Capturing the Interconnected Development of Whole Number Arithmetic Operations Using a Network Approach

Chang Xu[a]*, Sabrina Di Lonardo Burr[b], Shuyuan Yu[cd], Jo-Anne LeFevre[ce], Hongxia Li[f], Jiwei Si[f], Yudan Wang[f], & Siyao Wu[f]

[a] School of Psychology, Queen's University Belfast, United Kingdom
[b] School of Psychology, University of Sheffield, United Kingdom
[c] Department of Cognitive Science, Carleton University, Canada
[d] School of Education, University of California, Irvine, the United States
[e] Department of Psychology, Carleton University, Canada
[f] Shandong Provincial Key Laboratory of Brain Science and Mental Health, Faculty of Psychology, Shandong Normal University

*Co-corresponding Author:
Chang Xu, Ph.D.
Email Address: c.xu@qub.ac.uk
School of Psychology
Queen's University Belfast
David Keir Building, 18-30 Malone Road, Belfast, BT9 5BN

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Authors' Note

Chang Xu: https://orcid.org/0000-0002-6702-3958
Sabrina Di Lonardo Burr: https://orcid.org/0000-0001-6338-9621
Shuyuan Yu: https://orcid.org/0000-0002-5993-1681
Jo-Anne LeFevre: https://orcid.org/0000-0002-1927-7734
Hongxia Li: https://orcid.org/0000-0002-5052-2742
Jiwei Si: https://orcid.org/0000-0003-1342-7348
Yudan Wang: N/A

Siyao Wu: N/A

Authors' contributions

Chang Xu – conceptualization; data curation; formal analysis; methodology; visualization; writing – original draft; writing – review & editing

Sabrina Di Lonardo Burr – conceptualization; formal analysis; methodology; writing – original draft; writing – review & editing

Shuyuan Yu – conceptualization; writing – review & editing

Jo-Anne LeFevre – conceptualization; writing – review & editing

Hongxia Li –funding acquisition; investigation; data curation; project administration; resources; writing – review & editing

Jiwei Si – data curation; resources; supervision; writing – review & editing

Yudan Wang – writing – review & editing

Siyao Wu – writing – review & editing

Data availability statement

To promote transparency and openness, the anonymized data, and code for the measures analyzed in the current study are available for download at the Open Science Framework (OSF): https://osf.io/2y7fe/

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Abstract

We used network analysis to examine the structure and development of arithmetic fluency in Chinese students from Grades 3 to 6—a critical period during which fluency across the four operations becomes increasingly integrated. In two preregistered studies, students completed timed fluency tasks in addition, subtraction, multiplication, and division. In Study 1, we compared network structures in Grade 3 (N = 1,072; M_{age} = 9.1 years) and Grade 6 (N = 1,160; M_{age} = 12.1 years). We found that students in Grade 6 demonstrated more strongly interconnected and uniformly structured networks than those in Grade 3. In Study 2, students (N = 1,055; M_{age} = 9.8 years) were assessed in a longitudinal design at four time points from Grades 4 to 5. Addition and subtraction consistently emerged as central operations, forming the foundational core of the arithmetic network. Division reflected significant integration of knowledge from other operations whereas multiplication generally showed weak connections with the other operations. Overall, development was highly interdependent with improvements in one operation closely linked to gains in others. This research provides empirical evidence that arithmetic knowledge evolves from a differentiated structure into a unified and interconnected system, highlighting the value of viewing arithmetic development as a dynamic network of associations that consolidate over time.

Abstract Word Count: 212

Public Significance Statement

We found that, during primary school, students’ development in arithmetic is highly interdependent, with progress in one operation closely linked to gains in others. Addition and subtraction emerged as core building blocks of arithmetic fluency, while division showed strong integration with other operations and multiplication showed weaker connections. These findings provide developmental evidence that arithmetic skills form increasingly cohesive and interrelated networks over time.

Capturing the Interconnected Development of Whole Number Arithmetic Operations Using a Network Approach

Arithmetic fluency – the ability to accurately and efficiently perform computations involving addition, subtraction, multiplication, and division – is a foundational skill in mathematics. This proficiency not only underpins academic achievement and predicts later success in advanced mathematics (McNeil et al., 2025), but also supports everyday decision-making in areas such as personal finances and health (Reyna et al., 2009; Sunderaraman et al., 2022). Developing arithmetic fluency involves more than mastering each operation in isolation; it requires the *integration* of arithmetic knowledge, that is, the process of building a more advanced understanding by connecting various number associations into a coherent mental representation (Clements et al., 2023; Hiebert, 1988; Siegler & Chen, 2008; Xu & LeFevre, 2021). Although researchers have extensively studied arithmetic fluency, both as an outcome and as a predictor of other mathematical skills (McNeil et al., 2025), few have explored the interconnections among arithmetic operations and how these connections develop over time. In the present research, using network analysis, we examined the structure of arithmetic networks in students from Grades 3 to 6, a critical period during which fluency across all four operations strengthens and becomes increasingly integrated.

Theoretical Models on the Integration of Arithmetic

The development of arithmetic fluency relies on the integration of a complex set of number associations. For example, the numbers 9 and 3 may be simultaneously linked to 12, 6, 27, and 3 through addition, subtraction, multiplication, and division. Early in learning, students undergo a process of *differentiation*, where they must recognize that solving 9 – 3 = 6 is conceptually different from solving 9 × 3 = 27. As students advance in their mathematical development, these associations no longer remain isolated. Instead, these associations begin to integrate and coexist within a shared mental network that becomes increasingly interconnected over time (Deacon, 1997; Hiebert, 1988; LeFevre & Bisanz, 1986; Werner & Kaplan, 1956). This progressive integration reflects a broader developmental process in which initially separate numerical concepts are coherently coordinated, supporting the emergence of more advanced mathematical understanding (Siegler & Lortie-Forgues, 2014).

Several theoretical frameworks, including the *Associative Network* model (Ashcraft, 1992), the *Distribution of Associations* model (Siegler, 1988), the *Network Interference* model (Campbell, 1995), the *Interacting Neighbours* model (Verguts & Fias, 2005), the *Identical Elements* model (Rickard, 2005), and the *Triple-Code* model (Dehaene, 1992) operate under the assumption that arithmetic knowledge is represented through interconnected mental codes or networks in which associations between numerals and operations vary in strength and accessibility. Stronger associations are more readily and accurately retrieved, facilitating more efficient problem solving. Thus, building a well-integrated network of arithmetic associations is essential for developing proficient arithmetic skills.

The Development of an Integrated Arithmetic Network

As number associations become more readily accessible, students' approaches to solving arithmetic problems shift: Less fluent students rely on less efficient strategies, such as counting and repeated addition for multiplication (LeFevre et al., 1996) whereas more fluent students, who are able to flexibly manipulate numbers, select more efficient and appropriate strategies for a given problem (e.g., $19 \times 4 = 20 \times 4 - 4$; Geary, 1994; Hickendorff et al., 2019; McMullen et al., 2017; Siegler, 1988; Torbeyns et al., 2004). With continued practice, these associations are further strengthened, enhancing accessibility and flexibility. Over time, this process leads to a denser and more integrated mental network that supports the use of efficient strategies across a wide range of arithmetic tasks (Siegler, 1996; Siegler & Lortie-Forgues, 2014).

Expanding beyond arithmetic, the *Hierarchical Symbol Integration* model (HSI; Xu et al., 2019, 2023) proposes a hierarchical structure of mathematical competence. Fundamental numeracy, including cardinal and ordinal associations, forms the foundation upon which arithmetic associations are built, progressively supporting the development of more advanced concepts such as rational numbers and algebra. Within these arithmetic associations, the HSI model emphasizes that additive associations (i.e., addition and subtraction) form the foundation for developing multiplicative associations (i.e., multiplication and division). Because addition and subtraction as well as multiplication and division are conceptually and procedurally complementary (Robinson, 2017), practicing one operation (e.g., addition) can strengthen its complementary counterpart (e.g., subtraction; Buckingham, 1927; Campbell & Agnew, 2009; Campbell & Alberts, 2009; De Brauwer & Fias, 2011). Moreover, multiplicative reasoning builds upon additive knowledge (Harel & Confrey, 1994; Steffe, 1992), suggesting that additive associations are essential components that students use to build multiplicative associations.

