

# Chain-Length-Dependent Correlated Molecular Motion in Polymers

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We show how dynamic heterogeneities (DHs), a hallmark of glass-forming materials, depend on chain flexibility and chain length in polymers. For highly flexible polymers, a relatively large number of monomers ( $N_c \sim 500$ ) undergo correlated motion at the glass transition temperature ( $T_g$ ), independent of molecular weight ( $M$ ). In contrast, less flexible polymers show a complex  $N_c(M)$  behavior divided into three regimes, consistent with observations in both  $T_g(M)$  and chain conformational structure. For short oligomers ( $\lesssim 2$  Kuhn steps), a transition from mainly intermolecular correlations and  $N_c \sim 200$  to strongly intramolecular correlations and  $N_c < 50$  (roughly the molecular size) is observed; for longer chains,  $N_c$  increases weakly, before saturating. For poly(methyl methacrylate), a remarkable similarity is found between  $N_c(M)$  and the  $M$ -dependent ratio of the activation barriers of the structural ( $\alpha$ ) and secondary ( $\beta$ ) relaxations; we present evidence that this relationship is a general feature of glass-forming polymers. Our results suggest a direct link between the DH length scale and the number of  $\beta$  relaxation events jointly activated to facilitate the  $\alpha$  relaxation.

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**Introduction**—Molecular motions in polymer melts slow down during cooling, eventually resulting in the formation of a disordered solid—a glass. The glass transition temperature  $T_g$  is defined as  $\tau_\alpha(T_g) \equiv 100$  s [1], where  $\tau_\alpha(T)$  is the temperature-dependent structural ( $\alpha$ ) relaxation time [2]. Near  $T_g$ , so-called dynamic heterogeneities (DHs), in which the local dynamics vary from place to place, become more prominent [3,4]. Their characteristic length scale  $\xi_{DH}$  has been determined for both polymeric and nonpolymeric liquids [5–9], using either experimental [9–16] or computational [17–23] approaches, with a resulting length scale  $\xi_{DH}(T_g) \sim 1\text{--}5$  nm [24], which for polymers correspond to 50 – 500 monomers [7,9,25]. However, very few studies have investigated how polymer chain length affects DH [9,26,27] and the entire range from short oligomers to long-chain polymers has rarely been explored [27].

The glass transition of polymers and its associated dynamics can typically be divided into three distinct regimes [28,29], corresponding to (I) short oligomers ( $\lesssim 2$  Kuhn steps), (II) chains with  $\sim 2\text{--}20$  Kuhn steps, and (III) long chains. These regimes are captured in the  $T_g(M)$  behavior (segmental dynamics) and in the chain structure [29].

The inset to Fig. 1 shows an example for PMMA, illustrating  $T_g(M)$  (open black symbols) and the  $M$ -dependent single-molecule aspect ratio  $\Lambda^2(M) = \lambda_3(M)^2/\lambda_1(M)^2$  (solid green symbols), where  $\lambda_3^2$  and  $\lambda_1^2$  are the largest and smallest eigenvalues, respectively, of the average conformational tensor [29]. The regime I-II boundary at  $M^*$  is clearly manifested as both a kink in  $T_g(M)$  and a peak in  $\Lambda^2$ , where the latter is due to chain folding occurring for  $M \gtrsim M^*$  [29]. For long chains in regime III ( $> M^{**}$ ),  $\Lambda^2$  approaches the Gaussian chain value of  $\approx 12$ .

To determine how chain length (or molecular weight  $M$ ) affects DHs in polymers, we study four different polymer chemistries: poly(methyl methacrylate) (PMMA); polystyrene (PS); poly(propylene glycol dimethyl ether) (PPG-DME), and poly(dimethyl siloxane) (PDMS). PMMA and PS show relatively high  $T_g$ , dynamic fragility  $m$  [the sensitivity of  $\tau_\alpha(T)$  to a  $T$  variation near  $T_g$ ], and chain stiffness [30], whereas PPG-DME and PDMS are significantly more flexible polymers characterized by lower  $T_g$  and  $m$ .

