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IFAC PapersOnLine 56-2 (2023) 4527–4532

Optimal Tracking & Evasive Algorithms For Fixed-Wing UAV & Target In 3-Dimensional Space

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Abstract: We present a three-dimensional optimal target tracking algorithm that enables an Unmanned Aerial Vehicle (UAV) to track a smart evasive ground-moving target. The UAV and target control algorithms are designed using two-step prediction schematics. For the UAV, the control strategy assumes a worst-case target evasion and derives a cost function to be continuously minimized. Conversely, the target evasion algorithm is developed by assuming a best-case UAV tracking scenario and establishing a cost function to persistently maximize their relative distance. In addition to acceleration and velocity constraints, the UAV and target control strategies are constrained by turn radius, turn rate, and bank angle limits. Simulations show that the UAV is able to pursue, maintain proximity and keep the target within its camera field of view despite frequent sharp evasive turns performed by the target.

Keywords: UAV, smart evasive target, optimal tracking control, 2D to 3D transformation.

1. INTRODUCTION

The Unmanned Aerial Vehicle (UAV) is increasingly being employed as an alternative to manned platforms for autonomous target tracking missions as it provides better manoeuvrability, reduced operational cost and less burden on human operators (Brown and Sun, 2019). To autonomously perform this role, a fixed-wing UAV must be capable of predicting and responding to evasive manoeuvres from agile targets. Several UAV control law designs for fixed-wing UAV tracking of manoeuvring targets exist. However, researchers commonly design non-smart manoeuvring targets to mimic evasive manoeuvres. To design a realistic UAV tracking engagement to continuously track a smart evasive target, the dynamic constraints of the UAV and target need to be taken into consideration. Additionally, the undulating nature of the ground terrain would require a 3-dimensional (3D) model that enables the UAV to adjust its altitude in response to target manoeuvre and terrain. This research addresses these design considerations.

Chen et al. (2019), implemented a 2-dimensional (2D) single UAV tracking with the target manoeuvre designed as a fixed velocity moving curve. Kim (2022) developed a 2D control strategy for fixed-wing UAV autonomous tracking of a randomly manoeuvring ground target while Yang et al. (2019) developed a reinforcement learning-based UAV tracking of an aerial evading target that uses a state-dependent statistical control policy. Although this strategy was implemented for 3D engagement, the control actions were heuristically determined. In Brown and Sun (2019), a control algorithm was designed to track a smart evading target that utilises a dipole-type vector field around the tracking UAV to execute evasive action. However, this was only implemented for 2D engagement dynamics.

Multiple UAV control for target tracking have also been designed and tested. For instance, Shi et al. (2021) designed a multi-UAV Lyapunov vector field cooperative UAV guidance to track an intelligent target that evades detection by mathematically maximizing the UAV estimation error. Quintero and Hespanha (2014) devolved an evasive target tracking algorithm using two dynamic control optimisation strategies for different target models of evasive and stochastic motion. However, the target control policies were essentially lookup tables of any combination of UAV and target engagement. Wolfe et al. (2022) also designed an Extended Kalman Filter (EKF) and T-Test selection model to track a target with random behaviour. Similarly, Kokolakis et al. (2020) developed a non-equilibrium game theoretic algorithm for tracking an active evasive target by two coordinated UAVs with the capability of estimating the level of target intelligence in order to deploy countermeasures. The pursuer-evader game was designed to minimise relative distance for the pursuing team of UAVs and maximize for the evading target. The UAVs in these researches were, however, assumed to fly at a constant speed and the pursuit-evasion game was implemented for only 2D scenarios.

Despite efforts in developing UAV control strategies to track evasive targets, limited attention has been paid to the implementation of smart evasive targets capable of initiating intelligent evasive manoeuvres against the tracking UAV. Furthermore, the dynamic nature of the pursuit-evasion requires that both platforms are designed with the capability to either accelerate or decelerate within design limits while allowing the UAV to adjust its altitude

10.1016/j.ifacol.2023.10.947

with changes in target states (Yao et al., 2015). This research aims to develop a 3D optimal control for a UAV tracking an evasive smart ground target, and accounting for the associated dynamic constraints. The contributions of this paper are as follows:

- The 2-step prediction fixed-wing UAV optimal control strategy by Kim (2022), is solved with the turn rate and bank angle constraints, enabling smoother target tracking while restricting excessive turns of the UAV.
- An evasive target control strategy is introduced by solving the maximization problem and providing realistic target movements.
- The 2D algorithm in (Kim, 2022) is extended to a 3D target tracking algorithm, taking into account the terrain changes.