Empirical evidence supporting this hierarchical view is limited. Cross-sectional studies show that the relations between operations vary across grade levels (Thevenot et al., 2023) and that these relations may be dependent on students' arithmetic skill level (Huber et al., 2013). To our knowledge, only one longitudinal study has examined the hierarchical development of arithmetic. Xu et al. (2021) found that addition and subtraction became increasingly interrelated from Grade 2 to Grade 3 (approximately aged 7-8 years) for Canadian students. Reciprocal relations were found, such that Grade 2 addition predicted growth in subtraction and vice versa, supporting the integration of these additive associations. Interestingly, they also found that subtraction, acquired later than addition, uniquely predicted multiplication performance in Grade 3, suggesting that subtraction captures a higher level of associative integration at this age. Although these prior studies provide insights into the interrelations among arithmetic operations, to fully understand how arithmetic knowledge becomes integrated, it is important to investigate all four operations together over a broad developmental span. Additionally, because instruction is strongly linked to mathematics outcomes, curriculum and educational experiences need to be considered.

The Chinese educational context is particularly well-suited for studying arithmetic integration due to its intensive early focus on the four operations. In Grade 1, students are introduced to addition and subtraction within 100. In the first half of Grade 2, students are introduced to multiplication (9 × 9 table), often taught through rhymed learning phrases designed to aid memorization (Zhang & Zhou, 2003). In the latter half of Grade 2, division is introduced as the inverse of multiplication, and students learn to compute quotients directly. By Grade 3, students are expected to have developed the skills necessary to accurately solve arithmetic problems for all four operations and are developmentally positioned to begin integrating them into a cohesive mental network through sustained practice and application.

The Present Research

Despite the numerous theoretical models of associative networks in arithmetic, there is little empirical information about the structure and characteristics of an integrated network. In the present preregistered studies (Xu et al., 2025), we examined the interconnections among arithmetic operations in Chinese students from Grades 3 to 6, using network analyses to examine patterns of intercorrelations among the four whole number arithmetic operations. Unlike traditional regression, network analysis accommodates multicollinearity by estimating partial associations, that is, the unique relations between each pair of variables while accounting for all others. This approach makes it well-suited for examining conceptually related constructs that are inherently intertwined, like arithmetic operations.

In network models we have nodes (i.e., observed variables – in this study, students' fluency in addition, subtraction, multiplication, and division) and edges (i.e., the strength of association between nodes; Borsboom et al., 2021). Each edge has an associated weight. In the present research, edge weights provide insights into integration, operationalized as the strength of the interconnections among nodes. More integrated networks are characterized by stronger interconnections among the four operations, whereas less integrated networks are characterized by weaker or absent connections between operations. Furthermore, longitudinal network models capture dynamic, rather than static, relations as they unfold over time, thereby offering insights into how integration develops. Thus, this method can be used to capture the developing structure of arithmetic fluency during the critical period when students' arithmetic skills are expected to become increasingly integrated. In the present research, we applied this framework to two datasets collected with students in China: A cross-sectional dataset with children in Grade 3 and Grade 6 (Study 1) and a longitudinal dataset in which children were assessed at six-month intervals starting in Term 1 of Grade 4 and continuing into Grade 5 (Study 2).

Study 1

Do students in Grade 3 possess less integrated arithmetic knowledge than students in Grade 6, as reflected in differences in network structures? To address this question, we compared the arithmetic network structures of students in Grade 3 and Grade 6, focusing on both the overall structure of each network and the strength of interconnections between operations. By Grade 3, Chinese students are expected to have received formal instruction on all four whole number arithmetic operations. Grade 3 thus provides insight into an early phase of arithmetic network development, characterized by limited experience, particularly with division, which is taught last among the four operations. In contrast, Grade 6 is the last year of elementary education, by which time students have had three additional years of practice. Several theoretical frameworks support the view that extended practice fosters both automatization and integration of skills (e.g., Anderson, 1982; Logan, 1988; Rickard, 2005; Shrager & Siegler, 1998). Within this broader literature, it has been proposed that fluency is not simply a matter of speed, but reflects qualitative shifts in cognitive processes from calculation, to retrieval, to automatic recognition of the answer, with practice leading to faster, more automatic execution and stronger interconnections among component processes (Anderson, 1982; Tenison & Anderson, 2016). More broadly, developmental theories, such as Siegler's overlapping waves framework, suggest that with increasing experience, students adopt more efficient strategies and strengthen the links among those strategies (Siegler, 1996). Taken together, these theoretical perspectives suggest that Grade 6 students should demonstrate more highly integrated arithmetic networks than students in Grade 3.

To examine whether the networks differed in their overall level of integration, we compared the global strength of the network structures between Grades 3 and 6 using *Multigroup Network Modelling*. Given the substantial differences in arithmetic experience, we expected students in Grade 6 would show stronger network structure among arithmetic operations compared to those in Grade 3, reflecting a higher degree of integration in the Grade 6 network (Hypothesis 1). We also expected the relative strength of connections between pairs of operations within each network to differ, with students in Grade 3 showing more variability in edge weights, reflecting a less integrated arithmetic structure, and students in Grade 6 showing more uniform edge strengths across operation pairs, reflecting a more coherent network (Hypothesis 2).

Study 2

Using longitudinal data collected at 6-month intervals over a two-year period, in Study 2 we investigated the developmental process of interconnections among the four operations from Grade 4 to Grade 5. During these years, formal instruction begins to focus on more advanced mathematical concepts such as fractions and decimals; however, these topics are deeply rooted in whole number arithmetic. For example, performing operations with fractions often relies on whole number arithmetic fluency, particularly in finding common denominators and understanding equivalence. This foundational reliance makes the upper elementary years a critical period for examining the continuing integration of core arithmetic operations. Following the framework proposed by Borsboom et al. (2021), we estimated three complementary network models: 1) *temporal*, which captures directional relations over time, allowing us to examine how performance in one arithmetic operation predicts, and is predicted by, performance in other operations at subsequent time points; 2) *contemporaneous*, which captures partial correlations among operations within the same time point, controlling for temporal influences; and 3) *between subjects*, which captures the partial correlations between operations based on individuals' average performance across time. Given the absence of prior studies examining the developmental integration of arithmetic using a network approach, our hypotheses were exploratory. Broadly, we hypothesized that as students consolidate and apply their arithmetic knowledge while learning more advanced mathematical concepts, the four operations (i.e., addition, subtraction, multiplication, and division) would increasingly support one another's development over time (Hypothesis 3). This hypothesis would be reflected in the temporal network, where growth in each operation is expected to predict, and be predicted by, growth in the other operations over time.

Method

Participants

Study 1

Participants in Study 1 were recruited from two public elementary schools located in a northern Chinese city with a population exceeding 6 million and an economic status approximately at the national average. Ethical approval for the study was obtained from the Institutional Review Board at Shandong Normal University and the local school board, with written informed consent collected from parents or guardians. Grade 3 students were recruited near the end of their first semester in December 2020 (N = 1,072; M_{age} = 9.1 years, SD = 0.6; 56% boys). Grade 6 students were recruited near the end of their first semester in December 2021 (N = 1,160; M_{age} = 12.1 years, SD = 0.6; 60% boys).

Study 2

Participants in Study 2 were recruited from three public elementary schools located in a northern Chinese city with a population exceeding 9 million and an economic status approximately at the national average. Ethical approval for the study was obtained from the Institutional Review Board at Shandong Normal University and the local school board, with written informed consent collected from parents or guardians. Participants (N = 1,055; M_{age} = 9.8 years, SD = 0.7; 52% boys) were assessed at four time points: the end of the first semester of Grade 4 in December 2021, the end of the second semester of Grade 4 in June 2022, the end of the first semester of Grade 5 in December 2022, and the end of the second semester of Grade 5 in June 2023.