DHs can be characterized by a “four-point” correlation function  $G_4(\mathbf{r}, t)$  that correlates the relaxation dynamics (probed by two-point correlators) in space and time. A space integration of  $G_4(\mathbf{r}, t)$  yields a dynamic susceptibility  $\chi_4(t)$ , which quantifies the fluctuations around the average dynamics [4,19,22,31,32].  $\chi_4(t)$  is typically a nonmonotonic function with a peak  $\chi_4^{\max}$  near  $t_{\max} \simeq \tau_\alpha(T)$ , where  $\chi_4^{\max}$  is proportional to the volume of correlated motions, or, correspondingly, the number of monomer units  $N_c^{(4)}$  that undergo correlated motions.

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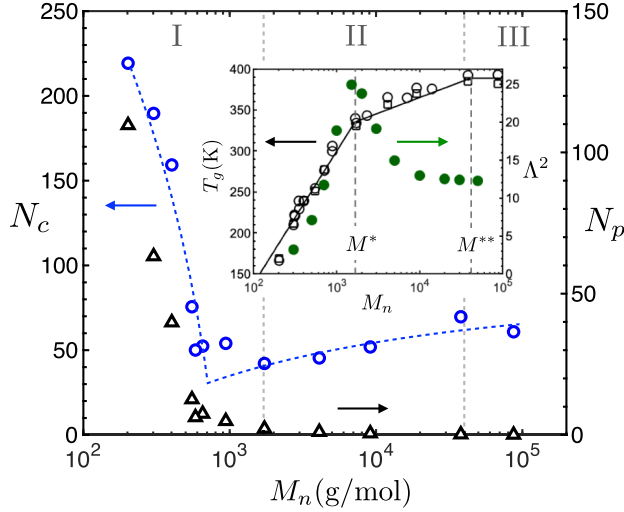


FIG. 1. (Left; blue circles) The number of monomers  $N_c$  involved in the  $\alpha$  relaxation at  $T_\alpha \simeq T_g$  for PMMA, determined using TMDSC. (Right; black triangles)  $N_p \equiv N_c/n$ , where  $n$  is the degree of polymerization. For short chains, where the behavior is intermolecular,  $N_p$  is roughly the number of molecules involved in the  $\alpha$  relaxation. In contrast,  $N_p \sim 1$  for  $M \sim M^*$ , which demonstrates the importance of intramolecular dynamics for  $M \geq M^*$ . The dashed (blue) lines are guides to the eye. The dotted vertical lines mark the regime boundaries at  $M^*$  and  $M^{**}$ , as shown in the inset. Inset: the  $M$  dependence of  $T_g$  (left) and the average polymer aspect ratio  $\Lambda^2$  (right), from Baker *et al.* [29]. The dotted vertical lines in the main panel and the inset mark the regime boundaries at  $M^*$  and  $M^{**}$ .

Direct determination of  $\chi_4(t)$  is difficult and has mainly been achieved in simulations [4,19,22] and for colloidal systems [33,34], as well as recently for a metallic glass former [35]. However, it has been demonstrated that  $\chi_4(t)$  can be estimated from the temperature dependence of a dynamic susceptibility [31,32], obtained from broadband dielectric spectroscopy (BDS), rheology, or scattering [31,32].

We employ two approaches to determine the number of dynamically correlated units involved in the  $\alpha$  relaxation [often referred to as a cooperatively rearranging region (CRR)]. The approach of Donth [11,36] yields the number of correlated units  $N_c(T \sim T_g)$  from the thermal fluctuations measured near  $T_g$  using temperature modulated differential scanning calorimetry (TMDSC). The approach of Berthier *et al.* [31] estimates  $\chi_4$  from the temperature dependence of the complex permittivity measured using BDS, which then yields  $N_c(T)$ . We use both methods to determine the  $M$  dependence of  $N_c$  for the four investigated polymers and show in the End Matter that both approaches give consistent quantitative results.

**Correlated dynamics**—The results for  $N_c(M)$  (at  $T_\alpha \simeq T_g$ ; see End Matter for a detailed discussion) are shown for PMMA in blue rings in Fig. 1. We find that  $N_c(M)$  falls dramatically in regime I from  $N_c \sim 220$  for

