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Model-Independent Test of Prerecombination New Physics: Measuring the Sound Horizon with Gravitational Wave Standard Sirens and the Baryon Acoustic Oscillation Angular Scale

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In a broad class of cosmological models where spacetime is described by a pseudo-Riemannian manifold, photons propagate along null geodesics, and their number is conserved, upcoming gravitational wave (GW) observations can be combined with measurements of the baryon acoustic oscillation (BAO) angular scale to provide model-independent estimates of the sound horizon at the baryon drag epoch. By focusing on the accuracy expected from forthcoming surveys such as the Laser Interferometer Space Antenna GW standard sirens and dark energy spectroscopic instrument (DESI) or Euclid angular BAO measurements, we forecast a relative precision of $\sigma_{r_\mathrm{d}}/r_\mathrm{d} \sim 1.5\%$ within the redshift range $z \lesssim 1$. This approach will offer a unique model-independent measure of a fundamental scale characterizing the early universe, which is competitive with model-dependent values inferred within specific theoretical frameworks. These measurements can serve as a consistency test for ΛCDM, potentially clarifying the nature of the Hubble tension and confirming or ruling out new physics prior to recombination with a statistical significance of $\sim 4\sigma$.

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Introduction—The disparity between the present-day expansion rate of the Universe—quantified by the Hubble parameter H_0—as determined by the SH0ES collaboration using type Ia supernovae ($H_0 = 73 \pm 1$ km/s/Mpc) [1–3], and inferred by the Planck Collaboration from measurements of the cosmic microwave background (CMB) temperature and polarization anisotropy angular power spectra, assuming a standard ΛCDM model of cosmology (67.04 ± 0.5 km/s/Mpc) [4], has been regarded with significant attention by the cosmology community.

Barring any potential systematic origin of the discrepancy [5], the Hubble tension might well stand as compelling evidence for the necessity of new physics beyond ΛCDM. While not aiming to offer a comprehensive overview of the debate that has heated up the community in the past few years [8–14], it is fair to say that attempts to tackle this issue have predominantly clustered around two distinct approaches: early time and late-time solutions. Broadly speaking, the former category entails proposals suggesting new physics acting prior to recombination, while the latter comprises models that seek to modify the expansion history of the Universe after recombination.

This dichotomy in approach stems from the highly precisely determined angular scale of the acoustic peaks in the CMB spectra, θ_s [4], which sets the ratio between the sound horizon at recombination $r_s(z_*)$ and the angular diameter distance to the last scattering surface $D_A(z_*)$. Increasing the value of H_0 without disrupting the acoustic scale requires either a reduction in the value of the sound horizon—a core tenet of early time solutions—or a distinct postrecombination expansion history of the Universe capable of compensating for a higher H_0 while preserving the angular diameter distance to the last scattering surface [15].

Despite the apparent simplicity of the goals characterizing these two approaches, neither has proven effective in resolving the H_0 tension thus far, leaving the problem widely open. The primary challenge in the current landscape of solutions lies in the fact that both early and late-time observations tightly constrain new physics at their respective cosmic epochs, posing significant hurdles for model building. Early time solutions typically act near the last scattering surface, a cosmic epoch severely constrained by CMB measurements. Consequently, a common issue with early time solutions is the need for a moderate level of fine-tuning to maintain a good fit to the CMB data, making it difficult to increase H_0 enough to match the SH0ES results [16–19]. On the other hand, late-time solutions are well constrained by observations of the local Universe, such as baryon acoustic oscillations (BAO) and supernovae

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(SN), which generally favor a ΛCDM-like cosmology at low redshifts [20–24].

Given the difficulties in constructing successful models, it seems natural to step back and consider independent methods to test signals of new physics without relying on any particular framework. For late-time new physics, this is typically achieved through model-independent reconstructions of available datasets using machine learning (ML) techniques such as artificial neural networks (ANN) or Gaussian processes (GPs). These approaches significantly limit theoretical assumptions in data analysis. Conversely, the state-of-the-art observational constraints on the early universe—particularly at the time of recombination—are largely dependent on the cosmological model assumed to analyze Planck CMB measurements. Therefore, a robust assessment of a fundamental physical quantity characterizing the early universe, not contingent on specific models, will certainly represent an important step forward for testing new physics before recombination and clarifying, once and for all, the intricate debate surrounding early and late-time solutions.

