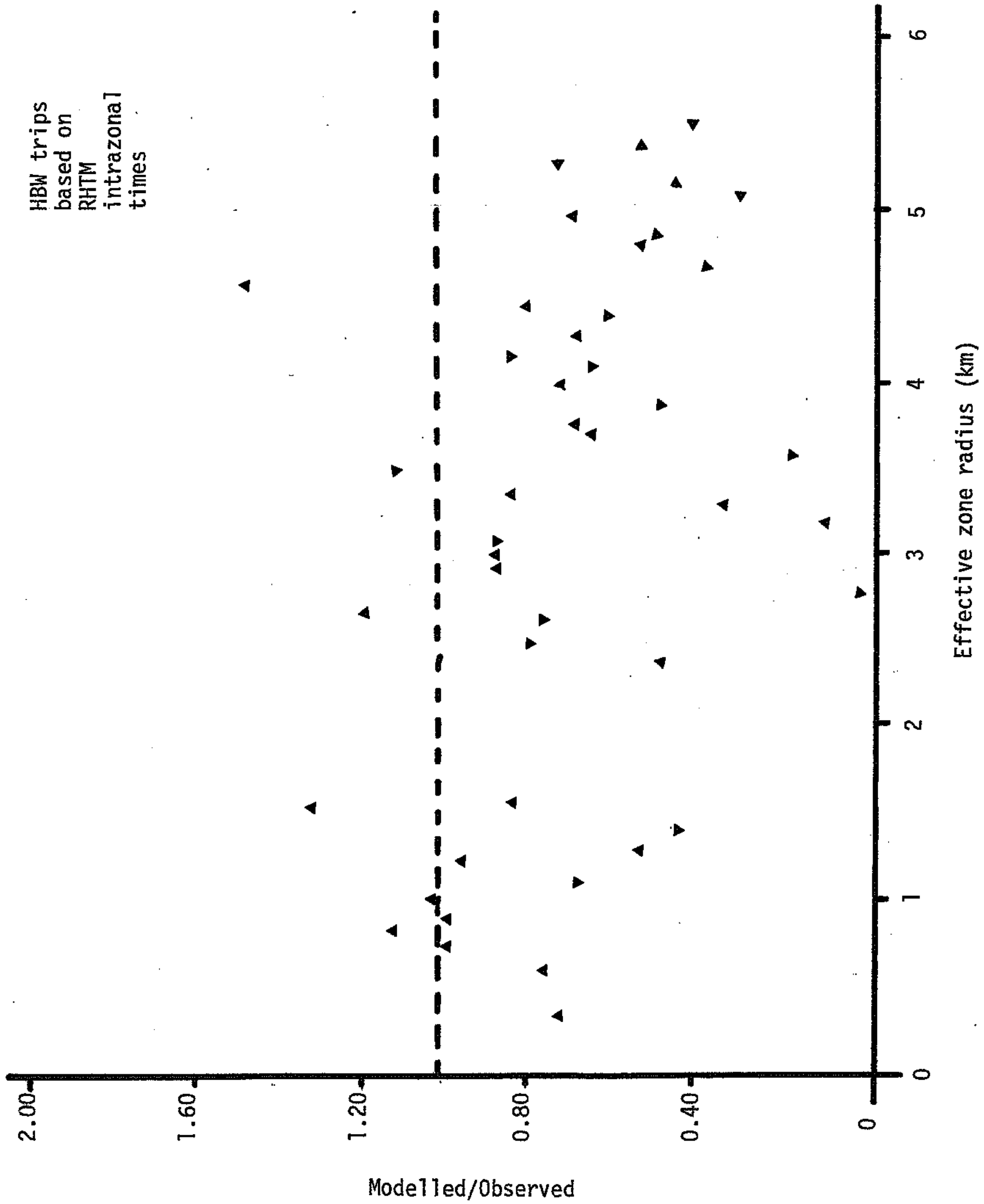

Figure 3.2(2) Modelled to observed number of intrazonal trips as a function of zone size, for rural zone types (prior to revision of intrazonal times).