Measures

For both Study 1 and Study 2, students completed paper-and-pencil whole number arithmetic tests based on a version of the standardized German *Heidelberg Rechen Test* (HRT; Haffner et al., 2005) adapted for use with Chinese students by Wu & Li (2006). The task consists of four subtests in addition, subtraction, multiplication, and division. For each subtest, students are presented with 40 problems in two columns on a single page, arranged in order of increasing difficulty. Students had one minute to solve as many problems as possible, in order. Scoring is the total number of correct responses, with a maximum possible score of 40. Test-retest reliability and Cronbach's α for each subtest have been found to exceed .70 for Chinese students from a large national assessment in China from Grades 1 through 6 (Wu & Li, 2006).

For the addition subtest, the left column includes a mixture of problems with single- and double-digit addends with no sums greater than 20 (e.g., 5 + 3 = __, 12 + 3 = __). The right column includes a mixture of problems with single-, double-, and triple-digit addends (e.g., 6 + 16 = __, 29 + 42 = __, 256 + 464 = __). For the subtraction subtest, the left column includes a mixture of problems with single- and double-digit minuends and subtrahends, with minuends no greater than 20 (e.g., 7 – 6 = __, 10 – 3 = __, 17 – 10 = __). The right column includes more complex problems with double- and triple-digit minuends and subtrahends (e.g., 27 – 8 = __, 55 – 25 = __, 120 – 22 = __, 452 – 395 = __). For the multiplication subtest, the left column includes a mixture of problems with single-digit multiplicands and multipliers (e.g., 4 × 2 = __, 9 × 6 = __). The right column includes a mixture of problems with single- and double-digit numbers, all less than 20 (e.g., 11 × 2 = __, 8 × 17 = __, 15 × 15 = __). For the division subtest, the left column includes a mixture of problems with single- and double-digit dividends and single-digit divisors (e.g., 6 ÷ 2 = __, 20 ÷ 4 = __). The right column includes problems with double- and triple-digit dividends (e.g., 56 ÷ 8 = __, 100 ÷ 5 = __, 450 ÷ 15 = __).

Procedure

For both Study 1 and Study 2, students completed testing during a single group session held in their classrooms during school hours. Testing was administered by two trained experimenters, each of whom either held or was pursuing a bachelor's degree in education. Data were independently entered and cross-checked by research assistants to ensure accuracy.

Transparency and Openness

For both Study 1 and Study 2, we adhered to the Journal Article Reporting Standards (JARS; Kazak, 2018). We report where and how the data were collected, justify any data exclusions, report all manipulations, and fully describe all measures used in the study. The present study was preregistered, with data and code available on the Open Science Framework (OSF; Xu et al., 2025). The data come from two larger projects focused on the relations between whole number arithmetic and fractions (see OSF for a full list of publications). The present study focused on a unique set of theoretical questions that have not been addressed in previous publications.

Analytical Plan

We followed the guidelines outlined by Burger et al. (2023) and Epskamp and Fried (2018) for implementing network analysis. To assess normality, we examined skewness and kurtosis (i.e., with values between -2 and 2 considered acceptable), identified potential outliers (i.e., using cutoff $|z| \geq 3.29$), and evaluated visualizations of the distributions of each arithmetic measure. The assumption of redundancy is not of concern in the present research because the four arithmetic operations were selected based on theoretical justification, with the understanding that they represent conceptually related yet distinct constructs. The accuracy of the edge estimates was evaluated via nonparametric bootstrapping with 2,000 samples, a procedure that allows us to determine how much the estimated connections between nodes (arithmetic operations) might vary due to sampling variability (Epskamp & Fried, 2018). The stability of the edge estimates was evaluated via the *Correlation Stability* (CS) Index using the case dropping method, which quantifies the proportion of the sample that can be removed while still maintaining a correlation of at least .70 between the edge weights estimated from the full sample and those from a subset, with 95% confidence (Epskamp & Fried, 2018). For the longitudinal data, stationarity (i.e., the assumption that the mean and variance of a variable remains stable over time) was assessed using the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) unit root test (Kwiatkowski et al., 1992).

Study 1

Network estimation for arithmetic measures in Grades 3 and 6 was performed using the *estimateNetwork* function from the *bootnet* package (Epskamp, 2023) in *R* (R Core Team, 2022), using the *psychonetrics* estimator (Epskamp, Borsboom, et al., 2018; Epskamp & Fried, 2018). There were no missing data in the Grade 3 dataset, and only an extremely small percentage of cases had missing data in Grade 6 (i.e., 0.1% for multiplication and division). Thus, missing data were unlikely to influence the interpretation of the results (Enders, 2010) and were handled using full information maximum likelihood.

For Hypothesis 1, we conducted multi-group network modelling using the fixed-effects meta-analytic Gaussian network aggregation framework to compare the global strength between the networks of Grades 3 and 6 (Epskamp, Isvoranu, et al., 2022). This method allows us to statistically assess whether the overall level of connectivity differs between Grades 3 and 6 by applying equality constraints across groups (i.e., constraining edge weights to be the same across groups). For Hypothesis 2, we compared edge weights between operation pairs within each network using the bootstrapped difference test, with Bonferroni corrections applied to reduce the risk of Type I error. Beyond the primary hypotheses, we also measured centrality, with a focus on interpreting centrality strengths (Bringmann et al., 2019) to explore whether certain nodes are more central than others.

Study 2

Following the guidelines outlined by Blanchard et al. (2022), we examined whether the data met the core assumptions for conducting network analyses using panel data. We built three networks using panel data: temporal, contemporaneous, and between-person. All models were implemented using multilevel vector autoregressive models via the mlVAR package in *R* (Epskamp, Deserno, et al., 2022). This framework incorporates both fixed effects (i.e., capturing group-level variability) and random effects (i.e., accounting for individual-level variability; Epskamp, Waldorp, et al., 2018). For Hypothesis 3, the temporal network analysis allowed us to examine the directional relations among operations over time, capturing how they dynamically influence each other's development.

Missing data were present for a small percentage of cases: 2% at Time 1, 5% at Time 2, and 14% at both Time 3 and Time 4. To determine whether there were differences between participants who completed all four waves of testing ($n = 911$) and those who missed at least one wave ($n = 144$), independent *t*-tests and χ^2 tests were conducted on students' gender and age. No significant differences were found between the complete and incomplete data groups, *p*s > .05. Thus, we were confident that data were missing at random. Missing values were estimated by multiple imputation with 20 datasets generated via the mice package in *R*, using predictive mean matching (Buuren & Groothuis-Oudshoorn, 2011).

Results

Study 1

Descriptive Statistics

An examination of the skewness and kurtosis (Table S1) revealed negative skewness in multiplication scores in Grade 3, and positive kurtosis in multiplication scores for Grades 3 and 6, and in division scores for Grade 6 (see Supplementary Materials for detail). Sensitivity analyses conducted with and without the outliers showed generally similar results; however, differences emerged in the comparisons of edge weights within each network. Given these discrepancies, we removed the identified outliers (≤1% for each operation in both grades). After removing these outliers, no concerns regarding normality remained, and we therefore report the results based on the dataset with outliers excluded in subsequent analyses. Descriptive statistics for the arithmetic measures for students in Grades 3 and 6 are shown in Table 1. All variables had positive, strong correlations, ranging from .55 to .76 in Grade 3 and from .65 to .75 in Grade 6 (see Table S2), suggesting that the arithmetic measures were strongly interrelated, reflecting a shared underlying conceptual structure. Across all operations, students in Grade 6 outperformed students in Grade 3 (see Table 1).

Table 1

Descriptives Statistics and Comparisons of the Arithmetic Measures for Students in Grade 3 (G3) and Grade 6 (G6)

Variable	N		M(*SD*)		Skewness		Kurtosis		G3 vs. G6 Performance		
	G3	G6	G3	G6	G3	G6	G3	G6	*t*	df	Cohen's *d*
Addition	1,068	1,155	23.3(*4.4*)	31.6(*4.8*)	0.2	-0.2	0.1	-0.3	-42.3	2221.0	-1.7
Subtraction	1,068	1,158	23.3(*4.7*)	30.1(*5.0*)	0.2	-0.3	-0.1	0.0	-33.1	2221.4	-1.4
Multiplication	1,061	1,152	28.4(*2.9*)	33.6(*2.8*)	-0.9	0.0	2.0	0.6	-43.3	2171.5	-1.8
Division	1,070	1,148	21.6(*6.4*)	34.9(*4.4*)	-0.5	-0.9	-0.2	0.5	-57.1	1876.8	-2.5

Global Strength Comparison Across Grades

Accuracy and stability checks (see Supplementary Materials) suggested that the edge weights were estimated with a high degree of precision, were robust to sampling variation, and had high stability. Figure 1 presents the estimated network structures showing the relations among arithmetic operations in Grades 3 and 6. In these networks, edges represent partial correlations between pairs of operations, controlling for all other nodes. The thickness of each edge reflects the strength of the partial correlation, ranging from .09 to .54 in Grade 3 and from .23 to .44 in Grade 6 (see Table 2). As shown in Figure 1, all nodes were interconnected in both Grades 3 and 6, with the strength of these connections varying across edges.