In this Letter, we demonstrate that geometric distance measurements in the local Universe can be used to independently measure the sound horizon. By using forecast measurements of gravitational waves (GWs), standard sirens from future surveys such as LISA [37,38] for GWs, in conjunction with the angular scale of baryon acoustic oscillations (BAOs) anticipated by ongoing and forthcoming experiments such as DESI [39,40] and Euclid [41–43], we argue that it is possible to extrapolate the value of the sound horizon at the baryon drag epoch with a relative uncertainty of ∼1.5%, making no assumption about the cosmological model. Since the sound horizon encapsulates information about the Universe's expansion history from (soon after) the hot Big Bang singularity all the way up to recombination, this estimate can gauge early time solutions, potentially confirming or ruling out new physics beyond ΛCDM with a statistical significance approaching four standard deviations.

Methodology—To obtain a model-independent estimate of the sound horizon at the baryon drag epoch [44], we propose using the very simple relationship that links r_d and the angular scale of baryon acoustic oscillations:

$$\theta_\mathrm{BAO}(z) = \frac{r_\mathrm{d}}{(1+z)D_\mathrm{A}(z)} = \frac{(1+z)r_\mathrm{d}}{D_\mathrm{L}(z)}, \tag{1}$$

where, in the second equality, we have assumed the distance duality relation (DDR), $D_\mathrm{L}(z) = (1+z)^2 D_\mathrm{A}(z)$, which connects the luminosity distance $D_\mathrm{L}(z)$ to the angular diameter distance $D_\mathrm{A}(z)$ at any redshift z [47].

Starting from this relation, we can isolate r_d and express it in terms of $\theta_\mathrm{BAO}(z)$ and $D_\mathrm{L}(z)$. In principle, both of these quantities are directly measurable from current and future probes. So the next step involves identifying the most suitable datasets for estimating them. (i) Acquiring a dataset capable of providing standard candles for measuring the luminosity distance, $D_\mathrm{L}(z)$, independently of any calibration methodologies or cosmological model assumptions, requires careful consideration. Although type Ia Supernovae serve as "standardizable" candles, they require calibration. Recent discussions have emphasized that the best achievable inference from supernovae data is the quantity $r_\mathrm{d}h$ (where $H_0 = 100h$ km/s/Mpc) [50], which does not provide sufficient information to address the nature of the Hubble tension and test new physics prior to recombination. Therefore, we suggest leveraging future gravitational wave observations as standard sirens to estimate $D_\mathrm{L}^\mathrm{GW}(z_\mathrm{GW})$. Planned experiments such as the Einstein Telescope (ET) and the Laser Interferometer Space Antenna (LISA) are expected to accurately measure the luminosity distance across a wide redshift range $z_\mathrm{GW} \in [z_\mathrm{GW}^\mathrm{min}, z_\mathrm{GW}^\mathrm{max}]$ that, depending on the specific experiment, can range from $z_\mathrm{GW}^\mathrm{min} \sim 0.1$ up to $z_\mathrm{GW}^\mathrm{max} \sim 8$ [51]. For both ET and LISA, gravitational wave standard sirens can achieve a few-percent precision on $D_\mathrm{L}(z)$ at $z \lesssim 0.5$. For LISA, favorable source orientations can yield relative uncertainties as low as ∼1–2%, increasing up to ≳10–30% at higher redshifts or for less favorable orientations. For ET, relative uncertainties at low redshift are typically around 10%, with few-percent precision achievable depending on redshift and inclination. However, they grow to ∼20–30% at higher redshifts due to weak lensing and detector limitations. For more discussions we refer to Supplemental Material [52]. (ii) Measurements of the angular scales of BAOs have been released by several independent groups, extracting $\theta_\mathrm{BAO}(z)$ from diverse catalogues, including those provided by the Baryon Oscillation Spectroscopic Survey (BOSS) and extended Baryon Oscillation Spectroscopic Survey (eBOSS). surveys [80–82]. However, ongoing and forthcoming large-scale structure experiments, such as Euclid and DESI, are poised to significantly enhance the precision in estimating $\theta_\mathrm{BAO}(z)$ across a redshift window $z_\mathrm{BAO} \in [z_\mathrm{BAO}^\mathrm{min}, z_\mathrm{BAO}^\mathrm{max}]$, spanning from $z_\mathrm{BAO}^\mathrm{min} \sim 0.1$ to $z_\mathrm{BAO}^\mathrm{max} \sim 2$, contingent upon the specifics of the experiment [83]. To avoid mixing up current and forecast datasets, we focus on the anticipated improvements brought forth by DESI and Euclid-like experiments on $\theta_\mathrm{BAO}(z)$, generating mock datasets for both surveys by following the methodology outlined in the Supplemental Material [52]. Note that, in both cases, the expected relative precision on $\theta_\mathrm{BAO}(z)$ falls in the range of 1–4%, depending on the redshift bin.