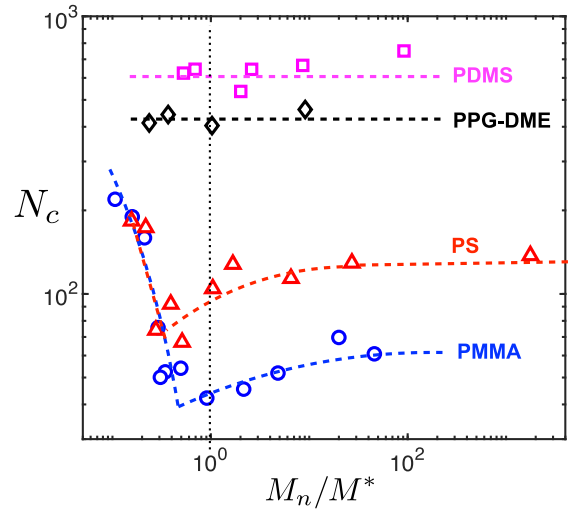


FIG. 2. Comparison of the number of correlated monomers  $N_c$  involved in the  $\alpha$  relaxation for four different polymers as a function of molecular weight  $M_n$ , scaled by the molecular weight  $M^*$  separating regimes I and II.

the dimer to  $N_c < 50$  as the regime II boundary at  $M^*$  is approached. This demonstrates that near  $T_g$  the  $\alpha$  relaxation dynamics for short oligomers (within regime I) are highly correlated, and since these molecules are very short, the dynamics are strongly intermolecular. This is further illustrated in black triangles in Fig. 1, which shows  $N_p(M) = N_c(M)/n(M)$ , where  $n$  is the degree of polymerization. For short chains  $N_p(M)$  is roughly the  $M$ -dependent number of correlated molecules. We observe a drop from  $N_p \sim 110$  for the dimer to  $N_p \sim \mathcal{O}(1)$  near  $M^*$ , reflecting strongly intramolecular behavior for  $M \gtrsim M^*$ . Figure 1 shows that  $N_c$  increases slightly with  $M$  in regime II to reach  $N_c \sim 60$  in regime III. Thus, the number of correlated monomers  $N_c(M)$ , as well as  $T_g(M)$  and the chain structure  $\Lambda^2(M)$  (inset of Fig. 1) all show distinct changes in behavior for similar characteristic molecular weights.

We compare  $N_c(M)$  for all four polymers in Fig. 2. The stiffer polymers (PMMA and PS) show a sharp decrease of  $N_c(M)$  in regime I, followed by a weak increase in regime II. However, the more flexible polymers (PPG-DME and PDMS) show a roughly constant  $N_c(M)$ , which exceeds that of the shortest PMMA and PS oligomers. For PPG-DME and PDMS,  $N_c$  is larger than the degree of polymerization  $n$ , i.e.,  $N_p > 1$ , until regime III is reached.

These results suggest that intramolecular constraints are much weaker for PDMS and PPG-DME, so that chain connectivity and chain length do not significantly influence their  $\alpha$  relaxation behavior, as demonstrated by the nearly  $M$ -independent  $N_c \sim 300 - 700 \gg n$ . Consistent with this, the rotational dihedral barriers of PPG-DME and PDMS (respectively,  $\lesssim 1.0$  [37] and  $0.6$  kcal/mole [38]) are

significantly smaller than those of PMMA and PS (respectively, 2.8 [39] and 3 kcal/mole [38]), and the O-Si-O bending energy for PDMS is notoriously weak [40].

For nonpolymeric glass formers and for short oligomers (in regime I), the molecular motions linked to the structural  $\alpha$  relaxation are dominated by translational degrees of freedom (DOF). As chains with relatively high dihedral barriers (such as PMMA or PS) grow, the number of DOF available for molecular motions is reduced, by exchanging three intermolecular translational DOF for one dihedral intramolecular DOF (per additional degree of polymerization) [41]. This is accompanied by a change of the  $\alpha$  relaxation character from mainly intermolecular to highly intramolecular. This is directly illustrated near the regime I-II crossover ( $M \sim M^*$ ), where chain folding takes place [29] and  $N_c \sim n$ . Correspondingly, as chains grow within regime I, less cooling is required for dynamic arrest to occur, meaning that fewer monomers are involved in correlated motions at  $T_g$ , as shown in Fig. 2.

**Activation barriers**—To further investigate the  $M$  dependence of  $N_c$  observed for PMMA and PS (Fig. 2), we first note that, in addition to the structural  $\alpha$  relaxation, glass formers generally also show a faster  $\beta$  relaxation corresponding to more “local” cooperative motions [29]; there is significant evidence that the two relaxations are intrinsically linked [29,42].

