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Placement of dedicated lanes for autonomous vehicles considering the changes of urban spatial structure

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ABSTRACT

This paper investigates the placement schemes of dedicated lanes for autonomous vehicles (AVs) considering the changes of urban spatial structure in a closed city, where AVs and regular vehicles (RVs) coexist. An urban spatial equilibrium model for a monocentric and circular city is formulated, in which the travel and residential location choice are interdependent, considering the variable road capacity and reduced value of travel time brought by AVs. The theoretical boundary for the AV proportion in flow with the positive and negative influence of dedicated AV lanes on traffic efficiency are given. Then, the optimized placement of dedicated lanes for AVs, with and without consideration of residential relocation, are examined. First, the short-term effects of a dedicated AV lane are examined under a fixed residential distribution, with the objective of minimizing total travel time. The results indicate that a dedicated AV lane improves traffic efficiency only when the market penetration rate (MPR) of AVs is at a moderate level. Second, the long-term effects of a dedicated AV lane are considered, allowing for changes in residential location choices. In this case, the design objective is to maximize social welfare, and the optimal solution requires a shorter dedicated AV lane compared to the short-term scenario. The findings reveal the opposite spatial changes, that is, dedicated AV lanes encourage AV users to relocate farther from the city center, while RV users tend to move closer. Furthermore, neglecting the long-term relocation behavior of residents leads to inaccurate travel time estimates and excessive infrastructure investments.

1. Introduction

With the sustained development of autonomous driving technology, the traffic flow on urban roads will gradually change from only regular vehicles (RVs) to a mixed state of autonomous vehicles (AVs) and RVs. The global autonomous vehicle market amounted to almost 17,000 vehicles in 2022. It is projected that the market will grow and reach around 127,000[1] vehicles in 2030. However, the rapid popularity of the current AVs does not mean that the RV traffic flow will soon be transformed into a completely unmanned traffic flow. It will be a long process for a full market penetration rate (MPR) of AVs (Mahmassani, 2016).

New features of a mixed traffic flow can be reflected in three aspects: vehicles, roads, and travelers. Firstly, the reaction time of AV is much shorter than that of RV (Ye and Yamamoto, 2019). The rapid development and application of advanced sensors, high-performance computing units, and vehicular communication such as vehicle-to-vehicle and vehicle-to-infrastructure in autonomous driving technology have combined to improve the speed of AV reaction to the behavior of other drivers or unexpected road events (Rydzewski and Czarnul, 2021). Secondly, the road capacity with mixed traffic flow becomes elastic (Van den Berg and Verhoef, 2016; Ramezani et al., 2017; Mohajerpoor and Ramezani, 2019; Chen et al., 2022). While AVs can safely follow another AV at short distances with short reaction time resulting in increased road capacity (Sheu, 2007; Levin and Boyles, 2016), AVs following RVs do not yield such advantage (Ghiasi et al., 2017). Thirdly, travelers become less time-sensitive when taking the AV than the RV (Childress et al., 2015; Auld et al., 2017), because AVs free their users from controlling the vehicles while traveling and therefore afford them time to do other things which an RV driver could not (Krueger et al., 2016).

The above characteristics will not only change the travel cost and travel behavior of commuters in the short term but also influence their long-term choices such as residential location choices. For AV commuters, reduced sensitivity

[1] https://www.nextmsc.com/report/wireless-electric-vehicle-charging-market

to travel time will make them accept longer travel distances and live further away from the CBD to benefit from lower residential cost (Zakharenko, 2016). For RV commuters, however, the uncertainty in road capacity and travel cost changes their long-term residential location choices. These changes in residential behavior collectively influence the urban spatial structure, which in turn alters the spatial distribution of travel demand. Therefore, it is necessary to analyze the mixed transportation system from the perspective of urban spatial equilibrium.

From the infrastructure planning perspective, it is natural to consider creating a "fully automated" environment, i.e. dedicated AV lanes, in order to take advantage of AVs in reduced headway and increased road capacity. This is akin to giving dedicated lanes to high occupancy vehicles (Xiao et al., 2021; Boysen et al., 2021) or to public transportation (Shen et al., 2019; Cheng et al., 2024). Nevertheless, it is not possible to deploy dedicated lanes in all locations. The reason is that the deployment of dedicated lanes also incurs costs, and in addition, dedicated lanes occupy some road resources, which may have a negative impact on the passage of some vehicles. The key question lies in determining where to set dedicated AV lanes in urban space. In this paper, we consider the design of dedicated AV lanes that is both cost- and performance-effective. In addition, we consider the long-term effect of having the dedicated AV lane on individual residential choice and the resulting change in the spatial distribution of travel demand.

Based on the classic analytical framework of monocentric city model (Alonso, 1964; Muth, 1969; Mills, 1967), giving the fixed road capacity, the impacts of AV introduction on transportation system and urban structure were studied in discrete zones (Liu et al., 2021) and continuous urban space (Zakharenko, 2016; Larson and Zhao, 2020). There is a wealth of literature pertinent to dedicated lane management on a single highway with a bottleneck. Examples include design of time-dependent capacity allocation for high-occupancy vehicle lanes (Xiao et al., 2021) and fast lanes (Fosgerau, 2011). Other studies have analyzed and compared different deployment strategies for dedicated lanes (Ghiasi et al., 2017; Mohajerpoor and Ramezani, 2019; Jiang et al., 2023). At the network level, most of the research has been focused on the deployment of dedicated lanes, specifically the strategic decision of where to establish AV dedicated lanes, i.e. on which road segment(s) in the network (Movaghar et al., 2020; Madadi et al., 2021; Zhang et al., 2023; Ngoduy et al., 2024; Wang et al., 2024) or in which designated areas of the network (Chen et al., 2017). In these studies, the OD travel demand is assumed to be fixed or elastic with a given demand function (Zhang et al., 2023), a given diffusion function (Chen et al., 2016), and a given distribution of random demand (Chen et al., 2025). However, the interactive relationship between transportation planning and urban land use has been widely concerned (Leibowicz, 2020; Zhong et al., 2022), which should be taken into account in the placement of dedicated AV lanes.

Summarizing the existing literature, we show two significant research gaps in Table 1. First, the current articles that study the impact of AV introduction using city models ignore the spatial variation of road capacity with the mixed traffic flow. Second, existing studies on the design of dedicated lanes in a bottleneck or a single road segment have neglected spatial variation in travel demand, while those concerning network-wide dedicated lanes do not tend to consider the long-term endogenous change of travel demand distribution with residential relocation.

In order to fill these gaps, this paper proposes a spatial equilibrium model of a closed monocentric city with a mixed AVs and RVs flow, considering the variable road capacity and reduced value of time (VOT) caused by AVs. We explore the long-term endogenous change of travel demand with introduction of AVs and dedicated lane for AVs. Furthermore, we examine the spatial configuration of the dedicated AV lanes and propose a method for the design optimal placement of AV lanes including the starting and ending points of the dedicated lane in the network. The spatial impacts of dedicated AV lanes on lane speed and efficiency of the mixed traffic system are explored analytically. We

Table 1

Related studies

Citation	Traffic flow	Road capacity	Travel demand		Dedicated lane
			Spatiality	Redistribution	
Liu et al. (2021)	AVs	Fixed	No	Long term	-
Zakharenko (2016)	AVs	Fixed	Yes	Long term	-
Larson and Zhao (2020)	AVs	Fixed	Yes	Long term	-
Fosgerau (2011)	RVs	Variable	No	Exogenous	Time window assignment
Xiao et al. (2021)	RVs	Variable	No	Exogenous	Time window assignment
Ghiasi et al. (2017)	AVs & RVs	Variable	No	Exogenous	Number of lanes
Jiang et al. (2023)	AVs & RVs	Variable	No	Exogenous	Number of lanes
Chen et al. (2017)	AVs & RVs	Fixed	Yes	Exogenous	Zone designation
Madadi et al. (2021)	AVs & RVs	Variable	Yes	Exogenous	Link selection
Zhang et al. (2023)	AVs & RVs	Variable	Yes	Exogenous	Link selection
Wang et al. (2024)	AVs & RVs	Variable	Yes	Exogenous	Link selection
Chen et al. (2016)	AVs & RVs	Fixed	Yes	Endogenous MPR	Link selection
Zhang et al. (2020)	AVs & RVs	Fixed	Yes	Elastic function	Link selection
Chen et al. (2025)	AVs & RVs	Variable	Yes	Random MPR	Link selection
This study	**AVs & RVs**	**Variable**	**Yes**	**Long term**	**Start-end determination**

Note: the last column represents the different decisions of dedicated lane management. "Time window assignment" is assigning time windows during which dedicated lanes are closed to all but specified traffic. "Number of lanes" is deciding whether to implement dedicated lanes or the optimal number of dedicated lanes. "Link selection" is selecting specific road links for deploying dedicated lanes. "Zone designation" is designating specific zones only be used for specific traffic flow. "Start-end determination" is determining the starting and ending points of dedicated lanes.

show that at low AV penetration, the speed of AV lane is always higher than that of the ordinary lane. However, when AV penetration exceeds a certain threshold, the speeds of dedicated AV lanes and ordinary lanes converges. We derive analytically the conditions for increasing, decreasing, or maintaining traffic efficiency after placing dedicated AV lanes, and develop a two-layer iterative algorithm for solving the numerical solution of urban spatial equilibrium. We propose two optimization models for the placement of dedicated AV lanes: (a) a short-term one where the demand distribution remains unchanged and the objective for the optimal lane placement is to minimize total travel time; and (b) a long-term scenario where the travel demand is endogenous and is caused by residence relocation. A pattern search algorithm is developed for solving the two optimization models. We compare the differences between the long-term and short-term optimal placement schemes of dedicated AV lanes and analyze the respective impacts on the equilibrium of travel and residence.

The remainder of this paper is organized as follows. Section 2 reviews the related literature. Section 3 describes the spatial equilibrium model with a mixed transportation system. Section 4 gives some analytical properties and the simulation solution procedure for the urban spatial equilibrium. Section 5 proposes the optimization models and algorithms for the placement of dedicated AV lanes in the short and long term. In Section 6, we relax several of the underlying assumptions. Specifically, we demonstrate the following extensions: (i) variable radial roads, (ii) endogenous market penetration rate, (iii) income heterogeneity and (iv) polycentric city. Section 7 concludes this study with a summary of the results and potential directions for future research.

2. Literature review

2.1. Transportation system and urban spatial structure

This study builds upon urban spatial equilibrium models developed in the urban economics literature. Alonso (1964), Muth (1969), and Mills (1967) developed the original monocentric city model, and laid a crucial groundwork for analyzing the relationship between urban spatial structure and transportation system. Based on the fundamental models neglecting vehicle types (Arnott et al., 1991; Anas and Kim, 1996; Arnott and MacKinnon, 1977; Anas and Xu, 1999), many scholars have further extended it to multi-class transportation city models, but most separated different types of vehicles on isolated paths to focus on travel mode choice (Capozza, 1973; Franco, 2017; Xu et al., 2018).

In a traffic environment where multiple types of vehicles compete for road capacity, several studies investigated the intricate effects of mixed traffic flow on urban performance. Normalizing road capacity to unity, Hirte and Tscharaktschiew (2013) modeled a spatial equilibrium between two urban zones where households choose between conventional fuel-powered vehicles and electric vehicles for their commuting, to explore how this affects externalities, tax interactions, and the optimal policy for subsidizing or taxing electric power usage. The results showed that electric vehicles should not be subsidized but rather taxed, because the social costs of subsidizing electric vehicles exceed the benefits from reduced CO2 emissions. Considering the coexistence of single occupant vehicles and HOVs in the roads where the capacity is a fixed fraction of the land area, Zhao (2020) developed a general spatial equilibrium model in continuous urban space to examine the effects of HOV lanes on urban form, energy consumption, and emissions. It was found that HOV lanes reduce traffic congestion and improve welfare, but the fall in transportation cost leads to urban sprawl. Wang and Huang (2024) presented a two-zone monocentric city, where traffic congestion depends on the sum of electric vehicles and gasoline vehicles on the cross-zone road with a given capacity, to explore the effects of diverse government subsidy strategies for the construction of charging facilities on urban spatial structure, travel modes, and environmental outcomes. Dividing the road capacity into HOV lanes and other lanes, Konishi and Mun (2010) constructed a two-zone city model of commuters using a highway with multiple lanes, in which commuters are heterogeneous in their carpool organization costs, and later extended it to a continuous urban space. In Konishi and Mun (2010), the capacity of the lane with mixed traffic flow does not change with the composition of the traffic flow structure, and the total capacity of all lanes is fixed. However, these city models without AV introduction are not suitable for urban systems with mixed AV and RV traffic, in which the AV proportion in flow changes the headway time of car following and the road capacity varies with location. In addition, numerous studies have investigated shared autonomous vehicles (SAVs) as an emerging form of AV operation. These studies often integrate SAVs with parking management, dynamic lane allocation, advanced relocation strategies, and pricing mechanisms. Tang et al. (2021) examined morning commute dynamics under parking constraints, showing that optimal SAV penetration levels depend on transit fares and parking availability. Their findings suggested that SAVs and public transit tend to dominate when parking is removed from the system. Ji et al. (2024) focused on infrastructure planning, proposing optimal allocation strategies for dedicated AV lanes in mixed traffic environments. They emphasized that user preference for SAVs stems from cost and time savings, and that lane deployment should evolve in line with SAV adoption. Li and Liao (2020) introduced a bi-level optimization framework with a novel hub-based relocation strategy for SAVs. Their approach significantly improved system efficiency by aligning vehicle redistribution with users' multi-activity travel chains. Kaddoura et al. (2020) advocated for welfare-optimal SAV operation through congestion pricing applied equitably to both SAVs and conventional cars, aiming to manage modal shifts and mitigate externalities such as congestion and emissions. However, these studies are also limited to exploring the impacts and operational strategies within the transportation system, and do not consider the impacts of endogenous changes in the urban spatial structure.

To our knowledge, Zakharenko (2016), Larson and Zhao (2020) and Liu et al. (2021) are the only studies that theoretically analyze the impact of AV on urban spatial equilibrium. Taking into account the characteristics of AVs in terms of time value, fixed costs, and parking behavior, Zakharenko (2016) explored the effect of AVs on urban form in a monocentric two-dimensional city without road congestion. Considering that the continuous land used for AV parking and housing is determined endogenously, but the road area and capacity are given, Larson and Zhao (2020) examined the impacts of fully AVs on urban sprawl, energy consumption, and housing affordability. However, these two studies did not pay attention to the impact of AVs on headway time and road capacity. In a closed discrete two-zone city where on-demand and frequency-based AVs are used for commute, Liu et al. (2021) investigated the effect of the automation level and policy of right-of-way on traffic flow distribution and urban spatial structure. It is the only study that simultaneously considers the changes in VOT and road capacity due to AV introduction by adopting functions of VOT and road capacity with respect to automation level rather than physical car-following characteristics. To fill these shortcomings, this paper innovatively incorporates the characteristics of AV to reduce car-following headway time and VOT into the analysis framework of a monocentric continuous city. The proposed model of this paper can well describe the spatial distribution of mixed AV and RV traffic, especially the variable road capacity, and facilitates the analysis of its impact on urban spatial structure, housing affordability and social welfare of the city system. Besides, the variable road capacity in a continuous urban space also poses complexity and challenges to the spatial placement method of transportation infrastructure.

2.2. Dedicated lane management

As an important measure of lane management, there has been a wealth of literature exploring the dedicated lanes for buses (Cherry et al., 2005; Xu et al., 2013) , HOV (Dahlgren, 2002; Menendez and Daganzo, 2007; Kwon and Varaiya, 2008; Chu et al., 2012) and truck(Rakha et al., 2005; Abdelgawad et al., 2011; Cherry and Adelakun, 2012; Rudra and Roorda, 2014), which have been widely applied in traffic practices. These studies provided the methodological foundation for improving the road capacity and system performance of the mixed traffic system.

Neglecting the spatial distribution of travel demand, many scholars focused on the issues of assigning time windows for dedicated lanes at a traffic bottleneck (Fosgerau, 2011; Xiao et al., 2021) and how many dedicated lanes should be implemented at a road segment(Ghiasi et al., 2017; Menendez and Daganzo, 2007; Jiang et al., 2023). Fosgerau (2011) considered a congested bottleneck, where a fast lane reserves a more than proportional share of capacity to a designated group of travelers, and revealed that the fast lane scheme is welfare improving when demand is elastic. Employing a single Vickrey's bottleneck dynamic setting, Xiao et al. (2021) modeled a one-to-one highway corridor problem with two commute modes, solo driving and carpooling, to investigate the impacts of a temporal allocation of bottleneck capacity, when carpool lane is available only within a reserved time window, and a joint temporal-spatial capacity allocation, on morning commute patterns. Ghiasi et al. (2017) proposed an analytical capacity model for highway with mixed traffic flow based on a Markov chain representation of spatial distribution of heterogeneous and stochastic headways, and efficiently determined the optimal number of AV exclusive lanes to maximize throughput of mixed traffic flow with different demand levels, MPRs, platooning intensities, and technology scenarios. Menendez and Daganzo (2007) used a simulation to study how HOV lanes affect the performance of adjacent general purpose lanes and nearby bottlenecks, to examine a dynamic operating strategy for HOV lanes that increases a bottleneck's total discharge rate by exploiting the smoothing effect. Jiang et al. (2023) proposed a mixed capacity and lane management model considering platoon size and platoon intensity of connected AVs, to explore the optimal lane management strategy under different market penetration rate and traffic demand size. However, these lane management studies are unable to deal with the spatial changes in road capacity, which is caused by the characteristic of car-following in the mixed AV and RV traffic.

