On Biased Harmonic Signal Estimation: Application to Electric Power Grid Monitoring

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***Abstract*—Parametric estimation of a biased harmonic signal is a significant technical challenge for many engineering applications. Such a problem is particularly important for electric utility grid-connected power electronic converters. This paper utilizes a linear regression model of the signal to solve this interesting practical problem. A continuous-time dynamic regressor extension and mixing (DREM) based approach is then applied for parameter estimation. For practical implementation, continuous-time estimators are discretized using implicit and explicit Euler methods. We then prove that the implicit discretization can achieve fixed-time convergence for the unknown frequencies estimation. Thanks to the estimated frequencies, another DREM-based linear regression problem is solved for the parameter estimation purpose. The overall order of the proposed technique is the same as the number of unknown parameters, making the estimator suitable for real-time implementation in embedded devices. Theoretical results are validated through extensive comparative experimental studies.**

***Index Terms*—Frequency estimation, DREM, fixed-time convergence.**

I. Introduction

Many engineering applications require fast, accurate, and online parameter estimation of a biased harmonic signal. These parameters are required to ensure maximum power transfer through grid-connected converters (GCCs) [1], [2], reject harmonic disturbance in a DC motor [3], enhance ride comfort through semi-active suspension system [4], monitor the status of a power grid [5], reconstruct the material surface topology by atomic force microscopy [6], estimate the rotational speed of turbocharger [7], measure the wheel speed through a vibration signal [8], to name a few. These applications require fast convergent algorithms that are not computationally demanding, easy to implement, and robust to measurement noise and disturbances.

Estimating the harmonic signal parameters such as amplitudes, bias, and phases becomes a simple linear regression problem when the frequencies are known. However, estimating parameters becomes complicated in the unknown frequencies case as frequencies appear nonlinearly in the regression problem. This paper considers this challenging case, *i.e.*, parameter estimation of an unknown-frequencies biased harmonic signal.

Several estimators are available in the literature due to the importance of the harmonic signal parameter estimation problem. Some of the existing continuous-time techniques are: adaptive observer [9], algebraic estimator [10], demodulation [11], adaptive notch filter (ANF) [12], second-order generalized integrator frequency-locked loop (SOGI-FLL) [13], enhanced phase-locked loop [14]. Many of these techniques have local stability properties. Even if a global stability result is available for the continuous-time algorithms, their real-time implementation requires discretization. Frequently, the discretized algorithms do not hold the same stability and convergence properties as that of the continuous-time counterpart.

Moreover, high-order explicit discretization can be computationally expensive for real-time embedded systems. In addition to continuous-time estimators, discrete-time estimators are also popular in the literature. In [15], a discrete-time frequency locked loop has been proposed for a single-frequency sinusoid, however, it works only with unbiased measurements. Results of [15] are extended to multi-frequency sinusoidal signals in [16]. In [17], a state-dependent Riccati equation filter is proposed for a single-frequency unbiased sinusoidal signal without any convergence analysis. Linear filters with linear adaptation law are proposed in [18]. However, the estimator order is higher than the number of parameters.

Recently several finite-time convergent algorithms [19]–[21] have been proposed in the literature. Nonetheless, these techniques are only available in continuous time. Out of these techniques, the results proposed in [19] are interesting. In that work, a linear regression model of an unbiased harmonic signal is obtained. Then, the dynamic regressor extension and mixing (DREM) method [22]–[24] is applied for frequency estimation purposes. The advantages of the DREM approach include the possibilities of separating the estimation for each parameter, ensuring monotone and accelerated convergence with a tight evaluation of gains with respect to the measurement noise, while the excitation requirement can be relaxed [25].

Motivated by the results presented in [19], a linear regression model of the biased harmonic signal is studied in this work. Then, applying the DREM framework, parameter estimation algorithms are proposed. Both implicit and explicit discretizations of the continuous-time estimators are considered. An extension of the implicitly discretized estimation algorithm is presented having the fixed-time convergence property. Finally, comparative experimental results are provided to show the practical feasibility of the proposed estimators.

Our method has the same order as the number of unknown parameters, easy to implement and tune. Fixed-time frequency estimation has been achieved in [26] with a fifth-order algorithm and has numerous parameters to tune. In contrast, our frequency estimation algorithm is of first-order. These are the main contributions of this work.

The outline of this paper is as follows. A brief description of DREM is introduced in Section II. The convergence and robustness conditions of estimation algorithms in discrete time are established in Section III. The problem statement is given in Section IV. The estimation algorithms are presented in Section V. The application of these estimators is considered in Section VI. Finally, Section VII concludes this paper.

Notation

- Let $\mathbb{Z}$ denote the set of integers and $\mathbb{R}$ the set of real numbers, $\mathbb{R}_+ = \{t \in \mathbb{R} : t \geq 0\}$.
- For $x \in \mathbb{R}$, $|x|$ denotes the absolute value (or a vector norm for $x \in \mathbb{R}^n$). The induced matrix norm for a matrix $A \in \mathbb{R}^{n\times n}$ is denoted by $\|A\|$.
- For a Lebesgue measurable and essentially bounded function $x : \mathbb{R} \to \mathbb{R}^n$, denote $\|x\|_\infty = \operatorname{ess\,sup}_{t\in\mathbb{R}} |x(t)|$, and define

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by $\mathcal{L}_\infty(\mathbb{R},\mathbb{R}^n)$ the set of all such functions with finite norms $\|\cdot\|_\infty$. If

$$\int_{-\infty}^{+\infty} |x(t)|^2 dt < +\infty,$$

then this class of functions is denoted by $\mathcal{L}_2(\mathbb{R},\mathbb{R}^n)$.

