The value of partial and full pre-trip information under stochastic demand and bottleneck capacity in the morning commute

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This paper studies the welfare effects of providing pre-trip information to morning commuters in a single-bottleneck model, where both bottleneck capacity and travel demand are exogenously stochastic and assumed to follow an arbitrary joint distribution. We first derive the equilibrium travel costs under varying levels of information completeness, and then examine how information completeness influences travel costs and the key factors driving the welfare outcomes of information provision. We find that the welfare effects of providing pre-trip information are associated with the information completeness, the degree of correlation between bottleneck capacity and demand, and the frequency and amplitude of bottleneck capacity and demand changes. Although providing full information is never welfare-reducing, providing partial information can increase travel costs compared to no information (i.e., information paradox) when demand and bottleneck capacity are moderately correlated. Nevertheless, transitioning from partial to full information consistently leads to a reduction in travel costs. Our numerical examples further confirm the theoretical results and highlight the necessity of accounting for uncertainties in both supply and demand when developing traveler information systems.

Keywords: Morning commute problem, stochastic supply and demand, advanced traveler information system, information completeness, information value

1. Introduction

Nowadays, more than 50% of the world's population lives in cities (Goetz, 2019). Although the agglomeration effect of cities can bring benefits to people's lives, such as high-quality medical care and education, it also leads to many problems. One of the formidable problems in many cities, especially big ones, is traffic congestion (Small and Verhoef, 2007). In particular, many commuters living in big cities often experience severe traffic congestion during peak hours. The 2021 Urban Mobility Report estimated that the price of congestion in the U.S. was up to $190 billion in 2019, resulting from 8.7 billion hours of travel delay

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and 3.5 billion gallons of additional fuel (Schrank et al., 2021). From 1989 to 2019, the delay hours for each commuter in the most populated areas in the U.S. increased from 27 to 54 hours per year, nearly doubling over three decades (Schrank et al., 2021). It is expected that the urban population will continue to grow in recent years, so the problems caused by traffic congestion may worsen as the urban population grows (Goetz, 2019). The ultimate reason for traffic congestion is the demand-supply imbalance. Moreover, various unpredictable events, such as traffic accidents, adverse weather, and unannounced road works, as well as individual constraints that prevent commuters from driving, such as residential relocations, vacations, and other personal circumstances, further exacerbate the uncertainty in both travel demand and network capacity. This uncertainty intensifies the supply-demand imbalance, making traffic congestion more severe and unpredictable. Therefore, reducing the supply and demand uncertainty in transportation is one feasible way to reduce traffic congestion and additional travel costs.

In recent years, the development of advanced traveler information systems (ATIS), particularly the widespread use of smartphone navigation applications, can better collect and deliver travel information to commuters (Ben-Elia and Avineri, 2015). The advent of ATIS inspires us to understand how commuters respond to the provided information and what factors and how these factors affect the performance of ATIS so that it better develops. Providing information about traffic states to commuters before they depart (i.e., pre-trip information) is a common way to reduce uncertainty in transportation (Lindsey et al., 2014; Han et al., 2021). Previous studies have demonstrated that the welfare effects of pre-trip information in the morning commute are related to many factors, such as information accuracy (Arnott et al., 1999; Yu et al., 2021), unpredictable fluctuations in road capacities (Arnott et al., 1991; Khan and Amin, 2018; Han et al., 2021), commuter heterogeneity (Khan and Amin, 2018; Yu et al., 2021), historical knowledge (Zhu et al., 2019), and pricing schemes (Yu et al., 2023). These studies did good work in understanding the welfare effects of pre-trip information; however, most of them only considered the uncertainty in supply (i.e., stochastic bottleneck capacity). Although some studies, such as Arnott et al. (1999), consider uncertainty in both supply and demand, the underlying factors influencing the welfare effects of pre-trip information remain insufficiently explored and understood.

Uncertainty is ubiquitous in both supply and demand of transportation systems, often leading to adverse effects such as increased costs and traffic congestion. These negative impacts, however, are typically believed to be alleviated through the provision of traveler information. However, when supply and demand are both stochastic, it is still unclear about the welfare effects of information provision and what factors and how these factors influence the welfare effects of this pre-trip information in the morning commute. In this paper, we investigate the welfare effects of providing pre-trip information to morning commuters in a single-bottleneck model where bottleneck capacity (i.e., supply) and the number of commuters (i.e., demand) are both stochastic. We first investigate travel costs at user equilibrium under varying levels of pre-trip information provision, specifically focusing on information completeness. We distinguish between three levels of information completeness: (1) no information, where commuters make decisions without any prior knowledge of the stochastic conditions affecting their journey; (2) partial information, where commuters have access to limited pre-trip details, such as either demand or capacity forecasts; and (3) full information, where commuters are fully informed about the joint realization of both demand and capacity before departure. Next, we evaluate the value of providing different levels of pre-trip information (i.e., information value) by comparing the equilibrium travel costs under varying levels of information completeness. The value of pre-trip information is measured by the change in travel costs when providing one level of information completeness to commuters, compared to another level of completeness. Specifically, we focus on the value of pre-trip information by comparing partial information with no information, full information with no information, and full information with partial information, as well as examining the effects of two types of partial information, namely demand information and bottleneck information. Pre-trip information is considered welfare-improving (or welfare-reducing) if it results in a decrease (or increase) in travel costs relative to the scenario with lower completeness or no information. However, if the pre-trip information does not affect travel costs compared to scenarios with lower completeness or no information, it is welfare-neutral (Lindsey et al., 2014; Han et al., 2021). Also, we assume the information provided to commuters before departure is one hundred percent accurate.

Our study differs from the previous ones about the value of pre-trip information in the morning commute in at least two aspects. First, the stochastic demand and bottleneck capacity are assumed to follow an arbitrary joint distribution, and the degree of correlation between supply and demand is introduced to describe the relationship between bottleneck capacity and demand caused by unpredictable events. Second, the welfare effects of information completeness are considered by providing different amounts of pre-trip information. We derive the expected travel costs under user equilibrium in the three regimes regarding the amounts of information provision: zero-information, partial-information, and full-information. Two scenarios in the partial-information regime, i.e., only providing demand information and only providing supply information, are considered.

Our study quantifies the welfare effects of varying levels of information completeness in a correlated stochastic environment, where demand and bottleneck capacity are jointly distributed. By introducing a flexible correlation structure between supply and demand, we develop a analytical framework that captures how different degrees of information completeness interact with underlying system uncertainties to influence equilibrium travel costs and commuter welfare. Our study makes several contributions to the literature on the morning commute problem with stochastic demand and bottleneck capacity as well as ATIS. First, we theoretically demonstrate that providing full pre-trip information consistently improves welfare compared to no information, as it simultaneously eliminates uncertainty on both the supply and demand sides. Second, the welfare effects of providing partial information, relative to no information, depend on the correlation between bottleneck capacity and demand, as well as the frequency and magnitude of their fluctuations. Notably, when supply and demand are uncorrelated, partial information is either welfare-neutral or beneficial. However, when they are moderately correlated, partial information may lead to higher travel costs compared to no information—a phenomenon known as the information paradox. Third, we find that the type of partial information matters: bottleneck capacity information generally yields better performance than demand information, especially under moderate correlation scenarios. Fourth, the transition from partial to full information consistently reduces travel costs, even though partial information alone may trigger the information paradox over zero information.

The rest of the paper is organized as follows. Section 2 reviews the related literature. Section 3 presents the model. Section 4 derives the equilibrium solutions when providing different amounts of pre-trip information. Section 5 investigates the welfare effects of pre-trip information. Section 6 provides numerical examples to confirm the theoretical findings. Section 7 concludes the paper and discusses potential research directions.

2. Literature review

For decades, many transport researchers and policymakers have paid attention to the morning commute problem since commuters during peak hours usually experience more severe congestion than in other traveling periods, thereby leading to many adverse effects, such as additional travel costs and greenhouse gas emissions (Small, 2015; Li et al., 2020). The single-bottleneck model proposed by Vickrey (1969) has been a classical theoretical base to investigate the morning commute problem, in which commuters need to balance the trade-off between travel time and schedule delay. Many studies since then have extended the model in various directions, such as stochastic capacity (Xiao et al., 2015; Long et al., 2022), elastic demand (Arnott et al., 1999; Liu et al., 2025), information provision (Arnott et al., 1991; Khan and Amin, 2018; Han et al., 2021; Yu et al., 2023), the value of reliability (Fosgerau and Karlström, 2010; Liu et al., 2020), congestion pricing and metering (De Palma and Lindsey, 2011), and so forth.

One non-negligible reason for traffic congestion and the difficulties of traffic prediction is uncertainty. In the morning commute, uncertainty significantly influences departure time choice behavior and changes departure flow patterns and travel costs. For example, Xiao et al. (2015) found that uncertainty in bottleneck capacities would cause commuters to depart earlier and spread departure flows over a longer period compared to a deterministic bottleneck setting. Also, the departure flow patterns became more complex under a stochastic bottleneck setting than a deterministic bottleneck setting. For example, Long et al. (2022) identified five possible departure flow patterns formed in different queuing types and schedule delay types when the bottleneck capacity was assumed to follow a general continuous probability distribution; in contrast, only one pattern emerges when capacity is deterministic. Furthermore, Fosgerau (2008) found that the stochastic of demand and bottleneck capacity could increase congestion costs by up to 50% compared to the case where demand and bottleneck capacity were deterministic.

Providing information is a practical and effective strategy to influence travelers' behavior in transportation systems. This, in turn, can help alleviate congestion and optimize system efficiency. Ben-Elia and Avineri (2015) reviewed information classification in travel behavior research. Information sources are typically categorized into three fundamental types: experiential, descriptive, and prescriptive. Experiential information is acquired through continuous learning from historical experience feedback. Descriptive information describes current or predicted travel conditions, while prescriptive information includes recommendations, guidelines, or alternative suggestions. Pre-trip information, which is provided before the journey begins, falls under the category of descriptive information as it helps to eliminate uncertainty about the trip, providing travelers with critical insights into conditions they may face. Many studies have investigated the welfare effects of information provision in the morning commute. For example, Arnott et al. (1991) used a simple model with departure time and route choices, where bottleneck capacity was assumed to follow the Bernoulli distribution. They found that providing imperfect information might increase travel costs compared to no pre-trip information. Khan and Amin (2018) studied the effects of heterogeneous information on departure choice behavior in a bottleneck model with stochastic capacity, where information heterogeneity was described using a Bayesian game with two asymmetrically informed commuter populations, providing evidence for the importance of the degree of information penetration. Zhu et al. (2019) studied the impact of long-term historical knowledge and real-time information provision on the bounded rational commuters in a bottleneck corridor with stochastic capacity. They found that the convergence of bounded rational user equilibrium was influenced by information perceptions. Yu et al. (2021) investigated the joint effects of inaccurate pre-trip information and commuters' responses and heterogeneity on morning commute behavior under stochastic bottleneck capacity. They found that the welfare effects of inaccurate information were significantly influenced by commuters' responses and heterogeneity. Inaccurate information might be better than accurate information when commuters complied with the provided inaccurate information. Han et al. (2021) studied the value of pre-trip information in the morning commute with departure time and route choices in which bottleneck capacity was assumed to follow a general probability distribution. They found that information accuracy and the uncertainty of the free-flow travel time significantly influenced the welfare effects of pre-trip information. Full and accurate pre-trip information might be welfare-reducing when free-flow travel time and bottleneck capacity were both stochastic. Yu et al. (2023) investigated the effects of information provision and congestion pricing on social welfare and travel costs in the morning commute under price-sensitive demand and stochastic bottleneck capacity. They found that responsive pricing performed better than habitual pricing, especially when high-quality information was provided.

Furthermore, in the context of information provision, the completeness of information is crucial in shaping the performance of transportation systems. Previous studies have investigated the influence of information complete (or information integrity) on travel behavior and system efficiency. For example, Peeta et al. (2004) quantified information completeness as the perceived information reliability (PIR) metric. They found that when PIR was small, the convergence of Wardrop equilibrium decreased. Abdel-Aty and Yuan (2010) developed a robust static traffic assignment model that accounted for incomplete information, demonstrating that the absence of information results in increased total system travel time. Avitabile et al. (2018) discovered that information completeness had an inverted U-shaped relationship with cognitive load, as demonstrated through eye-tracking behavioral experiments. They explained that providing too much information can actually reduce the quality of travel decisions. Lu et al. (2020) proposed an information integrity compensation framework and developed the information integrity index. They demonstrated that when data completeness was low, prediction errors increased nonlinearly. Furthermore, some studies investigated the welfare effects of pre-trip information in the morning commute under the uncertainty of demand and supply. Arnott et al. (1988) investigated the impact of information on the time-of-use decisions of commuters in a congestible facility where demand and capacity were stochastic. They further analyzed a simple case of partial information where demand was fixed and capacity followed the Bernoulli Distribution, finding that complete information was better than zero information. Arnott et al. (1999) showed that information might reduce social welfare when demand was isoelastic in price and bottleneck capacity was stochastic. They also found that imperfect information might negatively affect social welfare compared to no information.

From the literature mentioned above, we find that most studies only investigate the impact of pre-trip information in the morning commute under the stochastic bottleneck capacity. Relatively few studies have investigated the welfare effects of pre-trip information when both travel demand and bottleneck capacity are stochastic. Furthermore, although Arnott et al. (1988) investigated the impact of partial information on commuting costs, they only studied a simple case where demand was assumed to be fixed. [1] Also, Khan and Amin (2018) argued that additional insights about the value of information could be gained by assuming that the demand and bottleneck capacity followed an arbitrary joint distribution. Motivated by these gaps, this paper presents a general analysis of the impact of partial and full information on the morning commute, where demand and bottleneck capacity are both exogenously stochastic and are assumed to follow an arbitrary joint distribution. Partial information refers to scenarios in which commuters are informed about either demand or bottleneck capacity prior to departure, while full information provides both. We pay special attention to analyzing the impact of information completeness, the degree of correlation between demand and bottleneck capacity, and the frequency and amplitude of bottleneck capacity and demand changes.

3. The model

3.1. Assumptions and notations

In the model, we assume a highway with a bottleneck connecting a residential district (RD) and a central business district (CBD). Unlike the deterministic setting of the classical bottleneck model, there is uncertainty in demand and bottleneck capacity. The uncertainty in bottleneck capacity can arise from factors such as adverse weather, accidents, roadwork, or special events. Furthermore, we incorporate uncertain travel demand to account for commuters who do not drive to work every weekday. This uncertainty in travel demand is exogenous, primarily driven by external factors such as adverse weather, residential relocations, personal vacations, and other circumstances that prevent commuting. Previous theoretical studies on stochastic demand have typically modeled the variability in travel demand as either exogenous (Zhong et al., 2014; Pedroso et al., 2024) or price-sensitive (Arnott

[1] Arnott et al. (1988) argued that a general analysis of partial information was conceptually and analytically difficult; therefore, they only investigated a simplified situation where demand was fixed.

et al., 1993b; van den Berg, 2012; Liu et al., 2025). In this study, we assume that both travel demand and bottleneck capacity follow exogenously given probability distributions.

We denote the set of possible bottleneck capacity states as Ω and the possible demand states as Ψ. Let s_ω denote the bottleneck capacity in state ω, where $\omega \in \Omega$, and N_ψ denote the demand in state ψ, where $\psi \in \Psi$. Let $\underline{N}$ and $\overline{N}$ denote the minimum and maximum demand, respectively, and let $\underline{s}$ and $\overline{s}$ denote the lower and upper bounds of the bottleneck capacity, respectively. Then, the relationship between $\underline{N}$ and $\overline{N}$ can be described as $\underline{N} = \pi_N \overline{N}$, where $0 < \pi_N < 1$, and the relationship between $\underline{s}$ and $\overline{s}$ can be described as $\underline{s} = \pi_s \overline{s}$, where $0 < \pi_s < 1$. Without loss of generality, we assume Ω and Ψ are continuous. Let $j(N_\psi, s_\omega)$ denote the joint probability density function of a commuter departing from RD to CBD under demand N_ψ and bottleneck capacity s_ω, and the corresponding joint cumulative distribution function can be described as $J(N_\psi, s_\omega) = \int_{\underline{N}}^{N_\psi} \int_{\underline{s}}^{s_\omega} j(N_\psi, s_\omega) \mathrm{d}s_\omega \mathrm{d}N_\psi$. Then, the probability density function and cumulative distribution function of s_ω are denoted as $f(s_\omega)$ and $F(s_\omega) = \int_{\underline{s}}^{s_\omega} f(s_\omega) \mathrm{d}s_\omega$. The probability density function of a commuter who needs to commute in condition ω is denoted as $g(N_\psi)$, and the corresponding cumulative distribution function is $G(N_\psi) = \int_{\underline{N}}^{N_\psi} g(N_\psi) \mathrm{d}N_\psi$.

We use correlation to denote the relationship between demand and bottleneck capacity. Further, referring to the rule of thumb, we classify the strength of the correlation between demand and bottleneck capacity into five levels: independent ($r = 0$), weak, moderate, strong, and complete ($r = \pm 1$). It should be noted that the weak, moderate, and strong correlations are relative divisions. When the correlation coefficient is close to ± 1, we refer to the case as a strong correlation. When the correlation coefficient is close to 0, we refer to the case as a weak correlation. The strength of the relationship between the weak and strong correlations is regarded as a moderate correlation.

Following the previous studies related to the nature of non-recurrent congestion caused by unpredictable events (Arnott et al., 1988, 1991; Khan and Amin, 2018; Han et al., 2021; Yu et al., 2023), the following assumptions are adopted in our model:

Assumption 1. Commuters are risk-neutral to travel costs.

Assumption 2. Commuters are homogeneous regarding the shadow values of travel time and schedule delay.

Assumption 3. The bottleneck capacity and demand are constant within a day but may fluctuate from day to day.

Assumption 4. The distributions of demand and bottleneck capacity are stationary and commonly known to all commuters.

Assumption 3 reflects the practical observation that demand and bottleneck capacity tend to remain stable within a single day (e.g., during morning peak hours), while exhibiting variability across days due to external factors such as adverse weather, traffic incidents, or day-specific travel patterns. It allows us to capture meaningful uncertainty without introducing within-day complexity. Assumption 4 underpins the formulation of an equilibrium concept in which commuters optimize their departure time choices based on expected travel costs without full information. This expected equilibrium captures long-run behavioral adaptation to a system characterized by stochastic yet predictable dynamics.

The notations used throughout the paper are listed in Table 1.

3.2. The zero-information regime

In the zero-information regime, commuters do not obtain the pre-trip information, and they are assumed to be aware of the joint probability distribution of the bottleneck capacity and demand based on their long-time experiences. The number of commuters and bottleneck capacity may fluctuate from day to day, and the commuters who choose the same departure time in their commuting days may experience different queue lengths and schedule delays. Let t^* denote the work start time. In this regime, the expected travel cost of a commuter departing at time t is:

$$E[C^Z(t)] = E[\alpha T_{\psi\omega}(t) + \beta \mathrm{SDE}_{\psi\omega}(t) + \gamma \mathrm{SDL}_{\psi\omega}(t)] \tag{1}$$

where $\mathrm{SDE}_{\psi\omega}(t)$ and $\mathrm{SDL}_{\psi\omega}(t)$ are the schedule delay early and schedule delay late costs for the commuter departing at time t in bottleneck capacity state ω and demand state ψ, which can be expressed as $\mathrm{SDE}_{\psi\omega}(t) = \max\{(t^* - t - T_{\psi\omega}(t)), 0\}$ and $\mathrm{SDL}_{\psi\omega}(t) = \max\{0, (t + T_{\psi\omega}(t) - t^*)\}$, in which $T_{\psi\omega}(t)$ is the travel time in bottleneck capacity state ω and demand state ψ, $t \in [t_0, t_e]$, and t_0 and t_e denote the earliest and latest departure times, respectively. Let $h(t)$ denote the departure rate at time t. A queue develops in bottleneck capacity state ω and demand state ψ when arrival rate exceeds bottleneck capacity s_ω, and the queuing length is $Q_{\psi\omega}(t) = \max\{H(t) - s_\omega(t - t_0), 0\}$, where $H(t)$ is the cumulative departures at t and $H(t) = \int_{t_0}^{t} h(x)\mathrm{d}x$. The travel time at t in bottleneck capacity state ω and demand state ψ is $T_{\psi\omega}(t) = Q_{\psi\omega}(t)/s_\omega + T^f$, where T^f is the free-flow travel time. Without loss of generality, we set the free-flow travel time $T^f = 0$, indicating that a commuter arrives at the bottleneck immediately after he/she departs from RD.

3.3. The partial-information regime

3.3.1. Only providing bottleneck information

When providing bottleneck information, commuters will know the bottleneck condition before departure. In this case, the conditional density function of N_ψ at a given s_ω is $f(N_\psi|s_\omega) = \frac{\partial}{\partial N_\psi} J(N_\psi, s_\omega)$, and the corresponding conditional cumulative distribution function is $F(N_\psi|s_\omega) = \int_{\underline{N}}^{N_\psi} f(N_\psi|s_\omega)\mathrm{d}N_\psi$. When only providing bottleneck information, the expected travel cost of a commuter departing at time t is:

$$E[C^B_{\psi|\omega}(t)] = E[\alpha T_{\psi|\omega}(t) + \beta \mathrm{SDE}_{\psi|\omega}(t) + \gamma \mathrm{SDL}_{\psi|\omega}(t)] \tag{2}$$

where $\mathrm{SDE}_{\psi|\omega}(t)$, $\mathrm{SDL}_{\psi|\omega}(t)$, and $T_{\psi|\omega}(t)$ denote the schedule delay early costs, schedule delay late costs, and travel time costs for the commuter departing at time t in demand state ψ at a given bottleneck capacity state ω, which can be expressed as $\mathrm{SDE}_{\psi|\omega}(t) = \max\{(t^* - t - T_{\psi|\omega}(t)), 0\}$, $\mathrm{SDL}_{\psi|\omega}(t) = \max\{0, (t + T_{\psi|\omega}(t) - t^*)\}$ and $T_{\psi|\omega}(t) = Q_{\psi|\omega}(t)/s_\omega + T^f$. The queuing length is $Q_{\psi|\omega}(t) = \max\{\int_{t_0}^{t} h(x)\mathrm{d}x - s_\omega(t - t_0), 0\}$, where $h(t)$ denotes the departure rate at time t under the given bottleneck capacity state ω.