The rest of the paper is outlined as follows. In section 2, the UAV target tracking problem is formulated using simplified dynamics and mathematical models of constraints considered in the development of optimal target tracking. In section 2.2, the solution for the UAV optimal target tracking problem and its cost function development is presented while Section, 3 discusses the target manoeuvre control design, incorporating vehicle constraints and cost function evaluation. A method for extending from 2D to 3D target tracking algorithm is presented in Section 4 along with simulation results and a discussion of sample engagement scenarios. Section 5 presents concluding remarks and future plans.

2. UAV 2D TARGET TRACKING

Consider a UAV with the task of tracking an evasive ground target in a 3D engagement space. The UAV is able to accelerate or decelerate within specified bounds and is constrained by a maximum turn radius. We assume that the UAV is able to identify the target in its camera field of view at all times.

2.1 Dynamics

The UAV and target dynamics are represented as shown in Fig. 1. The UAV positions are represented by x_a, y_a and z_a. and the dynamic equation of the UAV is represented by

$$\dot{x}_a = v_{ax},\ \dot{y}_a = v_{ay},\ \dot{z}_a = v_{az}, \tag{1a}$$

$$\phi_a = \tan^{-1}\left(\frac{v_{ay}}{v_{ax}}\right) \tag{1b}$$

$$\sigma_a = \tan^{-1}\left(\frac{v_{az}}{\sqrt{(v_{ax})^2 + (v_{ay})^2}}\right) \tag{1c}$$

$$v_{ax} = \mathbf{v}_a \cos\sigma_a \sin\phi_a \tag{1d}$$

$$v_{ay} = \mathbf{v}_a \sin\sigma_a \cos\phi_a \tag{1e}$$

$$v_{az} = \mathbf{v}_a \sin\sigma_a \tag{1f}$$

$$\dot{v}_{ax} = u_{ax},\ \dot{v}_{ay} = u_{ay},\ \dot{v}_{az} = u_{az} \tag{1g}$$

where $\dot{(\ )}$ is the derivative with respect to time, $\mathbf{v}_a$ is the UAV velocity vector with respective components as v_{ax}, v_{ay} and v_{az} in the global reference frame, indicated by x-y-z in Fig. 1. Furthermore, σ_a and ϕ_a are the flight path and heading (course) angles respectively (Hou et al., 2019), while u_{ax}, u_{ay} and u_{az} are the control acceleration input

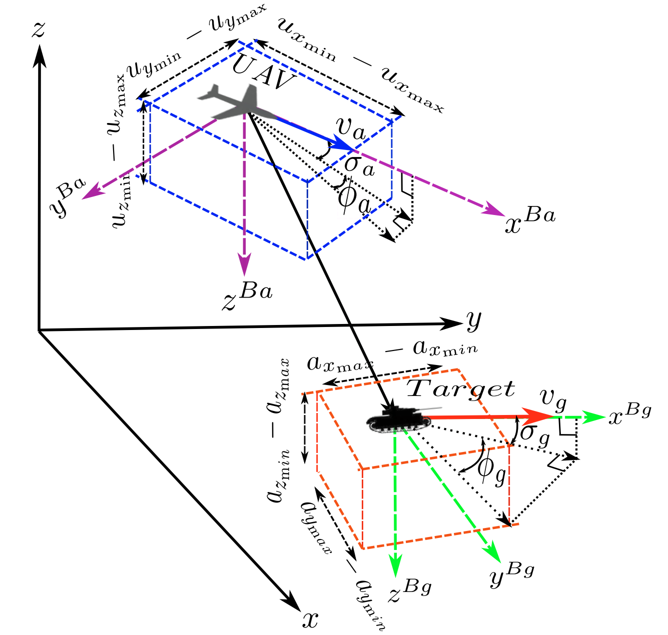


Fig. 1. UAV and target engagement dynamics. The coordinates (x,y,z) are global while (x^{Ba}, y^{Ba}, z^{Ba}) and (x^{Bg}, y^{Bg}, z^{Bg}) are local. The doted boxes around the UAV and target indicate their respective control input magnitude constraints.

of the UAV. The body frame is defined by $\mathbf{x}^{Ba}$-$\mathbf{y}^{Ba}$-$\mathbf{z}^{Ba}$ as shown in Fig. 1, where $\mathbf{x}^{Ba}$ is aligned with the UAV velocity vector, $\mathbf{y}^{Ba}$ is towards the right-hand-side of the wing, and $\mathbf{z}^{Ba}$ is given by the cross product of $\mathbf{x}^{Ba}$ and $\mathbf{y}^{Ba}$.