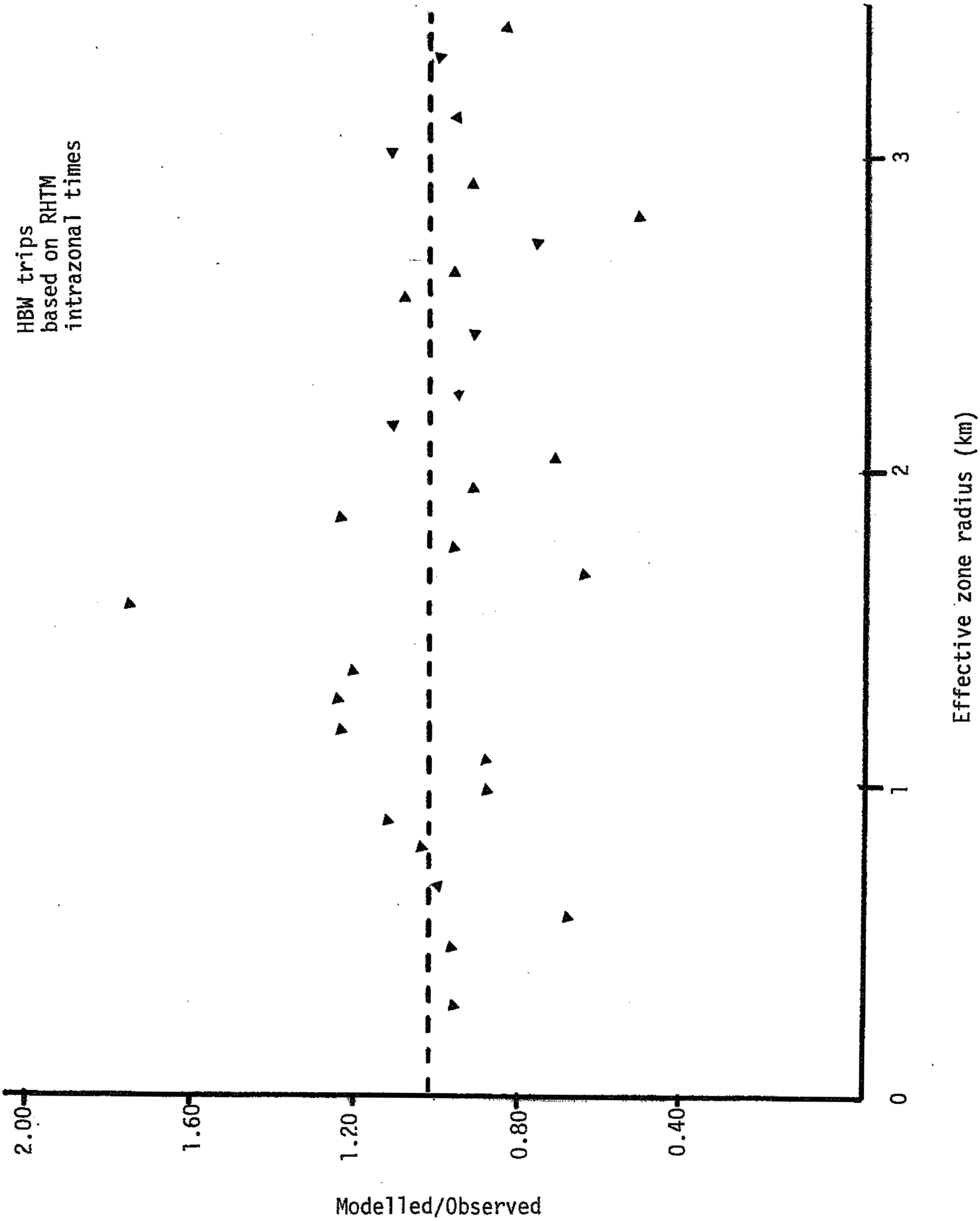

Figure 3.2(3) Modelled to observed numbers of intrazonal trips as a function of zone size, for urban zone types (prior to revision of intrazonal times).

