PAPER • **OPEN ACCESS**

Examining optimum prediction time of rainfall dynamics based on chaotic perspective at different temporal scales: a case study in Bojonegoro, Indonesia

To cite this article: A M Hidayat *et al* 2021 *IOP Conf. Ser.: Earth Environ. Sci.* **893** 012024

Examining optimum prediction time of rainfall dynamics based on chaotic perspective at different temporal scales: a case study in Bojonegoro, Indonesia

A M Hidayat[1*], U Efendi[2], R H Virgianto[3*] and H A Nugroho[4]

[1] Weather Forecaster, Meteorological Station of Ahmad Yani, Semarang, Indonesia
[2] Weather Forecaster, Maritime Meteorological Station of Tanjung Emas, 58 Yos Sudarso street, Semarang, Indonesia
[3] Department of Climatology, State College of Meteorology Climatology and Geophysics, 5 Perhubungan I street, South Tangerang, Indonesia
[4] Department of Instrumentation, State College of Meteorology Climatology and Geophysics, 5 Perhubungan I street, South Tangerang, Indonesia

anistia.malinda@gmail.com; rista.virgianto@stmkg.ac.id

Abstract. As the driving force of the hydrological system, rain has severe impact when dealing with petroleum mining activities, especially in protecting assets and safety. Rainfall is one of the meteorological factors characterized by high spatial and temporal variability (chaotic data). Due to this reason, long-term forecasting can only be done in a stochastic way. The highly nonlinear relationships on rainfall dynamics then examined using the Lyapunov exponent method to analyze the chaotic behavior on rainfall time series data. The study of rainfall dynamics has been done in three different temporal scales, i.e., daily data, 5-day, and 10-day, observed over 6 years daily observed rainfall data at one of the largest petroleum mining sites in Bojonegoro, Indonesia. The time delay (τ) was obtained by using the Average Mutual Information (AMI) method for the three-rainfall series (3, 2, 3, respectively). The finite correlation dimensions obtained for all three-rainfall series data is around m=4, indicate a possible existence of chaotic behavior in rainfall observed in Bojonegoro at the three scales. The maximum Lyapunov exponent λ_{max} for each of three-rainfall series in Bojonegoro is 0.111, 0.057, 0.062, respectively. These values were analyzed to find the optimum prediction time of rainfall occurrence to perform better forecasting. The result shows that the optimum range of prediction time for daily, 5-day, and 10-day have 9, 18, and 16 times longer than their temporal scale.

Keywords: rainfall, chaotic, optimum time, Lyapunov exponent

1. Introduction

Weather scenario, such as rainfall, presents some challenging hazards into mining activities. In many cases, it can cause excess runoff to tailings impoundments, overwhelm site drainage, diversion structures, and damage on pipeline systems [1]. Millions of dollars are lost, and mining production is drop. In 2008, the Ensham Mining Operation suffered a significant loss due to the flood and storm in Queensland. Total damages were reported by the company as being $270 million in May 2008 [2], whereas its total costs were estimated at over $300 million by July 2008. Heavy rainfall in early June 2012 caused the artificial banks on a diversion of the Morwell River across the Yallourn Mine to collapse

[3]. As one of the largest oil production areas in Indonesia, Bojonegoro has the same potential loss due to the extreme rainfall phenomenon.

Bojonegoro regency is situated in East Java, Indonesia. It is located in the inland part of northern Java plain, on the banks of the Solo River, the longest river in Java. Bojonegoro becomes the new focus of attention in Indonesia after a new oil field was found in this area. This oil production is the most extensive oil discovery in Indonesia in the last three decades and one of the most significant reserves in Indonesia [4]. Based on the classification of three dominant rainfall region, Bojonegoro classified into A region which has one peak and one trough and strongly influences by the two monsoons, namely the wet northwest (NW) monsoon from November to March and the dry southeast (SE) monsoon from May to September [5]. Besides monsoon activity, there are many meteorological factors, both locally and globally, which affect the variability of rainfall in Indonesia, including Bojonegoro, such as Indian Ocean Dipole (IOD), El Nino Southern Oscillation (ENSO), Madden Julian Oscillation (MJO), or the topography condition. This complexity triggers the chaotic characteristics of rainfall that happen over the research region.