The state space representation is given by

$$\dot{\mathbf{x}}_a = A_a\mathbf{x}_a + B_a\mathbf{u}_a = \begin{bmatrix} 0_3 & I_3 \\ 0_3 & 0_3 \end{bmatrix}\mathbf{x}_a + \begin{bmatrix} 0_3 \\ I_3 \end{bmatrix}\mathbf{u}_a \tag{2a}$$

$$\mathbf{y} = C_a\mathbf{x}_a = [I_3\ 0_3]\mathbf{x}_a \tag{2b}$$

where 0_3 is the 3×3 zero matrix, I_3 is the 3×3 identity matrix, A_a, B_a and C_a are defined appropriately in the above equation, $\mathbf{x}_a = [x_a, y_a, z_a, v_{ax}, v_{ay}, v_{az}]^T$ and $\mathbf{u}_a = [u_{ax}, u_{ay}, u_{az}]^T$.

Similarly, the target dynamics is given by

$$\dot{x}_g = v_{gx},\ \dot{y}_g = v_{gy},\ \dot{z}_g = v_{gz}, \tag{3a}$$

$$v_{gx} = \mathbf{v}_g \cos\sigma_g \sin\phi_g \tag{3b}$$

$$v_{gy} = \mathbf{v}_g \sin\sigma_g \cos\phi_g \tag{3c}$$

$$v_{gz} = \mathbf{v}_g \sin\sigma_g \tag{3d}$$

$$\dot{v}_{gx} = a_{gx},\ \dot{v}_{gy} = a_{gy},\ \dot{v}_{gz} = a_{gz} \tag{3e}$$

where x_g, y_g and z_g represent the position of the target and v_{gx}, v_{gy} and v_{gz} represent the respective components of the target velocity vector $\mathbf{v}_g$. The target path angle is σ_g, and its heading angle is ϕ_g. Additionally, the target acceleration components are a_{gx}, a_{gy} and a_{gz} respectively while its state space representation is given by

$$\dot{\mathbf{x}}_g = A_g\mathbf{x}_g + B_g\mathbf{u}_g = \begin{bmatrix} 0_3 & I_3 \\ 0_3 & 0_3 \end{bmatrix}\mathbf{x}_g + \begin{bmatrix} 0_3 \\ I_3 \end{bmatrix}\mathbf{a}_g \tag{4a}$$

$$\mathbf{z} = C_g\mathbf{x}_g = [I_3\ 0_3]\mathbf{x}_g \tag{4b}$$

where, A_g, B_g and C_g are defined appropriately in the above equation, $\mathbf{x}_g = [x_g, y_g, z_g, v_{gx}, v_{gy}, v_{gz}]^T$ and

$\mathbf{a}_g = [a_{gx}, a_{gy}, a_{gz}]^T$.

We discretize the governing differential equation with the time step, Δt, for the UAV and target as follows:

$$\mathbf{x}_a(k+1) = F_a\mathbf{x}_a + G_a\mathbf{u}_a(k) \quad (5a)$$

$$\mathbf{x}_g(k+1) = F_g\mathbf{x}_g + G_g\mathbf{a}_g(k) \quad (5b)$$

$$\mathbf{y}(k) = C_a\mathbf{x}_a(k) \quad (5c)$$

$$\mathbf{z}(k) = C_g\mathbf{x}_g(k) \quad (5d)$$

where

$$F_a = \begin{bmatrix} I_3 & \Delta t I_3 \\ 0_3 & I_3 \end{bmatrix}, G_a = \begin{bmatrix} 0_3 \\ \Delta t I_3 \end{bmatrix}, F_g = F_a, G_a = G_g \quad (6)$$

The control input space of the UAV is confined by

$$u_{ax_{\min}} \le u_{ax}^B \le u_{ax_{\max}} \quad (7a)$$

$$u_{ay_{\min}} \le u_{ay}^B \le u_{ay_{\max}} \quad (7b)$$

$$u_{az_{\min}} \le u_{az}^B \le u_{az_{\max}} \quad (7c)$$

where u_{ax}^B, u_{ay}^B, and u_{az}^B are the control input of UAV in the UAV's body coordinates. The control input acceleration of the ground vehicle is given by

$$a_{gx_{\min}} \le a_{gx}^B \le a_{gx_{\max}} \quad (8a)$$

$$a_{gy_{\min}} \le a_{gy}^B \le a_{gy_{\max}} \quad (8b)$$

$$a_{gz_{\min}} \le a_{gz}^B \le a_{gz_{\max}} \quad (8c)$$

where a_{gx}^B, a_{gy}^B, and a_{gz}^B are the control input of the target in the body coordinates.