Exploring various combinations of mock datasets from future BAO and GW surveys, we aim to forecast the precision achievable in determining r_d independently of any specific cosmological model. One significant challenge underlying the analysis arises from the fact that gravitational wave and BAO measurements will collect data at different redshifts, $z_\mathrm{GW} \neq z_\mathrm{BAO}$. To overcome this difficulty, we propose employing ML regression techniques, specifically ANN. As outlined in Supplemental Material [52], ANN can be trained to reconstruct the luminosity distance function, $D_L^{GW}(z)$, using gravitational wave observations $D_L^{GW}(z_{GW})$. Subsequently, these trained ANNs can extrapolate the luminosity distance to the same redshifts as BAO measurements: $D_L^{GW}(z_{BAO})$ [84]. In this way, we can obtain several independent estimates of r_d—each corresponding to a BAO measurement:

$$r_d = \frac{\theta_{BAO}(z_{BAO}) D_L^{GW}(z_{BAO})}{1 + z_{BAO}}. \quad (2)$$

Based on the specifications of the various experiments under consideration, we identify a redshift range for each combination of datasets wherein both uncertainties and systematic errors are well under control. Sticking to these redshift ranges, we extract conservative yet informative estimates regarding the forecast precision of our measurement of the sound horizon, σ_{r_d}. For further technical details on this matter, we refer to Supplemental Material [52].

Results—In Table I, we summarize the relative precision we forecast on the sound horizon by combining measurements of gravitational waves and baryon acoustic oscillations from various surveys at different redshifts. To obtain these results, we assume conservative yet realistic forecasts for the number of gravitational wave events with electromagnetic counterparts detectable by future observatories. For LISA, we follow the projections of Ref. [85] under the N2A5M5L6 configuration, which account for the expected capabilities of EM follow-up facilities. For the Einstein Telescope, we conservatively assume $\sim 10^3$ binary neutron star events with identified EM counterparts over several years of observation, consistent with cautious estimates in the literature [86]. For a more detailed discussion of our setup, as well as an extensive set of consistency tests and analyses covering different configurations and event numbers, we refer to Supplemental Material [52].

At first glance, we note that when dealing with combinations involving ET, the uncertainties are quite substantial. For ET + Euclid, the percentage relative precision σ_{r_d}/r_d consistently exceeds 10%, reaching nearly 20% at $z \sim 1$. These significant uncertainties arise from the fact that Euclid will be able to collect BAO measurements primarily at high redshifts $z \gtrsim 0.6$, where both the error bars of ET and the scatter of the mock data around the fiducial cosmology notably increase. This leads to broader uncertainty in the ML reconstruction of $D_L^{GW}(z)$ and, consequently, in the inferred value of r_d. On the other hand, combining DESI and ET allows for a significant reduction in uncertainties. This is because DESI is able to gather BAO measurements at lower redshifts where ET will observe more GW events with higher precision, resulting in a more accurate reconstruction of $D_L^{GW}(z)$. However, even with this overall improvement, the most optimistic estimate still yields $\sigma_{r_d}/r_d \gtrsim 4\%$, which remains comparable to the changes needed in r_d to resolve the Hubble tension [87]. Moreover, uncertainties increase rapidly within the redshift range $z \sim 0.25$–0.55, far exceeding $\sigma_{r_d}/r_d \gtrsim 10\%$. Therefore, the primary lesson we can glean is that the combinations of datasets involving ET are not ideal for achieving an informative model-independent measurement of r_d. Such an estimate would carry uncertainties so significant that it would be practically unusable for assessing new physics at early times.

Looking at the brighter side, a substantial improvement in constraining the value of the sound horizon occurs when we focus on LISA. Despite the fewer numbers of GW events expected from this survey, the error bars will be significantly reduced compared to ET, leading to a more precise reconstruction of the luminosity distance across the redshifts probed by DESI and Euclid. In this case, the constraining power achieved by combining LISA with either DESI or Euclid is similar: in both cases, we can select a notable redshift window ($z \lesssim 1$) where the uncertainties in the inferred value of the sound horizon remain $\sigma_{r_d}/r_d \lesssim 2\%$. In the most optimistic scenario, the uncertainties can be as small as $\sigma_{r_d}/r_d \lesssim 1.5\%$.