Giving a fixed spatial distribution of travel demand, a wealth of literature was concerned with selecting specific road links or zone for deploying dedicated lanes at network level (Chen et al., 2016, 2017; Liu and Song, 2019; Kumar et al., 2020; Madadi et al., 2021; Zhang et al., 2023). For example, Chen et al. (2017) designed specific areas of road network dedicated exclusively to AVs, where the route choices of AVs are fully controlled to enhance the performance of the network. Amirgholy et al. (2020) designed an optimal lane management strategy for corridors with a heterogeneous demand of human-driven, autonomous, and communicant autonomous vehicles, to study the impacts of the rise in the penetration rate of AVs on the performance of the system. Then it minimizes a weighted summation of the experienced delay in the corridors and the total travel time in the urban network by optimizing the number of lanes of each type and the dynamic size of the communicant AV platoons. Madadi et al. (2021) proposed a unified formulation for combining AV-ready subnetworks for mixed traffic, dedicated AV links, and dedicated AV lanes, to optimize the costs and benefits of deploying using a bilevel modeling framework. The upper level decides on AV-ready network links and lanes, while the lower level models travelers' route choices and traffic flow in the network. Addressing the uncertain road capacity caused by the different AV and RV fleet mix and sequencing, Zhang et al. (2023) developed a robust and optimal integrated placement solution for the design of AV-dedicated lane and roadside unit. Incorporating that AVs have the potential to alter the routing equilibrium principles during a trip, Wang et al. (2024) proposed an dedicated AV lane design problem which is formulated as a bi-level programming model, and developed a heuristic nested algorithm for large-scale scenarios. The only three exceptions are Chen et al. (2016), Zhang et al. (2020) and Zhang et al. (2023) studied lane management strategies under elastic travel demand at the network level. In considering evolution of AV MPR with a given diffusion function, Chen et al. (2016) optimized a time-dependent placement plan of AV lanes with mixed traffic flow. With a given travel demand function, Zhang et al. (2020) proposed a novel lane management of AV and credit charging links with endogenous MPR of AV, and a nested-logit model was applied to describe travelers' choices of vehicle types and routes under tradable credit scheme. Incorporating uncertainties in AV market share which is assumed to follow a given distribution, Chen et al. (2025) developed a optimal network design model for dedicated AV lanes by introducing a chance constrained programming approach, to harmonize network efficiency and social equity. Nevertheless, all the above studies failed to consider the long-term residential relocation during AV introduction, which has been verified by empirical research (Krueger et al., 2019; Carrese et al., 2019; Kim et al., 2020). It will cause the long-term endogenous changes in the spatial distribution of travel demand, and may invalidate the strategy of short-term dedicated lane deployment under a fixed spatial distribution of travel demand (or a given form of elastic demand).

To our best knowledge, only Chu et al. (2012) explored the dedicated lane placement considering the long-term residential relocation. Dividing the road capacity into HOV lanes and other lanes, it optimized the farthest location of a HOV lane for a linear monocentric urban area, to maximize social welfare of the transportation system with an endogenous demand distribution along a continuum corridor. However, the model proposed by Chu et al. (2012) cannot handle capacity changes in the lanes with mixed AV and RV traffic, and assumed that dedicated lanes can only be built from the CBD and can only determine the farthest range of dedicated lane placement. The traffic flow is very high near the CBD, where placing dedicated lanes will reduce the supply of road resources for the vehicles that are not allowed to enter the dedicated lanes, which may exacerbate traffic congestion. In order to address these shortcomings, we propose a novel optimization method for lane placement that determines the starting and ending points of dedicated AV lanes under urban spatial equilibrium, to improve the operational efficiency of transportation system and performance of urban system.

3. Model

We consider a closed and monocentric urban space, and the total population N is exogenously given (Alonso, 1964; Muth, 1969; Mills, 1967; Leibowicz, 2020). The residential distribution and transportation conditions of any radial urban space are symmetrical, therefore a representative linear corridor can be used to represent the circle urban space (Salop, 1979; Vickrey et al., 1999). All jobs are concentrated within the Center Business District (CBD) within a radius of $\underline{x}$ in the city center, where the spatial differences of economic production and traffic conditions are overlooked for simplification (Anas and Moses, 1979; Zakharenko, 2016; Larson and Zhao, 2020). All residents live between $\underline{x}$ and the city boundary $\bar{x}$, which is determined endogenously by the results of the decisions on the location and area of all individual residences (Li et al., 2013; Dong et al., 2022).

On this urban space, we consider two types of lanes: ordinary lane (denoted by O) and dedicated AV lane (denoted by D), as shown in Fig. 1. It is assumed that only AVs can freely enter the dedicated lane without considering friction from changing lanes (Ghiasi et al., 2017). Both RVs and AVs can use the ordinary lanes, but the reaction time of AVs during driving is much smaller than that of RVs, and conservative driving strategies have to be adopted by AVs in order to drive safely (Li et al., 2020). As a result, the capacity of the ordinary lanes is not only determined by physical properties, but also by the proportion of AVs in the mixed traffic flow. Setting up dedicated AV lanes isolates AVs from mixed traffic flow, thereby creating a fully automated driving environment and reducing disruptions caused by RVs.

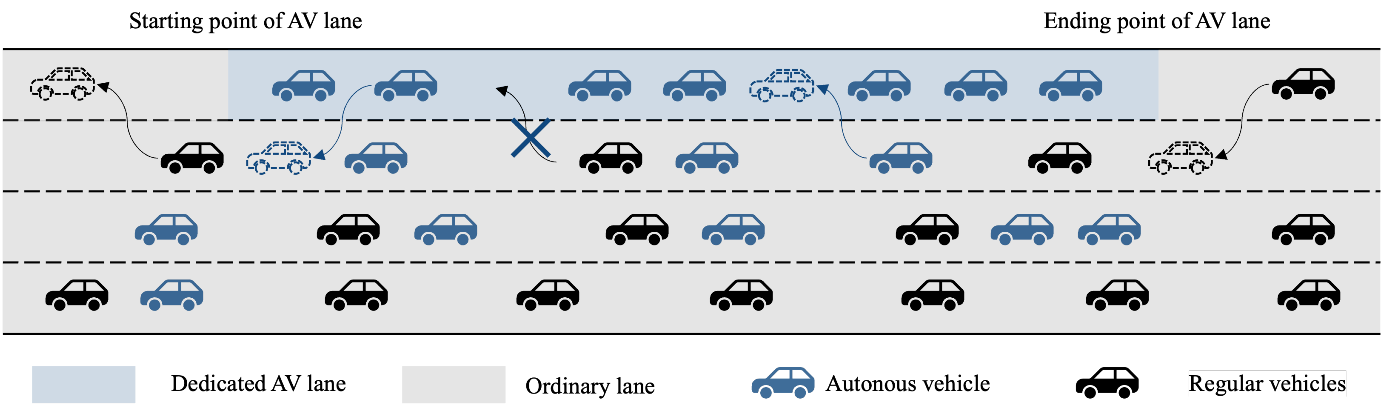

Figure 1: Dedicated AV lane and ordinary lane

There are two types of commuters: RV users and AV users (label as $k = \{RV, AV\}$), who commute using own vehicles, and the AV penetration rate is exogenously given η. The model depicts two characteristics of AV travel: the purchasing cost and the VOT. While AVs are more expensive to buy (Tian et al., 2021), AV users can rest or work during AV travel, leading to a lower VOT compared to RV users (Fagnant and Kockelman, 2015).

We introduce an urban spatial equilibrium consisting of user equilibrium in the mixed AV and RV traffic and residential equilibrium. The interactions between each component are illustrated in Fig. 2. The user travel equilibrium takes the residential distribution derived from the residential equilibrium as input, and AV users select their driving lane to minimize their travel time at equilibrium. Taking the daily commuting cost as input from the user travel equilibrium with mixed traffic flow, all residents decide their residential location and consumption to maximize their utility at equilibrium. Besides, the urban spatial equilibrium is influenced by the introduction of AVs and placement of dedicated AV lanes.

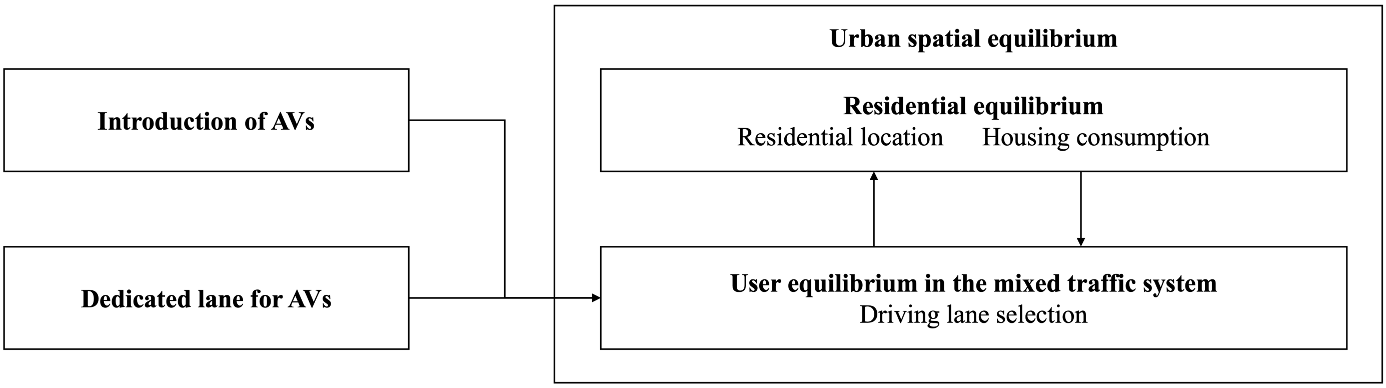

Figure 2: Interactions in urban spatial equilibrium with AV introduction and placement of dedicated AV lane.

3.1. User equilibrium in a mixed AV and RV traffic system

This subsection focuses on the travel behavior of two types of commuters in the mixed traffic system under a given residential distribution. AV commuters can choose freely between ordinary lanes and dedicated AV lanes, directly influencing the driving speeds of both AVs and RVs. The collective interactions of all rational commuters endogenously determine the traffic dynamics and equilibrium status of the two types of lanes.

3.1.1. Speed and capacity of different lanes

Firstly, we analyze the traffic states on both ordinary lanes and dedicated AV lanes along any radial road. Based on the assumption that traffic capacity is determined by the average reaction time of vehicles (Wang et al., 2019; Zhang et al., 2023), the capacity of a single lane depends on the proportion of AVs in traffic flow , can be expressed as:

$$K^O(x) = v_f \frac{1}{v_f \left(p^{AV}\delta^{AV} + (1-p^{AV})\delta^{RV}\right) + l} \tag{1}$$

$$K^D(x) = v_f \frac{1}{v_f \delta^{AV} + l}, \tag{2}$$

where v_f is the free driving speed, $p^{AV} = \frac{Q^{AV,O}(x)}{Q^{RV}(x)+Q^{AV,O}(x)}$ is the proportion of AVs among the vehicles traveling in that lane , $Q^{RV}(x)$ is the traffic volume of RVs passing location x, $Q^{AV,D}(x)$ and $Q^{AV,O}(x)$ is the traffic volumes of AVs on dedicated AV lanes and ordinary lanes passing location x, respectively. l is the average vehicle length, δ^{RV} and δ^{AV} are respectively the reaction times of RV and AV, and $\delta^{AV} < \delta^{RV}$ holds.

Then we give the general expressions for the travel speed [2] of two types of lanes, which are related to traffic volume and road capacity, $v^O\left(Q^{RV}(x), Q^{AV,O}(x), n^O(x)K^O(x)\right)$ and $v^D\left(Q^{AV,D}(x), n^D(x)K^D(x)\right)$, where $n^O(x)$ and $n^D(x)$ are the numbers of ordinary lanes and dedicated AV lanes at location x. The total number of lanes of any radial road is assumed to be a constant, i.e. $n^O(x) + n^D(x) = n_l$. Lane speed should be negatively correlated with traffic volume and positively correlated with road capacity, mathematically $\frac{\partial v^M}{\partial Q^{RV}} < 0$, $\frac{\partial v^O}{\partial Q^{AV,O}} < 0$, $\frac{\partial v^D}{\partial Q^{AV,D}} < 0$, $\frac{\partial v^O}{\partial K^O} > 0$ and $\frac{\partial v^D}{\partial K^D} > 0$ hold.

Since AVs are free to switch lanes and all travelers are entirely rational, AVs always choose the path that minimizes their total travel time. In other words, at any given time, AVs will select the fastest available lane unless the two types of lanes have equal speeds. Based on this behavior, we can derive the relationship between the speeds of dedicated AV lanes and ordinary lanes under different traffic scenarios, as follows:

Lemma 1. After placing dedicated AV lanes, there only exist four cases regarding the traffic flows and speeds of different lanes:
i) If $Q^{AV,D}(x) = 0$, $Q^{AV,O}(x) = 0$ and $Q^{RV}(x) > 0$ hold, $v^D(x) > v^O(x)$;
ii) If $Q^{AV,D}(x) > 0$, $Q^{AV,O}(x) > 0$ and $Q^{RV}(x) = 0$ hold, $v^D(x) = v^O(x)$;
iii) If $Q^{AV,D}(x) > 0$, $Q^{AV,O}(x) = 0$ and $Q^{RV}(x) > 0$ hold, $v^D(x) > v^O(x)$;
iv) If $Q^{AV,D}(x) > 0$, $Q^{AV,O}(x) > 0$ and $Q^{RV}(x) > 0$ hold, $v^D(x) = v^O(x)$.

Proof. If $Q^{AV,D}(x) > 0$, $Q^{AV,O}(x) = 0$ and $Q^{RV}(x) > 0$, there are no AVs driving on ordinary lanes. This implies that ordinary lanes are not part of the optimal paths for AVs at location x. Therefore, we obtain $v^D(x) > v^O(x)$. If $Q^{AV,D}(x) = 0$, $Q^{AV,O}(x) = 0$ and $Q^{RV}(x) > 0$, any AV entering a dedicated AV lane with zero traffic will have the free flow speed, which is greater than the driving speed on ordinary lanes with some traffic flow, then we obtain $v^D(x) > v^O(x)$. If $Q^{AV,D}(x) > 0$, $Q^{AV,O}(x) > 0$ and $Q^{RV}(x) = 0$, there are AVs driving on both two types of lanes, that is, ordinary lane and AV lane must be both on one of the optimal paths for AVs at location x, then we obtain $v^D(x) = v^O(x)$. It is also similar to the situation with $Q^{AV,D}(x) > 0$, $Q^{AV,O}(x) > 0$ and $Q^{RV}(x) > 0$, and $v^D(x) = v^O(x)$ holds.

Enumerate all other situations about the traffic flows of lanes as follows: If $Q^{AV,D}(x) > 0$, $Q^{AV,O}(x) = 0$ and $Q^{RV}(x) = 0$, there are only AVs driving on dedicated AV lanes, in other words, ordinary lane must not be on the optimal paths for AVs at location x, but any vehicle entering a ordinary lane with zero traffic will have the maximum speed, which contradicts under rational driving; If $Q^{AV,D}(x) = 0$, $Q^{AV,O}(x) > 0$ and $Q^{RV}(x) = 0$, there are only AVs driving on ordinary lanes, in other words, dedicated AV lanes are not on the optimal paths for AVs at location x, but any AV entering dedicated AV lanes with zero traffic will have the maximum speed, which contradicts under rational driving, and it is similar to the situation with $Q^{AV,D}(x) = 0$, $Q^{AV,O}(x) > 0$ and $Q^{RV}(x) > 0$. This completes the proof. □

Thus, the speed relationship between the two types of lanes can be established as follows: the speed of dedicated AV lanes is always greater than or equal to that of ordinary lanes. Equality holds if and only if AVs are simultaneously driving on both types of lanes.

Mathematically expressed as:

$$v^D(x)\begin{cases} = v^O(x), & \text{if } Q^{AV,D}(x) > 0 \text{ and } Q^{AV,O}(x) > 0 \\ > v^O(x), & \text{otherwise} \end{cases}, x \in \left[x^D_{start}, x^D_{end}\right). \tag{3}$$

[2] The specific expressions can be found in Section 4

Eq. 3 identifies two possible scenarios after the implementation of dedicated lanes for AVs: (1) AVs experience no difference in travel speed regardless of lane choice, as they are allowed to freely enter dedicated lanes; and (2) AVs prioritize driving on dedicated lanes, which are not yet fully utilized, while ordinary lanes become exclusively occupied by RVs controlled by human drivers.

3.1.2. Daily commuting cost

Next, we analyze the driving speeds of the two types of vehicles during commuting. For RVs, they travel exclusively in ordinary lanes, and their driving speed satisfying $v^{RV}(x) = v^{O}(x)$. On a road without dedicated AV lanes, AVs also drive in ordinary lanes with the same speed $v^{O}(x)$. However, on a road with dedicated AV lanes, these lanes are always part of the optimal route for AVs. The travel speed of AVs at the location x is then denoted as,

$$v^{AV}(x) = \begin{cases} v^{D}(x), \text{if } x \in \left[x^{D}_{start}, x^{D}_{end}\right) \\ v^{O}(x), \text{otherwise} \end{cases} . \tag{4}$$

Combining with Eq. 3, it is easy to get $v^{AV}(x) \geq v^{RV}(x)$.

The generalized daily commuting cost consists of three parts: the fixed cost of purchase and maintenance of vehicle c^k_f, the variable monetary cost associated with travel distance c^k_m, and the variable cost related to travel time c^k_t. The daily generalized commuting cost of RV users and AV users is then expressed as,

$$c^k(x) = c^k_f + c^k_m(x) + c^k_t(x), k \in \{RV, AV\}. \tag{5}$$

Assuming that AVs have a higher purchase price, the fixed cost of AV commuting is greater than that of RVs, i.e. $c^{RV}_f < c^{AV}_f$. The monetary cost, such as the fuel cost or charging cost that changes with distance, is given by,

$$c^k_m(x) = 2m\left(x - (1-\rho)\underline{x}\right), k \in \{RV, AV\}, \tag{6}$$

where "2" means that the commuting to work and the commuting back home are completely symmetrical, m is the variable cost per unit of driving distance. To simplify the model, specific driving paths within the CBD are ignored, all vehicles are assumed to travel at a given average speed v_C over a given average distance $\rho\underline{x}$.

The travel time costs for RV commuting and AV commuting are expressed as:

$$c^k_t(x) = 2\tau^k \frac{w}{8\lambda}\left(\int_0^x \frac{1}{v^k(\omega)}\mathrm{d}\omega + \frac{\rho\underline{x}}{v_C}\right), k \in \{RV, AV\}, \tag{7}$$

where τ^k is the ratio of VOT to hourly wage (Small, 2012), w is the annual income of all commuters, "8" means that everyone works eight hours per working day, and λ is the average amount of work days in a year. $\frac{w}{8\lambda}$ is the hourly wage of all commuters. According to the assumption, AV users can rest or work during driving, making them less sensitive to travel time, i.e. $\tau^{RV} > \tau^{AV}$. As residential locations move farther away from the city center, the generalized travel costs for both AV and RV commuting increase monotonically, as shown by $\frac{\mathrm{d}c^k(x)}{\mathrm{d}x} = 2\left(m + \frac{\tau^k w}{v^k(x)}\right) > 0$ holds.

Lemma 2. The slope of the generalized travel cost of AV commuting is strictly less steep than that of RV commuting.