Table 1: Notational glossary.

Notation	Description	Notation	Description
Scenarios			
Z	Zero-information scenario	F	Full-information scenario
D	Demand-information scenario	B	Bottleneck-information scenario
Parameters			
α	Shadow value of travel time	Ψ	Set of possible demand states, $\Psi = \{H, L\}$
β	Shadow value of schedule delay early	Ω	Set of possible bottleneck states, $\Omega = \{G, B\}$
γ	Shadow value of schedule delay late	T^f	Free-flow travel time
t^*	Work start time		
Variables			
r	The degree of correlation between the random variables N_ψ and s_ω	$g(s_\omega\|N_\psi)$	The conditional density function of s_ω at a given N_ψ
$\rho_{\psi\omega}$	The correlation parameter between the random variables N_ψ and s_ω	$G(s_\omega\|N_\psi)$	The conditional cumulative distribution function of s_ω at a given N_ψ
s_ω	Bottleneck capacity in condition ω	$\overline{s}$	Maximum bottleneck capacity
N_ψ	Demand in condition ψ	$\underline{s}$	Minimum bottleneck capacity, $\underline{s} = \pi_s \overline{s}$
$\overline{N}$	Maximum demand	ω	Possible states of a bottleneck
$\underline{N}$	Minimum demand, $\underline{N} = \pi_N \overline{N}$	ψ	Possible states of a demand
$\overline{\theta}$	Upper bound of $\theta_{\psi\omega}$	$f(s_\omega)$	Probability density function of s_ω
$\underline{\theta}$	Lower bound of $\theta_{\psi\omega}$, $\underline{\theta} = \pi_\theta \overline{\theta}$	$F(s_\omega)$	Cumulative distribution function of s_ω
p_N	Probability of demand under the high-level $(0 < p_N < 1)$	p_s	Probability of capacity in the good-condition $(0 < p_s < 1)$
$g(N_\psi)$	Probability density function of N_ψ	$G(N_\psi)$	Cumulative distribution function of N_ψ
$k(\theta_{\psi\omega})$	Probability density function of $\theta_{\psi\omega}$	$K(\theta_{\psi\omega})$	Cumulative distribution function of $\theta_{\psi\omega}$
$j(N_\psi, s_\omega)$	Joint probability density function under N_ψ and s_ω	$J(N_\psi, s_\omega)$	Joint cumulative distribution function under N_ψ and s_ω
$P(s_\omega)$	Probability distribution of s_ω	$P(N_\psi\|s_\omega)$	Conditional probability of N_ψ at a given s_ω
$f(N_\psi\|s_\omega)$	Conditional density function of N_ψ at a given s_ω	$F(N_\psi\|s_\omega)$	Conditional cumulative distribution function of N_ψ at a given s_ω
t_0	Earliest departure time	$T_{\psi\|\omega}(t)$	Travel time at time t under N_ψ at a given s_ω
t_e	Latest departure time	$T_{\psi\omega}(t)$	Travel time at time t under N_ψ and s_ω
$\hat{\theta}$	Pseudo travel time, $\hat{\theta} = t_e - t_0$	$T_{\omega\|\psi}(t)$	Travel time at time t under s_ω at a given N_ψ
$P(N_\psi)$	Probability distribution of N_ψ	$P(s_\omega\|N_\psi)$	Conditional probability of s_ω at a given N_ψ
π_s	Capacity degradation rate $(0 < \pi_s < 1)$	π_N	Demand degradation rate $(0 < \pi_N < 1)$
$\text{SDE}_{\psi\omega}(t)$	Schedule delay early at time t under N_ψ and s_ω	$C^D_{\omega\|\psi}(t)$	Travel cost at time t when only providing demand information N_ψ
$\text{SDE}_{\psi\|\omega}(t)$	Schedule delay early at time t under N_ψ at a given s_ω	$C^Z(t)$	Travel cost at time t in the zero information scenario
$\text{SDE}_{\omega\|\psi}(t)$	Schedule delay early at time t under s_ω at a given N_ψ	$C^D(t)$	Travel cost at time t in the demand information scenario
$\text{SDL}_{\psi\omega}(t)$	Schedule delay late at time t under N_ψ and s_ω	$C^F(t)$	Travel cost at time t in the full-information s-cenario
$\text{SDL}_{\psi\|\omega}(t)$	Schedule delay late at time t under N_ψ at a given s_ω	$C^B(t)$	Travel cost at time t in the bottleneck capacity-information scenario
$\text{SDL}_{\omega\|\psi}(t)$	Schedule delay late at time t under s_ω at a given N_ψ	$C^B_{\psi\|\omega}(t)$	Travel cost at time t when only providing bottleneck capacity information s_ω
$H(t)$	Cumulative departures at time t	$h(t)$	Departure rate at time t
$P(\theta_{\psi\omega})$	Joint probability of N_ψ and s_ω	$Q_{\psi\omega}(t)$	Queue length at the bottleneck at time t under N_ψ and s_ω
$\theta_{\psi\omega}$	$\theta_{\psi\omega} = N_\psi / s_\omega$		

3.3.2. Only providing demand information

When providing demand information, commuters will know the number of commuters before departure. In this case, the conditional density function of s_ω at a given N_ψ is $g(s_\omega|N_\psi) = \frac{\partial}{\partial s_\omega} J(N_\psi, s_\omega)$, and the corresponding conditional cumulative distribution function is $G(s_\omega|N_\psi) = \int_{\underline{s}}^{s_\omega} g(s_\omega|N_\psi)\mathrm{d}s_\omega$. When only providing demand information, the expected travel cost of a commuter departing at time t is:

$$E[C^D_{\omega|\psi}(t)] = E[\alpha T_{\omega|\psi}(t) + \beta \mathrm{SDE}_{\omega|\psi}(t) + \gamma \mathrm{SDL}_{\omega|\psi}(t)] \tag{3}$$

where $\mathrm{SDE}_{\omega|\psi}(t)$, $\mathrm{SDL}_{\omega|\psi}(t)$, and $T_{\omega|\psi}(t)$ denote the schedule delay early costs, schedule delay late costs, and travel time costs for the commuter departing at time t in bottleneck capacity state ω at a given demand state ψ, which can be expressed as $\mathrm{SDE}_{\omega|\psi}(t) = \max\left\{(t^* - t - T_{\omega|\psi}(t)), 0\right\}$, $\mathrm{SDL}_{\omega|\psi}(t) = \max\left\{0, (t + T_{\omega|\psi}(t) - t^*)\right\}$ and $T_{\omega|\psi}(t) = Q_{\omega|\psi}(t)/s_\omega + T^f$. The queuing length is $Q_{\omega|\psi}(t) = \max\{\int_{t_0}^{t} h(x)\mathrm{d}x - s_\omega(t - t_0), 0\}$, where $h(t)$ denotes the departure rate at time t under the given demand state ψ.

3.4. The full-information regime

In this full-information regime, commuters are provided with both demand and bottleneck information before departure. In this regime, the travel cost in bottleneck capacity state ω and demand state ψ degrades into the classical bottleneck model under a deterministic setting. The travel cost of a commuter departing at time t is:

$$C^F_{\psi\omega}(t) = \alpha T_{\psi\omega}(t) + \beta \mathrm{SDE}_{\psi\omega}(t) + \gamma \mathrm{SDL}_{\psi\omega}(t) \tag{4}$$

where $\mathrm{SDE}_{\psi\omega}(t)$, $\mathrm{SDL}_{\psi\omega}(t)$, and $T_{\psi\omega}(t)$ denote the schedule delay early costs, schedule delay late costs, and travel time costs for the commuter departing at time t at given bottleneck capacity state ω and demand state ψ, which can be expressed as $\mathrm{SDE}_{\psi\omega}(t) = \max\left\{(t^* - t - T_{\psi\omega}(t)), 0\right\}$, $\mathrm{SDL}_{\psi\omega}(t) = \max\left\{0, (t + T_{\psi\omega}(t) - t^*)\right\}$ and $T_{\psi\omega}(t) = Q_{\psi\omega}(t)/s_\omega + T^f$. The queuing length is $Q_{\psi\omega}(t) = \max\{\int_{t_0}^{t} h(x)\mathrm{d}x - s_\omega(t - t_0), 0\}$, where $h(t)$ denotes the departure rate at time t under the given bottleneck capacity state ω and demand state ψ.

4. Equilibrium analysis

In this section, we derive the expected travel costs under user equilibrium (UE) in the three information regimes. We first derive the general formulations of expected travel costs when the stochastic demand and bottleneck capacity are assumed to follow an arbitrary joint distribution. Then, we provide an example by assuming that the stochastic bottleneck capacity and demand follow Bernoulli distributions. The Bernoulli distribution allows us to capture key dynamics while maintaining analytical tractability. Moreover, in practice, bottleneck failures often occur suddenly and discretely, such as due to signal failures or lane closures. The capacity of bottlenecks may completely fail at certain times, while remaining normal at others. This "all or nothing" randomness is well-suited to the Bernoulli distribution (Lindsey et al., 2014; Han et al., 2021). Also, fluctuations in travel demand, particularly in contexts where demand is subject to abrupt changes, may also exhibit binary characteristics, making the Bernoulli distribution a reasonable choice (Albareda-Sambola et al., 2011; Ghaffarinasab, 2022). Furthermore, due to the simplicity of the Bernoulli distribution, its parameters can be estimated using empirical data or historical observations. This involves fitting the data to estimate the probabilities of high and low demand and capacity states, ensuring that the model parameters are both realistic and representative of the observed system behavior. [2]

4.1. The zero-information regime

4.1.1. General results

Per the definition of user equilibrium (UE), the expected travel costs at UE under stochastic bottleneck capacity and demand can be obtained from

$$\mathrm{d}E[C^Z(t)]/\mathrm{d}t = 0, \quad \text{if } h(t) > 0 \tag{5}$$

Previous studies typically derived the expected travel costs at UE under stochastic capacity by analyzing departure patterns. Han et al. (2021) developed a simple method to obtain the expected travel costs at UE under stochastic capacity without analyzing the departure patterns. We extend the method proposed by Han et al. (2021) to derive the expected travel costs at UE under stochastic bottleneck capacity and demand. To this end, the following proposition is first proved.

Proposition 1. *The latest departure time t_e at UE is never earlier than the work start time t^*, i.e., $t_e \geq t^*$.*

Proof: Assume this proposition is false, and all commuters depart before t_e. In this case, commuters departing at t_e may encounter three different scenarios: schedule delay early without congestion, schedule delay early with congestion, schedule delay late with congestion. In this case, delaying departure until t^* is always better than departing at t_e because the schedule delay cost and/or queuing cost is reduced, we have $E[C^Z(t_e)] > E[C^Z(t^*)]$. Therefore, the proposition $t_e \geq t^*$ is true. □

According to Proposition 1, we have two cases, i.e., Case I ($t_e > t^*$) and Case II ($t_e = t^*$), based on the relationship between t_e and t^*. Furthermore, let $\theta_{\psi\omega} = N_\psi/s_\omega$, where $\theta_{\psi\omega} \in [\underline{\theta}, \overline{\theta}]$. Then, the probability density function of $\theta_{\psi\omega}$ can be obtained from $k(\theta_{\psi\omega}) = \int_{\underline{s}}^{\overline{s}} s_\omega j(s_\omega, s_\omega\theta_{\psi\omega})\mathrm{d}s_\omega$, and the corresponding cumulative probability distribution of $\theta_{\psi\omega}$ is $K(\theta_{\psi\omega}) = \int_{\underline{\theta}}^{\theta_{\psi\omega}} k(\theta_{\psi\omega})\mathrm{d}\theta_{\psi\omega}$. Also, we define the pseudo travel time $\hat{\theta}$ under stochastic bottleneck capacity and demand, where $\hat{\theta} = t_e - t_0$. In what follows, we derive the expected travel costs at UE in the two cases.

[2] It is worth noting that calibrating the model parameters is a challenging yet essential step for translating the theoretical framework into practical applications. However, this process involves substantial empirical analysis and is beyond the scope of the present study.

(Case I). Commuters departing at t_0 will not experience congestion and will arrive at the CBD before t^*. Therefore, the expected travel costs at t_0 under the stochastic bottleneck capacity and demand is

$$E[C^Z(t_0)] = \beta(t^* - t_0) \tag{6}$$

Unlike commuters departing at t_0, commuters who depart at t_e will experience congestion if $\theta_{\psi\omega} > \hat{\theta}$ or not experience congestion otherwise. If $\theta_{\psi\omega} > \hat{\theta}$, the travel time of commuters who depart at t_e is $\theta_{\psi\omega} - \hat{\theta}$. Therefore, the expected travel cost at t_e under stochastic bottleneck capacity and demand without pre-trip information is:

$$\begin{aligned} E[C^Z(t_e)] &= \int_{\underline{\theta}}^{\hat{\theta}} k(\theta_{\psi\omega})\gamma(t_e - t^*)d\theta_{\psi\omega} + \int_{\hat{\theta}}^{\overline{\theta}} k(\theta_{\psi\omega}) \left\{\alpha(\theta_{\psi\omega} - \hat{\theta}) + \gamma[t_e + (\theta_{\psi\omega} - \hat{\theta}) - t^*]\right\} \mathrm{d}\theta_{\psi\omega} \\ &= \gamma(t_e - t^*) + (\alpha + \gamma) \int_{\hat{\theta}}^{\overline{\theta}} k(\theta_{\psi\omega})(\theta_{\psi\omega} - \hat{\theta})\mathrm{d}\theta_{\psi\omega} \end{aligned} \tag{7}$$

Since $t_e = \hat{\theta} + t_0$, the expected travel costs at t_e can be denoted as:

$$E[C^Z(\hat{\theta})] = \gamma(\hat{\theta} + t_0 - t^*) + (\alpha + \gamma) \int_{\hat{\theta}}^{\overline{\theta}} k(\theta_{\psi\omega})(\theta_{\psi\omega} - \hat{\theta})\mathrm{d}\theta_{\psi\omega} \tag{8}$$

The first partial derivative of $E[C^Z(\hat{\theta})]$ to $\hat{\theta}$ is:

$$\frac{\partial E[C^Z(\hat{\theta})]}{\partial \hat{\theta}} = \gamma - (\alpha + \gamma)[1 - K(\hat{\theta})] \tag{9}$$

where $K(\hat{\theta})$ is a non-decreasing and right-continuous function. Let $\hat{\theta}^*$ denote the pseudo travel time that minimizes the expected travel costs of the last commuter. Setting $\partial E[C^Z(\hat{\theta})]/\partial\hat{\theta} = 0$, we have $K(\hat{\theta}^*) = \alpha/(\alpha + \gamma)$.

Per the definition of UE, $E[C^Z(t_e)] = E[C^Z(t_0)]$. Then we have the expected travel costs of each commuter at UE under stochastic bottleneck capacity and demand without pre-trip information in Case I:

$$E[C^Z] = \frac{\beta(\alpha + \gamma)}{\beta + \gamma} \int_{\hat{\theta}^*}^{\overline{\theta}} k(\theta_{\psi\omega})\theta_{\psi\omega}\mathrm{d}\theta_{\psi\omega} \tag{10}$$

where $\hat{\theta}^* = K^{-1}(\alpha/(\alpha + \gamma))$.

(Case II). The expected travel costs at t_0 and $t_e(t^*)$ under stochastic bottleneck capacity and demand without pre-trip information can be formulated as follows:

$$\begin{cases} E[C^Z(t_0)] = \beta\hat{\theta} \\ E[C^Z(t_e)] = (\alpha + \gamma) \displaystyle\int_{\hat{\theta}}^{\overline{\theta}} k(\theta_{\psi\omega})(\theta_{\psi\omega} - \hat{\theta})\mathrm{d}\theta_{\psi\omega} \end{cases} \tag{11}$$

Letting $E[C^Z(t_e)] = E[C^Z(t_0)]$, we can have the pseudo travel time $\hat{\theta}^{**}$ by solving $\beta\hat{\theta}^{**} = (\alpha + \gamma) \int_{\hat{\theta}^{**}}^{\overline{\theta}} k(\theta_{\psi\omega})(\theta_{\psi\omega} - \hat{\theta}^{**})\mathrm{d}\theta_{\psi\omega}$. The expected travel cost of each commuter at UE under the stochastic demand and capacity without pre-trip information in Case II is

$$E[C^Z] = \beta\hat{\theta}^{**} \tag{12}$$

The boundary condition between Case I and Case II can be obtained by solving $K(\hat{\theta}^{**}) = \alpha/(\alpha+\gamma)$. The derivation of this boundary condition can be found in the Appendix A.1. If the two random variables s_ω and N_ψ follow a joint discrete probability distribution, the above method can also be used to derive the expected travel cost per commuter at UE (see Appendix A.2 for details).

4.1.2. Results for the Bernoulli distribution

To simplify analysis and without loss of generality, many previous studies assumed that the stochastic bottleneck capacity followed the Bernoulli distribution (Arnott et al., 1991; Khan and Amin, 2018; Han et al., 2021). Here, we further derive the formulations of expected travel costs at UE by assuming that the bottleneck capacity and demand follow Bernoulli distributions. Let $\omega \in \Omega = \{G, B\}$ denote the set of possible bottleneck states, and $\psi \in \Psi = \{H, L\}$ denote the set of possible demand states. We assume the bottleneck capacity in bad condition (i.e., $s_B = \underline{s}$) with probability $P(s_B) = 1 - p_s$ and bottleneck capacity in good condition (i.e., $s_G = \overline{s}$) with probability $P(s_G) = p_s$. The relationship between s_G and s_B can be expressed as $s_B = \pi_s s_G$, where π_s is the degradation amplitude of bottleneck capacity in bad condition over good condition and $0 < \pi_s < 1$. Furthermore, we set a commuter's commuting probabilities under the high-level demand (i.e., $N_H = \overline{N}$) and the low-level demand (i.e., $N_L = \underline{N}$) as $P(N_H) = p_N$ and $P(N_L) = 1 - p_N$, respectively. The relationship between N_G and N_B can be expressed as $N_B = \pi_N N_G$, where π_N is the degradation amplitude of the number of commuters in low-level demand over high-level demand and $0 < \pi_N < 1$.

Therefore, we have four possible state combinations under stochastic demand and bottleneck capacity, i.e., HG, LG, HB, and LB, and the joint probability under a possible state combination is $P(\theta_{\psi\omega})$. In transportation systems, demand and bottleneck capacity are often influenced by common external factors. For example, adverse weather conditions, such as heavy rain or snow, can simultaneously reduce bottleneck capacity and increase travel demand. Therefore, it is reasonable to expect a correlation between demand and capacity in many real-world scenarios. Capturing this interdependence is crucial for accurately modeling system performance and for assessing the effectiveness of information provision strategies under uncertainty. In this study, we apply the Pearson correlation coefficient r as a tractable and interpretable metric to quantify the linear relationship between stochastic demand and bottleneck capacity under Bernoulli distributions:

$$r = \frac{P(\theta_{HG})P(\theta_{LB}) - P(\theta_{HB})P(\theta_{LG})}{\sqrt{P(N_H)P(N_L)P(s_G)P(s_B)}}, \tag{13}$$

where $P(\theta_{\psi\omega}) = P(N_\psi, s_\Omega)$ and $-1 \le r \le 1$. If $r = 0$, demand and bottleneck capacity are uncorrelated. If $r > 0$, demand and bottleneck capacity are positively correlated, indicating that high and low demand levels are more likely to correspond to good and bad bottleneck conditions, respectively. If $r < 0$, demand and bottleneck capacity are negatively correlated, indicating that high and low demand levels are more likely to correspond to bad and good bottleneck conditions, respectively. The relationships among the values of $\theta_{\psi\omega}$ under the four possible state combinations can be described as $\theta_{LG} < \{\theta_{HG}, \theta_{LB}\} < \theta_{HB}$. To simplify the following description, we let $\theta_1 = \theta_{LG}$, $\theta_2 = \min\{\theta_{HG}, \theta_{LB}\}$, $\theta_3 = \max\{\theta_{HG}, \theta_{LB}\}$, and $\theta_4 = \theta_{HB}$ to make sure $\theta_1 < \theta_2 < \theta_3 < \theta_4$. When $\pi_N > \pi_s$, we have $\theta_2 = \theta_{HG}$ and $\theta_3 = \theta_{LB}$, otherwise, we have $\theta_2 = \theta_{LB}$ and $\theta_3 = \theta_{HG}$. The expressions of $E[C^Z]$ under Bernoulli distributions can be found in the Appendix A.3.

4.2. The partial-information regime

We adopt a similar method for deriving the expected travel costs without pre-trip information to obtain the expected travel costs when providing partial pre-trip information.

4.2.1. Only providing bottleneck information

When the bottleneck capacity and demand follow an arbitrary joint distribution, the expected travel cost of each commuter at UE at a given bottleneck state ω is:

$$E[C^B_{\psi|\omega}] = \begin{cases} \dfrac{(\alpha+\gamma)\beta}{(\beta+\gamma)s_\omega} \displaystyle\int_{\hat{N}^*}^{\overline{N}} f(N_\psi|s_\omega) N_\psi \mathrm{d}N_\psi & \text{if } t_e > t^* \\ \hat{N}^{**}\beta/s_\omega & \text{if } t_e = t^* \end{cases} \tag{14}$$

where $\hat{N}^* = F^{-1}(\alpha/(\alpha+\gamma))$ and $\hat{N}^{**}$ can be obtained by solving the nonlinear equation $\hat{N}^{**}\beta = (\alpha+\gamma)\int_{\hat{N}^{**}}^{\overline{N}} f(N_\psi|s_\omega)[N_\psi - \hat{N}^{**}]\mathrm{d}N_\psi$.