To demonstrate that a model-independent estimate of the sound horizon with a precision $\sigma_{r_d}/r_d \sim 1.5\%$ could decisively clarify the debate on early and late-time solutions while confirming or ruling out new physics beyond the standard cosmological model with high statistical significance, we propose the following conceptual exercise as a proof of concept. First, we suppose that the Hubble

TABLE I. Forecasted relative precision σ_{r_d}/r_d on the sound horizon, derived from gravitational wave and baryon acoustic oscillation angular scale measurements by different combinations of surveys including ET and LISA for GWs, and DESI and Euclid for BAO.

Redshift	DESI + ET	DESI + LISA	EUCLID + ET	EUCLID + LISA
0.15	4.3%	3.5%	⋯	⋯
0.25	7.5%	2.2%	⋯	⋯
0.35	9.7%	1.7%	⋯	⋯
0.45	11.5%	1.5%	⋯	⋯
0.55	13.1%	1.4%	⋯	⋯
0.65	14.8%	1.5%	14.2%	1.6%
0.75	16.2%	1.6%	15.5%	1.5%
0.85	17.5%	1.8%	16.9%	1.7%
0.95	18.6%	2.0%	18.2%	1.9%
1.05	19.6%	2.2%	19.5%	2.3%
1.15	20.4%	2.5%	20.6%	2.7%
1.25	21.2%	2.7%	21.5%	3.1%
1.35	21.8%	2.9%	22.2%	3.4%
1.45	22.4%	3.2%	22.6%	3.8%
1.55	22.9%	3.6%	22.8%	4.2%
1.65	23.5%	4.2%	23.0%	4.8%
1.75	24.0%	5.2%	23.2%	5.8%
1.85	24.6%	6.2%	23.3%	5.4%
1.95	⋯	⋯	23.5%	6.0%
2.05	⋯	⋯	24.12%	7.9%