Proof. We have known that $v^{AV}(x) \geq v^{RV}(x)$ and $\tau^{RV} > \tau^{AV}$, then it can be obtained that $\frac{\tau^{AV} w}{v^{AV}(x)} < \frac{\tau^{RV} w}{v^{RV}(x)}$. Making a difference between the derivatives of the generalized travel costs for commuting by AV and RV, it can be seen that $\frac{\mathrm{d}c^{AV}(x)}{\mathrm{d}x} < \frac{\mathrm{d}c^{RV}(x)}{\mathrm{d}x}$ holds. This completes the proof. □.

3.2. Residential location choice equilibrium

This subsection further analyzes the residential location choice equilibrium resulting from households' residential distribution. Each household maximizes their utility level by deciding the housing and non-housing consumption subject to a budget constraint. The outcome of residence transactions in all locations is determined by the rental bids of the AV and RV users.

The objective of households owning k type vehicle and living at location x is to maximize their own utility, which which follows the Cobb-Douglas function. Their decisions consist of the consumption of composite non-housing goods $z^k(x)$ and the housing space $q^k(x)$. For simplicity, it is assumed that residences consume land directly (Borck and Tabuchi, 2019). All income is allocated to non-housing goods, residential land rent, and commuting cost.Using a similar residents' decision-making form in (Li et al., 2013; Dong et al., 2022), the individual utility maximization problem is mathematically expressed as :

$$\max U^k(x) = z^k(x)^{\beta} q^k(x)^{1-\beta}, 0 < \beta < 1, \underline{x} < x < \bar{x}, \tag{8}$$

$$\text{s.t. } w = z^k(x) + q^k(x) r^k(x) + \lambda c^k(x), \tag{9}$$

where β is a given parameter of the CD function and represents the share of goods in non-commuting consumption, $r^k(x)$ indicates the bid rent of k type of residents. In equilibrium, the utility of each type of residents is equalized, i.e. $U^k(x) = u^k$. Using the first-order optimal condition for utility maximization, the housing demand $q^k(x)$ and housing bidding rent $r^k(x)$ are derived as,

$$q^k(x) = \beta^{\frac{-\beta}{1-\beta}} \left(w - \lambda c^k(x)\right)^{\frac{-\beta}{1-\beta}} \left(u^k\right)^{\frac{1}{1-\beta}}, \tag{10}$$

$$r^k(x) = (1-\beta)\, \beta^{\frac{\beta}{1-\beta}} \left(w - \lambda c^k(x)\right)^{\frac{1}{1-\beta}} \left(u^k\right)^{\frac{-1}{1-\beta}}. \tag{11}$$

The transaction rent of any residential land equals the highest bidding rent, i.e. $r(x) = \max\limits_k r^k(x)$. Then the relationship between the bidding rent and the residential density is expressed as,

$$r^k(x) \begin{cases} = r(x), \text{if } D^k(x) > 0 \\ < r(x), \text{if } D^k(x) = 0 \end{cases}, \tag{12}$$

where $D^k(x)$ is the density of k type residents at location x and equals the number of residents per unit of land. The housing bidding rent for each type of residents decreases monotonically with the location away from the city centre, because $\frac{\mathrm{d}r^k(x)}{\mathrm{d}x} = -\lambda \beta^{\frac{\beta}{1-\beta}} \left(w - \lambda c^k(x)\right)^{\frac{\beta}{1-\beta}} \left(u^k\right)^{\frac{-1}{1-\beta}} \frac{\mathrm{d}c^k(x)}{\mathrm{d}x} < 0$ holds.

Then the property regarding the residential segregation of the two types of individuals can be stated as,

Proposition 1. There exists a location $\hat{x}$ where $r^{AV}(\hat{x}) = r^{RV}(\hat{x})$. All RV users reside within $\hat{x}$, and all AV users reside outside $\hat{x}$.

Proof. The bid rent functions of AV users and RV users change continuously with residential location, and there must be a location $\hat{x}$ where both types of residents bid the same land rent, i.e. $r^{AV}(\hat{x}) = r^{RV}(\hat{x})$. Otherwise, the residential bidding rent of one type would always dominate the other. From conservation relationships, the following equation holds

$$\frac{w - \lambda c^{AV}(\hat{x})}{u^{AV}} = \frac{w - \lambda c^{RV}(\hat{x})}{u^{RV}}. \tag{13}$$

Taking the derivative of the bid rent functions for the two user types, we obtain:

$$\begin{aligned}\frac{\mathrm{d}r^{AV}(x)}{\mathrm{d}x}-\frac{\mathrm{d}r^{RV}(x)}{\mathrm{d}x}=&-\lambda\beta^{\frac{\beta}{1-\beta}}\left(w-\lambda c^{AV}(x)\right)^{\frac{\beta}{1-\beta}}\left(u^{AV}\right)^{\frac{-1}{1-\beta}}\frac{\mathrm{d}c^{AV}(x)}{\mathrm{d}x}\\&+\lambda\beta^{\frac{\beta}{1-\beta}}\left(w-\lambda c^{RV}(x)\right)^{\frac{\beta}{1-\beta}}\left(u^{RV}\right)^{\frac{-1}{1-\beta}}\frac{\mathrm{d}c^{RV}(x)}{\mathrm{d}x}.\end{aligned} \tag{14}$$

Substituting $x=\hat{x}$ into the above equation yields:

$$\left.\frac{\mathrm{d}r^{AV}(x)}{\mathrm{d}x}\right|_{\hat{x}}-\left.\frac{\mathrm{d}r^{RV}(x)}{\mathrm{d}x}\right|_{\hat{x}}=-\lambda\beta^{\frac{\beta}{1-\beta}}\left(\frac{w-\lambda c^{AV}(\hat{x})}{u}^{k}\right)^{\frac{\beta}{1-\beta}}\left(\left.\frac{\mathrm{d}c^{AV}(x)}{\mathrm{d}x}\right|_{\hat{x}}\frac{1}{u^{AV}}-\left.\frac{\mathrm{d}c^{RV}(x)}{\mathrm{d}x}\right|_{\hat{x}}\frac{1}{u^{RV}}\right). \tag{15}$$

Combining with Lemma 2, it follows that $\left.\frac{\mathrm{d}c^{AV}(x)}{\mathrm{d}x}\right|_{\hat{x}}<\left.\frac{\mathrm{d}c^{RV}(x)}{\mathrm{d}x}\right|_{\hat{x}}$ and $u^{AV}>u^{RV}$. Therefore, Eq. (15) is positive, implying that the derivative of the AV bidding rent at $\hat{x}$is strictly greater than that of RV users. Consequently, there is only one intersection point between the bid rent curves. At this intersection point, all RV users reside within $\hat{x}$, and all AV users reside outside $\hat{x}$. This completes the proof. □

Based on the result of residential separation, the transaction land rent at location x can be rewritten by a piece-wise function

$$r(x)=\begin{cases}r^{RV}(x), & \text{if } \underline{x}<x<\hat{x},\\ r^{AV}(x), & \text{if } \hat{x}\le x<\bar{x}.\end{cases} \tag{16}$$

RV users prefer to reside near the CBD, willing to pay higher rents for the convenience of reduced travel distances. In contrast, AV users tend to settle on the outskirts of the city, given their lower sensitivity to travel time and costs, and to benefit from reduced housing expenses. This spatial segregation reflects the equilibrium outcome in the residential land market, where individuals choose their residential locations based on travel preferences and housing costs.

3.3. Urban spatial equilibrium conditions

This subsection analyzes the urban spatial equilibrium in a closed economy, where the utility levels of each resident type and the city boundary are endogenously determined. Drawing on classic urban models of residential decision-making (Alonso, 1964; Muth, 1969; Mills, 1967), achieving urban spatial equilibrium requires satisfying two types of conditions.

The first condition is that the land rent at the city boundary is equal to the exogenous given agricultural land rent, and can be expressed by

$$r\left(\bar{x}\right)=R_A. \tag{17}$$

The second condition is that in a closed city all residents live between the CBD edge and city boundary, and it can be expressed by

$$N=N^{RV}+N^{AV}=\int_{\underline{x}}^{\hat{x}}2\pi xD^{RV}\left(x\right)\mathrm{d}x+\int_{\hat{x}}^{\bar{x}}2\pi xD^{AV}\left(x\right)\mathrm{d}x. \tag{18}$$

In addition, the traffic flow of AVs at each location equals the total number of AV users residing outside that location. The same condition applies to RVs. These equality constraints are formulated as

$$n_r Q^{RV}(x) = \int_x^{\bar{x}} 2\pi\omega D^{RV}(\omega)\, d\omega, \tag{19}$$

$$n_r(Q^{AV,D}(x) + Q^{AV,O}(x)) = \int_x^{\bar{x}} 2\pi\omega D^{AV}(\omega)\, d\omega, \tag{20}$$

where n_r represents the total number of radial roads. Based on the above conditions, all the endogenous variables can be obtained by solving these equations simultaneously.

4. Analytical derivations and solution procedures

This section analytically examines the impacts of placing dedicated AV lanes on traffic speed and traffic efficiency under varying AV proportions at a single location.It then summarizes the spatial effects of different AV market penetration rates (MPRs) across the entire urban area. Finally, the solution procedures for the user equilibrium and urban spatial equilibrium are provided.

Travel time is adopted by the form of the Bureau of Public Roads (BPR), and specific expressions of speed on the ordinary lanes and dedicated AV lanes are $v^O(x) = v_f\left[1 + 0.15\left(\frac{Q^{RV}(x)+Q^{AV,O}(x)}{n^O(x)K^O(x)}\right)^4\right]^{-1}$ and $v^D(x) = v_f\left[1 + 0.15\left(\frac{Q^{AV,D}(x)}{n^D(x)K^D(x)}\right)^4\right]^{-1}$.

4.1. Impact of placing dedicated AV lanes on on traffic speed and traffic efficiency

In the mixed traffic system, road capacity at a single location depends on the proportion of AVs. Before analyzing the impact of placing dedicated AV lanes, we define the share of AVs in total traffic passing location x as $\mathrm{H}(x) = \frac{Q^{AV,D}(x)+Q^{AV,O}(x)}{Q^{AV,D}(x)+Q^{AV,O}(x)+Q^{RV}(x)}$, which varies with MPR of AVs. It is known from Proposition 1 that the residential locations of AV and RV users are spatially separate. Outside the residential separation location, only AV flows exist and are evenly distributed across all lanes at the same speed. In this case, placing dedicated AV lanes will not alter traffic conditions. It is got that $\mathrm{H}(x) = 1$ holds for $x \in [\hat{x}, \bar{x}]$. Within the residential separation location, the AV flow remains constant at $Q^{AV}(x) = N^{AV}$, while the RV flow increases as the distance to the CBD decreases. Correspondingly, $\mathrm{H}(x)$ decreases from 1 to the MPR of AV η. And $\eta \leq \mathrm{H}(x) < 1$ holds for $x \in [\underline{x}, \hat{x})$.

To quantify the technical difference between AVs and RVs, we introduce the AV technology coefficient, σ, defined as $\sigma = 1 - \frac{v_f\delta^{AV}+l}{v_f\delta^{RV}+l}$. The higher the value of σ indicates a shorter reaction time for AVs, reflecting more advanced autonomousdriving technology. Proposition 2 further explores the impact of placing dedicated AV lanes on lane speeds and traffic flow dynamics.

Proposition 2. If $\mathrm{H}(x) \geq \frac{n^D}{n_l-(n_l-n^D)\sigma}$, the speed of dedicated AV lanes is equal to that of the ordinary lane; otherwise, the speed of dedicated AV lanes is higher than that of ordinary lanes.

Proof. Combining Lemma 1 and Proposition 1, AV users always live outside the residential separation location, so we know case (i) with only RV flow and no AV flow does not exist in the mixed traffic system.

In case (ii), $Q^{AV,D}(x) > 0$, $Q^{AV,O}(x) > 0$, $Q^{RV}(x) = 0$ and $v^D(x) = v^O(x)$, there are only AVs driving in roads, which corresponds to the area outside the residential separation location. Then we obtain that the equivalent condition of case (iii) is $\mathrm{H}(x) = 1$, $x \in (\hat{x}, \bar{x})$.

In case (iii), $Q^{AV,D}(x) > 0$, $Q^{AV,O}(x) = 0$, $Q^{RV}(x) > 0$ and $v^D(x) > v^O(x)$, after combining and rearranging, we can obtain that the equivalent condition of case (iii) is $\mathrm{H}(x) < \frac{n^D}{n_l-(n_l-n^D)\sigma}$.

In case (iv), $Q^{AV,D}(x) > 0$, $Q^{AV,O}(x) > 0$, $Q^{RV}(x) > 0$ and $v^D(x) = v^O(x)$, after combining and rearranging, the AV flows in two types of lanes satisfy the following equations,

$$Q^{AV,D}(x) = \frac{(1-\sigma)n^D N^{AV} + n^D Q^{RV}(x)}{(1-\sigma)n_l}, \tag{21}$$

$$Q^{AV,O}(x) = \frac{(1-\sigma)n^O N^{AV} - n^D Q^{RV}(x)}{(1-\sigma)n_l}. \tag{22}$$

There are no other cases, and it is easy to get that the equivalent condition of case (iv) is $\frac{n^D}{n_l-(n_l-n^D)\sigma} \le \mathrm{H}(x) < 1$, by taking the complement of the equivalent conditions of cases (ii) and (iii). This completes the proof. □

Proposition 2 examines the impact of dedicated AV lanes on traffic speeds under varyingAV proportions in the traffic flow. It concludes that when the market penetration rate (MPR)of AVs exceeds $\frac{n^D}{n_l-(n_l-n^D)\sigma}$, the speeds of dedicated AV lanes and ordinary lanes become identical.

Next, we turn to the impacts of placing dedicated AV lanes on traffic efficiency. The total travel time for all vehicles passing through location x can be expressed by,

$$T(x) = \frac{Q^{RV}(x)}{v^{RV}(x)} + \frac{Q^{AV}(x)}{v^{AV}(x)}. \tag{23}$$

Eq. (23) serves as a measure of traffic efficiency at each location, where a higher $T(x)$ indicates lower traffic efficiency. To further evaluate the effect of dedicated AV lanes, we define a piece-wise function based on the share of AVs in traffic flow and the AV technology coefficient. This function captures the positive or negative impact of dedicated AV lanes on traffic efficiency at any given location. It is expressed by

$$\Omega(\cdot) = \begin{cases} 0, & \text{if } \mathrm{H}(x) \ge \frac{n^D}{n_l-(n_l-n^D)\sigma} \\ \left(\frac{1-\mathrm{H}(x)\sigma}{n_l}\right)^4 - \frac{(1-\mathrm{H}(x))^5}{(n_l-n^D)^4} - \mathrm{H}(x)^5\left(\frac{1-\sigma}{n^D}\right)^4, & \text{if } \mathrm{H}(x) < \frac{n^D}{n_l-(n_l-n^D)\sigma} \end{cases}. \tag{24}$$

Based on Eq.(24), when $\mathrm{H}(x) < \frac{n^D}{n_l-(n_l-n^D)\sigma}$, the value of $\Omega(\cdot)$ depends on the interaction of three components: (1) The first term $\left(\frac{1-\mathrm{H}(x)\sigma}{n_l}\right)^4$ represents baseline congestion in a mixed-lane setting; (2) The second term $-\frac{(1-\mathrm{H}(x))^5}{(n_l-n^D)^4}$ reflects increased congestion for RVs caused by the reallocation of lanes, which reduces RV lane capacity. This is a negative effect; and (3) The third term $-\mathrm{H}(x)^5\left(\frac{1-\sigma}{n^D}\right)^4$ represents efficiency gains from dedicated AV lanes, which is a positive effect. Then we obtain the Proposition 3.

Proposition 3. The impact of placing dedicated AV lanes on traffic efficiency at a given location depends on the value of Ω:

- If $\Omega > 0$, placing dedicated AV lanes improves the traffic efficiency.
- If $\Omega = 0$, placing dedicated AV lanes doesn't change the traffic efficiency.
- If $\Omega < 0$, placing dedicated AV lanes reduces the traffic efficiency.

Proof. Before placing dedicated AV lanes, Eq. (23) can be rewritten by

$$T_0(x) = \frac{Q^{AV}(x) + Q^{RV}(x)}{v_f} \left[1 + 0.15 \left(\left(v_f \delta^{RV} + l \right) \frac{Q^{AV}(x)(1-\sigma) + Q^{RV}(x)}{n_l v_f} \right)^4 \right]. \tag{25}$$

Equation (25) represents the total travel time per unit distance for all vehicles passing through location x before the implementation of dedicated AV lanes. It adopts a BPR-type formulation to capture congestion effect. The term $\frac{Q^{AV}(x)+Q^{RV}(x)}{v_f}$ calculates the travel time if all vehicles traveled at free-flow speed v_f. The expression within the brackets quantifies the impact of congestion on travel time. The term $(v_f\delta^{RV} + l)\frac{Q^{AV}(x)(1-\sigma)+Q^{RV}(x)}{n_l v_f}$ calculates the effective volume-to-capacity ratio. This term converts AV flows into RV-equivalent units using the technology factor $(1-\sigma)$, which captures the increased capacity of AVs due to the shorter reaction time and more efficient spacing.