Molecular relaxations in the glassy state typically follow the Arrhenius law,  $\tau_i = \tau_{0i} \exp[\Delta H_i/RT]$  ( $i = \alpha, \beta$ ), where  $\tau_{0i} \sim 0.1$  ps is a microscopic relaxation time and  $R$  is the gas constant. Since  $\tau_\alpha(T_g) \equiv 100$  s,  $\Delta H_\alpha(M)|_{T=T_g} = RT_g \ln(100 \text{ s}/\tau_0)$ , while  $\Delta H_\beta$  is  $T$  independent. Thus,  $\Delta H_\alpha(M)|_{T=T_g}$  is proportional to  $T_g(M)$ , and both show three regimes in  $M$  [29] (inset to Fig. 1). While  $\Delta H_\alpha(M)$  follows the  $M$  dependence of  $T_g(M)$ ,  $\Delta H_\beta(M)$  for PMMA increases with  $M$  in regime I and is nearly  $M$  independent in regimes II and III [29]. Hence, for PMMA the ratio  $\mathcal{R}(M)$  obeys the  $M$  dependencies of both barriers in regime I, but mainly that of the  $\alpha$  barrier in regimes II and III. Remarkably, as shown in Fig. 3, the ratio  $\mathcal{R}(M) \equiv \Delta H_\alpha(M)/\Delta H_\beta(M)$  between the two activation barriers has an  $M$  dependence very similar to that of  $N_c(M)$ . Notably,  $\mathcal{R}(M) \approx 1$  for  $M \sim M^*$ , which corresponds to the onset of strongly intramolecular behavior in  $N_c$  (Fig. 1) and suggests a close correspondence between the properties of the  $\alpha$  and the more local  $\beta$  relaxation (see a detailed discussion in [29]).

As discussed in Supplemental Material (Sec. E [43]), the similarities between the  $M$  dependence of  $N_c(M)$  and the barrier ratio  $\mathcal{R}(M)$  extend beyond PMMA. By combining our own data with the small amount of literature data, we find that  $\mathcal{R}(M)$  for PS and poly( $\alpha$  methyl styrene) behave similar to  $\mathcal{R}(M)$  for PMMA. Moreover, for the highly flexible polymers PDMS and PPG-DME, distinct  $\beta$  relaxations are not observed, but these polymers instead show

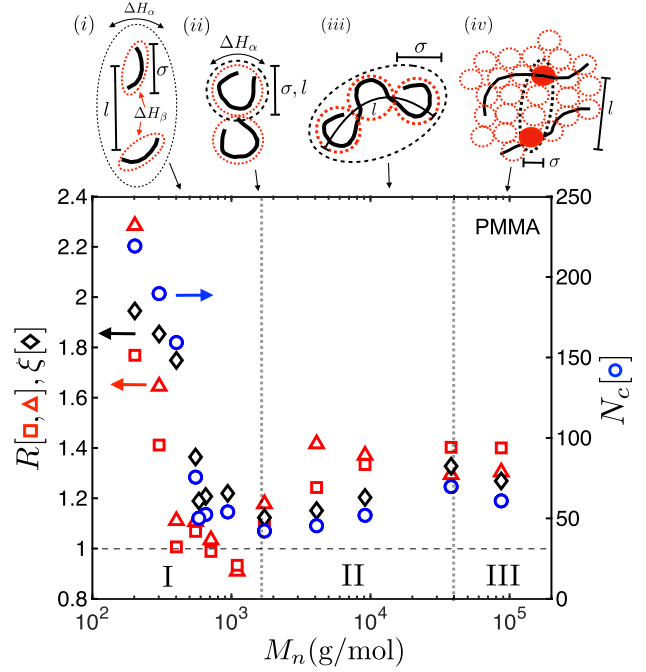


FIG. 3. Comparison between the molecular-weight-dependent ratio  $\mathcal{R} = \Delta H_\alpha/\Delta H_\beta$  of  $\alpha$  and  $\beta$  activation enthalpies of PMMA (left axis, red squares and triangles), characteristic length scale  $\xi_{DH}$  (black diamonds), and the number of correlated monomers  $N_c$  associated with the  $\alpha$  relaxation (right axis, blue circles). Here,  $\Delta H_\alpha(M) \equiv \Delta H_\alpha(M)|_{T=T_g}$  was estimated from  $\tau_\alpha(T)$  using two different approaches (squares and triangles) following [29] and Supplemental Material [43]. The dotted vertical lines mark the regime boundaries at  $M^*$  and  $M^{**}$ . The cartoons (i)–(iv) illustrate the proposed relationship between the activation enthalpies for the  $\alpha$  and  $\beta$  relaxations within the different regimes and are described in the text.