Rainfall is one of the nonlinear meteorological parameters which possess high variability of data. This condition might be explaining why rainfall prediction is such a complex process to be done. Andrian and Wayahdi [6] stated that rainfall is one of the most challenging parameters to predict accurately in Indonesia. Rainfall forecasting is one of the most challenging parts and is still under progressive study. Several studies have been successfully improved the accuracy predictions on weather parameters such as temperature, pressure, and wind, but it did not occur in the case of prediction of rainfall [7]. Meanwhile, the rainfall prediction process plays a significant role in specific sectors, such as mining industries. In this case, treating the rainfall data as a chaotic system is way better than treating them in a stochastic system. This method may create a better understanding of the underlying dynamics [8].

The theory of chaos depends sensitively on the initial condition, which means that a slight change in it might result in the significantly different outcomes. Literature shows many studies of deterministic chaos, and most of these studies analyzed the existence of chaos in rainfall [9-15]. A method called Lyapunov has the capability to handle this chaotic series of data, including rainfall data. It can give the average information of the divergence of a system and its unpredictability. This paper aims to analyze the chaotic behavior of rainfall series data by using Lyapunov and determine the optimum prediction time of rainfall occurrence to perform better weather forecasting, especially on the petroleum mining sites in Bojonegoro. The data used in this study was obtained from an observational rainfall post near the mining sites.

2. Methods employed

The importance of this work is to analyze the chaotic behavior of rainfall in Bojonegoro, using rainfall data for the last six years from 2012 – 2017. The daily rainfall data in millimeters (mm) was obtained from the climatological station of Karang Peloso Malang as the coordinating station of who is in charge of measuring rainfall amount in Bojonegoro, Indonesia. The location of the rainfall observational post is Ngempak Dalem village, sub-district of Dander, Bojonegoro, while its exact coordinate location is 7,11 S and 111,49 E. While there are several petroleum mining activities were conducted, such as Banyu Urip (48,6 km from the observation-rainfall post), Sukowati (14.2 km from the observation-rainfall post), Wonocolo (34.8 km from the observation-rainfall post), and Kedewan (43.3 km from the observation-rainfall post). The chaotic behavior of rainfall series data in this study was identified using the Lyapunov exponent. This method consists of few steps, such as determining the time delay (τ), estimating the embedding dimension (m), reconstructing phase space, and calculating maximum Lyapunov exponent λ_{max}. All these steps are calculated with the help of the R Studio algorithm.

2.1. Time Delay

The time delay for reconstructing the phase space in this study is presented mathematically by Taken [16]. This method is applicable to obtain the requested information about time series. Time delay τ is chosen as the optimum delay where the mutual information takes on the first minimum value. This value is used for the best representation in phase space. If the value of τ is too small, the phase space

coordinates will not be independent. That condition causes some information about the characteristics of the attractor structure loss. An attractor can be understood as a subset of trajectories that originate from different initial conditions, and eventually converge. This subset of trajectories attracts all other trajectories in the phase space; therefore, it is called the system's attractor [8]. In the chaotic system, the attractor might be characterized by a non-integer dimension. On the other hand, if the value of τ is too large, as the consequences, there will be no dynamic correlation of the state vectors because the neighboring trajectories diverge. Some information of the original system can be lost. The Average Mutual Information (AMI) method, mathematically presented in equation 1, is popularly used to determine the optimum value of time delay τ [17]. AMI function uses to select the τ value in a nonlinear system and presented as I in equation 1. If $x(t_i)$ is a set of measured values, $x(t_i + \tau)$ are the measurement after a time delay for τ, the mutual average function $I_{(\tau)}$ information between $x(t_i)$ and $t_i + \tau$ is $x(t_i + \tau)x(t_i)$ [18,19] will be:

$$I(\tau) = \sum_{m=1}^{N} P(x(t_i), x(t_i + \tau)) log_2 \left[\frac{P(x(t_i), x(t_i+\tau))}{P(x(t), x(t+\tau))}\right] \quad \tau > 0 \tag{1}$$

The mutual information $I_{(\tau)}$ measures how information can be derived from one point of a time series given complete information about the other [20,21]. Based on this approach, for a scalar time series X_i, where $i = 1,2,\ldots,N$, the dynamics of the chaotic time series can be fully embedded in m-dimensional phase space represented by a vector as showed in equation 2,

$$Y_j = X_j, X_{j+\tau}, X_{j+2\tau}, \ldots, X_{j+(m-1)\tau} \tag{2}$$

where j=1,2, …, N – (m–1)τ; m is the embedding dimension, and τ is the time delay.