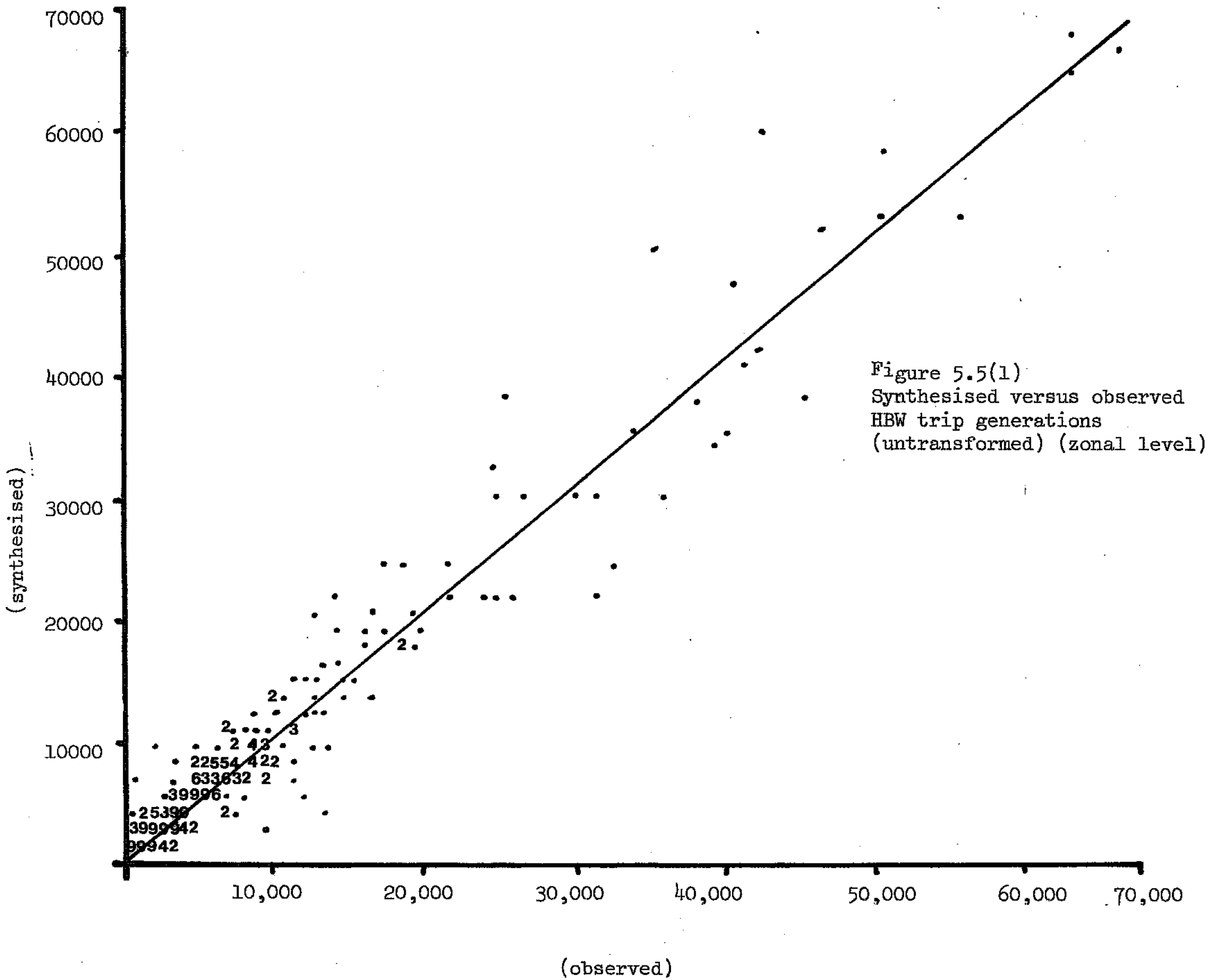

Figure 5.5(1)
Synthesised versus observed
HBW trip generations
(untransformed) (zonal level)