After placing dedicated AV lanes, $T(x)$ in cases (ii), (iii), and (iv) can be expressed by, respectively

$$T_1(x) = \frac{Q^{AV}}{v_f} \left[1 + 0.15 \left(\left(v_f \delta^{RV} + l \right) \frac{Q^{AV}(x)(1-\sigma)}{\left(n_l - n^D\right) v_f} \right)^4 \right], \tag{26}$$

$$\begin{aligned} T_2(x) =& \frac{Q^{RV}(x)}{v_f} \left[1 + 0.15 \left(\left(v_f \delta^{RV} + l \right) \frac{Q^{RV}(x)}{\left(n_l - n^D\right) v_f} \right)^4 \right] + \\ & \frac{N^{AV}}{v_f} \left[1 + 0.15 \left(\left(v_f \delta^{RV} + l \right) \frac{N^{AV}(1-\sigma)}{n^D v_f} \right)^4 \right], \end{aligned} \tag{27}$$

$$T_3(x) = \frac{N^{AV} + Q^{AV,O}(x)}{v_f} \left[1 + 0.15 \left(\left(v_f \delta^{RV} + l \right) \frac{Q^{AV,O}(x)(1-\sigma) + Q^{RV}(x)}{\left(n_l - n^D\right) v_f} \right)^4 \right]. \tag{28}$$

Subtract Eqs. (26)-(28) from Eq. (25), and substitute the conditions of Lemma 1 into these equations, then we obtain

$$T_1(x) - T_0(x) = 0, \tag{29}$$

$$\begin{aligned} T_2(x) - T_0(x) =& \left(v_f \delta^{RV} + l \right)^4 \left(\frac{N^{AV} + Q^{RV}(x)}{v_f} \right)^5 * \\ & \left[\frac{(1-\mathrm{H}(x))^5}{\left(n_l - n^D\right)^4} + \mathrm{H}(x)^5 \left(\frac{1-\sigma}{n^D} \right)^4 - \left(\frac{1-\mathrm{H}(x)\sigma}{n_l} \right)^4 \right], \end{aligned} \tag{30}$$

$$T_3(x) - T_0(x) = 0. \tag{31}$$

The above results indicates that in cases (ii) and (iv), the placement of dedicated AV lanes does not affect the total travel time for all vehicles passing through location x. In case (iii), the impact depends solely on the sign of the terms in the square brackets on the right-hand side of Eq. (30), as all other components are obviously positive. By combining the flow conditions outlined in Proposition 2, the impacts of placing dedicated AV lanes on traffic efficiency can be summarized using the piece-wise function $\Omega(\cdot)$ defined in Eq. (24). This completes the proof. □

The sign of Ω varies with σ and $\mathrm{H}(x)$, as illustrated in Fig. 3, using the parameter values $n_l = 4$ and $n^D = 1$. In the figure, brighter colors represent higher values of Ω. The value of the function is divided into three zones by two curves, from the right to the left, $\Omega = 0$, $\Omega > 0$, and $\Omega < 0$, as shown in Fig. 3(a). For clarity, regions with $\Omega < 0$ are represented in the same dark blue color, and the specific figures in this area are shown in the Fig. 3(b) and 3(c). The curve on the right is $\mathrm{H}(x) - \frac{n^D}{n_l-(n_l-n^D)\sigma} = 0$, and the curve on the left has an unknown specific expression and is denoted as $\mathrm{H}(x) - G(\sigma) = 0$. Under a given AV technology coefficient σ, if $\mathrm{H}(x) \geq \frac{n^D}{n_l - (n_l - n^D)\sigma}$ holds, Ω equals zero; if $G(\sigma) \leq \mathrm{H}(x) < \frac{n^D}{n_l - (n_l - n^D)\sigma}$ holds, Ω is positive and increases first and then decreases with the reduction of AV proportion in the traffic flow; if $\mathrm{H}(x) < G(\sigma)$ holds, Ω is negative and decreases with reduction of AV proportion in the traffic flow. In addition, as the AV technology coefficient σ increases, the width of the area with $\Omega > 0$, i.e. the horizontal distance between the two curves increases. The above results hold for various values of n^D and n_l, as demonstrated in Appendix B.

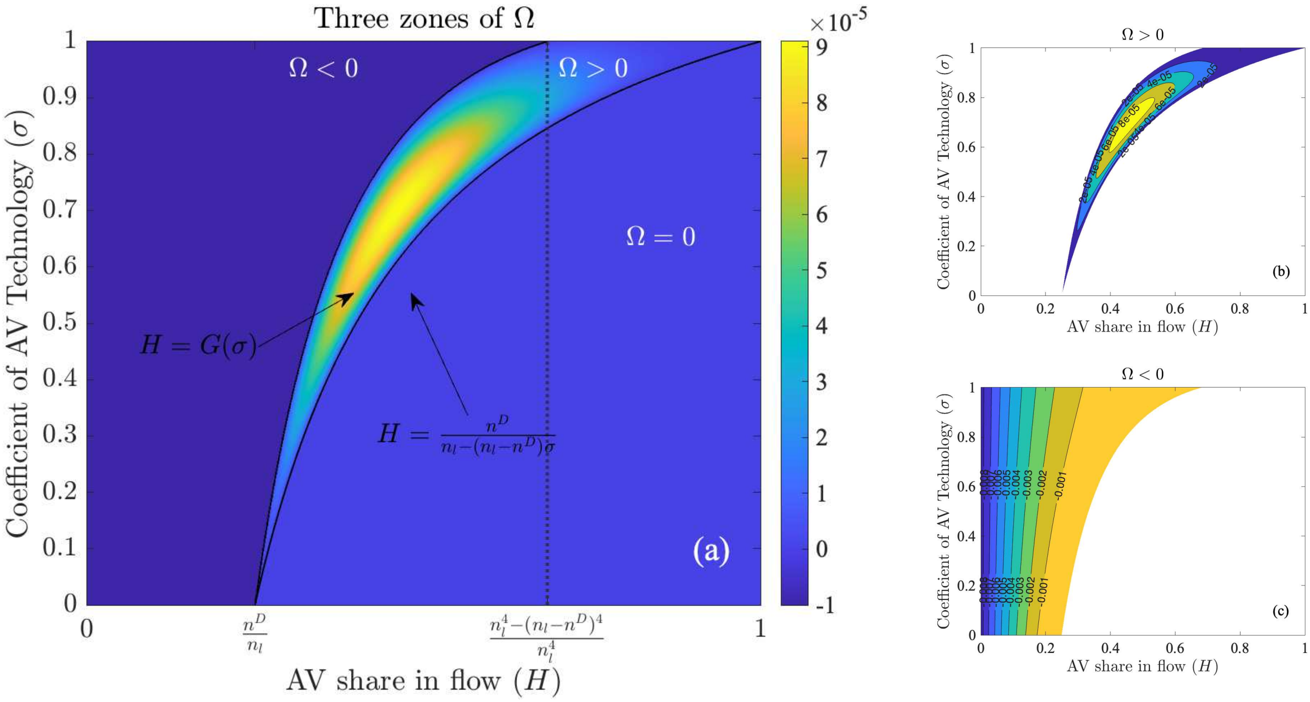

Figure 3: Changes of Ω with coefficient of AV technology and AV share in traffic flow

From the perspective of traffic efficiency in a single location, when AV proportion in the traffic flow is very high, the traffic efficiency of ordinary lanes is already maximized. In this case, placing dedicated AV lanes does not further improve traffic efficiency. When the AV proportion is within a certain range, dedicated AV lanes allow AVs to pass quickly without being restricted by RVs, improving both traffic capacity and efficiency. When the AV proportion is low, dedicated AV lanes benefit only a few AVs while reducing the capacity for the majority of traffic (RV users), resulting in an overall decline in traffic efficiency.

From the perspective of the whole urban space, the AV proportion in the traffic flow remains at 100% outside the residential separation point $\hat{x}$, and decreases to the MPR of AVs η within the point, and the corresponding values of Ω change accordingly. The spatial difference of the impact of placing dedicated AV lanes on traffic efficiency across the whole urban space depends on the MPR of AVs. Let the solutions of $\mathrm{H}(x) - \frac{n^D}{n_l - (n_l - n^D)\sigma} = 0$ and $\mathrm{H}(x) - G(\sigma) = 0$ be denoted by $\dot{x}$ and $\ddot{x}$, respectively. The spatial influence of placing dedicated AV lanes under different MPRs of AVs can be summarized as follows

Remark 1. If the MPR of AVs exceeds $\frac{n^D}{n_l - (n_l - n^D)\sigma}$, the placement of dedicated AV lanes does not affect the traffic efficiency at any location. If the MPR of AVs is between $G(\sigma)$ and $\frac{n^D}{n_l - (n_l - n^D)\sigma}$, the placement of dedicated AV lanes improves the traffic efficiency in the interval $(\underline{x}, \dot{x})$, while having no effect in the interval $(\dot{x}, \bar{x})$. If the MPR of AVs is less than $G(\sigma)$, the placement of dedicated AV lanes reduces the traffic efficiency in the interval $(\underline{x}, \ddot{x})$, improves the traffic efficiency in the interval $(\ddot{x}, \dot{x})$, and does not affect the traffic efficiency in the interval $(\dot{x}, \bar{x})$.

This remark reveals that dedicated AV lanes can only enhance traffic efficiency in specific locations and under appropriate conditions. Designing optimal placement for dedicated AV lanes requires strategic planning, as the boundaries of their impact depend on $\mathrm{H}(x)$ and σ, but not directly on specific spatial locations. Since $G(\sigma)$ lacks a closed-form expression, analytical solutions are not feasible. Therefore numerical optimization methods are employed in subsequent sections to explore the optimal placement of dedicated AV lanes under various objectives.

4.2. Solution procedures of equilibrium

This subsection introduces the solution procedures to solve the proposed user equilibrium and the urban equilibrium model. To solve these continuous models numerically, we discretize the urban space to sufficiently small segments and represent spatial locations as arrays of segments.

Firstly, we use the Method of Successive Averages (MSA) to solve the user equilibrium in the mixed AVs and RVs traffic system under a given residential distribution, i.e. solving Eqs. (1)-(3), (19) and (20), as depicted in Algorithm 1. To begin, let $\mathbf{Q}_i := \left[\mathbf{Q}_i^{AV,D}, \mathbf{Q}_i^{AV,O}, \mathbf{Q}_i^{RV}\right]^{\intercal}$(line 1). According to Eqs. (19)-(20), assign RVs into the ordinary lanes, and assign AVs into the dedicated AV lanes, then we obtain the initial distribution of traffic flows (line 2). During ith iteration, update the capacity of the ordinary lanes $\mathbf{K}_i^M$ using Eq. (1), then the velocity distributions on the dedicated AV lanes and the ordinary lanes can be obtained. Assign all RVs into ordinary lanes, and assign all AVs into the fastest lane at each location, then obtain the traffic flow of the all-or-nothing method $\tilde{\mathbf{Q}}_i$ (line 7). Then update the traffic flow with the given step size (line 8). In the end, the algorithm returns the distributions of traffic flow and lane speeds at the user equilibrium.

Algorithm 1: Solution procedure for solving the user equilibrium

Input: Placement of dedicated AV lanes, MPR of AVs, residential distribution and all other parameters

Output: Distributions of traffic flow and lane speed at user equilibrium

1 let $\mathbf{Q}_i := \left[\mathbf{Q}_i^{AV,D}, \mathbf{Q}_i^{AV,O}, \mathbf{Q}_i^{RV}\right]^{\intercal}$;

2 assign all RVs into ordinary lanes and all AVs into the dedicated AV lanes, get $\mathbf{Q}_1$;

3 $i \leftarrow 1$;

4 **repeat**

5   update the capacity of ordinary lanes;

6   calculate the distribution of lane speed;

7   assign all RVs into ordinary lanes and all AVs into the fastest lane, get an auxiliary vector $\tilde{\mathbf{Q}}_i$;

8     $\mathbf{Q}_{i+1} \leftarrow \mathbf{Q}_i + \frac{1}{i}(\tilde{\mathbf{Q}}_i - \mathbf{Q}_i)$;

9     $i \leftarrow i + 1$;

10 **until** $\frac{||\mathbf{Q}_i - \mathbf{Q}_{i-1}||}{||\mathbf{Q}_{i-1}||}$ *is less than the acceptable level*;

Then we propose a two-stage algorithm for solving the urban spatial equilibrium with endogenous residential distribution, i.e. solving Eqs. (1)-(20), as depicted in Algorithm 2. To begin, initialize the residential distribution according to uniform distribution and let $\mathbf{y}_j := \left[u_j^{AV}, u_j^{RV}, \bar{x}_j, \hat{x}_j\right]^{\intercal}$ (lines 1-2). In the first stage, the MSA is used to solve the user equilibrium in the mixed traffic system with AVs and RVs as Algorithm 1, then we calculate the daily commuting costs for AV commuters and RV commuters $\left\{\mathbf{c}_{i+1}^{AV}, \mathbf{c}_{i+1}^{RV}\right\}$ by Eqs. (5)-(7) (line 6). In the second stage, the Newton-Raphson method is used to solve the nonlinear equations of the residential equilibrium with AV users and RV users. During jth iteration of the second stage, the residential separation location is determined by the current residential distribution. Initialize the values of utility levels and city boundary(line 8), and rewrite $r^{AV}(\hat{x}) = r^{RV}(\hat{x})$ and Eqs. (17)-(20) to the equation of vector function $\mathbf{F}(\mathbf{y_j}) = 0$. Update the utility, residential separation location and city boundary according to $\mathbf{y}^{j+1} = \mathbf{y}_j + \mathbf{J}^{-1}(\mathbf{y}_j)\mathbf{F}(\mathbf{y_j})$, where $\mathbf{J}$ is the Jacobi matrix of $\mathbf{F}$ (line 10). In the end of the second stage, the residential density of AV commuters and RV commuters at each residential location $\left\{\mathbf{D}_{i+1}^{AV}, \mathbf{D}_{i+1}^{RV}\right\}$ can be calculated (line 13). Repeat the two stages until both the distributions of residential density and traffic costs meet the convergence criteria. Finally, the distributions of speed, flow, and residence in the urban spatial equilibrium are returned as the output.

Algorithm 2: Solution procedure for solving the urban spatial equilibrium

Input: Placement of dedicated AV lanes, MPR of AVs, and all other parameters

Output: Distributions of residence, traffic flow and speed

1 initialize the distributions of residence;
2 let $\mathbf{y}_j := \left[u_j^{AV}, u_j^{RV}, \bar{x}_j, \hat{x}_j\right]^{\intercal}$;
3 $i \leftarrow 1$;
4 **repeat**
5   solve the user equilibrium in the mixed traffic system by using Algorithm 1;
6   calculate $\mathbf{c}_{i+1}^{AV}$ and $\mathbf{c}_{i+1}^{RV}$;
7     $j \leftarrow 1$;
8   initialize the values of utility levels and city boundary, then get $\mathbf{y}_1$;
9   **repeat**
10         $\mathbf{y}_{j+1} \leftarrow \mathbf{y}_j + \mathbf{J}^{-1}(\mathbf{y}_j)\mathbf{F}(\mathbf{y_j})$;
11         $j \leftarrow j + 1$;
12   **until** $\frac{||\mathbf{y}_j - \mathbf{y}_{j-1}||}{||\mathbf{y}_{j-1}||}$ *is less than the acceptable level*;
13   calculate $\mathbf{D}_{i+1}^{AV}$ and $\mathbf{D}_{i+1}^{RV}$;
14     $i \leftarrow i + 1$;
15 **until** $\frac{||\mathbf{c}_i^{AV} - \mathbf{c}_{i-1}^{AV}||}{||\mathbf{c}_{i-1}^{AV}||}$, $\frac{||\mathbf{c}_i^{RV} - \mathbf{c}_{i-1}^{RV}||}{||\mathbf{c}_{i-1}^{RV}||}$, $\frac{||\mathbf{D}_i^{AV} - \mathbf{D}_{i-1}^{AV}||}{||\mathbf{D}_{i-1}^{AV}||}$ *and* $\frac{||\mathbf{D}_i^{RV} - \mathbf{D}_{i-1}^{RV}||}{||\mathbf{D}_{i-1}^{RV}||}$ *are all less than the acceptable level and* $i > 2$;

5. Placement of dedicated AV lanes

This section further explores the impacts of dedicated AV lanes and the optimal placement schemes in the short and long term. "Short term" means that the distribution of residential density is fixed, and the placement of dedicated AV lanes only changes the travel costs and behavior of commuters, without affecting the individual residential choice, which is relaxed in the "long term". Travel behavior is made daily and is easily changed immediately with transportation infrastructure, while residential decision is made every several years or more, and there is a lag effect in residential distribution when placing dedicated AV lanes. In the following subsections, we propose the programming model for optimizing the placement dedicated AV lanes to minimize the total travel time in the short term, under the fixed residential distribution, then compare it to the traffic before placement. Then, we propose the programming model

Table 2

Baseline value of parameters

Parameter	Value	Description
N	1.5×10^5	Population
n_r	10	Amount of radial roads
n_l	4	Amount of lanes of each radial road
n^D	1	Amount of dedicated AV lanes
β	0.75	Share of non-housing consumption in budget
λ	200	Annual average number of working day
v_f	50	Free-flow speed (km/h)
m	1	Monetary cost of driving per unit distance (CNY/km)
c_f^{AV},c_f^{RV}	10, 5	Daily fixed cost of AV and RV (CNY)
$\underline{x}$	3	Boundary of the CBD (km)
v_C	20	Average speed of driving in the CBD (km/h)
ρ	1	Coefficients converting $\underline{x}$ to the average travel distance in the CBD
δ^{AV}, δ^{RV}	0.1, 1	Reaction time of AV and RV (s)
l	4	Average length of vehicles (m)
w	10^5	Annual income (CNY/year)
τ^{AV}, τ^{RV}	0.5, 1	Ratio of the VOT in AV and RV to hourly wage
R_A	10^6	Agricultural land rent (CNY/km^2)
ζ	100	Parameter that converts the utility into the equivalent monetary units
c_{const}	10^6	Deployment cost of dedicated AV lane per unit distance (CNY/km)

of optimal placement dedicated AV lanes to maximize social welfare in the long term, with endogenous residential distribution, and then compare changes in urban spatial structure and traffic efficiency.

Before calculating simulations, we give the specification of formulas and the baseline value of parameters. The total travel time of all users' one-trip commuting can be expressed by

$$TTT = n_r \int_{\underline{x}}^{\bar{x}} T(x)\mathrm{d}x + \frac{N\rho\underline{x}}{v_c}. \tag{32}$$

Using a similar mathematical form in (Li et al., 2013), social welfare consists of three parts: the sum of all residents' utility, the aggregate rent received by the absentee landlords, and the negative part of the construction cost of the dedicated AV lanes, and it can be formulated as

$$SW = \zeta\left(u^{AV}N^{AV} + u^{RV}N^{RV}\right) + \int_{\underline{x}}^{\bar{x}} 2\pi x\left(r(x) - R_A\right)\mathrm{d}x - c_{const}n_r\left(x_{end}^D - x_{start}^D\right). \tag{33}$$

where ζ is the parameter that converts the utility into the equivalent monetary units, c_{const} is the construction cost of placing dedicated AV lanes per unit distance.

Unless otherwise specified, the parameters of all the simulations are based on the values in Table 2.

5.1. Optimal placement of dedicated AV lanes in the short term

Based on the analytical properties in Section 4.1, this subsection further explores the optimal schemes of placing dedicated AV lanes in the short term. Given the residential distribution without dedicated AV lanes, we propose the programming model of minimizing the total travel time of all users' one-trip commuting by determining the starting point x^D_{start} and ending point x^D_{end} of placing one dedicated AV lane. The minimization problem of total travel cost in the short term can then be expressed as

$$\min_{x^D_{start}, x^D_{end}} TTT + M\left(x^D_{end} - x^D_{start}\right), \tag{34}$$

$$\text{s.t.} \begin{cases} \text{Eqs. (1)-(3), (19) and (20),} \\ x^D_{end} - x^D_{start} \geq 0. \end{cases} \tag{35}$$

where M is a penalty coefficient to avoid the excessive length of dedicated AV lane.