2.2. Embedding Dimension

Embedding dimension was introduced and mathematically determined using the False Nearest Neighbors (FNN) method. It is one of the most popular methods used for estimating the embedding dimension. This method enables the determination of the dimension in which the attractor is unfolded [22]. Therefore, the dimension m can be defined as the minimum number of state variables required to describe the chaotic system. The basic idea of FNN is searching all the nearest data points (neighbors) in a particular embedding dimension m and which changes upon increasing the embedding dimension to $m+1$, then compute the ratio of these two distance points. If this ratio is more significant than a specific threshold f, then the neighbors are false. When the ratio falls to near-zero or its minimum value, then embedded dimension m is considered good enough to represent the dynamic of the chaotic system [8]. For achieving so, the threshold f should be sufficiently large. Hegger and Kantz [23] suggested the minimal reasonable threshold of exp (λ_{max} τ), where λ_{max} is maximal Lyapunov exponents (explained in the next section), and τ is the time delay. Based on this method, *Euclidian distance* is defined as the difference between pairs a nearest neighborhood on the $y_i(m)$, and $y_i(m + 1)$. Euclidian distance is defined as following equation 3:

$$a(i, m) = \frac{\|y_i(m+1) - y_{n(i,m)}(m+1)\|}{\|y_i(m) - y_{n(i,m)}(m)\|} \quad ; \; i = 1,2, \ldots, N - m\tau \tag{3}$$

where $\|\cdot\|$ is the method to calculate the Euclidian distance and its maximum norm (equation 4) defined by Cao [24], i.e.,

$$\left\| y_k(m) - y_l(m) = \max_{0 \le j \le m-1} |x_{k+jr} - x_{l+jr}| \right\| \tag{4}$$

Any two points are close to each other in the m dimension and the reconstructed phase space of the $m + 1$ dimension if the m value is determined appropriately [24]. The $y_i(m + 1)$ in equation 3 is the *i-th* reconstructed vector with embedding dimension $m+1$, and an integer, $y_{n(i,m)}(m)$ is the nearest neighbour of $y_i(m)$ in the m-dimensional reconstructed phase space in the distance $\|\cdot\|$, while $n(i,m)$ depend on i and m. The shape of pairs of points are called "right" neighbors, and otherwise, they are called "wrong" neighbors. It is said to be a perfect embedding if there is no false neighbour exist in the system. Nevertheless, it is difficult and nearly impossible to give a reasonable threshold value f.

Different time series data may have different threshold values. To avoid this problem, Cao [24] proposed the following quantity as showed in equation 5, the mean value of all *a(i,m)*,

$$E(m) = \frac{1}{(N-m\tau)} \sum_{i=1}^{N-m\tau} a(i,m) \quad (5)$$

The value of *E(m)* is only dependent on the *m* dimension and time delay *τ*. Equation 6 shows the analysis of its variation from *m* to *m+1*.

$$E1(m) = \frac{E\,(m_0+1)}{E(m_0)} \quad (6)$$

If the time series data comes from an attractor, the value of *E1(m)* will stop changing when *m* is more significant than some value of m_0, then m_0+1 is the minimum embedding dimension needed. Calculation of *E1(m)* and *E2(m)* are suggested to determine the minimum embedding dimension of scalar time series data and distinguish it from random data [24].

2.3. Reconstruction Phase Space

Since the dynamic of a chaotic system cannot be determined, the phase space is reconstructed using the scalar series. Phase space is reconstructed by the method of time delay from single time series of rainfall data. Phase space theory considered that the reconstruction phase space with embedding dimension *m* and delay time could represent the character of the whole system. The phase space of observation sequence and recovery the form of chaotic attractors in high-dimensional phase space need to be reconstructed to analyze and predict chaotic dynamical system [25]. The estimation of embedding dimension and time delay determined before are used to study the chaotic dynamical system, and for the next step, embedding dimension and delay time are used to estimating the largest Lyapunov exponent of rainfall data in Bojonegoro as well. We assume that the time series of the rainfall data is $\{x(t_i), i = 1,2,\ldots,N\}$. The embedding dimension is *m* and the delay time is τ, then the reconstructed phase space defined as equation 7:

$$X(t_i) = (x(t_i), x(t_i+\tau), x(t_i+2\tau)\ldots, x(t_i+(m-1)\tau)), i = 1,2,\ldots,M \quad (7)$$

where, $x(t_i)$ is the phase point in m-dimension phase space, *M* is the number of phase points, $M = N - (m-1)\,\tau$, and the time series set, $\{x(t_i), i = 1,2,\ldots,N\}$, τ is the delay time, and *m* is the embedding dimension respectively.