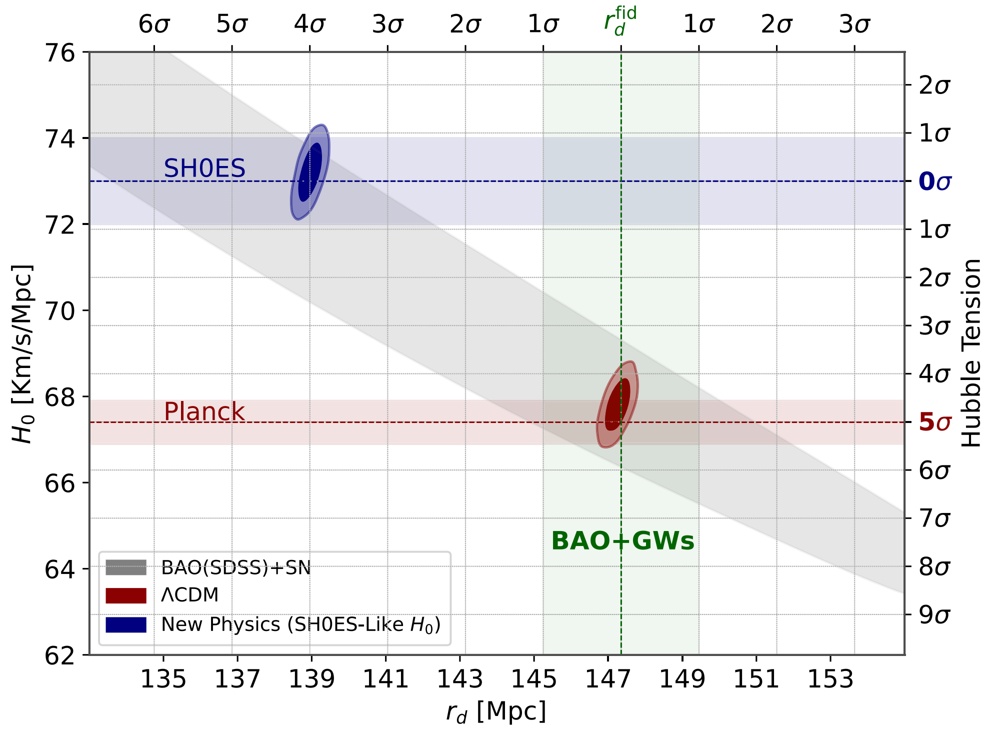

FIG. 1. Illustrative plot in the $r_{\rm d}$-H_0 plane of the consistency test proposed to assess the possibility of new physics prior to recombination for solving the Hubble constant tension. The red band represents the present value of H_0 measured by the Planck collaboration within a standard ΛCDM model of cosmology, whereas the 2D contours represent the marginalized 68% and 95% CL constraints obtained from the Planck-2018 data. The gray band represents the 95% CL region of the plane identified by analyzing current BAO measurements from the SDSS collaboration and type Ia supernovae from the Pantheon + catalogue. The horizontal blue band represents the value of the Hubble constant measured by the SH0ES collaboration. In order to reconcile all the datasets, a potential model of early time new physics should shift the ΛCDM red contours along the gray band until the gray band overlaps with the SH0ES result. This scenario is depicted by the 2D blue contours obtained under the assumption that the model of new physics does not increase uncertainties on parameters compared to ΛCDM. The green vertical band represents the model-independent value of the sound horizon we are able to extract from combinations of GW data from LISA and BAO measurements (either from DESI-like or Euclid-like experiments) assuming a fiducial ΛCDM baseline cosmology. As is clear from the top x axis, this value would be able to confirm or rule out the possibility of new physics at about 4σ.

tension could indeed indicate the presence of new physics beyond the standard cosmological model. Second, we suppose the existence of an effective realization of early time new physics—though currently unknown—that can reconcile the discrepancy. While remaining agnostic about the specific solution, we can outline the characteristics it must satisfy: (1) Maintaining consistency with BAO and SN: The solution must be consistent with all current cosmological datasets, including measurements of BAO and SN. As discussed in the literature [15], the combination of SN and BAO data defines a band in the $r_{\rm d}$-H_0 plane. In Fig. 1, we show in gray the 95% confidence level (CL) region of the parameter space defined by combining the SN gathered from the Pantheon Plus sample with BAO measurements released by the SDSS collaboration. Maintaining consistency with SN and BAO requires moving along this gray band. (2) Consistency with SH0ES: The solution must provide an H_0 value in agreement with the current local distance ladder estimate reported by the SH0ES collaboration, represented by the blue horizontal band in Fig. 1. Therefore, as we move along the gray band, we must intersect the blue band. A viable solution should lie at the intersection of the blue and gray bands in Fig. 1, implying a significant reduction in the sound horizon, as has been argued in the literature. (3) Genuine solution: The solution must fit well with CMB data and genuinely resolve the tension by shifting the CMB contours for ΛCDM in the direction indicated by other datasets, ideally maintaining uncertainties in the Bayesian inference of cosmological parameters comparable to that achieved within the standard ΛCDM scenario.

By imposing these three conditions, we can anticipate where the 2D probability contours that any hypothetical effective solution to the Hubble tension should place in the $r_{\rm d}$-H_0 plane. Our ideal scenario is depicted in Fig. 1 with the blue 2D contours for the model that could resolve the Hubble tension. In the figure, we remain somewhat conservative and impose that the blue contours lie as close as possible to those obtained from the standard cosmological model—that is, we consider the *smallest* possible reduction in the value of the sound horizon that can fully satisfy these requirements [89]. Starting from this ideal scenario, we pose the following questions: supposing a theoretical physicist somewhere in the world, perhaps in the (near) future, realizes the solution represented by the blue contours in Fig. 1(i) is it possible to independently test this solution using the methodology introduced in this Letter, and (ii) to what level of significance can we confirm or reject the hypothesis of new physics?

To address our questions, we can refer back to Fig. 1, where we illustrate the forecast obtained by assuming a standard ΛCDM cosmological model for the sound horizon, derived from a combination of data involving LISA, with a relative uncertainty of 1.5%. As evident from the figure, a simple independent estimate of the sound horizon would represent a remarkable consistency test for ΛCDM, dismissing the hypothesis of new physics at a statistical significance exceeding four standard deviations (refer to the x axis at the top of the figure). At this point, only two possibilities would remain on the table: the first would be to assume that the Hubble tension is due to systematics. This possibility, in turn, can be tested with gravitational waves as standard sirens, which will precisely measure H_0 [90,91], confirming or rejecting the measurement obtained from SH0ES. If the Hubble tension were confirmed, then this would necessarily imply the need to resort to a different physical mechanism than the one proposed by early time solutions. In our ideal scenario to resolve the problem, we will need to shift the red contours in Fig. 1 vertically and force the cosmological model to lie at the intersection between the green and blue horizontal bands. This possibility would be in strong tension with the gray contours derived from current SN and BAO measurements; thus, it appears less plausible.