- A continuous function $\alpha:\mathbb{R}_+\to\mathbb{R}_+$ belongs to the class $\mathcal{K}$ if it is strictly increasing and $\alpha(0)=0$. A function $\alpha\in\mathcal{K}$ belongs to the class $\mathcal{K}_\infty$ if it increases to infinity. A function $\beta:\mathbb{R}_+\times\mathbb{R}_+\to\mathbb{R}_+$ belongs to the class $\mathcal{KL}$ if $\beta(\cdot,t)\in\mathcal{K}$ for each fixed $t\in\mathbb{R}_+$ and $\beta(s,\cdot)$ is decreasing with $\lim_{t\to+\infty}\beta(s,t)=0$ for each fixed $s\in\mathbb{R}_+$. A function $\beta:\mathbb{R}_+\times\mathbb{R}_+\to\mathbb{R}_+$ belongs to the class $\mathcal{GKL}$ if $\beta(s,0)\in\mathcal{K}$, $\beta(s,\cdot)$ is decreasing for each fixed $s\in\mathbb{R}_+$ and there is $T_s\in\mathbb{R}_+$ such that $\beta(s,t)=0$ for all $t\geq T_s$.
- The identity matrix of order n is denoted by I_n.
- A sequence of positive integers $1,2,...,n$ is denoted by $\overline{1,n}$.
- Denote the exponential function by $\mathbf{e}:\mathbb{R}\to\mathbb{R}$ and define $\lceil s\rfloor^\alpha := |s|^\alpha \text{sign}(s)$ for any $s\in\mathbb{R}$ and $\alpha\in\mathbb{R}_+$.

II. Dynamic Regressor Extension and Mixing Method

Consider a parameter estimation problem (linear regression):

$$\begin{aligned} x(t) &= \Omega^\top(t)\theta,\ t\geq t_0\in\mathbb{R}, \\ y(t) &= x(t)+w(t), \end{aligned} \tag{1}$$

where $x(t)\in\mathbb{R}$ is the model output, $\theta\in\mathbb{R}^n$ is the vector of unknown constant parameters that is necessary to estimate, $\Omega:\mathbb{R}\to\mathbb{R}^n$ is the regressor function assumed to be known, $y(t)\in\mathbb{R}$ is the signal available for measurement with measurement noise $w:\mathbb{R}\to\mathbb{R}$. Introduce the following hypothesis:

Assumption 1. Let $\Omega\in\mathcal{L}_\infty(\mathbb{R},\mathbb{R}^n)$ and $w\in\mathcal{L}_\infty(\mathbb{R},\mathbb{R})$.

As proposed in [24], to relax the requirements on the excitation of Ω and also to improve the transient performance, the DREM procedure transforms (1) into n new one-dimensional regression models, allowing a decoupled estimation of θ_i, $i=\overline{1,n}$ to be performed with an adjustable convergence speed.

In the DREM procedure, a choice of a linear operator of the form $H:\mathcal{L}_\infty(\mathbb{R},\mathbb{R})\to\mathcal{L}_\infty(\mathbb{R},\mathbb{R})$ has to be done. This linear operator can be, for instance, a stable linear time-invariant filter with the transfer function $W(s)=\frac{\alpha}{s+\beta}$, where $s\in\mathbb{C}$ is a complex variable, and $\alpha\neq 0$, $\beta>0$ are selected to filter the noise w in (1); or it can realize the delay operator with the transfer function $W(s)=e^{-\tau s}$ for $\tau>0$. For the system (1), $n-1$ linear operators $H_j:\mathcal{L}_\infty(\mathbb{R},\mathbb{R})\to\mathcal{L}_\infty(\mathbb{R},\mathbb{R})$ are introduced for $j=\overline{1,n-1}$. Note that $y\in\mathcal{L}_\infty(\mathbb{R},\mathbb{R})$ under Assumption 1, then these operators are applied to the measured output y of (1), and using the superposition principle, we obtain:

$$\begin{gathered} \tilde{y}_j(t)=H_j(y(t))=\tilde{\Omega}_j^\top(t)\theta+\tilde{w}_j(t),\ j=\overline{1,n-1}; \\ \tilde{\Omega}_{j,i}(t)=H_j(\Omega_{j,i}(t)),\ j=\overline{1,n-1},\ i=\overline{1,n}, \\ \tilde{\Omega}_j^\top(t)=[\tilde{\Omega}_{j,1}(t)\dots\tilde{\Omega}_{j,n}(t)], \end{gathered}$$

where $\tilde{y}_j(t)\in\mathbb{R}$ is the j^{th} operator output, $\tilde{\Omega}_j:\mathbb{R}\to\mathbb{R}^n$ is the j^{th} filtered regressor function and $\tilde{w}_j(t):\mathbb{R}\to\mathbb{R}$ is the new j^{th} noise signal, which is composed by the transformation of the noise w by H_j and other exponentially converging components related to the initial conditions of the filters (if they exist). By construction, $\tilde{\Omega}_j\in\mathcal{L}_\infty(\mathbb{R},\mathbb{R}^n)$ and $\tilde{w}_j\in\mathcal{L}_\infty(\mathbb{R},\mathbb{R})$, for all $j=\overline{1,n-1}$. Define new vector variables

$$\begin{gathered} \tilde{Y}(t)=[y(t)\ \tilde{y}_1(t)\dots\tilde{y}_{n-1}(t)]^\top\in\mathbb{R}^n, \\ \tilde{W}(t)=[w(t)\ \tilde{w}_1(t)\dots\tilde{w}_{n-1}(t)]^\top\in\mathbb{R}^n \end{gathered}$$

and a time-varying matrix

$$M(t)=\left[\Omega(t)\ \tilde{\Omega}_1(t)\dots\tilde{\Omega}_{n-1}(t)\right]^\top\in\mathbb{R}^{n\times n}.$$

Stacking the original equation (1) with the $n-1$ filtered regressor models, we construct an extended regressor system:

$$\tilde{Y}(t)=M(t)\theta+\tilde{W}(t).$$

For any matrix $\mathbf{M}\in\mathbb{R}^{n\times n}$, the following equality holds:

$$\text{adj}\,(\mathbf{M})\,\mathbf{M}=\det(\mathbf{M})\,I_n,$$

even if $\mathbf{M}$ is singular, where $\text{adj}\,(\mathbf{M})$ is the adjugate matrix (or adjoint matrix) of $\mathbf{M}$ and $\det(\mathbf{M})$ is its determinant. Define

$$\begin{gathered} Y(t)=\text{adj}\,(M(t))\,\tilde{Y}(t),\ W(t)=\text{adj}\,(M(t))\,\tilde{W}(t), \\ \phi(t)=\det(M(t)). \end{gathered}$$