excess wings near  $T_g$  indicating highly “merged”  $\beta$  and  $\alpha$  relaxations [68,69]. As discussed in Supplemental Material [43], we expect that such behavior should result in an effectively  $M$ -independent  $\mathcal{R}(M)$ , which is consistent with the  $M$ -independent behavior of  $N_c(M)$  for PPG-DME and PDMS (Fig. 2). Hence, the observed correlation between  $\mathcal{R}(M)$  and  $N_c(M)$  appears to be general for polymers.

Both nonpolymeric and long-chain (regime III) polymeric glass formers often satisfy  $\Delta H_\beta \approx 24RT_g$  [29,42,70], which suggests a typical value of the ratio,  $\mathcal{R} \approx \frac{1}{24} \ln(100 \text{ s}/\tau_0) \approx 1.4$  and a direct link between the  $\alpha$  and  $\beta$  relaxations. In fact, as long ago as 1940, Kauzmann and Eyring [71] suggested that flow in polymers results from “flow segments” comprising  $\sim 5$ – $10$  bonds. Numerous studies have since tried to link local dynamics, typically on the scale of the flow segment, to the structural  $\alpha$  relaxation [72–77].

**Dynamic facilitation**—We recently suggested that the link between the  $\alpha$  and  $\beta$  relaxations could be explained by “dynamic facilitation” (DF) [23,78,79], whereby a local



relaxation facilitates adjacent relaxations [23,29]. In polymers near  $T_g$ , we can apply DF to two situations: (1) For oligomers of stiffer polymers (such as PMMA or PS) at  $T \sim T_g$ , dense packing and relatively large dihedral barriers restrict local intramolecular motion to arise from cooperative motion, involving a few adjacent dihedral angles. These local cooperative rearrangements can propagate along the chain (intramolecular facilitation) and facilitate the rearrangement of the entire oligomer, resulting in the  $\beta$  relaxation and its associated enthalpy  $\Delta H_\beta(M)$ . (2) This newly mobile oligomeric chain can facilitate the mobility of other oligomers through intermolecular facilitation, to yield the  $\alpha$  relaxation and its associated activation enthalpy  $\Delta H_\alpha(T_g)$ . For longer chains, chain folding divides the chain into  $\beta$  relaxation “beads,” leading to a near constant  $\Delta H_\beta(M^*)$  for  $M \geq M^*$ . Here, the structural  $\alpha$  relaxation (and thus  $T_g$ ) results from propagation of mobility through either intramolecular (along the chain) or intermolecular facilitation of the  $\beta$  beads; the nature of the facilitated dynamic coupling varies with chain length, thus separating  $T_g(M)$  into distinct dynamic regimes (see schematic illustration in Fig. 3).

A hallmark of DF is hierarchical relaxations [78,79], where relaxation on one length scale facilitates adjacent relaxation on a larger length scale, leading to a logarithmic relationship between the length scale  $\ell$ , separating fundamental excitations (relaxations) with energy  $\Delta E_\sigma$ , and the activation barrier  $\Delta E = \Delta E_\sigma[1 + \nu \log(\ell/\sigma)]$  of the resulting facilitated relaxation, where  $\nu \sim \mathcal{O}(1)$  is a constant,  $\ell(M)$  is the average length scale between fundamental relaxations, and  $\sigma(M)$  is their size [23]. By applying this reasoning to  $\Delta H$  data from BDS [29], we recently suggested that the ratio of  $\alpha$  and  $\beta$  barriers at  $T_g$  obeys

$$\mathcal{R}(M) \equiv \frac{\Delta H_\alpha(M)}{\Delta H_\beta(M)} = \left[ 1 + \nu \log\left(\frac{\ell(M)}{\sigma(M)}\right) \right], \quad (1)$$

which is proportional to the maximum number of  $\beta$  events that act in sync to facilitate the  $\alpha$  relaxation [29,79]. This is in the spirit of the suggestion by Fujimori and Oguni [80] that  $\mathcal{R}$  varies with the size of the correlation region in glass formers. The simple long-chain relation  $\mathcal{R} \simeq \frac{1}{24} \log(100 \text{ s}/\tau_0)$  breaks down at smaller molecular weights ( $M < M^{**}$ ) in stiff polymers, where intramolecular facilitation plays a role in the structural relaxation.