2.4. Lyapunov Exponent

The iconic feature of a chaotic system is their sensitive dependence on the initial conditions. A slight deviation in the initial condition may result in a significant change in its outcomes. This divergence will be exponentially fast in the case of a chaotic system. Lyapunov can give the average information of this divergence. It can also define as the exponent, which is essential to indicate the dynamic characteristic of the nonlinear system [8]. This exponent is used to recognize the dependency of a system into its initial condition and show the dynamical behavior of the system [26]. The maximum Lyapunov exponent concludes the predictability of a dynamical system and characterizes the separation rate of its trajectories. The value of the Lyapunov exponent implies the chaotic degree of a system. A system is considered to be chaotic, and the orbit is unstable if the maximum Lyapunov exponent has a positive value ($\lambda > 0$) in the spectrum of Lyapunov exponent [27]. Meanwhile, the negative Lyapunov exponent means that the system is dissipative and non-conservative. The orbits attract to a stable fixed point and periodic. The time delay information between embedding vectors is needed to estimate the Lyapunov exponent. In this study, Lyapunov exponent is calculated by looking at the exponential growth of the average orbital distance of neighbour on a logarithmic scale with the prediction error (*p*) for time (*k*) steps:

$$p(k) = \frac{1}{Nt_s} \sum_{n=1}^{N} log_2 \left(\frac{\left\| y_i(m+1) - y_{n(i,m)}(m+1) \right\|}{\left\| y_i(m) - y_{n(i,m)}(m) \right\|} \right) \quad (8)$$

$$p(k) = \frac{1}{Nt_s}\sum_{n=1}^{N} log_2(a(i,m))$$

where N is the number of data points, t_s is the sampling period, and $y_{n(i,m)}(m)$ is the nearest neighbor $y_i(m)$ in the m-dimensional reconstructed phase space. The maximum Lyapunov λ_{max} exponent values are determined by the slope of regression line using the least square method. If $p(k)$ exhibits a linear increase, then its slope can be taken as an estimate of the maximal Lyapunov exponent λ_{max}. Stehlik [18] mathematically define the maximum Lyapunov exponent in equation 9 as below:

$$\lambda_1 = \frac{1}{t_N - t_0}\sum_{n=1}^{N} log_2\left(\frac{\|y_i(m+1) - y_{n(i,m)}(m+1)\|}{\|y_i(m) - y_{n(i,m)}(m)\|}\right)$$

$$\lambda_1 = \frac{1}{t_N - t_0}\sum_{n=1}^{N} log_2(a(i,m)) \quad (9)$$

where N is the number of replacement steps, t_N is the period after sampling N, t_0 is the initial time, and $a(i,m)$ is the Euclidean distance as explained in equation 3 and equation 8.

3. Results and discussion

3.1. The time delay and embedding dimension

The time delay was determined using the Average Mutual Information (AMI) method in three different temporal scales, i.e., daily, 5-day, and 10-day. The corresponding figure 1, 2, and 3 showed how the value of AMI varies over the time delay (day) in different temporal scales daily (Figure 1.a), 5-day (Figure 1.b), and 10-day (Figure 1.c), respectively. The delay time for the phase space reconstruction is the first minimum value. This value of the time delay curve was the optimum time delay for calculating the Lyapunov exponent [8]. The range of time delay τ on each temporal scale of daily, 5-day, and 10-day rainfall series data is 3, 2, and 3 days consecutively. The time delays τ are determined separately for each time series using the mutual information method [28].