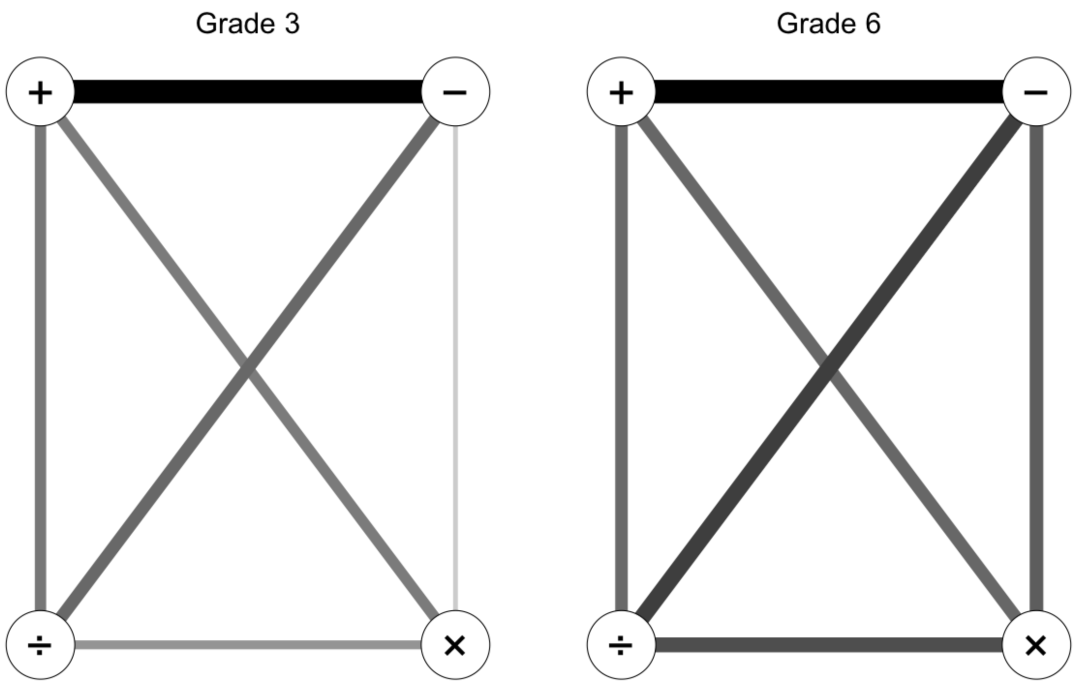
Table 2

Edge Weights Among the Arithmetic Measures for Students in Grade 3 (Below the Diagonal) and Grade 6 (Above the Diagonal)

Variable	Addition	Subtraction	Multiplication	Division
Addition	-	.44	.23	.23
Subtraction	.54	-	.25	.30
Multiplication	.24	.09	-	.27
Division	.25	.28	.19	-

Figure 1

Estimated Partial Correlation Network of Students' Arithmetic Fluency in Grades 3 and 6



Note. All relations in the networks are positive. Each node represents one of the arithmetic operations (i.e., addition, subtraction, multiplication, and division). Edge thickness reflects the strength of the partial correlation between two nodes, controlling for all other nodes in the network. No specific minimum/maximum/cut values have been used for network visualization.

To examine whether the overall strength of connections among arithmetic operations differed between grades, a multigroup network analysis was conducted. Specifically, we compared a model in which all edge weights were freely estimated across Grade 3 and Grade 6 to a model in which all edge weights were constrained to be equal across groups. The unconstrained model had a significantly better fit than the constrained model, $\Delta\chi^2(2) = 84.02$, *p*

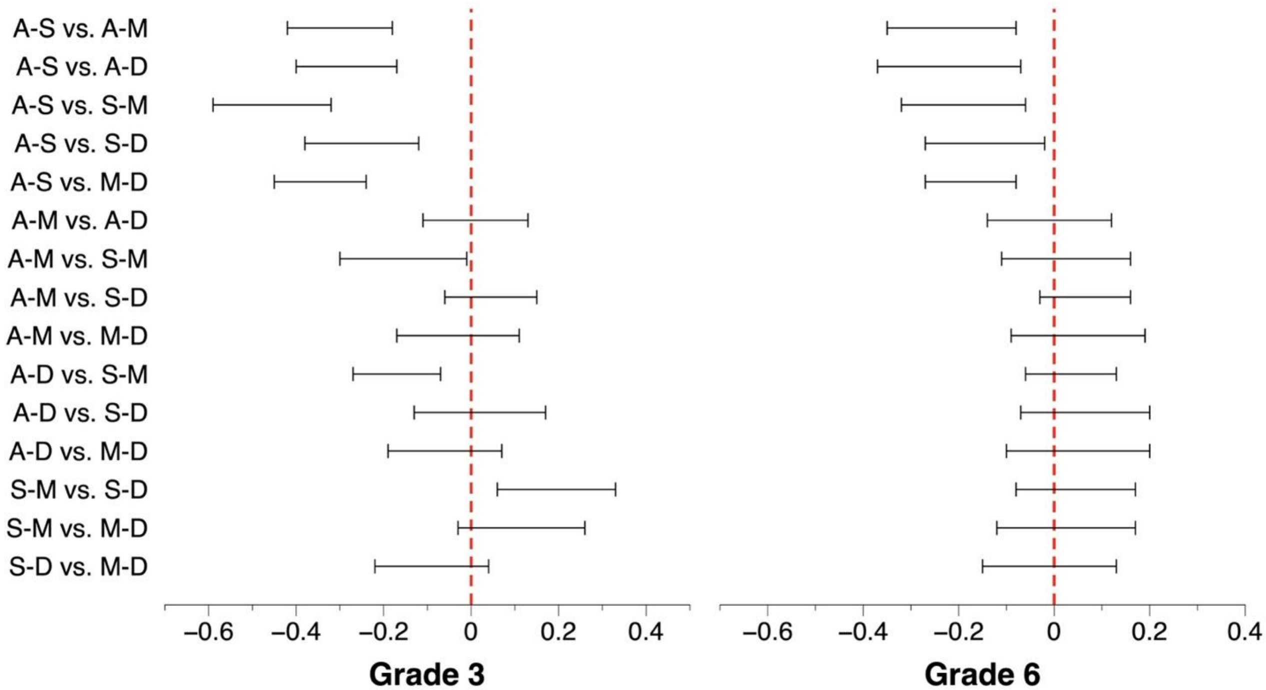
< .001. Specifically, the global strength of connections among arithmetic operations was higher in Grade 6 than in Grade 3 (1.72 vs. 1.60, $p < .001$), supporting Hypothesis 1.

Differences Between Edge Weights within Each Network

We compared edge weights between operation pairs within each network using the bootstrapped difference test, with Bonferroni correction applied for multiple comparisons ($p < .003$). Of the 15 pairwise comparisons of edge weights within each network, more than half were significant in Grade 3, whereas one-third were significant in Grade 6 (see Figure 2). Notably, the edge between addition and subtraction was consistently stronger than all other edges in both grade levels. In Grade 3, the edge between subtraction and multiplication was significantly weaker than all other pairs except for multiplication and division (see Figure S2 for estimated edge weights). In contrast, edge strengths in Grade 6 were not statistically different across operation pairs, except for the addition–subtraction edge, which was stronger than all other connections. Together, these results indicate that the network structure in Grade 3 is more differentiated, whereas in Grade 6, the network structure is more uniform, supporting Hypothesis 2.