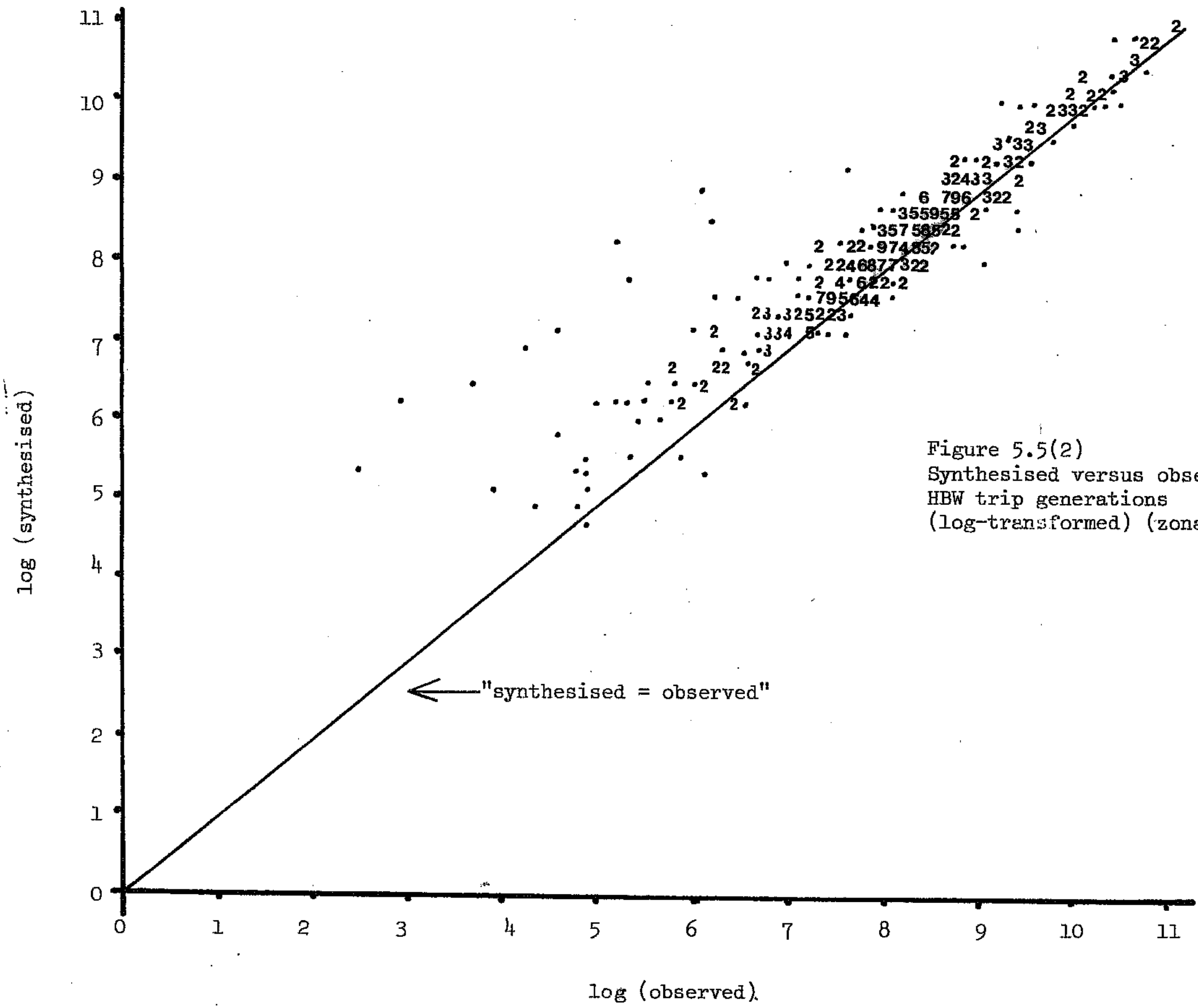

Figure 5.5(2)
Synthesised versus observed
HBW trip generations
(log-transformed) (zonal level)