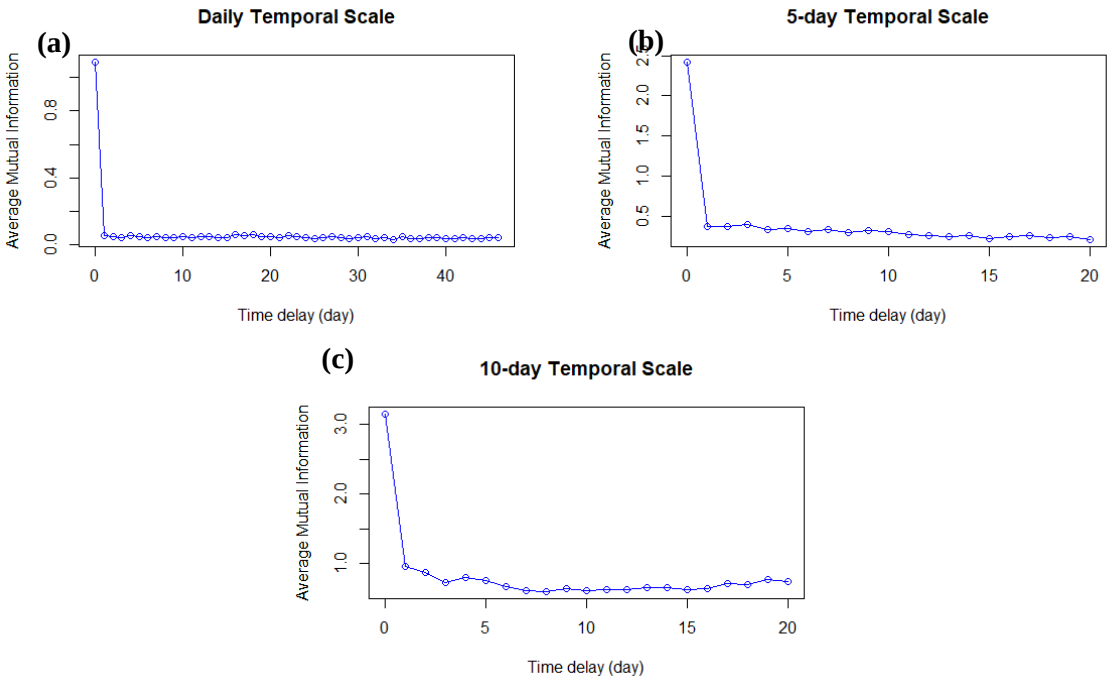

Figure 1. Time delay of rainfall series data from 2012 – 2017 in different temporal scale, daily (a), 5-day (b), and 10-day (c) in the Bojonegoro.

The complexity lies in the determination process of embedding dimension, especially when the data is multivariate. Every possible combination with a different value of *m* is needed to be tried out to determine the optimal combination of the embedding dimension [28]. FNN method was firstly introduced by Kennel et al. [22] as the concept if the dynamics in the phase space can be represented by a smooth vector field, resulting from a condition where the neighboring states would be subject to almost the same time evolution [29]. To avoid any spurious results due to noise, Hegger and Kantz [30] modified this algorithm in which the fraction of false nearest neighbors is computed in a probabilistic way. The total dimension *m* is determined from the individual embedding dimension m_i's for each component while dealing with multivariate time series data. The exact calculation of each individual embedded dimension m_i's will be computed by the method of Cao [24]. This modified algorithm was then applied to all rainfall time series data (daily, 5-day, and 10-day) to identify the total embedded dimension *m*. It can be seen in Figure 2, the value of each individual embedded dimension m_i's in every temporal scale of rainfall series data. The fraction of nearest neighbors is falling to a minimum value then deciding as the embedding dimension. The steep increase after each embedding dimension can be attributed to additive noise or the presence of a large amount of zero data (about 57%) in the whole-time series data [8]. This condition leads to high space dimensionality at smaller spaces. The selection of a suitable noise reduction is needed but not dealt with in this study. After the calculation process, it is known that the optimum embedding dimension in this study was adopted as 4 (*m*=4).