Figure 2

Differences Between Edge Weights within Grade 3 and Grade 6 Networks



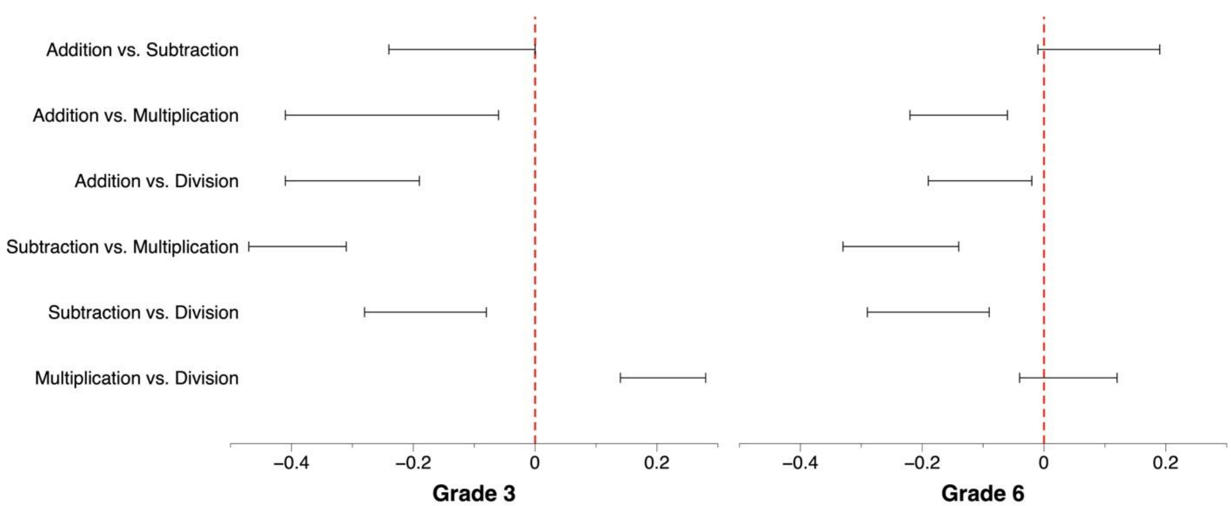
Note. Error bars represent bootstrapped 95% confidence intervals, with intervals excluding 0 indicating significant differences, with the Bonferroni correction applied ($p < .003$). A–S: Addition–Subtraction, A–M: Addition–Multiplication, A–D: Addition–Division, S–M: Subtraction–Multiplication, S–D: Subtraction–Division, M–D: Multiplication–Division.

Centrality Strengths within Each Network

In addition to comparing edge weights within each network, we examined centrality strengths in an exploratory analysis to determine whether certain nodes were more central. Central strengths in Grade 3 were 1.05, 0.52, −1.24, and −0.31 for addition, subtraction, multiplication, and division, respectively. Central strengths in Grade 6 were 0.37, 1.23, −0.99, and −0.61 for addition, subtraction, multiplication, and division, respectively. Statistical comparisons of these values, based on the bootstrapped difference test, are presented in Figure 3, with the Bonferroni correction applied ($p < .008$). In Grade 3 and Grade 6 both addition and subtraction were significantly more central when compared with multiplication and division, although addition and subtraction did not significantly differ from one another. This pattern suggests that these two operations were the most central. In Grade 3, multiplication was less central than addition and subtraction but more central than division, whereas in Grade 6 the centrality strengths of multiplication and division did not significantly differ.

Figure 3

Differences Between Centrality Strengths within Grade 3 and Grade 6 Networks



Note. Error bars represent bootstrapped 95% confidence intervals, with intervals excluding 0 indicating significant differences, with the Bonferroni correction applied ($p < .008$)

Study 2

Descriptive Statistics

An examination of the skewness and kurtosis (Table S3) revealed positive kurtosis in multiplication scores across the four time points (see Supplementary Materials for detail). Sensitivity analyses conducted with and without the outliers showed similar results; however, noticeable differences in the edge weight strengths emerged in the contemporaneous model. Thus, we removed the identified outliers ($\leq$1.5% for each operation across time points). After removing these outliers, no concerns regarding normality remained, and we therefore reported the results based on the dataset with outliers excluded in subsequent analyses. Table 3 shows the descriptive statistics for the arithmetic measures after removing the outliers. All variables had positive, strong correlations ranging from .60 to .81 (see Table S4). Students' arithmetic performance improved over the two-year period (see Supplementary Materials for ANOVAs). The KPSS test confirmed that the four arithmetic operations met the assumption of stationarity (ps > .05).

Table 3

Descriptive Statistics for Students in Grade 4 (T1 and T2), and Grade 5 (T3 and T4)

Variable	N				M(*SD*)				Skewness				Kurtosis			
	T1	T2	T3	T4	T1	T2	T3	T4	T1	T2	T3	T4	T1	T2	T3	T4
Addition	1,033	998	905	904	26.2(*4.8*)	28.1(*4.7*)	29.2(*4.6*)	29.5(*4.8*)	0.1	-0.2	-0.3	-0.5	-0.2	0.0	0.0	0.1
Subtraction	1,033	999	906	906	24.2(*5.1*)	25.8(*5.2*)	27.3(*5.2*)	27.3(*5.4*)	0.1	-0.3	-0.4	-0.6	-0.3	-0.2	0.0	0.3
Multiplication	1,023	985	897	898	29.5(*3.4*)	29.5(*3.6*)	31.5(*2.9*)	31.9(*3.1*)	-0.7	-0.9	-0.7	-0.6	0.6	1.0	1.4	1.1
Division	1,033	1,001	901	903	23.5(*7.3*)	24.6(*7.4*)	27.6(*7.1*)	30.1(*6.7*)	-0.4	-0.4	-0.8	-0.9	-0.5	-0.4	0.4	0.5

Note. T1–T4 represent Grade 4 Fall, Grade 4 Spring, Grade 5 Fall, and Grade 5 Spring, respectively.

Temporal Network

Accuracy and stability checks (see Supplementary Materials) suggested that the edge weights were estimated with a high degree of precision (see Figure S4), were robust to sampling variation, and had high stability. The estimated edge weights are shown in Table 4, and the temporal network is shown in the left panel of Figure 4. Interdependency is reflected in the presence of both incoming and outgoing arrows between operations in the temporal network, representing significant directional associations over time. These arrows are based on partial directed correlations that isolate the unique predictive contribution of one operation to another, after controlling for the influence of all other operations in the model. As seen in Figure 4, the development of arithmetic fluency appeared to be highly interconnected, with the growth of each operation statistically predicted by, and predictive of, the others. A notable exception, however, was multiplication, which showed a more limited role in the broader developmental network: It significantly predicted growth in division but did not contribute to the development of addition or subtraction, as indicated by the absence of significant outgoing edges to those nodes. Conversely, the development of division appeared to be strongly influenced by all three other operations—addition, subtraction, and multiplication—as reflected in the thicker arrows, suggesting that gains in division fluency may be particularly dependent on a well-integrated arithmetic network. Together, Hypothesis 3 was mostly supported: From Grades 4 to 5, the development of arithmetic operations was highly interdependent, with the exception of multiplication, which showed limited reciprocal connections.