On the flip side, it is worth noting that this argument works in reverse, too. If the hypothetical theoretical physicist happens to have stumbled upon the correct solution, our test would yield a model-independent measurement of the sound horizon consistent with that predicted by the model of new physics, thereby ruling out the value inferred within the standard cosmological framework at four standard deviations.

In closing, we would like to stress two key remarks. First, as shown in Supplemental Material [52], although the relative precision on r_d clearly depends on the number of gravitational wave detections, our methodology remains robust even under pessimistic assumptions: with as few as 7–10 low-redshift ($z < 0.5$) detections from LISA, the sound horizon can be measured at the 1.4–2.6% level, while with only 4–6 detections, we achieve 2–3.5% precision, demonstrating the resilience of the approach in low-statistics regimes. Second—and most importantly—the significance of our test lies in the fact that gravitational wave standard sirens will be able to extract model-independent measurements of the sound horizon with a precision competitive with current model-dependent estimates from early Universe probes. This will allow us to test whether the value of the sound horizon is consistent with those predicted by the standard cosmological model, providing an important tool to shed light on the nature of the Hubble tension. Among other things, this will clarify once and for all the debate on early vs late-time solutions and set clear model-building guidelines for assessing possible new physics beyond ΛCDM.

Conclusion—In this Letter, we propose a method for measuring the sound horizon at the baryon drag epoch, $r_{\rm d}$, independently of the cosmological model. Our methodology is based on the relationship linking the angular scale of baryon acoustic oscillations and the angular diameter distance. Assuming that spacetime is described by a pseudo-Riemannian manifold, photons propagate along null geodesics, and their number is conserved over time, we can use the distance duality relation to connect $\theta_{\rm BAO}(z)$ and $D_{\rm L}(z)$ by means of Eq. (1). Starting from this relation, we propose using future gravitational waves as standard sirens to gather precise measurements of the luminosity distance in the Universe, which are free from calibration methods compared to current "standardizable" candles such as type Ia supernovae.

We argue that by employing machine learning techniques, particularly artificial neural network linear regression, one can combine future gravitational wave observations from LISA with angular BAO measurements, either from DESI-like or Euclid-like surveys, enabling us to obtain model-independent estimates of the sound horizon. In the redshift range $z \lesssim 1$, we forecast that the relative precision of these estimates will be as small as $\sigma_{r_{\rm d}}/r_{\rm d} \sim 1.5\%$. This would offer a *unique* model-independent measure of a fundamental scale characterizing the early universe, with a precision competitive to values inferred within specific theoretical frameworks, including the standard cosmological model. Such measurements have several significant implications: (i) Providing a consistency test for the standard cosmological model: We can test the value of $r_{\rm d}$ predicted by the baseline cosmological model with high statistical significance, either confirming its predictions or providing hints of new physics beyond ΛCDM. (ii) Shedding light on the nature of the Hubble tension: If the tension is confirmed by gravitational wave standard sirens, our test could provide crucial information about its nature. In particular, we will be able to conclusively assess if the Hubble tension requires new physics acting prior to recombination to reduce the value of the sound horizon. As shown in this Letter, we can draw some general model-agnostic guidelines from current data that give solid grounds to believe that a compelling early time solution would require a reduction of about 5–6% in the value of the sound horizon. Such a hypothesis can be confirmed or ruled out at least at 4σ by our test; see also Fig. 1. (iii) Providing a definitive answer about the debate surrounding early vs late-time solutions of the Hubble tension: Should the Hubble tension be confirmed by gravitational wave standard sirens, leading to a necessary paradigm shift in cosmological modeling, our test can provide definitive guidelines. It will decisively address the ongoing debate between early and late-time solutions. On one hand, the detection of new physics from the early universe with high statistical significance could bolster early time solutions, challenging the late-time community. Conversely, if our test confirms the sound horizon value predicted by the ΛCDM model, it would undermine early time solutions, thus favoring alternative late-time proposals.

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