**Discussion**—The remarkable similarity between the  $M$  dependence of the barrier ratio  $\mathcal{R}(M)$  and  $N_c(M)$  is more difficult to interpret, and even though dynamic facilitation directly implies dynamic heterogeneities [23], DHs can exist without DF [19]. However, the growing evidence for DF in glass-forming systems [22,23,29,81–84] makes it increasingly important to identify links between DF properties, such as  $\mathcal{R}(M)$ , and the size of DHs, as reflected in

$N_c(M)$  or the corresponding characteristic length scale  $\xi_{\text{DH}}$  [85]. Computer simulations and experiments [22,87–89] have investigated the  $T$ -dependent geometry of DHs, with several studies finding that dynamic clusters (CRRs) become increasingly compact near  $T_g$ , so that the characteristic length scale  $\xi_{\text{DH}} \propto N_c^\gamma$  with  $\gamma \approx 1/3$ . If we use the simple approximation of compact CRRs, then the corresponding length  $\xi_{\text{DH}}(M) \sim N_c(M)^{-1/3}$  roughly scales with  $\mathcal{R}(M)$  (Fig. 3; see Fig. S8 of Supplemental Material [43] for more discussion). Following the DF interpretation [29,79], the ratio  $\mathcal{R}$  corresponds to the maximum number of  $\beta$  relaxation beads that need to be jointly activated to facilitate the structural  $\alpha$  relaxation.

Several recent computational studies have uncovered relations between DF and secondary relaxation contributions. Recent simulations by Guiselin *et al.* [82] and Scalliet *et al.* [83] used a swap Monte Carlo technique to access equilibrated temperatures near  $T_g$ , finding that slow regions relax by DF by spreading of mobility from more localized relaxations situated within an “excess wing” on the high-frequency side of the structural  $\alpha$  relaxation response (analogous to our picture of  $\beta$  relaxations facilitating the  $\alpha$  relaxation). Ortlieb *et al.* [22] combined simulations with experiments on a colloidal glass former [22], finding that each “particle” participating in a CRR takes part in many excitations (DF) during the lifetime ( $\sim \tau_\alpha$ ) of a CRR, leading the authors to speculate that CRRs form by accumulation of excitations [90]. Finally, Nishikawa and Berthier [84] demonstrated that, in a three-dimensional lattice glass model near  $T_g$ , structural relaxation is driven by a small population of mobile particles (with low activation barriers) acting as emerging quasi-particles that drive DH.

**Conclusion**—We demonstrate how chain length and chain flexibility affect dynamic heterogeneities in linear polymers. Highly flexible polymers show simpler behavior with chain-length-independent dynamic heterogeneity, whereas less flexible polymers show a complex chain-length dependence resulting from an interplay between intra- and intermolecular cooperativity. Our results provide a benchmark for new theories and models of glass formation in polymers and other glass formers with internal degrees of freedom coupled to molecular motion.

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**Data availability**—The data that support the findings of this article are openly available [91].

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- [95] Since  $C_p = C_V + TV\lambda^2/\kappa_T$ , where  $\lambda$  is the thermal expansion coefficient and  $\kappa_T$  is the isothermal bulk modulus, the approximation  $C_V \simeq c_p$  will slightly overestimate the number of the correlated monomers from Eq. (A1). Hempel *et al.* [7] calculated a correction factor based on data for both small molecular and polymeric glass formers, which if applied would increase the determined correlation volume values by  $\approx 3\%$ . However, since (i) the correction factor is small, (ii) we do not know how the correction varies for the different chemistries, or  $M$ , investigated here, and (iii) a correction will not affect the conclusions of this Letter, we follow the approach normally used in literature and approximate  $c_V \approx c_p$ .
- [96] The amplitude-renormalized  $c_p''(T)$  peaks (inset to Fig. 4) show a broadening  $\delta T$  with increasing  $M$ . The asymmetry for longer PMMA chains is due to vitrification effects below  $T_g$  [45], so the temperature range of the Gaussian fit was limited to  $T \gtrsim T_g$ . Qualitatively similar trends ( $T_\alpha$  and  $\delta T$  increasing with  $M$ ) are observed for PS and PDMS (see Fig. S3 in [43]). The increase in  $\delta T$  is more prominent in PMMA and PS than in the more flexible PPG-DME and PDMS.
- [97] The excess wing has commonly been attributed to a “hidden”  $\beta$  relaxation [12,62,92] or, alternatively, to an intrinsic feature of the  $\alpha$  relaxation [93].