Table 4

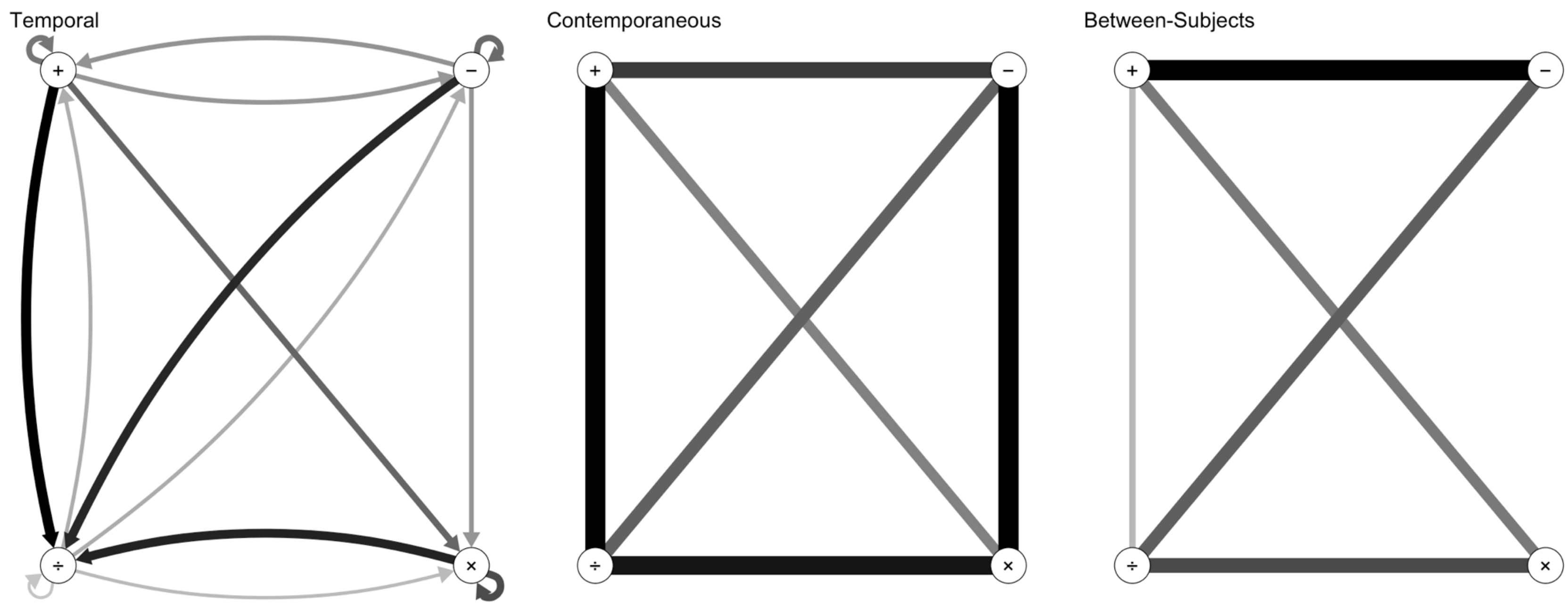
Edge Weights and Centrality Strengths of the Temporal, Contemporaneous, and Between-Subjects Networks

	Addition	Subtraction	Multiplication	Division	Temporal	
Temporal					In-strength	Out-strength
Addition →	-.16	.12	.27	.22	0.33	0.61
Subtraction →	.16	-.16	.22	.21	0.29	0.59
Multiplication →	NA	NA	-.18	.16	0.68	0.16
Division →	.16	.17	.19	-.08	0.69	0.52
	Contemporaneous, lower triangle; Between-Subjects, upper triangle				Contemporaneous	Between-Subjects
Addition	—	.58	.31	.16	0.43	1.05
Subtraction	.14	—	NA	.39	0.45	0.97
Multiplication	.09	.19	—	.39	0.47	0.70
Division	.20	.12	.19	—	0.51	0.94

Note. NA (not available) indicates the absence of an edge (i.e., partial correlation between the two nodes is not significantly different from zero, after controlling for other nodes in the network). For the temporal network, the operations in the columns and rows represent directional edge weights: The operations with → (rows) indicate nodes that predict operations (columns), with same-operation cells representing autoregressive loops. Negative autoregressive loops in the temporal network were observed, likely reflecting a regression-to-the-mean effect, where students who scored unusually high or low at one time point tended to return closer to their average level subsequently (Hamaker et al., 2021).

Figure 4

Temporal, Contemporaneous, and Between-Subjects Network



Note. Edge thickness indicates the strength of associations, controlling for other effects. In the temporal network (left), loops reflect autoregressive effects, while arrows indicate predictive relations between operations across time points. The contemporaneous network (middle) shows associations within the same time point, accounting for temporal effects. The between-subjects network (right) represents correlations between students’ average performance on each operation over time.

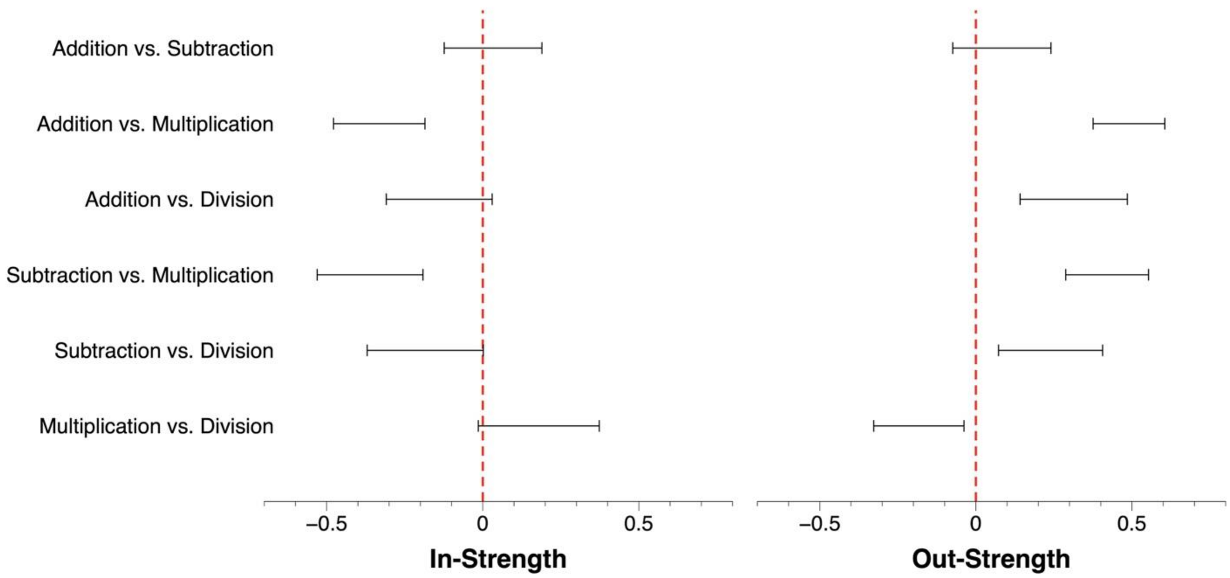
Centrality strengths, reported in Table 4, were compared using the bootstrapped difference test, with the Bonferroni correction applied ($p < .008$). As shown in Figure 5, addition and subtraction showed higher out-strength (i.e., the extent to which a node predicts other nodes over time) than multiplication and division, with no difference between addition and subtraction. Moreover, division had higher out-strength than multiplication. In contrast, multiplication and division showed higher in-strength (i.e., the extent to which a node is predicted by other nodes over time) than addition and subtraction. There were no in-strength differences between addition and subtraction, or between multiplication and division.

Contemporaneous Network

Next, we examined the contemporaneous network, which captures within-person associations between nodes measured at the same time point, after accounting for their relations with all other nodes at that time point, as well as temporal effects from the preceding time point. As shown in the middle panel of Figure 4, there were positive associations between all pairs of arithmetic operations within each time point, after controlling for all temporal effects, suggesting that arithmetic operations are not only longitudinally interconnected over time but also functionally linked within each measurement occasion, possibly reflecting shared cognitive processes or overlapping task demands. No significant differences in centrality strength were found among any pairs of operations within the contemporaneous network (see Figure 6).