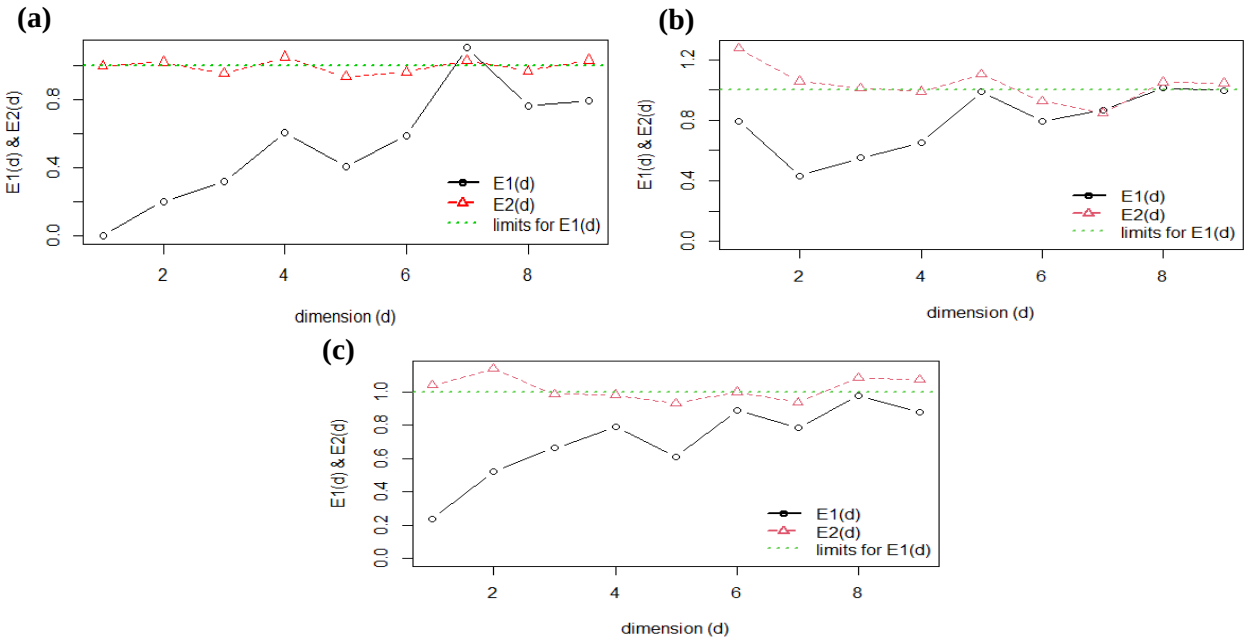

Figure 2. Embedding dimension of rainfall time series data from 2012 – 2017 in Bojonegoro in three temporal scale data of daily (a), 5-day (b), and 10-day (c).

3.2. Maximum Lyapunov exponent

The largest or the maximal Lyapunov exponent is calculated by employing equation 8. The variation of *p(k)* with the time, *t* is the time at dimension *m*=4, is shown in Figure 4. This exponent could be estimated after the time delay, and embedding dimension are determined. Maximum Lyapunov exponent is determined by the slope of regression line using the least square method. The red dash line in figure 4 shows the slope of the regression line using a least square method, and its value represents the maximum Lyapunov exponent. The maximum Lyapunov exponent for daily temporal data is around 0.111, 5-day temporal data is around 0.057, and 10-day temporal data is around 0.062. These exponents in all temporal scale in this study is more significant than zero, indicating that the daily rainfall series in Bojonegoro undergoes chaotic behavior.

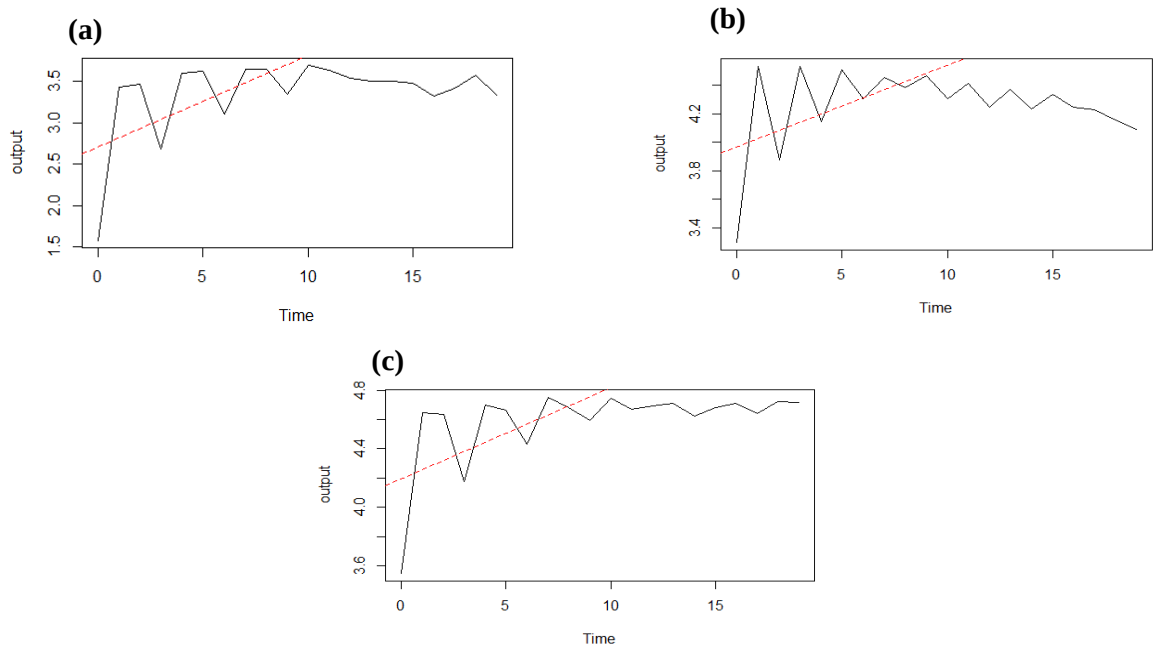

Figure 3. Maximum Lyapunov exponent of rainfall time series data in three temporal scale: daily (a), 5-day (b), and 10-day (c) in Bojonegoro. The red dash line shows the slope of regression line using least square method.