Then multiplying the adjugate matrix $\text{adj}\,(M(t))$ on the left of the extended regressor system, we get n scalar regressor models of the form:

$$Y_i(t)=\phi(t)\theta_i+W_i(t) \tag{2}$$

for $i=\overline{1,n}$. Again, by construction $Y=[Y_1\dots Y_n]^\top\in\mathcal{L}_\infty(\mathbb{R},\mathbb{R}^n)$, $W=[W_1\dots W_n]^\top\in\mathcal{L}_\infty(\mathbb{R},\mathbb{R}^n)$, and $\phi\in\mathcal{L}_\infty(\mathbb{R},\mathbb{R})$. For the scalar linear regression model (2), the conventional gradient estimation algorithm takes the form:

$$\dot{\hat{\theta}}_i(t)=\gamma_i\phi(t)\left(Y_i(t)-\phi(t)\hat{\theta}_i(t)\right),\ \gamma_i>0 \tag{3}$$

for all $i=\overline{1,n}$, where now the estimation processes for all components of θ are decoupled, and the adaptation gain γ_i can be adjusted separately for each element of θ. However, all these estimation algorithms are dependent on the same regressor ϕ (the determinant of M).

Define the parameter estimation error as $e(t)=\theta-\hat{\theta}(t)$, then its dynamics satisfies the differential equation:

$$\dot{e}_i(t)=-\gamma_i\phi(t)(\phi(t)e_i(t)+W_i(t)),\ i=\overline{1,n}, \tag{4}$$

and the following result can be proven for the DREM method:

Theorem 1. *[24], [27] Consider the linear regression system (1) under Assumption 1. Assume that for the selected operators $H_j:\mathcal{L}_\infty(\mathbb{R},\mathbb{R})\to\mathcal{L}_\infty(\mathbb{R},\mathbb{R})$, $j=\overline{1,n-1}$:*

$$\int_{t_0}^{+\infty}\phi^2(t)dt=+\infty \tag{5}$$

for any $t_0\in\mathbb{R}$. Then the estimation algorithm (3) has the following properties:

(A) If $\|W\|_\infty=0$, then the system (4) is globally uniformly asymptotically stable at the origin if and only if (5) is valid.

(B) For all $W\in\mathcal{L}_2(\mathbb{R},\mathbb{R}^n)$, we have $e\in\mathcal{L}_\infty(\mathbb{R},\mathbb{R}^n)$. In addition, for all $t_0\in\mathbb{R}$ and $e_i(t_0)\in\mathbb{R}$:

$$\begin{aligned} |e_i(t)| \leq\ & \mathbf{e}^{-\gamma_i\int_{t_0}^t\phi^2(\tau)d\tau}|e_i(t_0)| \\ & +\sqrt{\frac{\gamma_i}{2}}\sqrt{1-\mathbf{e}^{-2\gamma_i\int_{t_0}^t\phi^2(\tau)d\tau}}\sqrt{\int_{t_0}^t W_i^2(s)ds} \end{aligned}$$

for all $t\geq t_0$ and $i=\overline{1,n}$.

Obviously, if the signal ϕ is persistently excited (PE)[1], then the error dynamics is input-to-state stable (ISS) with respect to $W\in\mathcal{L}_\infty(\mathbb{R},\mathbb{R}^n)$ and an exponential convergence rate can be guaranteed [28]–[31]:

[1] A bounded signal $\psi:\mathbb{R}\to\mathbb{R}^n$ is *persistently excited* if there exist $\mu>0$ and $T>0$ such that $\int_t^{t+T}\psi(s)^\top\psi(s)ds\geq\mu I_n$, for all $t\in\mathbb{R}_+$.

Corollary 2. *Consider the linear regression system* (1) *under Assumption 1. Assume that for the selected operators* H_j : $\mathcal{L}_\infty(\mathbb{R},\mathbb{R}) \to \mathcal{L}_\infty(\mathbb{R},\mathbb{R})$*,* $j = \overline{1, n-1}$*:*

$$\int_{t_0}^{t_0+\ell} \phi^2(t)dt \geq \upsilon > 0 \tag{6}$$

for some $\ell > 0$ *and for any* $t_0 \in \mathbb{R}$*. Then, in* (3)*, for all* $t_0 \in \mathbb{R}$*,* $e_i(t_0) \in \mathbb{R}$ *and* $W \in \mathcal{L}_\infty(\mathbb{R},\mathbb{R}^n)$*, we have:*

$$\begin{aligned} |e_i(t)| \quad &\leq \quad \mathbf{e}^{\gamma_i \upsilon}(\mathbf{e}^{-\gamma_i \upsilon \frac{t-t_0}{\ell}}|e_i(t_0)| \\ &+\frac{\ell}{\upsilon}\|\phi\|_\infty(1-\mathbf{e}^{-\gamma_i \upsilon \frac{t-t_0}{\ell}})\|W\|_\infty) \end{aligned}$$

for all $t \geq t_0$ *and* $i = \overline{1, n}$*.*

As we can conclude from the estimates obtained for the error e in Theorem 1 and Corollary 2, increasing the value of adaptation gain γ_i makes it possible to accelerate the error rate convergence at the price of an augmented sensibility in the measurement noise.

Remark 3. Note that if we apply the algorithm (3) directly to the model (1) without application of DREM (without decoupling the estimation of different elements of θ), then it is impossible to arbitrarily accelerate the convergence speed by increasing the values of γ_i, and there is an optimal value of the adaptation gain dependent on the PE parameters ℓ and υ [29]–[31].