To solve this minimization problem of total travel cost, we further propose a pattern search algorithm for optimizing the placement of a dedicated AV lane. By combining the proposed two-stage algorithm with the Hooke-Jeeves method, the placement of the dedicated lane and the step size are adjusted iteratively to progressively approach the optimal solution, as depicted in Algorithm 3. The algorithm starts by defining the vector $\mathbf{x}$, which represents the start and end points of a dedicated AV lane (lines 1). It then records the objective value of Eq. (34) under the current placement $\mathbf{x}$ using Algorithm 1 as $Z(\mathbf{x})$ (line 2). The search area, two base vectors, initial starting and ending positions, and iteration counters are initialized (lines 3-6). The core of the algorithm is a repeated process that continues until the step size θ falls below an acceptable level. Within this process, a nested loop explores potential placements for the dedicated AV lane by considering variations of $\mathbf{y}_j$ based on the calculated objective value $Z(\mathbf{y}_j)$. For each $\mathbf{y}_j$, the algorithm compares the objective value of current placement with those of placements slightly adjusted in either direction by θ (lines 9 and 11). If an adjustment results in a better objective value, the algorithm updates $\mathbf{y}_j$ accordingly (lines 10 and 12). If no adjustment improves the objective value, $\mathbf{y_j}$ remains unchanged (line 14). After exploring all variations, if the objective value $Z(\mathbf{y}_3)$ is better than that of the previous best placement $\mathbf{x}_i$, the algorithm updates the placement scheme of the dedicated AV lane and adjusts the starting point for the next iteration (lines 19-20). Otherwise, it reduces the step size θ and maintains the previous scheme (lines 22-24). In the end, the optimal placement of the dedicated lane for AV is obtained, along with the corresponding minimum objective value and all other endogenous variables. The initial step size, reduction rate and acceleration coefficient are set as $\theta = 1$, $\gamma = 0.7$, $\alpha = 2$ in the following simulations.

The changes in total travel time due to the placement of a dedicated AV lane under different MPRs of AVs are shown in Fig. 4. Since the ending point of the dedicated lane for AVs must be greater than or equal to the starting point, each sub-figure only takes its value in the upper triangle area. When the MPR of AVs is 20% as shown in Fig. 4(a), placing a dedicated AV lane at any location increases the total travel time. When the MPR of AVs is 40% as shown in Fig. 4(b), placing a dedicated AV lane in the middle area between the CBD edge and the city boundary can reduce the total travel time to the greatest extent. When the MPR of AVs is 60% as shown in Fig. 4(c), the dedicated AV lane started from the CBD edge decreases the total travel time to the greatest extent. When the MPR of AVs is 80% as shown in Fig. 4(d), placing a dedicated AV lane at any location does not cause change. In summary, when MPR of AVs is low, dedicating any lane increases total travel time. And the introduction of dedicated AV lanes has no noticeable impact at high MPR level. However, at intermediate level of MPR, strategically placing a dedicated AV lane can reduce total travel time,

Algorithm 3: Pattern search for optimizing the placement of dedicated AV lane

Input: MPR of AVs, initial step size θ, acceleration coefficient γ and all other parameters

Output: The optimal placement of dedicated lane for AVs

let $\mathbf{x} := \left[x^D_{start}, x^D_{end}\right]^\intercal$;
record the objective value of minimizing problem under the placement scheme $\mathbf{x}$ as $Z(\mathbf{x})$;
$\Phi \leftarrow \left\{[\omega_1, \omega_2]^\intercal | \underline{x} < \omega_1 < \omega_2 < \bar{x}\right\}$;
$\mathbf{e}_1 \leftarrow [1, 0]^\intercal$, $\mathbf{e}_2 \leftarrow [0, 1]^\intercal$;
$\mathbf{x}_1 \leftarrow [\underline{x}, \underline{x}]^\intercal$ $\mathbf{y}_1 \leftarrow \mathbf{x}_1$;
$i \leftarrow 1, j \leftarrow 1$;
repeat
    repeat
        if $Z(\mathbf{y}_j + \theta \mathbf{e}_j) < Z(\mathbf{y}_j)$ *and* $\mathbf{y}_j + \theta \mathbf{e}_j \in \Phi$ **then**
            $\mathbf{y}_{j+1} \leftarrow \mathbf{y}_j + \theta \mathbf{e}_j$;
        else if $Z(\mathbf{y}_j - \theta \mathbf{e}_j) < Z(\mathbf{y}_j)$ *and* $\mathbf{y}_j - \theta \mathbf{e}_j \in \Phi$ **then**
            $\mathbf{y}_{j+1} \leftarrow \mathbf{y}_j - \theta \mathbf{e}_j$;
        else
            $\mathbf{y}_{j+1} \leftarrow \mathbf{y}_j$;
        end
        $j \leftarrow j + 1$;
    until $j > 2$;
    if $Z(\mathbf{y}_3) < Z(\mathbf{x}_i)$ **then**
        $\mathbf{x}_{i+1} \leftarrow \mathbf{y}_3$;
        $\mathbf{y}_1 \leftarrow \mathbf{x}_{i+1} + \alpha(\mathbf{x}_{i+1} - \mathbf{x}_i)$;
    else
        $\theta \leftarrow \gamma\theta$;
        $\mathbf{x}_{i+1} \leftarrow \mathbf{x}_i$;
        $\mathbf{y}_1 \leftarrow \mathbf{x}_i$;
    end
    $i \leftarrow i + 1$;
    $j \leftarrow 1$;
until θ *is less than the acceptable level*;

with the optimal location shifting from the middle region toward the edge of CBD. These results are consistent with Remark 1.

Then we compare the short-term traffic states under the optimal scheme and the absence of a dedicated AV lane, as shown in Table 3. Obviously, when the MPR of AVs is 20% or 80%, the optimal scheme is not to place any dedicated AV lane, and the traffic state remains unchanged. When the MPR of AVs is 40%, the optimal scheme is to place a dedicated AV lane on 4.8 km of the road in the middle area between the CBD edge and the city boundary (not starting from the CBD edge). This reduces the average travel time of all users by 1.16%, reduces the average travel time of AVs by 8.68%, but increases the average travel time of RVs by 5.74%. When the MPR of AVs is 60%, the optimal scheme

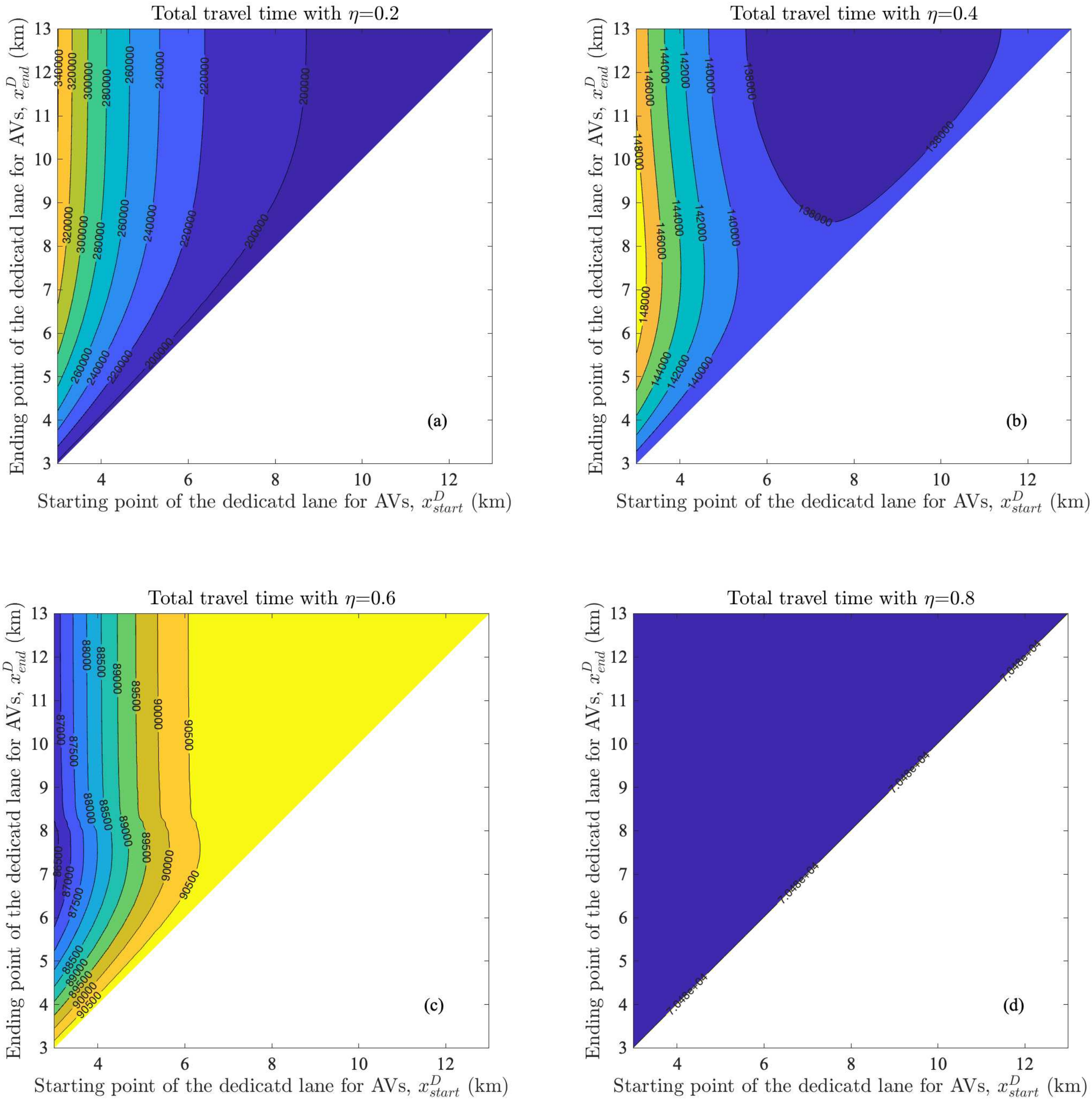

Figure 4: Changes of total travel time with the starting and ending points of the dedicated AV lane in the short term

is to place a dedicated AV lane on 4.3 km of the road starting from the CBD edge. This reduces the average travel time of all users by 4.95%, decreases the average travel time of AVs by 9.93%, but increases the average travel time of RVs by 6.79%. The trends of change in the average speeds are opposite to those in the average travel time. Notably, when the MPR of AVs is 60%, a shorter dedicated AV lane achieves a greater proportional reduction in the total travel time compared to when the MPR of AVs is 40%.

The results also reveal significant equity implications across different user groups. While dedicated AV lanes improve system efficiency (1.16-4.95% total travel time reduction) and substantially benefit AV users (8.68-9.93% time savings). In contrast, RV users are disadvantaged, facing travel time increase of 5.74-6.79%. This regressive effect occurs because reallocating lanes to AVs reduces capacity for RVs and concentrates them onto fewer ordinary

Table 3

Optimal lane placement with different MPRs of AVs in the short term

Variables	$\eta = 0.2$	$\eta = 0.4$	$\eta = 0.6$	$\eta = 0.8$
Placement of dedicated AV lane (km)	-	7.7, 12.5, 4.8	3.0, 7.3, 4.3	-
Total travel time (h)	1.95e+05	1.37e+05 (-1.16%)	8.63e+04 (-4.95%)	7.05e+04
Average travel time (h)	1.30	0.91(-1.16%)	0.58 (-4.95%)	0.47
Average travel time by AV (h)	1.61	1.01(-8.68%)	0.64 (-9.93%)	0.52
Average travel time by RV (h)	1.22	0.85 (5.74%)	0.48 (6.79%)	0.27
Average speed (km/h)	11.44	19.38 (1.17%)	33.96(5.21%)	43.40
Average speed of Avs (km/h)	13.73	23.60 (9.51%)	37.65 (11.03%)	43.89
Average speed of RVs (km/h)	10.68	16.03 (-5.43%)	26.64 (-6.36%)	39.59

Note: in the second row, the first number is the starting point of the dedicated lane for AVs, the second number is the ending point of the dedicated lane for AVs, the third number is the length of dedicated AV lane. The values between parentheses represent the percentage change compared to when no dedicated lane is placed.

lanes, thereby intensifying congestion. The disparity is most pronounced at $\eta = 60\%$ where the increases in RV travel time nearly offsets the gains for AV users (6.79% penalty vs 9.93% benefit), creating a mobility access gap that may potentially disadvantages lower-income travelers who cannot afford AV technology (which is further discussed in the model extension). These findings highlight the equity-efficiency tradeoff in AV infrastructure planning, and suggest that policy implementation should incorporate compensatory mechanisms for non-adopters.

In addition, we analyze the impact of AV reaction time and user VOT on the optimal placement for short-term dedicated AV lanes (see Table C1 in the Appendix for detail). The results show that the placement of dedicated AV lanes remains beneficial for reducing total system travel time only when the level of MPR is within a moderate range. However, this effective range would be influenced by these parameters. Shorter AV reaction time narrows the interval of MPR in which dedicated lanes yield benefits. This finding suggests that dedicated AV lanes are more effective during the early and middle phases of AV technology adoption, rather than when the technology reaches full maturity.

5.2. Optimal placement of dedicated AV lanes in the long term

As previously discussed, the placement of a dedicated AV lane influences individual residential location choices over a longer time scale, which, in turn, alters the urban spatial structure and reshapes the distribution of travel demand. This section relaxes the assumption of a fixed residential distribution, considers the impacts of placing a dedicated AV lane on residential choice behaviors and urban characteristics, and explores the optimal schemes under different MPRs of AVs in the long term.

We propose a programming model of maximizing social welfare by determining the starting point x^D_{start} and ending point x^D_{end} of placing one dedicated AV lane. The model can be expressed as:

$$\max_{x^D_{start},x^D_{end}} SW, \tag{36}$$

$$\text{s.t.} \begin{cases} \text{Eqs. (1)-(20)}, \\ x^D_{end} - x^D_{start} \geq 0. \end{cases} \tag{37}$$

Recording the value of social welfare under the dedicated lane placement $\mathbf{x}$ as $-Z(\mathbf{x})$, the maximization problem of social welfare can also be solved by Algorithm 3.

Fig. 5 illustrates the the changes in social welfare due to the placement of a dedicated AV lane under different MPRs of AVs. When the MPR of AVs is 20% as shown in Fig. 5(a), placing a dedicated AV lane at any location decreases social welfare. When the MPR of AVs is 40% as shown in Fig. 5(b), placing a dedicated AV lane in the middle area between the CBD edge and the city boundary (not starting from the CBD edge) increases social welfare to the greatest extent. When the MPR of AVs is 60% as shown in Fig. 5(c), a dedicated AV lane placed from the CBD decreases social welfare. When the MPR of AVs is 80% as shown in Fig. 5(d), the result is similar to that observed at $\eta = 0.2$, where placing a dedicated AV lane decreases social welfare. These results reveal that the placement of a dedicated AV lane improves social welfare only when AV penetration is at an intermediate level. When the MPR of AVs is either too high or too low, placing a dedicated AV lane does not yield any benefits and may even reduce social welfare.

Then we compare the long-term urban characteristics under the optimal placement of a dedicated AV lane and the absence of such lane, as shown in Table 4. When the MPR of AVs is 20% or 80%, the optimal scheme is not to place any dedicated AV lane, and no change occurs in urban characteristics. When the MPR of AVs is 40%, the optimal scheme is to place a dedicated AV lane on 3 km of the road in the middle area between the CBD edge and the city boundary (not starting from the CBD edge). This raises social welfare by 0.08%, reduces the average travel time and distance by 3.43% and 0.92%, respectively, and compacts the urban space with 0.16% decreasing in city size. At the same time, for AV users, the utility increases by 1.68%, the average travel time and distance decreases by 8.95% and 0.51%, respectively, and the average rent increases by 0.07%. For RV users, the utility decreases by 1.05%, the average travel time and distance increases by 1.63% and decreased by 1.40%, respectively, and the average rent increases by 2.29%. When the MPR of AVs is 60%, the optimal scheme is to place a dedicated AV lane on 3.1 km of the road starting from the CBD edge, which raises social welfare by 0.14%, reduces the average travel time and distance by 4.43% and 0.09%, respectively, and expands the urban space with 0.19% increasing in city size. At the same time, for AV users, the utility increases by 0.92%, the average travel time and distance decreases by 8.41% and 0.01%, respectively, and the average rent increases by 0.01%. For RV users, the utility decreases by 0.72%, the average travel time and distance increases by 4.97% and decreases by 0.33%, respectively, and the average rent increases by 0.22%. In summary, at both low and high MPR of AVs, the optimal strategy involves no dedicated AV lanes, resulting in no significant changes to urban changes. At intermediate MPR of AVs, strategically placed dedicated AV lanes enhance system efficiency—increasing social welfare and reducing average travel time. Yet, these efficiency gains come with equity concerns: while AV users benefit from substantial utility improvement and reduced travel time, RV users face utility loss, longer travel time, and increased housing rent. The results also reveal that the length of the dedicated AV lane under optimal placement in the long term is shorter than that in the short term, with less reduction in travel time but an increase in social welfare.