It is also possible to estimate the optimum time interval or the dynamic forward of rainfall prediction. Stehlik [18] mathematically define the dynamic forward of rainfall prediction and written as in equation 10:

$$\text{optimum time predictability} = \frac{T_s}{\lambda_{max}} \tag{10}$$

where T_s is the sampling interval of rainfall data in which it is set to be daily in this study. This method was applied to identify the optimum time predictability for each temporal scale that is used in this study. Therefore, the daily, 5-day, and 10-day temporal scale respectively calculated as follow:

$$\text{optimum time predictability of daily temporal scale} = \frac{1}{0.111} \approx 9$$

$$\text{optimum time predictability of 5-day temporal scale} = \frac{1}{0.057} \approx 18$$

$$\text{optimum time predictability of 10-day temporal scale} = \frac{1}{0.062} \approx 16$$

For daily temporal scale, it could be recognized that the optimum time predictability for rainfall prediction is nine times of their temporal scale (day). It means that the daily rainfall forecast will be optimum to predict within nine days ahead maximally. If it is over nine days, the result of the prediction or forecasting will not be good enough. Meanwhile, for 5-day and 10-day temporal scales are 18 times their temporal scale and 16 times their temporal scale, respectively. The results of this study can be used to improve the accuracy of weather forecasts by simulating the most effective time forecasting periods. In other words, weather predictions could not be better than the above-computed predictability horizon [18]. It is also recommended that petroleum mining companies in Bojonegoro should take this information into account in decision-making about design and planning consideration as an act of adaption and mitigation, to minimize loss due to weather hazards scenario, such as rainfall activities.

4. Conclusion

Many atmospheric parameters such as rainfall have been proven to possess a sensitive dependence on the initial condition. In other word, a slight change in the data may result in significant change in the outcomes, especially in the case of prediction. For mining activities, the accuracy of rainfall prediction is one of the most matter things. Based on chaotic theory, the rainfall time series data from 2012 – 2017 in the research area is chaotic due to the positive value of Lyapunov exponent in all the three-temporal scale: daily, 5-day, and 10-day. The maximum Lyapunov exponent λ_{max} for daily, 5-day, and 10-day are around 0.111, 0.057, and 0.062, respectively. The computed dynamics forward in time show that the optimum time intervals of rainfall predictability daily, 5-day, and 10-day are around 9, 18, and 16 times longer than their temporal scale, respectively. Understanding the optimum prediction time of rainfall occurrence may result in better performance of rainfall forecasting, especially related to the effort of improving the accuracy value. By doing this, a petroleum company, as one of the biggest in Bojonegoro, may perform effectively after considering the better rainfall prediction and its optimum time of rainfall predictability. As the consequences, the petroleum mining industry in the research area may have a better strategy to prepare the adaptation and mitigation action due to rainfall occurrence and minimize loss due to rainfall activities.

Acknowledgments

One of the authors wishes to acknowledge the Climatological Station of Karang Ploso Malang for fully supporting and providing the rainfall data series was used in this research.

References

[1] Pearce T D 2011 Climate change and mining in Canada *Mitig. Adapt. Strateg. Glob. Chang.* **16**(3) pp. 347–68.
[2] Porter L 2008 Ensham's flooded dragline high and dry *ABC Capricornia report* of 2 May 2008. http://www.abc.net.au/local/stories/2008/05/02/2233953.htm [accessed 5th March 2018].
[3] Gardiner A 2012 Flood shuts Yallourn power station mine for second time in a month *Herald Sun*, June 22, 2012 http://www.heraldsun.com.au/news/victoria/showers-on-the-way-after-chilly-night/storyfn7x8me2-1226382189892 [accessed 5th March 2018].
[4] Silaban M R 2016 Big oil, big impact: visiting the Cepu block *American Chamber of Commerce in Indonesia*. https://www.amcham.or.id/amcham/5434-big-oil-big-impact-visiting-the-cepu-block [accessed 20th April 2018].
[5] Aldrian E and Susanto R D 2003 Identification of three dominant rainfall regions within Indonesia and their relationship to sea surface temperature *Int. J. Climatol,* **23** pp. 1435–52.
[6] Andrian Y and Wayahdi M R 2014 Analisis algoritma inisialisasi Nguyen-Widrow pada proses prediksi curah hujan Kota Medan menggunakan metode backpropagation neural network *Proc. National Informatics* **1**(1) pp. 57-63.
[7] Gustari I Hadi T W Hadi S and Renggono F 2012 Akurasi prediksi curah hujan harian operasional di Jabodetabek: Perbandingan dengan model WRF *J. Meteorologi dan Geofisika* **13**(2) pp.119-30.
[8] Dhanya C T and Kumar N 2010 Nonliniear ensemble prediction of chaotic daily rainfall, *J. Advances in Water Resources* **33**, pp. 327-47
[9] Jayawardena AW and Lai F 1994 Analysis and prediction of chaos in rainfall and stream flow time series *J. Hydrol.* **153** pp. 23–52.
[10] Puente C E and Obregon N A 1996 A deterministic geometric representation of temporal rainfall: results for a storm in Boston *Water Resour. Res* **32**(9) pp. 2825–39.
[11] Rodriguez-Iturbe I De Power F B Sharifi M B and Georgakakos K P 1989 Chaos in rainfall *Water Resour. Res* **25**(7) pp. 1667–75.
[12] Sharifi M B Georgakakos K P and Rodriguez-Iturbe I 1990 Evidence of deterministic chaos in the pulse of storm rainfall. *J. Atmos. Sci.* **47** pp. 888–93.
[13] Sivakumar B 2001 Rainfall dynamics at different temporal scales: a chaotic perspective *Hydrol. Earth Syst. Sci.* **5**(4) pp. 645–51.
[14] Sivakumar B Liong S Y and Liaw C Y 1998 Evidence of chaotic behavior in Singapore rainfall