The expected travel cost of each commuter at UE under stochastic bottleneck capacity and demand with providing bottleneck information is:

$$E[C^B] = \int_{\underline{s}}^{\overline{s}} f(s_\omega) E[C^B_{\psi|\omega}] \mathrm{d}s_\omega \tag{15}$$

Furthermore, if the bottleneck capacity and demand follow Bernoulli distributions, we have the conditional probability of N_ψ at a given s_ω: $P(N_\psi|s_\omega) = P(N_\psi, s_\omega)/P(s_\omega)$. The expected travel cost of a commuter at UE with bottleneck capacity information is:

$$E[C^B] = p_s E[C^B_{\psi|G}] + (1-p_s)E[C^B_{\psi|B}] \tag{16}$$

where $E[C^B_{\psi|G}]$ and $E[C^B_{\psi|B}]$ denote the expected travel costs when bottleneck capacity in good and bad conditions, respectively. The expressions of $E[C^B_{\psi|G}]$ and $E[C^B_{\psi|B}]$ can be found in the Appendix A.4.

4.2.2. Only providing demand information

When the bottleneck capacity and demand follow an arbitrary joint distribution, the expected travel cost of each commuter at UE at a given demand state ψ is:

$$E[C^D_{\omega|\psi}] = \begin{cases} \dfrac{N_\psi(\alpha+\gamma)\beta}{\beta+\gamma} \displaystyle\int_{\underline{s}}^{\hat{s}^*} g(s_\omega|N_\psi)/s_\omega \mathrm{d}s_\omega & \text{if } t_e > t^* \\ N_\psi\beta/\hat{s}^{**} & \text{if } t_e = t^* \end{cases} \tag{17}$$

where $\hat{s}^* = G^{-1}(\gamma/(\alpha+\gamma))$ and $\hat{s}^{**}$ can be obtained by solving the nonlinear equation $\beta/\hat{s}^{**} = (\alpha+\gamma)\int_{\underline{s}}^{\hat{s}^{**}} g(s_\omega|N_\psi)[1/s_\omega - 1/\hat{s}^{**}]\mathrm{d}s_\omega$.

The expected travel cost of each commuter at UE under stochastic bottleneck capacity and demand with demand information is:

$$E[C^D] = \int_{\underline{N}}^{\overline{N}} g(N_\psi)E[C^D_{\omega|\psi}]\mathrm{d}N_\psi \tag{18}$$

Furthermore, if the bottleneck capacity and demand follow Bernoulli distributions, we have the conditional probability of s_ω at a given N_ψ: $P(s_\omega|N_\psi) = P(N_\psi, s_\omega)/P(N_\psi)$. The expected travel cost of a commuter at UE with bottleneck capacity information is:

$$E[C^D] = p_N E[C^D_{\omega|H}] + (1-p_N)E[C^D_{\omega|L}] \tag{19}$$

where $E[C^D_{\omega|H}]$ and $E[C^D_{\omega|L}]$ denote expected travel costs under high and low demand levels, respectively. The expressions of $E[C^D_{\omega|H}]$ and $E[C^D_{\omega|L}]$ can be found in the Appendix A.5.

4.3. The full-information regime

When the bottleneck capacity and demand follow an arbitrary joint distribution, the expected travel cost of a commuter at UE under stochastic bottleneck capacity and demand with full pre-trip information can be formulated as follows:

$$E[C^F] = \frac{\beta\gamma}{\beta+\gamma}\int_{\underline{\theta}}^{\overline{\theta}} \theta_{\psi\omega}k(\theta_{\psi\omega})\mathrm{d}\theta_{\psi\omega} \tag{20}$$

Furthermore, if the bottleneck capacity and demand follow Bernoulli distributions, the expected travel cost of a commuters at UE with full pre-trip information is:

$$E[C^F] = \frac{\beta\gamma}{\beta+\gamma}[P(\theta_{HG})\theta_{HG} + P(\theta_{HB})\theta_{HB} + P(\theta_{LG})\theta_{LG} + P(\theta_{LB})\theta_{LB}] \tag{21}$$

5. The value of pre-trip information

Up to now, we have derived the expected travel costs at UE in the three regimes, i.e., zero-information, partial-information and full-information. In this section, we analyze the value of providing different kinds of information. Figure 1 illustrates the three information regimes and the value of providing different information. In this section, we analyze the value of full information over zero information (i.e., G^{ZF}), the value of partial information over zero information (i.e., G^{ZB}, G^{ZD}, and G^{BD}), and the benefit gains/losses from partial information to full information (i.e., G^{BF} and G^{DF}).

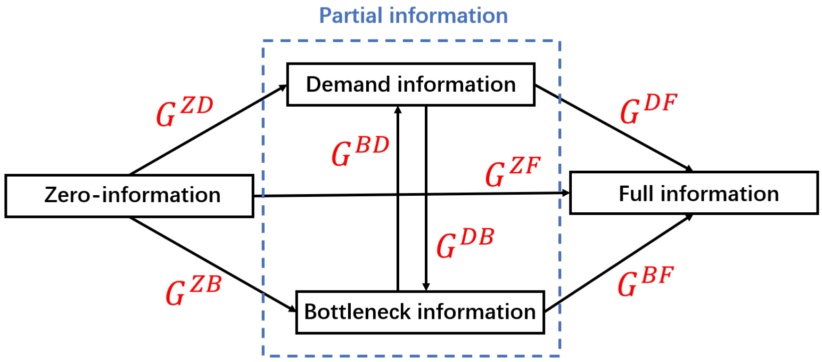

Figure 1: Three information regimes and the value of providing different kinds of pre-trip information.

5.1. The value of full pre-trip information

The benefit gains from providing full pre-trip information over zero information are:

$$G^{ZF} = E[C^Z] - E[C^F] \tag{22}$$

Proposition 2. *Let assumptions hold, then,*
(a) (General probability distribution) providing full pre-trip information does not increase travel costs compared to zero information (i.e, $G^{ZF} \geq 0$*).*
(b) (Bernoulli distribution) providing full pre-trip information is welfare-neutral (i.e, $G^{ZF} = 0$*) when the amplitude of bottleneck capacity drop is equal to the amplitude of demand drop (i.e.,* $\pi_s = \pi_N$*) and bottleneck capacity and demand are perfectly positively correlated (i.e.,* $r = 1$*).*
Proof: The proof can be found in Appendix A.6. □

Proposition 2 asserts that providing full pre-trip information never generates adverse effects on travel costs compared with no information provided when bottleneck capacity and demand are both stochastic. Therefore, considering uncertainty on both sides of supply and demand in the morning commute is a sound way of developing ATIS. Previous studies, such as Arnott et al. (1991) and Han et al. (2021), found that providing full pre-trip information is always welfare-improving over zero information when demand is fixed and bottleneck capacity is stochastic. However, Proposition 2(b) provides a special case that providing full information may be welfare-neutral, indicating the necessity of considering uncertainty in both supply and demand sides.

5.2. The value of partial information

The benefit gains/losses from providing partial information (i.e., bottleneck capacity or demand information) over zero information are:

$$\begin{cases} G^{ZB} = E[C^Z] - E[C^B] \\ G^{ZD} = E[C^Z] - E[C^D] \end{cases} \tag{23}$$

where G^{ZB} and G^{ZD} denote the welfare gains/losses from providing bottleneck capacity information and demand information over zero information, respectively.

The following corollary reveals the benefit effects of providing partial information over zero information when demand and bottleneck capacity are strongly correlated.

Corollary 1. *(General probability distribution). Providing partial pre-trip information is more likely to be welfare-improving (i.e, $G^{ZD} > 0$ and $G^{ZB} > 0$) when demand and bottleneck capacity are strongly correlated.*

Proof: Obviously. □

Commuters can infer the conditional probability of the other state after obtaining one kind of partial information. Corollary 1 indicates that when the changes in bottleneck capacity are strongly associated with the changes in demand, commuters are more likely to benefit from partial pre-trip information. When demand and bottleneck capacity are completely correlated, providing partial information is equivalent to providing full information. When demand and bottleneck capacity are strongly correlated, the effects of providing partial information are similar to providing full information. Per Proposition 2, providing full pre-trip information is never welfare-reducing. Therefore, providing partial information is welfare-improving when demand and bottleneck capacity are strongly correlated.

5.2.1. The value of bottleneck information

The following propositions reveal interesting properties about the welfare effects of providing bottleneck information compared to zero information.

Proposition 3. *When bottleneck capacity and demand are uncorrelated,*
(a)(General probability distribution) providing bottleneck capacity information does not increase travel costs compared to zero information (i.e, $G^{ZB} \geq 0$).
(b)(Bernoulli distribution) providing bottleneck capacity information is more likely to be welfare-neutral over zero information (i.e., $G^{ZB} = 0$) when the amplitude of bottleneck capacity drop is less than the amplitude of demand drop (i.e., $\pi_s > \pi_N$) and demand frequently experiences degradation (i.e., $p_N < \frac{\gamma}{\gamma+\alpha}$).

Proof: The proof can be found in the Appendix A.7. □

Proposition 4. *Let demand and bottleneck capacity follow Bernoulli distributions, then,*
(a) If the amplitude of bottleneck capacity drop is more than the amplitude of demand drop (i.e., $\pi_s < \pi_N$), providing bottleneck information is always welfare-improving(i.e., $G^{ZB} > 0$).
(b) If the amplitude of bottleneck capacity drop is less than the amplitude of demand drop (i.e., $\pi_s > \pi_N$), providing bottleneck information can be welfare-reducing(i.e., $G^{ZB} < 0$) when bottleneck capacity and demand are moderately correlated and bottleneck capacity rarely experiences degradation.

Proof: The proof can be found in the Appendix A.8. □

Propositions 3 and 4 indicate that the benefit gains/losses from bottleneck capacity information are associated with the correlation between capacity and demand and the frequency and severity of bottleneck capacity and demand reductions. Proposition 3 implies that providing bottleneck capacity information can be welfare-neutral over zero information if the amplitude of bottleneck capacity degradation is less than the amplitude of demand degradation and demand frequently experiences degradation. Proposition 4(a) asserts that commuters always benefit from providing partial pre-trip information over zero information when the amplitude of bottleneck capacity degradation is more than the amplitude of demand degradation (i.e., $\pi_s < \pi_N$). However, Proposition 4(b) indicates that providing bottleneck capacity information may induce paradox over zero information when bottleneck capacity and demand are moderately correlated and the amplitude of bottleneck capacity drop is less than the amplitude of demand drop (i.e., $\pi_s > \pi_N$). In this case, providing bottleneck information may induce concentration behavior, thereby possibly generating a deadweight loss.

5.2.2. The value of demand information

The following propositions reveal interesting properties about the welfare effects of providing demand information over zero information.

Proposition 5. *When bottleneck capacity and demand are uncorrelated,*
(a)(General probability distribution) providing demand information does not increase travel costs compared to zero information (i.e, $G^{ZD} \geq 0$).
(b)(Bernoulli distribution) providing demand information is more likely to be welfare-neutral over zero information (i.e., $G^{ZD} = 0$) when the amplitude of bottleneck capacity drop is larger than the amplitude of demand drop (i.e., $\pi_s < \pi_N$) and bottleneck capacity rarely experiences degradation (i.e., $p_s > \frac{\alpha}{\gamma+\alpha}$).
Proof: The proof can be found in the Appendix A.9. □

Proposition 6. *Let demand and bottleneck capacity follow Bernoulli distributions, then,*
(a) If the amplitude of bottleneck capacity drop is less than the amplitude of demand drop (i.e., $\pi_s > \pi_N$), providing demand information is always welfare-improving (i.e., $G^{ZD} > 0$) when bottleneck capacity and demand are negatively correlated.
(b) If the amplitude of bottleneck capacity drop is larger than the amplitude of demand drop (i.e., $\pi_s < \pi_N$), providing demand information may be welfare-reducing (i.e., $G^{ZD} < 0$) when bottleneck capacity and demand are moderately correlated and bottleneck capacity rarely experiences degradation.
(c) If the amplitude of bottleneck capacity drop is less than the amplitude of demand drop (i.e., $\pi_s > \pi_N$) , providing demand information may be welfare-reducing (i.e., $G^{ZD} < 0$) when bottleneck capacity and demand are moderately positively correlated and bottleneck capacity and demand both frequently experiences degradation.
Proof: The proof can be found in the Appendix A.10. □

Similar to providing bottleneck capacity information, Propositions 5 and 6 indicate that the benefit gains/losses from demand information over zero information are associated with the correlation degree between demand and bottleneck capacity and the frequency and severity of bottleneck capacity and demand changes. Proposition 5 asserts that providing demand information is also never welfare-reducing over zero information when demand and bottleneck capacity are uncorrelated. Also, providing demand information can be welfare-neutral over zero information when the amplitude of demand degradation is less than the amplitude of bottleneck capacity degradation and bottleneck capacity rarely experiences degradation.

Like the welfare effects caused by providing bottleneck capacity information, Proposition 6 implies that providing demand information may also induce information paradox (i.e., providing demand information may increase travel costs compared to zero information) when demand and bottleneck capacity are moderately correlated. However, different from providing bottleneck capacity information, there are two possible situations that may occur information paradox when providing demand information. Especially, Proposition 6(b) indicates that the paradox of providing demand information may occur when bottleneck capacity rarely experiences degradation and the amplitude of bottleneck capacity drop is larger than the amplitude of demand drop. Proposition 6(c) indicates that the paradox of providing demand information may occur when demand and bottleneck capacity both frequently experience degradation and the amplitude of bottleneck capacity drop is less than the amplitude of demand drop.

5.2.3. The comparison between bottleneck capacity information and demand information

The above analysis indicates that the welfare effects of providing bottleneck capacity and demand information are different. Previous studies about the welfare effects of providing pre-trip information under stochastic traffic state usually assume the demand is fixed (Lindsey et al., 2014; Khan and Amin, 2018; Han et al., 2021). Next, we analyze the benefit gains/losses from providing demand information over bottleneck capacity information to understand the differences in the welfare effects of providing the two kinds of partial information. The benefits gains/losses from providing demand information compared to bottleneck capacity information are:

$$G^{BD} = E[C^B] - E[C^D] \tag{24}$$

The following proposition reveals interesting properties about which partial information (i.e., demand information or bottleneck capacity information) is more valuable when demand and bottleneck capacity are both stochastic.

Proposition 7. *Let demand and bottleneck capacity follow Bernoulli distributions, then,*
(a) If the amplitude of bottleneck capacity drop is larger than the amplitude of demand drop ($\pi_s < \pi_N$), providing bottleneck capacity information is more valuable than providing demand information (i.e., $G^{BD} < 0$).
(b) If the amplitude of bottleneck capacity drop is less than the amplitude of demand drop ($\pi_s > \pi_N$), providing bottleneck capacity information is more likely to be more valuable than providing demand information (i.e., $G^{BD} < 0$) when bottleneck capacity and demand are not strongly correlated and bottleneck capacity rarely experience degradation.

Proof: The proof can be found in Appendix A.11 and Appendix A.12. □

Proposition 7(a) asserts when the amplitude of bottleneck capacity degradation is larger than the amplitude of demand degradation, providing bottleneck capacity must be better than providing demand information. However, Proposition 7(b) indicates that providing which kind of partial information is better depends on the frequency and severity of bottleneck capacity and demand changes when the amplitude of bottleneck capacity degradation is less than the amplitude of demand degradation. These results indicate that providing bottleneck information is more likely to be better than providing demand information when demand and bottleneck capacity are both stochastic.

Up to now, we have discussed the welfare effects of providing two kinds of partial information (i.e., demand information and bottleneck capacity information) over zero information and compared the welfare effects between demand information and bottleneck capacity information. We find that the welfare effects of partial information are significantly affected by the correlation relationship between demand and bottleneck capacity and the frequency and severity of demand and bottleneck capacity changes. When demand and bottleneck capacity are moderately correlated, providing partial information can be welfare-reducing over zero information (i.e., information paradox).

5.3. The benefit gains/losses from partial information to full information

The benefit gains/losses from providing full information over partial information are:

$$\begin{cases} G^{BF} = E[C^B] - E[C^F] \\ G^{DF} = E[C^D] - E[C^F] \end{cases} \tag{25}$$

where G^{BF} and G^{DF} denote the welfare gains/losses from full information compared to providing bottleneck capacity information and demand information, respectively.

Proposition 8. *Let demand and bottleneck capacity follow general probability distributions,*
(a) Providing full information does not increase travel costs compared to providing partial information (i.e., $G^{BF} \geq 0$ and $G^{DF} \geq 0$).
(b) The benefit gains from providing full information over partial information increase as α increases (i.e., $\partial G^{BF}/\partial \alpha > 0$ and $\partial G^{DF}/\partial \alpha > 0$).
Proof: The proof can be found in Appendix A.13. □

Proposition 8(a) asserts providing full information cannot be welfare-reducing over partial information, indicating that developing ATIS to provide information on both the supply and demand sides will not generate a deadweight loss for the morning commute. Proposition 8(b) indicates that traffic congestion and travel time caused by congestion play crucial roles in the welfare effects of providing full information over partial information. Providing full information can gain more benefits than providing partial information as traffic congestion becomes more severe. In other words, when traffic congestion is severe, providing full information can ease more congestion than providing partial information, thereby reducing travel costs and travel time caused by traffic congestion.

6. Numerical examples

In this section,we present numerical results to illustrate how pre-trip information affects benefit gains/losses under stochastic demand and bottleneck capacity. Unless otherwise specified, we adopt the following parameters based on the empirical findings in Small (1982): $\alpha = 6.4, \beta = 3.9$ and $\gamma = 15.21$. The other parameters are set as: $\overline{N} = 5000(veh), \overline{s} = 6000(veh/h)$. We assume the stochastic bottleneck capacity and demand follow Bernoulli distributions.

6.1. The benefit gains of providing full information over zero information

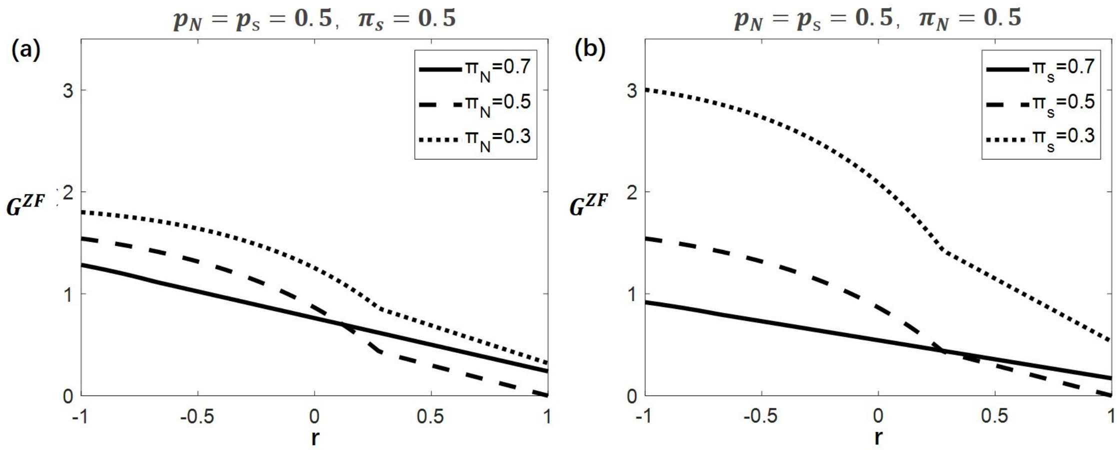

Figure 2: Benefit gains from full information for different r, π_N and π_s (i.e., $p_N = p_s = 0.5$).

Fig. 2 presents the benefit gains of providing full information over zero information (i.e., G^{ZF}) for different correlation coefficients (i.e., r) and the amplitudes of bottleneck capacity and demand degradations (i.e., π_s and π_N). We can see that the providing full information is usually welfare-improving over zero information. This result confirms Proposition 2, indicating that providing full information to reduce uncertainty on both the supply and demand sides is usually useful in reducing travel costs. Previous studies, such as Arnott et al. (1991) and Han et al. (2021), found that providing full and accurate information is always welfare-improving when bottleneck capacity is stochastic but demand is fixed. However, when demand and bottleneck capacity are both stochastic and completely positively correlated, providing full pre-trip information may be welfare-neutral. This result confirms Proposition 2(b). Furthermore, it should be noted that the benefit gains G^{ZF} are not necessarily monotonic in the correlation coefficient r and the amplitude of demand reduction π_N (see Figure 13 in the Appendix A.14 for more details).