To accelerate the estimation error convergence without degrading the noise robustness, consider a nonlinear adaptive estimation algorithm proposed in [25], [32]:

$$\begin{aligned} \dot{\hat{\theta}}_i(t) \quad &= \quad \phi(t)(\gamma_{1,i} \left\lceil Y_i(t) - \phi(t)\hat{\theta}_i(t) \right\rfloor^{1-\alpha} \\ &+\gamma_{2,i} \left\lceil Y_i(t) - \phi(t)\hat{\theta}_i(t) \right\rfloor^{1+\alpha}) \end{aligned} \tag{7}$$

for $\gamma_{1,i} > 0$, $\gamma_{2,i} > 0$ for all $i = \overline{1, n}$ and $\alpha \in [0, 1)$, with $\hat{\theta}(t_0) \in \mathbb{R}^n$, which admits the following properties:

Theorem 4. *[25] Let Assumption 1 be satisfied, and for given* $T^0 > 0$ *and* $T_f > 0$*,*

$$\int_t^{t+\ell} \min\left\{|\phi(s)|^{2-\alpha}, |\phi(s)|^{2+\alpha}\right\} ds \geq \upsilon > 0 \tag{8}$$

for all $t \in [-T^0, T^0 + T_f]$ *and some* $\ell \in \left(0, \frac{T_f}{2}\right)$*. Set*

$$\min\left\{\gamma_{1,i}, \gamma_{2,i}\right\} > \frac{2^{2+\frac{\alpha}{2}}}{\alpha\upsilon\left(\frac{T_f}{2\ell} - 1\right)}.$$

Then the estimation error $e(t) = \theta - \hat{\theta}(t)$ *dynamics of* (7) *is short-fixed-time ISS for* T^0 *and* T_f*,* i.e.*, there exist* $\beta \in \mathcal{GKL}$ *and* $\varrho \in \mathcal{K}$ *such that for all* $\hat{\theta}(t_0) \in \mathbb{R}^n$*, all* $W \in \mathcal{L}_\infty(\mathbb{R},\mathbb{R}^n)$ *and* $t_0 \in [-T^0, T^0]$*:*

$$|e(t)| \leq \beta(|e(t_0)|, t - t_0) + \varrho(\|W\|_\infty), \quad \forall t \in [t_0, t_0 + T_f]$$

and $\beta(|e(t_0)|, T_f) = 0$*.*

As it has been shown in [25], the function ϱ is proportional to $\gamma_{1,i}, \gamma_{2,i}$ and $\frac{\ell}{\upsilon}$, but for (7), there is no need to arbitrarily increase their values to have global uniform convergence of the estimation error to the origin due to the introduced nonlinearities. Also, only interval excitation is required by (8).

III. Discrete-Time Realization of Estimation Algorithms

According to theorems 1 and 4, algorithms (3) and (7) guarantee reliable and fast estimation of the vector of unknown parameters in (1) under Assumption 1 and different restrictions (5) and (8) on the excitation of ϕ (both restrictions are covered by PE, while the convergence rate in Theorem 4 is better than in Corollary 2 derived for the PE case and algorithm (3)). These algorithms are formulated in continuous time, and for implementation, their discretization is required. As it has been observed in the literature, for finite/fixed-time converging systems, the implicit Euler discretization method is preferable to the explicit one [33]: the former preserves the convergence rate and accuracy, while the latter suffers from various drawbacks for highly nonlinear systems as (7). The price of applying an implicit discretization method to (7) is that a nonlinear equation has to be numerically solved on each step. For linear dynamical systems, as in (3), the implicit Euler discretization method also has some advantages, *e.g.*, it provides the independence of the discretization step on the adaptation rate γ_i, hence avoiding expensive computations.

A. Implementation of (3)

Therefore, in this work the implicit Euler discretization of (3) is analyzed:

$$\hat{\theta}_i(k+1) = \hat{\theta}_i(k) + h\gamma_i\phi(k+1)(Y_i(k+1) - \phi(k+1)\hat{\theta}_i(k+1)),$$
$$i = \overline{1, n}, \ k \in \mathbb{Z}, \ k \geq k_0,$$

where $\hat{\theta}(k) = \hat{\theta}(t_k) \in \mathbb{R}^n$ is the value of the vector of estimates at discrete time instants $t_k = kh$ with $t_0 = k_0 h$, $k_0 \in \mathbb{Z}$, $\hat{\theta}(k_0) \in \mathbb{R}^n$, $h > 0$ is the constant discretization step, $\phi(k) = \phi(t_k)$ and $Y(k) = Y(t_k)$ similarly. Resolving the last equation with respect to $\hat{\theta}_i(k+1)$ we obtain:

$$\hat{\theta}_i(k+1) = \frac{\hat{\theta}_i(k) + h\gamma_i\phi(k+1)Y_i(k+1)}{1 + h\gamma_i\phi^2(k+1)}, \ i = \overline{1, n}, \tag{9}$$

which generates bounded trajectories for any non-zero ϕ and $h\gamma_i > 0$. Moreover,

$$e_i(k+1) = \frac{e_i(k) - h\gamma_i\phi(k+1)W_i(k+1)}{1 + h\gamma_i\phi^2(k+1)}, \ i = \overline{1, n}, \tag{10}$$

where $e_i(k) = \theta_i - \hat{\theta}_i(k)$, $k \geq k_0$, is the estimation error, and the following result can be obtained (it is a noise-case extension of the result from [22]):

Theorem 5. *For* $i = \overline{1, n}$*, assume* $\sup_{j \geq k_0} |Y_i(j)| < +\infty$*, for any* $k_0 \in \mathbb{Z}$*. Then* (9) *admits the following properties:*

1) *if* $\sum_{j=k_0}^{+\infty} |W_i(j)| < +\infty$ *and*

$$\lim_{k \to +\infty} \prod_{s=k_0}^{k} 1 + h\gamma_i\phi^2(s) = +\infty \tag{11}$$

for any $k_0 \in \mathbb{Z}$*, then*

$$|e_i(k)| \leq \frac{|e_i(k_0)|}{\prod_{s=k_0+1}^{k} 1 + h\gamma_i\phi^2(s)} + \frac{\sqrt{h\gamma_i}}{2} \sum_{j=k_0+1}^{k} |W_i(j)|$$

for all $k > k_0$*;*

2) *if* $\sup_{j \geq k_0} |W_i(j)| < +\infty$ *and there exists* $\ell \geq 1$ *and* $\upsilon > 1$ *such that*

$$\prod_{s=k_0}^{k_0+\ell} 1 + h\gamma_i\phi^2(s) \geq \upsilon \tag{12}$$

for any $k_0 \in \mathbb{Z}$*, then*

$$|e_i(k)| \leq \upsilon^{-\lfloor \frac{k-k_0-1}{\ell} \rfloor}|e_i(k_0)| + \frac{\sqrt{h\gamma_i}}{2}\frac{\ell\upsilon}{\upsilon - 1} \sup_{j \geq k_0+1} |W_i(j)|$$

for all $k > k_0$*, where* $\lfloor \cdot \rfloor$ *denotes the floor function.*

Proof. From (10) we get:

$$e_i(k) = \frac{e_i(k_0)}{\prod_{s=k_1}^{k} 1 + h\gamma_i\phi^2(s)} - \sum_{j=k_1}^{k} \frac{h\gamma_i\phi(j)W_i(j)}{\prod_{s=j}^{k} 1 + h\gamma_i\phi^2(s)}$$

for all $k \geq k_1 = k_0 + 1$ and any $k_0 \in \mathbb{Z}$, which implies:

$$|e_i(k)| \leq \frac{|e_i(k_0)|}{\prod_{s=k_1}^{k} 1 + h\gamma_i\phi^2(s)} + \sum_{j=k_1}^{k} \frac{h\gamma_i|\phi(j)||W_i(j)|}{\prod_{s=j}^{k} 1 + h\gamma_i\phi^2(s)}.$$