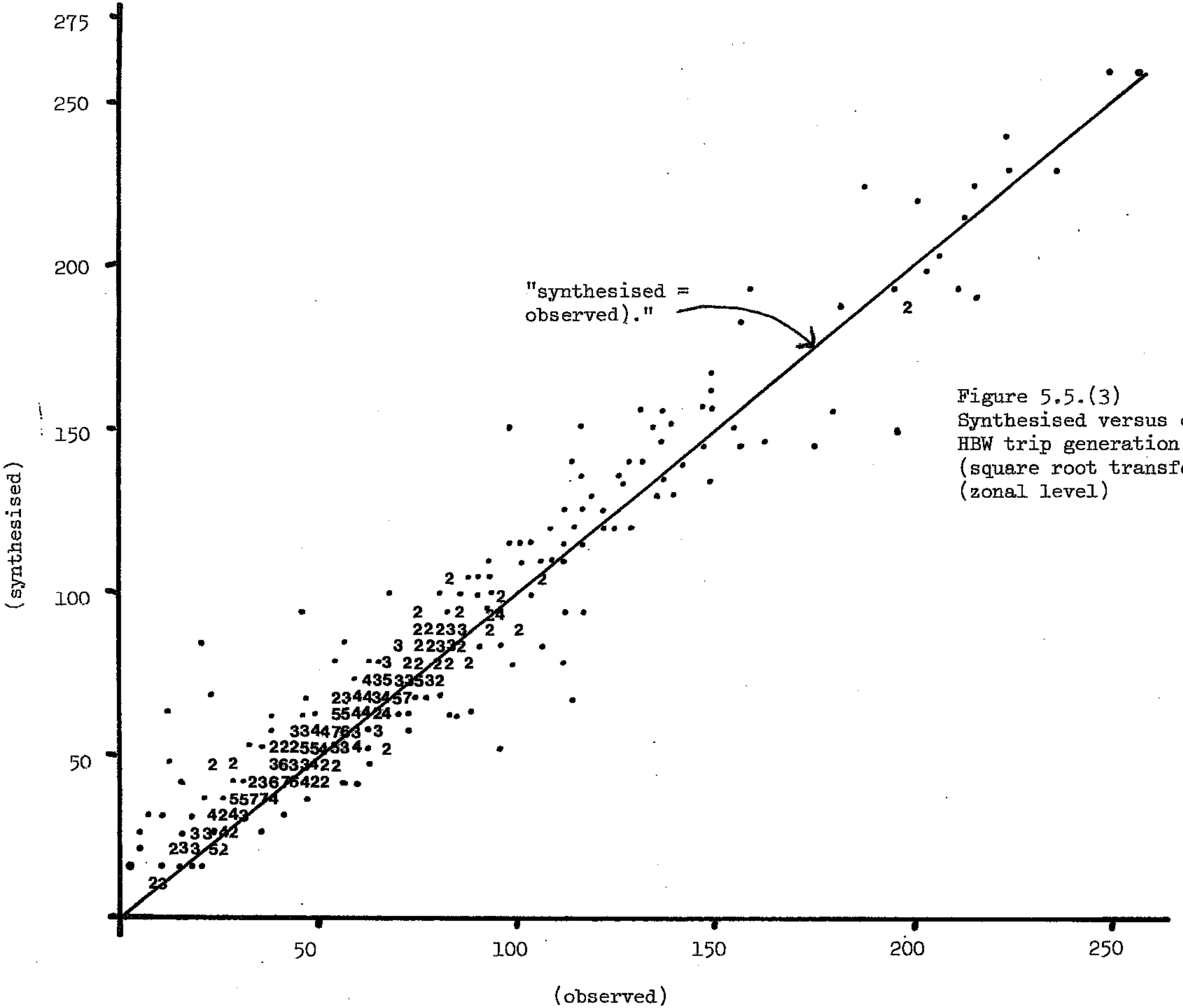

Figure 5.5.(3)
Synthesised versus observed
HBW trip generation
(square root transformed)
(zonal level)