To simplify the tracking optimization problem, Kim (2022) assumes that the altitude of UAV is fixed and the terrain where the ground vehicle moves is flat. Hence, the corresponding dynamics given by (1) is used excluding z_a and v_{az} making it a 2D tracking problem. The velocity and control vectors in the global coordinate must satisfy the constraint, while its turn radius must be larger than its minimum radius of turn given as $r_{\min}$. The curvature of the UAV flight path in 2D space must be smaller than the inverse of the minimum turn radius. These constraints are summarized as follows:

$$v_{a_{\min}} \le \sqrt{v_{ax}^2 + v_{ay}^2} \le v_{a_{\max}} \quad (9a)$$

$$u_{ax_{\min}} \le u_{ax}\cos\phi_a + u_{ay}\sin\phi_a \le u_{ax_{\max}} \quad (9b)$$

$$u_{ay_{\min}} \le -u_{ax}\sin\phi_a + u_{ay}\cos\phi_a \le u_{ay_{\max}} \quad (9c)$$

$$0 \le v_{gx}^2 + v_{gy}^2 \le v_{g_{\max}}^2 \quad (9d)$$

$$\frac{|v_{ax}u_{ay} - v_{ay}u_{ax}|}{(v_{ax}^2 + v_{ay}^2)^{(3/2)}} \le \frac{1}{r_{\min}} \quad (9e)$$

where $v_{a_{\min}}$ and $v_{a_{\max}}$ are the respective minimum and maximum allowed UAV velocities, while $v_{g_{\max}}$ is the maximum target velocity. Similarly, $u_{ax_{\min}}$, $u_{ax_{\max}}$, $u_{ay_{\min}}$ and $u_{ay_{\max}}$ are the minimum and maximum control inputs of the UAV along the x^{Ba}-y^{Ba} axes.

In addition, sharp turns by the UAV could result in high loading to the structure of UAV. To prevent excessive loading, the UAV bank angle, γ_a, and the turn rate, $\dot\psi$, are constrained as follows (Pothen and Ratnoo, 2017):

$$\gamma_{a_{\min}} \le \gamma_a \le \gamma_{a_{\max}} \quad (10a)$$

$$\dot\psi_{\min} \le \dot\psi \le \dot\psi_{\max} \quad (10b)$$

where the bank angle and the turn rate are related to the speed of UAV, $\|\mathbf{v}_a\|$, and $r_{\min}$ as follows (Machmudah et al., 2022):

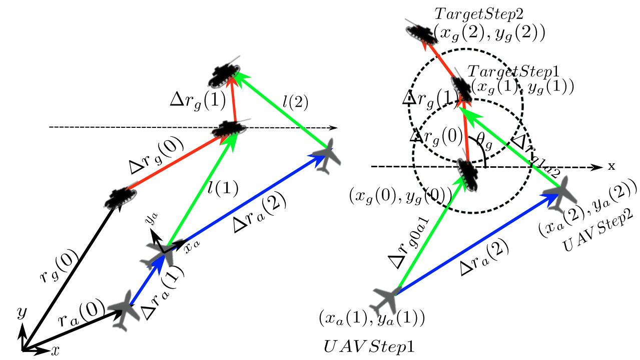


Fig. 2. UAV 2-Step tracking prediction

$$\gamma_a = \frac{\dot\psi\|\mathbf{v}_a\|}{g},\ \dot\psi = \frac{\|\mathbf{v}_a\|}{r_{\min}} \quad (11)$$

2.2 Target Tracking Algorithm

The two-step discretized tracking cost function is given by

$$\underset{\mathbf{v}_g(0),\mathbf{v}_g(1)\in\mathbb{V}_g}{\text{Maximize}}\ \underset{\mathbf{u}_a(0)\in\mathbb{U}_a}{\text{Minimize}}\ J = \sum_{k=1}^{2}[l(k)]^2 \quad (12a)$$

subject to (5), (7), (9) and (10), where $l(k)$ is equal to $\|\mathbf{y}(k) - \mathbf{z}(k)\|$, and $\mathbb{V}_g$ and $\mathbb{U}_a$ represent the feasible control input sets of the target and the UAV, respectively. Note that (8) is ignored in the UAV tracking algorithm design phase allowing the target to change its velocity instantaneously. This provides an advantage for the target to evade from the tracking algorithm's point of view. The two-step is chosen as it is the minimum number of steps for providing the control input in the cost function, i.e., the relative degree of the system.

To design a worst-case scenario, consider the problem from the target perspective and assume the UAV has an unknown optimal tracking algorithm. The best evasive option for the target is to maximise the sum of the relative distance from the UAV. As the UAV approaches to the target, the evading ground target moves with maximum speed providing the biggest advantage to maximizing the distance from the UAV.