J. Am. Water Resour. Assoc. **34**(2) pp. 301–10.
[15] Sivakumar B Liong S Y Liaw C Y and Phoon K K. 1999. Singapore rainfall behavior: chaotic? *J. Hydrol. Eng.* **4**(1) pp. 38–48.
[16] Takens F 1981 Detecting strange attractors in turbulence *Lecture Notes in Mathematics, Phys. Rev.* **898**, pp. 366-381 (Berlin: Springer).
[17] Frazer A M and Swinney H L 1986 Independent coordinates for strange attractors from mutual information *Phys. Rev.* A **33**(2) pp. 1134–40.
[18] Stehlík J 2006 Searching for chaos in rainfall and temperature records – a nonlinear analysis of time series from an experimental basin *J. of Hydrol. and Hydrodynamics* **48**(3) pp. 143-6.
[19] Uzunhisarcikli E 2010 Nonlinear dynamic analysis of mitral valve doppler signals: surrogate data analysis *Turkish J. of Electrical Engineering and Computer Sci.***18**(2), pp. 327-37.
[20] Fraser A M and Swinney H L 1996 Independent coordinates for strange attractors from mutual information *Phys. Rev.* A **33** pp. 1134–40.
[21] Shang P Li X and Kamae S 2005 Chaotic analysis of traffic time series *Chaos, Solitons and Fractals* **25**(1), pp. 121-28.
[22] Kennel M B Brown R and Abarbanel H D I 1992 Determining embedding dimension for phase space reconstruction using a geometric method *Phys. Rev.* A **45** pp. 3403–11.
[23] Hegger R and Kantz H 1999 Improved false nearest neighbour method to detect determinism in time series data. *Phys Rev* E **60** pp. 4970–3.
[24] Cao L 1997 Practical method for determining the minimum embedding dimension of a scalar time series *Physica* D **110**, pp. 43–50.
[25] Selvi T and Selvaraj S R 2015 Application of lyapunov exponent on the predictability of temperature over an equatorial station-Chennai *Bulletin of Environment Pharmacology and Life Science*, **4(**7) pp. 144–50.
[26] Triyatno M W Zebua O and Ginarsa I M 2015 Perkiraan kestabilan tegangan secara dinamis menggunakan eksponen lyapunov maksimum (studi kasus sistem kelistrikan wilayah Lampung) *National J. of Tech. Electro* **4** pp 101–09.
[27] Nugroho H. A Joelianto E and Puspito N T 2012 Characteristics of earthquake occurrences based on chaotic analysis and fractal dimension *Proc. IEEE Conf. Control. Syst. Ind. Informatics, ICCSII 2012* pp. 208–13.
[28] Dhanya C T and Kumar D 2011 Multivariate nonlinear ensemble prediction of daily chaotic rainfall with climate inputs *J. of Hydrol.* **403**(3-4) pp. 292–306.
[29] Kantz H Schreiber T 2004 Nonlinear Time Series Analysis, second ed. (Cambridge, UK: Cambridge University Press).
[30] Hegger R and Kantz H 1999 Improved false nearest neighbor method to detect determinism in time series data *Phys. Rev.* E **60** pp. 4970–73.