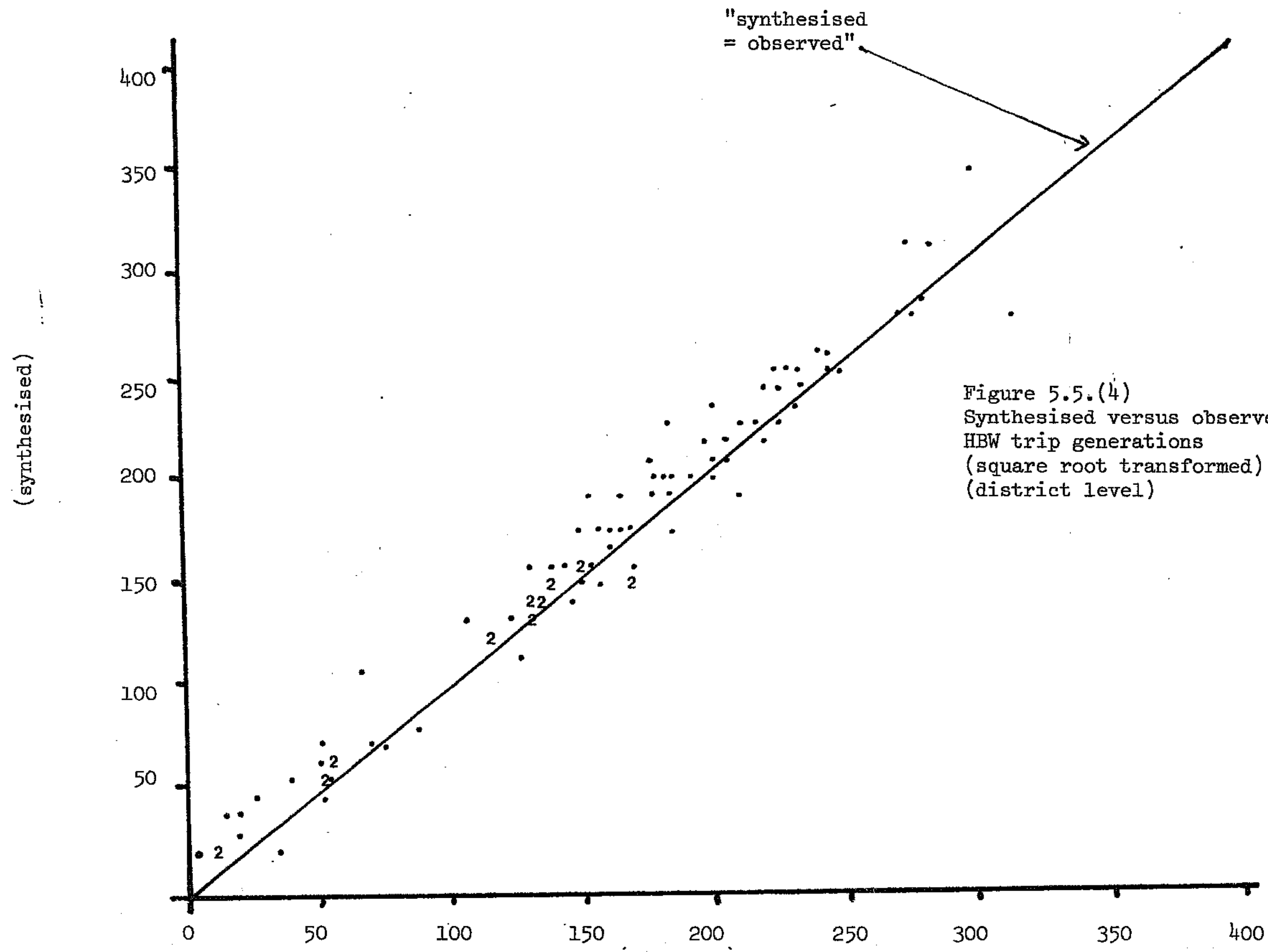

Figure 5.5.(4)
Synthesised versus observed
HBW trip generations
(square root transformed)
(district level)