6.2. The welfare effects of providing partial information

6.2.1. The benefit gains/losses of providing bottleneck information over zero information

Fig. 3 presents the benefit gains from providing bottleneck capacity information over zero information (i.e., G^{ZB}) for different frequency and severity of bottleneck capacity and

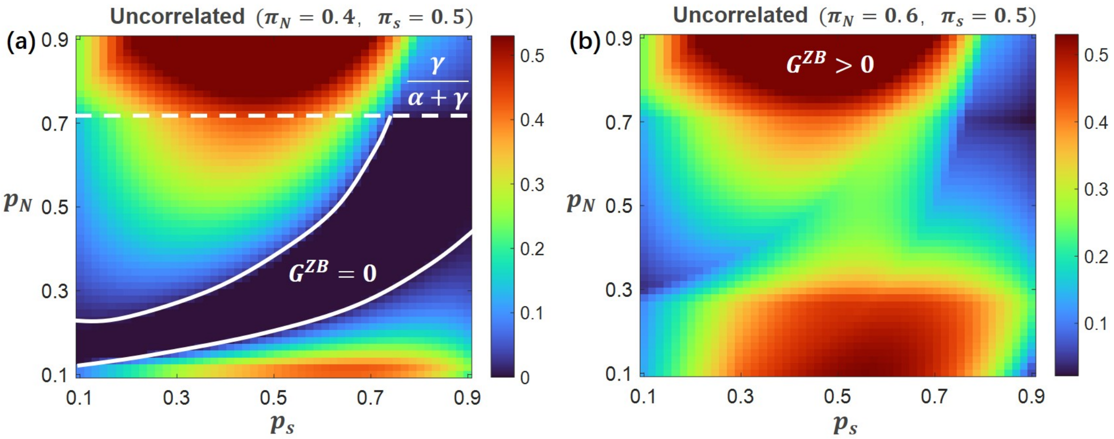

Figure 3: Benefit gains/losses from bottleneck capacity information over zero information for different p_s, p_N, π_s and π_N when demand and bottleneck capacity are uncorrelated (i.e., $r = 0$). The white solid lines indicate where $G^{ZB} = 0$, and the white dashed line represents the boundary defined by $p_N = \gamma/(\alpha + \gamma)$.

demand reductions when demand and bottleneck capacity are uncorrelated. We can see that providing bottleneck capacity information will not increase travel costs compared to zero information when demand and bottleneck capacity are uncorrelated, verifying Proposition 3(a). Furthermore, as shown in Fig. 3(a), providing bottleneck capacity information can be welfare-neutral when $\pi_s > \pi_N$ and demand frequently experience degradation (i.e., $p_N < \frac{\gamma}{\alpha+\gamma}$), which confirms Proposition 3(b).

Fig. 4 presents the benefit gains/losses of providing bottleneck capacity information over zero information (i.e., G^{ZB}) for different correlation coefficients r. We can see that providing bottleneck information is welfare-improving when demand and bottleneck capacity are strongly correlated (i.e., $|r|$ is large), which confirms Corollary 1. Furthermore, as shown in Fig. 4(a-b), providing bottleneck capacity information is always welfare-improving over zero information when the amplitude of demand degradation is larger than the amplitude of bottleneck capacity (i.e., $\pi_N > \pi_s$), which confirms Proposition 4(a). When bottleneck capacity rarely experience degradations (i.e., p_s is large) and the amplitude of demand degradation is less than the amplitude of bottleneck capacity (i.e., $\pi_N < \pi_s$), providing bottleneck capacity information can be (1) welfare-neutral if demand and bottleneck capacity are uncorrelated and (2) welfare-reducing if demand and bottleneck capacity are moderately correlated. These results confirm Proposition 3(b) and Proposition 4(b), indicating that only providing bottleneck capacity may even increase travel costs (i.e., the emergence of the information paradox) compared to zero information when demand and bottleneck capacity are both stochastic.

Fig. 5 presents the benefit gains/losses from bottleneck capacity information over zero information (i.e., G^{ZB}) for different p_N, π_N and r. As shown in Fig. 5, providing bottleneck capacity information can be likely welfare-reducing when the amplitude of bottleneck capacity drop is less than the amplitude of demand drop (i.e.,$\pi_s > \pi_N$), bottleneck capacity

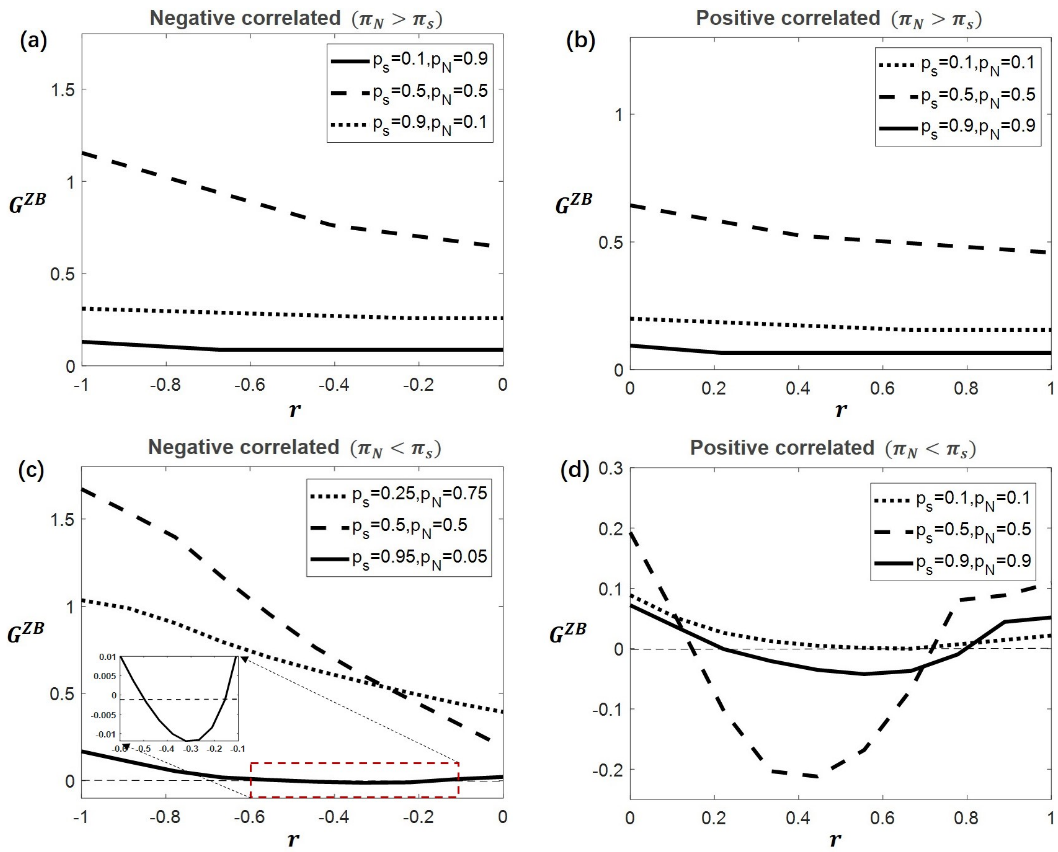

Figure 4: Benefit gains/losses from bottleneck capacity information over zero information for different r, p_s and p_N (i.e., (a-b) $\pi_s = 0.5$, $\pi_N = 0.8$; (c-d) $\pi_s = 0.5$, $\pi_N = 0.4$).

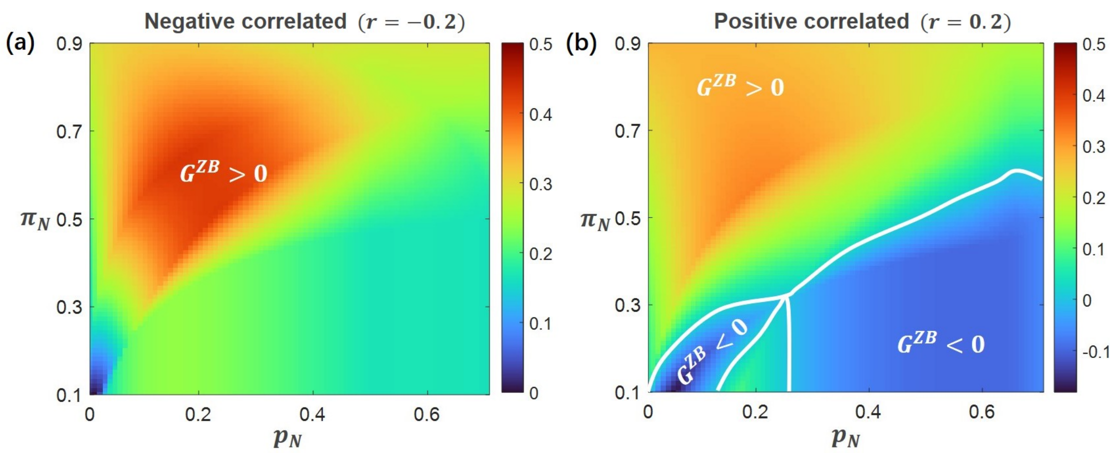

Figure 5: Benefit gains/losses from bottleneck information over zero information for different p_N, π_N and r (i.e., (a) $r = -0.2$; (b) $r = 0.2$), with fixed $\pi_s = 0.5$, $p_s = 0.8$. The white solid lines indicate where $G^{ZB} = 0$.

rarely experiences degradation, and travel demand and bottleneck capacity are moderately positively correlated. This result provides additional evidence that providing bottleneck information can lead to an information paradox compared to zero information when travel demand and bottleneck capacity are moderately correlated, thereby reaffirming Proposition 4(b).

6.2.2. The benefit gains/losses of providing demand information over zero information

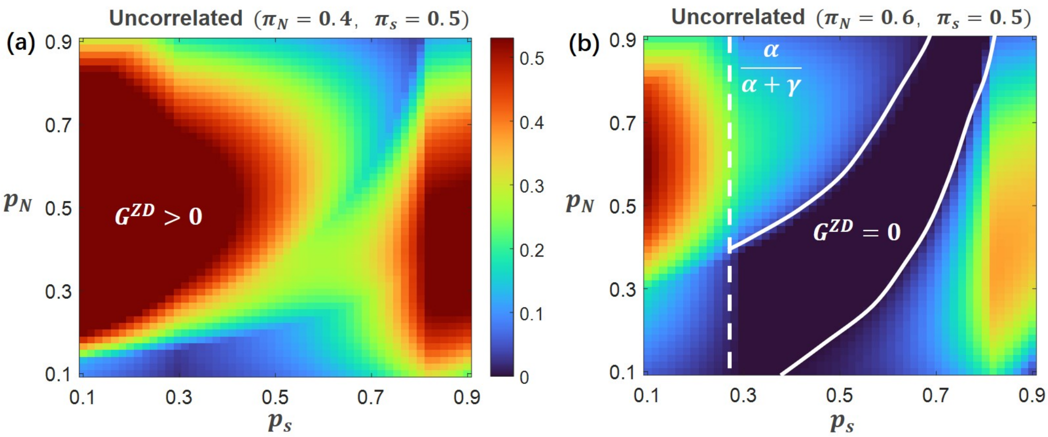

Figure 6: Benefit gains/losses from demand information over zero information for different p_N, p_s, π_s and π_N when demand and bottleneck capacity are uncorrelated (i.e., $r = 0$).

Fig. 6 presents the benefit gains of providing demand information over zero information (i.e., G^{ZD}) for different frequency and severity of bottleneck capacity and demand reductions when demand and bottleneck capacity are uncorrelated. Like providing bottleneck capacity information, we can see that providing demand information is also not welfare-reducing compared to zero information, verifying Proposition 5(a). Providing demand information can be welfare-neutral when $\pi_N > \pi_s$ and bottleneck capacity rarely experience degradation (i.e., $p_s > \frac{\alpha}{\alpha+\gamma}$), corresponding to Fig. 6(b), which confirms Proposition 5(b).

Fig. 7 presents the benefit gains/losses of providing demand information over zero information (i.e., G^{ZD}) for different correlation coefficients r. We can see that providing demand information is also welfare-improving when demand and bottleneck capacity are strongly correlated (i.e., $|r|$ is large), which reaffirms Corollary 1. Furthermore, as shown in Fig. 7(c), providing demand information is always welfare-improving when the amplitude of demand degradation is larger than the amplitude of bottleneck capacity degradation and bottleneck capacity and demand are negatively correlated, which confirms Proposition 6(a). As shown in Fig. 7(a-b), providing demand information may be welfare-reducing over zero information when the amplitude of bottleneck capacity degradation is larger than the amplitude of demand degradation and demand and bottleneck capacity are moderately correlated. These results confirm Proposition 6(b). Also, as shown in Fig. 7(d), providing demand information can be welfare-reducing over zero information when the amplitude of demand degradation is

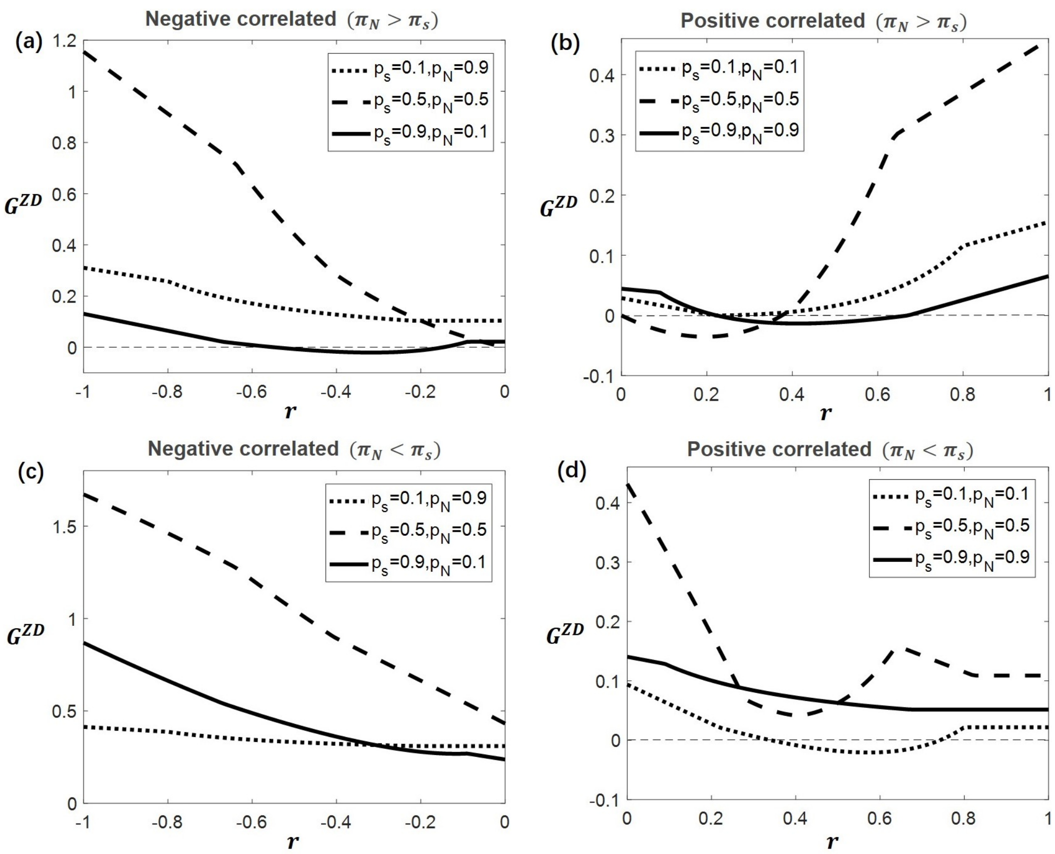

Figure 7: Benefit gains/losses from demand information over zero information for different r, p_s and p_N. (i.e., (a-b) $\pi_s = 0.5$, $\pi_N = 0.8$; (c-d) $\pi_s = 0.5$, $\pi_N = 0.4$).

larger than the amplitude of bottleneck capacity degradation, bottleneck capacity and demand are positively moderately correlated, and demand and bottleneck capacity frequently experience degradations. This result verifies Proposition 6(c). Compared to providing bottleneck capacity, providing demand information is more likely to be welfare-reducing over zero information.

Fig. 8 presents the benefit gains/losses G^{ZD} from demand information over zero information for different p_s, π_s and r. As shown in Fig. 8(b), providing demand information can be welfare-reducing compared to zero information when bottleneck capacity and travel demand are moderately positively correlated, particularly under the following two conditions: (1) when the amplitude of bottleneck capacity drop is less than the amplitude of demand drop (i.e.,$\pi_s > \pi_N$) and bottleneck capacity and demand both frequently experiences degradation; (2) when the amplitude of bottleneck capacity drop is larger than the amplitude of demand drop (i.e.,$\pi_s < \pi_N$) and bottleneck capacity rarely experiences degradation. These results provide additional evidence supporting Proposition 6(b) and (c).

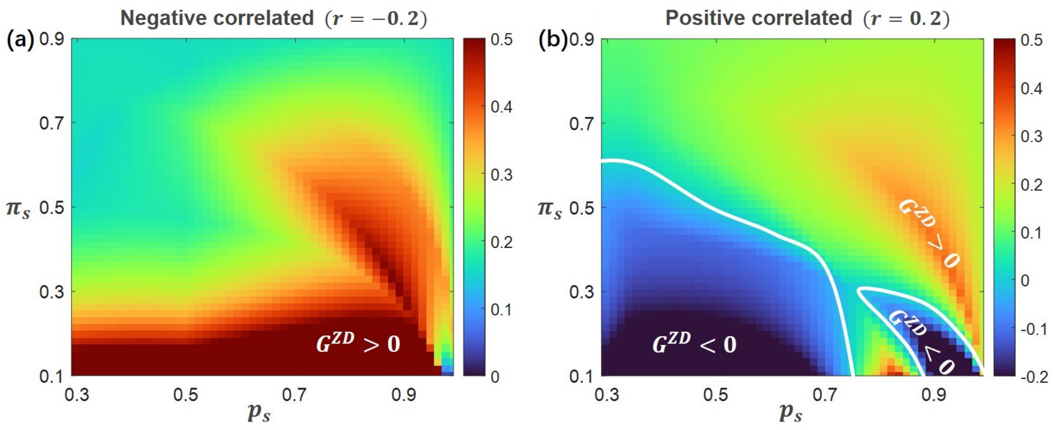

Figure 8: Benefit gains/losses from demand information over zero information for different p_s, π_s and r (i.e., (a) $r = -0.2$; (b) $r = 0.2$), with fixed $\pi_N = 0.5$, $p_N = 0.2$. The white solid lines indicate where $G^{ZD} = 0$.

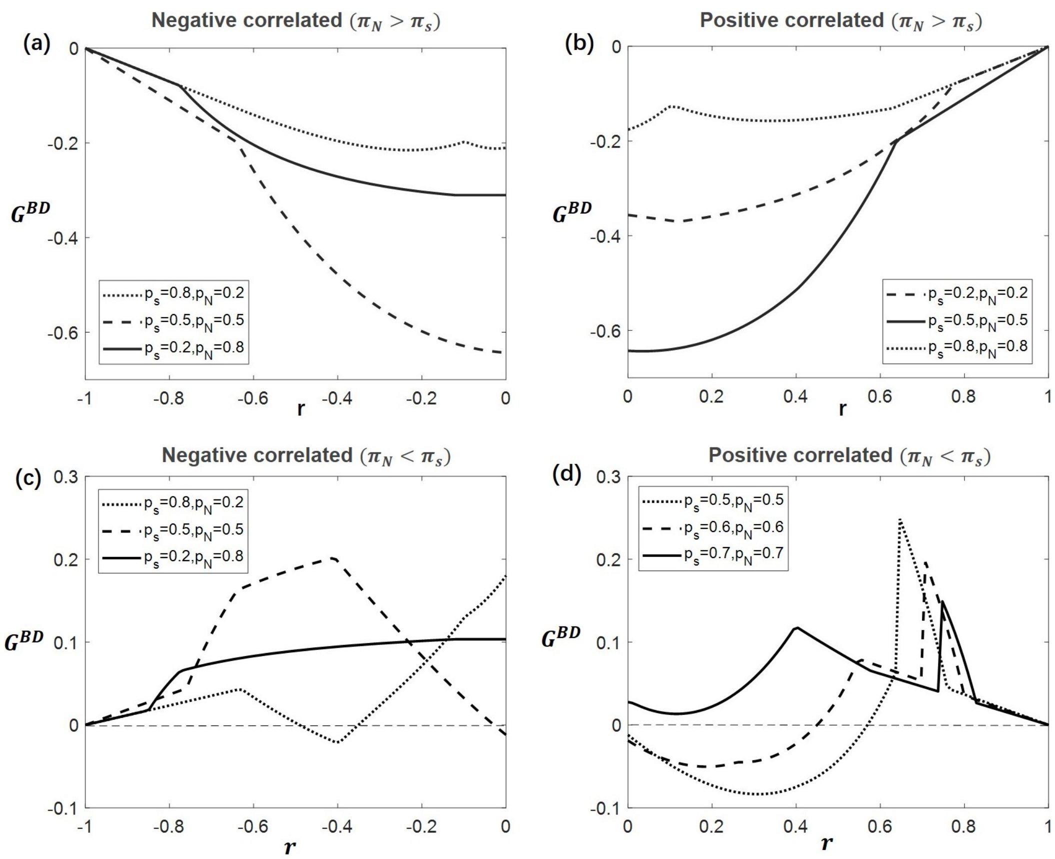

Figure 9: Benefit gains/losses from capacity information as compared to from demand information G^{BD} for different r, p_s and p_N (i.e., (a-b) $\pi_s = 0.5$, $\pi_N = 0.8$; (c-d) $\pi_s = 0.5$, $\pi_N = 0.4$).

6.2.3. The comparison between bottleneck capacity information and demand information

Fig. 9 shows the benefit gains from capacity information as compared to from demand information G^{BD} when demand and bottleneck capacity are correlated for different r, p_s, and p_N. As shown in Fig. 9(a-b), providing bottleneck capacity information is always better than providing demand information ($i.e., G^{BD} < 0$) when the amplitude of bottleneck capacity degradation is larger than the amplitude of demand degradation (i.e., $\pi_s < \pi_N$), which confirms Proposition 7(a). Also, as shown in Fig.9(c-d), providing bottleneck capacity information can still be more valuable than providing demand information when the amplitude of bottleneck capacity degradation is less than the amplitude of demand degradation (i.e., $\pi_s > \pi_N$), bottleneck capacity and demand are not strongly correlated, and bottleneck capacity rarely experience degradation. These results verify proposition 7(b).