## End Matter

We use two different methods for estimating the number of dynamically correlated units involved in the  $\alpha$  relaxation. In the first approach due to Donth [11], the mean square temperature fluctuations  $\delta T^2$  within a

rearranging region are related to the breadth  $\delta T$  of the calorimetric glass transition response in  $c_p''$  and, in turn, to the volume  $V_c$  of the correlated regions by  $V_c = k_B T_\alpha^2 \Delta c_V^{-1} / (\rho \delta T^2)$  [3,11].

The peak in  $c_p''(T)$  occurs at  $T_\alpha$ ,  $\rho$  is the mass density, and  $\Delta c_V^{-1} = c_{V,g}^{-1} - c_{V,l}^{-1}$  is the difference in reciprocal isochoric specific heat of the glassy and liquid states at  $T_\alpha$ . The number of monomers  $N_c = \rho V_c N_A / M_0$  taking part in correlated motion can be estimated as

$$N_c(M, P) = \frac{k_B N_A T_\alpha^2 \Delta c_V^{-1}}{M_0 \delta T^2}, \quad (\text{A1})$$

where  $N_A$  is Avogadro's number,  $M_0$  is the monomer molar mass, and  $P$  is the period of the TMDSC oscillation.

TMDSC measurements were performed on PMMA, PS, PPG-DME, and PDMS, as described in Supplemental Material [43], yielding the complex specific heat capacity  $c_p^*(T) = c_p'(T) - i c_p''(T)$ . The real component  $c_p'(T)$  has a step across the glass transition (see Fig. S1 of [43]), which yields  $\Delta c_p^{-1}$ . The imaginary component  $c_p''(T)$  shows a peak at the transition temperature  $T_\alpha$ ;  $T_\alpha$  and  $\delta T$  were calculated based on a Gaussian fit to  $c_p''(T)$  as in [45]. We follow the literature [7,9,94] and approximate  $c_V \approx c_p$ , which slightly overestimates  $N_c$  but does not affect our conclusions [95].

$c_p''(T; M)$  are shown for PMMA in Fig. 4, measured using a modulation period  $P = 60$  s, corresponding to  $\tau_\alpha \approx 10$  s. A clear increase in the peak temperatures  $T_\alpha$  and in the peak broadening  $\delta T$  [96] are observed with increasing  $M$ . As shown in Fig. S5 (Supplemental Material [43]), the variation with molecular weight is smaller for the flexible polymers PPG-DME and PDMS than for PMMA or PS, which is expected due the higher chain

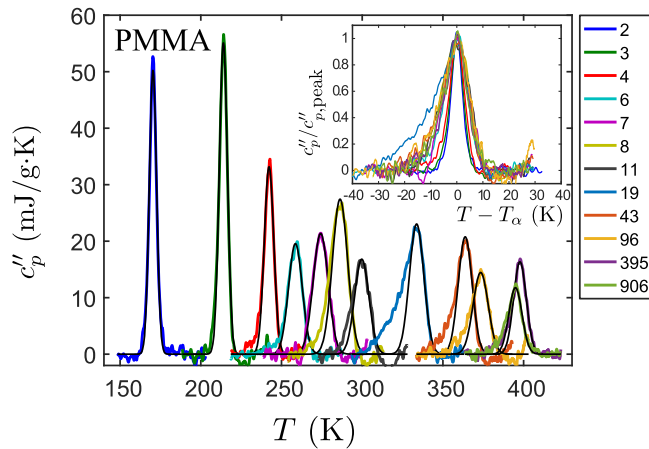


FIG. 4. The imaginary part of the  $T$ -dependent specific heat  $c_p''(T)$  for PMMA. The legend shows the degree of polymerization  $n$ . The peak in  $c_p''(T)$  corresponds to the response due to the structural  $\alpha$  relaxation, with Gaussian fits (black lines). The inset shows all  $c_p''(T)$  data normalized and centered on the peak temperature  $T_\alpha$ .

flexibility of PPG-DME and PDMS [29] that leads to a smaller variation in  $T_g$ .