Intuitively, placing a dedicated AV lane reduces the cost of AV travel, allowing AV users to accept longer commuting distances and live further away from the CBD, leading to an expansion of city size, as shown in the column $\eta = 0.6$ of Table 4. However, some results indicate the optimal dedicated AV lane deceases the city size as shown in the column $\eta = 0.4$ of Table 4. For both $\eta = 0.4$ and 0.6, the residential separation point moves closer to the CBD after placing a dedicated AV lane, indicating that the optimal scheme makes some AV users live nearer to the CBD. This occurs because the placement of a dedicated AV lane reduces the supply of road resources for RVs. As a result, RV users are compelled to choose locations closer to the CBD in their long-term residential decisions to avoid higher commuting

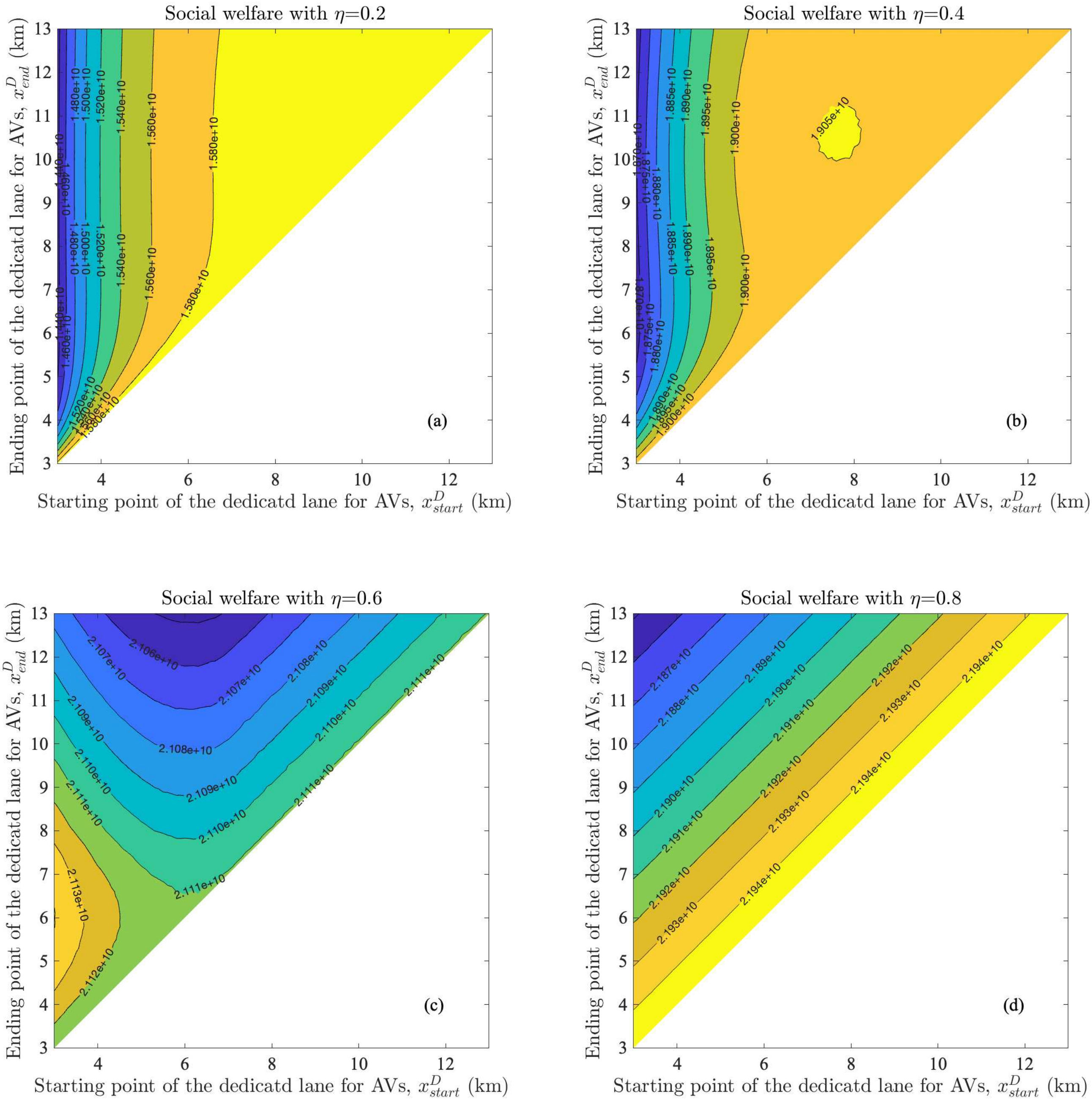

Figure 5: Changes of social welfare with the starting and ending points of the dedicated AV lane in the long term

costs, which frees up residential land near the CBD. For instance, when the MPR of AVs is 40%, the residential range of RV users contracts from the original (3.00, 16.79) to (3.00, 16.57) after implementing the optimal scheme. This denser concentration of RV users' residences releases some urban land, enabling AV users to relocate closer to the CBD. A trade-off emerges after placing the optimal dedicated AV lane between urban expansion caused by the reduction in AV travel costs and urban contraction caused by the denser residential patterns of RV users. When the MPR of AVs is 40%, the increased travel costs for RVs result in many RV users relocating closer to the CBD, compressing the urban space. When the MPR of AVs is 60%, RV users, as a minority group, exert less influence on urban spatial structure. Consequently, the change in city size is primarily driven by the reduction in AV travel costs, which encourages AV users to live further away from the CBD, leading to an expansion of city size.

Table 4

Optimal placement with different MPRs of AVs in the long term

Variables	$\eta = 0.2$	$\eta = 0.4$	$\eta = 0.6$	$\eta = 0.8$
Placement of dedicated AV lane (km)	-	7.7, 10.7, 3.0	3.0, 6.1, 3.1	-
Social welfare	1.59e+10	1.91e+10(0.08%)	2.11e+10(0.14%)	2.19e+10
Total travel time (h)	1.95e+05	1.33e+05 (-3.43%)	8.68e+04 (-4.43%)	7.05e+04
Average travel time (h)	1.30	0.89 (-3.43%)	0.58 (-4.43%)	0.47
Average travel distance (km)	14.87	17.49 (-0.92%)	19.53 (-0.09%)	20.40
Average speed (km/h)	11.44	19.65 (2.60%)	33.74 (4.53%)	43.40
Average land rent (CNY/km^2)	1.80e+06	1.53e+06 (0.79%)	1.40e+06 (-0.09%)	1.36e+06
Residential separation point (km)	16.96	16.57 (-1.31%)	14.84 (-0.40%)	10.93
City size (km)	21.07	24.47 (-0.16%)	26.62 (0.19%)	27.39
Average utility	986	1207(0.15%)	1353(0.30%)	1407
Utility of AV users	1243	1346 (1.68%)	1403 (0.92%)	1417
Average travel time by AV (h)	1.61	1.00 (-8.95%)	0.65 (-8.41%)	0.52
Average travel distance by AV (km)	22.05	23.64 (-0.51%)	24.02 (-0.01%)	22.85
Average speed by AV (km/h)	13.73	23.55 (9.27%)	37.02 (9.18%)	43.89
Average rent of AV users (CNY/km^2)	1.08e+06	1.14e+06 (0.07%)	1.20e+06 (0.01%)	1.26e+06
Utility of RV users	922	1114 (-1.05%)	1277 (-0.72%)	1366
Average travel time by RV (h)	1.22	0.81 (1.63%)	0.47 (4.97%)	0.27
Average travel distance by RV (km)	13.07	13.39 (-1.40%)	12.79 (-0.33%)	10.57
Average speed by RV (km/h)	10.68	16.45 (-2.99%)	27.01 (-5.05%)	39.59
Average rent of RV users (CNY/km^2)	2.21e+06	2.01e+06 (2.29%)	1.88e+06 (0.22%)	1.92e+06

We further examine the equity challenges faced by different user groups in the long term. At intermediate MPR levels (η = 40% and 60%), AV users experience utility gains (0.92-1.68%), while RV users suffer utility loss (0.72-1.05%), thereby widening the mobility advantage gap. This disparity arises from two compounding mechanisms: (i) RV users endure longer travel time (1.63-4.97% increase) despite shortening their commutes (0.33-1.40% distance reduction), indicating intensified congestion on ordinary lanes; (ii) RV users face disproportionate rent increases (0.22-2.29%) as they are pushed toward CBD-adjacent locations to offset rising travel cost.In contrast, AV users benefit from both travel time saving and relatively moderated rent impact (0.01-0.07% increase). The equity impact is most pronounced at η = 40%, where the RV users' utility loss (-1.05%) exceeds half the utility gain (+1.68%), indicating a significant welfare transfer from non-adopters to early technology adopters. These findings highlight that dedicated AV lanes generate systemic re-distributional effect beyond immediate traffic impact, potentially exacerbating socioeconomic divides in urban accessibility.

Besides, we apply the short-term optimal schemes to the long-term equilibrium case and compare the results with those in the short term, as shown in Table 5. When the MPR of AVs is 40%, under the short-term optimal scheme (placing a dedicated AV lane from location x = 7.7 to location x = 12.5), the long-term social welfare increases by

0.28% compared to the short-term social welfare, the long-term total travel time decreases by 2.72%, and the long-term average land rent increases by 0.89%. This means that when the MPR of AVs is 40% placing the short-term optimal scheme in the long term leads to overestimate the total travel time while underestimate social welfare and average land rent. Conversely, when the MPR of AVs is 60%, under the short-term optimal scheme (placing a dedicated AV lane from location $x = 3.0$ to location $x = 7.3$), the long-term social welfare increases by 0.31% compared to the short-term social welfare, the long-term total travel time increases by 0.17%, and the long-term average land rent decreases by 0.07%. This means that when the MPR of AVs is 60% placing the short-term optimal scheme in the long term leads to underestimate the total travel time and social welfare while overestimate the average land rent. The misestimation significantly interferes with the effectiveness of urban road planning, resulting in either a waste of resources or a failure to fully utilize road capacity. In the long run, the setting of a dedicated AV lane influences residents' utility and land rent. Therefore, the goal of minimizing the total travel time, , which is solely based on the traffic system is no longer applicable. Instead, the design of traffic infrastructure in the context of increasing AV adoption needs to pursue a more comprehensive objective-maximizing social welfare.

Table 5

Percentage change of variables under the short-term schemes in the long-term equilibrium

Variables	$\eta = 0.4$	$\eta = 0.6$
Social welfare	0.28%	0.31%
Total travel time (h)	-2.72%	0.17%
Average travel time (h)	-2.72%	0.17%
Average travel distance (km)	-1.00%	-0.11%
Average speed (km/h)	1.76%	-0.28%
Average land rent (CNY/km^2)	0.89%	-0.07%
Residential separation point (km)	-1.43%	-0.40%
City size (km)	-0.20%	0.19%
Average utility	0.18%	0.32%
Utility of AV users	1.84%	0.98%
Average travel time by AV (h)	-1.26%	1.02%
Average travel distance by AV (km)	-0.56%	-0.02%
Average speed by AV (km/h)	0.71%	-1.03%
Average rent of AV users (CNY/km^2)	0.07%	0.03%
Utility of RV users	-1.12%	-0.76%
Average travel time by RV (h)	-3.87%	-1.50%
Average travel distance by RV (km)	-1.52%	-0.36%
Average speed by RV (km/h)	2.45%	1.16%
Average rent of RV users (CNY/km^2)	2.55%	0.27%

In addition, we analyze the impact of AV reaction time, user VOT and deployment cost on the optimal placement for long-term dedicated AV lanes (see Table C2 in the Appendix for detail). The results show that both lower AV response time and VOT of AV users will eliminate the social welfare promoting effect of placing AV lanes, while lower AV lane construction cost will expand the length of dedicated AV lane and the range of applicable MPR level of placing dedicated AV lane. These illustrate that the long-term effect of the parameters on the optimal strategy for dedicated lane placement is more sensitive than the short-term effect.

6. Extensions

While the original framework provides critical insights into AV lane deployment in a monocentric city with homogeneous users, real-world applications require accounting for dynamic market adoption patterns, socioeconomic disparities, and evolving urban spatial structure. In this paper, we propose four model extensions to enhance practical relevance: (1) making the number of radial road variable and distance-dependent, (2) endogenously determining the market penetration rate of AVs, (3) incorporating income-based user heterogeneity, and (4) transitioning from a monocentric to a polycentric city structure.

6.1. Variable radial roads

So far we assumed a fixed number of radial roads in the city. Now we redefine this number as an increasing function of the distance toward the city center, which can capture the outward increase in road density observed in real cities. The function of the number of radial roads with respect to location can be expressed as

$$n_l(x) = \frac{2\pi x}{d_r}. \tag{38}$$

where$2\pi x$ is the circumference of the arc at location x, d_r denotes the arc distance between adjacent radial roads and set as $d_r = 3.5$ in simulations.

Table 6 and Table 7 compares the short-term traffic states and the long-term urban characteristics under optimal dedicated AV lane deployment versus no deployment considering variable radial roads, respectively. Both short-term and long-term scenarios demonstrate substantial traffic efficiency improvements through travel time reduction and speed enhancement when placing dedicated AV lanes at intermediate MPR levels. In the short term, when the MPR of AVs is 60%, implementing AV lanes from 3.0 km to 6.4 km (3.4 km length) starting from the CBD edge reduces the total travel time by 11.89%. In the long term, implementing AV lanes from 3.0 km to 6.2 km (3.2 km length) starting from the CBD edge raises social welfare by 1.05%, reduces the total travel time by 11.88%.

6.2. Endogenous market penetration rate of AVs

The basic model treats the MPR of AVs as an exogenous parameter, which fails to capture adoption inertia and market-driven behavioral shifts. In reality, car purchase decisions, like residential location choice, will be in the long-term decision-making content of households. Therefore, optimizing long-term dedicated AV lane placement requires endogenous modeling of vehicle market dynamics. This subsection endogenizes the MPR of AVs by integrating a binary logit model that correlates AV adoption decision with residents' utility. The endogenous MPR of AVs can be expressed as

$$\eta = \frac{e^{\mu u^{AV}}}{e^{\mu u^{AV}} + e^{\mu u^{RV}}}. \tag{39}$$

where μ is the coefficient of logit model and set as $\mu = 0.01$ in the following simulations.

Table 6

Optimal lane placement in the short term considering different MPRs and radial road variability

Variables	$\eta = 0.2$	$\eta = 0.4$	$\eta = 0.6$	$\eta = 0.8$
Placement of dedicated AV lane (km)	-	6.8, 8.2, 1.4	3, 6.4, 3.4	-
Total travel time (h)	2.73e+05	2.40e+05(-0.01%)	1.16e+05(-11.89%)	7.91e+04
Average travel time (h)	1.82	1.60(-0.01%)	0.77(-11.89%)	0.53
Average travel time by AV (h)	2.06	1.76(-0.84%)	0.71(-26.63%)	0.58
Average travel time by RV (h)	1.76	1.50(0.54%)	0.85(16.82%)	0.33
Average speed (km/h)	7.65	9.86(0.01%)	24.57(13.49%)	38.47
Average speed of Avs (km/h)	9.94	12.18(0.84%)	32.67(36.29%)	39.35
Average speed of RVs (km/h)	6.98	8.05(-0.54%)	14.56(-14.40%)	32.18

We analyze the impact of AV characteristics (e.g., reaction time and VOT of AV users) on the long-term optimal placement of dedicated AV lanes under endogenous MPR of AVs. Table 8 and Table 9 show that, for both reaction time (δ^{AV}) and VOT discount (τ^{AV}) of AV users, dedicated AV lanes are only beneficial beyond specific thresholds in the long run. From Table 8, it can be seen that the optimal scheme is not to place any dedicated AV lane when δ^{AV} is 0.05 or 0.1. When δ^{AV} is 0.15, the optimal scheme is to place a dedicated AV lane starting at 3 km and ending at 6.3 km, with a length of 3.3 km. Furthermore, higher δ^{AV} (i.e., longer reaction time) necessitates longer AV lanes starting from the CBD edge, reducing AV travel time significantly but increasing RV travel time. From Table 9, we find when the value-of-time discount factor of AVs τ^{AV} is 0.3 or 0.5, the optimal scheme is not to place any dedicated AV lane. while when τ^{AV} is 0.7, the optimal scheme is to place a dedicated AV lane starting at 3 km and ending at 4.8 km. In addition, higher τ^{AV} (i.e., greater in-vehicle time value) enables shorter lanes placed away from the CBD edge, simultaneously reducing travel times for both vehicle types. The above results suggest that well-developed AV technology and a very good AV ride can weaken the role of self-driving lanes.

6.3. Income heterogeneity

Building upon the endogenous MPR of AVs, this subsection incorporates income heterogeneity to better reflect how household vehicle adoption decision varies across socioeconomic groups. Recognizing that high-income (denoted as $i = H$) and low-income (denoted as $i = L$) residents exhibit distinct preferences and sensitivities to cost, the model segments the population. The adoption probability for each group is modeled using a binary logit structure. The endogenous MPR for AVs within income group i is given by:

$$\eta_i = \frac{e^{\theta u_i^{AV}}}{e^{\theta u_i^{AV}} + e^{\theta u_i^{RV}}}, \quad i = H, L \tag{40}$$

where u_i^{AV} and u_i^{RV} represent the utilities of choosing an AV or RV, respectively, specific to group i. In addition, the extended model distinguishes between MPR of AVs, travel cost, value of travel time, and housing consumption, all of which are calculated separately for the two groups by introducing the subscript i. We examine the optimal placement of dedicated AV lane and changes in urban characteristics under different proportions of high-income residents, and the heterogeneous incomes are set as $w_H = 1.2e5$ and $w_L = 1e5$.

Table 7

Optimal lane placement in the long term considering different MPRs and radial road variability

Variables	$\eta = 0.2$	$\eta = 0.4$	$\eta = 0.6$	$\eta = 0.8$
Placement of dedicated AV lane (km)	-	3.9, 7.2, 3.3	3, 6.2, 3.2	-
Social welfare	1.31e+10	1.57e+10(0.32%)	2.01e+10(1.05%)	2.17e+10
Total travel time (h)	2.73e+05	2.23e+05(-7.36%)	1.16e+05(-11.88%)	7.91e+04
Average travel time (h)	1.82	1.48(-7.36%)	0.77(-11.88%)	0.53
Average travel distance (km)	13.93	15.19(-3.78%)	18.88(-0.29%)	20.29
Average speed (km/h)	7.66	10.24(3.85%)	24.50(13.14%)	38.48
Average land rent (CNY/km^2)	1.74e+06	1.57e+06(2.00%)	1.40e+06(-0.55%)	1.36e+06
Residential separation point (km)	15.37	13.52(-6.05%)	13.98(-1.41%)	10.85
City size (km)	19.49	22.11(-0.14%)	26.01(0.74%)	27.26
Average utility	818	990(0.27%)	1289(1.23%)	1393
Utility of AV users	1153	1288(8.21%)	1388(3.86%)	1406
Average travel time by AV (h)	2.06	1.33(-24.81%)	0.73(-24.58%)	0.58
Average travel distance by AV (km)	20.47	21.01(-1.87%)	23.32(0.10%)	22.73
Average speed by AV (km/h)	9.95	15.76(30.51%)	31.81(32.72%)	39.37
Average rent of AV users (CNY/km^2)	1.08e+06	1.15e+06(0.25%)	1.20e+06(-0.14%)	1.27e+06
Utility of RV users	734	792(-7.13%)	1140(-3.24%)	1342
Average travel time by RV (h)	1.76	1.58(6.27%)	0.82(12.80%)	0.33
Average travel distance by RV (km)	12.29	11.32(-6.05%)	12.22(-1.41%)	10.50
Average speed by RV (km/h)	6.99	7.16(-11.59%)	14.87(-12.60%)	32.20
Average rent of RV users (CNY/km^2)	2.16e+06	2.31e+06(10.00%)	1.91e+06(0.87%)	1.93e+06

Table 10 compares the long-term traffic states and urban performances under the optimal placement of dedicated AV lanes for different high-income proportions (HIR). The results show that the optimal placement of dedicated AV lane is insensitive to the proportion of high-income residents. When HIR is 0.2, 0.4, or 0.6, the optimal dedicated AV lane placement is from 3 km to 8.1 km (length 5.1 km). When HIR is 0.8, the optimal placement shifts to 3 km to 9.6 km (length 6.6 km). As the proportion of high-income residents increases, accompanied by a rise in AV MPR for high-income groups and a decline for low-income groups. This placement shift contributes to a reduction in social welfare and total travel time, alongside expansions in city size and average traffic speed. However, average utility improves across all groups after placing the optimal scheme of dedicated AV lanes, with low-income residents experiencing relatively larger gains in utility regardless of vehicle type adoption.