6.3. The benefit gains from full information as compared to providing partial information

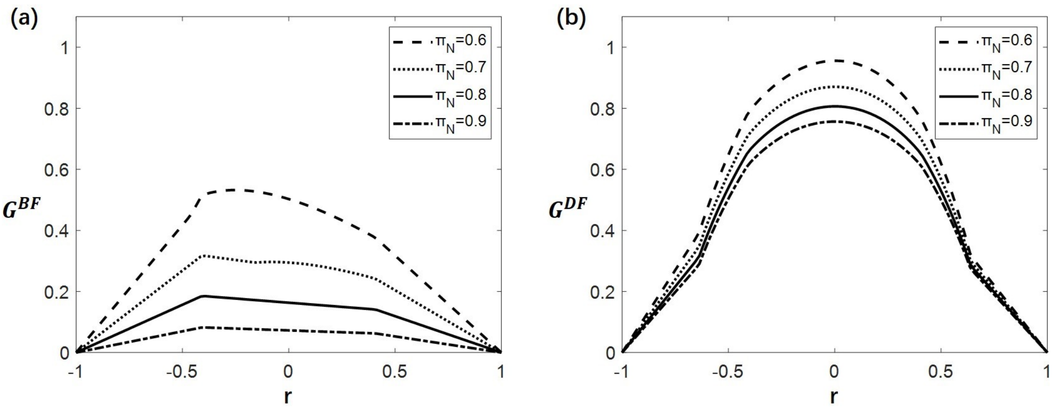

Figure 10: The benefit gains (i.e., G^{BF} and G^{DF} from providing full information over bottleneck information or demand information for different π_N and r, with fixed $p_N = p_s = 0.5$ and $\pi_s = 0.5$.

Fig. 10 presents the benefit gains from providing full information compared to providing only bottleneck information or demand information for different π_N and r. As shown in Fig. 10, the benefit gains from full information decrease as π_N increases. Furthermore, providing full information is always welfare-improving compared to partial information when demand and bottleneck capacity are not completely correlated, which provides evidence for supporting Proposition 8(a).

Fig. 11 presents the benefit gains from providing full information over partial information for different α and r. As shown in Fig. 11, providing full information is never welfare-reducing compared with providing partial information. This result reconfirms Proposition 8(a). Also, when demand and bottleneck capacity are not completely correlated, providing full information is always welfare-improving over partial information, indicating that developing an ATIS to reduce uncertainty in both the demand and supply sides is useful to reduce commuting costs. Furthermore, the benefit gains from partial information to full information increase as

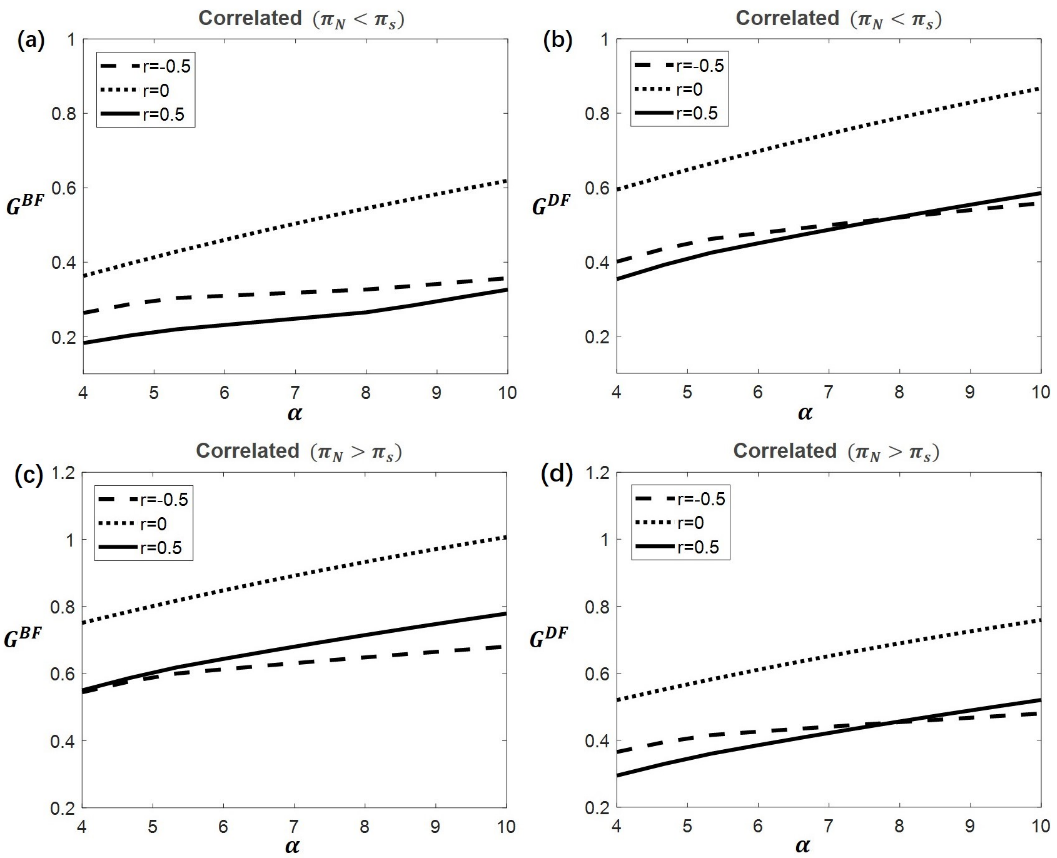

Figure 11: The benefit gains (i.e., G^{BF} and G^{DF}) from providing full information over bottleneck information or demand information for different α under different r with fixed $p_N = p_s = 0.5$. (a-b). $\pi_s = 0.5$, $\pi_N = 0.4$; (c-d). $\pi_s = 0.5$, $\pi_N = 0.8$.

α increases, indicating the necessity of reducing uncertainty in both the demand and supply sides when commuters are more averse to congestion. This result affirms Proposition 8(b).

Fig. 12 presents the benefit gains from providing full information over bottleneck information or demand information for different β and γ under different r. As shown in Fig. 12(a-b), the relation between G^{BF} and β as well as the relation between G^{DF} and β may be non-monotonic. When demand and bottleneck capacity are positively correlated, G^{BF} first increases and then decreases as β increases, while G^{DF} initially increases, then decreases, and finally increases again as β increases. As shown in Fig. 12(c-d), the relation between G^{BF} and γ as well as the relation between G^{DF} and γ may also be non-monotonic. When demand and bottleneck capacity are uncorrelated, both G^{BF} and G^{DF} first decrease and then increase as γ increases. However, when demand and bottleneck capacity are negatively correlated, both G^{BF} and G^{DF} increase as γ increases.

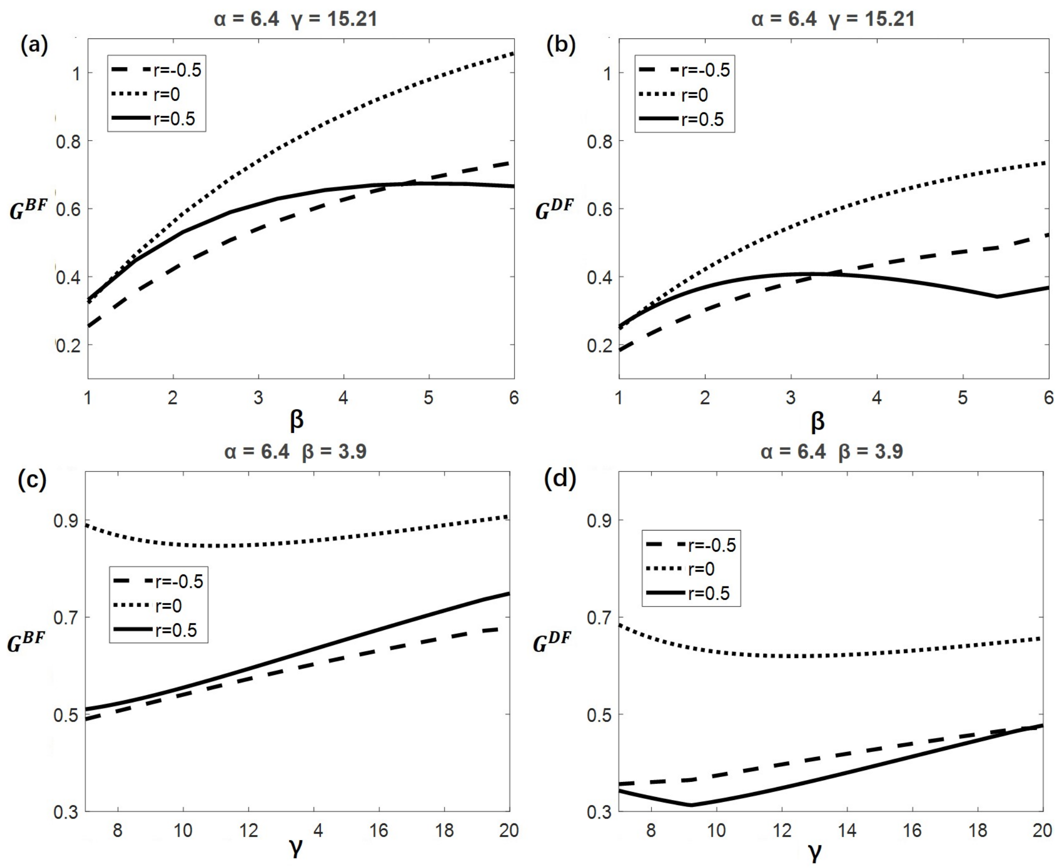

Figure 12: The benefit gains (i.e., G^{BF} and G^{DF}) from providing full information over capacity information or demand information under varying β or γ for different r, with fixed $\pi_N = 0.4$, $\pi_s = 0.5$, $p_N = p_s = 0.5$.

7. Conclusions and discussion

In this paper, we have investigated the welfare effects of partial and full pre-trip information on the morning commute behavior under stochastic demand and bottleneck capacity. The factors paid attention in the problem include information completeness, the degree of correlation between bottleneck capacity and demand, and the frequency and amplitude of bottleneck capacity and demand changes. The value of pre-trip information is reflected in the difference between the expected travel costs and different amounts of pre-trip information, including zero, partial, and full information.

We find that providing full pre-trip information does not increase travel costs compared to zero information (Proposition 2), indicating that simultaneously eliminating uncertainty on both sides of supply and demand can always bring positive benefits to the morning commute. However, the benefit gains/losses of providing partial information over zero information depend on the degree of correlation between bottleneck capacity and demand and the frequency and amplitude of bottleneck capacity and demand changes (Propositions 3 - 6). We find that providing partial pre-trip information does not increase travel costs compared to zero information when bottleneck capacity and demand are uncorrelated (Propositions 3 and 5). However, providing partial information can be welfare-reducing over zero information when bottleneck capacity and demand are moderately correlated (Propositions 4 and 6). Also, the welfare effects of the two kinds of partial information, demand information and bottleneck information, are different when demand and bottleneck capacity are not completely correlated. Which kind of partial information is more efficient depends on the degree of correlation between bottleneck capacity and demand and the frequency of demand and bottleneck capacity changes (Propositions 7). Providing bottleneck capacity information is more likely to have a better performance than providing demand information. Furthermore, although providing partial information may induce information paradox, the welfare effects from partial information to full information are always positive (Propositions 8).

Our study has practical implications, particularly for the design and implementation of ATIS. First, given the ubiquity of uncertainties on both the demand and supply sides, ATIS should deliver differentiated levels of pre-trip information based on the correlation between demand and bottleneck capacity, as well as the expected uncertainty in traffic conditions. This allows for targeted information provision that can help optimize commuter decision-making and reduce travel costs under varying conditions. Second, ATIS design should prioritize the provision of full pre-trip information in scenarios with high uncertainty to ensure better overall welfare, while partial information may be sufficient and beneficial when uncertainty is lower or when supply and demand are uncorrelated. Third, integrating pre-trip information with transport policies has the potential to significantly enhance their effectiveness. For example, providing pre-trip information about bottleneck capacity and demand can help commuters make more informed travel decisions, thereby improving the performance of policies such as congestion pricing and variable speed limits in managing demand and alleviating congestion. By dynamically adjusting pricing and speed limits based on real-time and predictive information, ATIS can serve as a key instrument for maximizing the effectiveness of these policies.

Our study can be extended in several directions for further research. First, the partial and full information concerned in our analysis is one hundred percent accurate. Previous studies have revealed that information accuracy is an important factor in affecting the performance of pre-trip information in the morning commute under stochastic bottleneck capacity (Arnott et al., 1999; Yu et al., 2021).Therefore, the first research direction is to understand the welfare effects of inaccurate information on the morning commute under stochastic demand and bottleneck capacity. Second, our model investigates the morning commute behavior in the classical single-bottleneck highway connecting one origin and one destination. However, previous studies have shown that the commuting behavior in multiple-bottleneck models and complex network structures, such as the Y-shaped networks, are distinct from the classical single-bottleneck model (Arnott et al., 1993a; Li et al., 2024). Therefore, whether the paradox of providing partial information still exists in multiple-bottleneck models and complex network structures should be further investigated. Third, we only consider the departure time choice under stochastic bottleneck capacity and demand in the morning commute; however, commuters usually face a series of choices, such as departure time, route, and mode, for each trip (Mannering et al., 1994). Therefore, the third direction is to investigate the value of partial and full information under uncertainty when commuters face multiple objectives. Fourth, in our model, travel demand is treated as exogenously given. However, demand may fluctuate in response to factors such as traffic conditions. Therefore, understanding the value of partial and full information under price-sensitive demand and stochastic bottleneck capacity is an important direction for future research. Fifth, the provision of information typically incurs costs, such as those associated with the development of ATIS. While our study focuses on the potential benefits of information provision, integrating the costs into a more comprehensive framework would be essential for assessing the net impact of information systems. Sixth our study primarily investigates the value of pre-trip information on travel costs under stochastic bottleneck capacity and demand. The interaction between pre-trip information and combination policies, such as congestion pricing and variable speed limits, could provide an effective strategy for reducing congestion and enhancing overall traffic flow. Exploring how these policies can be integrated with ATIS offers valuable insights into better demand management and system optimization, particularly during peak hours. Last but not least, we propose a general framework to evaluate the value of partial and full information by assuming that travel demand and bottleneck capacity follow a joint probability distribution. To facilitate analytical derivations, we adopt the Bernoulli distribution as a stylized example. However, it is worth noting that the Bernoulli distribution may not fully capture the complexities and nuances of real-world traffic systems. Therefore, when applying this framework to practical scenarios, the distributions of travel demand and bottleneck capacity should be carefully calibrated using empirical data to ensure the model's relevance and accuracy.

Acknowledgments

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Appendix

A.1. Boundary condition between Case 1 and Case 2.

In Case 2, assume that the expected travel costs at t_0 and $t_e(t^*)$ when the work start time t^* becomes $t^* + \delta$ are:

$$\begin{cases} E[C^Z(t_0)] = \beta\hat{\theta} + \beta\delta \\ E[C^Z(t_e)] = \gamma\delta + (\alpha+\gamma)\displaystyle\int_{\hat{\theta}}^{\overline{\theta}} k(\theta_{\psi\omega})(\theta_{\psi\omega} - \hat{\theta})\mathrm{d}\theta_{\psi\omega}. \end{cases} \tag{26}$$

Since $\hat{\theta}^* = t^* - t_0$ and $\hat{\theta} = t^* + \delta - t_0$, we can have $\hat{\theta} = \delta + \hat{\theta}^*$. When the system reaches user equilibrium, $E[C^Z(t^*+\delta)] > E[C^Z(t^*)]$, we have:

$$\gamma > \frac{\alpha+\gamma}{\delta}\int_{\hat{\theta}^*}^{\hat{\theta}} k(\theta_{\psi\omega})(\theta_{\psi\omega} - \hat{\theta}^*) + (\alpha+\gamma)[1 - K(\hat{\theta})]. \tag{27}$$

When $\lim \delta \to 0$, we have $K(\hat{\theta}) \approx K(\hat{\theta}^*) \geq \alpha/(\alpha+\gamma)$ and the condition $\underline{\theta} < \hat{\theta}^* < \overline{\theta}$ needs to be satisfied at this point. Therefore, the boundary condition between Case 1 and Case 2 can be obtained by solving $K(\hat{\theta}^*) = \alpha/(\alpha+\gamma)$.

A.2. Equilibrium solution under a general discrete probability distribution.

If stochastic bottleneck capacity s_ω and demand N_ψ follows a general discrete probability distribution, then we denote the probability in bottleneck capacity s_ω and demand N_ψ as $P(s_\omega)$ and $P(N_\psi)$, in which $\omega\psi \in \{1, 2, \cdots, k, \cdots, K\}$ denote all possible discrete conditions.

(1) When $t_e > t^*$, the expected travel cost per commuter at UE can be denoted as:

$$\phi(\hat{\theta}) = \frac{\gamma\beta}{\gamma+\beta}\hat{\theta} + \frac{(\alpha+\gamma)\beta}{\beta+\gamma}\sum_{\psi\omega=k}^{K} P(\theta_{\psi\omega})(\theta_{\psi\omega} - \hat{\theta}), \tag{28}$$

in which $\theta_{k-1} \leq \hat{\theta}$, and $\theta_k \geq \hat{\theta}$. The first partial derivative of $\Phi(\hat{\theta})$ to $\hat{\theta}$ is:

$$\frac{\partial\Phi(\hat{\theta})}{\partial\hat{\theta}} = \frac{\beta[(\alpha+\gamma)G(\theta_k) - \alpha]}{\beta+\gamma}. \tag{29}$$

Letting $G(\theta_k) = \sum_{\psi\omega=1}^{k} P(\theta_{\psi\omega})$,we assume that there are θ_{k-1}^* and θ_k^* which satisfies $G(\theta_{k-1}^*) < \frac{\alpha}{\alpha+\gamma} < G(\theta_k^*)$. The expected travel cost at UE is: $E[C^Z] = min\left\{\Phi(\theta_{k-1}^*), \Phi(\theta_k^*)\right\}$.

(2) When $t_e = t^*$, the expected travel cost per commuter at UE can be denoted as:

$$\phi(\hat{\theta}^{**}) = \beta\hat{\theta}^{**}, \tag{30}$$

in which $\hat{\theta}^{**}$ can be obtained by solving $\beta\hat{\theta}^{**} = (\alpha+\gamma)\sum_{\psi\omega=k}^{K} P(\theta_{\psi\omega})(\theta_{\psi\omega} - \hat{\theta}^{**})$. Besides, the boundary condition between Case 1 and Case 2 can be obtained by solving $G(\hat{\theta}^{**}) = \alpha/(\alpha+\gamma)$ and $\hat{\theta}^{**} \in \{\theta_1, \theta_2, \theta_3, \theta_4\}$.

A.3. Equilibrium solutions when demand and capacity follow Bernoulli distributions.

Let $\rho_{\psi\omega}$ be the correlation parameter between the random variables s_ω and N_ψ. Then, we have the joint probability distribution of the random variables N_ψ and s_ω: $P(\theta_{\psi\omega}) = \rho_{\psi\omega}P(N_\psi)P(s_\omega)$. r is the degree of correlation between the random variables N_ψ and s_ω. Therefore, we have the relationships between the degree of correlation r and the correlation parameter $\rho_{\psi\omega}$: $\rho_{HG} = 1 + r\sqrt{\frac{(1-p_N)(1-p_s)}{p_N p_s}}$, $\rho_{HB} = 1 - r\sqrt{\frac{(1-p_N)p_s}{p_N(1-p_s)}}$, $\rho_{LG} = 1 - r\sqrt{\frac{p_N(1-p_s)}{(1-p_N)p_s}}$, and $\rho_{LB} = 1 + r\sqrt{\frac{p_N p_s}{(1-p_N)(1-p_s)}}$.

We assume the demand in low level $\underline{N}$ with probability $1 - p_N$ and in high level $\overline{N}$ with probability p_N, and the bottleneck capacity in bad condition $\underline{s}$ with probability $1 - p_s$ and in good condition $\overline{s}$ with probability p_s. When $\pi_N \geq \pi_s$ is satisfied, congestion can definitely be alleviated by adjusting the bottleneck capacity. If $\pi_N \geq \pi_s$, then $\theta_2 = N_H/S_G$ and $\theta_3 = N_L/S_B$. We can have the specific expression of the joint probability distribution of the random variable s_ω and N_ψ is:

$$P(N_\psi, s_\omega) = \begin{cases} \rho_{HB} p_N (1-p_s), & \text{if } N_\psi = N_H, s_\omega = s_B \\ \rho_{LB}(1-p_N)(1-p_s), & \text{if } N_\psi = N_L, s_\omega = s_B \\ \rho_{HG} p_N p_s, & \text{if } N_\psi = N_H, s_\omega = s_G \\ \rho_{LG}(1-p_N) p_s, & \text{if } N_\psi = N_L, s_\omega = s_G \end{cases} \tag{31}$$

where $\rho_{LB} \in [0, \max\left\{\frac{1}{1-p_N}, \frac{1}{1-p_s}\right\}]$, $\rho_{LG} \in [0, \max\left\{\frac{1}{1-p_N}, \frac{1}{p_s}\right\}]$, $\rho_{HB} \in [0, \max\left\{\frac{1}{p_N}, \frac{1}{1-p_s}\right\}]$, and $\rho_{HG} \in [0, \max\left\{\frac{1}{p_N}, \frac{1}{p_s}\right\}]$.