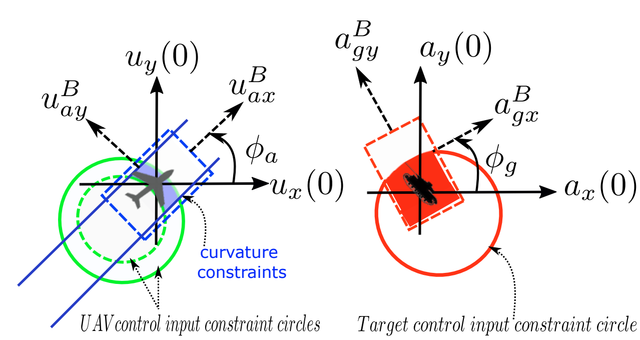


Fig. 3. UAV and target constraints and feasible control spaces

The two circles drawn around the target at $k = 1$ and $k = 2$ in Fig. 2, indicate that the next position of the target with the maximum speed can lie in any position at the boundary of the circles. At $k = 1$, the distance between UAV and target is a function of θ_g, the maximization parameter used in computing the worst-case scenario for the UAV tracking minimization, i.e.,

$$l(1) = \|\Delta \mathbf{r} g_o a_1 + v_{g_{\max}} \Delta t(\cos\theta_g \mathbf{i} + \sin\theta_g \mathbf{j})\| \quad (13)$$

At $k = 2$, the target simply tries to drive away from the UAV with maximum velocity in an opposite direction to the UAV velocity. The distance at $k = 2$ is given by

$$l(2) = \|\Delta \mathbf{r} g_1 a_1\| + v_{g_{\max}} \Delta t \quad (14)$$

Once the worst θ_g is determined by solving the maximization problem, the minimization problem for the UAV is obtained. The details of the 2D tracking algorithm are found in Kim (2022).

3. SMART TARGET MANOEUVRE DESIGN

The target cost function was designed using the min-max concept to maximise its distance from the best-case UAV minimisation effort. Similar to the UAV tracking algorithm design, the smart target is assumed to have the position and velocity information of the UAV. The target control cost function is given by (15).

$$\underset{\mathbf{v}_a(0),\mathbf{v}_a(1)\in\mathbb{V}_a}{\text{Minimize}} \; \underset{\mathbf{a}_g(0)\in\mathbb{U}_g}{\text{Maximize}} \; J = \sum_{k=1}^{2} [l(k)]^2 \quad (15)$$

subject to (5) and

$$v_{gx}^2 + v_{gy}^2 \le v_{g_{\max}}^2 \quad (16a)$$
$$a_{gx_{\min}} \le a_{gx}\cos\phi_g + a_{gy}\sin\phi_g \le a_{gx_{\max}} \quad (16b)$$
$$a_{gy_{\min}} \le -a_{gx}\sin\phi_g + a_{gy}\cos\phi_g \le a_{gy_{\max}} \quad (16c)$$
$$v_{a_{\min}}^2 \le v_{ax}^2 + v_{ay}^2 \le v_{a_{\max}}^2 \quad (16d)$$
$$\dot\psi_{a_{\min}} \le \dot\psi_g \le \dot\psi_{a_{\max}} \quad (16e)$$

where $\mathbb{V}_a$ and $\mathbb{U}_g$ represent the feasible control input sets of the UAV velocity and the target acceleration, $v_{g_{\max}}$ is the maximum velocities of the target, $a_{gx_{\min}}$, $a_{gx_{\max}}$, $a_{gy_{\min}}$ and $a_{gy_{\max}}$ are its respective minimum and maximum control input components and the turn rate, $\dot\psi_g$, is restricted by the minimum and the maximum bounds, $\dot\psi_{a_{\min}}$ and $\dot\psi_{a_{\max}}$.

In comparison to the tracking UAV, the target is designed with the consideration that it can stop, and move backward. Accordingly, the target is not restricted by curvature and minimum velocity constraints as shown by the shaded feasible control space for the UAV and target in Fig. 3.

Substituting and expanding all the expressions into the cost function for the target, the following compact function, $\bar{J}_g$, equivalent to the original cost function is obtained:

$$\bar{J}_g = a_{gx}^2(0) + \alpha_g a_{gx}(0) + a_{gy}^2(0) + \beta_g a_{gy}(0) + \gamma_g \quad (17)$$

where α_g, β_g and γ_g are functions of the UAV initial velocity and the initial positions of the UAV and the target. Similar to the worst-case scenario for the UAV tracking algorithm design, consider the worst-case scenario for the ground vehicle evasion manoeuvre. As shown in Fig. 4, the distance between UAV and target at $k = 1$, is equal to $l_g(1)$ and be calculated with respect to, θ_a, an optimization parameter introduced to obtain best-case UAV tracking minimization.

