Article:
Jimenez Varon, Cristian Felipe and KNIGHT, MARINA IULIANA (2026) Contribution to the Discussion of 'New tools for network time series with an application to COVID-19 hospitalisations' by Nason et al. Journal of the Royal Statistical Society: Series A (Statistics in Society). pp. 172-173. ISSN: 1467-985X

https://doi.org/10.1093/jrsssa/qnaf162

Cristian F. Jimenez-Varon and Marina I. Knight's contribution to the Discussion of New tools for network time series with an application to COVID-19 hospitalisations by Guy Nason, Daniel Salnikov and Mario Cortina-Borja

Journal:	*Journal of the Royal Statistical Society: Series A*
Manuscript ID	JRSSA-May-2025-0206
Manuscript Type:	Discussion Paper Contribution
Date Submitted by the Author:	28-May-2025
Complete List of Authors:	Jiménez Varón, Cristian Felipe; University of York, Knight, Marina Iuliana; University of York, Mathematics & Statistics;
Keywords:	

Cristian F. Jiménez-Varón and Marina I. Knight's contribution to the Discussion of "New tools for network time series with an application to COVID-19 hospitalisations" by Guy Nason, Daniel Salnikov and Mario Cortina-Borja

Cristian F. Jiménez-Varón[1], Marina I. Knight[1]

May 28, 2025

Discussion

First of all, we would like to congratulate the authors on their insightful paper (Nason et al., 2025) and valuable contribution to the analysis of network time series by developing new tools for studying such stochastic processes. At the heart of these developments sits the generalized network autoregressive (GNAR) model, originally proposed by Knight et al. (2020) and extended to e.g., incorporate covariates (Nason and Wei, 2022), model edge-based time series (Mantziou et al., 2023), forecast (Nason and Palasciano, 2025).

Through this work, Nason et al. (2025) not only introduce novel time-domain tools, namely the network and partial network autocorrelation functions and associated Corbit (correlation–orbit) visualisation plots, but the authors also build on graphical models for time series by theoretically connecting the GNAR formulation to a latent graphical process. Their Theorem 2 identifies two important structural properties. First, the connection to partial correlation graphs generalises earlier edge-based notions of dependence (Dahlhaus, 2000), where dependence is now captured via active (r-stage) neighbourhoods instead of direct connections. Second, the assignment of an r-stage GNAR adjacency is connected to a cross-spectral hierarchy, under the intuition that nodal dependence is strongest between first-stage neighbours and it weakens as r increases.

Regarding the first point, a key implication is that zeroes in the inverse spectral matrix no longer imply a direct node pair disconnection, but rather the absence of a path of length at most $2r^*$ between them, where r^* denotes the largest active r-stage neighbourhood regression of a particular GNAR model. With the increase of active neighbourhood regressions, the GNAR-induced adjacency becomes denser, thus potentially reducing interpretability in the Dahlhaus (2000) sense. When r^* exceeds the network diameter, the structural advantages of GNAR over standard VAR models may no longer be exploited via the adjacency sparsity. These observations raise a crucial question: can we still recover interpretable network structures from data when sparsity in the estimated inverse GNAR spectrum no longer directly maps to edge absence?

In the second part of Theorem 2, Nason et al. (2025) establish the existence of a cross-spectral hierarchy connected to the active neighbourhood regressions. The authors define an r-stage dependent inverse GNAR spectrum using a complex soft threshold operator (*sft*) underpinned by the property that the strength of nodal dependence decays with increasing r. Whilst insightful, this approach relies heavily on the set threshold levels, hence raising questions about their practical estimation and interpretation.

[1]Department of Mathematics, University of York, UK. E-mail: cristian.jimenezvaron@york.ac.uk; marina.knight@york.ac.uk

Finally, connecting these two points, the r-dependent inverse GNAR spectrum gives rise to an r-stage partial correlation graph $\mathcal{G}^{(r)} = (\mathcal{K}, \mathcal{E}^{(r)})$. Notably, once a pair of nodes becomes conditionally independent, the *sft* operator maps the corresponding entry to zero for all larger thresholds, allowing the latent graph to be pruned based on conditional cross-spectrum strength. This structure links edges in the underlying network to the intersection of r-dependent edge sets and the active node set ($\mathcal{P}_r$). Could the hierarchical structure that you defined for the second part of Theorem 2 provide a pathway for inferring underlying network structures when the network is unknown?

References

Dahlhaus, R. (2000). Graphical interaction models for multivariate time series. *Metrika 51*(2), 157–172.

Knight, M. I., K. Leeming, G. P. Nason, and M. A. Nunes (2020). Generalized network autoregressive processes and the GNAR package. *Journal of Statistical Software 96*, 1–36.

Mantziou, A., M. Cucuringu, V. Meirinhos, and G. Reinert (2023). The GNAR-edge model: a network autoregressive model for networks with time-varying edge weights. *Journal of Complex Networks 11*(6), cnad039.

Nason, G. P. and H. A. Palasciano (2025). Forecasting UK consumer price inflation with RaGNAR: Random generalised network autoregressive processes. *International Journal of Forecasting In Press.*

Nason, G. P., D. Salnikov, and M. Cortina-Borja (2025). New tools for network time series with an application to COVID-19 hospitalisations (with discussion). *J. R. Statist. Soc. A.* To appear.

Nason, G. P. and J. L. Wei (2022). Quantifying the economic response to COVID-19 mitigations and death rates via forecasting purchasing managers' indices using generalised network autoregressive models with exogenous variables (with discussion). *J. R. Statist. Soc. A 185*(4), 1778–1792.