The second method uses BDS to estimate the number of monomers within a dynamic correlation volume, using the fluctuation-dissipation-based approach of Berthier *et al.* [31,32]. They showed that a “three-point” dynamic susceptibility is a lower bound to  $\chi_4$ , so that the number of dynamically correlated monomers  $N_c^{(4)}(T)$  is

$$N_c^{(4)}(T) \approx \frac{k_B N_A}{m_0 \Delta c_p} T^2 \max_\omega \left\{ \left| \frac{d\chi(\omega, T)}{dT} \right| \right\}^2, \quad (\text{A2})$$

where  $m_0 \Delta c_p$  is the difference in isobaric monomer molar heat capacity between the liquid and glass, and  $\chi(\omega, T) = [\epsilon'(\omega, T) - \epsilon_\infty(T)] / [\epsilon'(0, T) - \epsilon_\infty(T)]$  is the normalized dynamic susceptibility [32]. Here,  $\epsilon'(\omega, T)$  is the real component of the complex permittivity and  $\epsilon_\infty(T)$  is its high-frequency limit. The structural relaxation times  $\tau_\alpha(T)$  associated with the response  $\chi(\omega, T)$  were determined using the fitting approach described in Supplemental Material [43], which allowed conversion from  $N_c^{(4)}(T)$  to  $N_c^{(4)}(\tau_\alpha)$ .

Figure 5 shows  $N_c^{(4)}(\tau_\alpha)$  for PMMA and PPG-DME from BDS data (open symbols) compared with  $N_c$  calculated for  $\tau_\alpha \approx 10$  s from TMDSC data (solid symbols). In both cases, extrapolation of the BDS data shows that  $N_c^{(4)}(10 \text{ s}) \simeq N_c$ . For PMMA,  $N_c^{(4)}(\tau_\alpha)$  was only calculated for oligomers ( $n = 2 - 7$ ), since for longer chains the  $\alpha$  relaxation response is increasingly obscured by a strong secondary

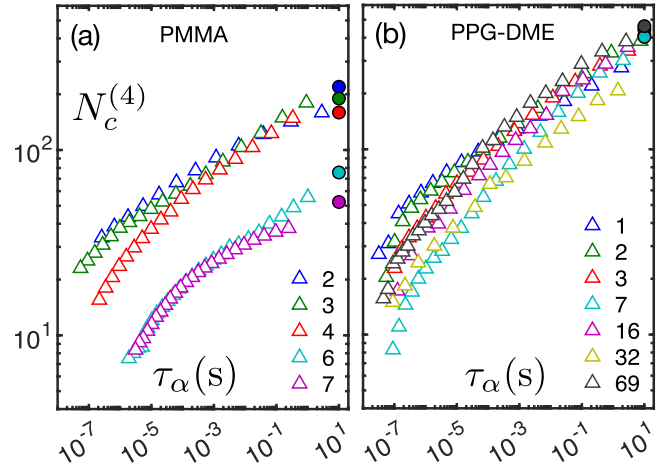


FIG. 5. The number of monomers  $N_c^{(4)}$  ( $\Delta$ ) undergoing correlated motion during the structural  $\alpha$  relaxation for (a) PMMA and (b) PPG-DME, determined using BDS, as a function of the  $\alpha$  relaxation time  $\tau_\alpha$ , for  $n$  noted in the legends. The extrapolation of  $N_c^{(4)}$  to  $\tau_\alpha = 10$  s agrees with the corresponding value  $N_c(\tau_\alpha = 10 \text{ s})$  from TMDSC ( $\bullet$ ).



$\beta$  relaxation [29], hindering an accurate determination of  $\chi(\omega, T)$ . The corresponding plots for PS and PDMS are shown in Fig. S7 of [43]. For PDMS, we generally find good correspondence between  $N_c^{(4)}(10 \text{ s})$  and  $N_c$ . For PS, a so-called excess wing [68,93] on the high-frequency

flank of the  $\alpha$  relaxation obscures the  $\alpha$  response [97], which increases the uncertainty in the absolute values of  $N_c^{(4)}(\tau_\alpha)$ . However,  $N_c^{(4)}$  increases monotonically with  $\tau_\alpha$ , and both  $N_c^{(4)}$  and  $N_c$  show a similar variation with chain length for  $\tau_\alpha \sim 10 \text{ s}$ .