Note that

$$\frac{h\gamma_i|\phi(j)|}{1 + h\gamma_i\phi^2(j)} \leq \frac{\sqrt{h\gamma_i}}{2}$$

and the maximum is reached for $|\phi(j)| = \frac{1}{\sqrt{h\gamma_i}}$. Hence, the latter estimate can be rewritten as

$$|e_i(k)| \leq \frac{|e_i(k_0)|}{\prod_{s=k_1}^{k} 1 + h\gamma_i\phi^2(s)} + \frac{\sqrt{h\gamma_i}}{2}\left(|W_i(k)| + \sum_{j=k_1}^{k-1} \frac{|W_i(j)|}{\prod_{s=j+1}^{k} 1 + h\gamma_i\phi^2(s)}\right).$$

If $\sum_{j=k_0}^{+\infty} |W_i(j)| < +\infty$, then the desired estimate follows immediately since by hypothesis, $\frac{1}{\prod_{s=j+1}^{k} 1+h\gamma_i\phi^2(s)} \leq 1$ for any $k > k_1$ and $j \in \overline{k_1, k-1}$.

If $\|W_i\|_\infty := \sup_{j\geq k_0} |W_i(j)| < +\infty$ and (12) is satisfied for some $\ell \geq 1$ and $\upsilon > 1$, then

$$|e_i(k)| \leq \frac{|e_i(k_0)|}{\prod_{s=k_1}^{k} 1 + h\gamma_i\phi^2(s)} + \frac{\sqrt{h\gamma_i}}{2}\|W_i\|_\infty\left(1 + \sum_{j=k_1}^{k-1} \frac{1}{\prod_{s=j+1}^{k} 1 + h\gamma_i\phi^2(s)}\right).$$

Note that

$$1 + \sum_{j=k_1}^{k-1} \frac{1}{\prod_{s=j+1}^{k} 1 + h\gamma_i\phi^2(s)} \leq \sum_{q=0}^{\lfloor \frac{k-k_0-1}{\ell} \rfloor} \frac{\ell}{\upsilon^q} \leq \sum_{q=0}^{+\infty} \frac{\ell}{\upsilon^q} \leq \frac{\ell\upsilon}{\upsilon - 1}$$

and $\prod_{s=k_1}^{k} 1 + h\gamma_i\phi^2(s) \geq \upsilon^{\lfloor \frac{k-k_0-1}{\ell} \rfloor}$, which leads to the estimate stated in the formulation of the theorem. □

As we can conclude from the result of Theorem 5, the implicitly discretized version (9) of the algorithm (3) recovers all properties stated in Theorem 1 and Corollary 2. For example, the condition (5) can be written in discrete time as $\sum_{j=k_0}^{+\infty} \phi^2(j) = +\infty$, which definitely implies (11). The noise gain multiplier $\sqrt{\gamma_i}$ is also presented in both theorems, 1 and 5, and an arbitrary increase of the estimation error convergence rate can be obtained by augmentation of the value of γ_i. Moreover, if the signal ϕ is bounded and $\|\phi\|_\infty := \sup_{j\geq k_0} |\phi(j)| \in (0, +\infty)$ is known for any $k_0 \in \mathbb{Z}$, then $\gamma_i < \frac{1}{h\|\phi\|_\infty^2}$ is the restriction on the adaptation gain to minimize the noise gain.

Remark 6. Note that it is not the case for the explicitly discretized version of (3) for $i = \overline{1, n}$:

$$\hat{\theta}_i(k+1) = (1 - h\gamma_i\phi^2(k))\hat{\theta}_i(k) + h\gamma_i\phi(k)Y_i(k),\ k \geq k_0,$$

where a straightforward and similar analysis shows that for stability, the adaptation gain γ_i should belong to the interval $\left(0, \frac{2}{h\|\phi\|_\infty^2}\right)$ (*i.e.*, the convergence rate regulation is limited), and the noise gain becomes proportional to $\gamma_i\|\phi\|_\infty$ (consequently, the signal ϕ has to be bounded and the gain is linear in γ_i, while in Theorem 1, the square root of γ_i is derived in this place).

B. Fixed-Time Estimation in Discrete Time

The convergence rate obtained in Theorem 5 is asymptotic or exponential. The approach of [34] can be used to guarantee a fixed-time convergence rate as in Theorem 4 (uniformity of the convergence time in the initial guesses $\hat{\theta}(k_0) \in \mathbb{R}^n$). Indeed, it is straightforward to adapt that method to the discrete-time setting and the algorithm (9). To this end, consider two versions of (9):

$$\hat{\theta}_{i,l}(k+1) = \frac{\hat{\theta}_{i,l}(k) + h\gamma_{i,l}\phi(k+1)Y_i(k+1)}{1 + h\gamma_{i,l}\phi^2(k+1)},$$
$$\hat{\theta}_{i,l}(k_0) = 0,\ i = \overline{1, n},\ l = 1, 2,\ k \in \mathbb{Z}$$

with adaptation gains $\gamma_{i,1} \neq \gamma_{i,2} \geq 0$. Then using the notation $e_{i,l}(k) = \theta_i - \hat{\theta}_{i,l}(k)$,