$$l_g(1) = \|\Delta \mathbf{r} a_o g_1 + v_{a_{\text{opt}}}(1)\Delta t(\cos\theta_a \mathbf{i} + \sin\theta_a \mathbf{j})\| \quad (18)$$

where $v_{a_{\text{opt}}}(1)$ is the UAV's optimal speed determined by the relative distance from the target. At $k = 2$, the UAV

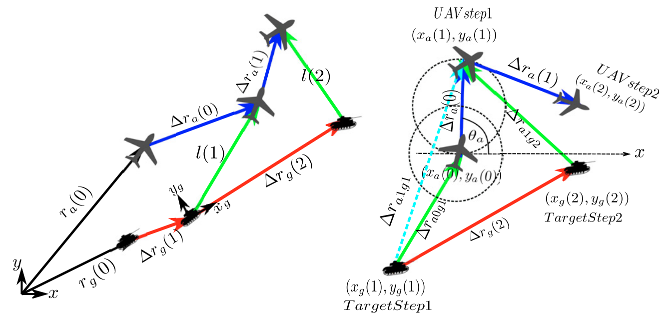


Fig. 4. Target 2-Step evasion manoeuvre

tries to close the relative distance between the two vehicles depicted as $l_g(2)$, which is calculated by

$$l_g(2) = \|\Delta \mathbf{r} a_1 g_1 + \Delta \mathbf{r}_a(1)\| = \|\Delta \mathbf{r} a_1 g_1\| + v_{a_{\text{opt}}}(2)\Delta t \quad (19)$$

where $v_{a_{\text{opt}}}(2)$ is the optimal speed applied by the UAV at $k = 2$ to close up with the target, dependent on the relative distance from the target given by $\|\Delta \mathbf{r}_a(1)\|$.

To simplify the worst case scenario for the target, we assume that both of the optimal UAV speeds are equal to the UAV's maximum speed. Then, the minimisation of $l_g(2)$ is equivalent to the minimising the following length:

$$\bar{l}_g(2) = \|\Delta \mathbf{r} a_1 g_1\| = \|\Delta \mathbf{r} a_0 g_1 - \Delta \mathbf{r}_g(2) + \Delta \mathbf{r}_a(0)\| \quad (20)$$

The original cost function can now be represented by the following minimization problem:

$$\bar{J}_g = [l_g(1)]^2 + [\bar{l}_g(2)]^2 \quad (21)$$

The evasive control input is obtained by solving the maximization of $\bar{J}_g$. The target optimal control is obtained using the same sampling or maximization approaches used for the UAV optimal acceleration, i.e., search the maximization solution at the boundary or the inside of the control input constraints.

Fig. 5 shows the cost function contours, constraints and the corresponding control inputs for the UAV and the target for a sample scenario.

4. EXTENSION TO 3D SCENARIO

4.1 3D Target Tracking Algorithm

The 2D target tracking in (Kim, 2022) is extended for 3D tracking scenarios. Considering the UAV and target velocity vectors $\mathbf{v}_a$ and $\mathbf{v}_g$ as shown in Fig. 1, where the angle between the two vectors is θ_{ag}, we assume that the angle is not equal to zero. An instantaneous moving frame, x_m-y_m is established with the unit vector in x_m-axis, aligned to the target velocity vector $\mathbf{v}_g$ while the unit vector towards $\mathbf{z}_m$-axis is orthogonal to the plane formed by the cross product, $\mathbf{v}_g \times \mathbf{v}_a$ as follows:

$$\mathbf{x}_m = \frac{\mathbf{v}_g}{\|\mathbf{v}_g\|}, \quad \mathbf{z}_m = \frac{\mathbf{v}_a \times \mathbf{v}_g}{\|\mathbf{v}_a \times \mathbf{v}_g\|} \quad (22)$$

The unit vector $\mathbf{y}_m$ is equal to $\mathbf{z}_m \times \mathbf{x}_m$. The direction cosine matrix, D_{Rm}, transforming the vectors in the moving frame to the reference frame is given by:

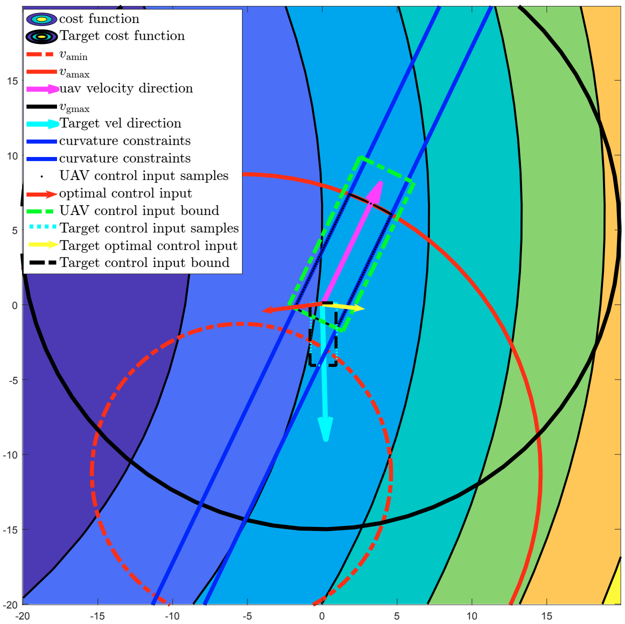


Fig. 5. Optimal control inputs and cost function contours for UAV minimisation and target maximisation

$$D_{Rm} = \begin{bmatrix} \mathbf{x}_m^R & \mathbf{y}_m^R & \mathbf{z}_m^R \end{bmatrix} \tag{23}$$

where $\mathbf{x}_m^R$, $\mathbf{y}_m^R$ and $\mathbf{z}_m^R$ are the moving frame unit vectors expressed in the reference frame and D_{Rm} is the orthonormal matrix satisfying $D_{Rm}D_{Rm}^T = I_3$, i.e., $D_{mR} = D_{Rm}^T$.

Once the 2D algorithm calculates the optimal tracking control input in the body frame, the following equation maps the 2D control input into the 3D space:

$$\mathbf{u}_a^m = [u_{ax}^B \;\; u_{ay}^B \;\; 0] \;\; , \;\; \mathbf{u}_a^R = D_{Rm}\mathbf{u}_a^m \tag{24}$$

where $\mathbf{u}_a^R$ is the UAV reference frame acceleration and $\mathbf{u}_a^m$ is the UAV body frame tracking command expressed as the moving frame. To complete the extension to 3D, similar computations used in (22) to (24) are used to compute the target control input and states. The 3D control inputs derived for the UAV and target are then used to compute their corresponding $x - y - z$ velocities and positions.

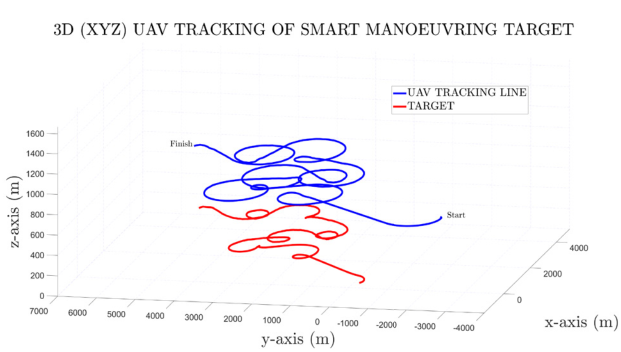


Fig. 6. 3D Plot of UAV Tracking a Manoeuvring Target

A pseudo-code detailing the 2D to 3D extension is summarized in Algorithm 1.

4.2 Simulations and Results

The UAV minimum turn radius, $r_{\min}$, is set at 400m with the velocity limits of $v_{a_{\min}}$ = 20m/s and $v_{a_{\max}}$ = 40m/s. Similarly, the respective minimum and maximum acceleration limits of the UAV along x, y, and z axes are as follows:

$$u_{ax_{\min}} = -10 \text{ [m/s}^2\text{]}, \;\; u_{ax_{\max}} = 8 \text{ [m/s}^2\text{]},$$
$$u_{ay_{\min}} = -4 \text{ [m/s}^2\text{]} \;, u_{ay_{\max}} = 4 \text{ [m/s}^2\text{]},$$
$$u_{az_{\min}} = -0.2 \text{ [m/s}^2\text{]}, \;\; u_{az_{\max}} = 0.5 \text{ [m/s}^2\text{]}$$

The target velocity limits is set to $v_{g_{\max}} = 16.7m/s$, while its respective acceleration limits along x, y,and z axes are as follows:

$$a_{gx_{\min}} = -2 \text{ [m/s}^2\text{]}, \;\; a_{gx_{\max}} = 4 \text{ [m/s}^2\text{]},$$
$$a_{gy_{\min}} = -2 \text{ [m/s}^2\text{]}, \;\; a_{gy_{\max}} = 2 \text{ [m/s}^2\text{]},$$
$$a_{gz_{\min}} = -0.2 \text{ [m/s}^2\text{]}, \;\; a_{gz_{\max}} = 0.4 \text{ [m/s}^2\text{]}$$

The initial UAV position is set to (2000m, -2000m, 500m) while the initial target position is set to at (50m,-50m, 50m) in x-y-z respectively.