Figure 5

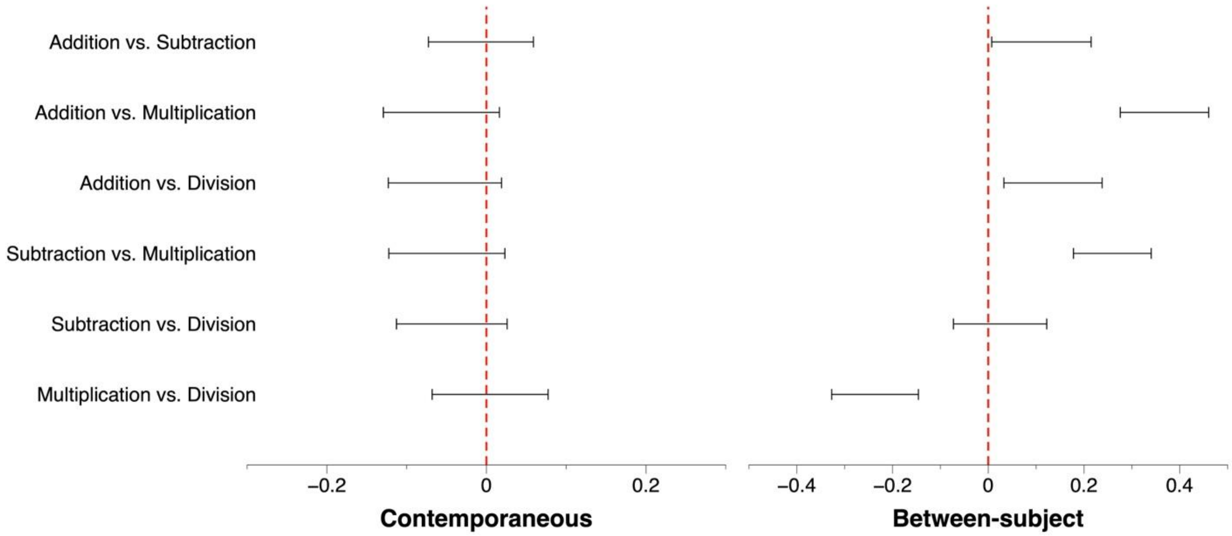
Differences Between Centrality Strengths for the Temporal Network



Note. Error bars represent bootstrapped 95% confidence intervals, with intervals excluding 0 indicating significant differences, with Bonferroni correction applied ($p < .008$). Out-strength refers to the extent to which a node predicts other nodes over time; in-strength refers to the extent to which node is predicted by other nodes over time.

Figure 6

Differences Between Centrality Strengths for the Contemporaneous and Between-Subjects Networks



Note. Error bars represent bootstrapped 95% confidence intervals, with intervals excluding 0 indicating significant differences, with the Bonferroni correction applied ($p < .008$).

Between-Subjects Network

Lastly, we examined the between-subjects network, which captures dependencies between nodes after controlling for all other nodes in the network. Unlike cross-sectional models that reflect relations at a single time point, the between-subjects network reflects how the variables are related on average across all time points. As shown in the right panel of Figure 4, the presence of undirected edges among most of the operations indicates conditional dependencies between nodes, after accounting for all other nodes in the network. Lastly, in terms of centrality strength within the between-subjects network, all pairs of operations differed significantly, except for subtraction and division. Specifically, addition was the most central node, followed by subtraction and division, with multiplication being the least central.

Discussion

Arithmetic fluency is a foundational skill that supports not only accurate and efficient computation but also facilitates conceptual understanding, flexible problem-solving, and the acquisition of advanced mathematical knowledge (McNeil et al., 2025). In the present research, we used network analysis to examine the interconnections among the four core arithmetic operations in Chinese students during a critical period of mathematical development. In Study 1, we found that the arithmetic networks of students in Grade 6, compared to those of students in Grade 3, were more strongly interconnected and had more consistent connections across operations. In Study 2, longitudinal analyses revealed that the development of these interconnections from Grades 4 to 5 was highly interdependent — improvement in one operation was predicted by advancements in the others. Furthermore, addition and subtraction formed the core building blocks of arithmetic fluency, whereas division reflected significant integration with the other operations, and multiplication generally showed weak connections with other operations. These findings offer the first developmental evidence for the integration of arithmetic operations, showing increasingly cohesive and interrelated arithmetic networks.

From Differentiation to Integration: Developing Interconnected Arithmetic Associations

Broadly, the results of the present research demonstrate a developmental progression from differentiated to integrated arithmetic knowledge between Grades 3 and 6. In Grade 3, the youngest and least experienced group in our sample, students demonstrated the most differentiated network, with strong connections between addition and subtraction and weaker or less consistent connections with multiplication and division. This pattern presumably reflects their limited exposure to the latter operations at this phase of learning. From Grade 4 to Grade 5, students' arithmetic operations became more strongly and evenly interconnected, with most operations developing interdependently, except for multiplication, which only predicted division. By Grade 6, the most experienced group in our sample, students demonstrated a uniform and highly interconnected network, with a particularly strong connection between addition and subtraction.

Our results suggest that from Grades 3 to 6, students show progress from a differentiated to a more integrated and coherent arithmetic network. The findings are consistent with prior research suggesting that the associative relations among numbers change with increased experience and exposure (LeFevre et al., 1991; LeFevre & Bisanz, 1987). Notably, these behavioural developments mirror patterns observed at the neural level, with research showing that brain regions involved in arithmetic processing become increasingly specialized and functionally segregated between the ages of 9 and 12 (Istomina & Arsalidou, 2024; Wang et al., 2022). Moreover, the absence of distinct neural signatures for individual arithmetic operations suggests that they have overlapping neutral networks (Istomina & Arsalidou, 2024), supporting the idea of integration at both cognitive and neural levels.

The development of an increasingly interconnected and largely uniform arithmetic network observed in the present research may reflect the instructional approach used in China. Arithmetic fluency is a strong pedagogical focus in the Chinese education system, where early instruction emphasizes mastery of basic operations as a foundation for developing deeper conceptual understanding (Dahlin & Watkins, 2000; Ma, 2010; Marton et al., 1996). Beginning in Grade 2, the national mathematics curriculum incorporates progressively complex mixed-operation problems that integrate addition, subtraction, multiplication, and division (e.g., 35 - 23 + 18; 63 ÷ 9 + 8 × 4; Ministry of Education of the People's Republic of China, 2022). These problems are designed to not only reinforce procedural fluency, but also to encourage conceptual integration across operations. Unlike single-operation problems, mixed-operation problems require students to distinguish between operations, understand their relational structures, and apply strategies based on the problem demands. Regular engagement with such problems may help students develop a deeper awareness of how operations are interconnected and promote flexible application of arithmetic knowledge. However, empirical research directly examining how mixed-operation practice supports arithmetic integration is still limited and represents a valuable direction for future research.

The Central Role of Additive Skills in Arithmetic Development

To further unpack arithmetic development, in the present study we closely examined the development of each operation in relation to the others. Across both studies, we observed a strong link between addition and subtraction. This strong link was expected, given that these operations are conceptually and procedurally complementary (Robinson, 2017), and are introduced together early in the Chinese elementary education. Prior research has shown that teaching and practicing addition facts enhances access to related subtraction facts (Buckingham, 1927; Campbell & Agnew, 2009). Thus, in the present study, students' extensive experience with additive operations likely contributed to the strengthened associative connections between these skills.

In relation to the other operations, addition and subtraction consistently emerged as the most central nodes in the arithmetic network, serving as critical hubs that support and integrate the development of other operations. This pattern aligns with the HSI model, which posits that arithmetic competence develops through a hierarchy of integrated associations, with additive knowledge forming the foundation from which multiplicative knowledge emerges (Xu et al., 2023). Our findings build on previous research demonstrating the predictive role of additive skills in multiplicative skills (Thevenot et al., 2023; Xu et al., 2021). Extending this line of work, we show that both addition and subtraction, which are conceptually and procedurally complementary, are equally central in supporting the development of multiplication and division. These results emphasize the view that additive operations precede and may play a predictive role in the integration of more complex multiplicative skills.

These findings underscore the critical importance of emphasizing addition and subtraction in early education, not simply as isolated skills, but as foundational and related building blocks for broader arithmetic development. Given their central role in supporting the integration of multiplication and division, instructional practices should prioritize helping students develop a deep conceptual understanding of how additive reasoning relates to multiplicative reasoning. While additive operations involve representing quantities as collections of individual units, multiplicative operations require students to conceptualize quantities as composed of equal groups of units (Clark & Kamii, 1996; Harel & Confrey, 1994; Nunes et al., 2016; Steffe, 1992). From this perspective, multiplicative representations are constructed on the foundation of existing additive understanding (Steffe, 1992). Thus, early mastery of additive associations may help support students' multiplicative reasoning by reinforcing the conceptual links between operations. Interventions targeting arithmetic fluency could benefit from ensuring that students develop a strong foundation in addition and subtraction to better support the acquisition of more advanced skills.