The expected travel costs in the four conditions can be denoted as:

$$(1)\ \frac{\alpha}{\alpha+\gamma} < G(\theta_1)(i.e., p_s \geq \frac{\alpha}{\alpha+\gamma}, r < \frac{(\alpha+\gamma)(1-p_N)p_s - \alpha}{(\alpha+\gamma)\sqrt{(1-p_N)(1-p_s)p_s p_N}}) : E[C^Z] = \Phi(\theta_1);$$

$$(2)\ G(\theta_1) \leq \frac{\alpha}{\alpha+\gamma} < G(\theta_2)(i.e., p_s \geq \frac{\alpha}{\alpha+\gamma}, \frac{(\alpha+\gamma)(1-p_N)p_s - \alpha}{(\alpha+\gamma)\sqrt{(1-p_N)(1-p_s)p_s p_N}} \leq r) :$$

$$E[C^Z] = min\{\Phi(\theta_1), \Phi(\theta_2)\} = \Phi(\theta_2) = \frac{\gamma\beta\theta_2}{(\gamma+\beta)} + \frac{(\gamma+\alpha)\beta}{(\gamma+\beta)}\{\rho_{HG} p_s \theta_3 + \rho_{HB}(1-p_s)\theta_4 - \theta_2\} p_N;$$

$$(3)\ G(\theta_2) \leq \frac{\alpha}{\alpha+\gamma} < G(\theta_3)(i.e., p_s < \frac{\alpha}{\alpha+\gamma}, r < \frac{\alpha - (\alpha+\gamma)(p_N p_s + 1 - p_N)}{(\alpha+\gamma)\sqrt{(1-p_N)(1-p_s)p_s p_N}}) :$$

$$E[C^Z] = min\{\Phi(\theta_2), \Phi(\theta_3)\} = \Phi(\theta_3) = \frac{\gamma\beta\theta_3}{(\gamma+\beta)} + \frac{(\gamma+\alpha)\beta}{(\gamma+\beta)}\rho_{HB} p_N (1-p_s)(\theta_4 - \theta_3);$$

$$(4)\ G(\theta_3) \leq \frac{\alpha}{\alpha+\gamma} < G(\theta_4)(i.e., p_s < \frac{\alpha}{\alpha+\gamma}, \frac{\alpha - (\alpha+\gamma)(p_N p_s + 1 - p_N)}{(\alpha+\gamma)\sqrt{(1-p_N)(1-p_s)p_s p_N}} \leq r) :$$

$$E[C^Z] = min\{\Phi(\theta_3), \Phi(\theta_4)\} = \Phi(\theta_4) = \frac{\gamma\beta\theta_4}{(\gamma+\beta)};$$

Then, we have the boundary condition separating Case 1 ($t_e > t^*$) and Case 2 ($t_e = t^*$). By solving $\beta\hat{\theta}^* = (\alpha+\gamma)\sum_{\hat{\theta}^*}^{\overline{\theta}} P(\theta_{\psi\omega})(\theta_{\psi\omega} - \hat{\theta}^*)$, we have $\hat{\theta}^*$.
(a) $\theta_1 < \hat{\theta}^* < \theta_2$, $\hat{\theta}_1^* = \frac{(\alpha+\gamma)[P(\theta_{HG})\theta_2 + P(\theta_{LB})\theta_3 + P(\theta_{HB})\theta_4]}{\beta + (\alpha+\gamma)[p_N + P(\theta_{LB})]}$ when $0 < \pi_N \leq \frac{[\beta+(\alpha+\gamma)(1-p_s)]\pi_s - (\alpha+\gamma)P(\theta_{HB})}{(\alpha+\gamma)(1-p_s) - (\alpha+\gamma)P(\theta_{HB})}$.
(b) $\theta_2 < \hat{\theta}^* < \theta_3$, $\hat{\theta}_2^* = \frac{(\alpha+\gamma)[P(\theta_{LB})\theta_3 + P(\theta_{HB})\theta_4]}{\beta+(\alpha+\gamma)(1-p_s)}$ when $\frac{(\alpha+\gamma)P(\theta_{HB})}{\beta+(\alpha+\gamma)P(\theta_{HB})} \leq \pi_N < 1$.
(c) $\theta_3 < \hat{\theta}^* < \theta_4$, $\hat{\theta}_3^* = \frac{(\alpha+\gamma)P(\theta_{HB})\theta_4}{\beta+(\alpha+\gamma)P(\theta_{HB})}$ when $\frac{[\beta+(\alpha+\gamma)(1-p_s)]\pi_s - (\alpha+\gamma)P(\theta_{HB})}{(\alpha+\gamma)(1-p_s) - (\alpha+\gamma)P(\theta_{HB})} < \pi_N < \frac{(\alpha+\gamma)P(\theta_{HB})}{\beta+(\alpha+\gamma)P(\theta_{HB})}$.

The expected travel cost when the system reaches the user equilibrium in Case 2 is $E[C^Z] = \beta\hat{\theta}^*$, otherwise, Case 1. The expected travel costs without information when $\pi_N \leq \pi_s$ as shown in Table A1.

In Table A1, $\pi_N^* = \frac{[\beta+(\alpha+\gamma)(1-p_s)]\pi_s - (\alpha+\gamma)P(\theta_{HB})}{(\alpha+\gamma)(1-p_s) - (\alpha+\gamma)P(\theta_{HB})}$ and $\pi_N^{**} = \frac{(\alpha+\gamma)P(\theta_{HB})}{\beta+(\alpha+\gamma)P(\theta_{HB})}$. π_N^* is a strictly monotonically increasing function of r, i.e. $\partial\pi_N^*/\partial r > 0$, but π_N^{**} is a strictly monotonically decreasing function of r, i.e. $\partial\pi_N^{**}/\partial r < 0$. $\phi(\theta_k) = \frac{\gamma\beta\theta_k}{\gamma+\beta} + \frac{(\alpha+\gamma)\beta}{\gamma+\beta}\sum_{\psi\omega=k}^{K} P(\theta_{\psi\omega})(\theta_{\psi\omega} - \theta_k)$ and $\beta\hat{\theta}_k^* = (\alpha+\gamma)\sum_{\psi\omega=k+1}^{K} P(\theta_{\psi\omega})(\theta_{\psi\omega} - \hat{\theta}_k^*)$, in which $\hat{\theta}_3^* > \hat{\theta}_2^* > \hat{\theta}_1^*$. From the Table A1, it can be seen that the correlation between demand and bottleneck capacity, the frequency

Table A1: The expected travel costs at UE when the stochastic demand and bottleneck capacity follow the Bernoulli distribution and $\pi_N > \pi_s$.

π_N	$0 \le p_s < \frac{\alpha}{\alpha+\gamma}$	
	$r_{\min} \le r < \frac{\alpha-(\alpha+\gamma)(p_N p_s+1-p_N)}{(\alpha+\gamma)\sqrt{p_N p_s(1-p_N)(1-p_s)}}$	$\frac{\alpha-(\alpha+\gamma)(p_N p_s+1-p_N)}{(\alpha+\gamma)\sqrt{p_N p_s(1-p_N)(1-p_s)}} \le r \le r_{\max}$
$0 < \pi_N < \pi_N^{**}$	$\Phi(\theta_4)$	$\beta\hat{\theta}_3^*$
$\pi_N^{**} < \pi_N < 1$	$\Phi(\theta_4)$	$\Phi(\theta_3)$
π_N	$\frac{\alpha}{\alpha+\gamma} \le p_s < 1$	
	$r_{\min} \le r < \frac{(\alpha+\gamma)(1-p_N)p_s-\alpha}{(\alpha+\gamma)\sqrt{p_N p_s(1-p_N)(1-p_s)}}$	$\frac{(\alpha+\gamma)(1-p_N)p_s-\alpha}{(\alpha+\gamma)\sqrt{p_N p_s(1-p_N)(1-p_s)}} \le r \le r_{\max}$
$0 < \pi_N \le \pi_N^*$	$\beta\hat{\theta}_1^*$	$\beta\hat{\theta}_3^*$
$\pi_N^* < \pi_N < \pi_N^{**}$	$\beta\hat{\theta}_3^*$	$\beta\hat{\theta}_3^*$
$\pi_N^{**} < \pi_N < 1$	$\beta\hat{\theta}_2^*$	$\beta\hat{\theta}_2^*$

and severity of demand and capacity reduction will significantly affect the expected travel costs and commuting patterns.

When this premise is not satisfied, congestion is also bound to occur by adjusting the capacity of bottlenecks. This means that $\pi_N \le \pi_s$, $\theta_2 = N_L/S_B$ and $\theta_3 = N_H/S_G$. The specific expression of the joint probability distribution of s_ω and N_ψ is same.

The expected travel costs in the four conditions can be denoted as:

$$(1)\ \frac{\alpha}{\alpha+\gamma} < G(\theta_1)(i.e., p_N \ge \frac{\gamma}{\alpha+\gamma}, r < \frac{(\alpha+\gamma)(1-p_N)p_s-\alpha}{(\alpha+\gamma)\sqrt{(1-p_N)(1-p_s)p_s p_N}}) : E[C^Z] = \Phi(\theta_1);$$

$$(2)\ G(\theta_1) \le \frac{\alpha}{\alpha+\gamma} < G(\theta_2)(i.e., p_N \ge \frac{\gamma}{\alpha+\gamma}, \frac{(\alpha+\gamma)(1-p_N)p_s-\alpha}{(\alpha+\gamma)\sqrt{(1-p_N)(1-p_s)p_s p_N}} \le r) :$$

$$E[C^Z] = min\{\Phi(\theta_1), \Phi(\theta_2)\} = \Phi(\theta_2) = \frac{\gamma\beta\theta_2}{(\gamma+\beta)} + \frac{(\gamma+\alpha)\beta}{(\gamma+\beta)}\{\rho_{HG}p_s\theta_3 + \rho_{HB}(1-p_s)\theta_4 - \theta_2\}p_N;$$

$$(3)\ G(\theta_2) \le \frac{\alpha}{\alpha+\gamma} < G(\theta_3)(i.e., p_N < \frac{\gamma}{\alpha+\gamma}, r < \frac{\alpha-(\alpha+\gamma)(p_N ps+1-p_N)}{(\alpha+\gamma)\sqrt{(1-p_N)(1-p_s)p_s p_N}}) :$$

$$E[C^Z] = min\{\Phi(\theta_2), \Phi(\theta_3)\} = \Phi(\theta_3) = \frac{\gamma\beta\theta_3}{(\gamma+\beta)} + \frac{(\gamma+\alpha)\beta}{(\gamma+\beta)}\rho_{HB}p_N(1-p_s)(\theta_4-\theta_3);$$

$$(4)\ G(\theta_3) \le \frac{\alpha}{\alpha+\gamma} < G(\theta_4)(i.e., p_N < \frac{\gamma}{\alpha+\gamma}, \frac{\alpha-(\alpha+\gamma)(p_N p_s+1-p_N)}{(\alpha+\gamma)\sqrt{(1-p_N)(1-p_s)p_s p_N}} \le r) :$$

$$E[C^Z] = min\{\Phi(\theta_3), \Phi(\theta_4)\} = \Phi(\theta_4) = \frac{\gamma\beta\theta_4}{(\gamma+\beta)}.$$

Then we solve the boundary condition separating Case 1($t_e > t^*$) and Case 2($t_e = t^*$). By solving $\beta\hat{\theta}^* = (\alpha+\gamma)\sum_{\hat{\theta}^*}^{\overline{\theta}} P(\theta_{\psi\omega})(\theta_{\psi\omega} - \hat{\theta}^*)$, we can be obtained $\hat{\theta}^*$.
(a)$\theta_1 < \hat{\theta}^* < \theta_2$, $\hat{\theta}_1^* = \frac{(\alpha+\gamma)[P(\theta_{LB})\theta_2+P(\theta_{HG})\theta_3+P(\theta_{HB})\theta_4]}{\beta+(\alpha+\gamma)[p_N+P(\theta_{LB})]}$ when $0 < \pi_s \le \frac{[\beta+(\alpha+\gamma)(1-p_s)]\pi_N-(\alpha+\gamma)P(\theta_{HB})}{(\alpha+\gamma)(1-p_s)-(\alpha+\gamma)P(\theta_{HB})}$.

(b)$\theta_2 < \hat{\theta}^* < \theta_3$, $\hat{\theta}_2^* = \frac{(\alpha+\gamma)[P(\theta_{HG})\theta_3+P(\theta_{HB})\theta_4]}{\beta+(\alpha+\gamma)(1-p_s)}$ when $\frac{(\alpha+\gamma)P(\theta_{HB})}{\beta+(\alpha+\gamma)P(\theta_{HB})} \leq \pi_s < 1$.
(c)$\theta_3 < \hat{\theta}^* < \theta_4$, $\hat{\theta}_3^* = \frac{(\alpha+\gamma)P(\theta_{HB})\theta_4}{\beta+(\alpha+\gamma)P(\theta_{HB})}$ when $\frac{[\beta+(\alpha+\gamma)(1-p_s)]\pi_N-(\alpha+\gamma)P(\theta_{HB})}{(\alpha+\gamma)(1-p_s)-(\alpha+\gamma)P(\theta_{HB})} < \pi_s < \frac{(\alpha+\gamma)P(\theta_{HB})}{\beta+(\alpha+\gamma)P(\theta_{HB})}$.

The expected travel cost when system at UE in case 2 is $E[C^Z] = \beta\hat{\theta}^*$, otherwise, Case 1. The expected travel costs without information when $\pi_N \leq \pi_s$ as shown in Table A2:

Table A2: The expected travel costs at UE when the stochastic demand and bottleneck capacity follow the Bernoulli distribution and $\pi_N \leq \pi_s$.

π_s	$0 \leq p_N < \frac{\gamma}{\alpha+\gamma}$	
	$r_{min} \leq r < \frac{\alpha-(\alpha+\gamma)(p_N p_s+1-p_N)}{(\alpha+\gamma)\sqrt{p_N p_s(1-p_N)(1-p_s)}}$	$\frac{\alpha-(\alpha+\gamma)(p_N p_s+1-p_N)}{(\alpha+\gamma)\sqrt{p_N p_s(1-p_N)(1-p_s)}} \leq r \leq r_{max}$
$0 < \pi_s < \pi_s^{**}$	$\Phi(\theta_4)$	$\beta\hat{\theta}_3^*$
$\pi_s^{**} < \pi_s < 1$		$\Phi(\theta_3)$
π_s	$\frac{\gamma}{\alpha+\gamma} \leq p_N < 1$	
	$r_{min} \leq r < \frac{(\alpha+\gamma)(1-p_N)p_s-\alpha}{(\alpha+\gamma)\sqrt{p_N p_s(1-p_N)(1-p_s)}}$	$\frac{(\alpha+\gamma)(1-p_N)p_s-\alpha}{(\alpha+\gamma)\sqrt{p_N p_s(1-p_N)(1-p_s)}} \leq r \leq r_{max}$
$0 < \pi_s \leq \pi_s^*$	$\beta\hat{\theta}_1^*$	$\beta\hat{\theta}_3^*$
$\pi_s^* < \pi_s < \pi_s^{**}$	$\beta\hat{\theta}_3^*$	
$\pi_s^{**} < \pi_s < 1$	$\beta\hat{\theta}_2^*$	

In Table A2, $\pi_s^* = \frac{[\beta+(\alpha+\gamma)p_N]\pi_N-(\alpha+\gamma)P(\theta_{HB})}{(\alpha+\gamma)p_N-(\alpha+\gamma)P(\theta_{HB})}$ and $\pi_s^{**} = \frac{(\alpha+\gamma)P(\theta_{HB})}{\beta+(\alpha+\gamma)P(\theta_{HB})}$. π_s^* is a strictly monotonically increasing function of r, i.e. $\partial\pi_s^*/\partial r > 0$, but π_s^{**} is a strictly monotonically decreasing function of r, (i.e. $\partial\pi_s^{**}/\partial r < 0$). $\phi(\theta_k) = \frac{\gamma\beta\theta_k}{\gamma+\beta} + \frac{(\alpha+\gamma)\beta}{\gamma+\beta}\sum_{\psi\omega=k}^{K} P(\theta_{\psi\omega})(\theta_{\psi\omega}-\theta_k)$ and $\beta\hat{\theta}_k^* = (\alpha+\gamma)\sum_{\psi\omega=k+1}^{K} P(\theta_{\psi\omega})(\theta_{\psi\omega}-\hat{\theta}_k^*)$, in which $\hat{\theta}_3^* > \hat{\theta}_2^* > \hat{\theta}_1^*$. From the Table A2, it can be seen that the correlation between demand and bottleneck capacity, the frequency and severity of demand and capacity reduction will significantly affect the expected travel costs and commuting patterns.

A.4. Equilibrium solutions with the bottleneck capacity information when demand and capacity follow Bernoulli distributions.

The probability of demand being in different state changes when commuters has acquired bottleneck capacity information before departure. When commuters are given information that the bottleneck capacity is in good condition for the day, the demand in bad condition $\underline{N}$ with probability $P'(\theta_{LG}) = (1-p_N)\rho_{LG}$ or in good condition $\overline{N}$ with probability $P'(\theta_{HG}) = p_N\rho_{HG}$. When commuters are given information that the bottleneck capacity is in bad condition for the day, the demand in bad condition $\underline{N}$ with probability $P'(\theta_{LB}) = (1-p_N)\rho_{LB}$ or in good condition $\overline{N}$ with probability $P'(\theta_{HB}) = p_N\rho_{HB}$. We can have the new joint probability distribution of the random variables is: $P'(N_\psi, s_\omega) = \rho_{N_\psi s_\omega}P(N_\psi)$.

Thus, we have the expected travel costs with bottleneck capacity information under the four possible states $C^B_{LG}, C^B_{HG}, C^B_{HB}$ and C^B_{LB} :

When the bottleneck capacity is in good condition for the day ($\theta_{\psi\omega} \in \{\theta_{LG}, \theta_{HG}\}$),
if $r < \frac{[\gamma-(\alpha+\gamma)p_N]p_s}{(\alpha+\gamma)\sqrt{(1-p_N)(1-p_s)p_sp_N}}$, $C^B_{LG} + C^B_{HG} = \Phi^B(\theta_{LG}) = \frac{\gamma\beta\theta_{LG}}{(\gamma+\beta)} + \frac{(\gamma+\alpha)\beta}{(\gamma+\beta)}\rho_{HG}p_N(\theta_{HG} - \theta_{LG})$;
If $r \geq \frac{[\gamma-(\alpha+\gamma)p_N]p_s}{(\alpha+\gamma)\sqrt{(1-p_N)(1-p_s)p_sp_N}}$, $C^B_{LG} + C^B_{HG} = min\left\{\Phi^B(\theta_{LG}), \Phi^B(\theta_{HG})\right\} = \Phi^B(\theta_{HG}) = \frac{\gamma\beta\theta_{HG}}{(\gamma+\beta)}$.

When the bottleneck capacity is in bad condition for the day ($\theta_\omega \in \{\theta_{LB}, \theta_{HB}\}$),
if $r < \frac{[(\alpha+\gamma)p_N-\gamma](1-p_s)}{(\alpha+\gamma)\sqrt{(1-p_N)(1-p_s)p_sp_N}}$, $C^B_{LB} + C^B_{HB} = min\left\{\Phi^B(\theta_{LB}), \Phi^B(\theta_{HB})\right\} = \Phi^B(\theta_{HB}) = \frac{\gamma\beta\theta_{HB}}{(\gamma+\beta)}$;
If $r \geq \frac{[(\alpha+\gamma)p_N-\gamma](1-p_s)}{(\alpha+\gamma)\sqrt{(1-p_N)(1-p_s)p_sp_N}}$, $C^B_{LB} + C^B_{HB} = \Phi^B(\theta_{LB}) = \frac{\gamma\beta\theta_{LB}}{(\gamma+\beta)} + \frac{(\gamma+\alpha)\beta}{(\gamma+\beta)}\rho_{HB}p_N(\theta_{HB} - \theta_{LB})$.

Then We solve the boundary condition separating Case 1($t_e > t^*$) and Case 2($t_e = t^*$). By solving $\beta\hat{\theta}^* = (\alpha+\gamma)\sum_{\hat{\theta}^*}^{\overline{\theta}} P(\theta_{\psi\omega})(\theta_{\psi\omega} - \hat{\theta}^*)$, we can obtain $\hat{\theta}^*$. When the bottleneck capacity is in good condition for the day, we can be obtained $\hat{\theta}^* = \frac{(\alpha+\gamma)\rho_{HG}p_N\theta_{HG}}{\beta+(\alpha+\gamma)\rho_{HG}p_N}$. We have that $\theta_{LG} < \hat{\theta}^* < \theta_{HG}$ always holds when $0 < \pi_N < \frac{(\alpha+\gamma)\rho_{HG}p_N}{\beta+(\alpha+\gamma)\rho_{HG}p_N}$. When the bottleneck capacity is in bad condition for the day, we can be obtained $\hat{\theta}^{**} = \frac{(\alpha+\gamma)\rho_{HB}p_N\theta_{HB}}{\beta+(\alpha+\gamma)\rho_{HB}p_N}$. We have that $\theta_{LB} < \hat{\theta}^{**} < \theta_{HB}$ always holds when $0 < \pi_N < \frac{(\alpha+\gamma)\rho_{HB}p_N}{\beta+(\alpha+\gamma)\rho_{HB}p_N}$. The expected travel cost when the system reaches the user equilibrium in case 2 is $C^B = \beta\hat{\theta}^*$, otherwise, Case 1.

By solving $E[C^B] = p_s(C^B_{LG} + C^B_{HG}) + (1-p_s)(C^B_{LB} + C^B_{HB})$, we can have the expected travel cost of a commuter at UE under stochastic conditions with capacity information.

A.5. Equilibrium solutions with the demand information when demand and capacity follow the Bernoulli distribution

The probability of bottleneck capacity being in different state changes when commuters has acquired demand information before departure.When commuters are given information that the demand is in good condition for the day, the bottleneck capacity in bad condition $\underline{s}$ with probability $P'(\theta_{HB}) = (1-p_s)\rho_{HB}$ or in good condition $\overline{s}$ with probability $P'(\theta_{HG}) = p_s\rho_{HG}$. When commuters are given information that the demand is in bad condition for the day, the bottleneck capacity in bad condition $\underline{s}$ with probability $P'(\theta_{LB}) = (1-p_s)\rho_{LB}$ or in good condition $\overline{s}$ with probability $P'(\theta_{LG}) = p_s\rho_{LG}$. We can have the new joint probability distribution of the random variables is: $P'(N_\psi, s_\omega) = \rho_{N_\psi s_\omega}P(s_\omega)$.