As shown in Fig. 6, the UAV and target manoeuvre trajectory indicate that the UAV responds to the target's evasive manoeuvres. The target performs an evasive manoeuvre when the UAV closes up to its location. Then, the evasive manoeuvre results in a corresponding change in the velocity and position of the UAV. When the target moves in a straight line, the UAV manoeuvre ensures the target is kept within favorable tracking distance.

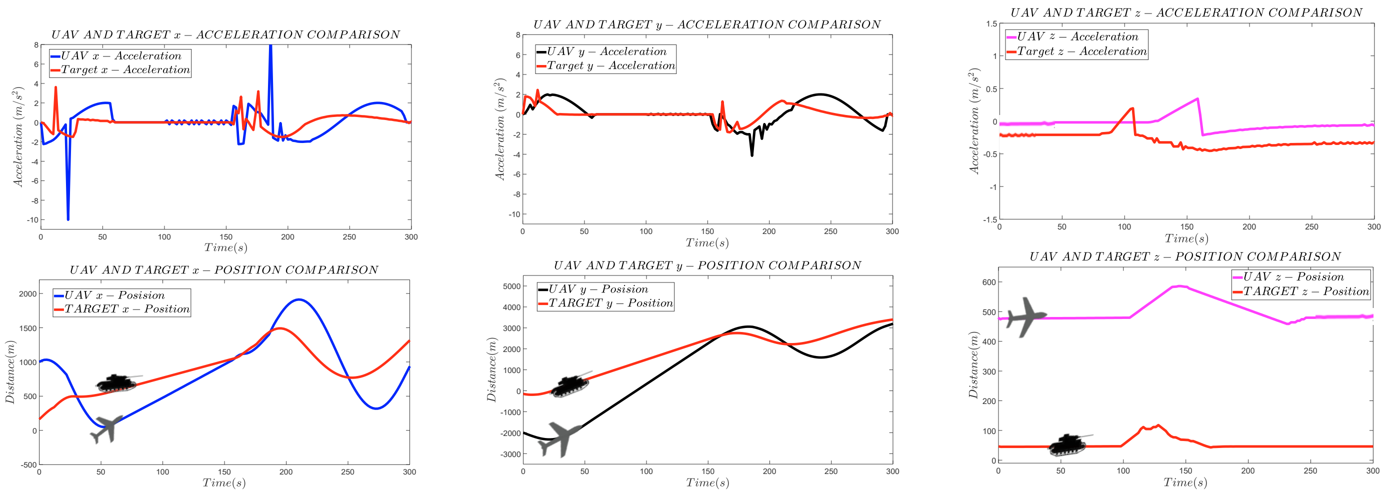
Comparing the x, y and z positions with the corresponding control input responses of the UAV and target for various engagement scenarios, we observe that the accelerations increase or decrease in response to manoeuvres by the other vehicle as shown in Figs. 7a, 7b and 7c. Sharp spikes resulting from sudden evasive target manoeuvres and a corresponding increase in UAV acceleration to close up with the target are shown in the figure. These abrupt UAV manoeuvres are restricted by the turn rate and bank angle constraints.

Algorithm 1 : 2D TO 3D ALGORITHM

Require: current position & velocity of UAV and target
for Every time step **do**
  Obtain the tracking commands as detailed in Sections 2 and 3
  Compute transformation matrices (23)
  Convert 2D control vector to 3D frame (24)
  Perform UAV & target manoeuvre given by the optimal acceleration commands
end for

5. CONCLUSIONS & FUTURE WORKS

This paper provides a fixed-wing UAV optimal target tracking strategy with the minimum turn radius varying with the bank angle, which enables the generation of realistic tracking paths for the UAV. In addition, we developed an evasive target control strategy by maximizing the same cost function, which produces smart evasive manouvres for the target. We extended the 2D target tracking algorithm to 3D cases using a simple plane mapping in the 3D spaces and two velocity vectors from the UAV and the target. The simulation results show that the UAV persistently tracked the evading target. The proposed tracking-evasion



(a) x-direction accelerations & positions (b) y-direction accelerations & positions (c) z-direction accelerations & positions

Fig. 7. UAV and Target Accelerations & Positions

strategy will be expanded to cooperative tracking using multiple UAVs while considering the effect of sensor noise in providing the target positions and the collision avoidance between the UAVs.

ACKNOWLEDGEMENTS

The research by the first author was funded by Petroleum Technology Development Fund, and supported by the Nigerian Air Force. And, the research by the second author was supported by Unmanned Vehicles Core Technology Research & Development Program (No. 2020M3C1C1A01--08316111) through the National Research Foundation of Korea and Unmanned Vehicle Advanced Research Center(UVARC) funded by the Ministry of Science and ICT, Republic of Korea.

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