$$Y_i(k) - \phi(k)\hat{\theta}_{i,l}(k) = \phi(k)e_{i,l}(k) + W_i(k)$$
$$= \frac{\phi(k)\theta_i}{\prod_{s=k_1}^{k} 1 + h\gamma_{i,l}\phi^2(s)} - \phi(k)\sum_{j=k_1}^{k} \frac{h\gamma_{i,l}\phi(j)W_i(j)}{\prod_{s=j}^{k} 1 + h\gamma_{i,l}\phi^2(s)} + W_i(k)$$

for $l = 1, 2$, by construction and the derivations made above. Calculating the difference of these two equations, we get:

$$\theta_i = \frac{(\hat{\theta}_{i,2}(k) - \hat{\theta}_{i,1}(k))}{\frac{1}{\prod_{s=k_1}^{k} 1+h\gamma_{i,1}\phi^2(s)} - \frac{1}{\prod_{s=k_1}^{k} 1+h\gamma_{i,2}\phi^2(s)}} + \frac{\sum_{j=k_1}^{k}\left(\frac{h\gamma_{i,1}\phi(j)W_i(j)}{\prod_{s=j}^{k} 1+h\gamma_{i,1}\phi^2(s)} - \frac{h\gamma_{i,2}\phi(j)W_i(j)}{\prod_{s=j}^{k} 1+h\gamma_{i,2}\phi^2(s)}\right)}{\frac{1}{\prod_{s=k_1}^{k} 1+h\gamma_{i,1}\phi^2(s)} - \frac{1}{\prod_{s=k_1}^{k} 1+h\gamma_{i,2}\phi^2(s)}}$$

provided that $\phi(k) \neq 0$. Therefore, the following estimate can be used:

$$\hat{\theta}_i(k) = \frac{\hat{\theta}_{i,2}(k) - \hat{\theta}_{i,1}(k)}{\frac{1}{\prod_{s=k_1}^{k} 1+h\gamma_{i,1}\phi^2(s)} - \frac{1}{\prod_{s=k_1}^{k} 1+h\gamma_{i,2}\phi^2(s)}} \quad (13)$$

for $\phi(k) \neq 0$, where it is better to call (13) at some instant of time that maximizes the denominator.

Theorem 7. *For the case $\gamma_{i,1} = 0$ and $\gamma_{i,2} > 0$, the worst case estimation error $e_i(k) = \theta_i - \hat{\theta}_i(k)$ in* (13) *is upper bounded as follows:*

$$|e_i(k)| \leq \left|\frac{\sum_{j=k_1}^{k} \frac{h\gamma_{i,2}\phi(j)W_i(j)}{\prod_{s=j}^{k} 1+h\gamma_{i,2}\phi^2(s)}}{1 - \frac{1}{\prod_{s=k_1}^{k} 1+h\gamma_{i,2}\phi^2(s)}}\right| \leq \frac{\sqrt{h\gamma_i}\ell\upsilon^2}{2(\upsilon-1)^2}\sup_{j\geq k_0} |W_i(j)|$$

for any $k > k_0 + \ell + 1$ provided that (12) *is satisfied.*

Proof. The result follows direct computations. □

Remark 8. Let us compare the noise gains for (9) and (13) given in theorems 5 and 7, respectively. Since $\upsilon > 1$ in (12), $\frac{\upsilon}{\upsilon-1} \leq \left(\frac{\upsilon}{\upsilon-1}\right)^2$, which implies that the fixed-time convergence can be obtained only at the price of degraded robustness. A possible solution is to use (13) eventually (or only once) to ensure uniformity in initial conditions, and cope permanently with (9).

IV. Problem Statement

Grid-connected single-phase inverter needs to operate synchronously with the grid voltage to ensure maximum power transfer. For this purpose, fast and accurate estimation of the grid voltage parameters is essential. The extracted parameters are used to generate the reference current for the current controller. In this work, we are interested in a particular form of the problem (1) dealing with the estimation of a biased harmonic signal representing a single-phase grid voltage signal:

$$y(t) = A_0 + \sum_{i=1}^{N} A_i \cos(\omega_i t + \phi_i) + \varpi(t), \quad (14)$$

where $A_0 \in \mathbb{R}$ is the unknown bias and $A_i \in \mathbb{R}_+$, $i = \overline{1, N}$ are the amplitudes of $N > 0$ harmonic grid-voltage signals with the unknown frequencies $\omega_i \in \mathbb{R}_+$ and the phases $\phi_i \in [-\frac{\pi}{2}, \frac{\pi}{2}]$; $\varpi \in \mathcal{L}_\infty(\mathbb{R}, \mathbb{R})$ is the measurement noise. The number N is supposed to be given, and $\omega_i \neq \omega_j$ for $i \neq j = \overline{1, N}$.

It is required to estimate all parameters of (14): the bias and the amplitudes, the frequencies and the phases, and it is desirable to make it reasonably fast (*e.g.*, for N=1, in less than 2 periods of the signal in the ideal case).

V. Design of Fast Estimation Algorithms

Due to the appearance of uncertain frequencies, we cannot present the signal (14) in the form (1) directly, and to overcome this obstacle, we use the approach of [19]. To this end, define a delay constant $\tau > 0$ and the corresponding delay operator $Z(y(t)) = y(t - \tau)$, for all $t \in \mathbb{R}_+$. Hence $Z^k(y(t)) = y(t - k\tau)$, for any integer $k \geq 0$.

A. Estimation of Frequencies

Following [19], a re-parametrization is introduced:

$$(Z - 1) \prod_{i=1}^{N} (Z^2 + 1 - 2c_i Z)(y(t)) = w'(t), \ \forall t \in \mathbb{R}_+ \quad (15)$$

where $c_i := \cos(\omega_i \tau)$, $\forall i \in \overline{1, N}$ and $w'(t) = (Z - 1) \prod_{i=1}^{N} (Z^2 + 1 - 2c_i Z)(\varpi(t))$ is the aggregated measurement noise. Notice that the symbol $\prod$ indicates the composition of operators. Indeed, the property $\prod_{i=1}^{N} (Z^2 + 1 - 2c_i Z) \left(\sum_{i=1}^{N} A_i \cos(\omega_i t + \phi_i) \right) = 0$ can be easily demonstrated by induction (*cf.* [19]), while the equality $(Z - 1)(A_0) = 0$ is trivial, the result then follows. Note that