Thus, we have the expected travel costs with demand information under the four possible states $C^D_{LG}, C^D_{HG}, C^D_{HB}$ and C^D_{LB} :

When the bottleneck demand is in good condition for the day ($\theta_{\psi\omega} \in \{\theta_{HG}, \theta_{HB}\}$),
if $r \geq \frac{[(\alpha+\gamma)(1-p_s)-\gamma]p_N}{(\alpha+\gamma)\sqrt{(1-p_N)(1-p_s)p_sp_N}}$, $C^D_{HB}+C^D_{HG} = \Phi^D(\theta_{HG}) = \frac{\gamma\beta\theta_{HG}}{(\gamma+\beta)}+\frac{(\gamma+\alpha)\beta}{(\gamma+\beta)}\rho_{HB}(1-p_s)(\theta_{HB}-\theta_{HG})$;
If $r < \frac{[(\alpha+\gamma)(1-p_s)-\gamma]p_N}{(\alpha+\gamma)\sqrt{(1-p_N)(1-p_s)p_sp_N}}$, $C^D_{HB}+C^D_{HG} = min\left\{\Phi^D(\theta_{HG}), \Phi^D(\theta_{HB})\right\} = \Phi^D(\theta_{HB}) = \frac{\gamma\beta\theta_{HB}}{(\gamma+\beta)}$.

When demand is in bad condition for the day ($\theta_{\psi\omega} \in \{\theta_{LG}, \theta_{LB}\}$),

if $r \geq \frac{[\gamma-(\alpha+\gamma)(1-p_s)](1-p_N)}{(\alpha+\gamma)\sqrt{(1-p_N)(1-p_s)p_s p_N}}$, $C_{LB}^D + C_{LG}^D = min\left\{\Phi^D(\theta_{LB}), \Phi^D(\theta_{LG})\right\} = \Phi^D(\theta_{LB}) = \frac{\gamma\beta\theta_{LB}}{(\gamma+\beta)}$;
If $r < \frac{[\gamma-(\alpha+\gamma)(1-p_s)](1-p_N)}{(\alpha+\gamma)\sqrt{(1-p_N)(1-p_s)p_s p_N}}$, $C_{LB}^D + C_{LG}^D = \Phi^D(\theta_{LG}) = \frac{\gamma\beta\theta_{LG}}{(\gamma+\beta)} + \frac{(\gamma+\alpha)\beta}{(\gamma+\beta)}\rho_{LB}(1-p_s)(\theta_{LB}-\theta_{LG})$.

Then We solve the boundary condition separating Case 1($t_e > t^*$) and Case 2($t_e = t^*$). By solving $\beta\hat{\theta}^* = (\alpha+\gamma)\sum_{\hat{\theta}^*}^{\overline{\theta}} P(\theta_{\psi\omega})(\theta_{\psi\omega} - \hat{\theta}^*)$, we can be obtained $\hat{\theta}^*$. When the demand is in good condition for the day, we can be obtained $\hat{\theta}^* = \frac{(\alpha+\gamma)\rho_{HB}(1-p_s)\theta_{HB}}{\beta+(\alpha+\gamma)\rho_{HB}(1-p_s)}$. We have that $\theta_{HG} < \hat{\theta}^* < \theta_{HB}$ always holds when $0 < \pi_s < \frac{(\alpha+\gamma)\rho_{HB}(1-p_s)}{\beta+(\alpha+\gamma)\rho_{HB}(1-p_s)}$. When the demand is in bad condition for the day, we can be obtained $\hat{\theta}^{**} = \frac{(\alpha+\gamma)\rho_{LB}(1-p_s)\theta_{LB}}{\beta+(\alpha+\gamma)\rho_{LB}(1-p_s)}$. We have that $\theta_{LG} < \hat{\theta}^{**} < \theta_{LB}$ always holds when $0 < \pi_s < \frac{(\alpha+\gamma)\rho_{LB}(1-p_s)}{\beta+(\alpha+\gamma)\rho_{LB}(1-p_s)}$. The expected travel cost when the system reaches the user equilibrium in case 2 is $C^D = \beta\hat{\theta}^*$, otherwise, Case 1.

By solving $E[C^D] = p_N(C_{HG}^D + C_{HB}^D) + (1-p_N)(C_{LG}^D + C_{LB}^D)$, we can have the expected travel cost of a commuter at UE under stochastic conditions with demand information.

A.6. Proof of Proposition 2.

Part (a): If $t_e > t^*$, $G^{ZF} = \frac{\alpha\beta}{\beta+\gamma}\int_{\hat{\theta}^*}^{\overline{\theta}}[k(\theta_{\psi\omega})\theta_{\psi\omega} - k(\theta_{\psi\omega})\hat{\theta}^*]\mathrm{d}\theta_{\psi\omega} + \frac{\gamma\beta}{\beta+\gamma}\int_{\underline{\theta}}^{\hat{\theta}^*}[k(\theta_{\psi\omega})\hat{\theta}^* - k(\theta_{\psi\omega})\theta_{\psi\omega}]\mathrm{d}\theta_{\psi\omega} \geq 0$; otherwise, $G^{ZF} = \beta\hat{\theta}^{**} - \frac{\beta\gamma}{\beta+\gamma}\int_{\underline{\theta}}^{\overline{\theta}}\theta_{\psi\omega}k(\theta_{\psi\omega})\mathrm{d}\theta_{\psi\omega} > \frac{\alpha\beta}{\beta+\gamma}\int_{\hat{\theta}^*}^{\overline{\theta}}[k(\theta_{\psi\omega})\theta_{\psi\omega} - k(\theta_{\psi\omega})\hat{\theta}^*]\mathrm{d}\theta_{\psi\omega} + \frac{\gamma\beta}{\beta+\gamma}\int_{\underline{\theta}}^{\hat{\theta}^*}[k(\theta_{\psi\omega})\hat{\theta}^* - k(\theta_{\psi\omega})\theta_{\psi\omega}]\mathrm{d}\theta_{\psi\omega} > 0$.
Part (b): If $\pi_s = \pi_N$ and bottleneck capacity and demand are perfectly positive correlated, we have $\theta_2 = \theta_3 = \hat{\theta}^*$ and $P(\theta_1) = P(\theta_4) = 0$. When $t_e > t^*$, $G^{ZF} = 0$; otherwise, $G^{ZF} > 0$

A.7. Proof of Proposition 3.

Part (a): If $t_e > t^*$, using Eq.(10) and Eqs.(14)-(15), we can derive the expected benefit gains from providing bottleneck information over zero information:

$$G^{ZB} = E[C^Z] - E[C^B] = \frac{(\alpha+\gamma)\beta}{\beta+\gamma}\left\{\int_{\hat{\theta}^*}^{\overline{\theta}}\theta_{\psi\omega}k(\theta_{\psi\omega})\mathrm{d}\theta_{\psi\omega} - \int_{\underline{s}}^{\overline{s}}\frac{f(s_\omega)}{s_\omega}\int_{\hat{N}^*}^{\overline{N}} f(N_\psi|s_\omega)N_\psi \mathrm{dN}_\psi \mathrm{ds}_\omega\right\} \tag{32}$$

where $f(N_\psi|s_\omega) = \frac{\partial}{\partial N_\psi}J(N_\psi, s_\omega)$, $k(\theta_{\psi\omega}) = \int_{\underline{s}}^{\overline{s}} s_\omega j(\theta_{\psi\omega}s_\omega, s_\omega)\mathrm{ds}_\omega$. When bottleneck capacity and demand are uncorrelated, $f(N_\psi|s_\omega) = g(N_\psi)$, $j(\theta_{\psi\omega}s_\omega, s_\omega) = g(\theta_{\psi\omega}s_\omega)f(s_\omega)$.

$$\begin{aligned} G^{ZB} &= \frac{(\alpha+\gamma)\beta}{\beta+\gamma}\left\{\int_{\hat{\theta}^*}^{\overline{\theta}}\theta_{\psi\omega}\int_{\underline{s}}^{\overline{s}} s_\omega g(\theta_{\psi\omega}s_\omega)f(s_\omega)\mathrm{ds}_\omega \mathrm{d}\theta_{\psi\omega} - \int_{\underline{s}}^{\overline{s}}\frac{f(s_\omega)}{s_\omega}\int_{\hat{N}^*}^{\overline{N}} g(N_\psi)N_\psi \mathrm{dN}_\psi \mathrm{ds}_\omega\right\} \\ &= \frac{(\alpha+\gamma)\beta}{\beta+\gamma}\left\{\int_{\underline{s}}^{\overline{s}}\int_{\underline{s}\hat{\theta}^*}^{\overline{N}}\frac{N_\psi}{s_\omega}f(s_\omega)g(N_\psi)\mathrm{ds}_\omega \mathrm{dN}_\psi - \int_{\underline{s}}^{\overline{s}}\int_{\hat{N}^*}^{\overline{N}}\frac{N_\psi}{s_\omega}f(s_\omega)g(N_\psi)\mathrm{ds}_\omega \mathrm{dN}_\psi\right\} \end{aligned} \tag{33}$$

where $\int_{\hat{\theta}^*}^{\overline{\theta}} g(N_\psi)f(s_\omega)\mathrm{d}\theta_{\psi\omega} = \int_{\hat{N}^*}^{\overline{N}} g(N_\psi)\mathrm{dN}_\psi = \frac{\gamma}{\alpha+\gamma}$. Hence, $\underline{s}\hat{\theta}^* \leq \hat{N}^*$, $G^{ZB} \geq 0$.
Part(b): When two conditional variables are uncorrelated (i.e., $r = 0$), $\pi_s > \pi_N$ and $p_N < \frac{\gamma}{\alpha+\gamma}$, by combining Eq.(16), Tables A1 and A2, we can obtain the expected benefit from

bottleneck information to zero information:

$$G^{ZB} = E[C^Z] - p_s E[C^B_{\psi|G}] - (1-p_s)E[C^B_{\psi|B}] = \Phi(\theta_3) - p_s\beta\hat{\theta}_1^{**} - (1-p_s)\hat{\theta}_2^{**} \tag{34}$$

where the specific expressions of $\Phi(\theta_3)$, $\hat{\theta}_1^{**}$ and $\hat{\theta}_2^{**}$ can be found in Appendix A.3 and Appendix A.4. By substituting specific expressions and $r = 0$, and simplifying, we obtain:

$$G^{ZB} = \frac{\beta[(\gamma+\alpha)p_N - \gamma](N_H - p_s N_L)}{(\gamma+\beta)s_G} + \frac{\beta[(\gamma+\alpha)p_N N_L + (1-2p_N)N_H - \alpha N_L](1-p_s)}{(\gamma+\beta)s_B} \tag{35}$$

When $\pi_N = \pi_s$ and $p_N < \frac{\gamma}{\alpha+\gamma}$, we have $G^{ZB} = 0$.

In the above, we find a special case when demand varies slightly that satisfies $G^{ZB} = 0$, the proposition is true. So we can conclude that providing bottleneck information can be welfare-neutral over zero information when bottleneck capacity and demand are independent (i.e., $r = 0$). If demand and $\pi_s > \pi_N$ frequently experience drops, providing bottleneck information can be likely welfare-neutral (i.e,$G^{ZB} = 0$).

A.8. Proof of Proposition 4.

Part (a): If two conditional variables are moderately correlated and $\pi_N < \pi_s$:

(1) When p_s is large and p_N is small, the benefit gains from bottleneck information is

$$\begin{aligned} G^{ZB} &= \frac{(\gamma+\alpha)\beta[\rho_{HG}p_N p_s N_H s_B + \rho_{HB}p_N(1-p_s)N_H s_G]}{[\beta+(\gamma+\alpha)(1-p_s)]s_B s_G} - \frac{(\gamma+\alpha)\beta p_s \rho_{HG} p_N N_H}{[\beta+(\gamma+\alpha)\rho_{HG}p_N]s_G} - \frac{\gamma\beta(1-p_s)N_L}{(\beta+\gamma)s_B} \\ &\quad - \frac{(\alpha+\gamma)\beta(1-p_s)(N_H-N_L)\rho_{HB}p_N}{(\beta+\gamma)s_B}, \qquad \rho_{HG}p_N > (1-p_s), \\ G^{ZB} &< \frac{\beta(\alpha+\gamma)(1-p_s)}{s_B}\left\{\frac{\rho_{HB}p_N N_H}{\beta+(\gamma+\alpha)(1-p_s)} - \frac{\rho_{HB}p_N N_H + (\gamma-\rho_{HB}p_N)N_L}{\beta+\gamma}\right\} < 0 \end{aligned} \tag{36}$$

(2) When p_s and p_N are large, the benefit gains from bottleneck information is

$$G^{ZB} = \frac{\gamma\beta(s_B - p_s s_G)N_H}{(\gamma+\alpha)s_G s_B} + (\gamma+\alpha)\beta N_H p_N\left\{\frac{(1-p_s\rho_{HG})(1-\pi_s)}{(\gamma+\beta)s_B} - \frac{p_s\rho_{HG}}{[\beta+(\gamma+\alpha)\rho_{HG}p_N]s_G}\right\} \tag{37}$$

When bottleneck capacity rarely experiences degradation (i.e., $\pi_s < p_s$ and $p_s > \frac{(1-\pi_s)[\beta+(\gamma+\alpha)\rho_{HG}p_N]}{\rho_{HG}(1-\pi_s)[\beta+(\gamma+\alpha)\rho_{HG}p_N]+\rho_{HG}\pi_s(\gamma+\beta)}$), $G^{ZB} < 0$.

Therefore, we can conclude that when bottleneck capacity and demand have a moderately correlation, bottleneck capacity rarely experience degradation, providing bottleneck information is more likely to be welfare-reducing over zero information (i.e., $G^{ZB} < 0$) when $\pi_N < \pi_s$.

Part (b): When $\pi_s < \pi_N$ and bottleneck capacity and demand are negatively correlated:

$$\begin{aligned} G^{ZB} &= \frac{\gamma\beta N_H}{(\gamma+\beta)s_B} - p_s\left[\frac{\gamma\beta N_L}{(\gamma+\beta)s_G} + \frac{(\gamma+\alpha)\beta}{\gamma+\beta}\rho_{HG}p_N\left(\frac{N_H}{s_G} - \frac{N_L}{s_G}\right)\right] - (1-p_s)\frac{\gamma\beta N_H}{(\gamma+\beta)s_B} \\ \frac{\partial G^{ZB}}{\partial \rho_{HG}} &= -\frac{(\gamma+\alpha)\beta p_N p_s(N_H - N_L)}{(\gamma+\beta)s_G} < 0, \qquad \frac{\partial \rho_{HG}}{\partial r} > 0 \end{aligned} \tag{38}$$

When $\pi_s < \pi_N$ and bottleneck capacity and demand are positively correlated:

$$G^{ZB} = \frac{(\gamma+\alpha)\beta(1-p_s)[\rho_{LB}(1-p_N)N_L - \rho_{HB}p_N N_H]}{[\beta+(\gamma+\alpha)(1-p_s)]s_B} - \frac{(\gamma+\alpha)\beta p_s \rho_{HG} p_N N_H}{[\beta+(\gamma+\alpha)\rho_{HG}p_N]s_G} - \frac{\gamma\beta(1-p_s)N_H}{(\beta+\gamma)s_B}$$
$$\frac{\partial G^{ZB}}{\partial \rho_{HG}} = \frac{(\gamma+\alpha)\beta p_N p_s(N_L+N_H)}{[\beta+(\gamma+\alpha)(1-p_s)]s_B} - \frac{(\gamma+\alpha)\beta^2 p_s p_N N_H}{[\beta+(\gamma+\alpha)\rho_{HG}p_N]^2 s_G} > 0, \qquad \frac{\partial \rho_{HG}}{\partial r} > 0 \tag{39}$$

Therefore, the benefit gains from bottleneck information decreases as the value of the degree of correlation increases when $\pi_s < \pi_N$. When bottleneck capacity and demand are perfectly positive correlated, $G^{ZB} \geq 0$. So we can conclude that the benefit gains from bottleneck capacity information $G^{ZB} \geq 0$ when $\pi_s < \pi_N$.

A.9. Proof of Proposition 5.

Part (a): If $t_e > t^*$, using Eq.(10) and Eq.(17)-(18),we can derive the expected benefit from demand information to zero information:

$$G^{ZD} = E[C^Z] - E[C^D] = \frac{(\alpha+\gamma)\beta}{\beta+\gamma}\left\{\int_{\hat{\theta}^*}^{\overline{\theta}} \theta_{\psi\omega} k(\theta_{\psi\omega})\mathrm{d}\theta_{\psi\omega} - \int_{\underline{N}}^{\overline{N}} N_\psi g(N_\psi)\int_{\underline{s}}^{\hat{s}^*} \frac{g(s_\omega|N_\psi)}{s_\omega}\mathrm{ds}_\omega \mathrm{dN}_\psi\right\} \tag{40}$$

where $g(N_\psi|s_\omega) = \frac{\partial}{\partial s_\omega}J(N_\psi, s_\omega)$, $k(\theta_{\psi\omega}) = \int_{\underline{s}}^{\overline{s}} s_\omega j(\theta_{\psi\omega}s_\omega, s_\omega)\mathrm{ds}_\omega$. When bottleneck capacity and demand are uncorrelated, $g(s_\omega|N_\psi) = f(s_\omega)$, $j(\theta_{\psi\omega}s_\omega, s_\omega) = g(\theta_{\psi\omega}s_\omega)f(s_\omega)$.

$$\begin{aligned} G^{ZD} &= \frac{(\alpha+\gamma)\beta}{\beta+\gamma}\left\{\int_{\hat{\theta}^*}^{\overline{\theta}} \theta_{\psi\omega}\int_{\underline{s}}^{\overline{s}} s_\omega g(\theta_{\psi\omega}s_\omega)f(s_\omega)\mathrm{ds}_\omega \mathrm{d}\theta_{\psi\omega} - \int_{\underline{s}}^{\hat{s}^*} N_\psi g(N_\psi)\int_{\underline{N}}^{\overline{N}} \frac{f(s_\omega)}{s_\omega}\mathrm{dN}_\psi \mathrm{ds}_\omega\right\} \\ &= \frac{(\alpha+\gamma)\beta}{\beta+\gamma}\left\{\int_{\underline{s}\hat{\theta}^*}^{\overline{N}}\int_{\underline{s}}^{\overline{s}} \frac{N_\psi g(N_\psi)f(s_\omega)}{s_\omega}\mathrm{ds}_\omega \mathrm{dN}_\psi - \int_{\underline{N}}^{\overline{N}}\int_{\underline{s}}^{\hat{s}^*} \frac{N_\psi g(N_\psi)f(s_\omega)}{s_\omega}\mathrm{ds}_\omega \mathrm{dN}_\psi\right\} \end{aligned} \tag{41}$$

where $\int_{\hat{\theta}^*}^{\overline{\theta}} g(N_\psi)f(s_\omega)\mathrm{d}\theta_{\psi\omega} = \int_{\overline{s}}^{\hat{s}^*} \mathrm{f(s}_\omega)\mathrm{ds}_\omega = \frac{\gamma}{\alpha+\gamma}$. Hence, $\underline{s}\hat{\theta}^* \leq \underline{N}$ and $\hat{s}^* \leq \overline{s}$, $G^{ZD} \geq 0$.

Part (b): When $\pi_s \leq \pi_N$ and $p_s > \frac{\alpha}{\alpha+\gamma}$, the benefit gains from demand information to zero information is $G^{ZD} = \frac{(\gamma+\alpha)\beta(1-p_s)}{s_B}\left\{\frac{(1-p_N)\rho_{LB}N_L + p_N\rho_{HB}N_H}{\beta+(\alpha+\gamma)(1-p_s)} - \frac{p_N\rho_{HB}N_H}{\beta+(\alpha+\gamma)\rho_{HB}(1-p_s)} - \frac{(1-p_N)\rho_{LB}N_L}{\beta+(\alpha+\gamma)\rho_{LB}(1-p_s)}\right\}$. If two conditional variables are uncorrelated($r = 0$), $G^{ZD} = 0$.

In the above, we find a special case when bottleneck capacity and demand both rarely experience drops that satisfies $G^{ZD} = 0$, the proposition is true. So we can conclude that providing demand information does not necessarily improve welfare when bottleneck capacity and demand are independent of each other($r = 0$). If bottleneck capacity rarely experience drops, providing demand information can be likely welfare-neutral (i.e,$G^{ZD} = 0$).

A.10. Proof of Proposition 6.

Part (a): If two conditional variables are moderately correlated and $\pi_N \geq \pi_s$. When p_s is large, the benefit gains from bottleneck information over zero information is

$$
\begin{aligned}
G^{ZD} &= \frac{(\gamma+\alpha)\beta(1-p_s)}{s_B}\left\{\frac{N_L+p_N\rho_{HB}(N_H-N_L)}{\beta+(\alpha+\gamma)(1-p_s)}-\frac{p_N\rho_{HB}N_H}{\beta+(\alpha+\gamma)\rho_{HB}(1-p_s)}\right\}\\
&-\frac{(\gamma+\alpha)\beta(1-p_s)}{s_B}\frac{(1-p_N)\rho_{LB}N_L}{\beta+(\alpha+\gamma)\rho_{LB}(1-p_s)}\\
\frac{\partial G^{ZD}}{\partial \rho_{HB}} &= \frac{(\gamma+\alpha)\beta(1-p_s)}{s_B}\left\{\frac{p_N(1-\pi_N)}{\beta+(\alpha+\gamma)(1-p_s)}-\frac{p_N\beta}{[\beta+(\alpha+\gamma)\rho_{HB}(1-p_s)]^2}\right\}\\
&-\frac{(\gamma+\alpha)\beta(1-p_s)}{s_B}\frac{\pi_N[(1-p_N)\beta+(\alpha+\gamma)(1-p_s)]}{[\beta+(\alpha+\gamma)\rho_{LB}(1-p_s)]^2}\\
&< \frac{(\gamma+\alpha)\beta(1-p_s)}{s_B}\left\{\frac{p_N(1-\pi_N)}{\beta+(\alpha+\gamma)(1-p_s)}-\frac{(1-\pi_N)p_N-\pi_N}{\beta}-\frac{\pi_N(\alpha+\gamma)(1-p_s)}{\beta^2}\right\}<0
\end{aligned}
\tag{42}
$$

If bottleneck capacity and demand are uncorrelated, $G^{ZD}=0$. So G^{ZD} has a value less than 0 when capacity and demand have a moderate positive correlation.