The Differing Roles of Multiplication and Division in Arithmetic Development

Although addition and subtraction were shown to be central to the arithmetic network, the connections between multiplication and the other operations were less consistent. In Study 1, the connections between multiplication and the other operations were generally weak in Grade 3, with the connection between subtraction and multiplication being the weakest. This contrasts with findings from Xu et al. (2021), who found that for Canadian students, subtraction uniquely predicted multiplication performance in Grade 3. In Study 2, multiplication served primarily as a predictor of division, but did not contribute to the development of addition or subtraction and it was the least central operation. Moreover, the lack of association between subtraction and multiplication in the between-subjects network suggests there are weaker cognitive or instructional links between these two operations.

These findings may reflect the curriculum and instructional approach for learning multiplication facts in China. Specifically, students are expected to fully memorize the multiplication table, using rhyming phrases by the end of Grade 2, promoting automatic retrieval. In contrast, in Canada and other countries, full memorization is not typically expected until later grades. As a result, subtraction, which reflects integrated additive associations, may play a more prominent role in supporting the development of multiplicative skills in the Canadian context, where these operations are still being consolidated. In contrast, in the Chinese context, multiplication may be more independently developed through rote memorization, resulting in weaker integration with other operations. Across time points and studies, most Chinese students were able to solve problems beyond the basic 9×9 multiplication table within a brief one-minute time limit, suggesting some memorization and possibly the use of partial retrieval strategies (e.g., for 14×6, a student might recall $10 \times 6 = 60$ and $4 \times 6 = 24$, then combine: $60 + 24 = 84$). Taken together, these differences in curriculum and instruction likely account for the contrasting patterns of association observed between subtraction and multiplication.

In contrast to multiplication, division emerged as the most dependent operation, strongly associated with prior knowledge of addition, subtraction, and multiplication. Division is often considered the most conceptually demanding of the four arithmetic operations (Greer, 1992; Parmar, 2003; Thompson & Saldanha, 2003). In the early phase of mathematics education in China, division is introduced as the inverse of multiplication—requiring students to have already memorized multiplication facts and understand their relational structure (Ministry of Education of the People's Republic of China, 2022). For example, to comprehend that $12 \div 3 = 4$, a student must recall the corresponding multiplication fact $3 \times 4 = 12$ and apply it in reverse.

Building on this foundation, a more advanced understanding of division involves flexibly applying both additive and multiplicative reasoning. For example, solving $48 \div 4$ can be approached not only by recalling the related multiplication fact ($4 \times 12 = 48$), but also through repeated subtraction (i.e., determining how many times 4 can be subtracted from 48), or by decomposing the dividend into simpler parts (e.g., $40 \div 4 + 8 \div 4$). More complex division problems, such as dividing a three-digit number by a two-digit number using a standard algorithm, require the coordination of multiple arithmetic skills – multiplication to estimate partial quotients, subtraction to track remainders, and addition to verify or adjust intermediate results. These examples show that division is not an isolated skill – it draws on a broad foundation of arithmetic knowledge and highlights the importance of integrating addition, subtraction, and multiplication.

Our findings offer empirical support for the view that division functions as a capstone operation, appearing to build on earlier-learned arithmetic knowledge and emerging only when foundational operations are well integrated. This conclusion aligns with the HSI model, which emphasizes that arithmetic competence develops through successive integration of operations, with division positioned at the top of the arithmetic hierarchy (Xu et al., 2022, 2023). Practically, this finding reinforces the pedagogical principle that instruction should be scaffolded to move students beyond procedural fluency, guiding them to recognize the relational structure among operations to foster a more flexible and interconnected understanding of arithmetic (Parmar, 2003).

Implications for Theoretical Models of Arithmetic

Several theoretical models of arithmetic converge on the idea that arithmetic knowledge is organized as an interconnected mental network (Ashcraft, 1992; Campbell, 1995; Rickard, 2005; Siegler, 1988; Verguts & Fias, 2005;). A common challenge for these frameworks, however, is the lack of empirical evidence on the structure and developmental dynamics of such networks. By applying network analysis, the present study provides an early empirical account of how the four operations become integrated across late primary school. Although most frameworks assume that associative strength underpins fluency, they rarely specify which operations should emerge as central or how their roles may shift over time.

Our network analyses revealed a strong association between addition and subtraction, which contrasts with predictions of the *Triple-Code* Model (Dehaene, 1992). According to this model, addition and multiplication should be closely related, as both are assumed to rely on verbal retrieval and to share representational pathways. One possible explanation for our different finding is that subtraction is often conceptualized and taught as the inverse of addition, fostering stronger links between these operations. Developmental and instructional factors may also play a role: in Chinese classrooms, addition and subtraction are introduced and reinforced together well before multiplication and division, further consolidating their integration (Ministry of Education of the People's Republic of China, 2022). Right from the initial introduction in Grade 1, students are taught addition and subtraction in an inverse format (e.g., 3 + 5 = 8, 8 – 3 = 5), with workbook exercises further reinforcing the close connection between these operations. This instructional emphasis may help explain why we observed such a strong addition – subtraction association in our networks. Taken together, these results suggest that the structure of arithmetic networks is not static but dynamic, shaped by both cognitive mechanisms and cultural–instructional experiences. Future theoretical work should account for these developmental differences and more explicitly integrate the role of instructional practices in shaping network organization.

Limitations and Future Research

One limitation of the present research is that we were unable to capture the full developmental trajectory of arithmetic integration from Grades 3 to 6. Although our longitudinal study (Study 2) tracked students from Grades 4 to 5, this period likely reflects a phase in which much of their basic arithmetic fluency has already been established, as evidenced by the relatively modest gains observed between time points. A longer-term panel study spanning Grades 3 to 6 would provide a more comprehensive picture of how integration evolves over time and may reveal more substantial developmental changes between time points. Future research using such a design could offer valuable insights into how students build connections among different arithmetic operations and how these connections are consolidated over the course of elementary education. Temporal network analysis in particular holds promise for tracing these dynamic patterns longitudinally.

Additionally, our ability to examine the mechanisms of arithmetic integration was limited by the constraints of group testing, which precluded the collection of item-level data such as response time and accuracy. It is possible that the observed lack of dependency between multiplication and other operations is specific to single-digit problems, where rote memorization plays a central role. In contrast, multiple-digit multiplication may require greater integration with other operations, such as addition, particularly in problems that involve carrying digits (e.g., 17×4). Thus, future research distinguishing between single- and multi-digit multiplication problems and capturing item response time would add important information about development. Moreover, in addition to capturing item-level responses, future research should consider incorporating measures of students' problem-solving strategies—for instance, whether they relied on retrieval, decomposition, or compensation. These data would shed light on the cognitive processes that underlie network integration and show how students flexibly draw on arithmetic associations as their mental networks become more interconnected. Such research could expand the HSI model by showing how integration supports efficient arithmetic problem solving.

Finally, although our primary interpretation emphasizes mathematics-specific experience and instruction, another possibility is that the increased integration among arithmetic operations from Grades 3 to 6 may reflect not only mathematics-specific development but also broader cognitive mechanisms. Research on the positive manifold suggests that as development progresses, cognitive abilities across domains become increasingly interrelated (van der Maas et al., 2006), and these skills can mutually reinforce one another over time (van der Maas et al., 2017). From this perspective, the increasingly strong interconnections among arithmetic operations observed over time in the present study may partly reflect domain-general processes, rather than solely mathematics instruction and practice. Future work that incorporates both domain-general tasks (e.g., working memory, processing speed, reasoning) and domain-specific tasks (e.g., arithmetic operations) within the same network framework will help clarify whether the observed integration is mathematics-specific or reflects broader cognitive processes.

Conclusion

The present research is the first to provide empirical evidence that arithmetic knowledge becomes increasingly integrated, progressing from differentiated, operation-specific knowledge to a more unified and interconnected system. Using network analysis, which eliminates constraints on interpretation imposed by the influence of strong correlations amongst the operations, we investigated the connections and overlap between operations that support a unified arithmetic network. Building on theoretical models, the present research provides empirical support for the representation of arithmetic knowledge in an interconnected mental network. Furthermore, we show that additive associations form the foundation for the development of multiplicative associations. These findings highlight the value of viewing arithmetic as a dynamic network of interrelated associations that consolidate over time. Solid whole number arithmetic skills provide the foundation for learning more advanced mathematical concepts and thus are crucial for students' mathematical development and achievement.

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