$$\|w'\|_\infty \leq 2^{N+1} \prod_{i=1}^{N} (1 + |c_i|) \|\varpi\|_\infty \leq 2^{2N+1} \|\varpi\|_\infty,$$

where the worst case upper bounds are used. Repeating the computations similar to [19], we obtain a linear regression as in (1):

$$y'(t) = \Omega'^\top(t)\theta' + w'(t), \quad (16)$$

where

$$y'(t) = (Z - 1)(Z^2 + 1)^N (y(t)) \, ; \ \theta' = [\theta_1' \dots \theta_N']^\top,$$

$$\theta_i' = (-1)^{i+1} \sum_{k_1=1}^{N} \sum_{k_2=k_1+1}^{N} \cdots \sum_{k_i=k_{i-1}+1}^{N} c_{k_1} \dots c_{k_i}, \ i = \overline{1, N};$$

$$\Omega' = [\Omega_1' \dots \Omega_N']^\top, \ \Omega_i'(t) = (Z - 1)(2Z)^i (Z^2 + 1)^{N-i} (y(t)) \, , \quad i = \overline{1, N}.$$

As it is well-known [29], [35], if

$$\tau < \frac{\pi}{2N \min\{\omega_1, \dots, \omega_N\}}$$

and $\|\varpi\|_\infty = 0$ (or the noise is sufficiently small), then for $t > (2N + 1)\tau$ the regressor Ω' is PE. Therefore, the linear regression problem (16) can be solved applying the DREM method with delays as the auxiliary filters $H_j(s) = e^{-j\tau s}$ for $j = \overline{1, N - 1}$, as it has been explained in the previous section for (1).

Example 9. For $N = 1$, a simple calculation shows that

$$\Omega'(t) = 2A_1 \sin(0.5\omega_1\tau) \sin(\omega_1(t - 1.5\tau) + \phi_1),$$

and if $0.5\omega_1\tau < \pi$, then it is PE.

B. Estimation of other parameters

According to the results of theorems 1 and 4, the estimates of frequencies $\hat{\omega}_i$ can be obtained in a fixed-time using the DREM method and the algorithm (3) (with the use of the tool described in Subsection III-B) or (7). Then the following approximation of (14) can be used:

$$\hat{y}(t) = A_0 + \sum_{i=1}^{N} A_i \cos(\hat{\omega}_i(t)t + \phi_i),$$

which gives

$$y(t) = \hat{y}(t) + w(t),$$

where

$$w(t) = \varpi(t) - 2 \sum_{i=1}^{N} A_i \sin\left(\frac{\omega_i + \hat{\omega}_i(t)}{2} t + \phi_i \right) \sin\left(\frac{\omega_i - \hat{\omega}_i(t)}{2} t \right)$$

is the new noise, which includes also the frequency estimation error $\omega_i - \hat{\omega}_i(t)$. Clearly, even if the discrepancy $\omega_i - \hat{\omega}_i(t)$ is sufficiently small for all $i = \overline{1, N}$, the term $\sin\left(\frac{\omega_i - \hat{\omega}_i(t)}{2} t \right)$ can slowly grow with the time, which leads to the worst case noise amplitude bound:

$$\|w\|_\infty \leq \|\varpi\|_\infty + 2 \sum_{i=1}^{N} A_i,$$

then a rapid estimation of the remaining parameters becomes again of great importance. We can use now the representation (1) with

$$\Omega(t) = [1 \ \cos(\hat{\omega}_1(t)t) \ -\sin(\hat{\omega}_1(t)t) \dots \cos(\hat{\omega}_N(t)t) \ -\sin(\hat{\omega}_N(t)t)]^\top,$$

$$\theta = [A_0 \ A_1 \cos(\phi_1) \ A_1 \sin(\phi_1) \dots A_N \cos(\phi_N) \ A_N \sin(\phi_N)]^\top.$$

For $\hat{\omega}_i(t) = \omega_i$ and $\|\varpi\|_\infty = 0$ (for a small enough noise w), it is again known [29], [35] that the regressor $\Omega(t)$ is PE, and if $\hat{\theta}(t)$ is the estimate of θ obtained using the approach presented in the previous section, then

$$\hat{A}_i(t) = \sqrt{\hat{\theta}_{2i}^2(t) + \hat{\theta}_{2i+1}^2(t)}, \ \hat{\phi}_i(t) = \arctan\left(\frac{\hat{\theta}_{2i+1}(t)}{\hat{\theta}_{2i}(t)} \right)$$

for all $i = \overline{1, N}$ are the estimates of the amplitudes and the phases.

Example 10. For $N = 1$, a simple calculation shows that

$$\phi(t) = 4 \sin^2(0.5\omega_1\tau) \sin(\omega_1\tau),$$

and if $\omega_1\tau < \pi$, then it is PE.

It is to be noted here that for $N = 1$, our algorithm has two parameters to tune, namely the delay τ and the adaptation gain γ. The delay can be selected as a factor of the fundamental signal period. Through extensive numerical simulation, we found that $\tau = T/4$ can be an excellent choice. Due to the periodic nature of the grid voltage signal, numerical simulation has found that this value can reduce the effect of high-order harmonics and noise. A trial and error method has been chosen to tune the adaptation gain.

VI. Experimental Validation

In this Section, experimental validation of the proposed parameter estimation algorithm is considered, and the experimental setup is shown in Fig. 1. A pulse width modulated (PWM) voltage source inverter with LCL-filter is used to emulate the grid voltage signal. Using GDP-100 differential probe, the voltage is measured at the load-side. Parameters of the inverter are given in [36, Table III]. Experimental results are exported to Matlab for plotting.

We consider the grid voltage signal (14) with $N = 1$ and the nominal grid frequency is 50 Hz for experimental validation. In the real grid, the harmonic components are not known *a priori*. As such,

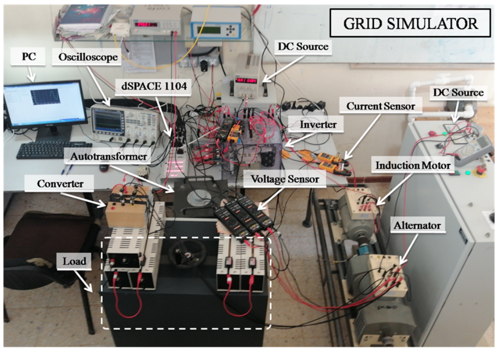

Figure 1. Grid voltage emulator setup [36].