In the above, we can conclude that providing demand information is more likely to be welfare-reducing(i.e., $G^{ZD}<0$) when capacity and demand have a moderately correlation, $\pi_N \geq \pi_s$, and bottleneck capacity rarely experience drops.

Part (b): bottleneck capacity and demand are negatively correlated.

(1) When $\pi_s \geq \pi_N$, p_s is small and p_N is large,

$$
\begin{aligned}
G^{ZD} &= \frac{(1-p_N)\gamma\beta N_H}{(\gamma+\beta)s_B}-\frac{(1-p_N)\beta}{\gamma+\beta}\left[\frac{\gamma N_L}{s_G}+\frac{(\gamma+\alpha)\rho_{LB}(1-p_s)N_L(s_G-s_B)}{s_Bs_G}\right]\\
\frac{\partial G^{ZD}}{\partial \rho_{LB}} &= -\frac{(\gamma+\alpha)\beta}{\gamma+\beta}(1-p_s)(1-p_N)\left(\frac{N_L}{s_B}-\frac{N_L}{s_G}\right)<0,\qquad \frac{\partial \rho_{HB}}{\partial r}>0
\end{aligned}
\tag{43}
$$

(2) When $\pi_s \geq \pi_N$, p_s is large and p_N is small,

$$
\begin{aligned}
G^{ZD} &= \frac{(\alpha+\gamma)p_N[s_B+\rho_{HB}(1-p_s)(s_G-s_B)]N_H}{[\beta+(\alpha+\gamma)(1-p_s)]s_Bs_G}-\frac{\gamma\beta(1-p_N)N_L}{(\gamma+\beta)s_G}-p_N\frac{(\alpha+\gamma)(1-p_s)\rho_{HB}N_H}{[\beta+(\alpha+\gamma)(1-p_s)\rho_{HB}]s_B}\\
&-\frac{(\gamma+\alpha)\beta(1-p_s)(1-p_N\rho_{HB})}{\gamma+\beta}\left(\frac{N_L}{s_B}-\frac{N_L}{s_G}\right),\qquad \frac{\partial \rho_{HB}}{\partial r}<0\\
\frac{\partial G^{ZD}}{\partial \rho_{HB}} &= \frac{(\alpha+\gamma)p_N(1-p_s)}{s_B}\left\{\frac{(1-\pi_s)N_H}{[\beta+(\alpha+\gamma)(1-p_s)]}-\frac{\beta N_H}{[\beta+(\alpha+\gamma)(1-p_s)\rho_{HB}]^2}+\frac{\beta(1-\pi_s)N_L}{(\gamma+\beta)}\right\}>0
\end{aligned}
\tag{44}
$$

Therefore, the benefit gains from demand information increases as the value of the degree of correlation decreases when bottleneck capacity and demand are negatively correlated and $\pi_s \geq \pi_N$. When bottleneck capacity and demand are uncorrelated, $G^{ZD} \geq 0$. So we can conclude that the benefit gains from demand capacity information $G^{ZD} \geq 0$ when bottleneck capacity and demand are negative correlated ($r<0$) and $\pi_s \geq \pi_N$.

part(c): If two conditional variables have a moderate positive correlation and $\pi_N < \pi_s$. When p_N and p_s are small, the benefit gains from demand information to zero information is

$$G^{ZD} = \frac{(\gamma+\alpha)\beta p_N N_H[\rho_{HG} p_s \pi_s + \rho_{HB}(1-p_s)]}{[\beta+(\gamma+\alpha)(1-p_s)]s_B} - \frac{\gamma\beta p_N N_H}{(\gamma+\beta)s_B} - \frac{\gamma\beta(1-p_N)N_L}{(\gamma+\beta)s_G} \\ - \frac{(\gamma+\alpha)\beta\rho_{LB}(1-p_N)(1-p_s)N_L(s_G-s_B)}{(\gamma+\beta)s_B s_G} \tag{45}$$

If demand frequently experience degradation (i.e., $p_N < \frac{\pi_N[\gamma\pi_s+(\gamma+\alpha)(1-p_s)(1-\pi_s)]}{(\gamma+\alpha)(\gamma+\beta)\pi_s-\gamma(1-\pi_N)[\beta+(\gamma+\alpha)(1-p_s)]}$), $G^{ZD} < 0$. So we can conclude that providing demand information is more likely to be welfare-reducing(i.e., $G^{ZD} < 0$) when capacity and demand have a moderate positive correlation, $\pi_N < \pi_s$, and bottleneck capacity and demand both frequently experience drops.

A.11. Lemma 1 and its proof.
If $\int_a^x f(t)\mathrm{dt} \geq \int_\mathrm{a}^\mathrm{x} \mathrm{g(t)dt}, \mathrm{x} \in [\mathrm{a,b}]$ and $\int_a^b f(t)\mathrm{dt} \geq \int_\mathrm{a}^\mathrm{b} \mathrm{g(t)dt}$, so $\int_a^b xf(x)\mathrm{dx} \geq \int_\mathrm{a}^\mathrm{b} \mathrm{xg(x)dx}$
The proof: let $\phi(x) = f(x) - g(x)$, The two conditions given above become $\int_a^x \phi(t)\mathrm{dt} \geq 0$ and $\int_a^b \phi(t)\mathrm{dt} = 0$. Let $\phi(x) = \Phi'(x)$, we have $\int_a^b x\phi(x)\mathrm{dx} = -\mathrm{x}\Phi(\mathrm{x}) - \phi(\mathrm{x})|_\mathrm{a}^\mathrm{b} \leq 0$.

A.12. Proof of Proposition 7.
Part (a): Assume bottleneck capacity rarely experience degradation (i.e., p_s is large) and $\pi_N < \pi_s$.

(1) If bottleneck capacity and demand have a negative correlation, the benefit gains in shifting from bottleneck information to demand information is

$$G^{BD} = \frac{\gamma\beta[N_H-(1-p_N)N_L]}{(\gamma+\beta)s_G} - \frac{p_N\gamma\beta N_H}{(\gamma+\beta)s_B} - \frac{(\gamma+\alpha)\beta[(1-p_N)-P(\theta_{LG})]}{\gamma+\beta}\frac{N_L}{s_B} \\ + \frac{(\gamma+\alpha)\beta[(1+p_s-p_N)N_L+p_sN_H-(2N_L+N_H)P(\theta_{LG})]}{(\gamma+\beta)s_G} \tag{46}$$

If $G^{BD} < 0$, p_N needs to satisfy the condition: $p_N \leq \frac{(1+\pi_s)\pi_N\rho_{LB}-(1+\pi_N)\pi_s(1-\rho_{LB})}{(1+\pi_s)\pi_N\rho_{LB}+(1+\pi_N)\pi_s\rho_{LB}}$.

(2) If bottleneck capacity and demand have a positive correlation, the benefit gains from providing bottleneck information over providing demand information is

$$G^{BD} = \frac{\gamma\beta[(p_s-p_N)\pi_sN_H-(1-p_N)N_L]}{(\gamma+\beta)s_B} + \frac{(\alpha+\gamma)\beta P(\theta_{HB})N_H}{(\gamma+\beta)s_B}[\frac{\gamma-(\alpha+\gamma)\rho_{HB}p_N}{\beta+(\alpha+\gamma)\rho_{HB}p_N} + \pi_s] \tag{47}$$

If $G^{BD} < 0$, p_N needs to satisfy the condition: $p_N \geq \frac{\gamma+\pi_s\beta}{(1-\pi_s)(\alpha+\gamma)\rho_{HB}}$.

So we can conclude that demand information can be more valuable (i.e., $G^{BD} > 0$) when the amplitude of bottleneck capacity drop is less than the amplitude of demand drop (i.e.,$\pi_N < \pi_s$) and bottleneck capacity rarely experience drops.
Part (b): Assume the amplitude of bottleneck capacity drop is larger than the amplitude of demand drop (i.e., $\pi_N > \pi_s$). The benefit gains from providing bottleneck information over

providing demand information are

$$G^{BD}=\frac{(\alpha+\gamma)\beta}{\gamma+\beta}\int_{\underline{s}}^{\overline{s}}\frac{f(s_\omega)}{s_\omega}\int_{\hat{N}^*}^{\overline{N}}N_\psi f(N_\psi|s_\omega)\mathrm{dN}_\psi\mathrm{ds}_\omega-\int_{\underline{N}}^{\overline{N}}g(N_\psi)N_\psi\int_{\underline{s}}^{\hat{s}^*}\frac{g(s_\omega|N_\psi)}{s_\omega}\mathrm{ds}_\omega\mathrm{dN}_\psi \tag{48}$$

Let $u=s_\omega/\overline{s}$ and $v=N_\psi/\overline{N}$, we assume $\hat{u}^*=\hat{s}^*/\overline{s}$ and $\hat{v}^*=\hat{N}^*/\overline{N}$. we can get it by substituting the definite integral:

$$\begin{aligned}G^{BD}&=\frac{(\alpha+\gamma)\beta\overline{N}^2}{\gamma+\beta}\left\{\int_{\hat{v}^*}^{1}vf(v\overline{N}|u\overline{s})\int_{\pi_s}^{1}\frac{f(u\overline{s})}{u}\mathrm{dudv}-\int_{\pi_N}^{1}vg(v\overline{N})\int_{\pi_s}^{\hat{u}^*}\frac{g(u\overline{s}|v\overline{N})}{u}\mathrm{dudv}\right\}\\&=\frac{(\alpha+\gamma)\beta\overline{N}^2}{\gamma+\beta}\left\{\int_{\hat{v}^*}^{1}vf(v\overline{N}|u\overline{s})\int_{\pi_s}^{1}\frac{f(u\overline{s})}{u}\mathrm{dudv}-\int_{\pi_N}^{1}vg(v\overline{N})\int_{\pi_s}^{1}\frac{g(u\overline{s}|v\overline{N})}{u}\mathrm{dudv}\right\}\\&\quad+\frac{(\alpha+\gamma)\beta\overline{N}^2}{\gamma+\beta}\int_{\pi_N}^{1}vg(v\overline{N})\int_{\hat{u}^*}^{1}\frac{g(u\overline{s}|v\overline{N})}{u}\mathrm{dudv}\end{aligned} \tag{49}$$

By lemma 1 in the Appendix A.11, $f(v\overline{N}|u\overline{s})\geq g(v\overline{N})$ and $g(u\overline{s}|v\overline{N})\geq f(u\overline{s})$, we have $\int_a^b vf(v\overline{N}|u\overline{s})\leq\int_a^b vg(v\overline{N})$ and $\int_a^b g(u\overline{s}|v\overline{N})/u\geq\int_a^b f(u\overline{s})/u$. It follows that:

$$\begin{aligned}G^{BD}\leq&\frac{(\alpha+\gamma)\beta\overline{N}^2}{\gamma+\beta}\int_{\pi_s}^{1}\frac{f(u\overline{s})}{u}\left\{\int_{\hat{v}^*}^{1}vf(v\overline{N}|u\overline{s})-\int_{\pi_N}^{1}vg(v\overline{N})\mathrm{dv}\right\}\mathrm{du}\\&+\frac{(\alpha+\gamma)\beta\overline{N}^2}{\gamma+\beta}\int_{\pi_N}^{1}vg(v\overline{N})\int_{\hat{u}^*}^{1}\frac{g(u\overline{s}|v\overline{N})}{u}\mathrm{dudv}\end{aligned} \tag{50}$$

So we can conclude that when providing bottleneck capacity information is more valuable than providing demand information when the amplitude of bottleneck capacity drop is larger than the amplitude of demand drop (i.e.,$\pi_N>\pi_s$).

A.13. Proof of Proposition 8.

Part (a): (From demand information to full information)

If $t_e>t^*$, the expected benefit gains from providing full information over demand information are

$$\begin{aligned}G^{DF}&=\frac{(\alpha+\gamma)\beta}{\beta+\gamma}\int_{\underline{N}}^{\overline{N}}N_\psi g(N_\psi)\int_{\underline{s}}^{\hat{s}^*}\frac{g(s_\omega|N_\psi)}{s_\omega}\mathrm{ds}_\omega\mathrm{dN}_\psi-\frac{\gamma\beta}{\beta+\gamma}\int_{\underline{\theta}}^{\overline{\theta}}\theta_{\psi\omega}k(\theta_{\psi\omega})\mathrm{d}\theta_{\psi\omega}\\&\geq\frac{\alpha\beta}{\beta+\gamma}\int_{\underline{N}}^{\overline{N}}N_\psi\int_{\underline{s}}^{\hat{s}^*}\frac{j(N_\psi,s_\omega)(\hat{s}^*-s_\omega)}{\hat{s}^*s_\omega}\mathrm{ds}_\omega\mathrm{dN}_\psi+\frac{\gamma\beta}{\beta+\gamma}\int_{\underline{N}}^{\overline{N}}N_\psi\int_{\hat{s}^*}^{\overline{s}}\frac{j(N_\psi,s_\omega)(s_\omega-\hat{s}^*)}{\hat{s}^*s_\omega}\mathrm{ds}_\omega\mathrm{dN}_\psi\end{aligned} \tag{51}$$

where $g(s_\omega|N_\psi)=\frac{\partial}{\partial s_\omega}J(N_\psi,s_\omega)$, $k(\theta_{\psi\omega})=\int_{\underline{s}}^{\overline{s}}s_\omega j(\theta_{\psi\omega}s_\omega,s_\omega)\mathrm{ds}_\omega$.

If $t_e=t^*$, the expected benefit gains from providing full information over demand information are

$$
\begin{aligned}
G^{DF} &= \beta\hat{\theta}^{**} - \frac{\gamma\beta}{\beta+\gamma}\int_{\underline{\theta}}^{\overline{\theta}} \theta_{\psi\omega} k(\theta_{\psi\omega})\mathrm{d}\theta_{\psi\omega} \\
&> \frac{(\alpha+\gamma)\beta}{\beta+\gamma}\int_{\underline{N}}^{\overline{N}} N_\psi g(N_\psi)\int_{\underline{s}}^{\hat{s}^*} \frac{g(s_\omega|N_\psi)}{s_\omega}\mathrm{ds}_\omega \mathrm{dN}_\psi - \frac{\gamma\beta}{\beta+\gamma}\int_{\underline{\theta}}^{\overline{\theta}} \theta_{\psi\omega} k(\theta_{\psi\omega})\mathrm{d}\theta_{\psi\omega} > 0
\end{aligned}
\tag{52}
$$

(From bottleneck information to full information):

If $t_e > t^*$, the expected benefit from bottleneck information to full information is

$$
\begin{aligned}
G^{BF} &= \frac{(\alpha+\gamma)\beta}{\beta+\gamma}\int_{\underline{s}}^{\overline{s}} \frac{f(s_\omega)}{s_\omega}\int_{\hat{N}^*}^{\overline{N}} f(N_\psi|s_\omega) N_\psi \mathrm{dN}_\psi \mathrm{ds}_\omega - \frac{\gamma\beta}{\beta+\gamma}\int_{\underline{\theta}}^{\overline{\theta}} \theta_{\psi\omega} k(\theta_{\psi\omega})\mathrm{d}\theta_{\psi\omega} \\
&\geq \frac{\alpha\beta}{\beta+\gamma}\int_{\underline{s}}^{\overline{s}} \frac{1}{s_\omega}\int_{\hat{N}^*}^{\overline{N}} j(N_\psi, s_\omega)(N_\psi - \hat{N}^*)\mathrm{dN}_\psi \mathrm{ds}_\omega + \frac{\gamma\beta}{\beta+\gamma}\int_{\underline{s}}^{\overline{s}} \frac{1}{s_\omega}\int_{\underline{N}}^{\hat{N}^*} j(N_\psi, s_\omega)(\hat{N}^* - N_\psi)\mathrm{dN}_\psi \mathrm{ds}_\omega
\end{aligned}
\tag{53}
$$

where $f(N_\psi|s_\omega) = \frac{\partial}{\partial N_\psi} J(N_\psi, s_\omega)$, $k(\theta_{\psi\omega}) = \int_{\underline{s}}^{\overline{s}} s_\omega j(\theta_{\psi\omega}s_\omega, s_\omega)\mathrm{ds}_\omega$.

If $t_e = t^*$, the expected benefit from bottleneck information to full information is

$$
\begin{aligned}
G^{BF} &= \beta\hat{\theta}^{**} - \frac{\gamma\beta}{\beta+\gamma}\int_{\underline{\theta}}^{\overline{\theta}} \theta_{\psi\omega} k(\theta_{\psi\omega})\mathrm{d}\theta_{\psi\omega} \\
&> \frac{(\alpha+\gamma)\beta}{\beta+\gamma}\int_{\underline{s}}^{\overline{s}} \frac{f(s_\omega)}{s_\omega}\int_{\hat{N}^*}^{\overline{N}} f(N_\psi|s_\omega) N_\psi \mathrm{dN}_\psi \mathrm{ds}_\omega - \frac{\gamma\beta}{\beta+\gamma}\int_{\underline{\theta}}^{\overline{\theta}} \theta_{\psi\omega} k(\theta_{\psi\omega})\mathrm{d}\theta_{\psi\omega} > 0
\end{aligned}
\tag{54}
$$

Hence $G^{DF} \geq 0$ and $G^{BF} \geq 0$, we have providing full information does not increase travel costs compared to providing partial information.

Part (b):Take the derivative of G^{BF} and G^{DF} with respect to α:

$$
\begin{aligned}
\frac{\partial G^{BF}}{\partial \alpha} &= \frac{\beta}{\beta+\gamma}\int_{\underline{s}}^{\overline{s}} \frac{f(s_\omega)}{s_\omega}\int_{\hat{N}^*}^{\overline{N}} f(N_\psi|s_\omega) N_\psi \mathrm{dN}_\psi \mathrm{ds}_\omega > 0 \\
\frac{\partial G^{DF}}{\partial \alpha} &= \frac{\beta}{\beta+\gamma}\int_{\underline{N}}^{\overline{N}} N_\psi g(N_\psi)\int_{\underline{s}}^{\hat{s}^*} \frac{g(s_\omega|N_\psi)}{s_\omega}\mathrm{ds}_\omega \mathrm{dN}_\psi > 0
\end{aligned}
\tag{55}
$$

So we can conclude that the benefit gains from full information as compared to providing only partial information are a increasing function of α.

A.14. The benefit gains G^{ZF} from complete information for different p_s, p_N, r and π_N when demand and bottleneck capacity follow the Bernoulli distribution

As shown in Fig. 13, the welfare gains G^{ZF} are not necessarily monotonic with respect to the correlation coefficient r or the amplitude of demand reduction π_N. When both bottleneck capacity and demand are stochastic and correlated, the relation between G^{ZF} and r becomes significantly more complex.

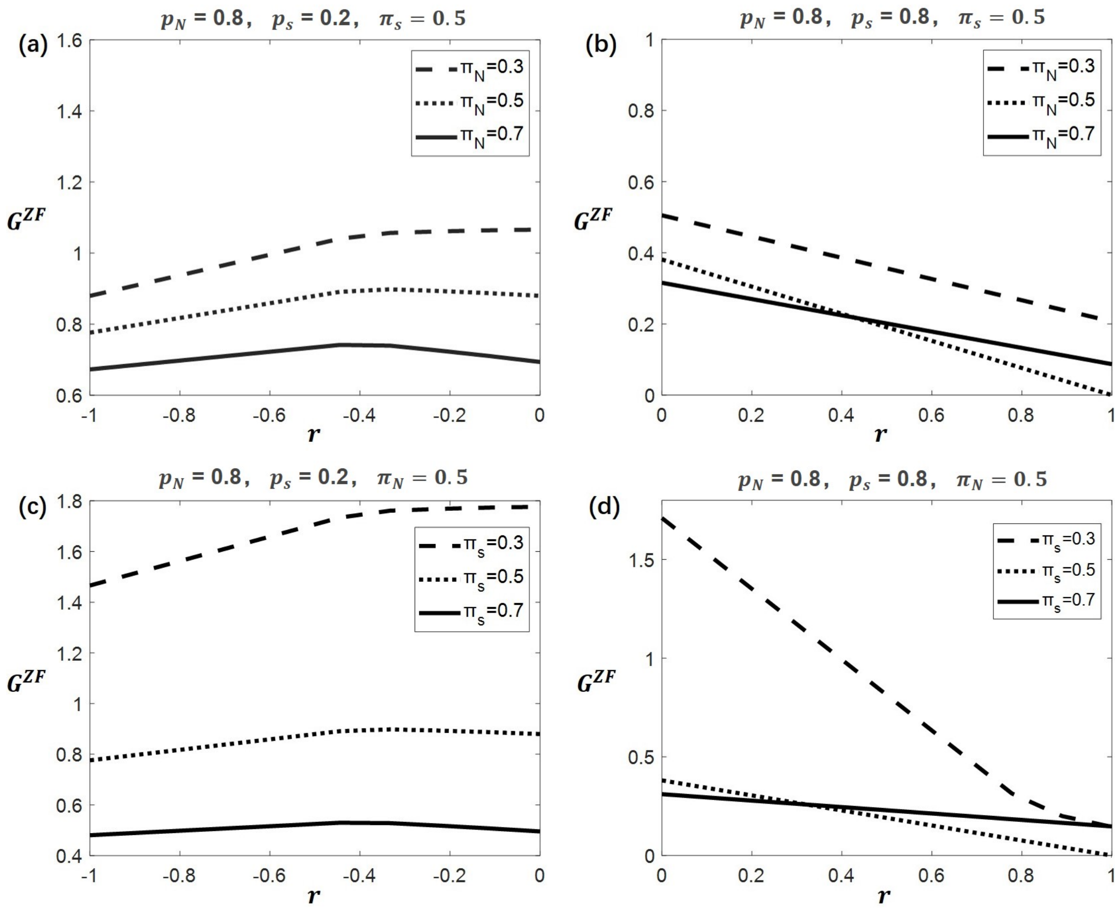

Figure 13: Benefit gains from providing full information over zero information under varying r for different p_s, p_N, π_s and π_N.

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