6.4. Polycentric city structure

In this subsection, the model is extended to a polycentric city to consider the spatial characteristics of the mixed traffic flow of AVs and RVs on the multi-directional road. $\underline{x}$ no longer represents the edge of CBD and is redefined as the geometric center of the city. Employment distribution are modeled as a series of concentric rings, denoted as $W(y)$, where $y \in (0, \bar{x})$. Employment and total population are conserved, i.e., $\int_0^{\bar{y}} W(y) = N$. The direction from city boundary

Table 8

Long-term optimal lane placement considering AV reaction time and endogenous MPR of AVs

Variables	$\delta^{AV} = 0.05$	$\delta^{AV} = 0.1$	$\delta^{AV} = 0.15$	$\delta^{AV} = 0.2$
Placement of dedicated AV lane (km)	-	-	3, 6.3, 3.3	3, 8.1, 5.1
Social welfare	2.17e+10	2.16e+10	2.15e+10(0.09%)	2.15e+10(0.46%)
Total travel time (h)	7.58e+04	7.90e+04	8.10e+04(-2.25%)	8.20e+04(-6.38%)
Average travel time (h)	0.51	0.53	0.54(-2.25%)	0.55(-6.38%)
Average travel distance (km)	20.06	20.00	19.96(0.12%)	19.93(0.45%)
Average speed (km/h)	39.68	37.96	36.96(2.42%)	36.46(7.30%)
Average land rent (CNY/km^2)	1.38e+06	1.38e+06	1.38e+06(0.00%)	1.39e+06(-0.17%)
Residential separation point (km)	13.73	13.49	13.31(0.99%)	13.20(2.80%)
City size (km)	27.10	27.04	26.99(0.11%)	26.96(0.37%)
Average utility	1387	1382	1379(0.24%)	1377(0.72%)
Utility of AV users	1411	1406	1403(0.22%)	1402(0.63%)
Average travel time by AV (h)	0.58	0.61	0.62(-2.18%)	0.62(-5.93%)
Average travel distance by AV (km)	23.81	23.67	23.58(0.34%)	23.52(0.96%)
Average speed by AV (km/h)	40.71	39.05	38.14(2.57%)	37.65(7.32%)
Average rent of AV users (CNY/km^2)	1.22e+06	1.22e+06	1.23e+06(-0.17%)	1.23e+06(-0.51%)
Utility of RV users	1337	1329	1324(0.40%)	1322(1.20%)
Average travel time by RV (h)	0.34	0.35	0.36(-3.33%)	0.36(-8.31%)
Average travel distance by RV (km)	12.22	12.05	11.94(0.75%)	11.88(2.18%)
Average speed by RV (km/h)	35.95	33.95	33.18(4.22%)	32.97(11.44%)
Average rent of RV users (CNY/km^2)	1.86e+06	1.88e+06	1.90e+06(-0.34%)	1.91e+06(-1.35%)

toward center is labeled as toC, and the opposite direction is labeled as toB, and all road transportation related variables will be introduced into these two subscripts. Then the placement of dedicated AV lane is considered separately in each direction.

Let $f^k(x, y)$ denote the travel demand for vehicle type$k$ between residential location x and job location y. This demand is assumed to be proportional to the number of residents at location by a binary function , which is proportional to the number$x$, the number of jobs at location y,and inversely proportional to the square of the generalized travel cost between the two locations for vehicle type k. Drawing on the network equilibrium with doubly-constrained gravity equations (Boyce et al., 1988) , $f^k(x, y)$ can be expressed as

$$f^k(x, y) = A^k(x)B(y)\frac{4\pi^2 xyD^k(x)W(y)}{C^k(x, y)^2} \tag{41}$$

with $A^k(x) = \left(\int_0^{\bar{y}} 2\pi yB(y)W(y)(C^k(x, y))^{-2}\mathrm{d}y\right)^{-1}$ and $B(y) = \left(\int_0^{\bar{x}} \sum_k 2\pi xA^k(x)D^k(x)(C^k(x, y))^{-2}\mathrm{d}x\right)^{-1}$, where $A^k(x)$ and $B(y)$ are intermediate computational variables that are not predetermined and can be solved iteratively from each other during numerical calculation. This formulation guarantees the conservation for both residents and jobs at each location, i.e., $\int_0^{\bar{x}} \Sigma_k f^k(x, y)\mathrm{d}x = 2\pi y$ and $\int_0^{\bar{y}} f^k(x, y)\mathrm{d}y = 2\pi xD^k(x)$ hold.

Table 9

Long-term optimal lane placement considering user VOT variability and endogenous MPR of AVs

Variables	$\tau^{AV} = 0.3$	$\tau^{AV} = 0.5$	$\tau^{AV} = 0.7$	$\tau^{AV} = 0.9$
Placement of dedicated AV lane (km)	-	-	3, 4.8, 1.8	3, 6.4, 3.4
Social welfare	2.24e+10	2.16e+10	2.08e+10(0.40%)	2.01e+10(3.58%)
Total travel time (h)	7.65e+04	7.90e+04	8.24e+04(-3.17%)	8.34e+04(-17.11%)
Average travel time (h)	0.51	0.53	0.55(-3.17%)	0.56(-17.11%)
Average travel distance (km)	20.77	20.00	19.28(0.29%)	18.67(2.31%)
Average speed (km/h)	40.74	37.96	35.10(3.57%)	33.60(23.43%)
Average land rent (CNY/km^2)	1.31e+06	1.38e+06	1.45e+06(-0.24%)	1.52e+06(-1.98%)
Residential separation point (km)	12.83	13.49	14.05(-1.33%)	14.21(-6.57%)
City size (km)	28.07	27.04	26.02(0.35%)	25.15(2.65%)
Average utility	1440	1382	1324(0.51%)	1277(3.98%)
Utility of AV users	1467	1406	1344(0.61%)	1292(4.80%)
Average travel time by AV (h)	0.58	0.61	0.63(-4.62%)	0.62(-21.87%)
Average travel distance by AV (km)	24.05	23.67	23.30(-0.14%)	22.87(-0.46%)
Average speed by AV (km/h)	41.53	39.05	37.01(4.70%)	37.14(27.40%)
Average rent of AV users (CNY/km^2)	1.20e+06	1.22e+06	1.24e+06(0.33%)	1.27e+06(2.26%)
Utility of RV users	1365	1329	1290(0.22%)	1253(2.60%)
Average travel time by RV (h)	0.32	0.35	0.41(-1.62%)	0.46(-14.34%)
Average travel distance by RV (km)	11.68	12.05	12.35(-0.86%)	12.44(-3.86%)
Average speed by RV (km/h)	36.72	33.95	30.30(0.78%)	26.98(12.23%)
Average rent of RV users (CNY/km^2)	1.78e+06	1.88e+06	1.98e+06(0.16%)	2.07e+06(0.60%)

The corresponding travel cost between the two locations can be expressed as

$$C^k(x,y) = \begin{cases} c_f^k + 2m(x-y) + 2\tau^k \frac{w}{8\lambda}\int_y^x \frac{1}{v_{toB}^k(\omega)}\mathrm{d}\omega, & \text{if } x >= y \\ c_f^k + 2m(y-x) + 2\tau^k \frac{w}{8\lambda}\int_x^y \frac{1}{v_{toC}^k(\omega)}\mathrm{d}\omega, & \text{otherwise} \end{cases}. \tag{42}$$

Then the conservation between traffic flow and travel demand can be expressed as

$$n_r Q_{toC}^k(x) = \int_x^{\bar{x}} \int_0^x f^k(\omega, v)\mathrm{d}\omega\mathrm{d}v \tag{43}$$

$$n_r Q_{toB}^k(x) = \int_0^x \int_x^{\bar{y}} f^k(\omega, v)\mathrm{d}\omega\mathrm{d}v \tag{44}$$

Besides, the long-term decision of residents are based on the average travel cost incurred by all residents of the same vehicle type at a given residential location. This average cost can be expressed as

$$c^k(x) = \frac{\int_0^{\bar{y}} C^k(x,y) f^k(x,y)\mathrm{d}y}{D^k(x)}. \tag{45}$$

Table 10

Optimal placement in the long term with varying high-income user proportions

Variables	$HIR = 0.2$	$HIR = 0.4$	$HIR = 0.6$	$HIR = 0.8$
Placement of dedicated AV lane (km)	3, 8.1, 5.1	3, 8.1, 5.1	3, 8.1, 5.1	3, 9.6, 6.6
MPR of AVs in high-income group	2.24e+10(0.40%)	2.34e+10(0.37%)	2.43e+10(0.36%)	2.52e+10(0.35%)
MPR of AVs in low-income group	8.15e+04(-5.97%)	8.16e+04(-5.71%)	8.22e+04(-5.44%)	8.31e+04(-5.64%)
Social welfare	7.61e-01(-1.77%)	7.51e-01(-1.74%)	7.42e-01(-1.70%)	7.32e-01(-1.87%)
Total travel time (h)	6.84e-01(-1.91%)	6.78e-01(-1.86%)	6.72e-01(-1.84%)	6.65e-01(-2.03%)
Average travel time (h)	0.54(-5.97%)	0.54(-5.71%)	0.55(-5.44%)	0.55(-5.64%)
Average travel distance (km)	20.09(0.37%)	20.35(0.36%)	20.71(0.34%)	21.16(0.39%)
Average speed (km/h)	36.95(0.37%)	37.41(0.36%)	37.77(0.34%)	38.19(0.39%)
Average land rent (CNY/km^2)	1.38e+06(-0.18%)	1.37e+06(-0.17%)	1.37e+06(-0.16%)	1.37e+06(-0.18%)
City size (km)	27.62(0.33%)	28.22(0.28%)	28.78(0.31%)	29.29(0.31%)
Average utility	1440(0.72%)	1501(0.69%)	1560(0.66%)	1617(0.71%)
Utility of high-income group with AV	1722(0.58%)	1716(0.55%)	1710(0.53%)	1704(0.58%)
Utility of high-income group with RV	1400(0.57%)	1399(0.55%)	1397(0.52%)	1397(0.56%)
Utility of low-income group with AV	1606(1.10%)	1605(1.05%)	1604(1.00%)	1604(1.07%)
Utility of low-income group with RV	1323(1.09%)	1324(1.04%)	1326(0.99%)	1328(1.07%)

The endogenous residential distribution of the polycentric city can be obtained by substituting the updated travel cost function Eq. (45) into the resident's utility maximization of Eqs. (8) and (9). Employment distribution is set to be evenly distributed in a circular space with a radius of 10 km, and the total population is set as 500,000 to avoid polycentric urban structure and bi-direction traffic causing extremely low traffic demand.

By incorporating the above formulas into the proposed basic model, we can construct travel equilibrium and residential equilibrium under a poly-center urban structure. Here, we analyze the impact of introducing dedicated AV lanes on the the travel efficiency and overall urban system performance under two scenarios: one in which dedicated lanes are deployed at all locations, and one in which no dedicated lanes are deployed. Table 11 and 12 show the long-term traffic states and urban performances, as well as the corresponding percentage changes after the introduction of dedicated AV lanes under different AV MPRs. When the MPR of AVs is extremely low ($\eta = 1\%$), the introduction of dedicated AV lanes reduces social welfare and increase total travel time. At low to moderate AV MPRs ($\eta = 20\%$ and $\eta = 40\%$), the introduction of dedicated AV lanes increases social welfare and reduces the total travel time. In terms of utility, the utility of AV users increases, while that of RV users decreases. When the MP of AVs is high (e.g., $\eta = 60\%$ and $\eta = 80\%$), the introduction of dedicated AV lanes reduces social welfare and increases total travel time. In these cases, the utility of AV users and RV users both decrease.

Besides, the algorithms of optimizing the starting and ending points of dedicated AV lanes remain applicable to this extended model in mathematical form, by independently optimizing placement of dedicated AV lanes for roads in both the "toB" and "toC" directions. However, due to the complexity of the spatial equilibrium of the polycentric city, the optimal placement scheme will be left for future research.

Table 11

Traffic states and urban performances after the introduction of dedicated AV lanes at all locations in the polycentric city

Variables	$\eta = 0.01$	$\eta = 0.2$	$\eta = 0.4$	$\eta = 0.6$	$\eta = 0.8$
Social welfare	4.75e+10	5.43e+10	5.57e+10	5.78e+10	5.90e+10
Total travel time (h)	2.23e+05	2.06e+05	3.81e+05	4.03e+05	3.97e+05
Average travel time (h)	0.45	0.41	0.76	0.81	0.79
Average travel distance (km)	2.32	4.84	7.40	9.07	10.82
Average speed (km/h)	5.22	11.74	9.71	11.26	13.61
Average land rent (CNY/km^2)	8.19e+06	5.36e+06	4.55e+06	4.05e+06	3.61e+06
City size (km)	16.10	25.40	28.70	26.80	27.00
Average utility	759	913	955	1007	1041
Utility of AV users	2001	1349	1065	1029	1040
Average travel time by AV (h)	0.25	0.46	1.35	1.10	0.87
Average travel distance by AV (km)	12.54	14.30	12.92	11.64	11.59
Average speed by AV (km/h)	49.65	31.01	9.60	10.62	13.25
Average rent of AV users (CNY/km^2)	1.15e+06	1.83e+06	2.70e+06	3.67e+06	3.79e+06
Utility of RV users	747	804	881	973	1044
Average travel time by RV (h)	0.45	0.40	0.37	0.37	0.48
Average travel distance by RV (km)	2.22	2.47	3.73	5.21	7.75
Average speed by RV (km/h)	4.97	6.19	10.00	14.14	16.28
Average rent of RV users (CNY/km^2)	9.78e+06	1.05e+07	7.56e+06	4.71e+06	3.06e+06

7. Conclusions

This paper formulates a spatial equilibrium model of a monocentric city with a mixed AV and RV traffic system, considering the characteristics of AV that reduce car-following headway time and value of travel time. It analyzes the impacts of dedicated AV lanes and explores the optimal placement schemes based on the proposed model, which is solved using a two-stage solution procedure. Through theoretical derivation, the spatial differences in the impacts of dedicated AV lanes on lane speed and traffic efficiency are analyzed. Specifically, once the proportion of AVs in the traffic flow exceeds a certain threshold, the speed on the dedicated AV lane and the conventional lane become equivalent. The boundary conditions for enhancing, diminishing, or preserving traffic efficiency under different placements of dedicated AV lanes have been identified. Then two optimization models for the placement of dedicated AV lanes are proposed. The short-term model, assuming fixed travel demand, aims to minimize total travel time, while the long-term model, which considers endogenous travel demand due to residence relocation, aims to maximize social welfare. Further, the long-term and short-term optimal placements are compared, and their impacts on travel user equilibrium and residence equilibrium are examined. Finally, we extend the proposed model by considering variable radial road configuration, endogenous AV market penetration rate, income heterogeneity, and the polycentric nature of urban structure.

Table 12

Percentage change of variables after the introduction of dedicated AV lanes at all locations in the polycentric city

Variables	$\eta = 0.01$	$\eta = 0.2$	$\eta = 0.4$	$\eta = 0.6$	$\eta = 0.8$
Social welfare	-0.72%	4.42%	1.08%	-0.51%	0.64%
Total travel time (h)	-11.08%	-40.87%	-11.29%	5.81%	0.95%
Average travel time (h)	-11.08%	-40.87%	-11.29%	5.81%	0.95%
Average travel distance (km)	-18.75%	-4.71%	-1.50%	1.23%	8.55%
Average speed (km/h)	-8.63%	61.14%	11.04%	-4.33%	7.52%
Average land rent (CNY/km^2)	-8.84%	1.60%	12.15%	-4.85%	-9.51%
City size (km)	-32.92%	-1.55%	11.67%	-6.94%	-1.46%
Average utility	-1.41%	4.20%	-0.08%	-0.02%	1.81%
Utility of AV users	72.12%	20.72%	0.11%	0.04%	1.79%
Average travel time by AV (h)	-82.01%	-73.05%	-10.19%	5.20%	2.77%
Average travel distance by AV (km)	16.90%	-1.40%	0.44%	0.82%	11.85%
Average speed by AV (km/h)	549.87%	265.87%	11.84%	-4.16%	8.83%
Average rent of AV users (CNY/km^2)	-4.74%	9.87%	10.31%	-3.79%	-13.38%
Utility of RV users	-2.54%	-1.46%	-0.23%	-0.12%	1.90%
Average travel time by RV (h)	-9.03%	-9.84%	-13.82%	8.64%	-10.68%
Average travel distance by RV (km)	-20.14%	-9.12%	-5.70%	2.63%	-7.72%
Average speed by RV (km/h)	-12.22%	0.80%	9.42%	-5.53%	3.31%
Average rent of RV users (CNY/km^2)	3.04%	11.81%	16.35%	-6.99%	3.93%

The main findings of this paper are as follows. First, whether placing dedicated AV lanes improves traffic efficiency depends on the share of AVs in the traffic flow. The length and location of the area where dedicated AV lanes positively impact traffic efficiency vary with the MPR of AVs. Second, when the AV MPR is either too high or too low, dedicated AV lanes should not be placed. For intermediate MPR values, there are two types of optimal placement schemes: starting from the CBD or starting from the middle area between the CBD boundary and the city boundary, depending on the specific value of the AV MPR. Third, the length of the long-term optimal placement is shorter than that of the short-term optimal scheme. Fourthly, in the long-term, placing dedicated AV lanes encourages AV users to live farther away from the city center, while RV users tend to live closer to it. This results in opposite trends in urban spatial changes under different AV MPRs. Fifth, ignoring the relocation behavior of the residents and adopting the short-term optimal scheme can lead to incorrect estimation of travel time and result in excessive infrastructure investments. The main findings of this paper are as follows. First, whether placing dedicated AV lanes improves traffic efficiency depends on the share of AVs in the traffic flow. The length and location of the area where dedicated AV lanes positively impact traffic efficiency vary with the MPR of AVs. Second, when the AV MPR is either too high or too low, dedicated AV lanes should not be placed. For intermediate MPR values, there are two types of optimal placement schemes: starting from the CBD or starting from the middle area between the CBD boundary and the city boundary, depending on the specific value of the AV MPR. Third, the length of the long-term optimal placement is shorter than that of the short-term optimal scheme.