Table I
COMPARATIVE TIME DOMAIN PERFORMANCE SUMMARY.

Test-I: -0.5 p.u.Voltage drop	SOGI-FLL	EPLL	Proposed
Frequency settling time* (msec.)	49	69	15
Frequency ripple (p-p) (Hz)	0.6	0.3	0.14
Amplitude ripple (p-p) (p.u.)	0.019	0.018	0.01
Test-II: Distorted grid			
Frequency Settling Time* (msec.)	49	11	5
Frequency ripple (p-p) (Hz)	1.56	0.47	0.11
Amplitude ripple (p-p) (p.u.)	0.1	0.04	0.018
Unit sine distortion (%)	2.63	1.86	1.09

* Settling time is calculated by considering the stead-state ripple band of SOGI-FLL as the error band.

the harmonic components are not taken into account in the modeling. As comparison techniques, two well-known power grid parameter estimation algorithms, SOGI-FLL [13] and EPLL [14], are selected. All the techniques considered in this section have the same order. The parameter estimation algorithms are implemented in Matlab/Simulink with a sampling frequency of 10 kHz. Using code generation, the algorithms are integrated into the dSPACE system. The proposed algorithm is implemented through a fixed-step discrete solver. SOGI-FLL and EPLL are discretized using a fixed-step ode4 (Runge-Kutta) solver in Matlab/Simulink. Tuning gains for the selected techniques are− the proposed algorithm: $\tau = 5$ msec, $\gamma = 22.5$; SOGI-FLL: K_s=1, $\gamma_s = 50$; and EPLL: $\mu_1 = \mu_3 = 60\pi$, $\mu_2 = 16000$. Tuning parameters of the comparative techniques are selected in a fair way to ensure that all the techniques have a similar convergence time when the grid voltage experience a $+2$Hz frequency jump. Frequency and amplitude estimation are considered as performance indicators.

In the first test, a sudden voltage drop of -0.5 per unit (p.u.) is considered. Many power grid interconnection standards [37] require a renewable energy-interfaced GCC system to remain connected to the grid even when the grid voltage is very low. This is commonly known as low voltage ride through (LVRT). The considered voltage sag is very suitable to test the proposed technique for LVRT capability. As summarized in Table I, the proposed technique has very fast frequency estimation convergence. Moreover, the peak-to-peak estimation ripple for frequency and amplitude is significantly lower for our method than SOGI-FLL and EPLL. This makes the proposed technique highly suitable as the grid-synchronization technique for GCCs.

In the second test, a heavily distorted grid is considered. To generate harmonic signal, a full-bridge diode-rectifier load is connected in parallel to the original load. As shown in Fig. 3, experimental results demonstrate that the proposed technique is robust to harmonic disturbance. As the harmonic components are not modeled, estimation ripple is unavoidable. However, the proposed technique has the lowest peak-to-peak (p-p) estimation ripple. The total harmonic distortion (THD) of the measured grid voltage is 15.83%. The THD of the estimated unit reference signal, $\cos(\hat{\theta}_g)$ is 2.63%,1.86%, and 1.09% for SOGI-FLL, EPLL, and the proposed technique, respectively. As a result, if the proposed technique is used inside the control system of GCCs, this will improve the power quality of the electric power grid. A comparative summary of the THD profile by individual techniques is given in Fig. 4.

A comparative time-domain summary of the three tests presented in this section is given in Table I. The time-domain summary clearly shows that the proposed technique outperformed EPLL and SOGI-FLL in every indicator.

VII. CONCLUSIONS AND FUTURE WORKS

This paper studied the parameter estimation problem of a biased harmonic signal. A linear regression model of the signal was obtained using delays. Then, the DREM method was applied for the frequency and parameter estimation purpose. It has been shown that thanks to DREM and the implicit discretization, the proposed frequency estimator has fixed-time convergence property. The implicit discretized estimators are applied for power grid parameter estimation. Comparative experimental studies showed that the proposed technique was very fast to detect a change in the grid voltage parameters, with excellent harmonic robustness properties. In this work, the adaptation gain has been chosen through trial and error. Developing a constructive gain tuning method for the adaptation gain will be considered in future works. In addition, reducing the algorithm's computational complexity will also be considered for efficient, practical implementation.

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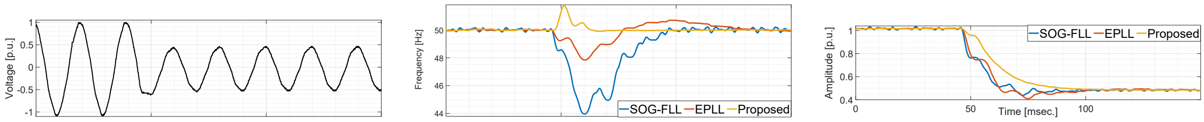
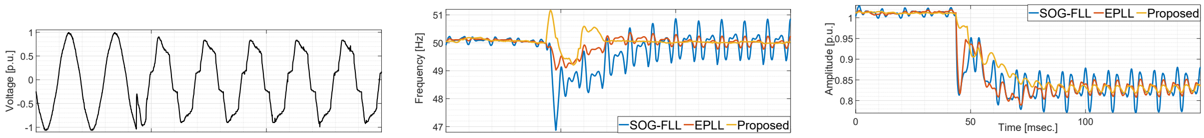


(a) Measured voltage signal (b) Estimated frequency (c) Estimated amplitude

Figure 2. Comparative experimental results for -0.5 p.u. amplitude step change.



(a) Measured voltage signal (b) Estimated frequency (c) Estimated amplitude

Figure 3. Comparative experimental results for harmonically distorted grid voltage signal.

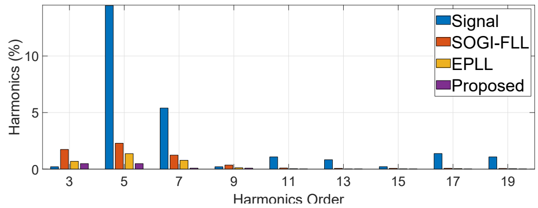


Figure 4. Comparative THD Summary.

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