Fourthly, in the long-term, placing dedicated AV lanes encourages AV users to live farther away from the city center, while RV users tend to live closer to it. This results in opposite trends in urban spatial changes under different AV MPRs. Fifth, ignoring the relocation behavior of the residents and adopting the short-term optimal scheme can lead to incorrect estimation of travel time and result in excessive infrastructure investments.

The contributions of this paper are manifested in three key aspects. First, we integrate the characteristics of AVs, specifically their reduced car-following headway time and VOT, into the analysis framework of a monocentric continuous city. The formulated model accurately captures the spatial distribution of mixed AV and RV traffic and enables the analysis of its impacts on urban spatial structure, housing affordability, and social welfare. Second, we propose a novel optimization method for determining the placement of dedicated AV lanes, identifying their optimal starting and ending points under both short-term user equilibrium and long-term urban spatial equilibrium. Then the long-term and short-term optimal placements are compared to examine their effects on travel and residence equilibrium. Third, we confirm that dedicated AV lanes are not always effective—consistent with findings in HOV lane management research Dahlgren (1998)—and derive the theoretical conditions for mixed traffic flow. Additionally, through numerical examples, we link these conditions to urban spatial changes driven by endogenous shifts in residential distribution.

This paper provides a framework for modeling the interactions between residential location choice equilibrium and user equilibrium in the mixed AV and RV traffic system. Several potential extensions can be explored in future research. First, our simulations focus solely on the placement of a single dedicated AV lane. Future studies could investigate the placement of multiple AV lanes and their combined impact on road capacity and traffic efficiency. Second, a continuous urban structure with with concentric circles is considered in this paper. It will be meaningful to explore the placement of the dedicated lanes in a more complex urban structure based on the real road network. Third, This paper considers only commuters using personal AVs or RVs. Future research could incorporate the impacts of public transportation and other modes of travel on the traffic efficiency of mixed traffic systems, paving the way for collaborative planning between public transportation and AV infrastructure. Fourth, this paper focuses exclusively on the optimal placement of dedicated AV lanes. Future research could further explore the integration of optimization and management strategies for implementing dedicated lanes under diverse real-world conditions. For instance, redistributing AV lane access fees as rebates for regular vehicle users may serve as a promising approach to address equity concerns in a mixed traffic system.

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Appendix A List of notations

Table A1

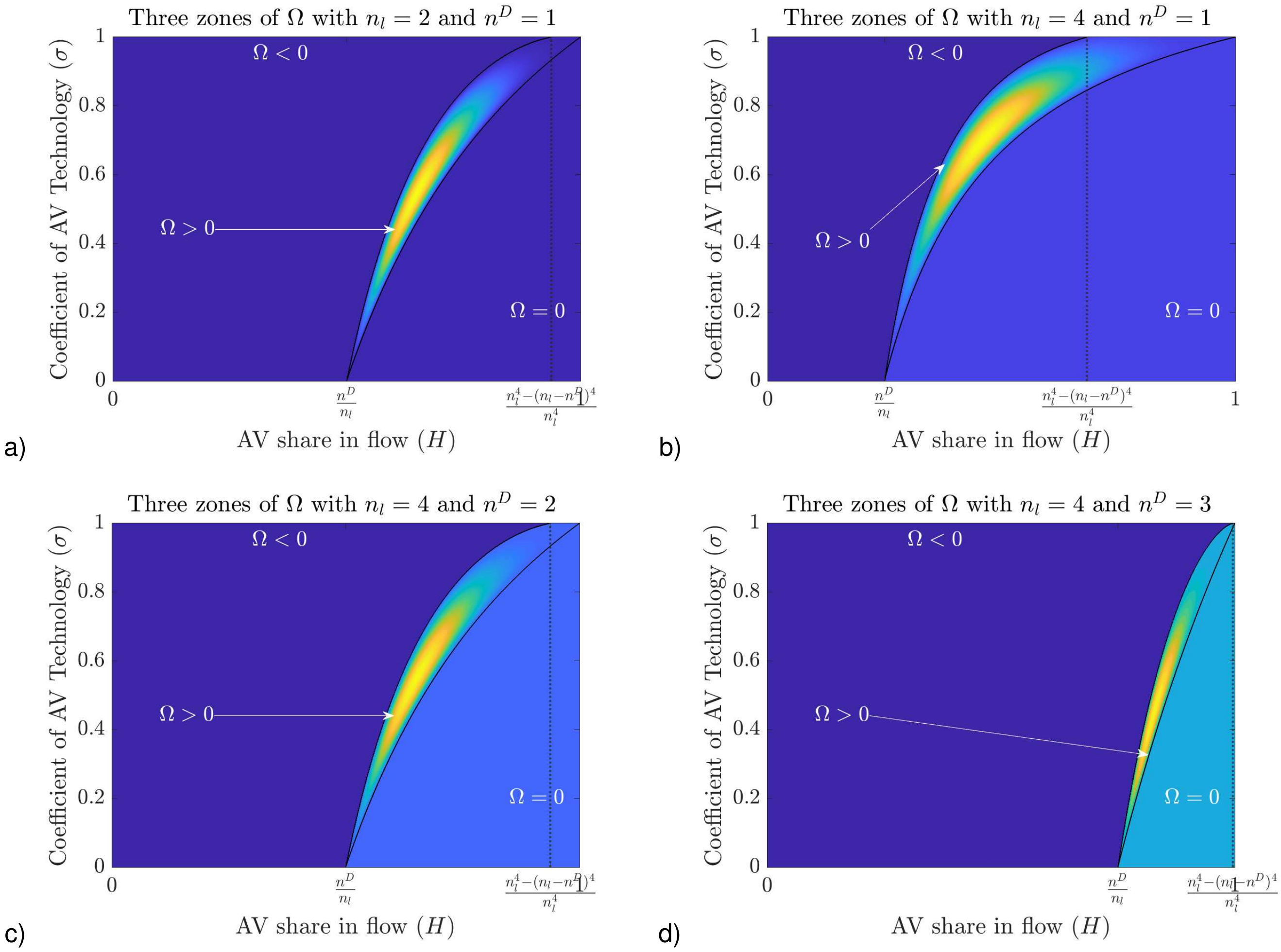
List of notations

Notation	Description
η	Market penetration rate of AV
k	Vehicle type, RV represents regular vehicle, AV represents autonomous vehicle
K^D, K^O	Capacity of a single dedicated AV lane and ordinary lane
v_f	Free-flow speed in road
$Q^{RV}(x)$	RV traffic flow passing location x
$Q^{AV,D}(x)$, $Q^{AV,O}(x)$	AV traffic flow on dedicated AV lanes and ordinary lanes at x
δ^{AV}, δ^{RV}	Reaction time of AV and RV
l	Average length of vehicles
$v^D(x)$, $v^O(x)$	Speed on dedicated AV lanes and ordinary lanes at location x
$v^k(x)$	Travel speed of AVs and RVs at location x
$c^k(x)$	Daily generalized commuting cost of k-type commuters who live at location x
c_f^k	Fixed cost of purchase and maintenance of k-type commuters
$c_m^k(x)$	Variable monetary cost associated with travel distance of k-type commuters who live at location x
$c_t^k(x)$	Variable cost related to travel time of k-type commuters who live at location x
m	Monetary cost of driving per unit distance
ρ	Coefficients converting $\underline{x}$ to the average travel distance in the CBD
$\underline{x}$	Boundary of the CBD
τ^k	Ratio of the VOT in AV and RV to hourly wage
w	Annual income
λ	Annual average number of working days
v_C	Average speed of driving in the CBD
$U^k(x)$	Utility level of k-type commuters who live at location x
$z^k(x)$	Consumption of composite non-housing goods of k-type commuters who live at location x
$q^k(x)$	Housing space of k-type commuters who live at location x
$r^k(x)$	Housing bidding rent of k-type commuters who live at location x
β	Share of non-housing consumption in budget
u^k	Equilibrium utility level of k-type commuters
$D^k(x)$	Residential density of k-type commuters at location x
$\hat{x}$	Residential segregation boundary between AV/RV users
$r(x)$	Highest bidding rent at location x
R_A	Agricultural land rent
$\bar{x}$	City boundary
N	Total population
N^k	Amount of k-type commuters
n_r	Amount of radial roads
n_l	Amount of lanes of each radial road
n^D	Amount of dedicated AV lanes

Continued on next page

Notation	Description
$H(x)$	Share of AVs in total traffic passing location x
σ	AV technology coefficient, quantifies the technical difference between AVs and RVs
$T(x)$	The total travel time for all vehicles passing through location x
$\Omega(\cdot)$	Indicator that captures the positive or negative impact of dedicated AV lanes on traffic efficiency at any given location
$G(\sigma)$	$H - G(\sigma) = 0$ represents the left curve in Fig. 3(a), where $\Omega = 0$ holds
TTT	Total travel time of all commuters
M	Penalty coefficient of the length of dedicated AV lane
SW	Social welfare of the urban system
ζ	Parameter that converts the utility into the equivalent monetary units
c_{const}	Deployment cost of dedicated AV lane per unit distance
$f^k(x, y)$	Traffic demand from residential location x to work location y
$C^k(x, y)$	Travel cost from residential location x to work location y
d_r	Constant arc width between adjacent radial roads
θ	Coefficient in the logit model
$A^k(x), B(y)$	Intermediate variables in
$W(y)$	Employment distribution at location y

Appendix B Changes of Ω with coefficient of AV technology and AV share in traffic flow under different number of lanes


Three zones of Ω with $n_l = 2$ and $n^D = 1$
Three zones of Ω with $n_l = 4$ and $n^D = 1$
Three zones of Ω with $n_l = 4$ and $n^D = 2$
Three zones of Ω with $n_l = 4$ and $n^D = 3$
$\Omega < 0$
$\Omega > 0$
$\Omega = 0$
Coefficient of AV Technology (σ)
AV share in flow (H)
$\frac{n^D}{n_l}$
$\frac{n_l^4-(n_l-n^D)^4}{n_l^4}$
0
0.2
0.4
0.6
0.8
1
a)
b)
c)
d)

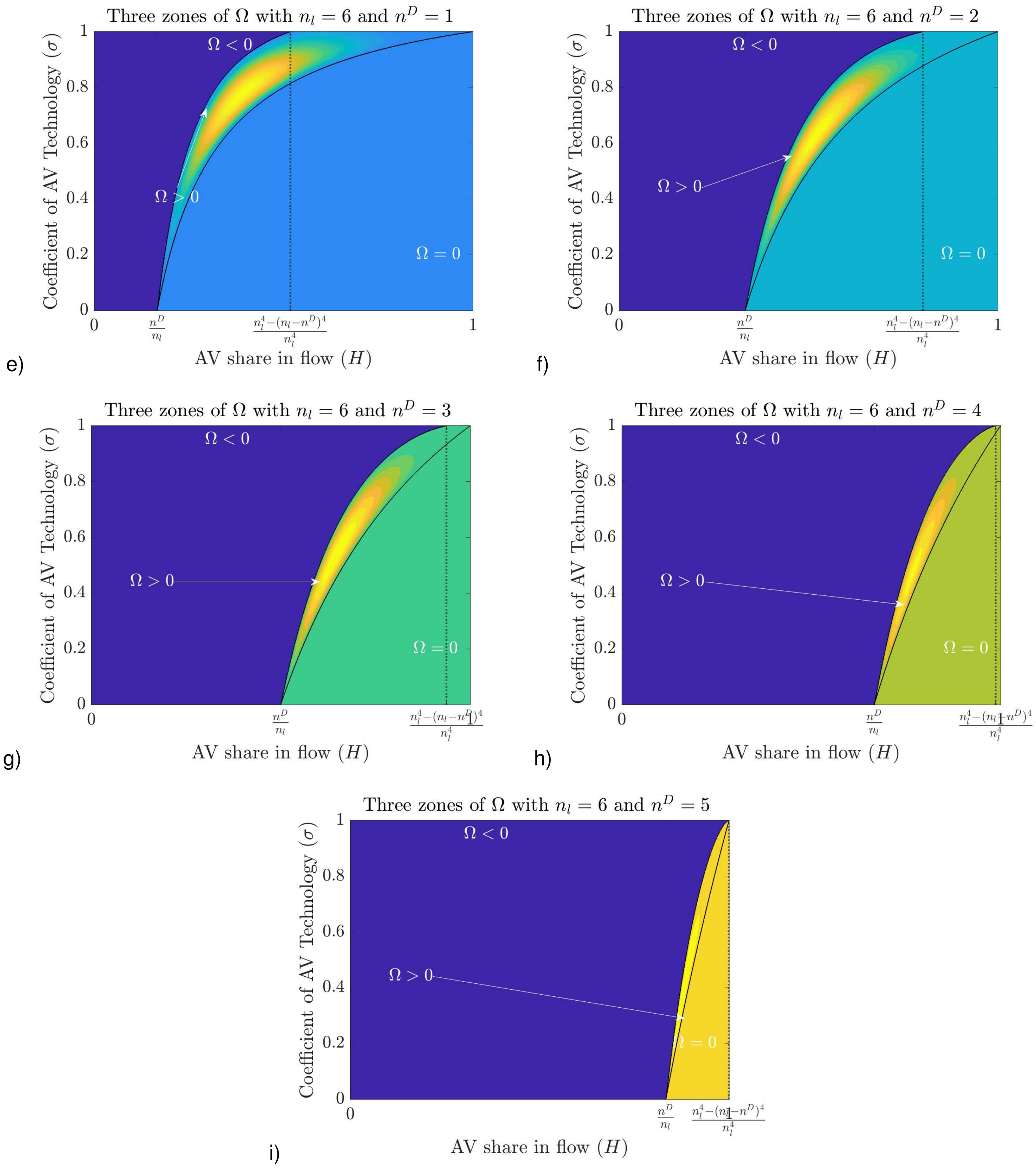


Figure B1: Changes of Ω with coefficient of AV technology and AV share in traffic flow

Appendix C Influence of AV reaction time, VOT of AV users and lane construction cost

Table C1

Optimal lane placement considering AV reaction time and user VOT across different MPR scenarios in the short term

Variables	$\eta = 0.2$	$\eta = 0.4$	$\eta = 0.6$	$\eta = 0.8$
$\delta^{AV} = 0.15$				
Placement of dedicated AV lane (km)	–	5.4, 14.3, 8.9	3, 11.4, 8.4	3, 8.1, 5.1
Total travel time (h)	1.98e+05	1.40e+05 (-3.42%)	8.69e+04 (-10.69%)	7.22e+04 (-1.48%)
$\delta^{AV} = 0.05$				
Placement of dedicated AV lane (km)	–	–	3, 4.6, 1.6	–
Total travel time (h)	1.92e+05	1.31e+05	8.50e+04 (-0.26%)	6.86e+04
$\tau^{AV} = 0.7$				
Placement of dedicated AV lane (km)	–	7.6, 12.3, 4.7	3, 7.9, 4.9	–
Total travel time (h)	1.94e+05	1.35e+05 (-0.83%)	8.49e+04 (-4.28%)	6.85e+04
$\tau^{AV} = 0.3$				
Placement of dedicated AV lane (km)	–	7.7, 12.7, 5	3, 7.9, 4.9	–
Total travel time (h)	1.96e+05	1.39e+05 (-0.83%)	8.88e+04 (-4.44%)	7.25e+04

Table C2

Optimal lane placement considering AV reaction time, user VOT and dedicated lane deployment cost across different MPR scenarios in the long term

Variables	$\eta = 0.2$	$\eta = 0.4$	$\eta = 0.6$	$\eta = 0.8$
$\delta^{AV} = 0.15$				
Placement of dedicated AV lane (km)	9, 11.1, 2.1	5.7, 11, 5.3	3, 9.6, 6.6	3, 4.6, 1.6
Social welfare	1.58e+10(0.08%)	1.89e+10(0.40%)	2.11e+10(0.82%)	2.19e+10(0.01%)
Total travel time (h)	1.94e+05(-1.88%)	1.34e+05(-7.64%)	8.78e+04(-9.70%)	7.28e+04(-0.73%)
$\delta^{AV} = 0.05$				
Placement of dedicated AV lane (km)	-	-	-	-
Social welfare	1.60e+10	1.93e+10	2.13e+10	2.20e+10
Total travel time (h)	1.92e+05	1.31e+05	8.52e+04	6.85e+04
$\tau^{AV} = 0.7$				
Placement of dedicated AV lane (km)	7.9, 11.4, 3.5	5.5, 10.8, 5.3	3, 6.4, 3.4	-
Social welfare	1.55e+10(0.26%)	1.86e+10(0.52%)	2.06e+10(0.35%)	2.14e+10
Total travel time (h)	1.87e+05(-3.39%)	1.27e+05(-6.51%)	8.50e+04(-4.11%)	6.85e+04
$\tau^{AV} = 0.3$				
Placement of dedicated AV lane (km)	-	-	-	-
Social welfare	1.63e+10	1.96e+10	2.17e+10	2.25e+10

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Variables	$\eta = 0.2$	$\eta = 0.4$	$\eta = 0.6$	$\eta = 0.8$
Total travel time (h)	1.96e+05	1.40e+05	9.29e+04	7.25e+04
$c_{const} = 1.2e + 6$				
Placement of dedicated AV lane (km)	-	7.9, 10.3, 2.4	3, 6.2, 3.2	-
Social welfare	1.59e+10	1.90e+10(0.01%)	2.11e+10(0.11%)	2.19e+10
Total travel time (h)	1.95e+05	1.34e+05(-2.48%)	8.71e+04(-4.05%)	7.04e+04
$c_{const} = 0.8e + 6$				
Placement of dedicated AV lane (km)	9.5, 11.4, 1.9	7.4, 10.9, 3.5	3, 6.3, 3.3	-
Social welfare	1.59e+10(0.08%)	1.91e+10(0.08%)	2.11e+10(0.17%)	2.19e+10
Total travel time (h)	1.92e+05(-1.58%)	1.33e+05(-3.60%)	8.70e+04(-4.14%)	7.05e+04